Lectures on the Cosmic Microwave Background

Raphael M. Flauger
The University of Texas

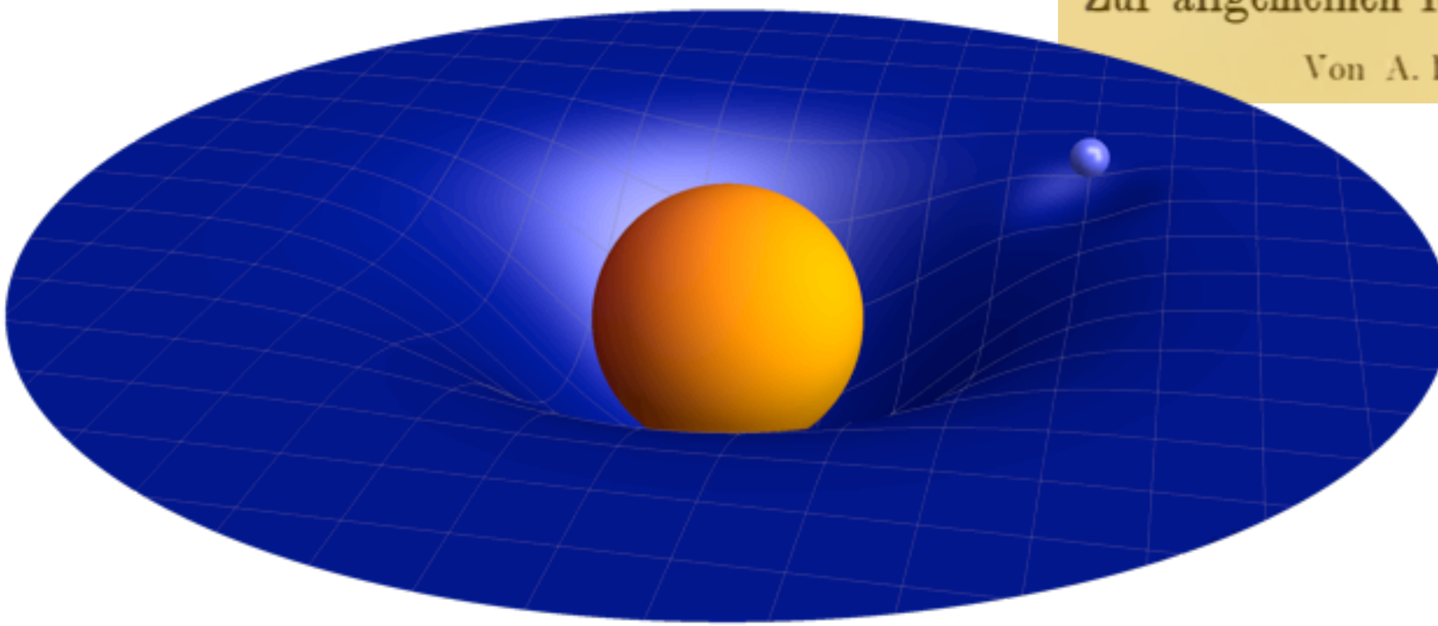
Summer School on Cosmology, ICTP, June 2016

Lecture I

Prediction and Early CMB Measurements

- General Relativity - Part I
- Prediction of the CMB
- Discovery of the CMB
- Spectrum of the CMB

General Relativity - Part I

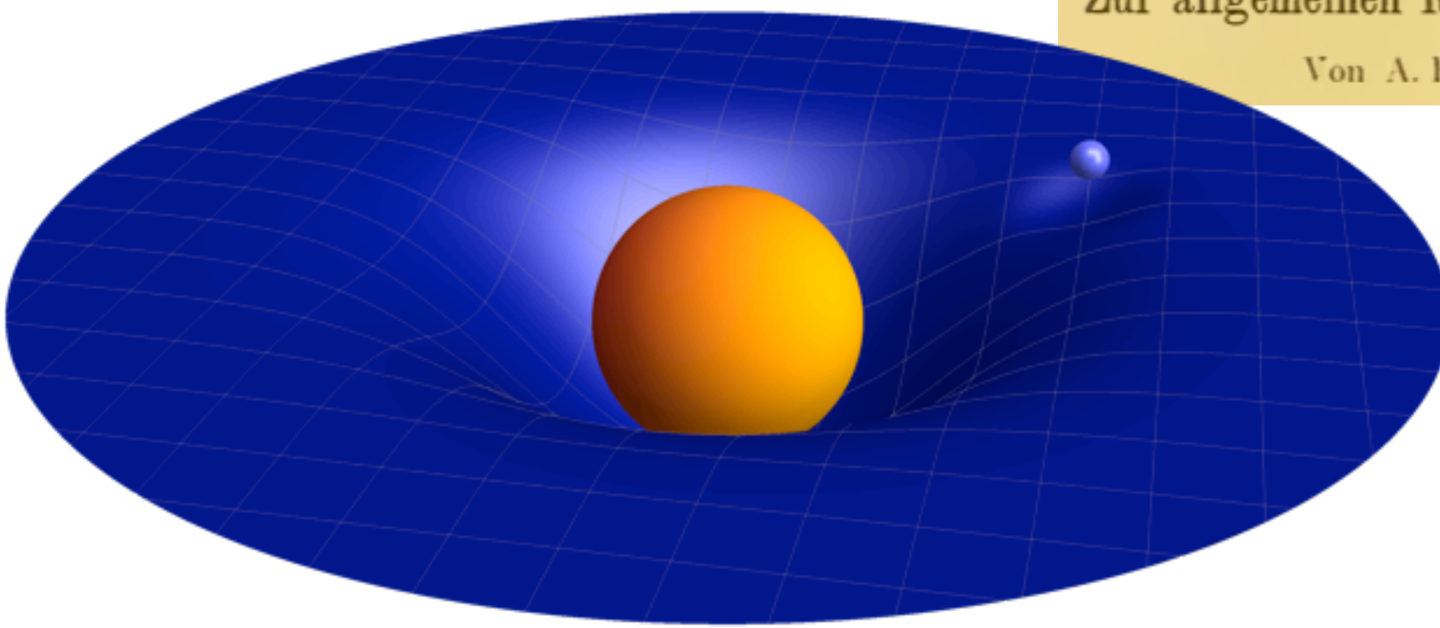


Zur allgemeinen Relativitätstheorie.

Von A. Einstein.

(Einstein 1915)

General Relativity - Part I




Zur allgemeinen Relativitätstheorie.

Von A. Einstein.

(Einstein 1915)

- Gravitational attraction arises because spacetime is curved.
- The geometry of spacetime is determined by the matter distribution.

General Relativity - Part I

The geometry of space is encoded by the line element

$$ds^2 = \sum_{ij} g_{ij} dx^i dx^j \equiv g_{ij} dx^i dx^j$$

Ueber

die Hypothesen, welche der Geometrie zu Grunde liegen.

Von

B. Riemann.

(Riemann 1854)

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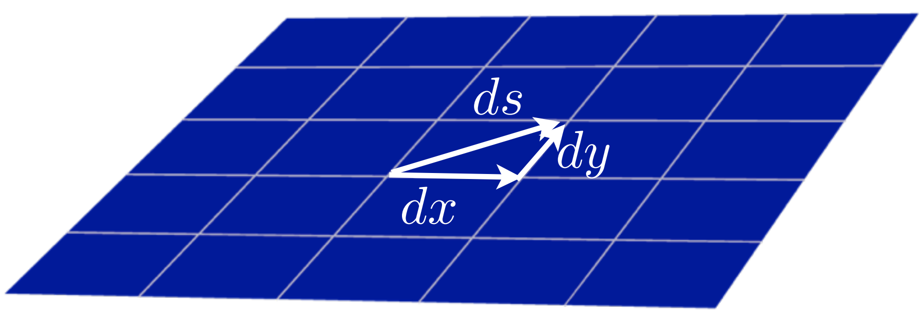

$$ds^2 = dx^2 + dy^2$$

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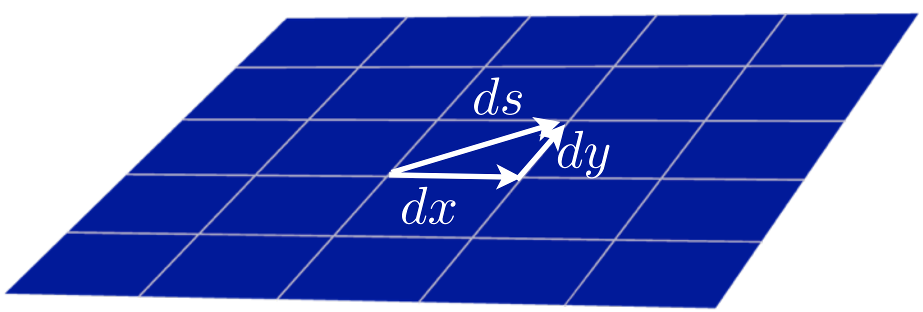

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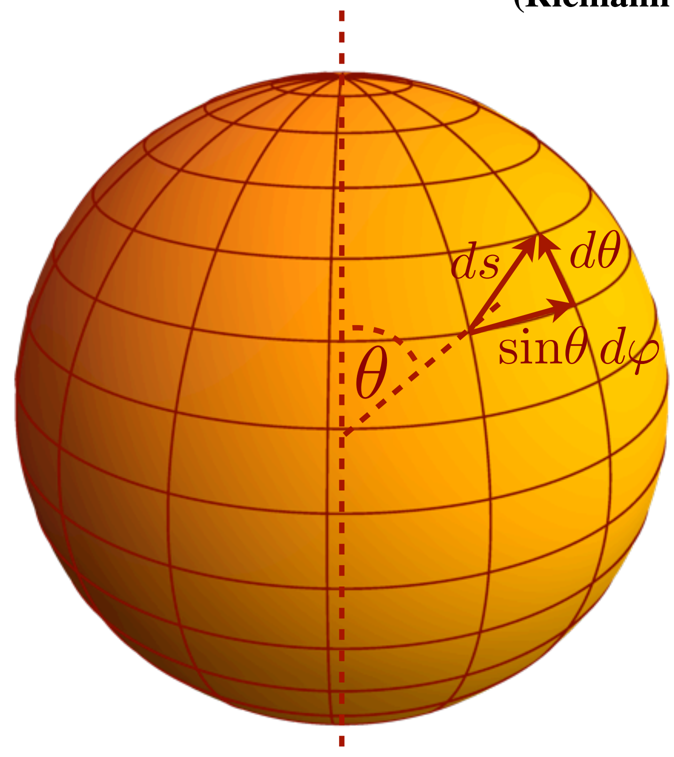


$$ds^2 = d\theta^2 + \sin^2\theta d\varphi^2$$

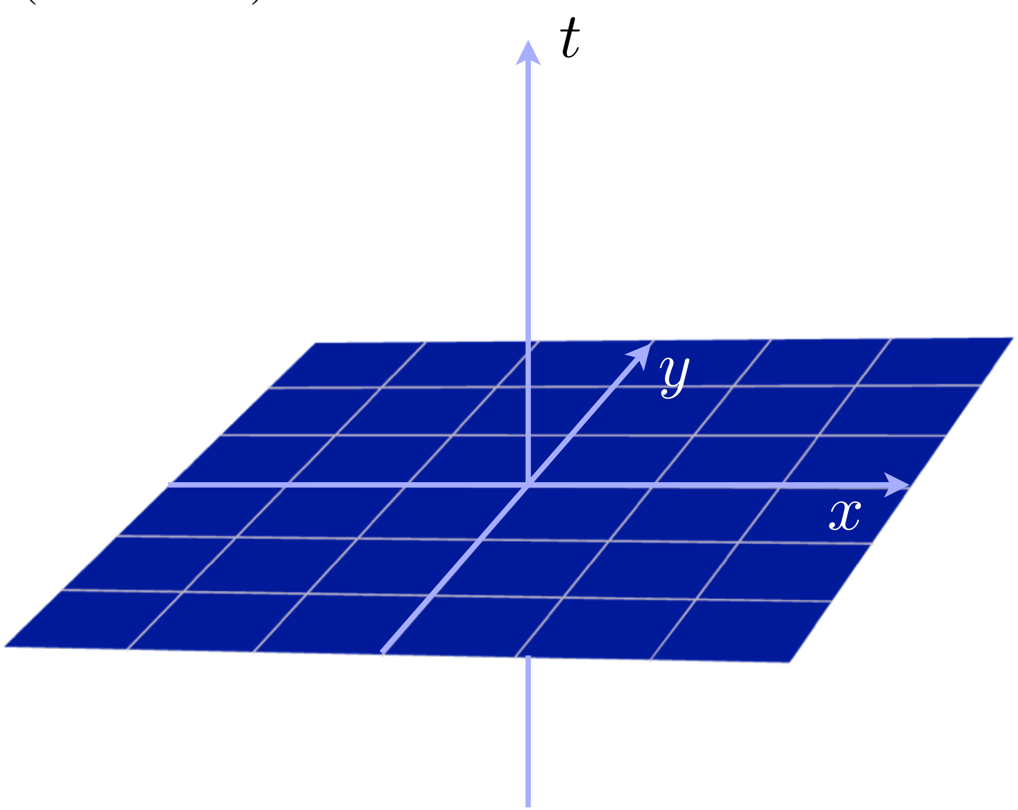
General Relativity - Part I

1. *Die Grundlage der allgemeinen Relativitätstheorie;*
von A. Einstein.

(Einstein 1916)

The geometry of spacetime is encoded by the line element

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu$$



$$ds^2 = -dt^2 + d\vec{x}^2$$

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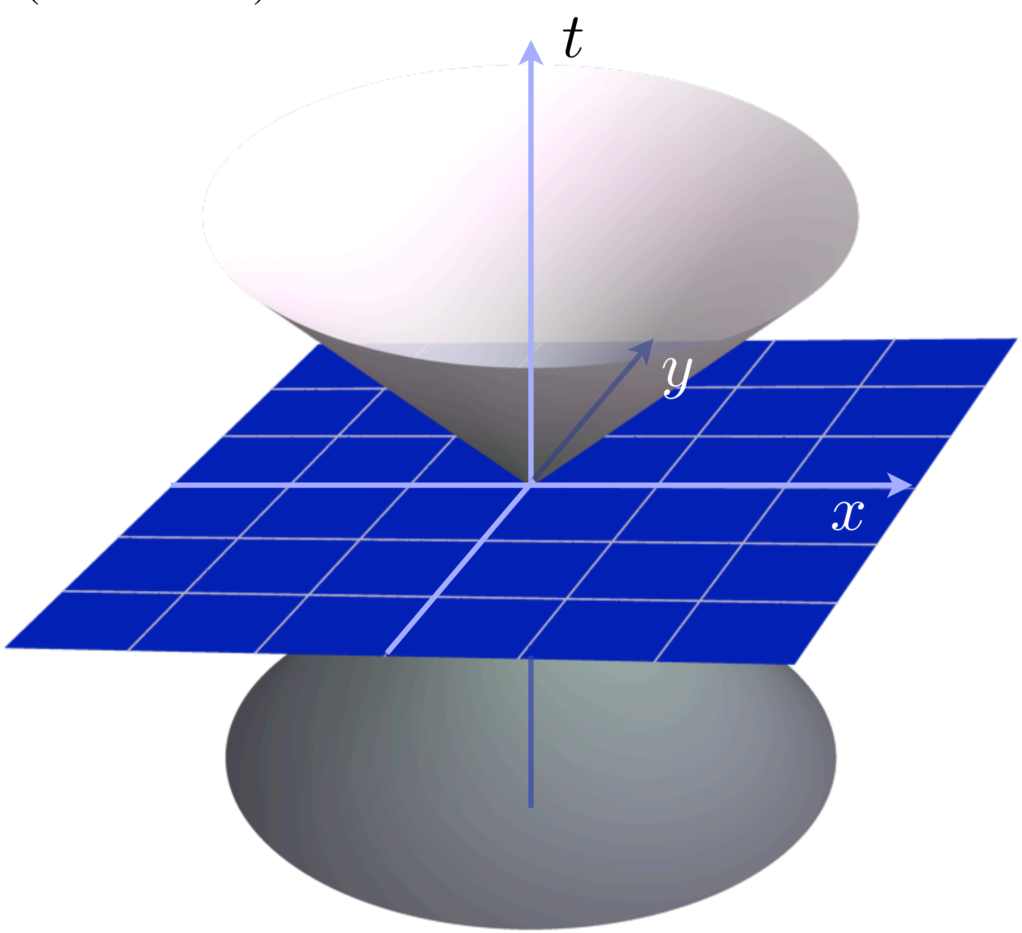

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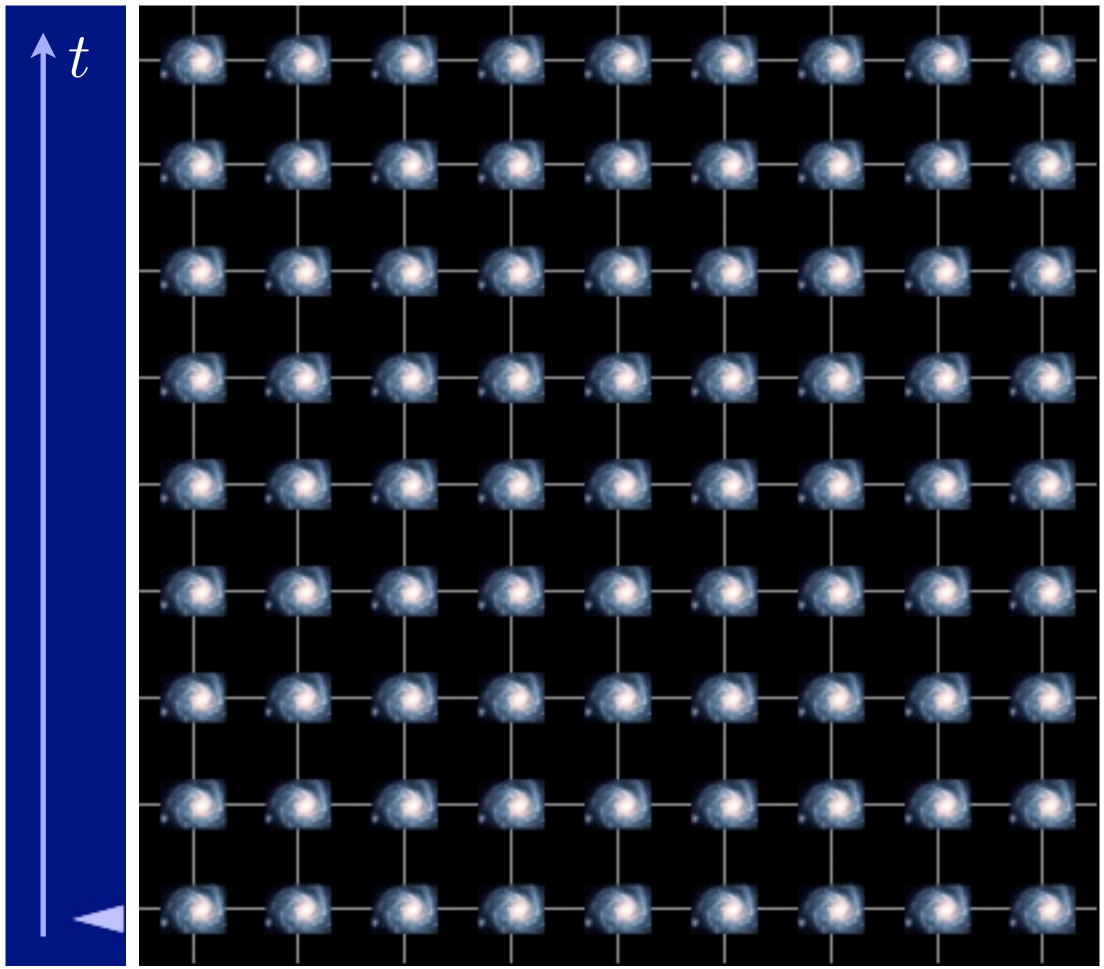
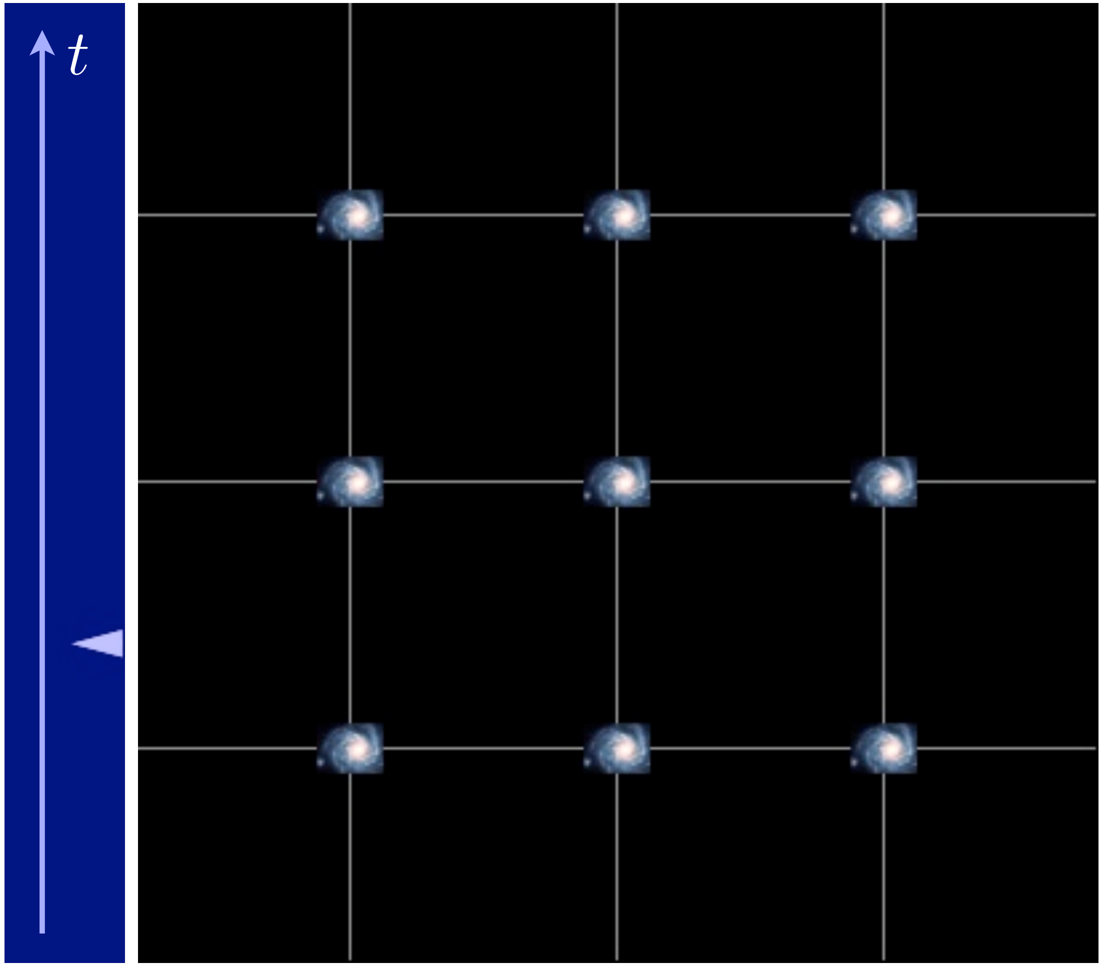
General Relativity - Part I



At $t = t_1$: $ds^2 = a_1^2 d\vec{x}^2$

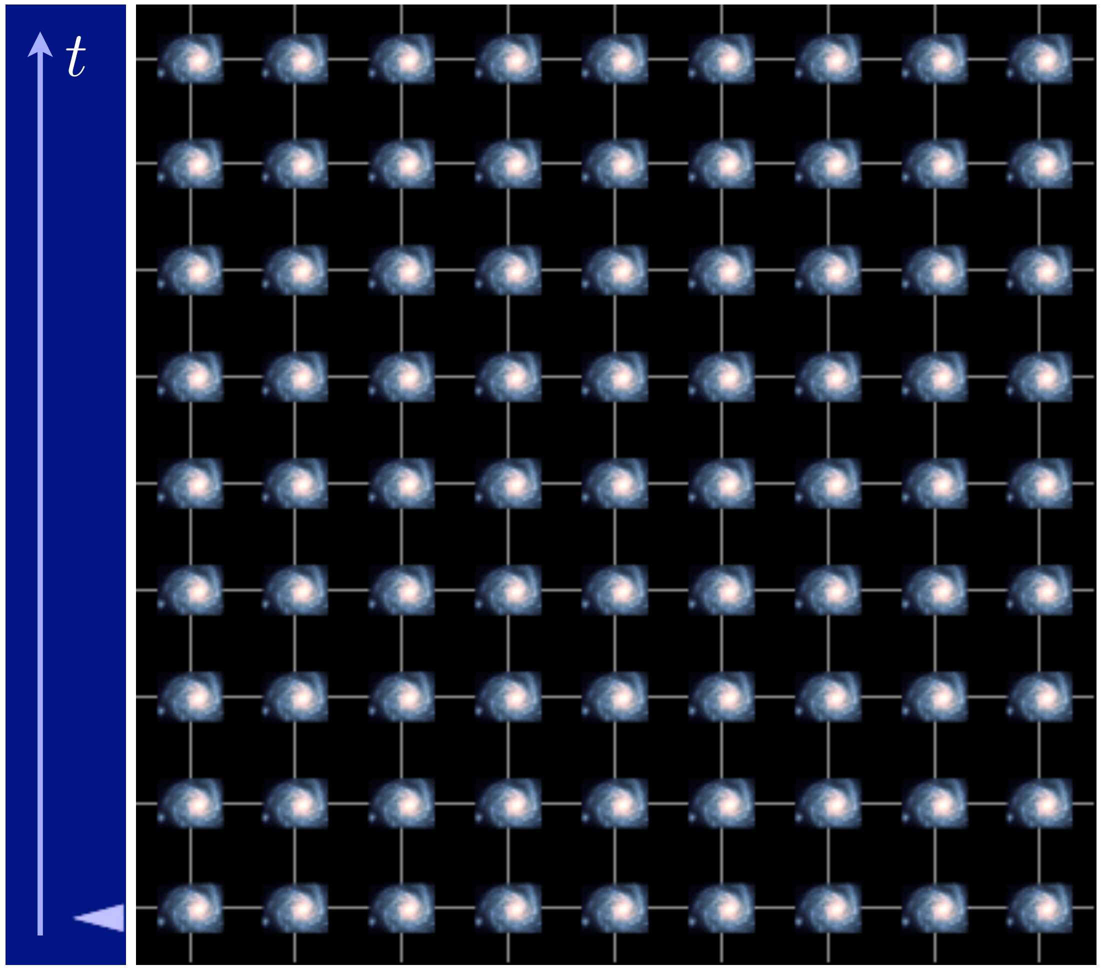
General Relativity - Part I



At $t = t_2$:

$$ds^2 = a_2^2 d\vec{x}^2$$

General Relativity - Part I



$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

General Relativity - Part I

More generally

$$ds^2 = -dt^2 + a^2(t)\left[d\vec{x}^2 + K\frac{\vec{x}\cdot d\vec{x}}{1 - K\vec{x}^2}\right]$$

$$K = \begin{cases} 1 & \text{closed} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$$

Currently all data is consistent with a flat universe

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

General Relativity - Part I

The geometry of our universe is thus encoded by the "scale factor" $a(t)$.

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In a spatially flat universe, physical quantities are independent of its normalization:

$$\frac{a(t)}{a(t_0)} = \frac{1}{1+z}$$

redshift ← (z)

General Relativity - Part I

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In a spatially flat universe, physical quantities are independent of its normalization:

$$\frac{a(t)}{a(t_0)} = \frac{1}{1+z}$$ ← redshift

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$ expansion or Hubble rate

General Relativity - Part I

In general relativity the dynamics of the metric is determined by the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

with

$$R_{\mu\nu} = \frac{\partial \Gamma^{\rho}_{\mu\nu}}{\partial x^{\rho}} - \frac{\partial \Gamma^{\rho}_{\rho\mu}}{\partial x^{\nu}} + \Gamma^{\rho}_{\mu\nu} \Gamma^{\sigma}_{\rho\sigma} - \Gamma^{\rho}_{\mu\sigma} \Gamma^{\sigma}_{\nu\rho} \qquad R = g^{\mu\nu} R_{\mu\nu}$$

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\frac{\partial g_{\sigma\mu}}{\partial x^{\nu}} + \frac{\partial g_{\sigma\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} \right)$$

and stress tensor $T_{\mu\nu}$

General Relativity - Part I

These follow from the Einstein-Hilbert action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\mathrm{matter}}$$

Just as an aside and without proof:

Any theory that contains a massless spin-2 particle will at low energies/long distances be of the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\mathrm{matter}} + \dots$$

higher derivative corrections negligible at long distances

General Relativity - Part I

For the homogeneous and isotropic FLRW universe and stress tensor the Einstein equations become

$$H^2 = \frac{8\pi G}{3}\rho \qquad \text{(00)}$$

$$3H^2 + 2\dot{H} = -8\pi G p \qquad \text{(ij)}$$

These imply a conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0$$

for $p = w\rho$, we have $\rho(t) = \rho(t_0)\left(\frac{a(t_0)}{a(t)}\right)^{3(1+w)}$

General Relativity - Part I

For a situation with more than one component

$$H^2 = \frac{8\pi G}{3}\left(\rho_M + \rho_R + \rho_\Lambda\right)$$

with

$$\rho_M = \Omega_M \rho_{\mathrm{crit},0} \left(\frac{a_0}{a}\right)^3 \qquad (p = 0) \qquad \text{matter}$$

$$\rho_R = \Omega_R \rho_{\mathrm{crit},0} \left(\frac{a_0}{a}\right)^4 \qquad (p = \frac{1}{3}\rho) \qquad \text{radiation}$$

$$\rho_\Lambda = \Omega_\Lambda \rho_{\mathrm{crit},0} \qquad (p = -\rho) \qquad \text{cosmological constant}$$

General Relativity - Part I

The motion of particles is governed by the action

$$S = -m \int d\tau \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$$

It leads to

$$\frac{d^2 x^\mu}{d\tau} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{d\tau} \frac{dx^\rho}{d\tau} = 0$$

In the FLRW universe

$$\frac{d}{dt}\frac{dx^i}{d\tau} + 2H\frac{dx^i}{d\tau} = 0 \qquad \text{or} \qquad \frac{dx^i}{d\tau} \propto \frac{1}{a(t)^2}$$

General Relativity - Part I

The momentum of a particle the behaves as

$$p = m\sqrt{g_{ij}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}} \propto \frac{1}{a(t)}$$

This remains true for massless particles

A particle emitted at time t with momentum p is redshifted to

$$p_0 = p\frac{a(t)}{a(t_0)} = \frac{p}{1+z} \quad \text{today.}$$

Prediction of the CMB

What is the origin of chemical elements and how can one explain their relative abundances?

Extrapolating the expansion rate backwards to energy densities necessary for element formation, Gamow in 1946 writes:

> Returning to our problem of the formation of elements, we see that *the conditions necessary for rapid nuclear reactions were existing only for a very short time*, so that it may be quite dangerous to speak about an equilibrium-state which must have been established during this period.

casting doubt on the previously held idea that the chemical elements formed in an equilibrium process.

Prediction of the CMB

Based on this observation, Alpher, Bethe, Gamow in 1948 propose that elements formed by neutron capture

$$\frac{dn_i}{dt} = f(t)(\sigma_{i-1}n_{i-1} - \sigma_i n_i) \quad i = 1, 2, \cdots 238,$$

With cross sections, and assuming all elements are created through this process, one can fit the observed abundances to determine

$$\int_{t_0}^{t_1} n_n dt$$

or equivalently $\int_{t_0}^{t_1} \rho_n dt$ using $\rho_n = m n_n$

Prediction of the CMB

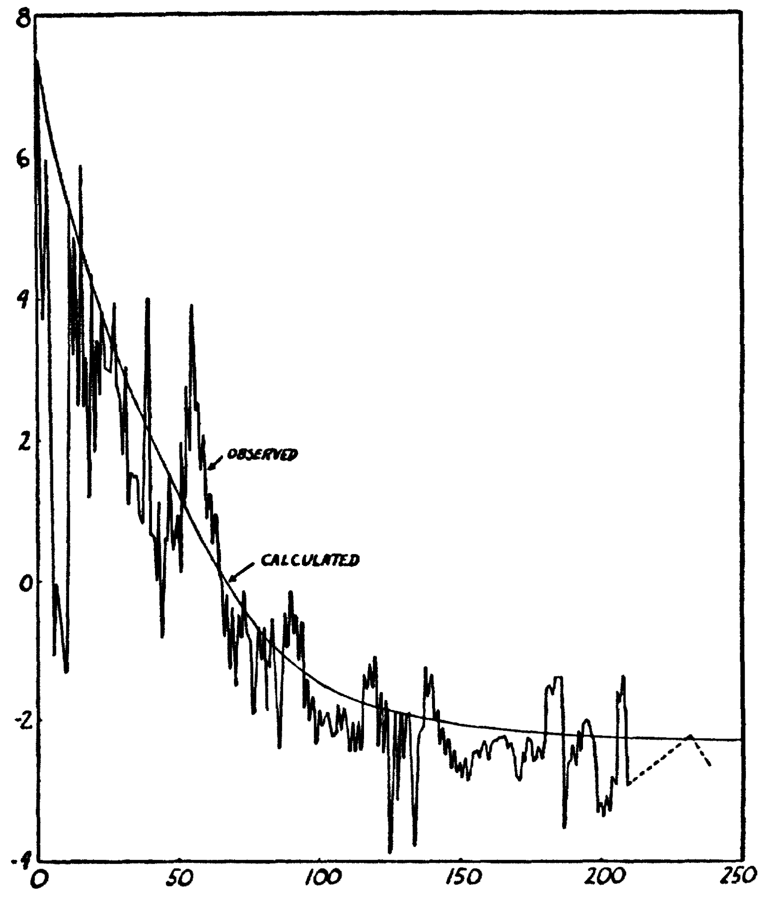

Fig. 1.

Alpher, Bethe, Gamow give a value that is wrong (by 10 orders of magnitude)

Prediction of the CMB

Alpher corrected the mistake in 1948, and finds

$$\int_{t_0}^{t_1} n_n dt = 0.81 \times 10^{18} \frac{s}{cm^3}$$

using this procedure.

If the universe were only filled with nucleons at this time, one would have

$$n_n \approx \frac{\rho e^{-t/\tau_n}}{m_n} \quad \text{with} \quad \rho = \frac{3H^2}{8\pi G} = \frac{1}{6\pi G t^2}$$

$$\text{and} \quad \int_{t_0}^{t_1} n_n dt = \int_{t_0}^{t_1} \frac{e^{-t/\tau_n}}{6\pi G m_n t^2}$$

Prediction of the CMB

Assuming the process takes a time comparable to the neutron lifetime, the observed abundances imply a start time

$$t_0 \approx 10^4 s \gg \tau_n$$

The universe would consist of only hydrogen!

As Alpher points out, a hot big bang in which the universe is filled with black body radiation in addition to matter at the time of element formation provides a way out.

Prediction of the CMB

A flaw with these estimates is that the gap at A=5,8 implies that the heavy elements cannot be formed by neutron capture in the early universe.

Gamow 1948 provides an alternative estimate that is on the right track.

Before heavy elements can form, deuterium must form.

$$n_{n,p}\sigma_n v \sim H$$

Prediction of the CMB

with the known capture cross section for fast neutrons on hydrogen

$$\sigma_n \sim 4 \times 10^{-29} cm^2$$

and velocity

$$v \sim 10^9 cm/s$$

this implies

$$n_n t \sim \frac{1}{\sigma_n v} \sim 10^{20} \frac{s}{cm^3}$$

In a matter dominated universe this again implies a start time

$$t_0 \approx 10^4 s \gg \tau_n$$

and a universe filled only with hydrogen.

Prediction of the CMB

Based on this both Alpher and Gamow consider a hot big bang with a universe dominated by black radiation at early times.

An estimate of the temperature of this radiation today is also given

In fact, we find that the value of $\rho_{r''}$ consistent with Eq. (4) is

$$\rho_{r''} \cong 10^{-32}\ \mathrm{g/cm^3}, \qquad (12d)$$

which corresponds to a temperature now of the order of 5°K.

Prediction of the CMB

In a hot big bang photons dissociate deuterons

$$p + n \leftrightarrow d + \gamma$$

The beginning of nucleosynthesis takes place when photo-dissociation becomes inefficient enough for deuterons to capture additional neutrons.

Prediction of the CMB

The first careful study of the formation of light elements in a hot big bang was by Fermi and Turkevich (but not published).

No.	Reaction	Specific reaction rates	Term in rate equations, $\mathcal{R}'$ [See Eq. (132)]
1	$N = H + e^-$	10^{-3} sec.$^{-1}$	$10^{-3}\mathbf{x}_N$
2	$N + H = D + h\nu$	6.6×10^{-20} sec.$^{-1}$	$6.6\times10^{-20}q_0\mathbf{x}_N\mathbf{x}_H t^{-3/2}$
3	$N + D = T + h\nu$	2.0×10^{-22} sec.$^{-1}$	$2.0\times10^{-22}q_0\mathbf{x}_N\mathbf{x}_D t^{-3/2}$
4	$N + D = N + N + H$	Negligible (see reaction 18)	0
5	$N + He^3 = He^4 + h\nu$	10^{-21} sec.$^{-1}$ (estimated)	$10^{-21}q_0\mathbf{x}_N\mathbf{x}_{He^3} t^{-3/2}$
6	$N + He^3 = T + H$	1.5×10^{-15} sec.$^{-1}$	$1.5\times10^{-15}q_0\mathbf{x}_N\mathbf{x}_{He^3} t^{-3/2}$
7	$H + H = D + e^+$	$a_1 = 2\times10^{-39}$; $a_2 = 3.16$	$7.0\times10^{-41}q_0(\mathbf{x}_H)^2 t^{-7/6}10^{-0.592t^{1/6}}$
8	$H + D = He^3 + h\nu$	$a_1 = 8.6\times10^{-21}$; $a_2 = 3.48$	$3.0\times10^{-22}q_0\mathbf{x}_H\mathbf{x}_D t^{-7/6}10^{-0.652t^{1/6}}$
9	$H + D = H + H + N$	Negligible (see reaction 18)	0
10	$H + T = He^4 + h\nu$	$a_1 = 1.5\times10^{-19}$; $a_2 = 3.62$	$5.3\times10^{-21}q_0\mathbf{x}_H\mathbf{x}_T t^{-7/6}10^{-0.678t^{1/6}}$
11	$H + T = He^3 + N$	$1.5\times10^{-15}\times10^{-36.8/T_8}$ sec.$^{-1}$	$1.5\times10^{-15}q_0\mathbf{x}_H\mathbf{x}_T t^{-3/2}10^{-0.242t^{1/2}}$
12	$D + D = He^4 + h\nu$	$a_1 = 3.07\times10^{-19}$; $a_2 = 3.99$	$1.08\times10^{-20}q_0(\mathbf{x}_D)^2 t^{-7/6}10^{-0.747t^{1/6}}$
13	$D + D = He^3 + N$	$a_1 = 3.0\times10^{-15}$; $a_2 = 3.99$	$1.1\times10^{-16}q_0(\mathbf{x}_D)^2 t^{-7/6}10^{-0.747t^{1/6}}$
14	$D + D = H + T$	$a_1 = 3.0\times10^{-15}$; $a_2 = 3.99$	$1.1\times10^{-16}q_0(\mathbf{x}_D)^2 t^{-7/6}10^{-0.747t^{1/6}}$
15	$D + T = He^4 + N$	$a_1 = 5.0\times10^{-13}$; $a_2 = 4.24$	$1.8\times10^{-14}q_0\mathbf{x}_D\mathbf{x}_T t^{-7/6}10^{-0.794t^{1/6}}$
16	$D + He^3 = He^4 + H$	$a_1 = 1.5\times10^{-12}$; $a_2 = 6.72$	$5.3\times10^{-14}q_0\mathbf{x}_D\mathbf{x}_{He^3} t^{-7/6}10^{-1.259t^{1/6}}$
17	$D + He^4 = Li^6 + h\nu$	$a_1 = 1.4\times10^{-21}$; $a_2 = 6.96$	$4.9\times10^{-23}q_0\mathbf{x}_D\mathbf{x}_{He^4} t^{-7/6}10^{-1.304t^{1/6}}$
18[a]	$D + h\nu = H + N$	$5.9\times10^{12}T_8^{3/2}10^{-110/T_8}$ sec.$^{-1}$	$1.1\times10^{+16}\mathbf{x}_D t^{-3/4}10^{-0.723t^{1/2}}$
19	$T = He^3 + e^-$	1.8×10^{-9} sec.$^{-1}$	$1.8\times10^{-9}\mathbf{x}_T$
20	$T + T = He^4 + N + N$	$a_1 = 2.6\times10^{-13}$; $a_2 = 4.57$	$9.1\times10^{-15}q_0(\mathbf{x}_T)^2 t^{-7/6}10^{-0.856t^{1/6}}$
21	$T + T = He^6 + h\nu$	$a_1 = 2.6\times10^{-19}$; $a_2 = 4.57$	$9.1\times10^{-21}q_0(\mathbf{x}_T)^2 t^{-7/6}10^{-0.856t^{1/6}}$
22	$T + He^3 = He^4 + N + H$	$a_1 = 1.5\times10^{-12}$; $a_2 = 7.24$	$5.3\times10^{-14}q_0\mathbf{x}_T\mathbf{x}_{He^3} t^{-7/6}10^{-1.356t^{1/6}}$
23	$T + He^3 = He^4 + D$	$a_1 = 1.0\times10^{-13}$; $a_2 = 7.24$	$3.5\times10^{-15}q_0\mathbf{x}_T\mathbf{x}_{He^3} t^{-7/6}10^{-1.356t^{1/6}}$
24	$T + He^3 = Li^6 + h\nu$	$a_1 = 3.1\times10^{-18}$; $a_2 = 7.24$	$1.1\times10^{-19}q_0\mathbf{x}_T\mathbf{x}_{He^3} t^{-7/6}10^{-1.356t^{1/6}}$
25	$T + He^4 = Li^7 + h\nu$	$a_1 = 5.5\times10^{-19}$; $a_2 = 7.56$	$1.9\times10^{-20}q_0\mathbf{x}_T\mathbf{x}_{He^4} t^{-7/6}10^{-1.416t^{1/6}}$
26	$He^3 + He^3 = Be^6 + h\nu$	$a_1 = 1.4\times10^{-17}$; $a_0 = 11.49$	$4.9\times10^{-19}q_0(\mathbf{x}_{He^3})^2 t^{-7/6}10^{-2.151t^{1/6}}$
27	$He^3 + He^3 = He^4 + H + H$	$a_1 = 1.4\times10^{-11}$; $a_2 = 11.49$	$4.9\times10^{-13}q_0(\mathbf{x}_{He^3})^2 t^{-7/6}10^{-2.151t^{1/6}}$
28	$He^3 + He^4 = Be^7 + h\nu$	$a_1 = 1.7\times10^{-19}$; $a_2 = 12.01$	$6.0\times10^{-21}q_0\mathbf{x}_{He^3}\mathbf{x}_{He^4} t^{-7/6}10^{-2.250t^{1/6}}$

[a] The photon concentration is included in the constant.

Prediction of the CMB

As in previous studies the universe was assumed to begin filled with neutrons.

As mentioned by Gamow in 1949 and shown by Hayashi

$$p + e^- \leftrightarrow n + \nu_e$$
$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$
$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

maintain thermal equilibrium between n and p

Prediction of the CMB

So we have

$$\frac{n_n}{n_N} = \frac{\exp(-t/\tau_n)}{1+\exp(\Delta m/T)} \approx 0.16\exp(-t/\tau_n)$$

until

$$d + d \to {}^3H + p$$
$$d + d \to {}^3He + n$$
$$d + {}^3H \to {}^4He + n$$
$$d + {}^3He \to {}^4He + p$$

become efficient, which occurs around $T \approx 10^9 K$ or around three minutes after the hot big bang.

$$Y_{He} = \frac{4n_{He}}{4n_{He} + n_H} = 2\frac{n_n}{n_N} \approx 0.26$$

Prediction of the CMB

The predictions were largely forgotten for the next decade, likely because

- it became clear that heavy elements could not have formed in this way
- nucleosynthesis in stars became better understood and was able explain the heavy elements

Prediction of the CMB

The irony perhaps is that there was evidence for radiation at a few K from 1941

MOLECULAR LINES FROM THE LOWEST STATES OF DIATOMIC MOLECULES COMPOSED OF ATOMS PROBABLY PRESENT IN INTERSTELLAR SPACE

BY ANDREW McKELLAR

Thus from (3) we find, for the region of space where the CN absorption takes place, the "rotational" temperature,

$$T = 2^{\circ}\!.3K.$$

Prediction of the CMB

Hoyle 1964:

nucleosynthesis in stars can explain abundances of heavy elements, but not of helium

This brings us back to our opening remarks. There has always been difficulty in explaining the high helium content of cosmic material in terms of ordinary stellar processes. The mean luminosities of galaxies come out appreciably too high on such a hypothesis. The arguments presented here make it clear, we believe, that the helium was produced in a far more dramatic way. Either the Universe has had at least one high-temperature, high-density phase, or massive objects must play (or have played) a larger part in astrophysical evolution than has hitherto been supposed.

Wagoner, Fowler, Hoyle 1966 began one of the first modern BBN computations

Discovery of the CMB

Dicke 1964:

Could a bounce set up a fireball, a universe filled with hot and dense radiation left over and detectable today?

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Jim Peebles working on the theory

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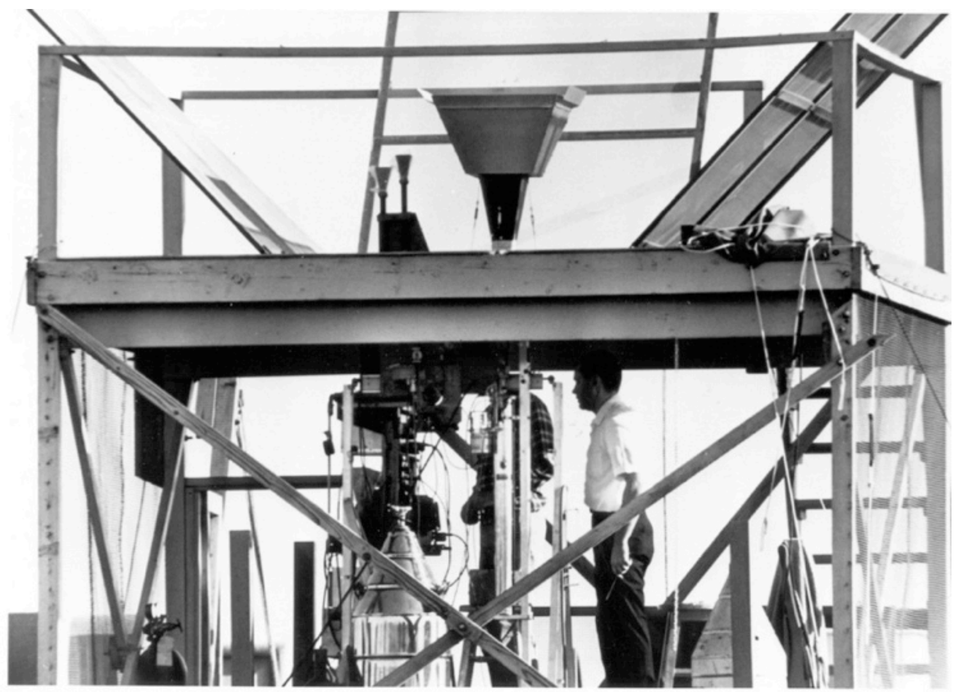
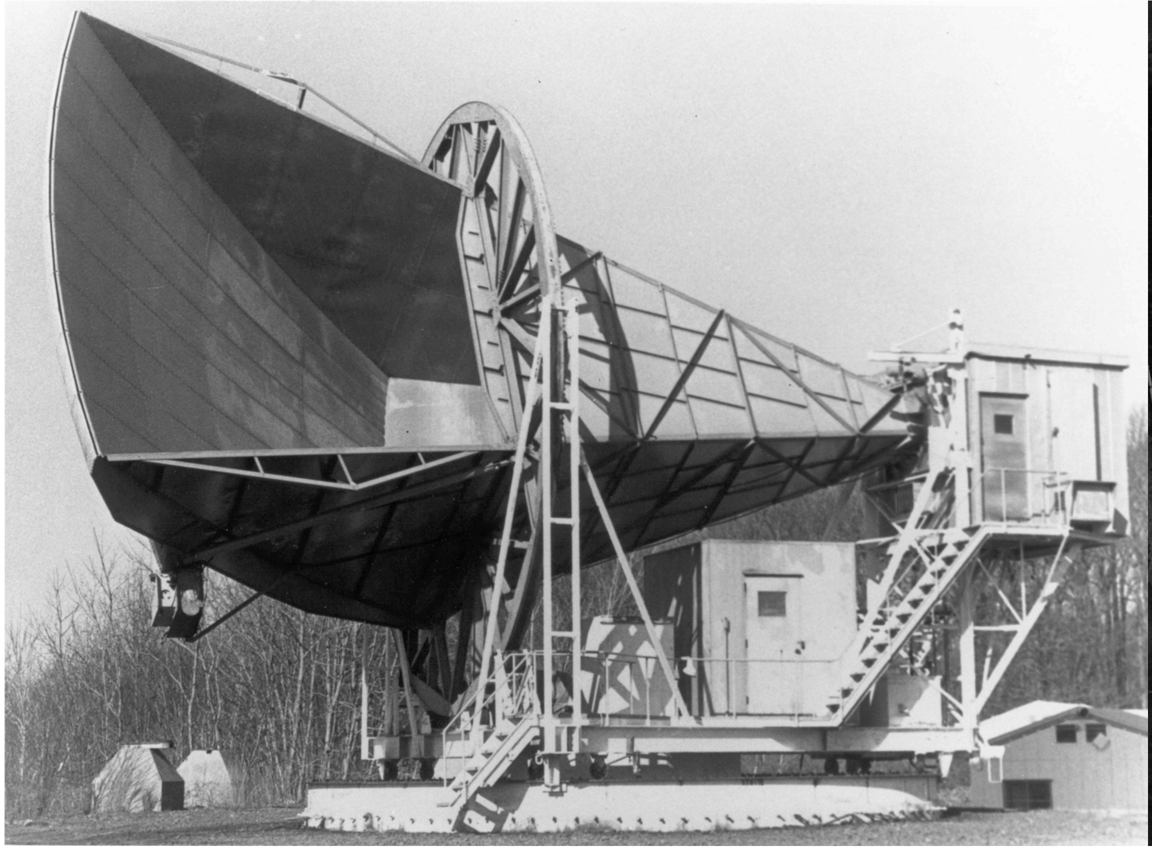
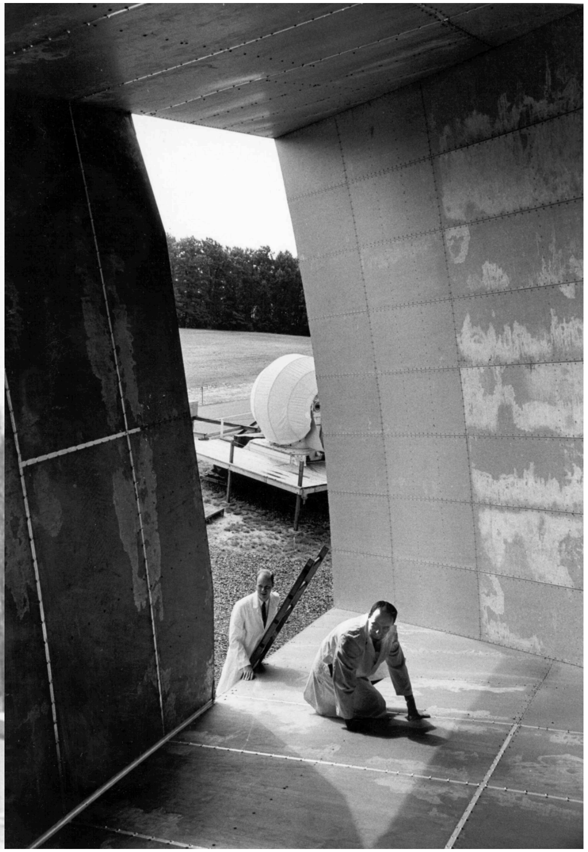


Roll and Wilkinson with the microwave radiometer

Discovery of the CMB

Meanwhile 30 miles away:

Penzias and Wilson are troubled by noise in their experiment

Discovery of the CMB

Penzias and Wilson are informed by Bernie Burke who is informed by Ken Turner of a talk given by Jim Peebles

COSMIC BLACK-BODY RADIATION*

R. H. Dicke
P. J. E. Peebles
P. G. Roll
D. T. Wilkinson

May 7, 1965
Palmer Physical Laboratory
Princeton, New Jersey

A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

A. A. Penzias
R. W. Wilson

May 13, 1965
Bell Telephone Laboratories, Inc
Crawford Hill, Holmdel, New Jersey

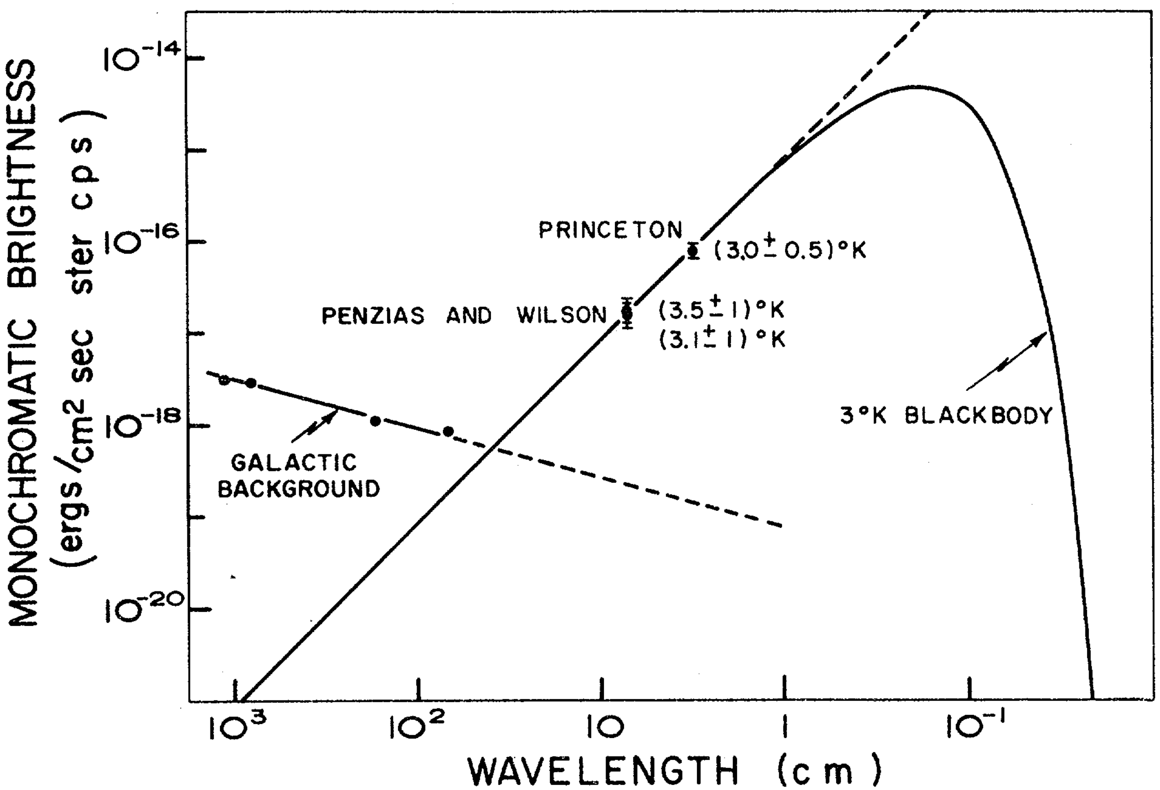
Discovery of the CMB

Additional measurements are required to confirm the interpretation

COSMIC BACKGROUND RADIATION AT 3.2 cm—SUPPORT FOR COSMIC BLACK-BODY RADIATION*

P. G. Roll† and David T. Wilkinson

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 27 January 1966)



Discovery of the CMB

COSMOLOGICAL BACKGROUND RADIATION SATELLITE

J. Mather
P. Thaddeus
Goddard Institute for Space Studies

R. Weiss
D. Muehlner
Massachusetts Institute of Technology

D. T. Wilkinson
Princeton University

M. G. Hauser
R. F. Silverberg
Goddard Space Flight Center

OCTOBER 1974

Discovery of the CMB

A PRELIMINARY MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE *COSMIC BACKGROUND EXPLORER* (*COBE*)[1] SATELLITE

J. C. Mather,[2] E. S. Cheng,[2] R. E. Eplee, Jr., [3] R. B. Isaacman,[3] S. S. Meyer,[4] R. A. Shafer,[2] R. Weiss,[4] E. L. Wright,[5] C. L. Bennett, N. W. Boggess,[2] E. Dwek,[2] S. Gulkis,[6] M. G. Hauser,[2] M. Janssen,[6] T. Kelsall,[2] P. M. Lubin,[7] S. H. Moseley, Jr.,[2] T. L. Murdock,[8] R. F. Silverberg,[2] G. F. Smoot,[9] and D. T. Wilkinson[10]

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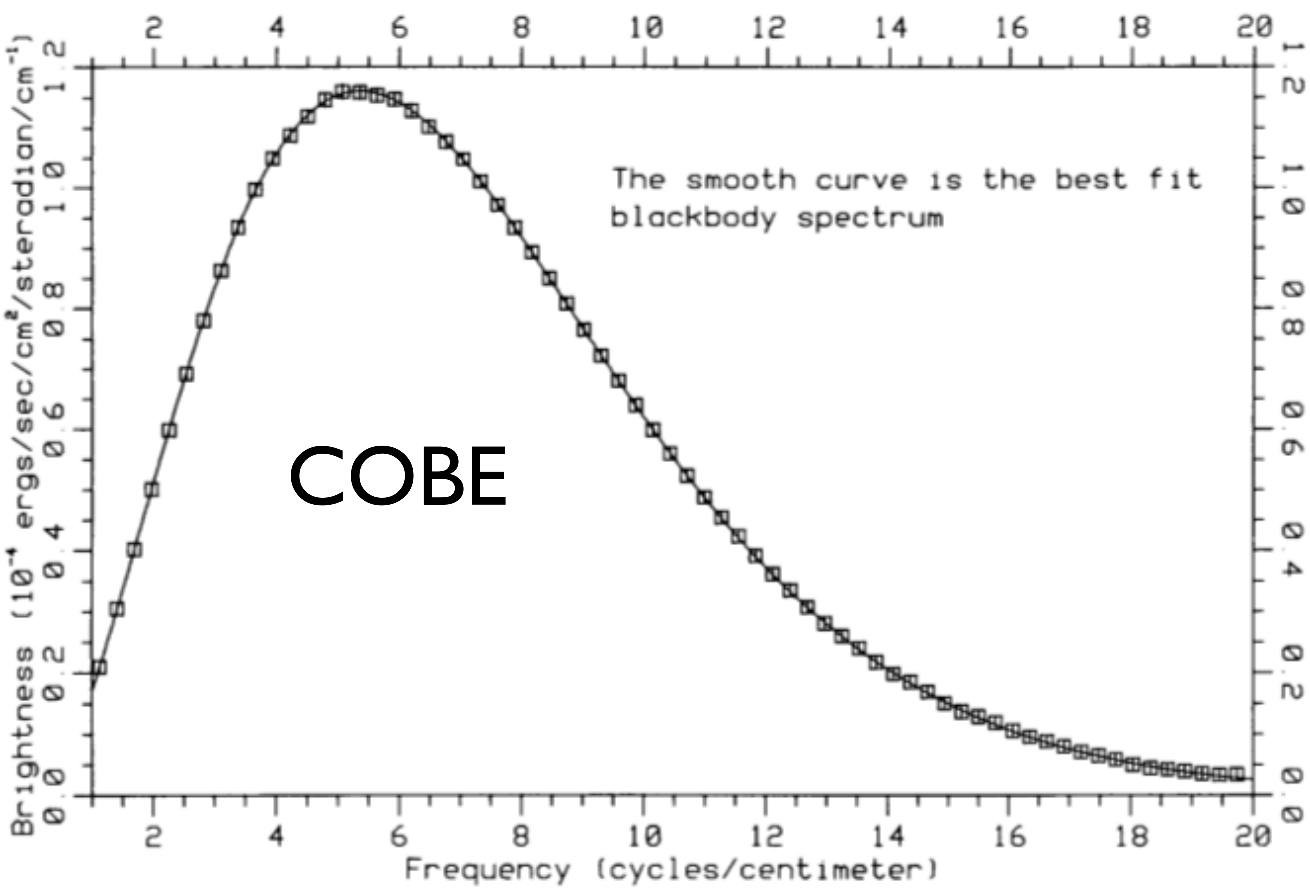

Discovery of the CMB

An Attempt to Measure the Far Infrared Spectrum of the Cosmic Background Radiation

H. P. GUSH

Department of Physics, University of British Columbia, Vancouver, British Columbia

Received August 13, 1973

A liquid helium cooled two-beam far infrared interferometer has been successfully flown in a Black Brant III B rocket. The detector was a germanium bolometer cooled to a temperature of 0.37 K by a liquid He^3 refrigerator. The sensitive range was between approximately 5 and 50 cm^{-1}. Satisfactory cosmic spectra were not obtained because of contamination by radiation from the earth.

Discovery of the CMB

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Rocket Measurement of the Cosmic-Background-Radiation mm-Wave Spectrum

H. P. Gush, M. Halpern, and E. H. Wishnow
Department of Physics, University of British Columbia, Vancouver, Canada V6T 2A6
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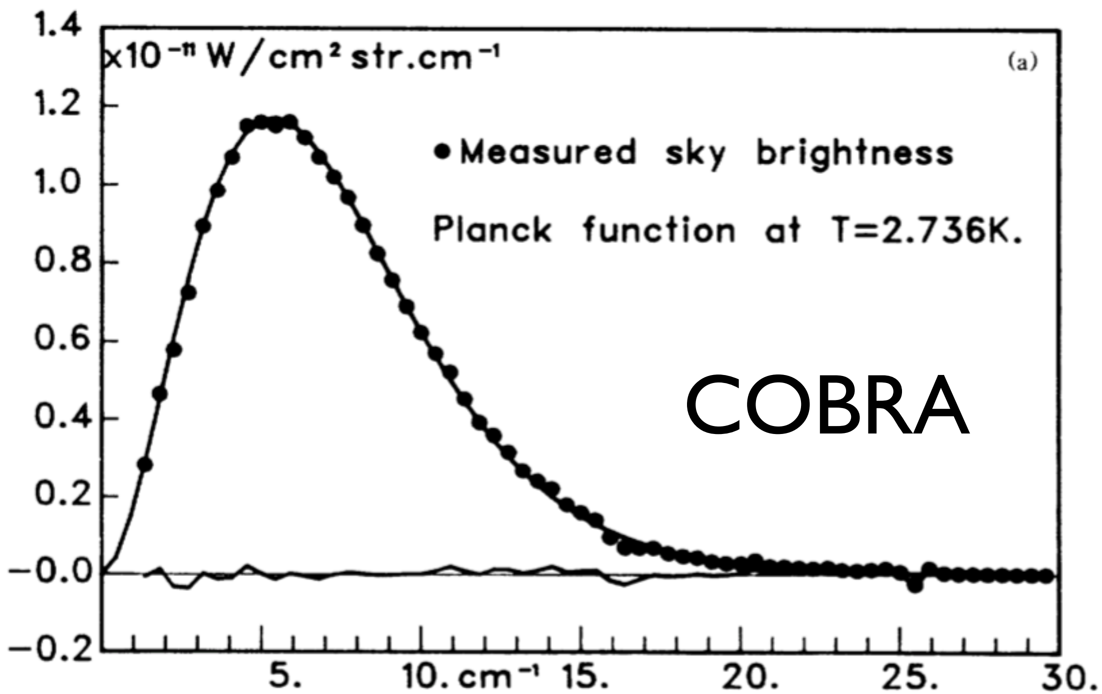

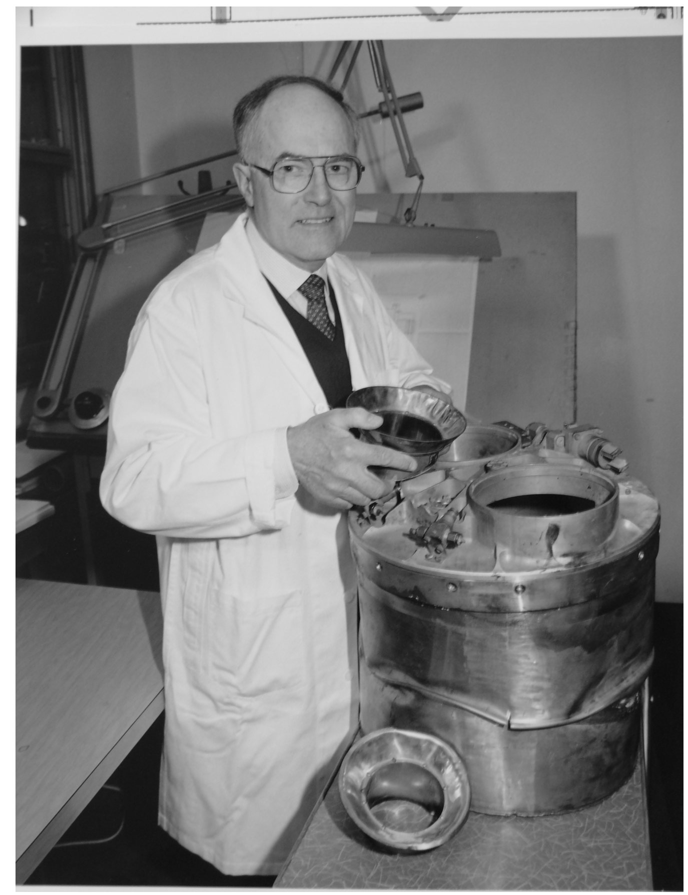

Spectrum of the CMB

At early times, (mostly)

Compton scattering $e^- + \gamma \to e^- + \gamma$

Double Compton scattering $e^- + \gamma \to e^- + \gamma + \gamma'$

Bremsstrahlung $e^- + p \to e^- + p + \gamma$

keep matter and radiation in thermal equilibrium and lead to a black body spectrum for the photons.

$$n_{T(t)}(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{\exp\left(h\nu/kT(t)\right) - 1}$$

Spectrum of the CMB

At some point radiation no longer efficiently scatters off matter and thermal equilibrium is no longer maintained.

So (why) do we expect to observe a black body spectrum today?

Consider an idealization:

- All photons last scatter at same time
- Black body spectrum until last scattering
- Ignore processes that inject photons

Spectrum of the CMB

Or put differently, how does the expansion affect the spectrum

$$n_{T(t)}(\nu)d\nu = \left(\frac{a(t_L)}{a(t)}\right)^3 n_{T(t_L)}\left(\nu a(t)/a(t_L)\right) d\left(\nu a(t)/a(t_L)\right)$$

or

$$n_{T(t)}(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{\exp\left(h\nu/kT(t)\right) - 1}$$

with

$$T(t) = T(t_L)\frac{a(t_L)}{a(t)}$$

For massless quanta the expansion preserves a black body distribution after last scattering

Spectrum of the CMB

This remains true if last scattering is not instantaneous provided scattering events around last scattering do not change the photon energies

When does last scattering occur?

Photons will scatter efficiently as long as

$$n_e \sigma_T c \gtrsim H$$

Spectrum of the CMB

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If there were no recombination and

$$n_e \approx n_b = \frac{\rho_{b,0}}{m_p} \left(\frac{a(t_0)}{a(t)} \right)^3$$

this would happen at temperatures around 100K.

Spectrum of the CMB

When does (re)combination occur?

In thermal equilibrium

$$\frac{n_{1s}}{n_e n_p} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(B/kT\right)$$

Neutrality implies $n_e = n_p$ (after Helium recombination)

The free electron fraction $x_e = \dfrac{n_e}{n_p + n_{1s}}$

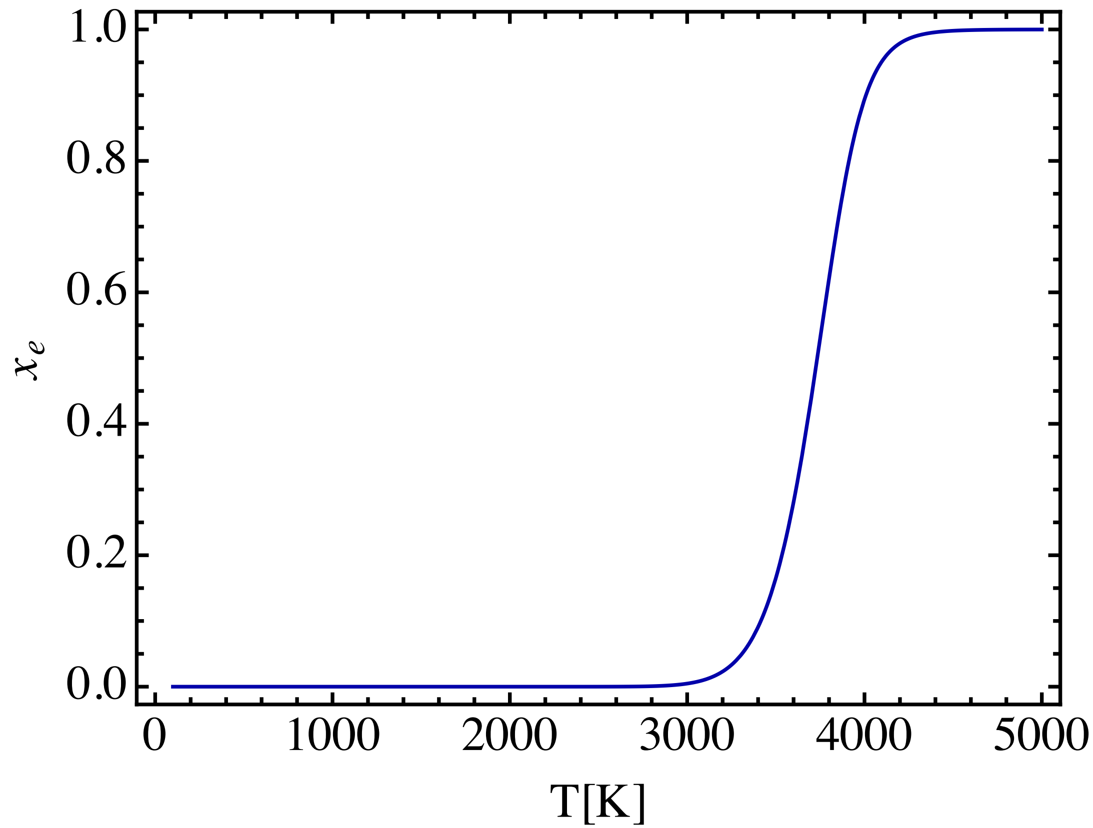
then satisfies the Saha equation

$$\frac{1 - x_e}{x_e^2} = (1 - Y_{He}) n_b \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(B/kT\right)$$

Spectrum of the CMB

When does (re)combination occur?

In thermal equilibrium between 3000K and 4000K



Spectrum of the CMB

However, recombination occurs out of equilibrium

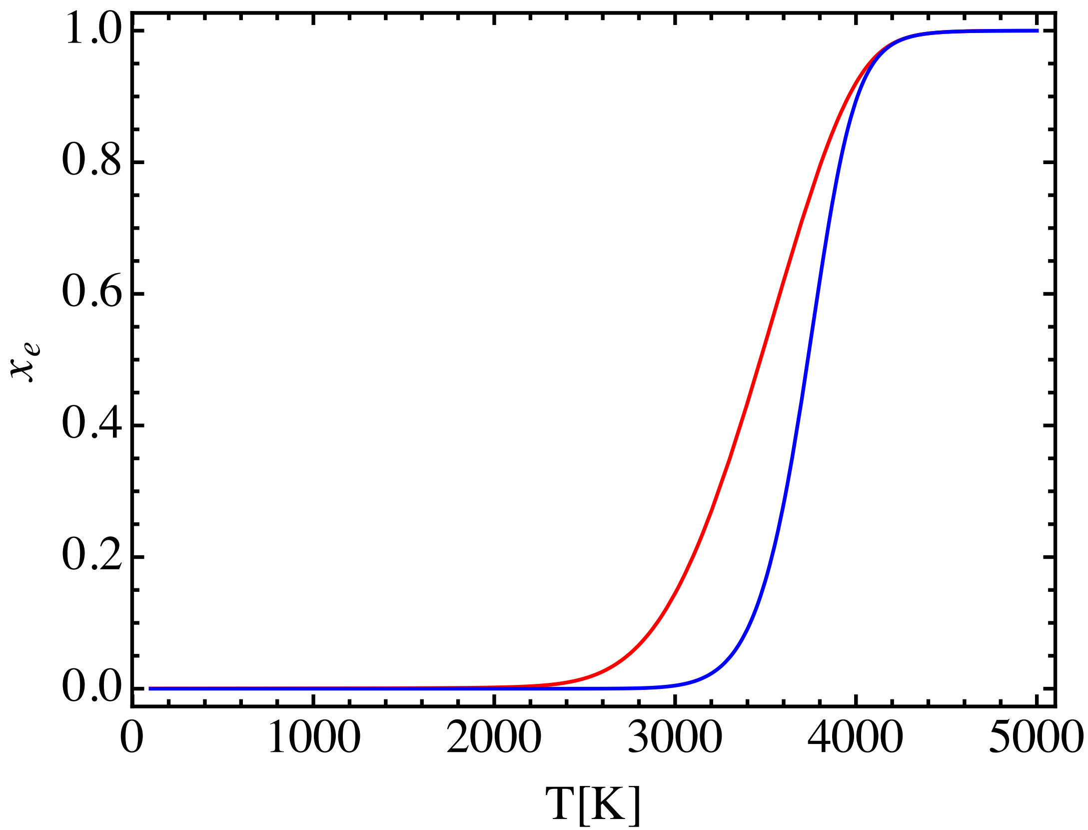
- photons emitted when electrons are captured into low lying energy levels ionize other atoms
- photons emitted in transitions from highly excited states to low lying states excite other atoms
- Ly-α photons excite other atoms from the ground state, making $2p \to 1s$ recombination inefficient so that $2s \to 1s$ is relevant

Peebles and independently Zel'dovich, Kurt, Sunyaev in 1968 derived

$$\frac{dx_e}{dt} = -C\left[\alpha n_p x_e - (1 - x_e)\beta \exp(E_{12}/kT)\right]$$

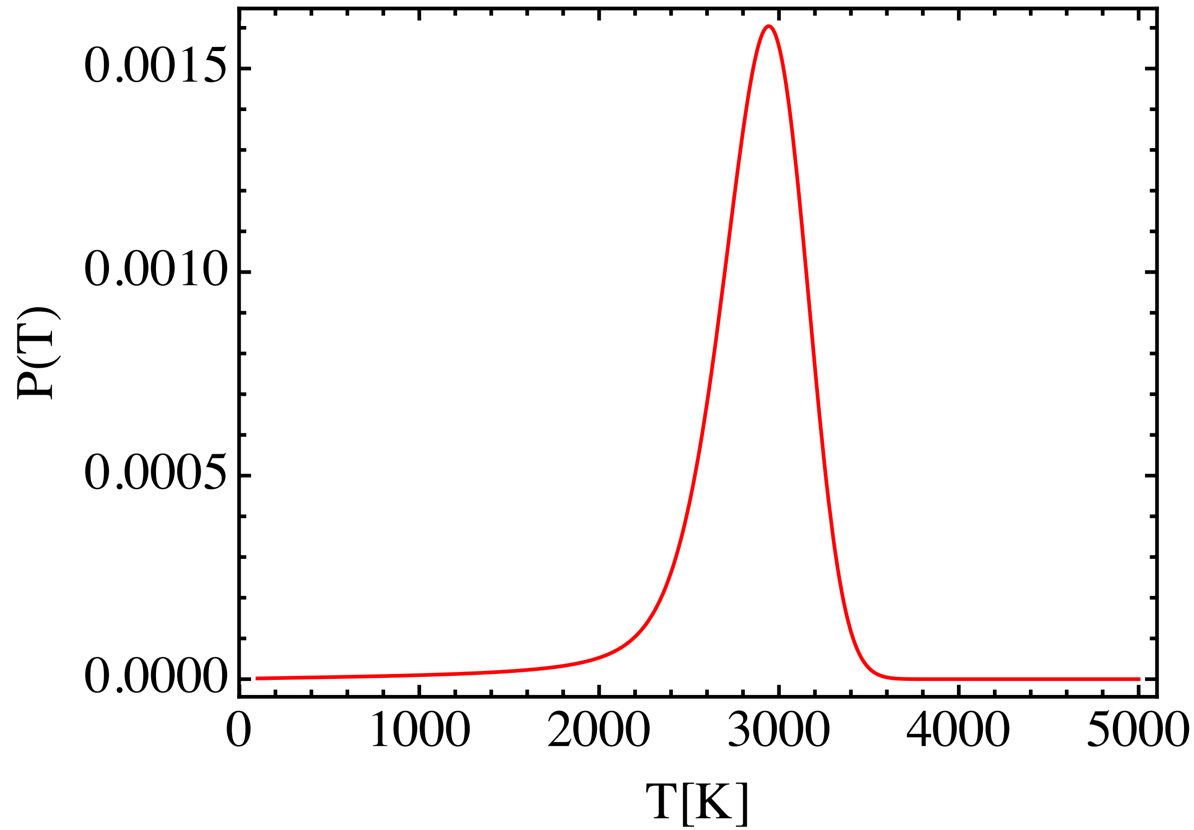
Spectrum of the CMB

Including departures from equilibrium delay recombination



Spectrum of the CMB

Last scattering probability peaks near 3000K



Spectrum of the CMB

So photons last scatter around 3000K.
Is energy still exchanged efficiently then?

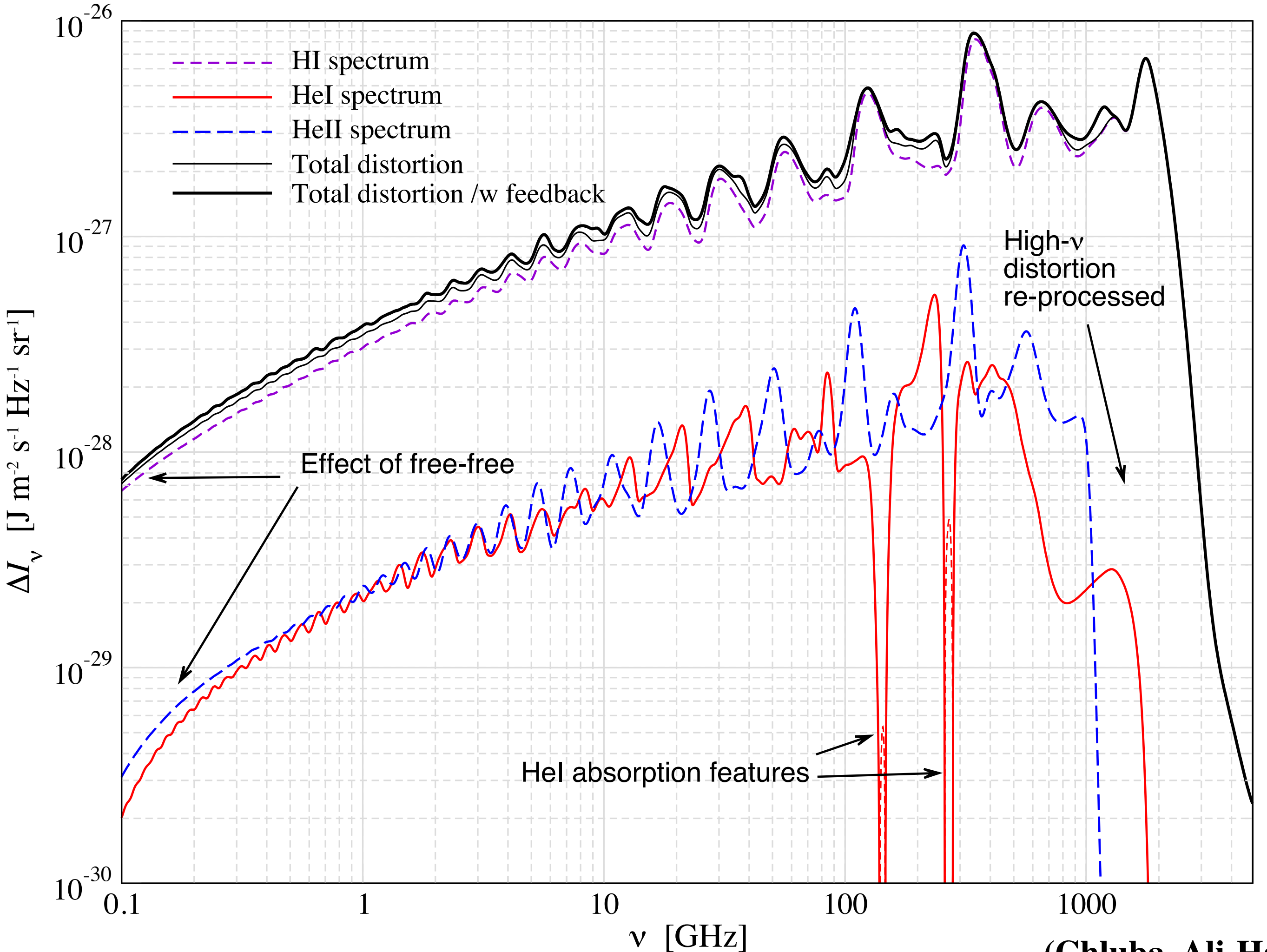
$$\frac{kT}{m_e} n_e \sigma_T c < H \quad \text{below } 10^5 K$$

Thomson scattering can only modify the spectrum at temperatures above $10^5 K$, not around last scattering.

So the spectrum is preserved even if not all photons last scatter at the same instant.

Spectrum of the CMB

However, if a process injects photons around recombination, we expect small spectral distortions



(Chluba, Ali-Haimoud 2015)

Spectrum of the CMB

Above $10^5 K$ energy is exchanged efficiently, but until when are photons efficiently produced?

Double Compton scattering is inefficient when

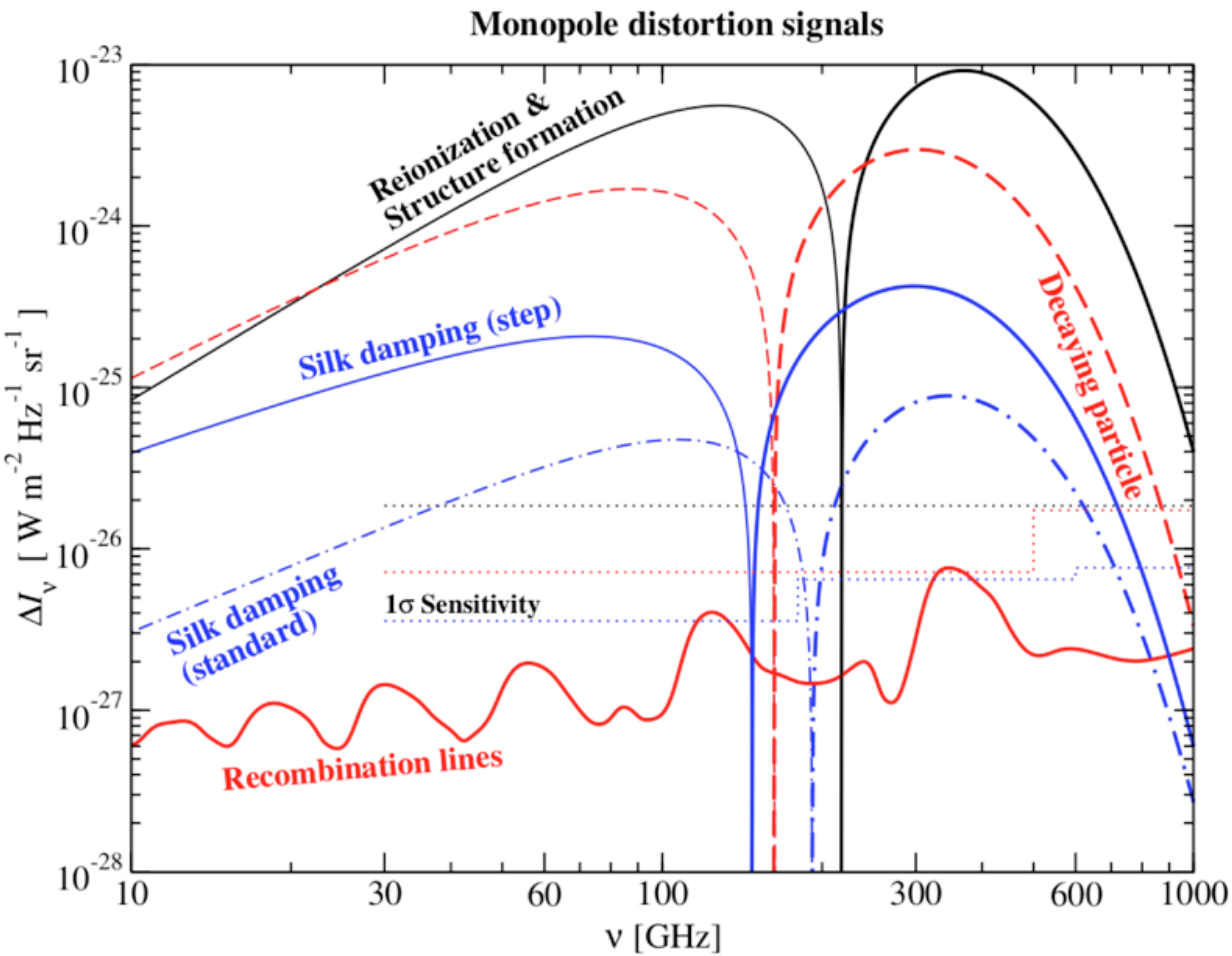
$$\alpha \left(\frac{kT}{m_e}\right)^2 n_e \sigma_T c < H$$ i.e. below $6 \times 10^6 K$

So

$T > 6 \times 10^6 K$	black body
$10^5 K < T < 6 \times 10^6 K$	μ-era
$T < 10^5 K$	y-era

Spectrum of the CMB

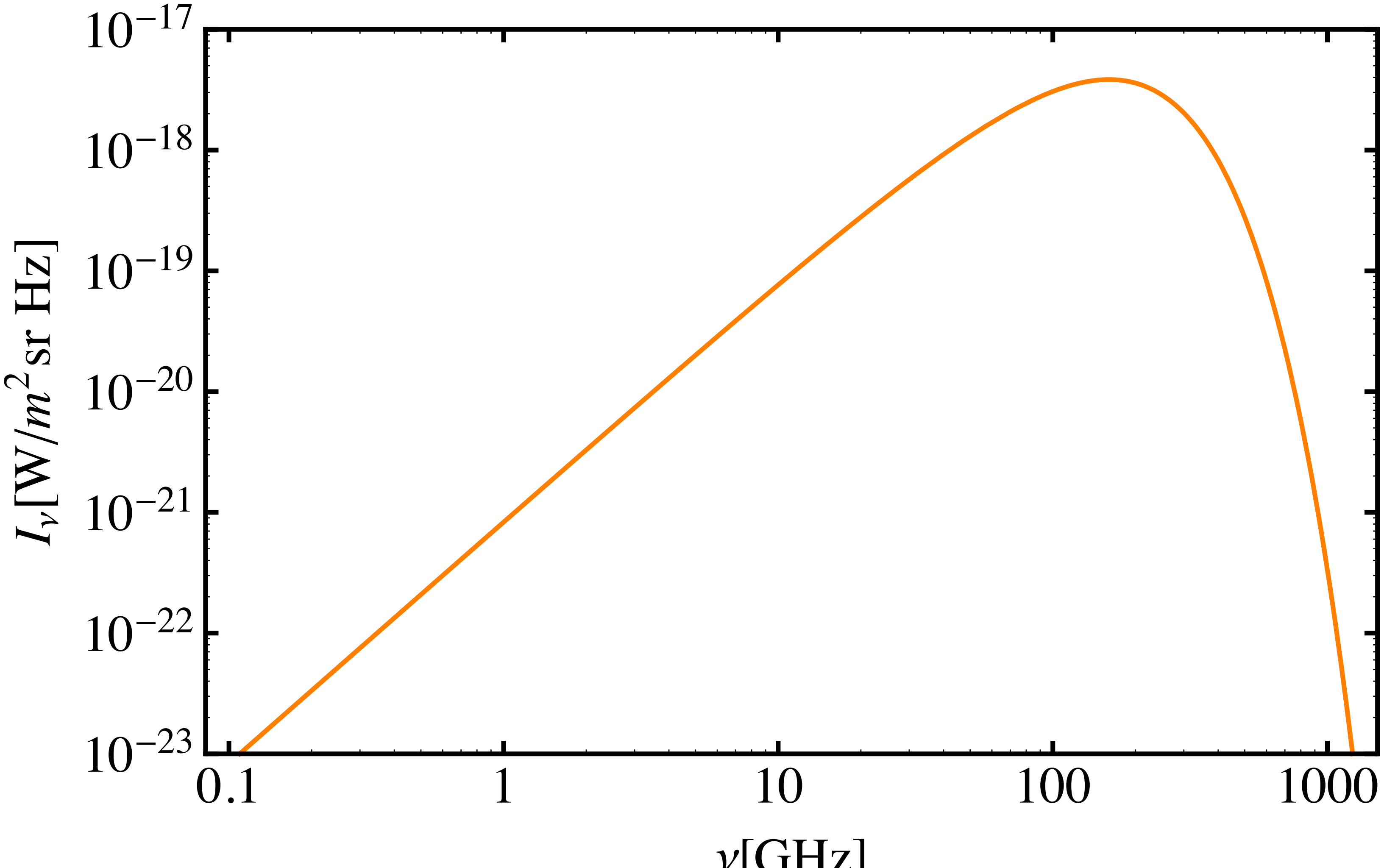
Potential distortions



(Andre et al. 2013)

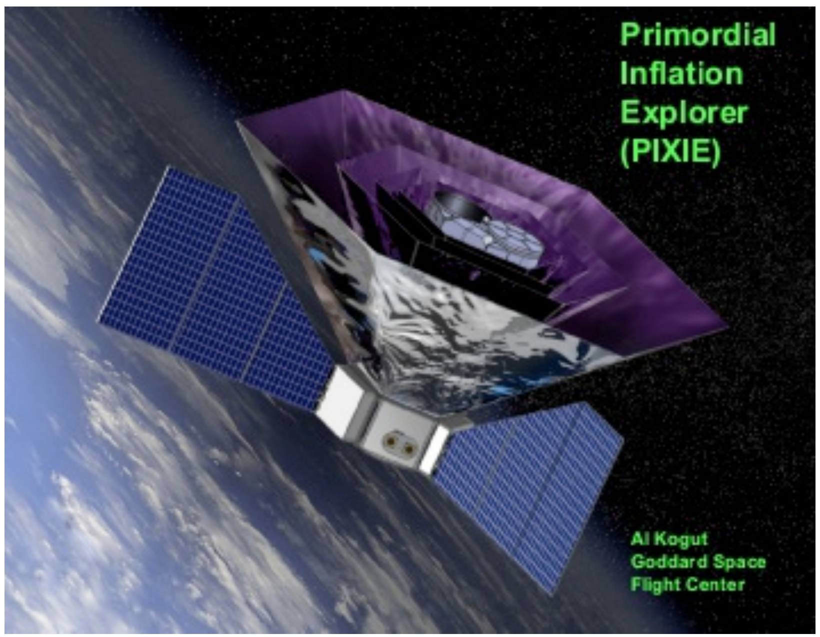
Spectrum of the CMB

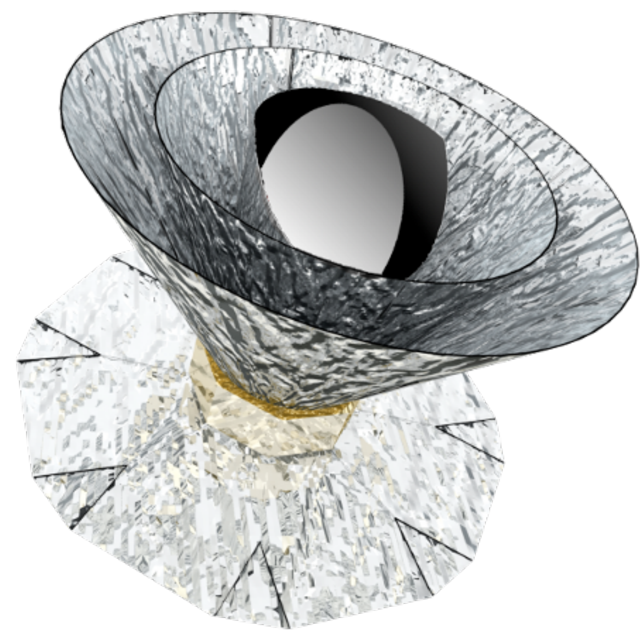
Note the scale



Spectrum of the CMB

Rather remarkably, potential small distortions may be detectable in future experiments





PRISM