PLAN OF LECTURES — ICTP 1.1

1. Motivations, dS, new problems, infl bkgd
2. slow-roll, multifield. PT & gauge-inv pert's
3. Weinberg's adiabatic mode. Scalar & Tensor power spect.
4. Advanced: Non-G, consistency relations, phenom.

MOTIVATIONS TO STUDY INFL.

- Unique bridge across scales: 0.2 meV $\leftrightarrow$ 10^{12} GeV & more
- Theoretical frontier: QFT in curved spacetime. ST?
- New UV-physics! New particle & $\underline{\underline{3}}$ new scales BSM.

★ OLD BIG BANG PROBLEMS:

$$ds^2 = -dt^2 + a^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \quad \& \ \Lambda CDM$$

$$3M_p^2\left(H^2 + \frac{k}{a^2}\right) = \sum_i \rho_i \qquad \Omega_\Lambda = 0.7,\ 0.3\ \Omega_{m,0}$$

- FLATNESS PROBLEM: $\Omega_K = \frac{k}{a^2H^2} = 1-\Omega_{tot} = 0.000 \pm 0.005$ [Planck]

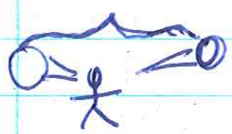
COMOVING HUBBLE RAD. $\chi_H = \frac{1}{aH} = H_0^{-1}\, a^{(3w-1)/2} \Rightarrow$ grows $w > -\frac{1}{3}$

Either $w < -1/3 \Rightarrow$ inflation or $K=0$. Latter is delicate

- PARTICLE HORIZON PROBLEM:

COMOVING DISTANCE $\chi(\tau_1,\tau_2) = \int_1^2 d\tau = \ldots = \frac{1}{(aH)_2}\frac{2}{3w+1}\left[\left(\frac{a}{a_2}\right)^{\frac{3w+1}{2}}\right]_1^2$

PARTICLE HORIZON $\chi(\tau_{-\infty},\tau) = \begin{cases} \infty & \text{accelerated } w<-1/3 \\ \frac{1}{(aH)}\frac{2}{3w+1} \end{cases}$



$$\frac{\chi(z)}{\chi_{ph}} \simeq 2\sqrt{1+z} > 1 \ . \text{ Need } w < -\tfrac{1}{3}$$

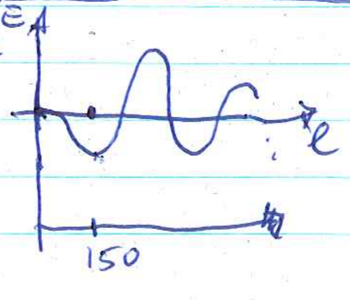
★ • COHERENT SUPERHUBBLE PERT'S: C_ℓ^{TE}

$\ell_{Hubble} = \chi_0\, K_{Hubble} \simeq 70$

Last scattering $z \simeq 1100$

TE

TT



$\Delta T \sim \delta_\gamma$, $E \sim \partial v_\gamma$. ICTP 1.2

$\langle a^T_{\ell m} a^E_{\ell m} \rangle \sim \langle \delta(\bar{k})\, k^i v^i(\bar{k}) \rangle$ consider very rough toy model $\dot{\delta} + \partial v = 0$

$\delta = A \cos[\omega t + \phi]$

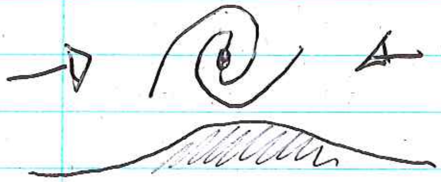
$\partial_i v^i = -\dot{\delta} = \omega A \sin[\omega t + \phi] \Rightarrow \langle \delta\, \partial_i v^i \rangle \propto$

If every mode has random in-coherent phases:

$$\langle \delta\, \partial_i v^i \rangle = \omega \langle A A^* \rangle \int_0^{2\pi} d\phi \, \sin[\omega t + \phi] \cos[\omega t + \phi] \overset{!}{=} 0$$

So ~~the~~ cosmo. pert's are COHERENT AROUND $\ell \sim 100$
We conclude that they must have been coherent on superHubble scales.

They were coherently in the GROWING MODE!



$\delta > 0 \Rightarrow \partial_i v^i \sim -\dot{\delta} < 0$ (flow in)

$\delta < 0 \Rightarrow \partial_i v^i < 0$ (flow out)

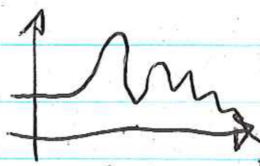
- APPROX SCALE INVARIANCE: $\langle R(x_1) \ldots R(x_n) \rangle = \langle R(\lambda x_1) \ldots \rangle$ in Fourier space

$$\langle R(\bar{k}_1)\, R(\bar{k}_2) \rangle = (2\pi)^3 \delta_D^3(\bar{k}_1 + \bar{k}_2) \frac{C}{k_1^3}$$

from HOMOG from ISOTRO. from ~~DILATIONS~~ DILATIONS

OBS: $TT \sim C_\ell^{TT}$

$$\frac{\Delta T(\hat{n})}{T} \sim -\frac{1}{5} R(\chi_{LS} \hat{n})$$



$$\langle R(\chi_{LS} \hat{n})\, R(\chi_{LS} \hat{n}') \rangle = \int d^2\ell\, d^2\ell'\, e^{i(\ell \hat{n} + \ell' \hat{n}')} \langle \theta(\ell)\, \theta(\ell') \rangle$$

$$= \int d^2\ell \, e^{i \hat{\ell} (\hat{n} - \hat{n}')} \, C_\ell$$ ISOTROPY

On large scales $C_\ell \sim 1/\ell(\ell+1) \sim \ell^{-2}$ [That's why we plot it as D_ℓ]

$\Rightarrow \langle R^2 \rangle$ independent of distance.

Follows from dS symmetry ICTP 1.3

$$ds^2 = \frac{-d\tau^2 + dx^2}{\tau^2 H^2} \Rightarrow \tau \to \lambda\tau \;\&\; \bar{x} \to \lambda \bar{x} \text{ inv.}$$

$$\langle R(\bar{x})\, R(\bar{0})\rangle = \int_{k_1 k_2} e^{ik_1 x} \langle R(k_1) R(k_2)\rangle$$
→ some fixed time $\quad \propto (2\pi)^3 \delta_D^3(\bar{k}_1 + \bar{k}_2)\, P(k)$

$$= \int_{\bar{k}} e^{ikx} P(k) \stackrel{!}{=} \int_k e^{ik\bar{x}\lambda} P(k)$$

$$\Rightarrow \lambda^{-3} P(k/\lambda) = P(k) \Rightarrow P(k) = \frac{c}{k^3}$$ SCALE INV!

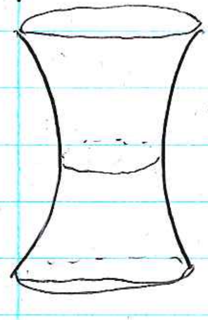
☆ Summary: need $w < -1/3$, need primordial dS ⇒ INFLATION

$$w < -1/3 \Leftrightarrow \ddot{a} > 0 \Leftrightarrow \partial_t\left(\frac{k}{aH}\right) < 0$$ ACCELERATION

$\boxed{dS_d}$ $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(R - 2\Lambda) = 0$ COSMO CONT. "Biggest blunder"

Einstein manifold $R_{\mu\nu} = \frac{2\Lambda}{d-2} g_{\mu\nu}$

Hyperboloid in Mink_{d+1} $\quad -(X^0)^2 + \sum_1^d X^a X^a = L^2$



- Maximally sym. $SO(d+1,1) \Leftrightarrow$ Lorentz in M_{d+1}
- dS isom: d translations, d-1 boosts, SO(d-1) rot same as in Mink_d = Poincaré.
- time translations ⇔ dilations

Coords: open, flat & closed, slicing w/ plane.

FLAT $\quad ds^2 = -dt^2 + e^{Ht2} dx^2 = \frac{-d\tau^2 + dx^2}{\tau^2 H^2}$

Invariant DIST. $\quad D(\tau, x; \tau' x') = \frac{\tau^2 + \tau'^2 - |\bar{x} - \bar{x}'|^2}{2\tau^2 \tau^2}$

INFLATION use a CC? does not end.

Λ w/ a clock ⇒ role it $V(\phi)$.

① Invent some $\phi(t)$. EFT.

② Derive it dynamically: Single Field toy

$$GR+SF = -\int d^4x \sqrt{g}\, \frac{1}{2}\left[M_P^2 R + (\partial\phi)^2 + 2V\right]$$ ICTP 1.4

~~QUASI DS~~ ~~...~~ Same as a SUPERFLUID

$$T_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\left[\frac{(\partial\phi)^2}{2} + V(\phi)\right]$$

EOM $\Box\phi = \frac{1}{\sqrt{g}}\partial_\mu\left(g^{\mu\nu}\sqrt{g}\,\partial_\nu\phi\right) + V_{,\phi} = 0$

HOMOG BACK: $\begin{cases} \rho = \dot\phi^2/2 + V \\ p = \dot\phi^2/2 - V \end{cases} \Rightarrow w = \frac{p}{\rho} = \frac{\dot\phi^2 - 2V}{\dot\phi^2 + 2V}$

$$\boxed{\text{EOM} \begin{cases} \ddot\phi + 3H\dot\phi + V_{,\phi} = 0 \\ 3H^2M_P^2 = \dot\phi^2/2 + V = \rho \end{cases}}$$

New H-friction

Coupled non-lin ode's

no general sol (H-Jacobi)

RECAP: Want Quasi dS $\frac{\ddot{a}}{a} = H^2(1-\varepsilon)$ ICTP 2.1

EOM. ($\varepsilon<1 \Leftrightarrow \ddot{a}>0$, $\varepsilon=0 \Leftrightarrow dS$)

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \ll 1$$ so H const — H FIRST S.R. ~~PARAM~~ PARAM. but we observe +7 efolds.

$d\log a = dN = H\,dt$ as measure of time

$$\eta \equiv \frac{\varepsilon_{,N}}{\varepsilon} = \frac{\dot{\varepsilon}}{H\varepsilon} \ll 1 \quad \left[\text{generalis. } \xi_{n\geq 3} = \partial_n \log \xi_{n-1}\right]$$

This means ε does not evolve much

$$\varepsilon(N) = \varepsilon(N_*) + \varepsilon_N (N-N_*) + \varepsilon_{NN}\frac{(N-N_*)^2}{2} + \dots$$
$$= \varepsilon(N_*)\left[1 + \eta(N-N_*) + \eta\xi(N-N_*)^2/2 + \dots\right]$$

If $\varepsilon_* \ll 1$ & $\xi\, \Delta N \ll 1 \Rightarrow \eta \sim \frac{1}{N}$ duration of I.

POTENTIAL SR PARAM'S

V resembles Λ if it's constant, so tentatively

$$\varepsilon_V \equiv \left(\frac{V'}{V}\right)^2 \frac{M_p^2}{2}, \quad \eta_V \equiv \frac{V''}{V} M_p^2, \quad \xi_{3V} \equiv M_p^4 \frac{V'V'''}{V^2}, \dots$$

Differentiate repetetively $V = (3-\varepsilon)H^2 M_p^2$ $\left[\text{derivative using } -\dot{H}^2 = \frac{\dot{\phi}^2}{2M_p^2}\right.$

$$\varepsilon_V = \frac{\varepsilon(\eta - 2\varepsilon + 6)^2}{4(\varepsilon-3)^2}; \quad \eta_V = \frac{\eta(\eta + \xi 2 + 6) - 2\varepsilon + \dots}{4(\varepsilon - 3)}$$

$\Rightarrow \varepsilon \simeq \varepsilon_V$ & $\eta \simeq 4\varepsilon_V - 2\eta_V$ much easier to use

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SLOW_ROLL INFLATION

$$X \equiv -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi \xrightarrow{\text{bkg}} X = +\frac{1}{2}\dot{\phi}^2; \quad \dot{X} = \ddot{\phi}\dot{\phi}$$

$$\rho = X + V, \quad p = X - V \quad \& \quad \dot{\rho} + 3H(\rho+p) = \dot{X} + 6HX + V'\dot{\phi} = 0$$

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{X}{H^2} = \frac{3X}{V+X} \ll 1 \Rightarrow X \ll V \Rightarrow 3H^2 \simeq V$$

$$a(t) \simeq e^{\int H dt} \simeq e^{\sqrt{V/3M_p^2}\, t}$$

$\eta = 2\epsilon + \frac{\dot X}{XH} \ll 1 \Rightarrow \dot X \ll XH$ ICTP 2.2

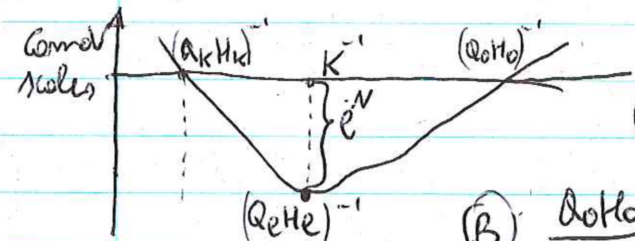
Cont. eq $\cancel{\dot X}_0 + 6HX = -V'\dot\phi \Leftrightarrow \dot\phi = -\frac{V'}{3H}$

- Second order ode ⇒ first order. Easier to solve
- lhs whole dynamics vs rhs only $V(\phi) \Rightarrow$ Hubble ⇔ pot.
- Unique property of single field

Play around w/ $V(\phi) = \phi^\lambda \mu$ or $(1-\phi^\lambda/\mu^\lambda)^2 V_0$

END OF INFL. $\ddot a \lessgtr 0 \Rightarrow \epsilon > 1$, $\boxed{\epsilon = 1 \text{ is END}}$

$N = \int d\log a = \int H\,dt = \int \frac{H}{\dot\phi}\,d\phi \simeq \int \frac{V}{V'}\,d\phi$ How long?



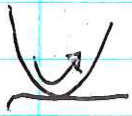
(A) $\left(\frac{k}{a_eH_e}\right)^{-1} \simeq \left(\frac{a_kH_k}{a_eH_e}\right)^{-2} \simeq e^{N(k)}$

(B) $\frac{a_0H_0}{a_eH_e} \simeq \frac{a_e}{a_0} = \left(\frac{\rho_0}{\rho_e}\right)^{1/4}$

assume rad domination $aH \simeq 1/a$

log(A) = log(B)

$N(k_0) \simeq 62 + \frac{1}{4}\ln\left(\frac{10^{16}\,GeV}{\rho_e^{1/4}}\right)$ typically $25 < N < 60$



- Infl. ends as ϕ reaches minimum
- $\phi(t)$ breaks both pert & non-pert into $\delta\phi(k,t)$
- $\delta\phi$ couples to SM & reheats the universe
- bath of SM part's at same freq.

MULTIFIELD INFL: no reason to have only 1 clock

- RG indicates more fields at high energies
- All ST realizations have $10^2 - 10^4$ moduli
- eg $\mathbb{R}^4 \times S_1 \Rightarrow g_{55} \sim R^2$ is a scalar field
- Perhaps this is the only class of theories we will be able to rule out using f_{NL}^{loc}

$\phi \to \Phi \in \mathbb{R}^n$ $X \equiv -\frac{1}{2} g^{\mu\nu}\partial_\mu X^I \partial_\nu X^J G_{IJ}$

Background $X = +\frac{1}{2}\dot\phi^I \dot\phi_I$ ICTP 2.3

EOM: $D_t \dot\phi^I + 3H\dot\phi^I + V_{,J} G^{IJ} = 0$

same as Geodesic eq of part in $\mathbb{R}^n$! can easily remove $3H\dot\phi^I$. 1D always flat.

new term $\ddot\phi^I + \Gamma^I_{JK}\dot\phi^J\dot\phi^K$
Target space "gravity"!
All result must be covariant.

Hubble slow-roll
$\epsilon = -\dot H/H^2 \ll 1$ same
$\eta = \dot\epsilon/\epsilon H \ll 1$ same!

Pot - slow-roll
$\epsilon = \frac{V_{,I} V_{,J} G^{IJ}}{V^2}$ not necessary
$\eta = \min$ evalue $V_{;IJ}/V$ ~~nec~~ suf

$$\eta = 2\epsilon + \frac{\dot X}{HX} \Rightarrow \dot X \ll 6HX = -V_{,K}\dot\phi^K \equiv -|V'||\dot\phi|\cos\theta$$

$$X_{sol} = -V + \sqrt{V^2 + \frac{2}{3}|V'|^2\cos^2\theta}$$

$$\epsilon = \frac{3X}{V+X} \simeq \frac{3X_{sol}}{V} = \boxed{\frac{M^2}{2}\frac{V_{,J}V_{,I}G^{IJ}}{V^2}\cos^2\theta = \epsilon_V}$$

Intuitively there are n-1 direct's $\perp$ to $V_{,I}$.
For $\cos\theta = 1$ recover usual story.
It's a SLOW DESCENT INFLATION.

$$\eta \simeq -\frac{\partial_t \cos\theta}{H\cos\theta} - \frac{\partial_t |V'|}{H|V'|} \xrightarrow{\cos\theta = 1} \frac{V''}{V} \text{ along direction}$$

PERTURBATIONS: QFT in curved spacetime generates pert's in our universe.
GR + Scalar is non-linear. Solve in PT around FLRW bk.
Symmetries:

- Homogeneity $\Rightarrow$ Fourier modes decouple at linear order

$$3\dot\delta(x) + \partial_{x_i}\delta(x) + \delta(x)^2 + \text{Back}(x)\,\delta(t)$$
$$\checkmark \quad \checkmark \quad \times \quad \times$$

- Isotropy $\Rightarrow$ Scalar - Vector - Tensor decouple at linear (under spatial rotations)

SVT decomp. of metric $g_{\mu\nu} = \bar{g}^{FLRW}_{\mu\nu} + h_{\mu\nu}$ ICTP 2.4

$h_{00} = -E$ $\qquad \phi = \phi + \varphi$ $\qquad$ Transverse

$h_{0i} = a\,[\partial_i F + G_i]$ $\qquad \partial_i G_i = \partial_i C_i = 0$

$h_{ij} = a^2 [A\,\delta_{ij} + \partial_{ij} B + \partial_{(i} C_{j)} + D_{ij}]$ $\qquad \partial_i D_{ij} = D_{ii} = 0$

$$\left(\begin{array}{c|c} S & S, V \\ \hline S, V & 2S, V, T \end{array}\right)$$

$4S + 2V(\times 2) + 1T(\times 2) = 10$ OK

Vectors decay. So study S & T separately.

GAUGE $\quad x'^\mu = x^\mu + \xi^\mu \Rightarrow \Delta\phi(x) = \phi'(x) - \phi(x) = \mathcal{L}_\xi \phi$

$$\Delta g_{\mu\nu} = \mathcal{L}_\xi h_{\mu\nu} = -\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \qquad = -\xi^\mu \partial_\mu \phi \simeq -\xi^0 \dot{\phi}$$

$\xi_\mu = \{\xi_0, \partial_i \xi^S + \xi^V_i\}$ can get rid of 2 scalars.

FLAT GAUGE $\quad A = B = 0$

COMOVING GAUGE $\quad B = \varphi = 0$.

GAUGE INV.: $\quad \mathcal{R} \equiv \frac{A}{2} - H \frac{\varphi}{\dot{\phi}} = \frac{A}{2} + H\delta u$

CURV. PERT'S ON $\begin{cases} \text{HOMOG } \zeta \\ \text{COMOV } \mathcal{R} \end{cases}$ $\quad \zeta \equiv \frac{A}{2} - \frac{\delta\rho}{\dot{\rho}} H \simeq \mathcal{R} + \mathcal{O}(k^2)$

WEINBERG'S THEOREM ON ADIABATIC MODES

ICTP 3.1

(Small gauge transf.)

R & ζ are gauge inv. if $\xi(x) \to 0$ for $\bar{x} \to \infty$

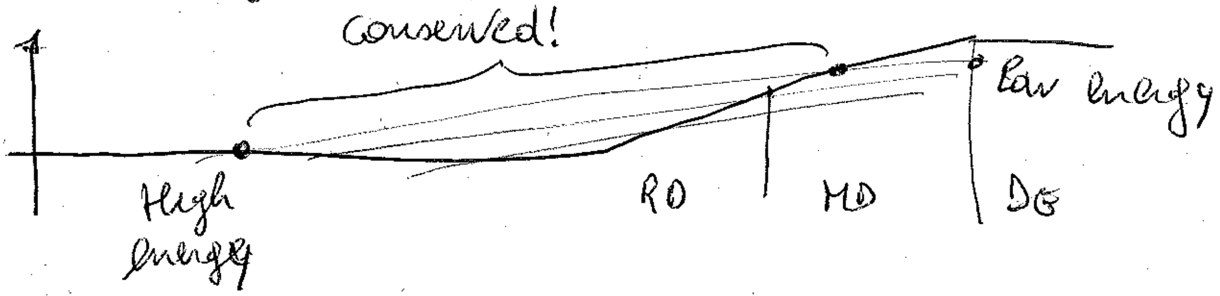
In many cases $\dot{R}, \dot{\zeta} \sim \frac{k^2}{(aH)^2}$ so conserved $k \ll aH$

<u>Th</u>: Whatever constituents of universe, $\exists$ solution (in Newt. gauge)

$$E = -A = 2R\left[-1 + \frac{H}{a}\int a\,dt\right]$$

$$\delta\,\text{scalar} = -\frac{R\dot{\bar{s}}}{a}\int a\,dt \quad \ldots \quad \text{w/ } R = -\frac{A}{2} - H\frac{\varphi}{\dot{\phi}} = \text{const}$$

Also $D_{ij} = \text{const}$.



Proof: • $R = \text{const} =$ large gauge transf. $= R(q=0)$
• $R(q \neq 0)$ exist only if some cond's are satisfied (Cfr w/ Goldstone theorem)

We measure precisely this ADIABATIC MODE (explained in single field)

$$\left(\frac{\delta\rho}{\dot{\rho}}\right)_i = \left(\frac{\delta\rho}{\dot{\rho}}\right)_j \quad \forall\, i,j = DM, b, \gamma, \nu, \ldots$$

Cannot predict $R(x)$ but can predict $\langle R(x)\, R(y)\rangle$.

Notation: homog. → isot.

$$\langle R(\vec{k})\, R(\vec{k}')\rangle = (2\pi)^3 \delta_D(\vec{k}+\vec{k}')\, P(k)$$

$$\langle R R\rangle' = P(k)$$

$$P(k) = \frac{2\pi^2 \Delta_R^2(k)}{k^3} \quad \leftarrow \text{CMB } 2.2\cdot 10^{-9}$$

Tensors $D_{ij}^{(TT)}$

$$D_{ij}(0,0,k) = \begin{pmatrix} D^\times & D^+ & 0 \\ D^+ & -D^\times & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

WARM UP: SHO $S=\int dt\left(\dot{x}^2-\frac{\omega^2}{2}x^2\right)$ ICTP 3.2

$\omega=\omega(t)$ EOM $\ddot{x}+\omega^2 x=0$ canonical Quant.

$p\equiv\frac{\partial\mathcal{L}}{\partial\dot{x}}$ $[\hat{x},\hat{p}]\stackrel{!}{=}i\hbar$ $\hat{x}=v(t)\hat{a}+v^*(t)\hat{a}^\dagger$

Wronsk $(v,v^*)\,[\hat{a},\hat{a}^\dagger]=1$ normalize $[\hat{a},\hat{a}^\dagger]=1$

$\frac{i}{\hbar}(v^*\dot{v}-\dot{v}^*v)$

Positive freq: $v(t)=\sqrt{\frac{\hbar}{2\omega}}\,e^{-i\omega t}$ $\Rightarrow\langle\hat{x}^2\rangle=|v|^2=\frac{\hbar}{2\omega}$

if $\omega(t)\sim$ const

for $t\to-\infty$

INFL P-SPEC $S=\int\frac{1}{2}\sqrt{g}\,d^4x\left[R-(\partial\phi)^2-2V\right]$

around $ds+\phi(t)$. Use ADM formalism [R contains $\ddot{g}$ so no variation swill des]

$ds^2=-N^2dt^2+h_{ij}(dx^i+N^idt)(dx^j+N^jdt)$

$\Rightarrow S=\frac{1}{2}\int dx^4\sqrt{h}\left[N\,{}^{(3)}R-2NV+N^{-1}(E_{ij}E^{ij}-E^2)+N^{-1}(\partial\phi)^2\right]$

Kinetic term only his

N, N_i are constrained not dynamical.

Solve for N, N_i & expand $\Rightarrow S_2=\int d^4x\,a^3\varepsilon\left[\dot{R}^2-\frac{(\partial_i R)^2}{a^2}\right]$

Can quantize it directly. or make it canonical (Mukh)

$v=a\sqrt{2\varepsilon}\,R\equiv zR$ $\Rightarrow S_2=\int d^3x\,d\tau\left[(v')^2-(\partial_i v)^2+\frac{z''}{z}v^2\right]$

$v(\bar{x})=\int_{\bar{k}}e^{i\bar{k}\bar{x}}v_{\bar{k}}(\tau)$ $\Rightarrow v_k''+\left(k^2-\frac{z''}{z}\right)v_k=0$

$v_{\bar{k}}(\tau)=v_k(\tau)\,\hat{a}_{\bar{k}}+v_k^*\,\hat{a}^\dagger_{\bar{k}}$

- real eq $\Rightarrow v$ & v^* are sol
- Romay b-rg $\Rightarrow v=v_{|\bar{k}|}$

$\omega^2(t)=k^2-\frac{z''}{z}\simeq k^2-2(aH)^2\left[1+\mathcal{O}(\varepsilon,\eta)\right]$

$\tau = -\frac{1}{aH} \Rightarrow k\tau = \frac{k}{aH}$

$v_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}$ for $k\tau \gg 1$ ICTP 33

Neglecting $\mathcal{O}(\epsilon, \eta) \Rightarrow v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v = 0$

$$\boxed{v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)}$$ dS mode functions

$\langle R(\vec{k})\, R(\vec{k}')\rangle = \langle a_{\vec{k}}\, v_k\, a^{\dagger}_{\vec{k}'}\, v^*_{k'}\rangle z^{-2} = (2\pi)^3 \delta(\vec{k} + \vec{k}') \left|\frac{v_k}{z^2}\right|^2$

$\langle R(\vec{k})\, R(\vec{k})\rangle' = \boxed{P_R(k) = \frac{H^2}{4\epsilon k^3}}\left[1 + \mathcal{O}\left(\cancel{\frac{k}{aH}}\right)^2\right]$ $\to 0$

$\Delta_R^2 = \frac{H_*^2}{8\pi^2 \epsilon_* M_p^2}$

- Since we neglected ϵ, η this does not go to a constant. So we compute it at Hubble cross for every mode $-k\tau_k = 1$
- It goes approx as k^{-3} as advertised
- Small tilt from time dep of $H(\tau_k)$ & $\epsilon(\tau_k)$.

$$n_s - 1 \equiv \frac{\partial \log \Delta_R^2}{\partial \log k} \simeq \frac{\partial \log \Delta_R^2}{\partial N} = \frac{\partial \log \Delta_R^2}{H \partial t} = -2\epsilon - \eta$$

- Power spec of a massless scalar in dS $\frac{H^2}{2M_p^2 k^3} = P_\phi(k)$

TENSORS: $S_2 = \frac{M_p^2}{8}\int d\tau\, d^3x\, a^2\left[(D_{ij}')^2 - (\partial_l D_{ij})(\partial^l D_{ij})\right]$

- Same action as massless scalars
- No need to solve constraints since N, N_i are not TT

$D_{ij} = \sum_{s=+,\times} \int_{\vec{k}} \epsilon^s_{ij}(\vec{k})\, D^s_{\vec{k}}(\tau)\, e^{i\vec{k}\vec{x}}$, quantize

$v^s_{\vec{k}} = \frac{M_p a}{2} D^s_k \Rightarrow \langle D^s(\vec{k})\, D^{s'}(\vec{k}')\rangle' = \delta_{ss'} \frac{H^2}{M_p^2}\frac{2}{k^3}$

$\Delta_T^2 = 2\Delta^2 = \boxed{\frac{2}{\pi^2}\left(\frac{H_*}{M_p}\right)^2} \Rightarrow n_T = \frac{\partial \log \Delta_T^2}{\partial \log k} = \boxed{-2\epsilon = n_T}$

- Different notation n_T

ICTP 3.4

- Single field relation
  $8 n_T = r$
- phenomenomen. parameterit.

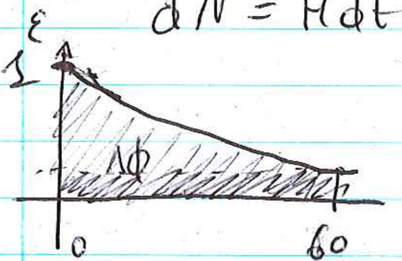
$$\Delta_R^2 = A_s \left(\frac{k}{k_*}\right)^{n_s-1} \qquad \Delta_T^2 = A_s r \left(\frac{k}{k_*}\right)^{n_T}$$

$A_s = 2.2\ 10^{-9}$ $\qquad$ $r < 0.07 = 16\,\epsilon$

$n_s - 1 \simeq 0.036 \pm 0.005$ $\qquad$ n_T ??

- $16\,\epsilon < 0.07 \to \epsilon < 0.005$ but $2\epsilon + \eta = 0.04$
  $\Rightarrow \epsilon \simeq \eta/3$ (so as long as no r) $\underline{\epsilon \ll \eta}$
- Lyth bound

$$dN = H\,dt = \frac{H}{\dot\phi}\,d\phi = \frac{d\phi}{M\sqrt{2\epsilon}} \Rightarrow \int \frac{d\phi}{M_p} = \int dN \sqrt{2\epsilon}$$



$$\frac{\Delta\phi}{M_p} \gtrsim 60\sqrt{2\epsilon_*} \simeq \sqrt{\frac{r}{0.01}}$$

More elaborate if we assume $\epsilon(N)$.

$\Delta\phi > M_p$ is LARGE FIELD e.g. $m^2\phi^2$ or $\lambda\phi^4$.

<u>Recap</u>: Inflation make a large homog & iso universe & scale inv. S & T pert's

$$\langle \mathcal{R}(\vec k)\,\mathcal{R}(\vec k')\rangle = (2\pi)^3\,\delta_D^3(\vec k+\vec k')\,P(k) \qquad \text{w/} \quad P(k) = \frac{H^2}{k^{3+(1-n_s)}\,4\epsilon M_p^2}$$

$$\langle D^s(\vec k)\,D^{s'}(\vec k)\rangle' = \delta_{ss'}\,P_T(k)$$

$$\text{w/} \quad P_T = 2\,\frac{H^2}{M_p^2} \Rightarrow r = \frac{\Delta_T}{\Delta_S} = 16\epsilon \qquad n_s - 1 \simeq -2\epsilon - \eta$$

CONNECTION TO CMB & LSS

$$\theta^T_{\ell m} \simeq \int_{\vec k} Y_{\ell m}(\hat k)\;\mathrm{TransFct}_\ell(|\vec k|)\;\mathcal{R}(\vec k)$$ see Raphael's lectures

$$\Rightarrow C_\ell^{TT} = \frac{2}{\pi}\int dk\,k^2\,P_{\mathcal{R}}(|\vec k|)\;\mathrm{TransFct}_\ell^2(k)$$

"linear" related to $P_{\mathcal{R}}(k)$!

similarly $C_\ell^{BB} \simeq \int dk\,k^2\,P_T(k)\;\mathrm{TransFct}_\ell^{\mathrm{Tensor}\,2}(k)$

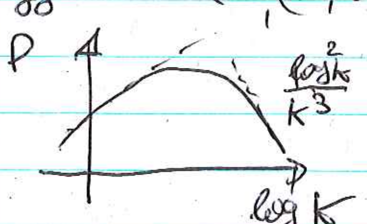
$\rho(\vec x, t) = E/V^3$ of DM & Baryons — see Shirley's lectures

$$\rho(\vec x,t) = \bar\rho(t)\left[1 + \delta(\vec x,t)\right]$$

DENSITY CONTRAST

$$\delta_{\substack{\text{Large}\\ \text{scales}}} = \delta_1 = -\frac{2}{5}\,\frac{k^2}{H^2\Omega_m}\,\mathrm{TrF}^\delta(k,t)\,\mathcal{R}(\vec k)$$ linear!

$$P_{\delta\delta} \equiv \langle \delta_1(\vec k,t)\,\delta_1(\vec k,t)\rangle' \simeq \frac{4}{10}\left(\frac{k^2 T_F}{H^2\Omega_m}\right)^2 \langle \mathcal{R}(\vec k)\,\mathcal{R}(\vec k)\rangle'$$



$$T(k \ll 0.01\,h/\mathrm{Mpc} \simeq k_{eq}) \simeq 1$$
$$\Rightarrow P_{\delta\delta} \simeq k^4 P_{\mathcal{R}} \simeq k.$$

$$T(k \gg k_{eq}) \simeq \frac{\log k}{k^2}$$

ISOCURVATURE VS ADIABATIC

$$\left(\frac{\delta\rho}{\dot\rho}\right)_i = \left(\frac{\delta\rho}{\dot\rho}\right)_j \quad \forall\, i,j = \gamma, DM, b, \nu, \ldots \qquad \delta u_i = \delta u_j$$ deviations

$$S_{ij} = \left(\frac{\delta\rho}{\dot\rho}\right)_i - \left(\frac{\delta\rho}{\dot\rho}\right)_j$$ e.g. DM − γ, bar − γ, (bar − ν), ⋮ — many possibilities all constrained

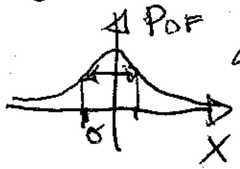
...isocurvature

$$\langle S(k) S(k) \rangle' = P_S(k) = \frac{\Delta_s}{k^3}$$

$$\Rightarrow \frac{\Delta_s}{\Delta_R} \equiv \frac{\alpha}{1-\alpha} \lesssim 3 \cdot 10^{-4} \ll 1$$ Very adiabatic (correlated isoc.)

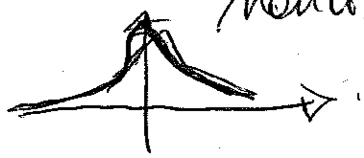
Why is our Univ. adiabatic? Single field (always been) VS Multifield (thermaliz.)

NON-GAUSSIANITY. Take simple RANDOM var X



Gaussian random var $\Rightarrow \begin{cases} \langle X^{2n+1} \rangle = 0 \\ \langle X^{2n} \rangle \simeq \langle X^2 \rangle^n \end{cases}$

$\langle X^2 \rangle = \sigma^2$



non-Gaussianity: $\langle X^{2n+1} \rangle \neq 0$, $\langle X^{2n} \rangle \neq \langle X^2 \rangle^n$

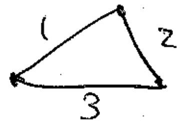
Our universe is Gauss on large scales:

bounds or $< 1‰ \sim 10^{-3}$

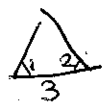
FIELD THEORY & $n=3$ SIZE SHAPE

$$\langle R(\vec{k}_1) R(\vec{k}_2) R(\vec{k}_3) \rangle = (2\pi)^3 \delta_D^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, f_{NL}\, B(k_1, k_2, k_3)$$

- 3×3 vars $= 9 - 3 - 3 = 3$



or



- Permutation inv! ISO. HOM.

Scale inv. $B(\lambda \vec{k}_1, \lambda k_2, \lambda k_3) = \lambda^{-6} B(\vec{k}_1, \vec{k}_2, \vec{k}_3)$

$$\Rightarrow B = \frac{S(k_2/k_1, k_3/k_3)}{k_1^6}$$ only two vars

TOY MODEL: (but relevant!) $R(x) = R_G(x) + f_{NL}\left(R_G(x)^2 - \langle R_G(x)^2 \rangle\right)$

LOCAL NG to ensure $\langle R \rangle = 0$

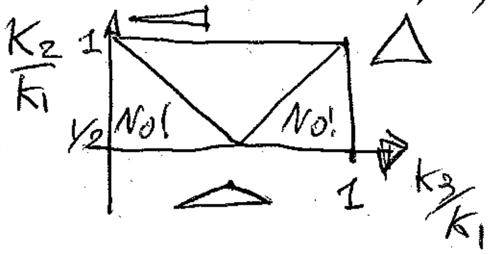
exercise $\Rightarrow B_{local}(k_1, k_2, k_3) = 2 f_{NL} \left(P(k_1) P(k_2) + \underbrace{\text{"23"} + \text{"13"}}_{2 \text{ perm's}} \right)$

Since $P(k) \simeq \frac{1}{k^3}$ thus peaks on $k_1 \ll k_2, k_3$

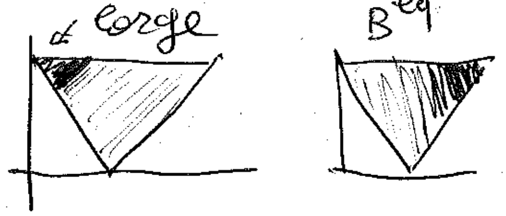
$$B^{local}(k_1 \ll k_2, k_3) \simeq 4 f_{NL} \frac{1}{k_1^3} \frac{1}{k_2^3}$$

To visualize plot ICTP 4.3

$$(k_1 k_2 k_3)^2 B(k_1,k_2,k_3) = k_1^6 \frac{k_2}{k_1}\frac{k_3}{k_1} B\left(1, \frac{k_2}{k_1}, \frac{k_3}{k_1}\right)$$



e.g. B^{local}



SINGLE FIELD SLOW-ROLL $GR + (\partial\phi)^2 + V(\phi)$

- NG from int in $V(\phi)$
- NG from int in $R\sqrt{g}$ & $\sqrt{g}[(\partial\phi)^2 + V(\phi)]$ ie from gravity

Sketch of calculation: choose a gauge $A=0$ or $\varphi=0$

- Solve constraints $N \simeq R + R^2$, $N_i \simeq \partial_i (R+R^2)$
- $S = S_2(R^2) + S_3(R^3) + \ldots$
- Compute CORRELATOR (not an in-out amplitude!)

$\langle \Omega | R^3 | \Omega \rangle$ w/ $|\Omega\rangle$ Vacuum of INTERACT. TH: not $|0\rangle$

$$|\Omega\rangle = T e^{-i\int_{t_0}^{t} H_{int} dt} |0\rangle \equiv U(t,t_0)|0\rangle$$

$$\Rightarrow \langle R^3 \rangle = \langle 0| U^{-1}(t,t_0) (R^{int})^3 U(t,t_0)|0\rangle$$

$$\simeq -i \int_{t_0}^{t} dt' \langle 0| [R^3, H_{int}(t')] |0\rangle$$

Maldacena's sol: $$\boxed{B = (-\varepsilon - 2\eta) B^{local} + \varepsilon B^{eq}} + \eta \xi_3 B^{eq}$$ "Higher order"

e.g. $\frac{k_1+k_2+k_3}{(k_1 k_2 k_3)^3}$

- Small! $f_{NL} \sim \mathcal{O}(\varepsilon, \eta) \ll 1$
- $R_{NG} \simeq R_G [1 + \underbrace{f_{NL} R_G}_{\sim f_{NL}\cdot 10^{-5} \ll 10^{-5}} + \ldots]$ small.
- $\varepsilon \ll \eta \Rightarrow$ only $B \simeq -2\eta B^{loc} + \eta \xi_3 B^{eq}$ even smaller

$\langle R^n \rangle$ must obey conf symm (non-linearly realiz).

LARGE NG: beyond SR single field   ICTP 4.4

(A) Multiple fields

- non-slow roll eg  or  $\Rightarrow$ [not robust b/c affect $P(k)$]

(B) non canonical

(B) $\mathcal{L} = P(X)$ w/ $X = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$ eg. $P(X) = X$ is canonical

(Actually the Low E. EFT of superfluids)

R & φ travels at $c_s^2 = \dfrac{P_{,X}}{P_{,X} + 2X P_{,XX}} \neq c^2$ (best studied in EFT of inflation)

$$S_2 = \int d^4x\, a^3 \epsilon \left[\dot{R}^2/c_s^2 - (\partial R)^2/a^2 \right]$$

$$S_3 \simeq \int d^4x\, a^3 \epsilon^2 \left[\dot{R}^2 R / c_s^2 + \dots \right] \Rightarrow f_{NL}^{EQ} \simeq \frac{1-c_s^2}{c_s^2} + \epsilon .$$