GW5—Data analysis (II) and tests of GR

Michele Vallisneri

ICTP Summer School on Cosmology 2016

GW detection in practice [see PRD 93, 122003 (2016)]

condition and calibrate detector output

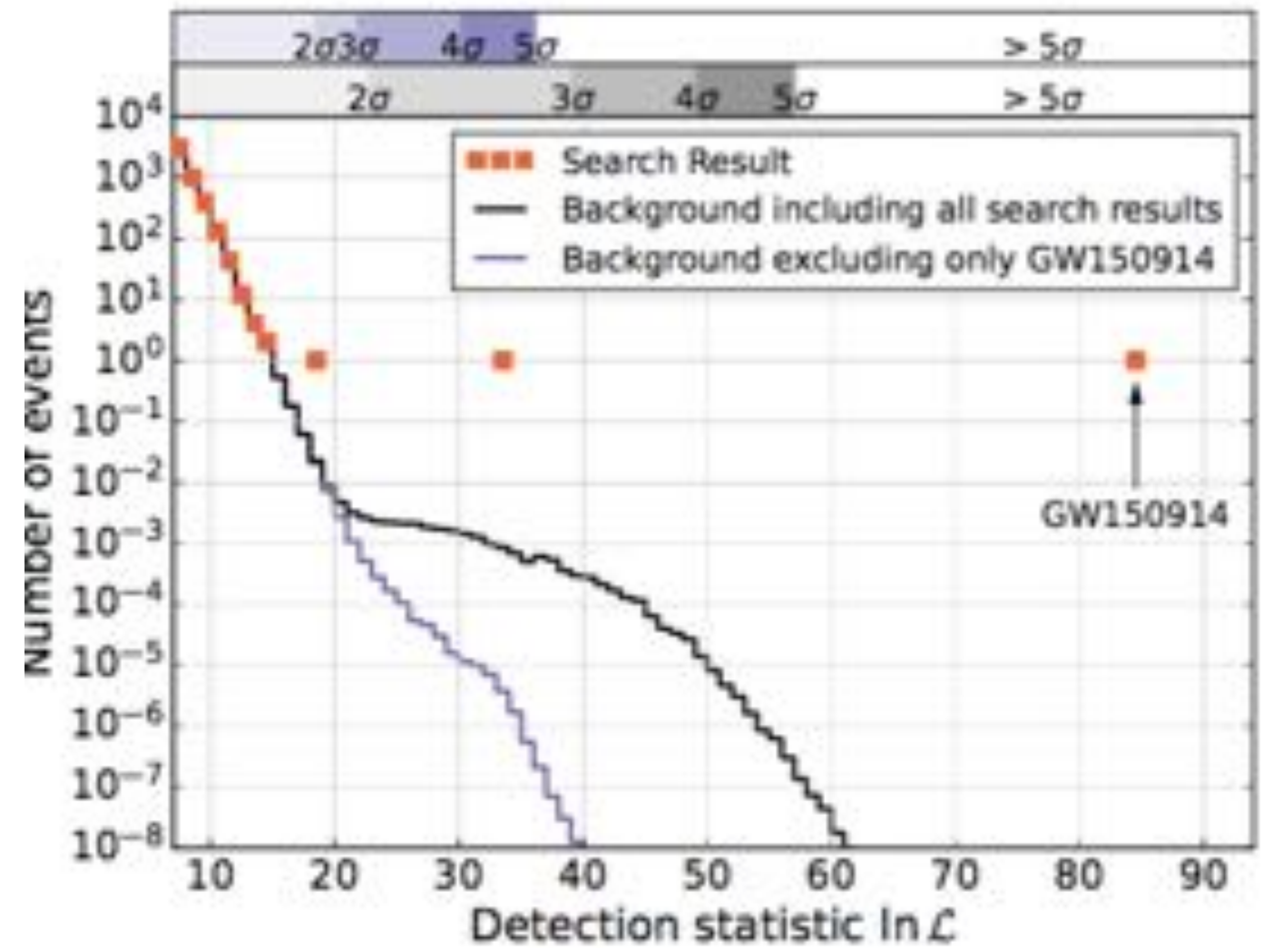
filter detector output with theoretical templates

request coincidence and consistency among detectors

apply data-quality cuts and signal vetos

estimate statistical significance

follow up candidates with detection checklist



(estimate background, using coincidence between time slides)

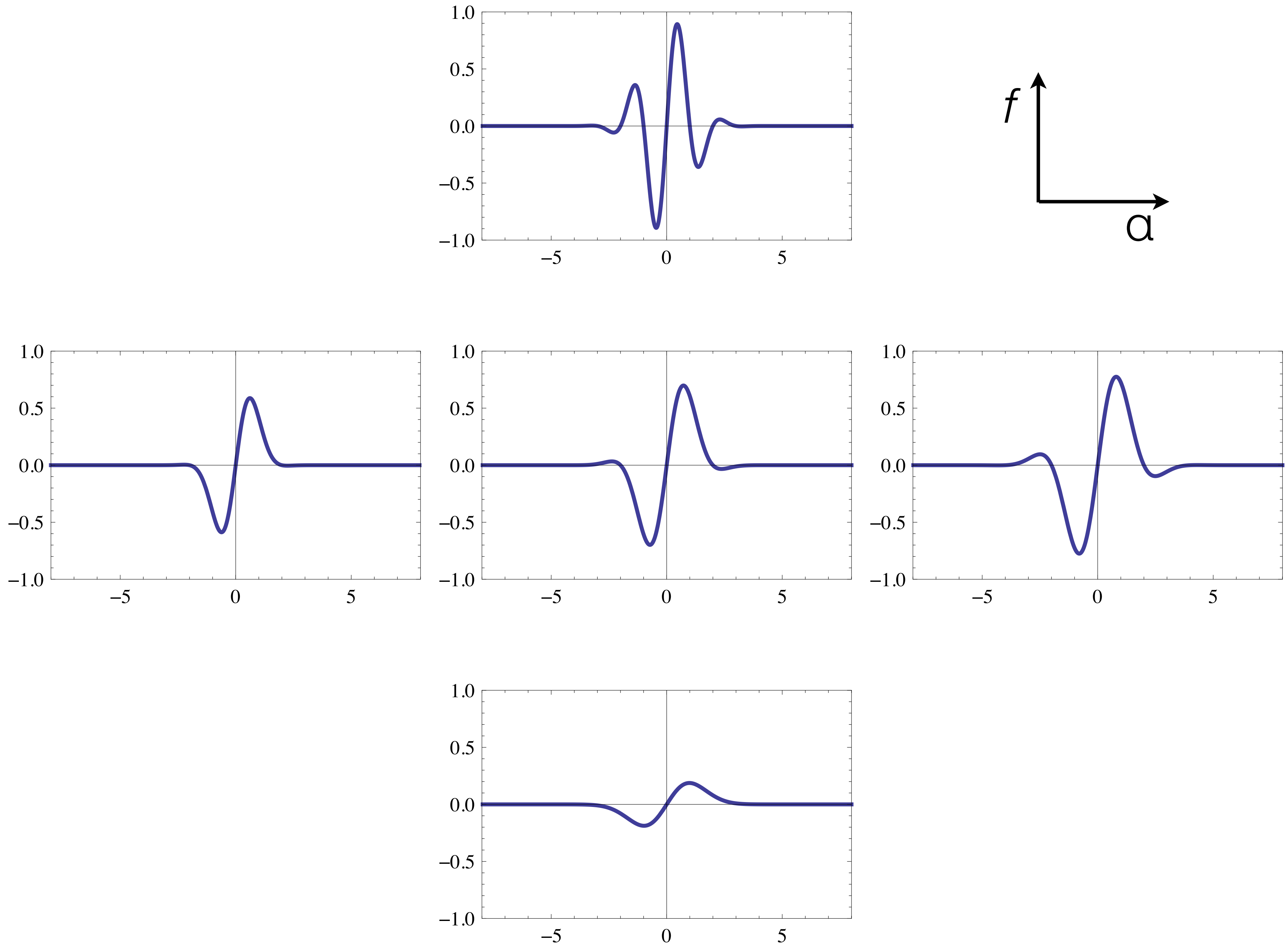
(estimate efficiency from injections, number of galaxies within horizon)

claim detection!

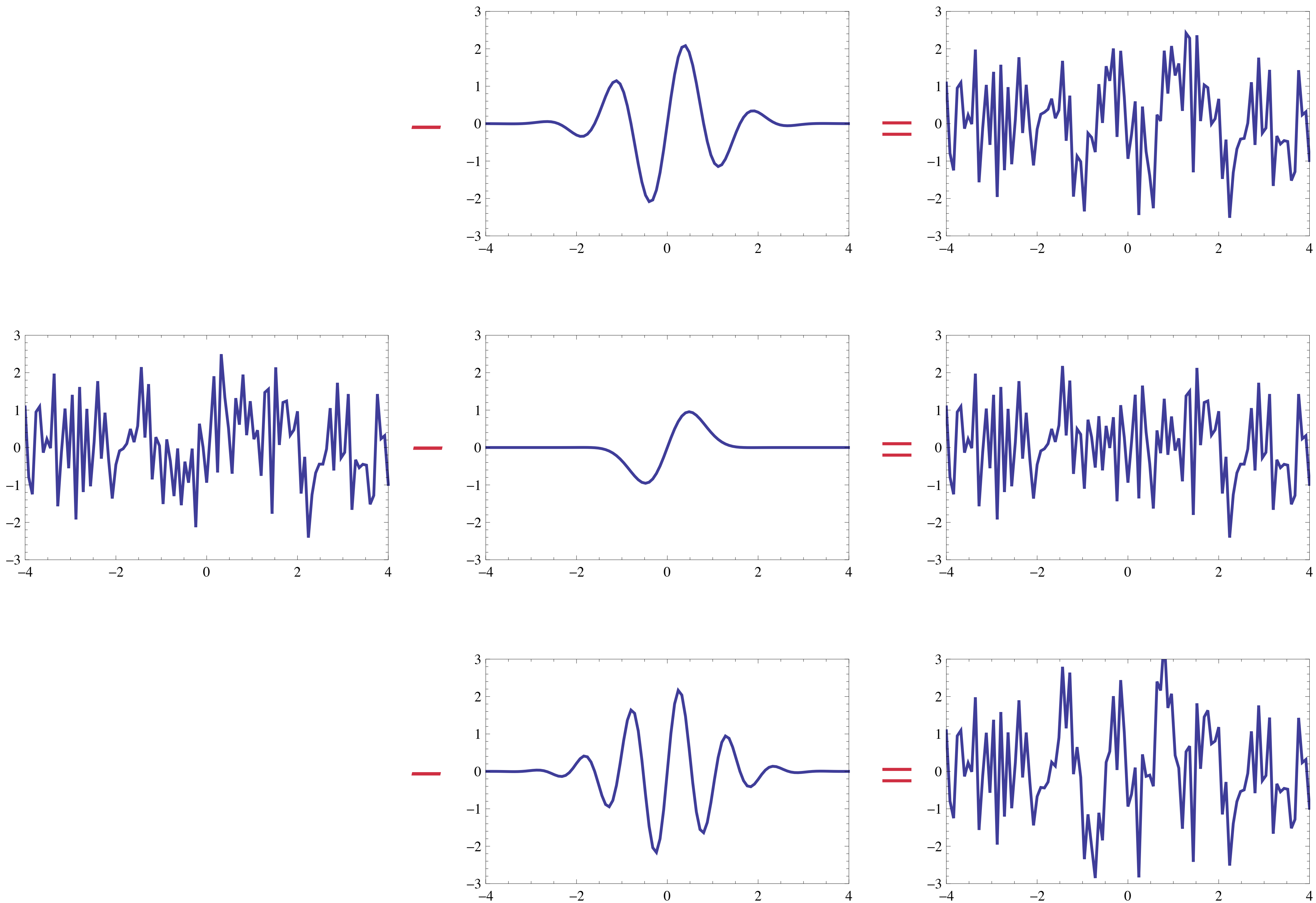
get upper limit

“science” ~ signal(parameters)

$$h(t; A, \alpha, f) = A\, e^{-\frac{t^2}{2\alpha^2}} \sin(2\pi f t)$$

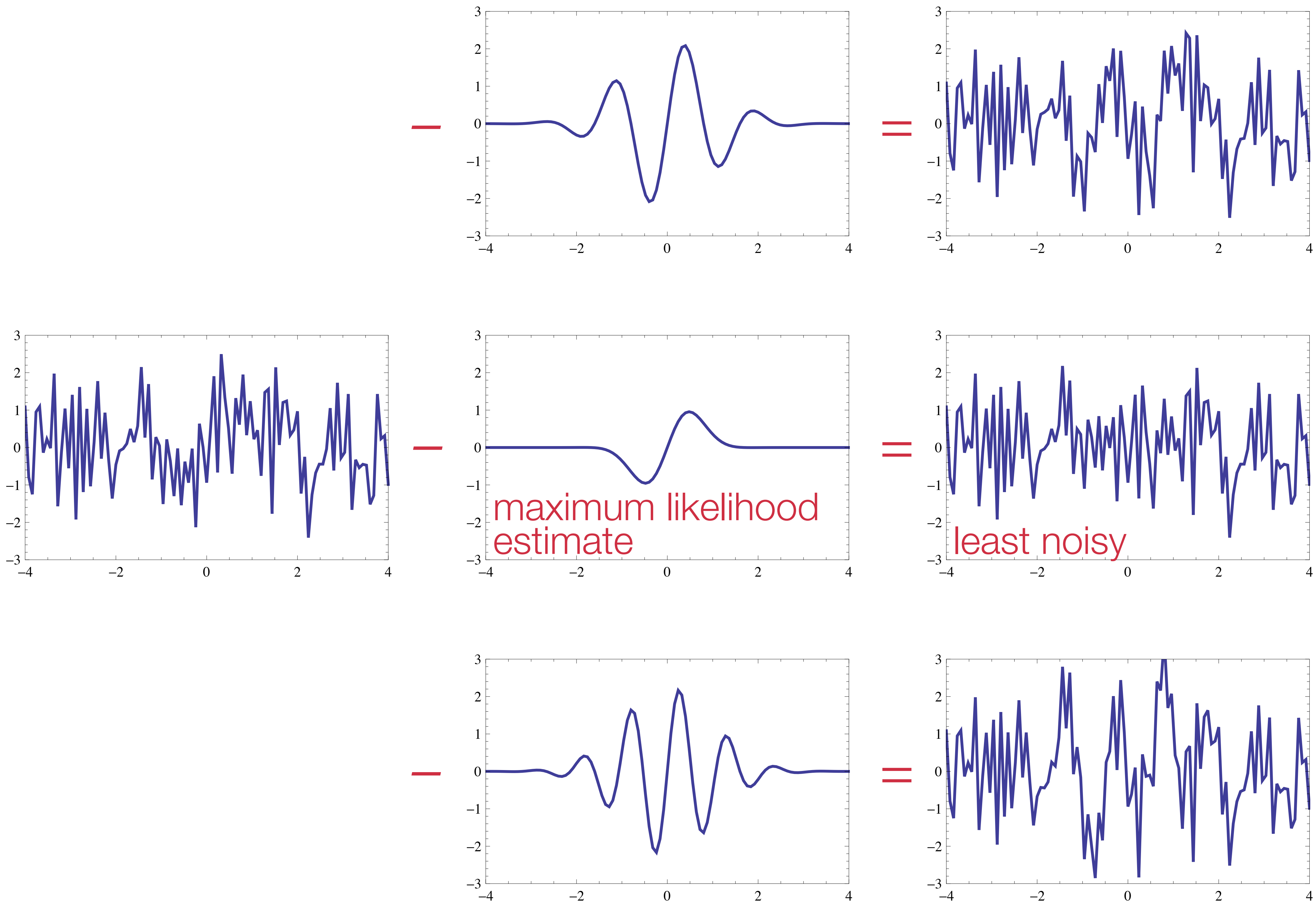


noise = data – signal



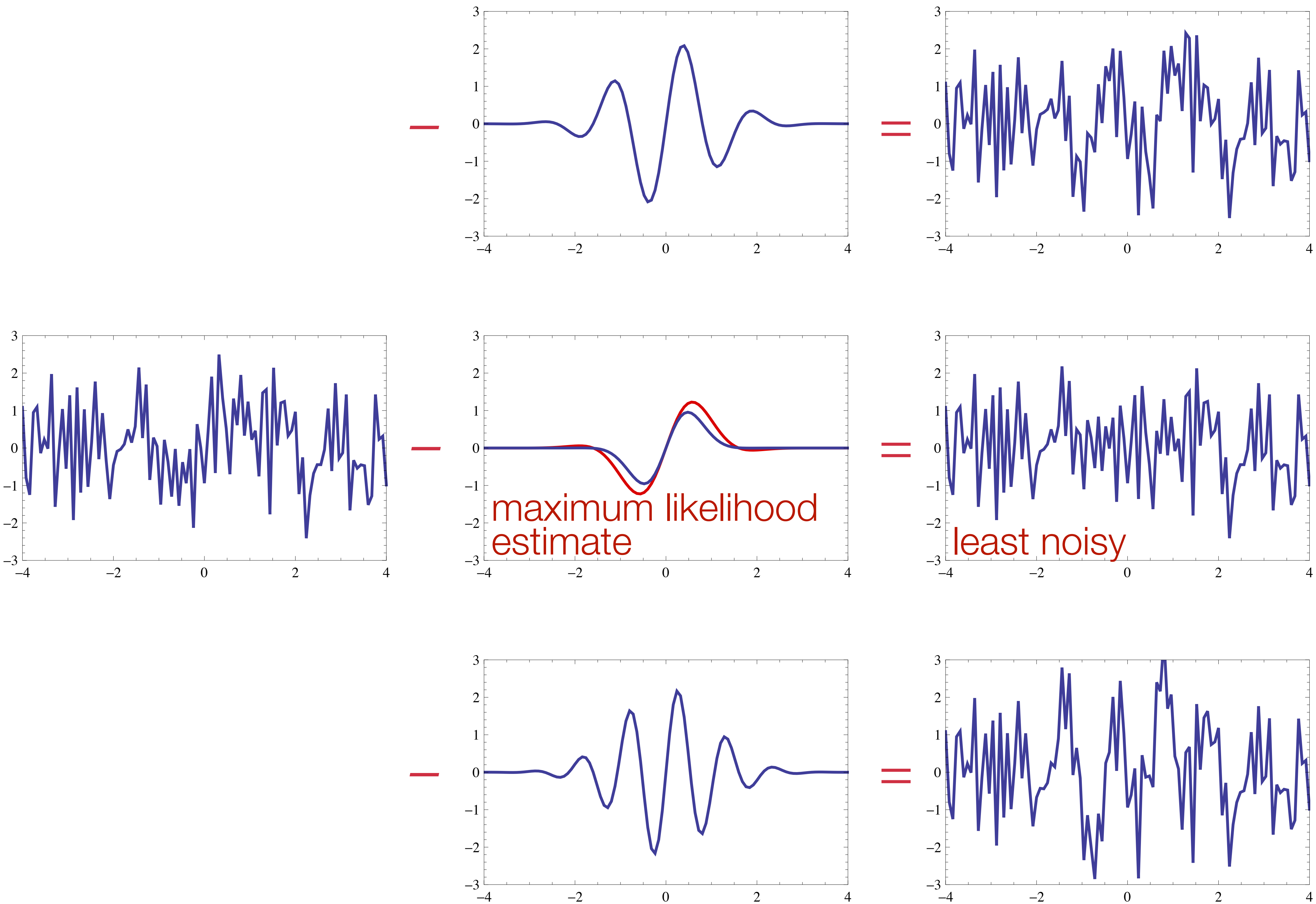
noise = data – signal

hence p(signal parameters) = p(noise residual)

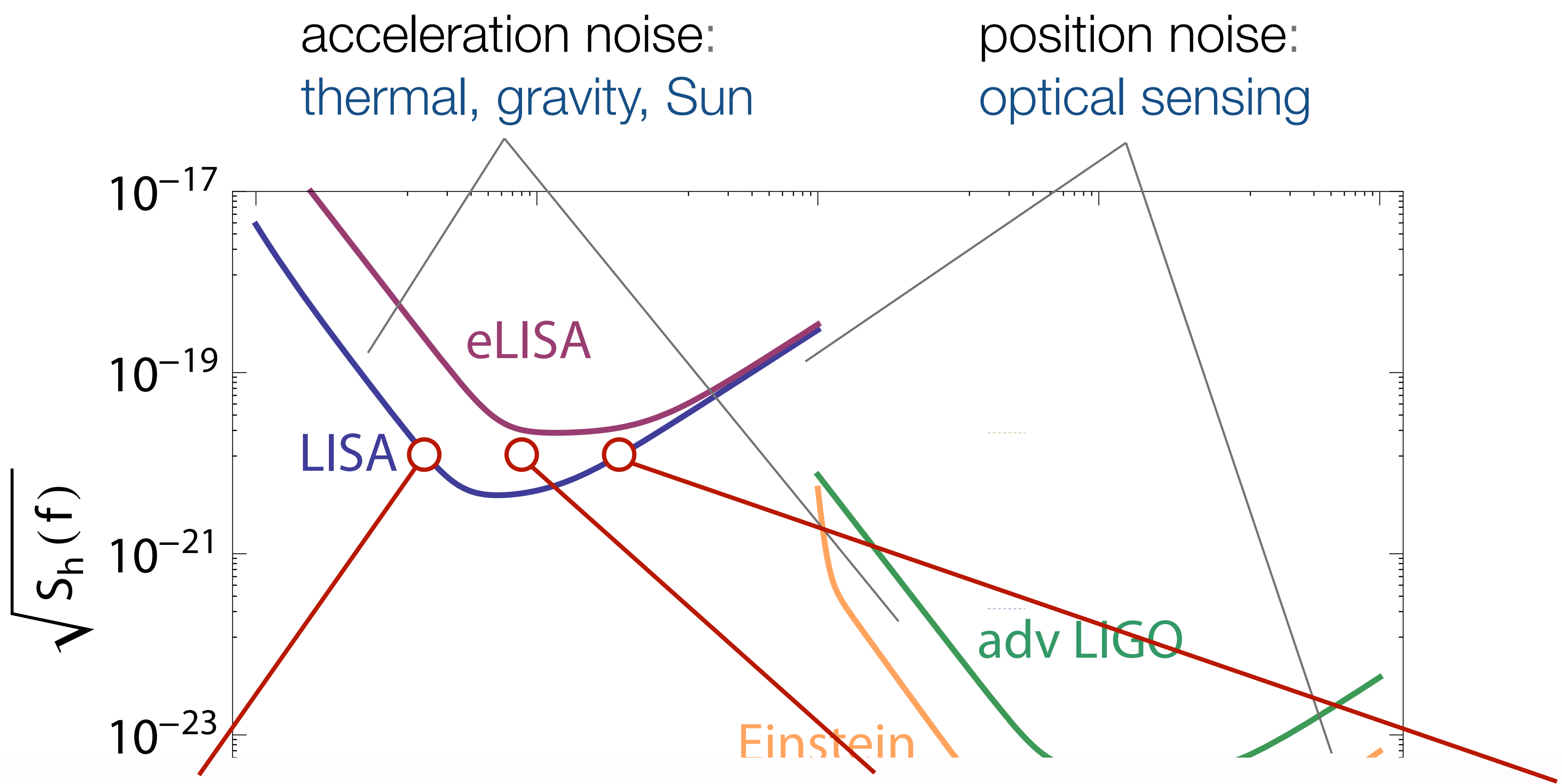


noise = data – signal

hence *p*(signal parameters) = *p*(noise residual)



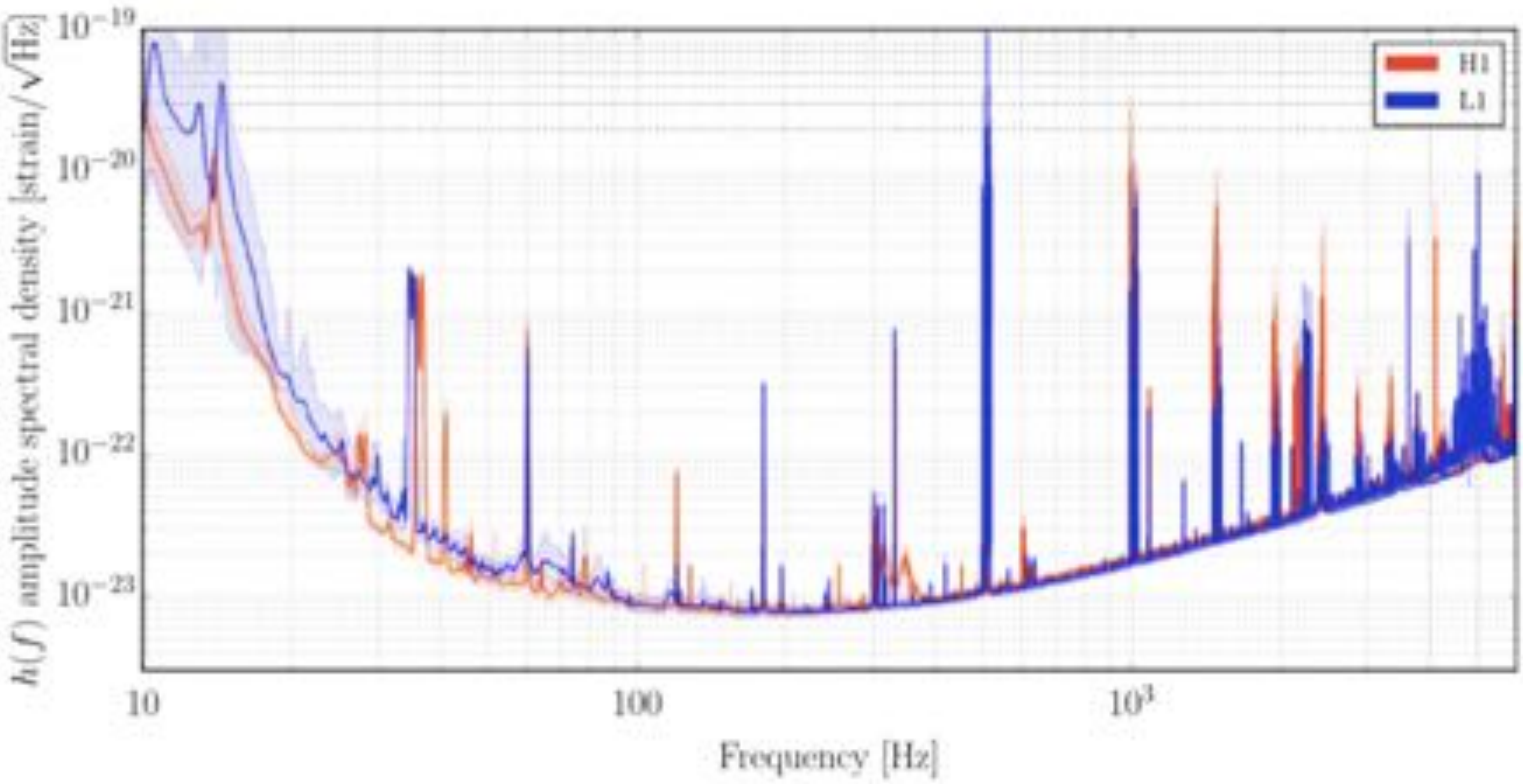
because of colored detector noise, detection and parameter estimation are sensitive to the frequency content of waveforms...



$s(t)$

$s(f)$

under the assumption of Gaussianity, the power spectral density yields the sampling distribution of noise



$$p(n) \propto \Pi_i \, e^{-n_i n_i^* / 2\sigma_i^2} = \mathrm{e}^{-2\int \frac{n(f) n^*(f)}{S(f)} df}$$

See “Data analysis recipes: Fitting a model to data”
Hogg, Bovy, and Lang 2010

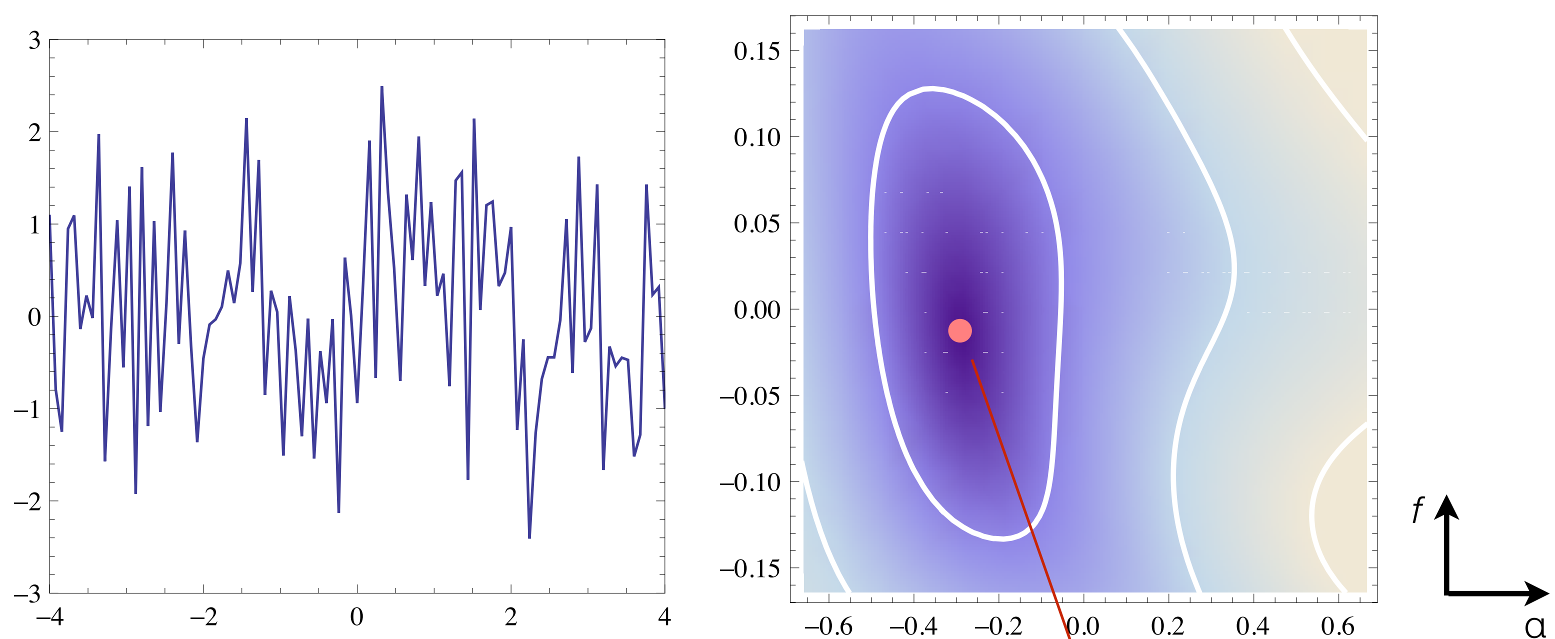
http://arxiv.org/abs/1008.4686

Bayesian inference: we update our prior knowledge of physical parameters using the likelihood of observed data

$$p(\theta_i|s) = \frac{p(\theta_i)p(s|\theta_i)}{p(s)}$$

$$p(s|\theta_i) = p(n = s - h(\theta_i)) = \mathcal{N}\mathrm{e}^{-(n,n)/2}$$

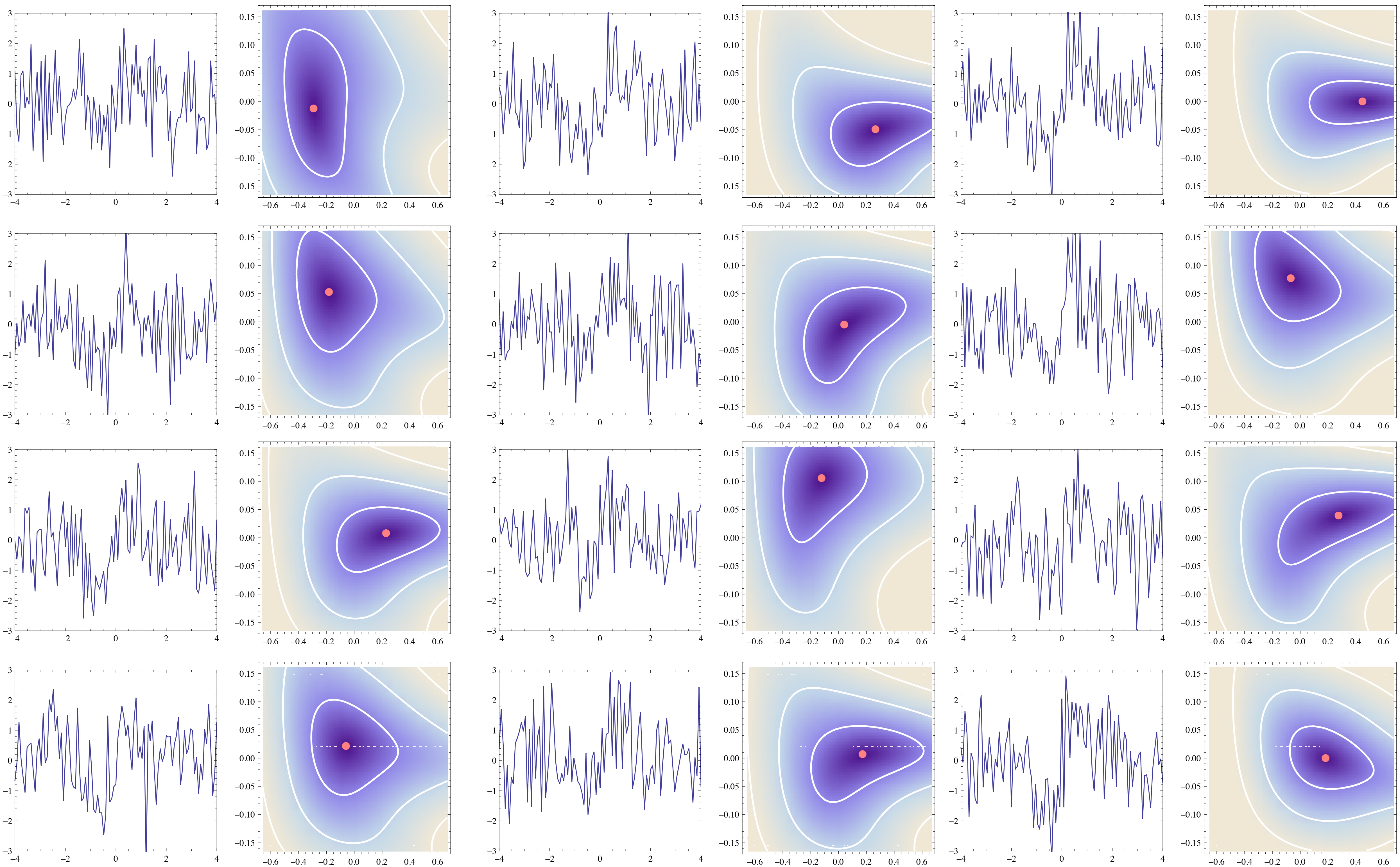
$$p(s) = \int p(\theta_i)p(s|\theta_i)\,\mathrm{d}\theta_\mathrm{i}$$



deviates from high-SNR covariance predicted with Fisher matrix

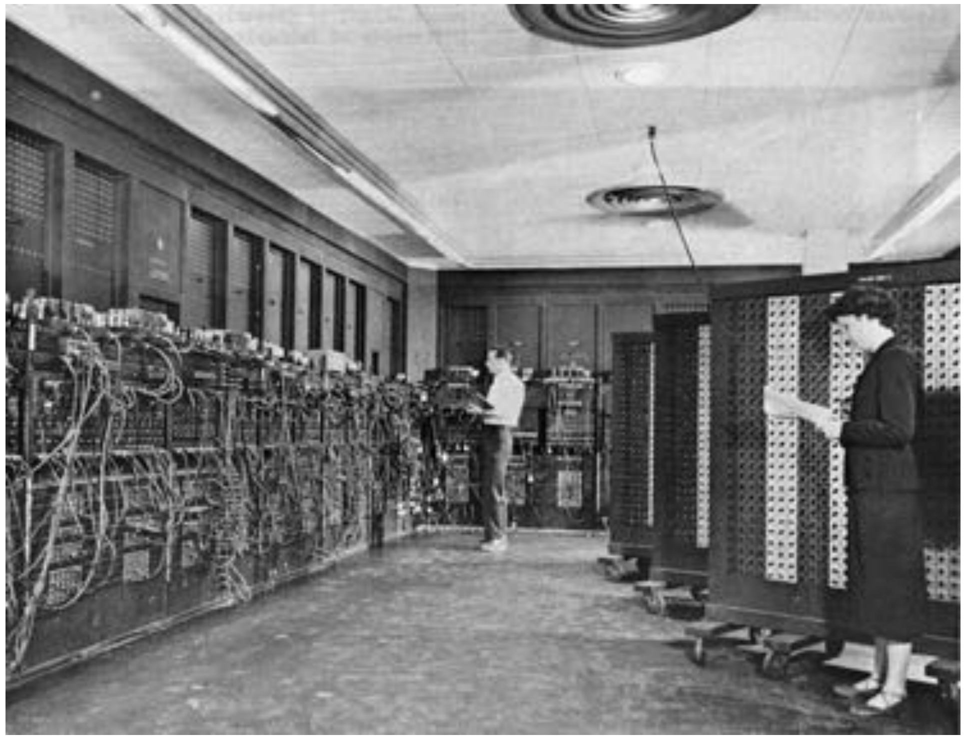
$$F_{ij} = \left(\frac{\partial h}{\partial \theta^i}\middle|\frac{\partial h}{\partial \theta^j}\right)$$

…but every noise realization will be different!



how to do this for many parameters?

Monte Carlo (Von Neumann and Ulam, 1946):
computational techniques that use random numbers



Monte Carlo (Von Neumann and Ulam, 1946):
computational techniques that use random numbers

$$\int \phi(x)dx \rightarrow \hat{\phi} = \frac{1}{R}\sum_r \phi(x^{(r)})$$

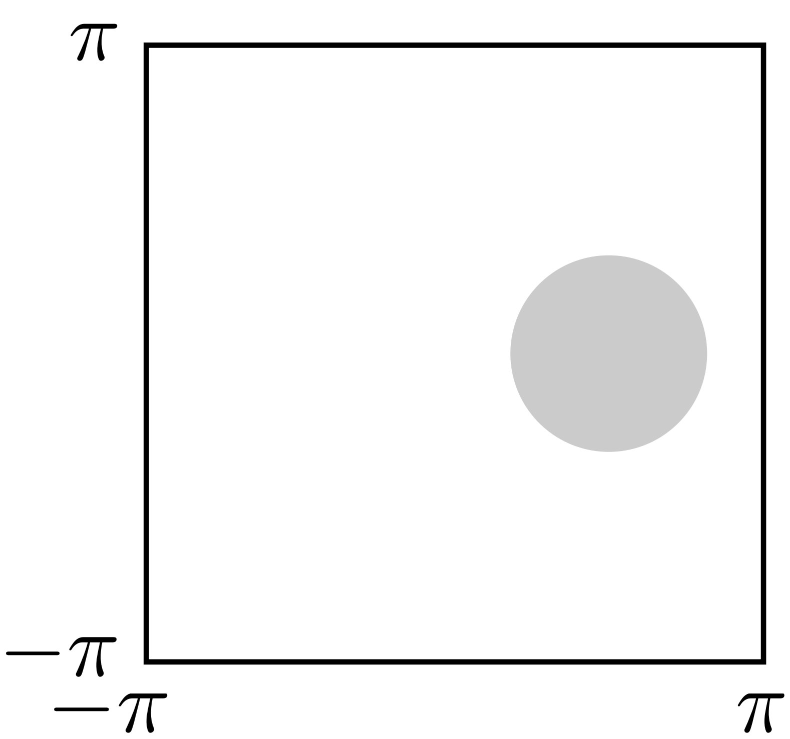
accuracy depends only on variance,
not on the number of dimensions

$$\int \phi(x)dx \rightarrow \hat{\phi} = \frac{1}{R}\sum_r \phi(x^{(r)})$$

$$\mathrm{var}\,\hat{\phi} = \frac{\mathrm{var}\,\phi}{R}$$

unfortunately uniform sampling is extremely inefficient in high-dimensional spaces

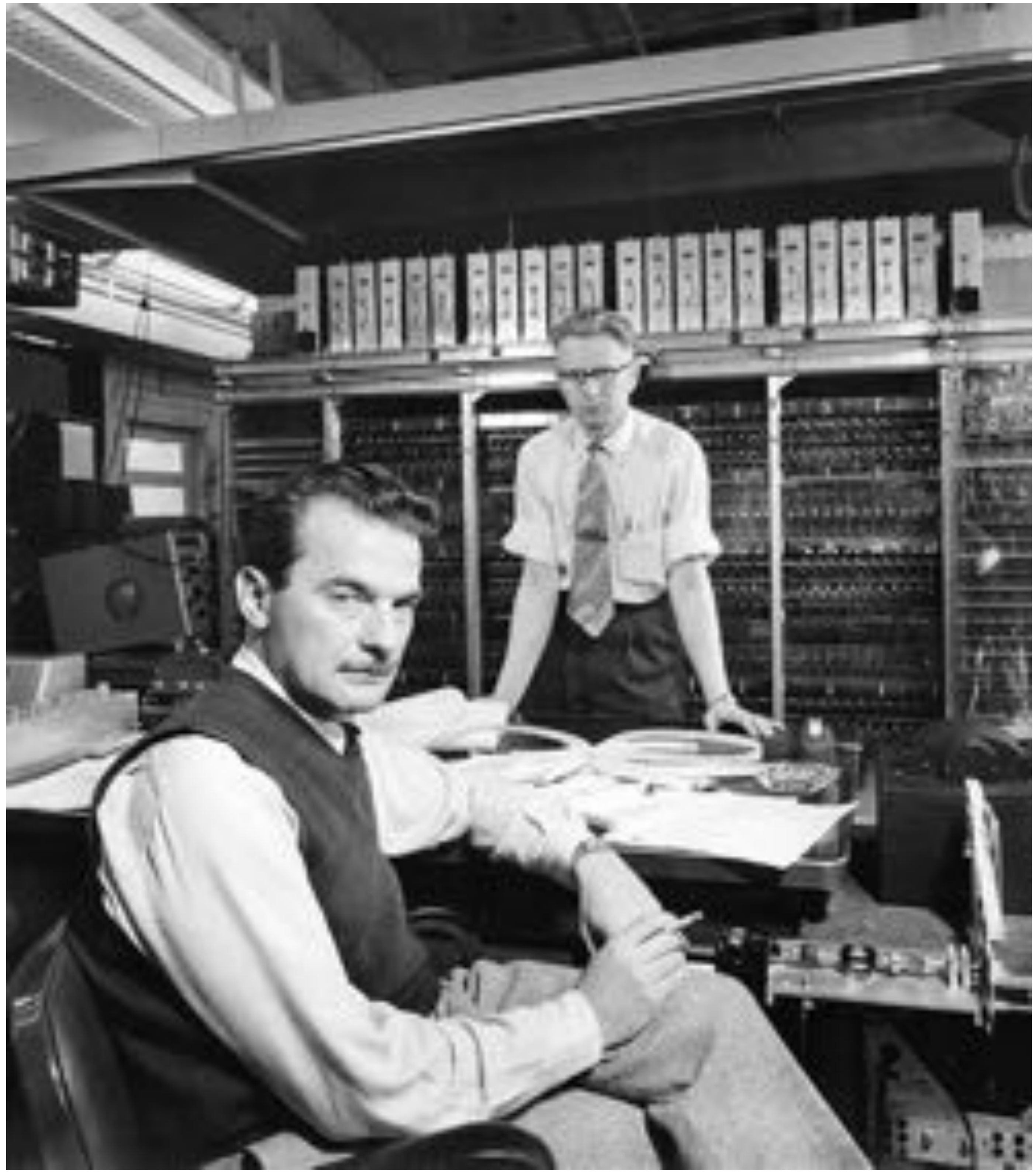
(and so are importance sampling and rejection sampling)



$$V_{\text{box}} = (2\pi)^d$$

$$V_{\text{ball}} = \frac{(\pi)^{d/2}}{\Gamma(n/2+1)}$$

$$\frac{V_{\text{box}}}{V_{\text{ball}}} \sim d^d$$



Nicholas Metropolis and his
Mathematical Analyzer Numerical Integrator And Calculator

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 21, NUMBER 6 JUNE, 1953

Equation of State Calculations by Fast Computing Machines

NICHOLAS METROPOLIS, ARIANNA W. ROSENBLUTH, MARSHALL N. ROSENBLUTH, AND AUGUSTA H. TELLER,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

AND

EDWARD TELLER,* *Department of Physics, University of Chicago, Chicago, Illinois*

(Received March 6, 1953)

A general method, suitable for fast computing machines, for investigating such properties as equations of state for substances consisting of interacting individual molecules is described. The method consists of a modified Monte Carlo integration over configuration space. Results for the two-dimensional rigid-sphere system have been obtained on the Los Alamos MANIAC and are presented here. These results are compared to the free volume equation of state and to a four-term virial coefficient expansion.

Marshall Rosenbluth and Edward Teller

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THE purpose of this paper is to describe a general method, suitable for fast electronic computing machines, of calculating the properties of any substance which may be considered as composed of interacting individual molecules. Classical statistics is assumed, only two-body forces are considered, and the potential field of a molecule is assumed spherically symmetric. These are the usual assumptions made in theories of liquids. Subject to the above assumptions, the method is not restricted to any range of temperature or density.

Teller's crucial suggestion: ensemble averaging...

$$\int \phi(x)p(x)dx, \quad \text{with} \quad p(x) \simeq e^{-E(x)/kT}$$

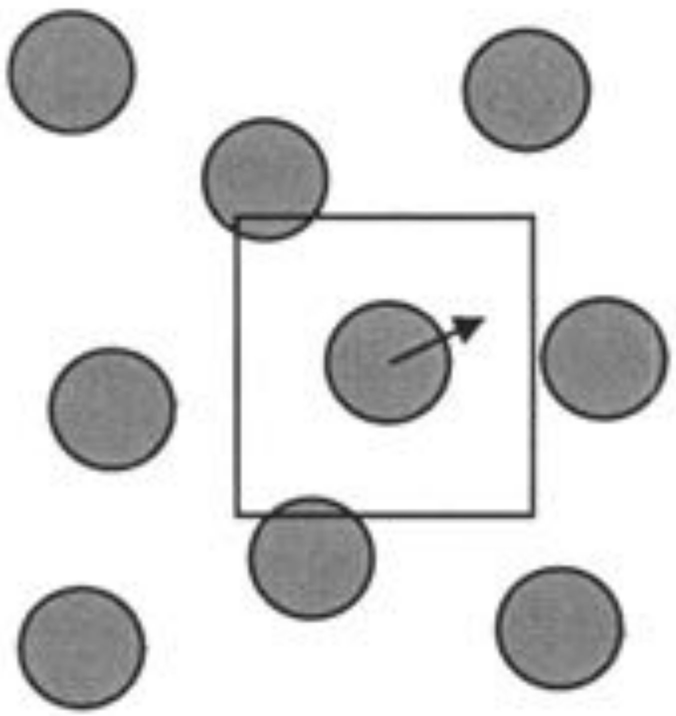
$$\Downarrow$$

$$\int \phi(x)dp(x) \simeq \frac{1}{R}\sum_{R}\phi(x^{(r)}) \quad \text{with} \quad \{x^{(r)}\}_P$$

Thus the most naive method of carrying out the integration would be to put each of the N particles at a random position in the square (this defines a random point in the $2N$-dimensional configuration space), then calculate the energy of the system according to Eq. (1), and give this configuration a weight $\exp(-E/kT)$. This method, however, is not practical for close-packed configurations, since with high probability we choose a configuration where $\exp(-E/kT)$ is very small; hence a configuration of very low weight. So the method we employ is actually a modified Monte Carlo scheme, where, instead of choosing configurations randomly, then weighting them with $\exp(-E/kT)$, we choose configurations with a probability $\exp(-E/kT)$ and weight them evenly.

§ It might be mentioned that the random numbers that we used were generated by the middle square process. That is, if ξ^u is an m digit random number, then a new random number ξ_{n+1} is given as the middle m digits of the complete $2m$ digit square of ξ_n.

...with samples generated by the "Metropolis" algorithm



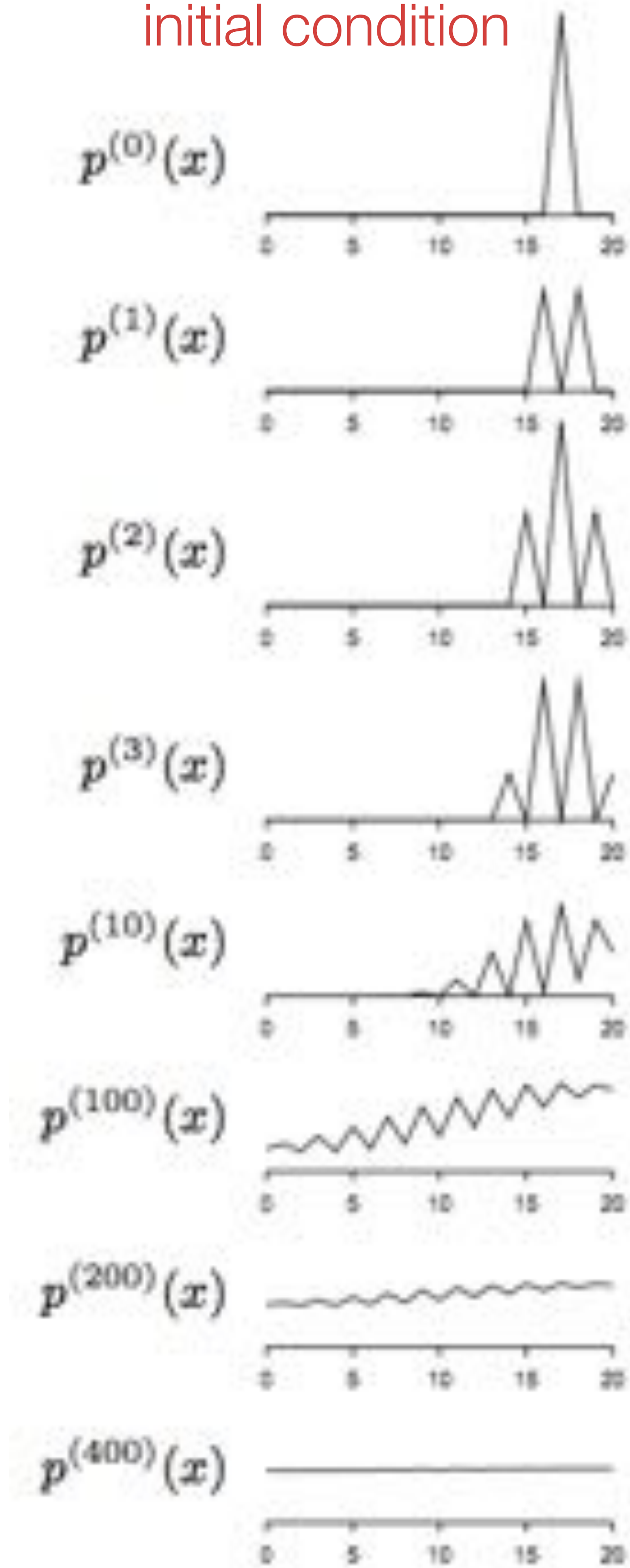
- given $x^{(r)}$, propose $x^{(r+1)}$ by random walk
- accept it if $\Delta E = E(x^{(r+1)}) - E(x^{(r)}) < 0$, or with probability $e^{-\Delta E/kT}$ if $\Delta E > 0$
- if not accepted, set $x^{(r+1)} = x^{(r)}$
- the resulting detailed balance guarantees convergence to P

We then calculate the change in energy of the system ΔE, which is caused by the move. If $\Delta E<0$, i.e., if the move would bring the system to a state of lower energy, we allow the move and put the particle in its new position. If $\Delta E>0$, we allow the move with probability $\exp(-\Delta E/kT)$; i.e., we take a random number ξ_3 between 0 and 1, and if $\xi_3<\exp(-\Delta E/kT)$, we move the particle to its new position. If $\xi_3 > \exp(-\Delta E/kT)$, we return it to its old position.

but why does it work?

- the Metropolis algorithm implements a Markov Chain $\{x^{(r)}\}$ with transition probability $T(x_i;x_j) = T_{ij}$
- T is set by the proposal distribution Q and the transition rule (e.g., Metropolis)
- if T_{ij} satisfies certain properties, its repeated application to any initial probability distribution ρ^0_j eventually yields the equilibrium distribution $\rho^*_i = P_i$

initial condition



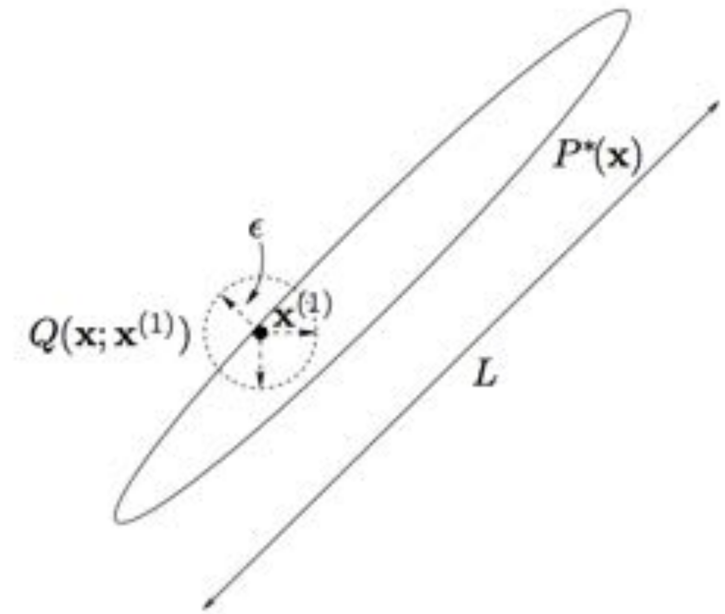
(MacKay 2003)

equilibrium distribution

- the Metropolis algorithm is very general and very easy to implement

but:

- convergence, while guaranteed, is hard to assess
- random-walk exploration is very inefficient



(MacKay 2003)

- need $(L/\varepsilon)^2 \sim (\sigma_{\text{max}}/\sigma_{\text{min}})^2$ steps to get independent sample

try:

- annealing, parallel tempering
- Hamiltonian MCMC
- affine-invariant samplers (**emcee**)
- the evidence/partition function is difficult to compute

$$Z = \int e^{-E(x)/kT} dx,$$

$$p(M) = \int p(\text{data}|x)p(x)dx$$

- thermodynamic integration
- reversible-jump MCMC
- nested sampling (**MultiNest**)

Testing GR: the standard hierarchy of theories of gravitation

WEP Newton's equivalence principle

10^{-13} $m_I = m_G$

EEP Einstein's equivalence principle

= WEP + local Lorentz invariance 10^{-22}

+ local position invariance 10^{-5}

metric theories (what fields?)

SEP EEP, but also for gravitational experiments

10^{-4}


The Confrontation between General Relativity and Experiment

Clifford M. Will

Living Reviews in Relativity

Abstract

1981→2006

Dicke: test of EPs + PPN tests of metric theories

the PPN formalism: metric and potentials

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2$$
$$+2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij}$$
$$+(2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U$$
$$-\alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).$$

$$U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$U_{ij} = \int \frac{\rho'(x - x')_i (x - x')_j}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x',$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|}\right) d^3x'\, d^3x'',$$

$$\mathcal{A} = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x',$$

$$\Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$\Phi_2 = \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$\Phi_3 = \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$\Phi_4 = \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$V_i = \int \frac{\rho' v'_i}{|\mathbf{x} - \mathbf{x}'|}\, d^3x',$$

$$W_i = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')](x - x')_i}{|\mathbf{x} - \mathbf{x}'|^3}\, d^3x'.$$

the PPN formalism: parameters

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories	
γ	How much space-curvature produced by unit rest mass?	1	γ	γ	10^{-5}
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β	10^{-4}
ξ	Preferred-location effects?	0	ξ	ξ	
α_1	Preferred-frame effects?	0	α_1	0	
α_2		0	α_2	0	
α_3		0	0	0	
α_3	Violation of conservation of total momentum?	0	0	0	
ζ_1		0	0	0	
ζ_2		0	0	0	
ζ_3		0	0	0	
ζ_4		0	0	0	

Gravitational radiation...

...is predicted in virtually any metric theory of gravity that embodies Lorentz invariance, but it may differ from GR in:

polarizations

speed of waves

radiation reaction (tested at low v with binary pulsars)

Unfortunately, no simple, principled framework like PPN exists for describing radiative systems or systems containing strong internal fields.

So we must consider individual alternative theories, or perform null tests of consistency.

Naïve and sentimental tests of GR consistency

In order of difficulty and un-likelihood:

- If we divide the waveform in segments, do individual SNRs pass a χ^2 test?
- Is there a coherent residual?
- What about the source parameters determined from each segment—are they consistent (within estimated errors) with the parameters determined from the entire waveform?
- Is the shape of the likelihood surface consistent with what's expected for this waveform family?

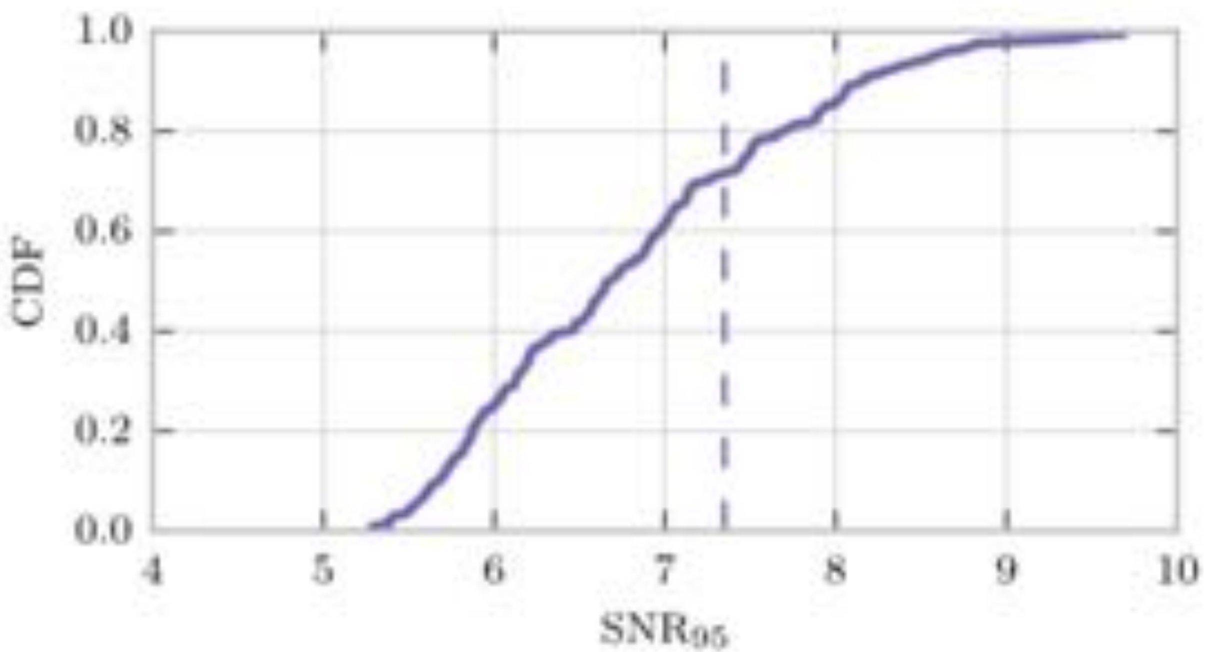
- But before we suspect general relativity: instrument systematics, modeling, data analysis, physical environments…

$$\left\langle \frac{\partial h}{\partial \lambda_{\mathrm{physical}}} \middle| \frac{\partial h}{\partial \lambda_{\mathrm{non\text{-}GR}}} \right\rangle \simeq 0$$

Selected for a Viewpoint in *Physics*
PRL **116,** 221101 (2016) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2016

Tests of General Relativity with GW150914

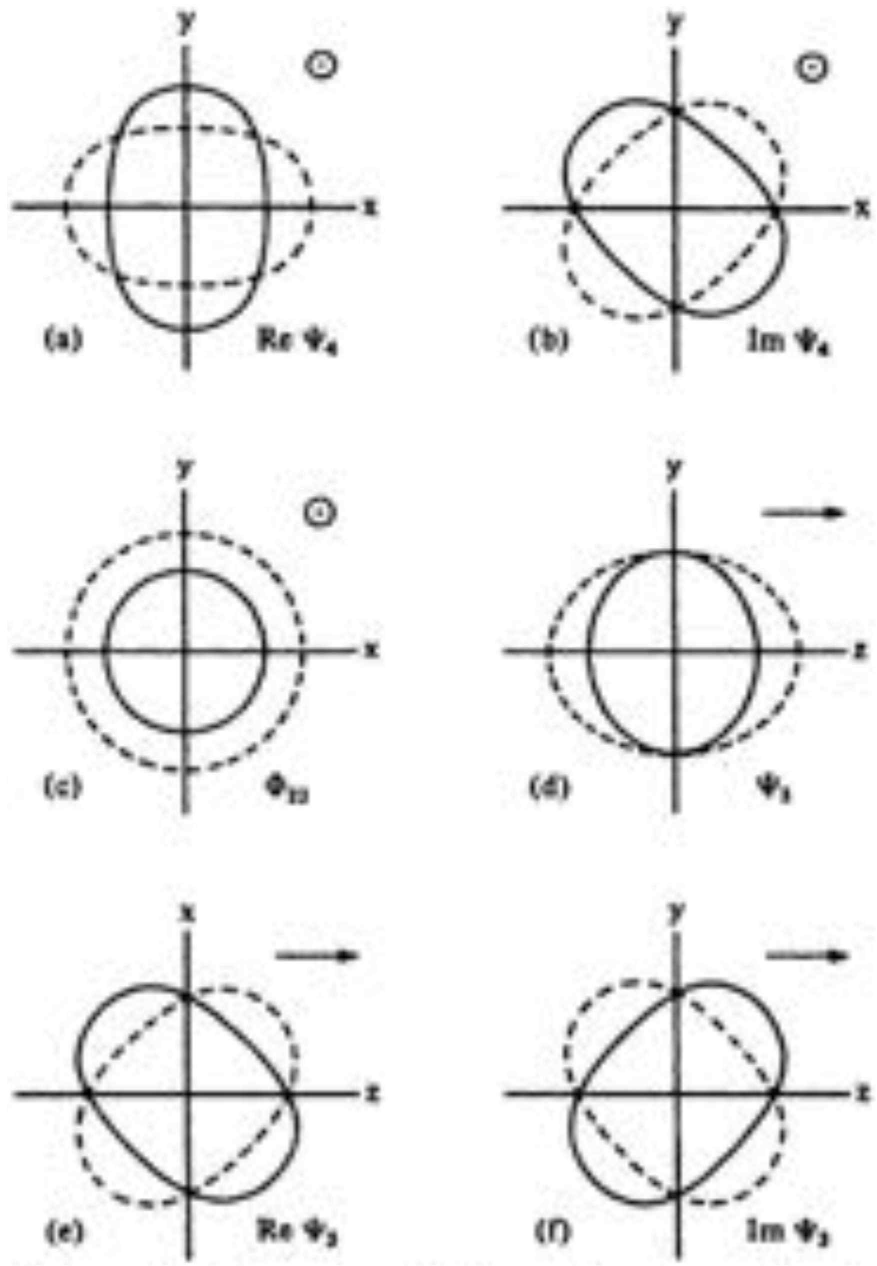
B. P. Abbott *et al.**
(LIGO Scientific and Virgo Collaborations)



SNR in coherent burst analysis of data residual after subtracting best-fit GW150914 waveform

If we assume that SNR_{res} is entirely due to the mismatch between the MAP waveform and the underlying true signal, and that the putative violation of GR cannot be reabsorbed in the waveform model by biasing the estimates of the physical parameters [54,55], we can constrain the minimum *fitting factor* (FF) [56] between the MAP model and GW150914. An imperfect fit to the data leaves $SNR_{res}^2 = (1 - FF^2)FF^{-2}SNR_{det}^2$ [57,58], where $SNR_{det} = 25.3^{+0.1}_{-0.2}$ is the network SNR inferred by LALInference [3]. $SNR_{res} \leq 7.3$ then implies $FF \geq 0.96$. Considering that, for parameters similar to those inferred for GW150914, our waveform models have much higher FFs against numerical GR waveforms, we conclude that the noise-weighted correlation between the observed strain signal and the true GR waveform is ≥96%. This statement can be read as implying that the GR prediction for GW150914 is verified to be better than 4%, in a precise sense related to noise-weighted signal correlation, and, conversely, that effects due to GR violations in GW150914 are limited to less than 4% (for effects that cannot be reabsorbed in a redefinition of physical parameters).

Polarization

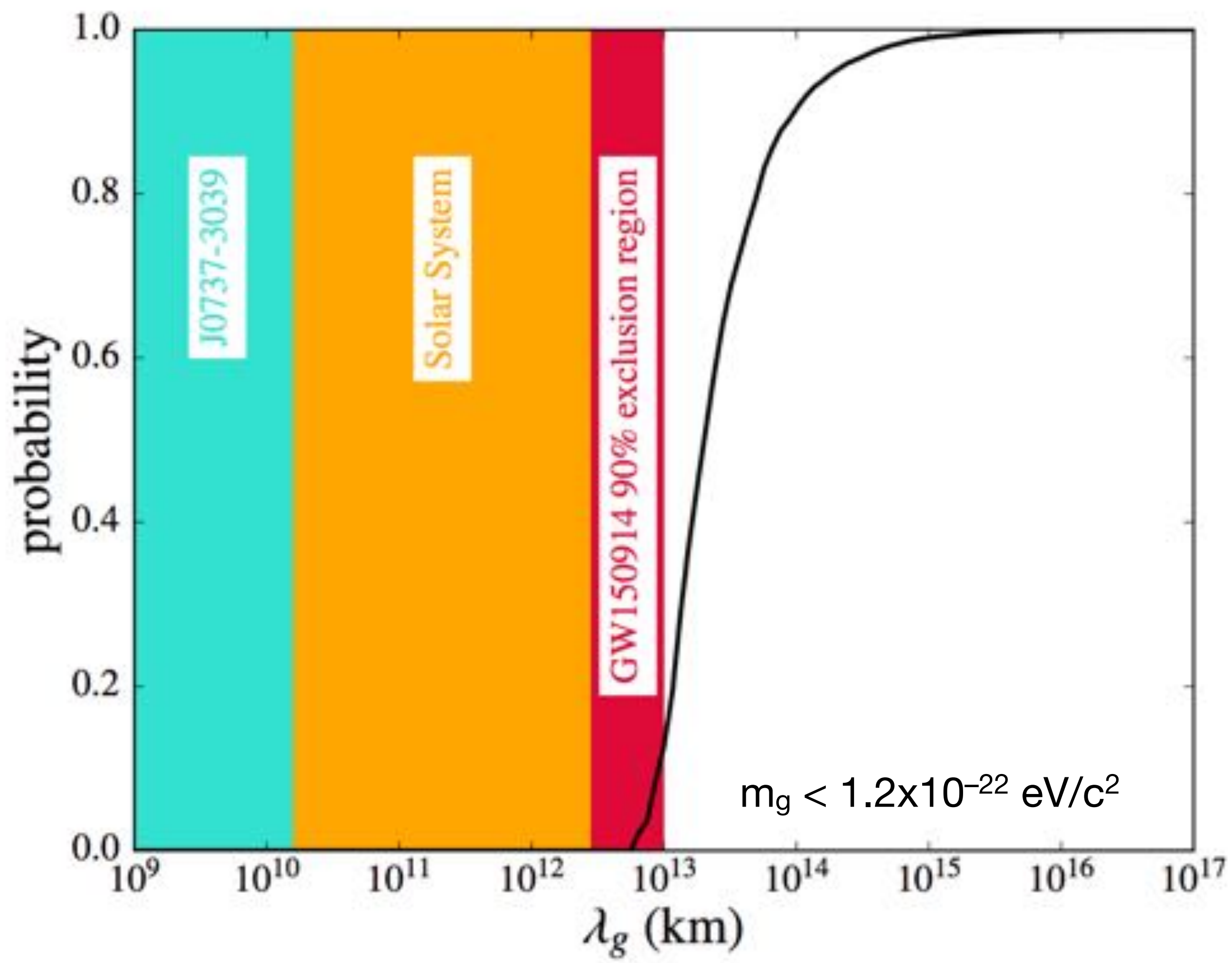


Tensor

Scalar

Vector

Speed of waves by dephasing in GW150914
(in future systems with counterparts: compare with EM!)



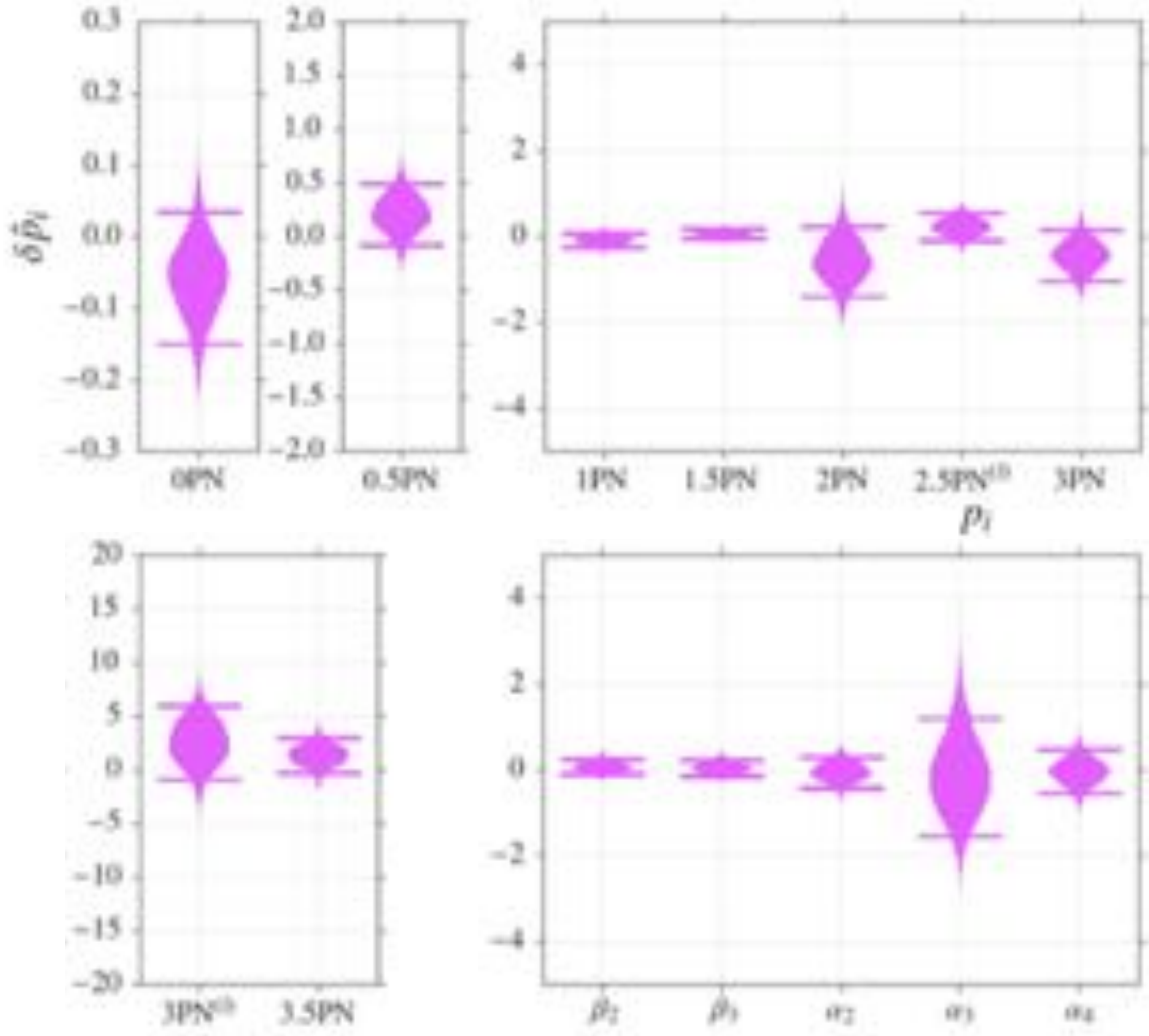
$$h(f) = \frac{1}{D}\frac{\mathcal{A}}{\sqrt{\dot{F}}} f^{2/3} e^{i\Psi(f)}$$

$$\Psi(f) = \sum_i \left[\psi_i + \psi_{il}\log f\right] f^{(i-5)/3} + \Phi^{\mathrm{MR}}[\beta_i, \alpha_i]$$

$$\frac{v_{\mathrm{g}}^2}{c^2} = 1 - \frac{m_{\mathrm{g}}^2 c^4}{E^2}$$

$$\delta\Psi(f) = \frac{\pi D c}{\lambda_g^2 (1+z) f}$$

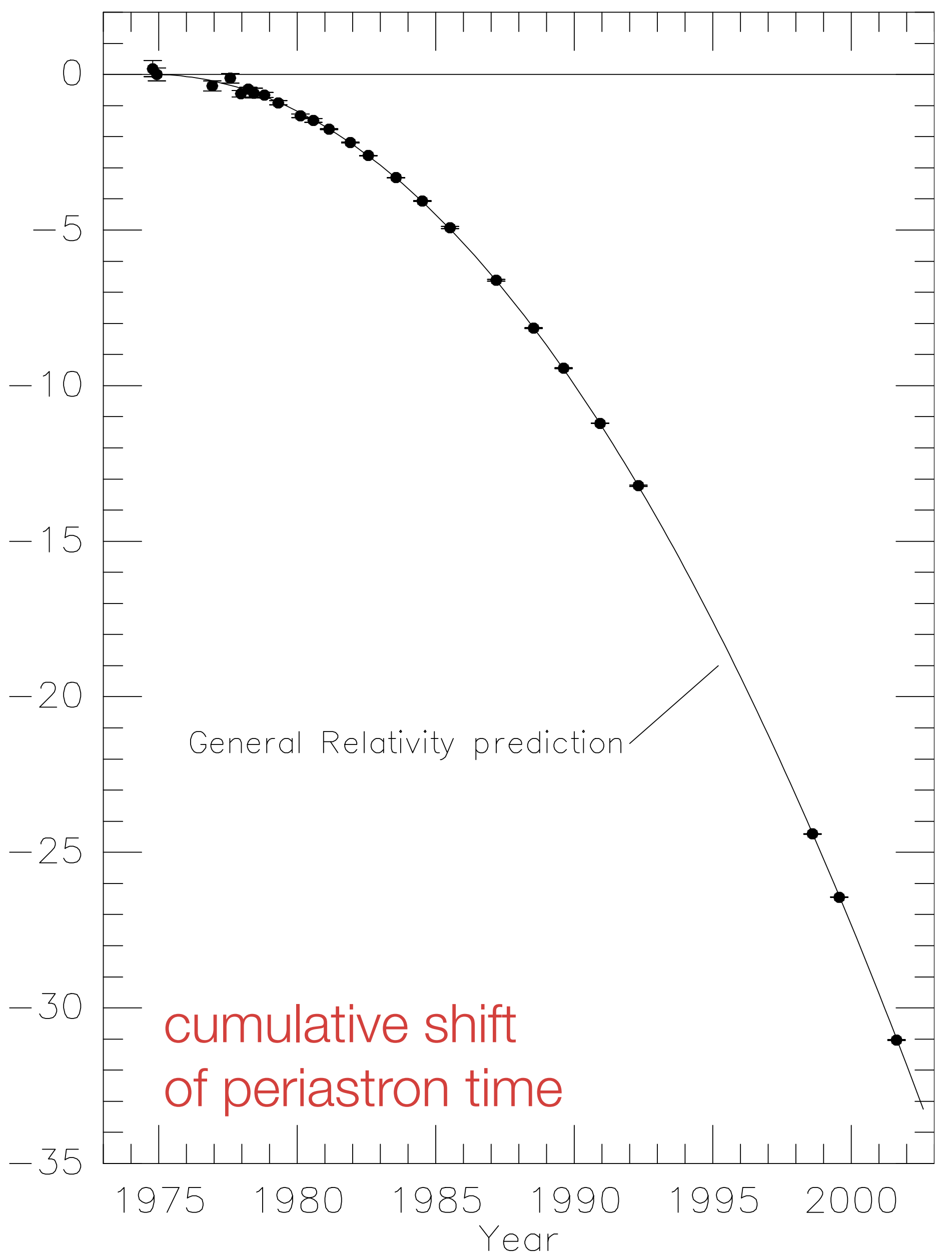
Radiation reaction by waveform coefficients in GW150914 and GW151226 (in NS binaries: dipolar radiation)



$$h(f) = \frac{1}{D}\frac{\mathcal{A}}{\sqrt{\dot{F}}} f^{2/3} e^{i\Psi(f)} \qquad \Psi(f) = \sum_i \left[\psi_i + \psi_{il}\log f\right] f^{(i-5)/3} + \Phi^{\mathrm{MR}}[\beta_i, \alpha_i]$$

For comparison: the timing of NS–NS pulsars allows accurate tests of GR in terms of easily interpreted parameters

PSR B1913+16
[Weisberg 2003]



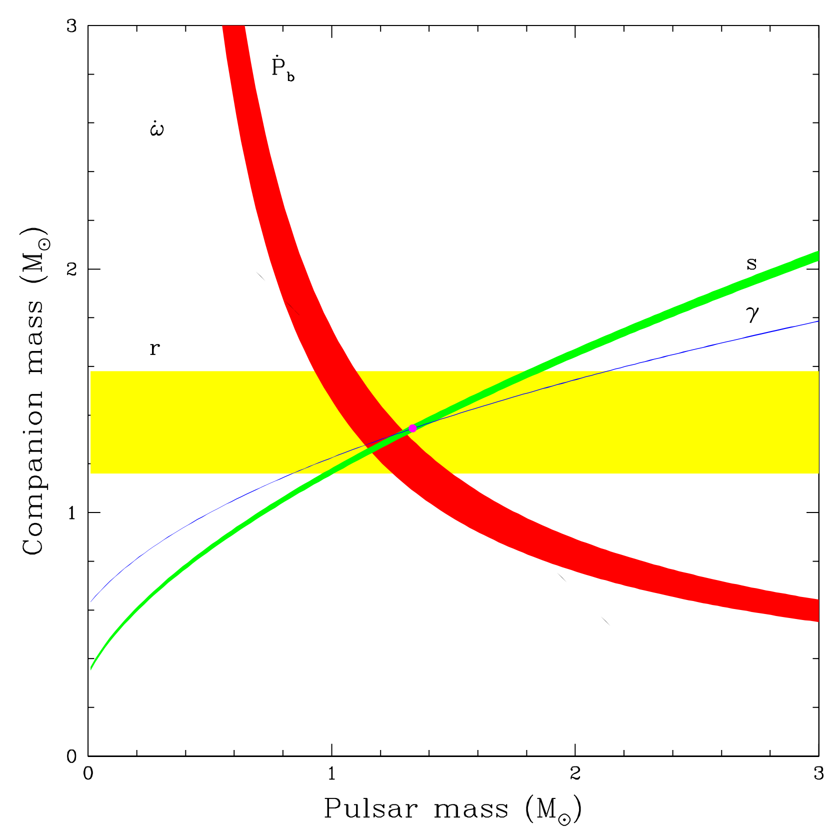
$$\dot{\omega} = 3\left(\frac{P_{\rm b}}{2\pi}\right)^{-5/3}(T_\odot M)^{2/3}\,(1-e^2)^{-1},$$ periastron advance

$$\gamma = e\left(\frac{P_{\rm b}}{2\pi}\right)^{1/3}T_\odot^{2/3}M^{-4/3}\,m_2(m_1+2m_2),$$ GR redshift

orb. period derivative
$$\dot{P}_{\rm b} = -\frac{192\pi}{5}\left(\frac{P_{\rm b}}{2\pi}\right)^{-5/3}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)(1-e^2)^{-7/2}\,T_\odot^{5/3}\,m_1 m_2\,M^{-1/3}$$

$$r = T_\odot\,m_2,$$ range of Shapiro delay

$$s = x\left(\frac{P_{\rm b}}{2\pi}\right)^{-2/3}T_\odot^{-1/3}\,M^{2/3}\,m_2^{-1}.$$ shape of Shapiro delay



PSR B1534+12
[Stairs et al. 2002]

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