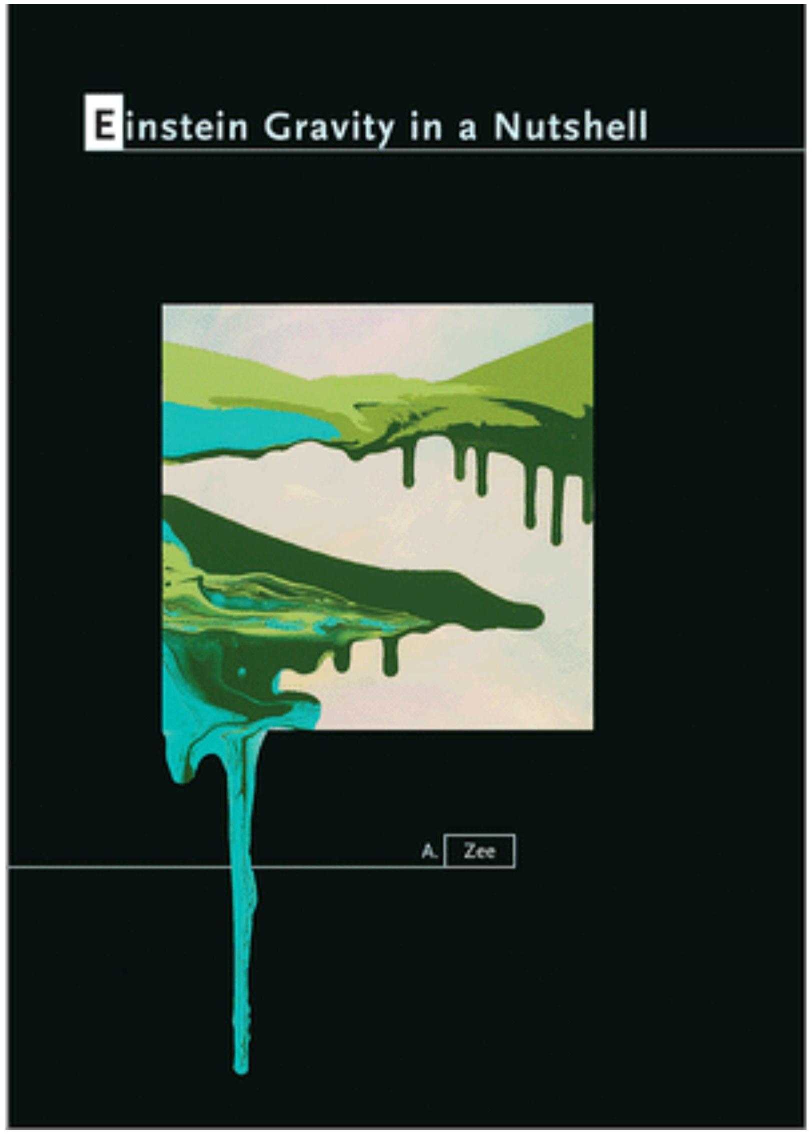
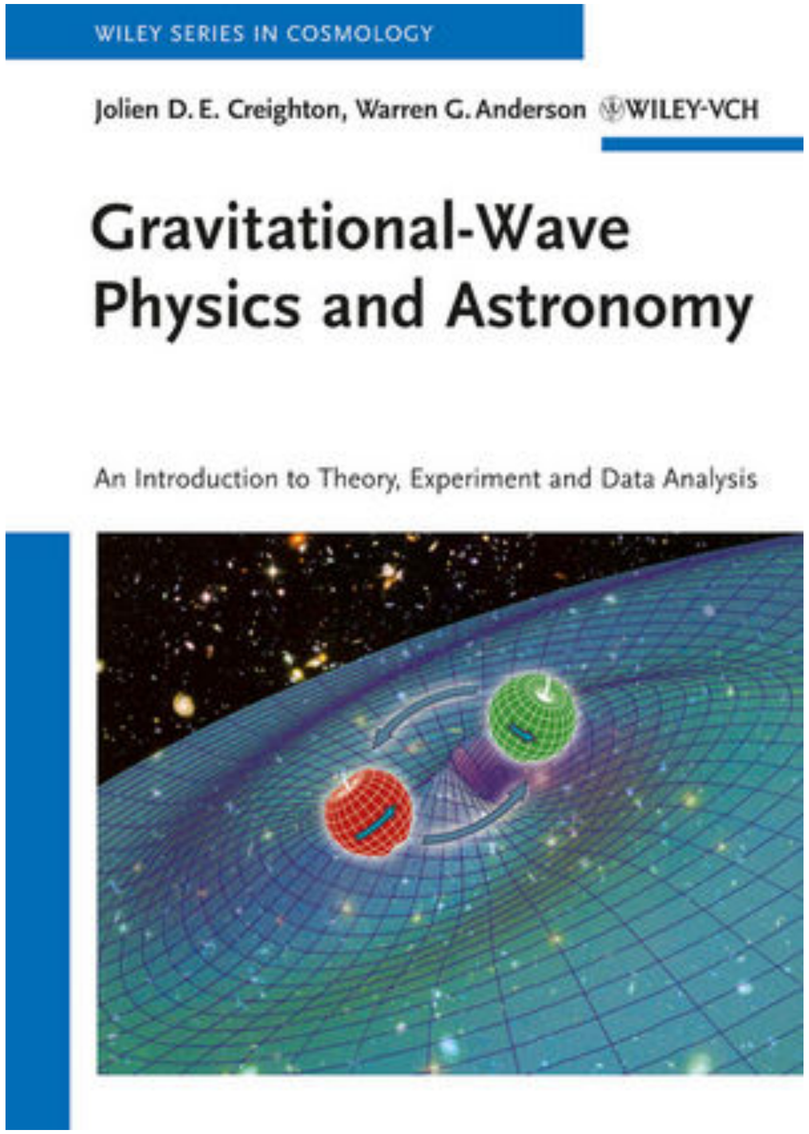
GW2—Propagation, detection, generation

Michele Vallisneri

ICTP Summer School on Cosmology 2016


Einstein Gravity in a Nutshell
A. Zee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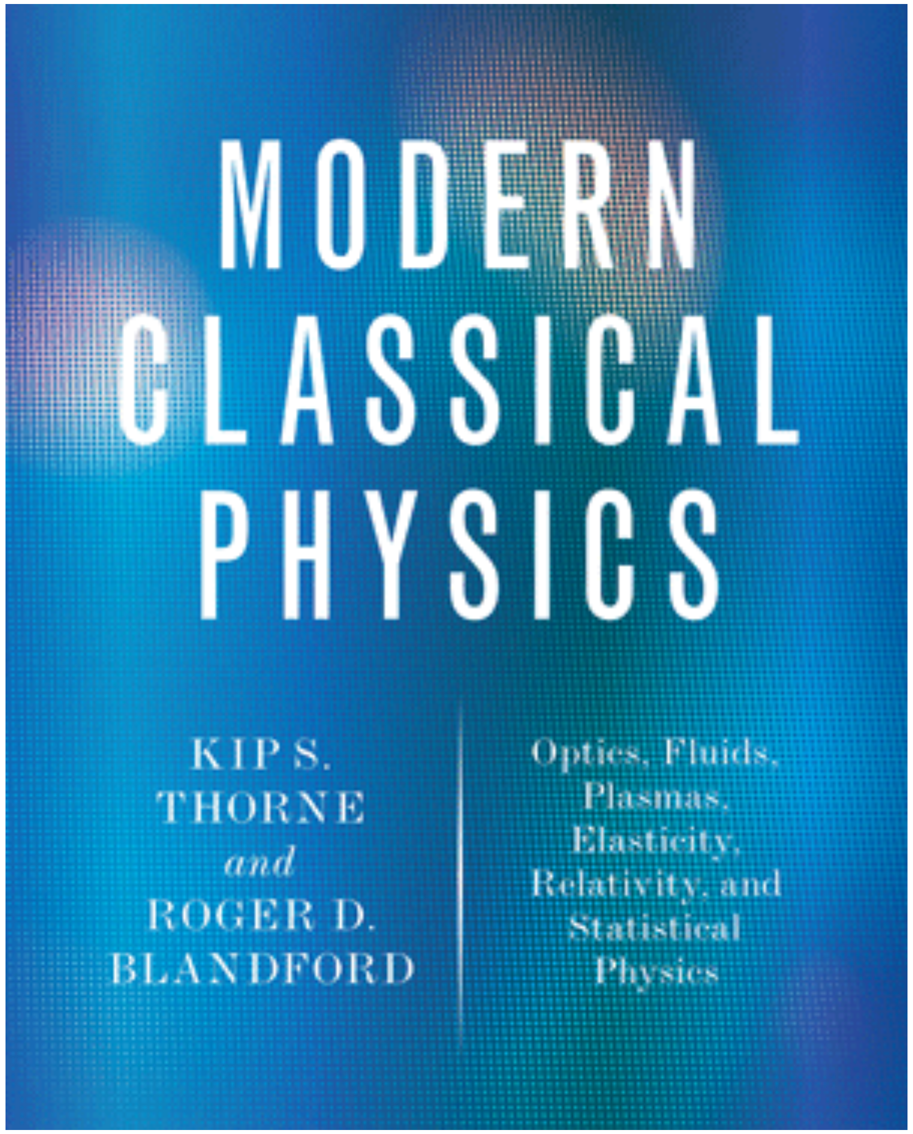
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Things to learn today

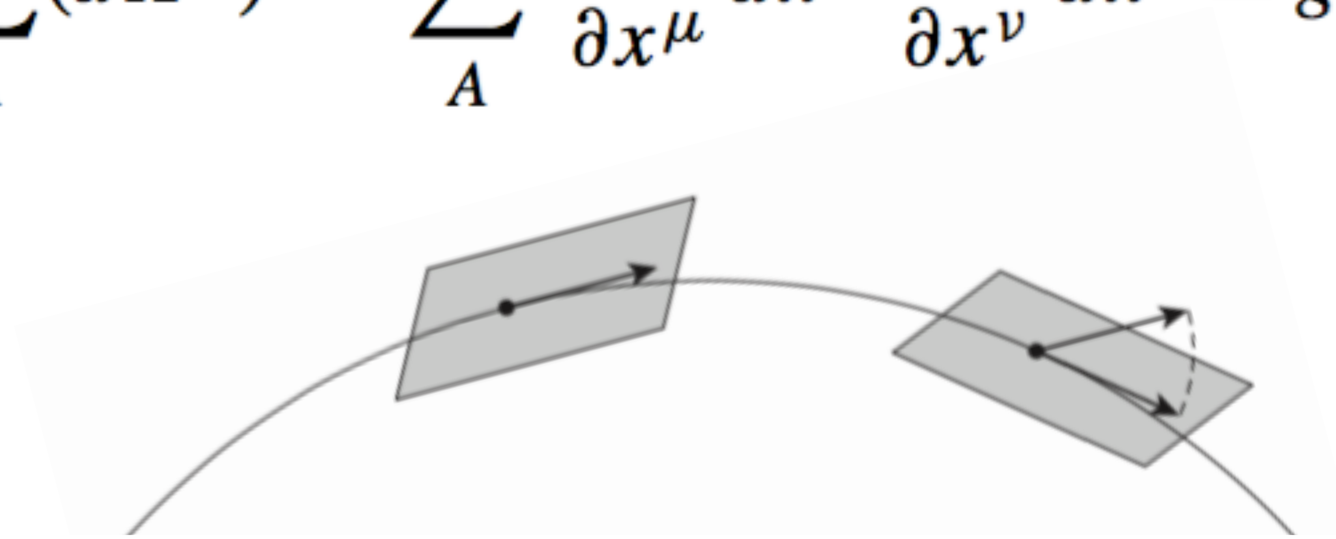
1. In general relativity, curvature propagates like a wave, at the speed of light
2. GWs act as tidal forces in a local Lorentz frame

   — OR, equivalently —

   GWs modulate the proper distance between nearby freely falling particles
3. GWs are transverse and quadrupolar
4. GWs carry energy–momentum, although it can be localized only approximately
5. GWs are emitted by time-dependent mass quadrupoles
6. GWs cause the accelerating inspiral of binary stars

Metric theories of gravity: parallel transport

$$ds^2 \equiv \sum_A (dX^A)^2 = \sum_A \frac{\partial X^A}{\partial x^\mu} dx^\mu \frac{\partial X^A}{\partial x^\nu} dx^\nu \equiv g_{\mu\nu} dx^\mu dx^\nu$$



$$\partial_\nu \vec{W}(x) = \partial_\nu (W^\mu(x) \vec{e}_\mu(x)) = (\partial_\nu W^\mu(x)) \vec{e}_\mu(x) + W^\mu(x) \partial_\nu \vec{e}_\mu(x)$$

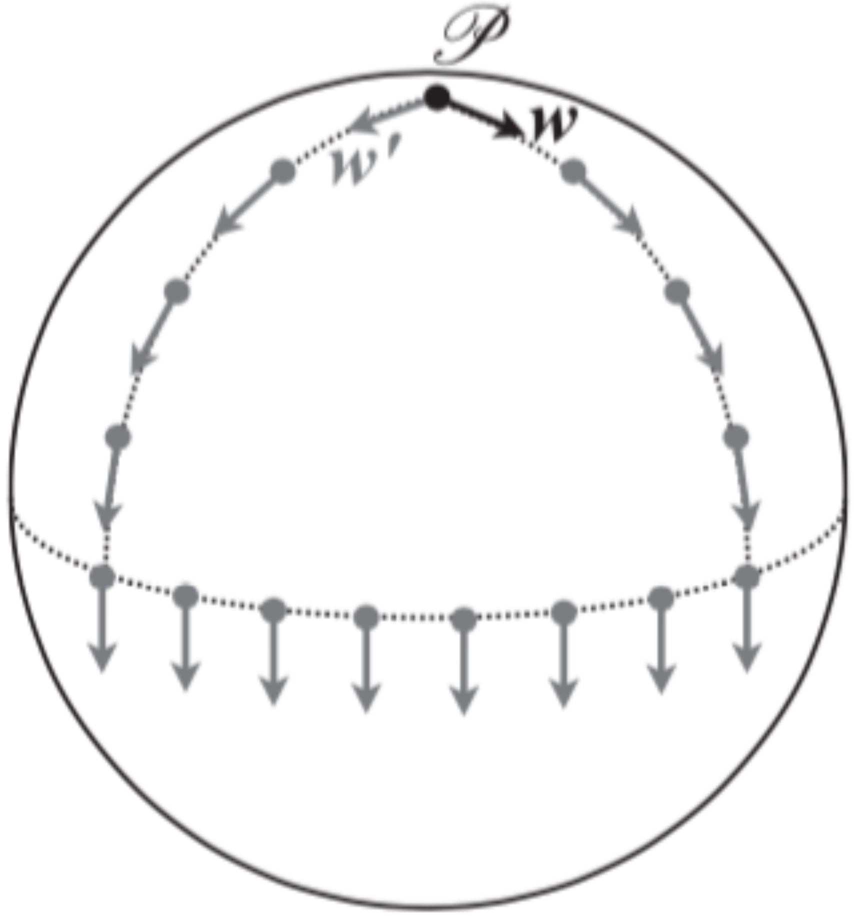
$$= (\partial_\nu W^\mu) \vec{e}_\mu + W^\lambda \Gamma^\mu_{\lambda\nu} \vec{e}_\mu + W^\mu K_{\mu\nu} \vec{n}$$

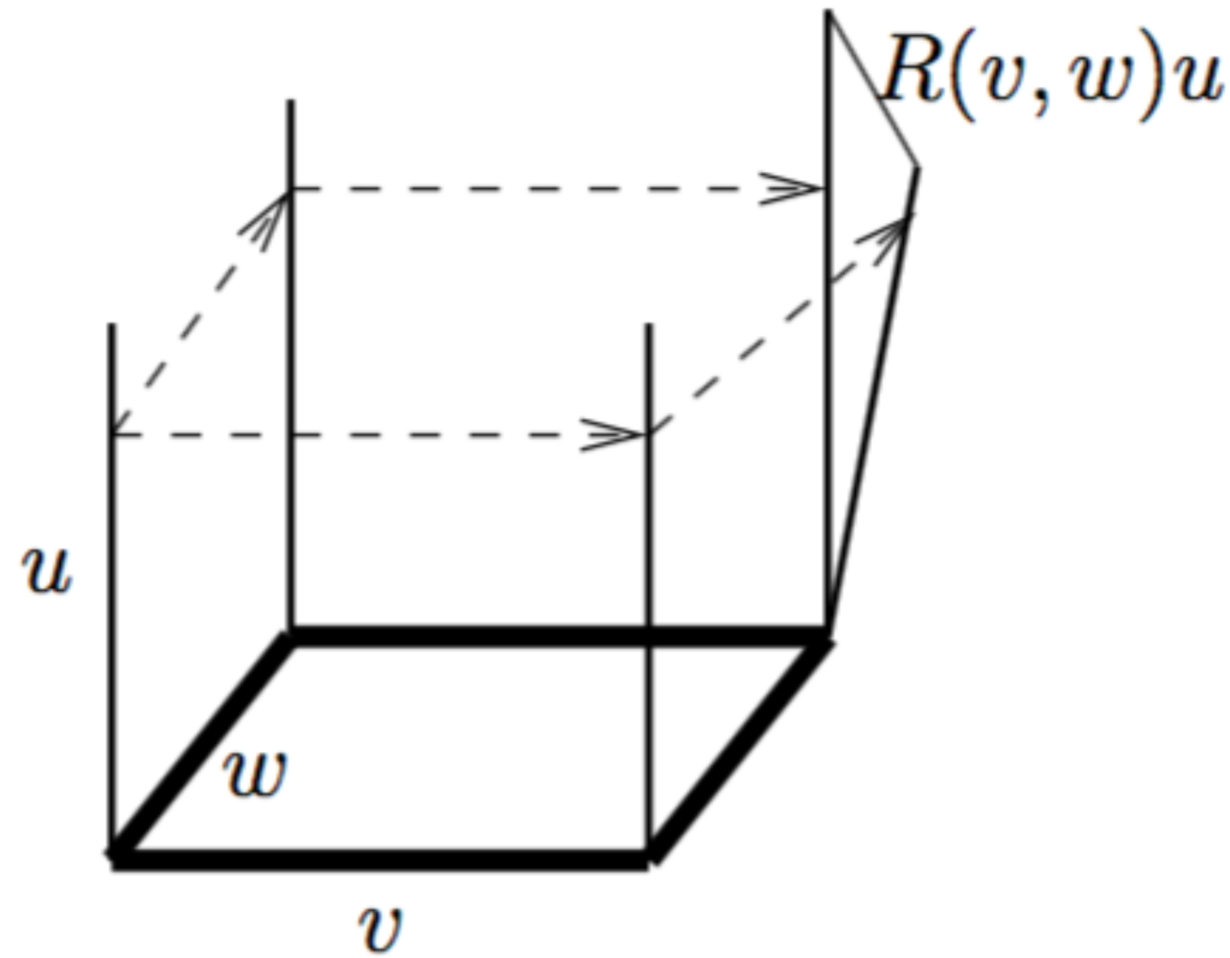
$$D_\nu \vec{W} \equiv (D_\nu W^\mu) \vec{e}_\mu$$

$$\Gamma^\rho_{\mu\nu}(x) \equiv \frac{1}{2} g^{\rho\sigma}(x) \left(\partial_\mu g_{\nu\sigma}(x) + \partial_\nu g_{\mu\sigma}(x) - \partial_\sigma g_{\mu\nu}(x) \right)$$

$$u^\alpha = dx^\alpha / dt \qquad u^\mu \nabla_\mu u^\alpha = 0$$

The Riemann tensor

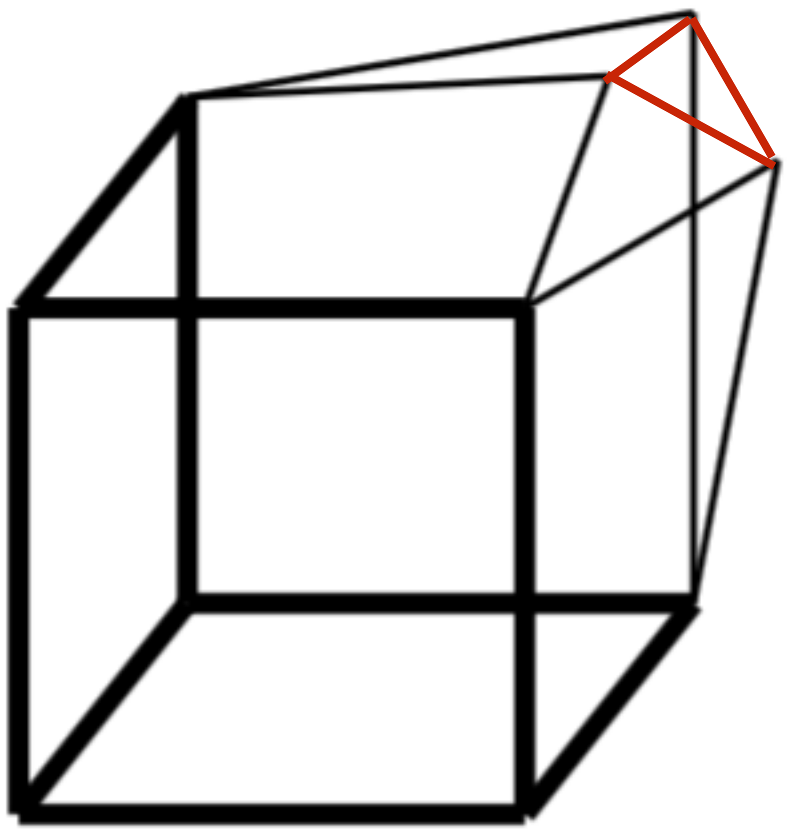




$$R_{\alpha\beta\gamma}{}^{\delta} w^{\gamma} := -(\nabla_{\alpha}\nabla_{\beta} - \nabla_{\beta}\nabla_{\alpha}) w^{\delta}$$

$$R_{\alpha\beta\gamma}{}^{\delta} = -\frac{\partial}{\partial x^{\alpha}} \Gamma^{\delta}_{\beta\gamma} + \frac{\partial}{\partial x^{\beta}} \Gamma^{\delta}_{\alpha\gamma} - \Gamma^{\delta}_{\alpha\mu}\Gamma^{\mu}_{\beta\gamma} + \Gamma^{\delta}_{\beta\mu}\Gamma^{\mu}_{\alpha\gamma}$$

The first Bianchi identity and other Riemann symmetries

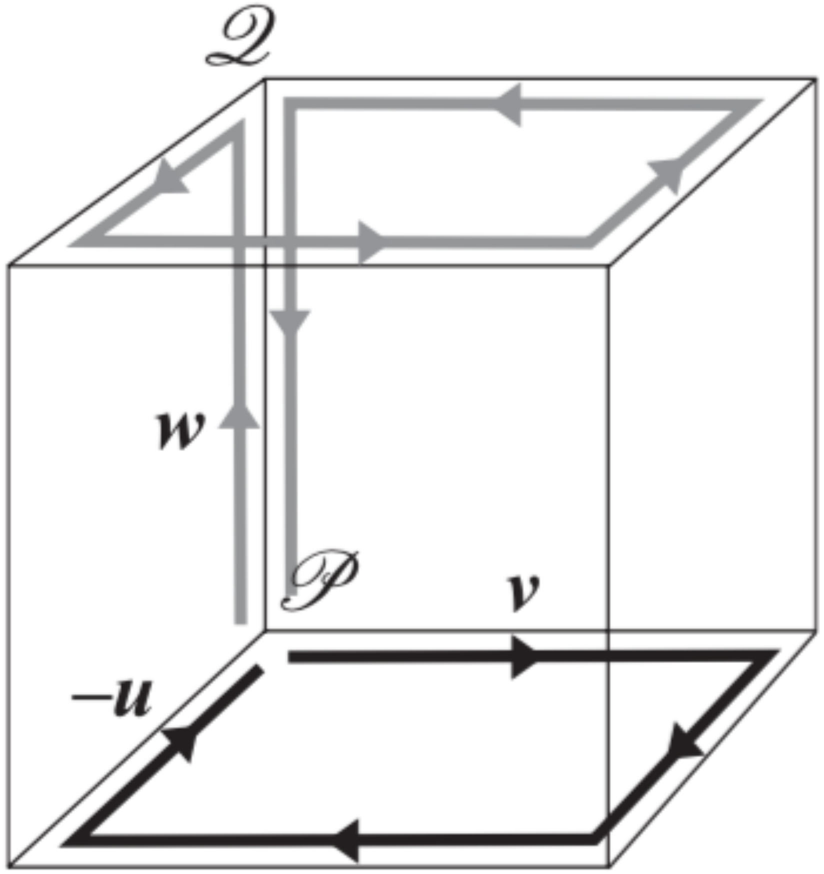


$$R_{\alpha\beta\gamma}{}^{\delta} + R_{\beta\gamma\alpha}{}^{\delta} + R_{\gamma\alpha\beta}{}^{\delta} = 0$$

$$R_{\alpha\beta\gamma\delta} = -R_{\beta\alpha\gamma\delta} \qquad R_{\alpha\beta\gamma\delta} = -R_{\alpha\beta\delta\gamma}$$

$$R_{\alpha\beta\gamma\delta} = R_{\gamma\delta\alpha\beta}$$

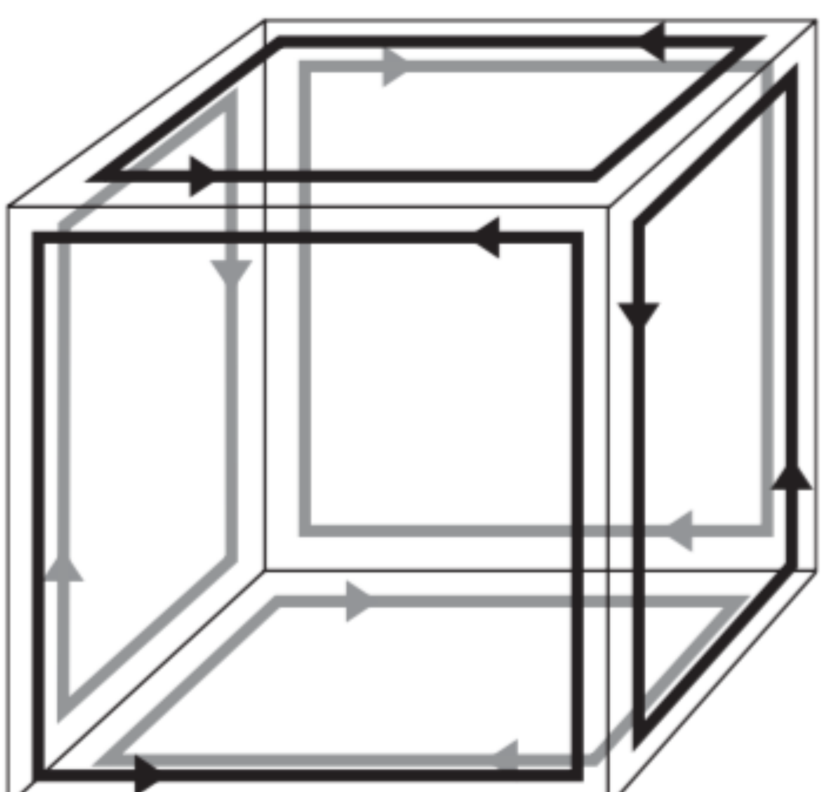
The second Bianchi identity



$$(\delta a^\alpha)_{\text{top}} = \epsilon^2 R_{\mu\nu\rho}{}^{\nu}(\mathcal{Q}) u^\mu v^\nu a^\rho$$

$$(\delta a^\alpha)_{\text{bot}} = -\epsilon^2 R_{\mu\nu\rho}{}^{\lambda}(\mathcal{P}) u^\mu v^\nu a^\rho$$

$$\begin{aligned}(\delta a^\alpha)_{\text{top+bot}} &= \epsilon^2 \left[R_{\mu\nu\rho}{}^{\alpha}(\mathcal{Q}) - R_{\mu\nu\rho}{}^{\alpha}(\mathcal{P}) \right] u^\mu v^\nu a^\rho \\ &= \epsilon^3 \left(w^\sigma \nabla_\sigma R_{\mu\nu\rho}{}^{\alpha} \right) u^\mu v^\nu a^\rho \,,\end{aligned}$$



$$\begin{aligned} 0 &= (\delta a^\alpha)_{\text{all faces}} \\ &= \epsilon^3 \boxed{\left(\nabla_\sigma R_{\mu\nu\rho}{}^{\alpha} + \nabla_\mu R_{\nu\sigma\rho}{}^{\alpha} + \nabla_\nu R_{\sigma\mu\rho}{}^{\alpha} \right)} u^\mu v^\nu w^\sigma a^\rho \end{aligned}$$

Ricci, Ricci, and Einstein

$$R_{\alpha\beta} := R_{\alpha\mu\beta}{}^{\mu} \qquad R := g^{\mu\nu} R_{\mu\nu}$$

$$G_{\alpha\beta} := R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$$

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Metric perturbation, linearized theory

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

infinitesimal coordinate transformation

$$\mathbf{x} \rightarrow \mathbf{x}' = \mathbf{x} + \boldsymbol{\xi} \qquad g'_{\alpha\beta} = g_{\alpha\beta} - \frac{\partial \xi_\beta}{\partial x^\alpha} - \frac{\partial \xi_\alpha}{\partial x^\beta} + O(h^2)$$

$$\Gamma^\gamma_{\alpha\beta} = \frac{1}{2}\eta^{\gamma\delta}\left(\frac{\partial h_{\beta\delta}}{\partial x^\alpha} + \frac{\partial h_{\alpha\delta}}{\partial x^\beta} - \frac{\partial h_{\alpha\beta}}{\partial x^\delta}\right) + O(h^2)$$

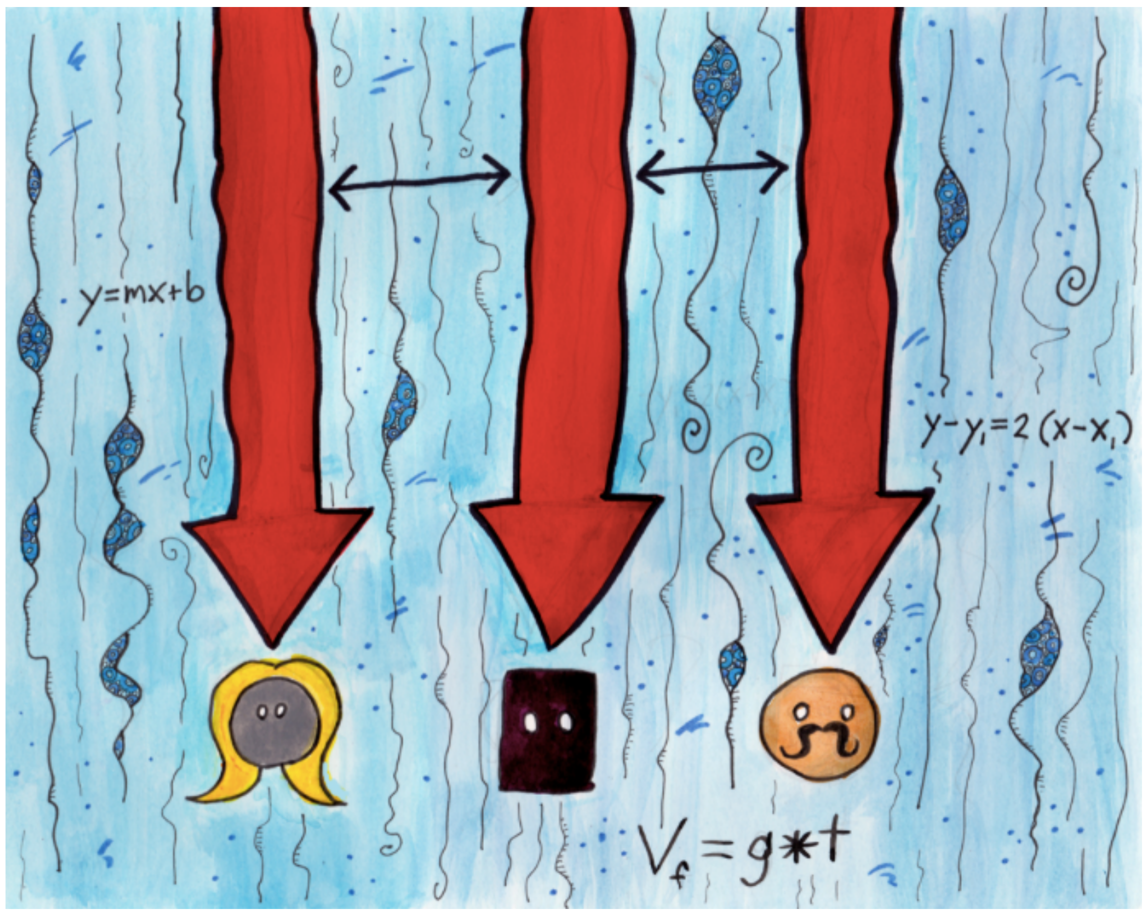
$$R_{\alpha\beta\gamma\delta} = \frac{1}{2}\left(-\frac{\partial^2 h_{\beta\delta}}{\partial x^\alpha \partial x^\gamma} + \frac{\partial^2 h_{\beta\gamma}}{\partial x^\alpha \partial x^\delta} + \frac{\partial^2 h_{\alpha\delta}}{\partial x^\beta \partial x^\gamma} - \frac{\partial^2 h_{\alpha\gamma}}{\partial x^\beta \partial x^\delta}\right) + O(h^2)$$

invariant!

Riemann propagates as a wave

$$\Box R_{\alpha\beta\gamma\delta} = 0$$

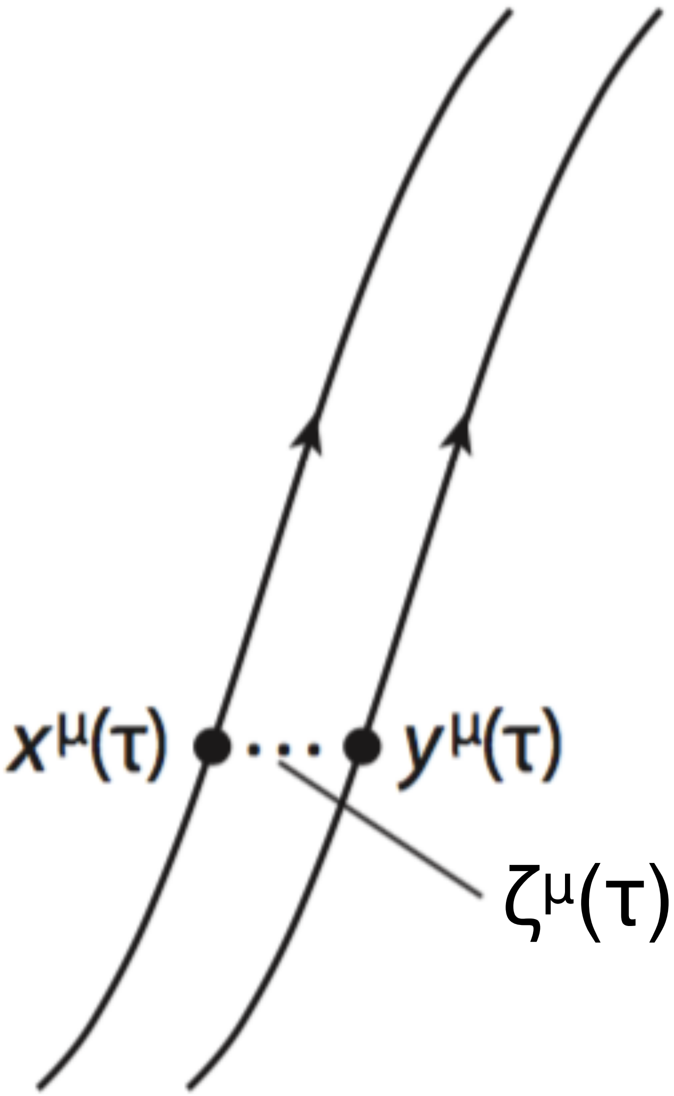
Geodesic deviation



You simply fell, indefinitely, for an indefinite length of time. I went down into the void, to the most absolute bottom conceivable, and once there I saw that the extreme limit must have been much, much farther below, very remote, and I went on falling, to reach it. Since there were no reference points, I had no idea whether my fall was fast or slow. Now that I think about it, there weren't even any proofs that I was really falling: perhaps I had always remained immobile in the same place, or I was moving in an upward direction; since there was no above or below these were only nominal questions and so I might just as well go on thinking I was falling, as I was naturally led to think. Assuming then that one was falling, everyone fell with the same speed and rate of acceleration; in fact we were always more or less on the same level: I, Ursula H'x, Lieutenant Fenimore.

Italo Calvino, "Cosmicomics" (1965)

Geodesic deviation



Newtonian

$$\boldsymbol{a} = \frac{d^2\boldsymbol{x}}{dt^2} = -\boldsymbol{\nabla}\Phi(\boldsymbol{x})$$

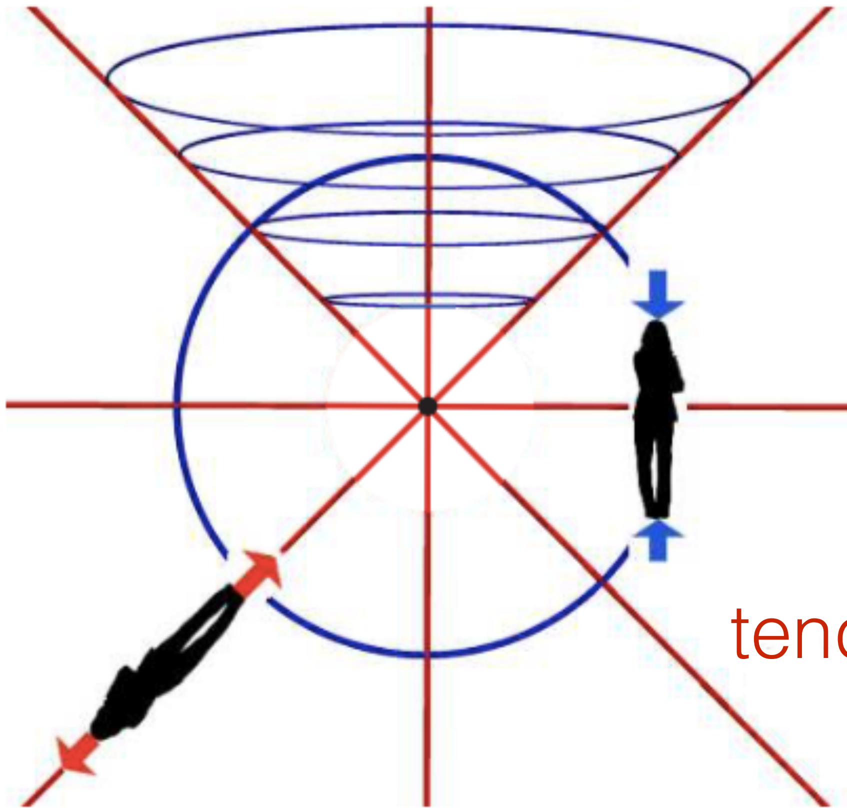
$$\Delta a_i = -\zeta^j \underbrace{\frac{\partial^2 \Phi}{\partial x^i \partial x^j}}_{\mathcal{E}_{ij}} = -\mathcal{E}_{ij}\zeta^j$$

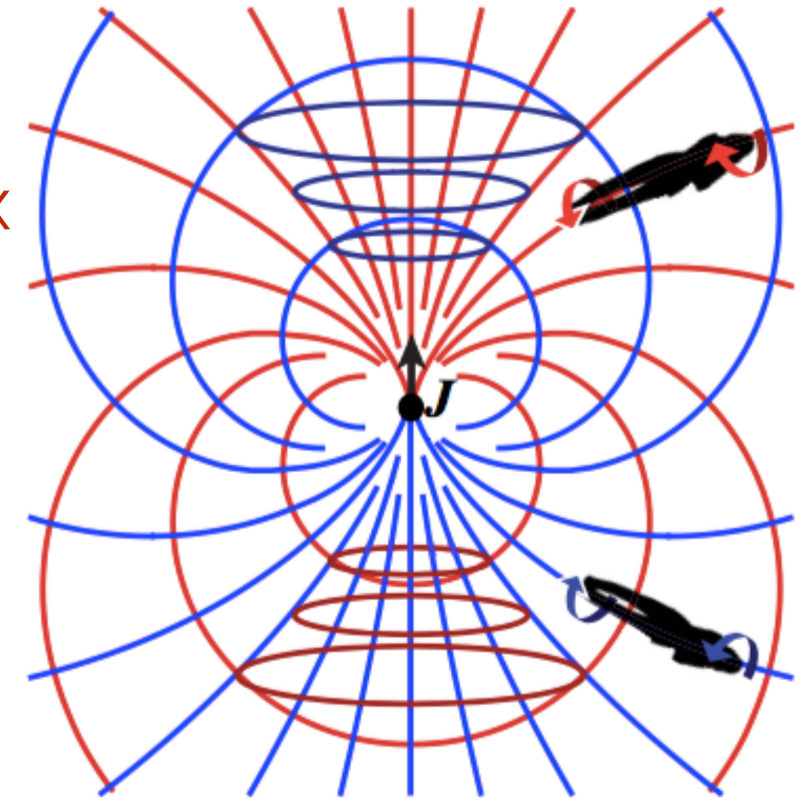
$$a^\alpha = -R_{\mu\rho\nu}{}^{\alpha} u^\mu \zeta^\rho u^\nu$$

$$\mathcal{E}_{ij} = R_{0i0j}$$

for particle at rest
in proper reference frame

Gravito-electromagnetism, tendexes, vortexes





$$\Delta a_i = -\mathcal{E}_{ij}\xi^j$$

$$\Delta\Omega_i = \mathcal{B}_{ij}\xi^j$$

$$\frac{1}{2}\epsilon_{ipq}R^{pq}{}_{j0}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{E}} = 0\,, \quad \boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{B}} = 0\,, \quad \frac{\partial\boldsymbol{\mathcal{E}}}{\partial t} - (\boldsymbol{\nabla}\times\boldsymbol{\mathcal{B}})^{\rm S} = 0\,, \quad \frac{\partial\boldsymbol{\mathcal{B}}}{\partial t} + (\boldsymbol{\nabla}\times\boldsymbol{\mathcal{E}})^{\rm S} = 0$$

Effect on particles (any metric theory)

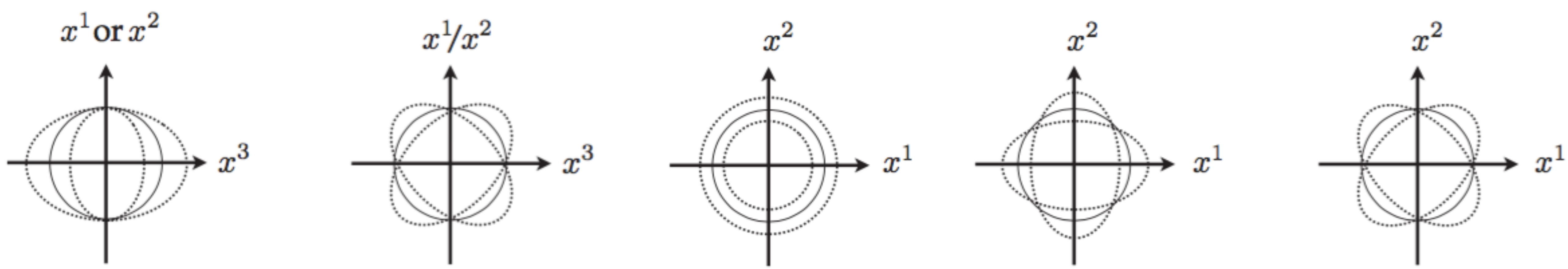
$$\frac{\partial^2 h^{\text{gw}}_{jk}}{\partial t^2} = -2R_{0j0k}$$

$$\delta x^j = \frac{1}{2} h^{\text{gw}}_{jk} x^k$$

local inertial frame

$$R_{0j0k}(t - z)$$

plane wave along *z*



longitudinal (spin 0) $\mathcal{E}_{33}$ (a)

longitudinal-transverse (spin 1) $\mathcal{E}_{31}/\mathcal{E}_{32}$ (b)

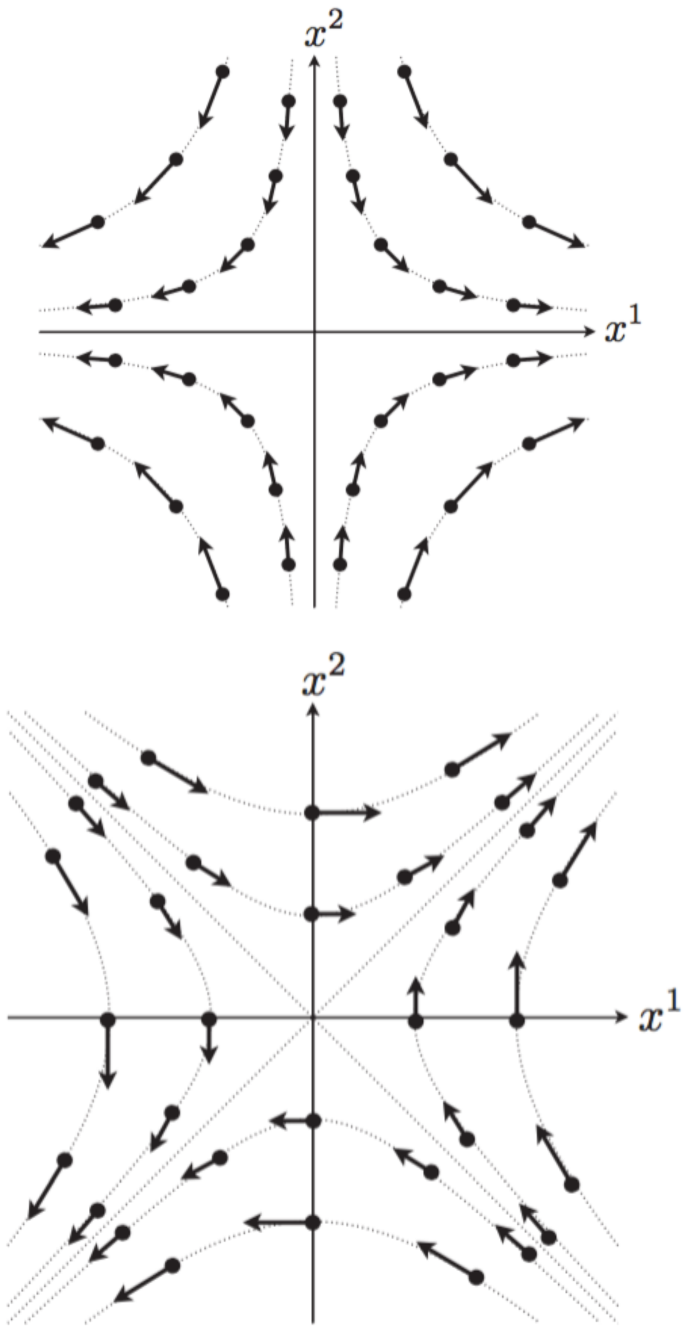
transverse (spin 0) $\mathcal{E}_{11} = \mathcal{E}_{22}$ (c)

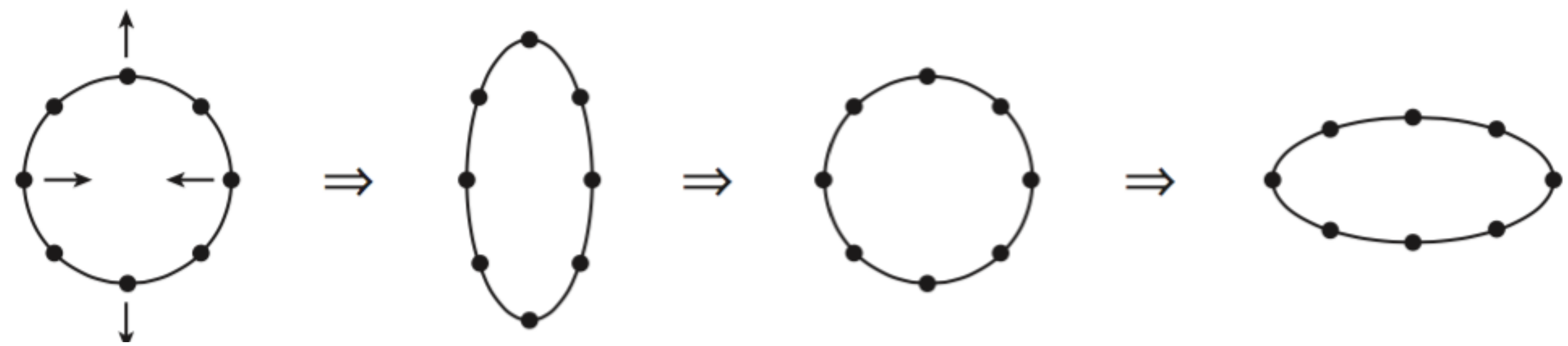
transverse (spin 2) "plus" $\mathcal{E}_{11} = -\mathcal{E}_{22}$ (d)

transverse (spin 2) "cross" $\mathcal{E}_{12}$ (e)

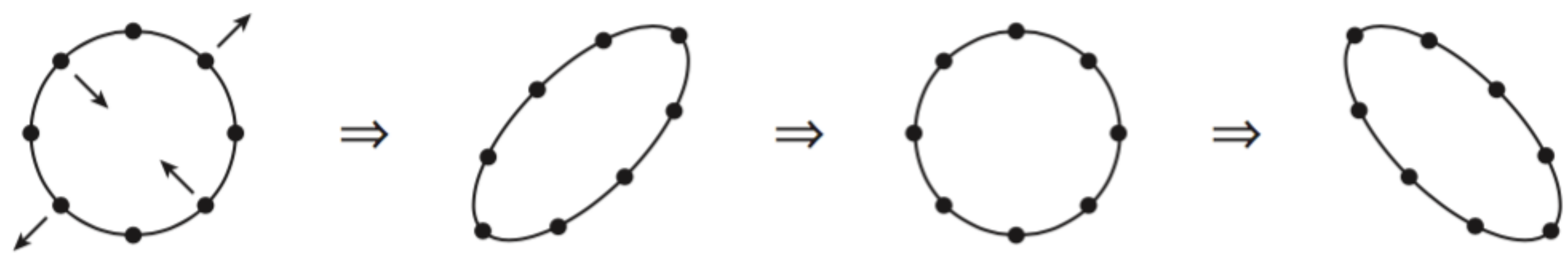
six independent degrees of freedom

Effect on particles (general relativity)





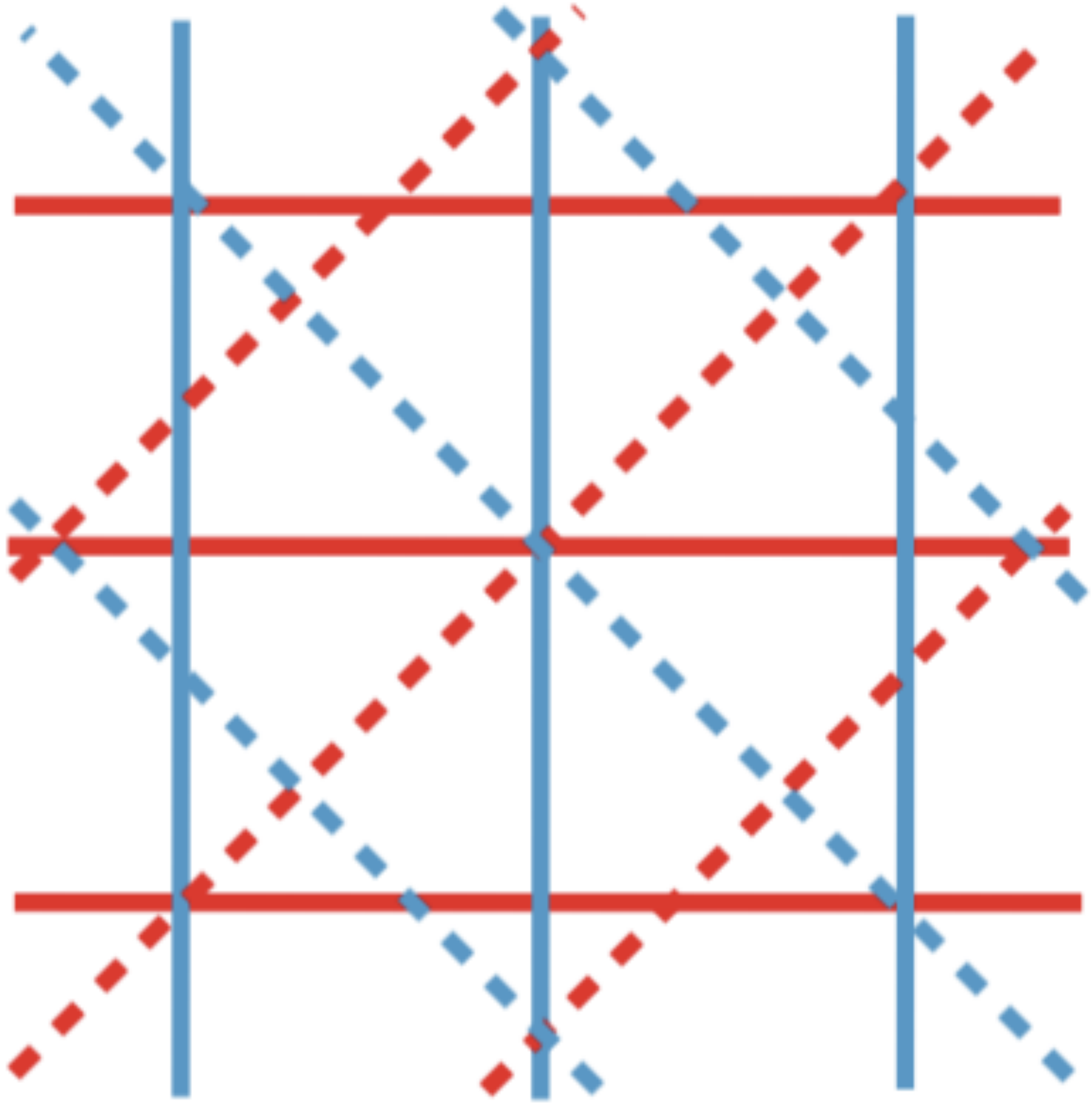
$$\mathcal{E}_{ij} = -\frac{1}{2}\ddot{h}_+ e^+_{ij} - \frac{1}{2}\ddot{h}_\times e^\times_{ij}$$



$$\mathbf{e}_+ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{e}_\times = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

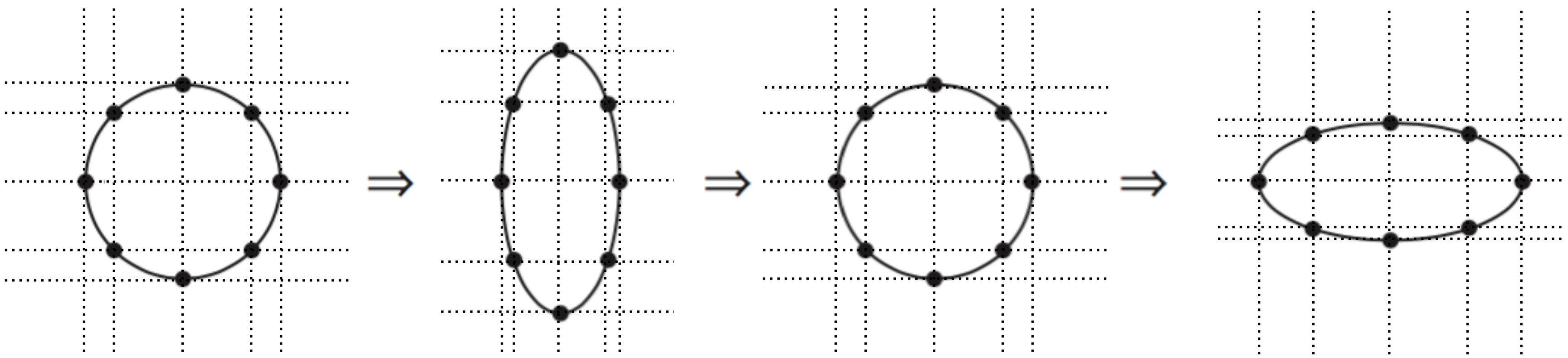
$$\mathbf{e}_+ \to \mathbf{e}'_+ = \mathbf{e}_+ \cos 2\phi - \mathbf{e}_\times \sin 2\phi$$
$$\mathbf{e}_\times \to \mathbf{e}'_\times = \mathbf{e}_\times \cos 2\phi + \mathbf{e}_+ \sin 2\phi$$

Effect on particles (general relativity)



Quasi-Lorentz TT frame (global!)

$$ds^2 = -dt^2 + (1+h_+)dx^2 + (1-h_+)dy^2 + 2h_\times dxdy + dz^2$$

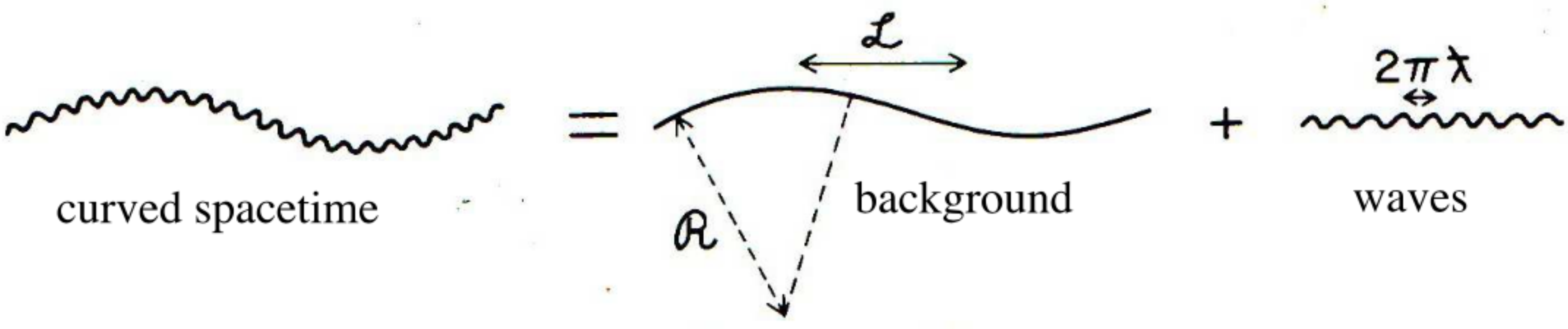


particles at rest have constant TT coordinates,
but proper distances change
(coordinates are stretched)

(compare with proper reference frame

$$ds^2 = -(1+R_{j0k0}x^j x^k)dt^2 - \frac{4}{3}R_{jkl0}x^k x^l dt dx^j + (\delta_{ij} - \frac{1}{3}R_{ijlm}x^l x^m)dx^i dx^j$$
)

GW energy-momentum can be defined in a two-lengthscale expansion



$$g_{\alpha\beta} = g^{\rm B}_{\alpha\beta} + h_{\alpha\beta}\ , \qquad R_{\alpha\beta\gamma\delta} = R^{\rm B}_{\alpha\beta\gamma\delta} + R^{\rm GW}_{\alpha\beta\gamma\delta}$$

$$g^{\rm B}_{\alpha\beta} \equiv \langle g_{\alpha\beta}\rangle \qquad R^{\rm B}_{\alpha\beta\gamma\delta} \equiv \langle R_{\alpha\beta\gamma\delta}\rangle$$

$$G_{\alpha\beta} = G^{\rm B}_{\alpha\beta} + G^{(1)}_{\alpha\beta} + G^{(2)}_{\alpha\beta} + \ldots.$$

$$G^{(1)}_{\mu\nu} = 0 \qquad G^{\rm B}_{\alpha\beta} + \langle G^{(2)}_{\alpha\beta}\rangle = 0$$

$$G^{\rm B}_{\alpha\beta} = 8\pi T^{\rm GW}_{\alpha\beta} \qquad T^{\rm GW}_{\alpha\beta} = -\frac{1}{8\pi}\langle G^{(2)}_{\alpha\beta}\rangle$$

GW energy-momentum can be defined in a two-lengthscale expansion

$$T_{\alpha\beta}^{\mathrm{GW}} = -\frac{1}{8\pi}\langle G_{\alpha\beta}^{(2)}\rangle \qquad R_{\alpha\beta} = \frac{\partial \Gamma^{\mu}_{\alpha\beta}}{\partial x^{\mu}} - \frac{\partial \Gamma^{\mu}_{\mu\beta}}{\partial x^{\alpha}} + \Gamma^{\mu}_{\alpha\beta}\Gamma^{\nu}_{\mu\nu} - \Gamma^{\mu}_{\nu\beta}\Gamma^{\nu}_{\alpha\mu}$$

$$T_{\alpha\beta}^{\mathrm{GW}} = \frac{1}{16\pi}\langle h_{+,\alpha}h_{+,\beta} + h_{\times,\alpha}h_{\times,\beta}\rangle$$

$$T^{\mathrm{GW}\;00} = T^{\mathrm{GW}\;0z} = T^{\mathrm{GW}\;z0} = T^{\mathrm{GW}\;zz} = \frac{1}{16\pi}\langle \dot{h}_+^2 + \dot{h}_\times^2\rangle$$

$$T^{\mathrm{GW}\,0z} \simeq \frac{\pi}{4}\frac{c^3}{G} f^2 h_{\mathrm{amp}}^2 \simeq 300\,\frac{\mathrm{ergs}}{\mathrm{cm}^2\ \mathrm{sec}}\left(\frac{f}{1\ \mathrm{kHz}}\right)^2\left(\frac{h_{\mathrm{amp}}}{10^{-21}}\right)^2$$

$= 0.3\ \mathrm{W/m^2}$

Quadrupole formula for slow-moving, non-selfgravitating system

$$\bar{h}_{\alpha\beta} := h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} h \qquad \partial \bar{h}^{\mu\alpha}/\partial x^{\mu} = 0$$

Lorenz gauge

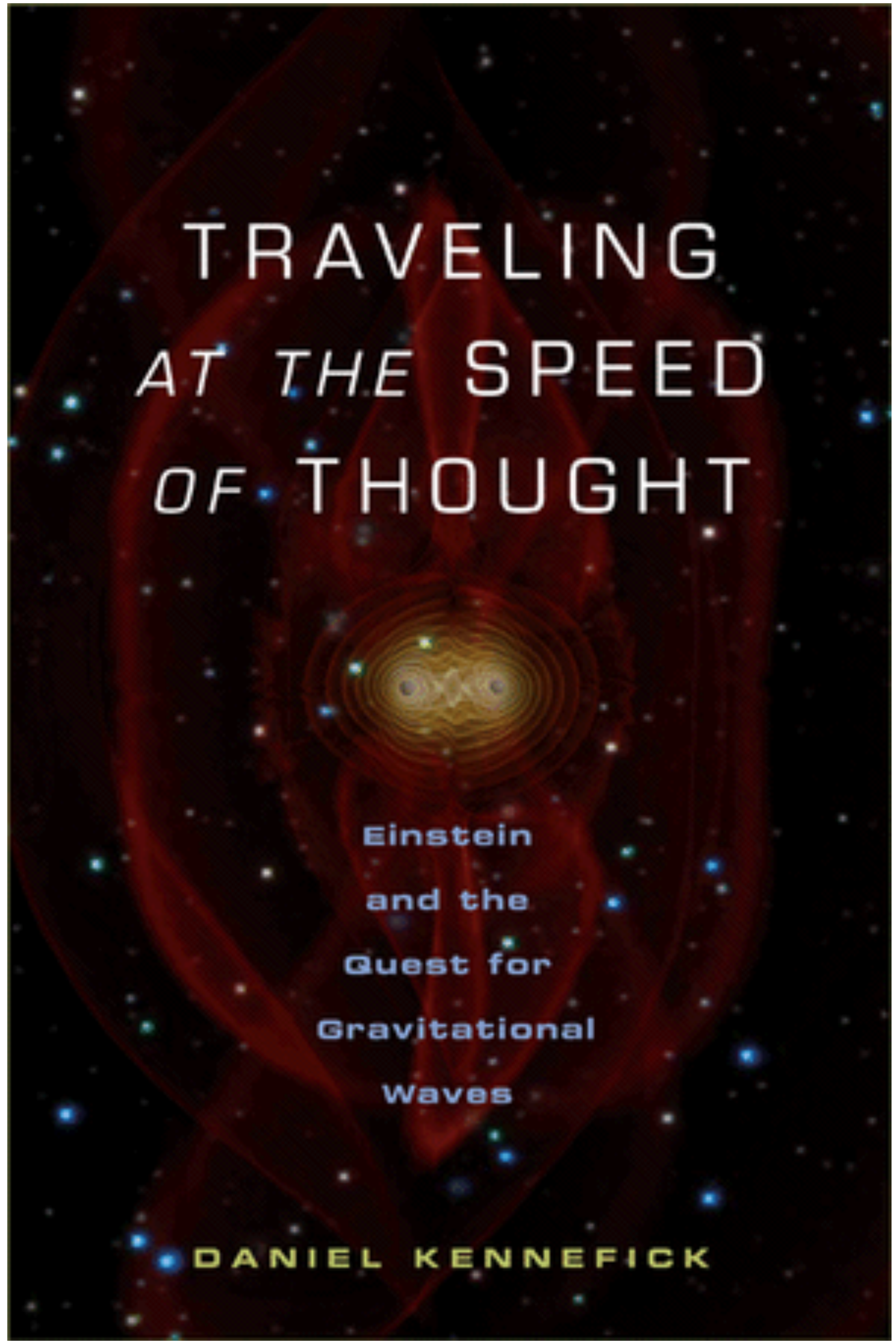
$$\Box \bar{h}^{\alpha\beta} = -\frac{16\pi G}{c^4}\tau^{\alpha\beta}$$

$$\bar{h}^{\alpha\beta}(t,\boldsymbol{x}) = \frac{4G}{c^4}\int \frac{\tau^{\alpha\beta}(t - \|\boldsymbol{x}-\boldsymbol{x}'\|/c, \boldsymbol{x}')}{\|\boldsymbol{x}-\boldsymbol{x}'\|} d^3x' \qquad \frac{\partial \tau^{\mu\alpha}}{\partial x^{\mu}} = 0$$

$$\tau^{ij} = \frac{1}{2}\frac{\partial^2}{\partial t^2}\left(x^i x^j \tau^{00}\right)$$

$$\bar{h}^{ij}(t,\boldsymbol{x}) \simeq \frac{2G}{c^4 r}\frac{\partial^2}{\partial t^2}\int x'^i x'^j \tau^{00}\left(t - r/c, \boldsymbol{x}'\right) d^3x' \simeq \frac{2G}{c^4 r}\ddot{I}^{ij}(t - r/c)$$

$$h_{ij}^{\mathrm{TT}}(t) \simeq \frac{2G}{c^4 r}\ddot{I}_{ij}^{\mathrm{TT}}(t - r/c)$$

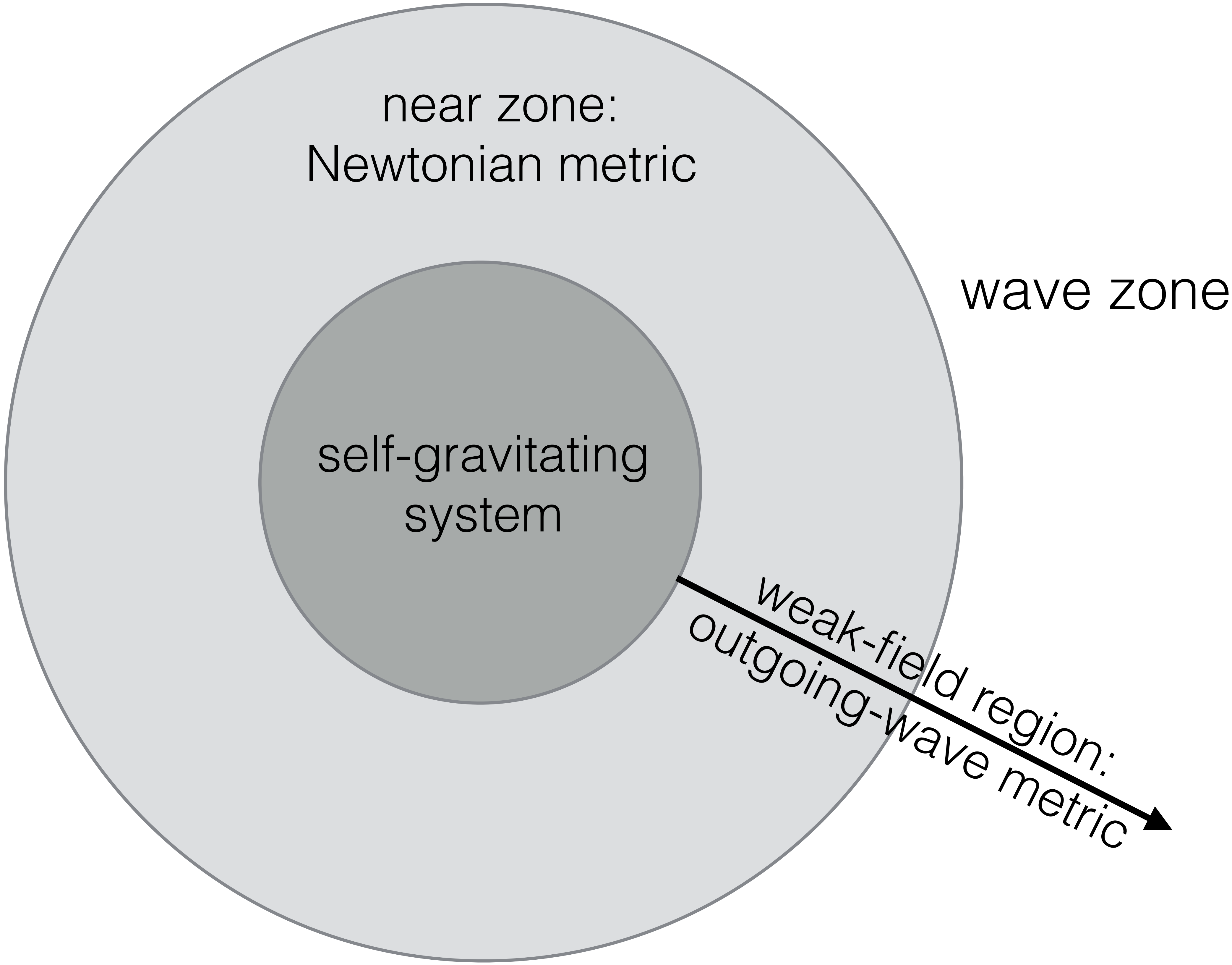

TRAVELING
AT THE SPEED
OF THOUGHT
Einstein
and the
Quest for
Gravitational
Waves
DANIEL KENNEFICK

Quadrupole formula for slow-moving, non-selfgravitating system

for a wave propagating along z, computing Riemann shows that the only tidal-field components are $h_{xx} = -h_{yy}$ and h_{xy}; then we obtain h^{TT} simply by projecting out all other elements

$$h_{ij}^{\mathrm{TT}}(t) \simeq \frac{2G}{c^4 r} \ddot{I}_{ij}^{\mathrm{TT}}(t - r/c)$$

Quadrupole formula for slow-moving, weakly/strongly gravitating system



Quadrupole formula for slow-moving, ~~weakly~~ strongly gravitating system

metric very accurately Newtonian in near zone

$$\Phi = -\frac{1}{2}c^4 h^{00} = -\frac{1}{4}c^4\left(\bar{h}^{00} + c^{-2}\delta_{ij}\bar{h}^{ij}\right)$$
$$= -G\int \frac{\tau^{00}(t, \boldsymbol{x}') + c^{-2}\delta_{ij}\tau^{ij}(t, \boldsymbol{x}')}{\|\boldsymbol{x} - \boldsymbol{x}'\|} d^3\boldsymbol{x}'$$
$$= -G\left[\frac{M}{r} + \frac{3}{2}\frac{I\!\!\!-_{ij}x^i x^j}{r^5} + \cdots\right]$$

~~$I\!\!\!-^{ij} := \int\left(x^i x^j - \frac{1}{3}r^2\delta^{ij}\right)\tau^{00}(\boldsymbol{x})d^3\boldsymbol{x}$~~

not valid for strongly gravitating systems; extract Newtonian potential in near zone

postulating an outgoing-wave solution in the weak-field region given by

$$\bar{h}_{00} = 2\left[\frac{\mathcal{I}_{jk}(t-r)}{r}\right]_{,jk}, \quad \bar{h}_{0j} = 2\left[\frac{\dot{\mathcal{I}}_{jk}(t-r)}{r}\right]_{,k}, \quad \bar{h}_{jk} = 2\frac{\ddot{\mathcal{I}}_{jk}(t-r)}{r}$$

matches the near-zone metric if quadrupoles match, hence

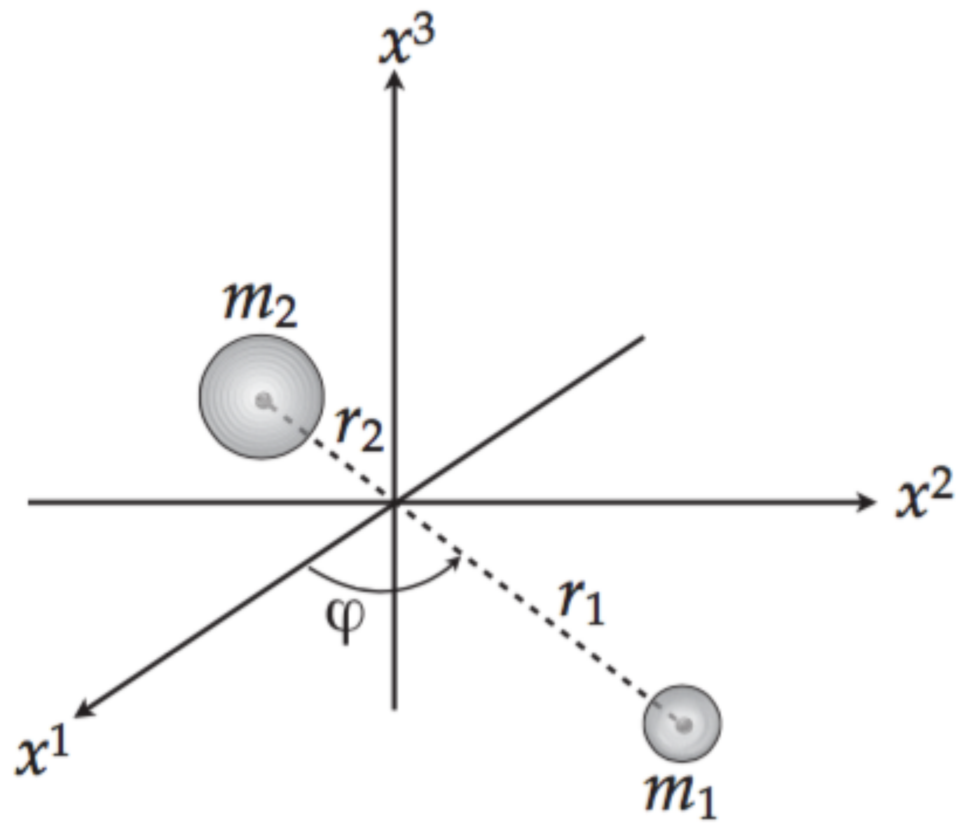
$$\boxed{h_{ij}^{\mathrm{TT}}(t) \simeq \frac{2G}{c^4 r}\ddot{I}_{ij}^{\mathrm{TT}}(t - r/c)}$$

GW emission from binary

$$I_{11} = m_1(r_1\cos\varphi)^2 + m_2\left[r_2\cos(\varphi+\pi)\right]^2$$
$$= \mu a^2\cos^2\varphi = \frac{1}{2}\mu a^2(1+\cos 2\varphi)$$

$$I_{22} = \frac{1}{2}\mu a^2(1-\cos 2\varphi)$$

$$I_{12} = \frac{1}{2}\mu a^2\sin 2\varphi$$



$$h_{ij}^{\rm TT}(t) \simeq \frac{2G}{c^4 r}\ddot{I}_{ij}^{\rm TT}(t-r/c)$$

$$\ddot{I}_{11} = -2\mu a^2\omega^2\cos 2\varphi$$

$$h_+ = -\frac{2G\mu}{c^2 r}(1+\cos^2\iota)\left(\frac{v}{c}\right)^2\cos 2\varphi$$

$$\ddot{I}_{22} = 2\mu a^2\omega^2\cos 2\varphi$$

$$h_\times = -\frac{4G\mu}{c^2 r}\cos\iota\left(\frac{v}{c}\right)^2\sin 2\varphi\ .$$

$$\ddot{I}_{12} = \ddot{I}_{21} = -2\mu a^2\omega^2\sin 2\varphi$$

$$v = a\omega \qquad GM = a^3\omega^2$$
$$GM\omega = v^3 \qquad v = (\pi G M f)^{1/3}$$

Adiabatic inspiral

2x10[53] W

$$\frac{dE}{dt\,dA} = -\frac{c^3}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle \qquad L_{GW} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\left\langle \dddot{I}_{ij}\dddot{I}^{ij}\right\rangle = \frac{32}{5}\frac{c^5}{G}\eta^2\left(\frac{v}{c}\right)^{10}$$

$$\eta = \mu/M$$

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{G m_1 m_2}{a} = -\frac{1}{2}\mu v^2$$

time to coalescence

$$\frac{d(v/c)}{dt} = \frac{32\eta}{5}\frac{c^3}{GM}\left(\frac{v}{c}\right)^9 \qquad \tau_c = \frac{5}{256\eta}\frac{GM}{c^3}\left(\frac{\pi G M f_0}{c^3}\right)^{-8/3}$$

energy balance

$$\frac{d\varphi}{dv} = \frac{d\varphi}{dt}\frac{dt}{dv} = -\omega\frac{GM}{c^3}\frac{1}{\mathcal{F}}\frac{d\mathcal{E}}{dv} = -\left(\frac{v}{c}\right)^3\frac{1}{\mathcal{F}}\frac{d\mathcal{E}}{dv}$$

$$L_{GW}(v) := \frac{c^5}{G}\mathcal{F}(v)$$

$$E(v) - Mc^2 =: Mc^2\mathcal{E}(v)$$

restricted waveform

$$h_+(t(v)) = -\frac{2G\mu}{c^2 r}(1+\cos^2\iota)\left(\frac{v}{c}\right)^2\cos 2\varphi(v)$$

$$h_\times(t(v)) = -\frac{4G\mu}{c^2 r}\cos\iota\left(\frac{v}{c}\right)^2\sin 2\varphi(v)$$

$$\varphi(v) = \varphi_c - \frac{1}{32\eta}\left(\frac{v}{c}\right)^{-5}$$

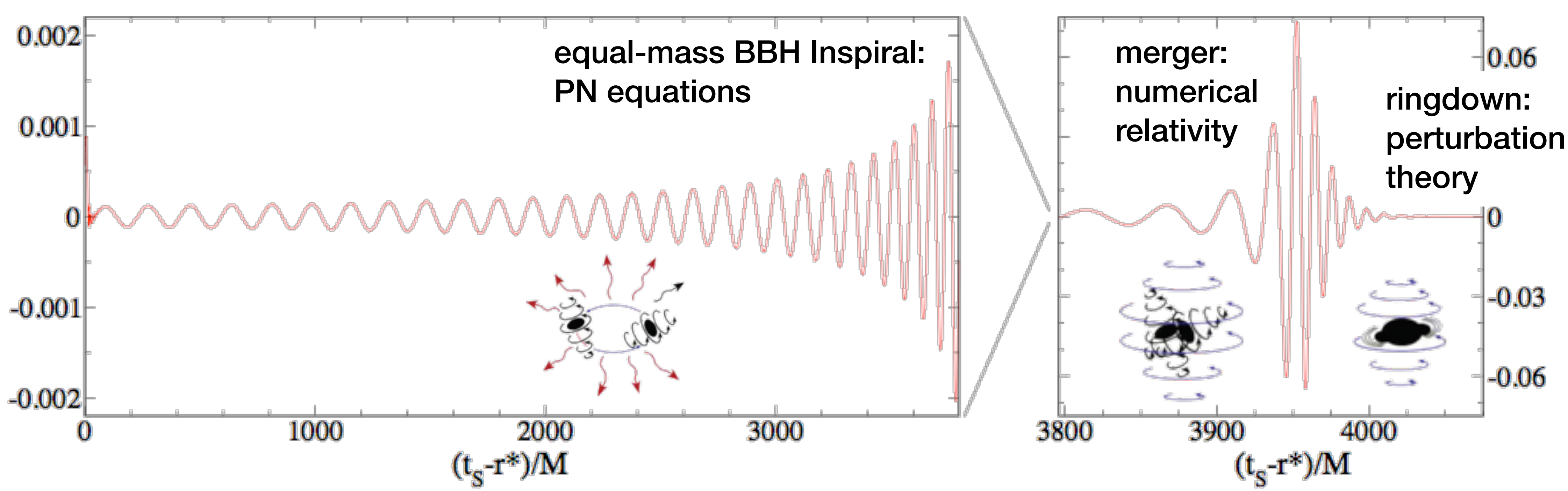
3.5PN waveform (circular, adiabatic)

$$\mathcal{E}(x) = -\frac{1}{2}\eta x \left\{1 - \left(\frac{3}{4} + \frac{1}{12}\eta\right) x - \left(\frac{27}{8} - \frac{19}{8}\eta + \frac{1}{24}\eta^2\right) x^2 - \left[\frac{675}{64} - \left(\frac{34\,445}{576} - \frac{205}{96}\pi^2\right)\eta + \frac{155}{96}\eta^2 + \frac{35}{5184}\eta^3\right] x^3\right\}$$

$$\begin{aligned}\mathcal{F}(x) = \frac{32}{5}\eta^2 x^5 \Bigg\{&1 - \left(\frac{1247}{336} + \frac{35}{12}\eta\right) x + 4\pi x^{3/2} \\ &- \left(\frac{44\,711}{9072} - \frac{9271}{504}\eta - \frac{65}{18}\eta^2\right) x^2 - \left(\frac{8191}{672} + \frac{583}{24}\eta\right)\pi x^{5/2} \\ &+ \left[\frac{6\,643\,739\,519}{69\,854\,400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E - \frac{856}{105}\ln(16x)\right. \\ &\left. + \left(\frac{41}{48}\pi^2 - \frac{134\,543}{7776}\right)\eta - \frac{94\,403}{3024}\eta^2 - \frac{775}{324}\eta^3\right] x^3 \\ &- \left(\frac{16\,285}{504} - \frac{214\,745}{1728}\eta - \frac{193\,385}{3024}\eta^2\right)\pi x^{7/2}\Bigg\},\end{aligned}$$

$$\begin{aligned}\varphi(x) = \varphi_c - \frac{1}{32\eta} x^{-5/2} \Bigg\{&1 + \left(\frac{3715}{1008} + \frac{55}{12}\eta\right) x - 10\pi x^{3/2} \\ &\left(\frac{15\,293\,365}{1\,016\,064} + \frac{27\,145}{1008}\eta + \frac{3085}{144}\eta^2\right) x^2 \\ &+ \left(\frac{38\,645}{1344} - \frac{65}{16}\eta\right)\ln\left(\frac{x}{x_0}\right)\pi x^{5/2} \\ &+ \left[\frac{12\,348\,611\,926\,451}{18\,776\,862\,720} - \frac{160}{3}\pi^2 - \frac{1712}{21}\gamma_E - \frac{856}{21}\ln(16x)\right. \\ &\left. + \left(-\frac{15\,737\,765\,635}{12\,192\,768} + \frac{2255}{48}\pi^2\right)\eta + \frac{76\,055}{6912}\eta^2 - \frac{127\,825}{5184}\eta^3\right] x^3 \\ &+ \left(\frac{77\,096\,675}{2\,032\,128} + \frac{378\,515}{12\,096}\eta - \frac{74\,045}{6048}\eta^2\right)\pi x^{7/2}\Bigg\}. \qquad \text{(B.9b)}\end{aligned}$$

$$x = v^2$$



Things we learned today

1. In general relativity, curvature propagates like a wave, at the speed of light
2. GWs act as tidal forces in a local Lorentz frame

   — OR, equivalently —

   GWs modulate the proper distance between nearby freely falling particles
3. GWs are transverse and quadrupolar
4. GWs carry energy–momentum, although it can be localized only approximately
5. GWs are emitted by time-dependent mass quadrupoles
6. GWs cause the accelerating inspiral of binary stars