Generalized Weyl anomalies in higher-spin theory

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Main question

- CFT has conformal anomaly
- CFT with higher-spin symmetry has generalized conformal anomalies (in addition to conformal anomaly)
- **Today:**
  compute boundary higher-spin conformal anomalies from bulk higher-spin theory

Reference

- Some aspects of holographic W-gravity

JHEP 1508, 035 (2015)

with <u>Stefan Theisen</u>

Plan

1. Generalized conformal anomaly in CFT with higher-spin symmetry
2. Bulk computation of boundary anomaly
3. Discussion

Conformal anomaly in CFT

Even dimensional CFT in curved background g_{ij}

$$\text{classical:} \qquad T^i{}_i = 0$$

$$\text{quantum mechanical:} \qquad \langle T^i{}_i \rangle \neq 0$$

Capper Duff '73

Generating function of conformal anomalies

- Integrate out CFT fields to obtain (non-local) effective action

$$e^{-W[g]} = \int D\Phi \, e^{-S_{\rm CFT}[\Phi, g]}$$

- Weyl transformation $\quad \delta_{\sigma_2} g_{ij} = 2\,\sigma_2\, g_{ij}$

$$\delta_{\sigma_2} W[g] = \int \sqrt{g}\, \sigma_2(x)\, \langle T^i{}_i \rangle \neq 0$$

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Conformal anomaly in CFT with higher-spin symmetry

Even dimensional CFT in curved background $g_{ij}, \varphi_{ijk}, \ldots$

$$\text{classical:} \qquad T^{i}{}_{i} = 0 \qquad W^{i}{}_{ij} = 0 \quad \ldots$$

$$\text{quantum mechanical:} \qquad \langle T^{i}{}_{i} \rangle \neq 0 \qquad \langle W^{i}{}_{ij} \rangle \neq 0 \quad \ldots$$

Generating function of conformal anomalies

- Integrate out CFT fields to obtain (non-local) effective action

$$e^{-W[g,\varphi]} = \int D\Phi \, e^{-S_{\mathrm{CFT}}[\Phi,g,\varphi]}$$

- Weyl transformation $\quad \delta_{\sigma_2} g_{ij} = 2\,\sigma_2\, g_{ij} \qquad \delta_{\sigma_2}\varphi_{ijk} = 4\,\sigma_2\,\varphi_{ijk}$

$$\delta_{\sigma_2} W[g,\varphi] = \int \sqrt{g}\,\sigma_2(x)\,\mathcal{A}_2 \neq 0$$

- Additional anomalous symmetry: $\mathcal{W}$-Weyl transformation

$$\delta_{\sigma_3} W[g,\varphi] = \int \sqrt{g}\,\sigma_3(x)\,\mathcal{A}_3 \neq 0$$

Weyl anomalies in 2D CFT
(from 2-point function in flat background)

Conserved currents

$$T_{ij}$$

Naively

$$\partial^i T_{ij} = 0 \qquad \text{and} \qquad \eta^{ij} T_{ij} = 0$$

Anomalous Ward Identity

1. Symmetry and conservation gives

$$\langle T_{ij}(p) T_{kl}(-p) \rangle = A(p^2) \left(p_i p_j - \eta_{ij} p^2 \right) \left(p_k p_l - \eta_{kl} p^2 \right)$$

2. Incompatible with conformal symmetry:

$$\eta^{ij} T_{ij} = 0 \quad \Longrightarrow \quad A(p^2) = 0 \quad \Longrightarrow \quad \langle T_{ij}(p) T_{kl}(-p) \rangle = 0$$

3. Give up conformal symmetry:

$$A(p^2) = \frac{c}{p^2}$$

$\mathcal{W}$-Weyl anomalies in 2D $\mathcal{W}$-CFT
(from 2-point function in flat background)

Conserved currents

$$T_{ij}(\equiv W^{(2)}_{ij}) \qquad W^{(3)}_{ijk} \qquad W^{(4)}_{ijkl} \qquad \cdots$$

Naively

$$\partial^i W_{i\cdots} = 0 \qquad \text{and} \qquad \eta^{ij} W_{ij\cdots} = 0$$

Anomalous Ward Identity

1. Symmetry and conservation gives

$$\langle W_{ijk}(p) W_{lmn}(-p)\rangle = A^{(3)}(p^2)\left[(p_i p_l - \eta_{il} p^2)(p_j p_m - \eta_{jm} p^2)(p_k p_n - \eta_{kn} p^2) + \dots\right]$$

2. Incompatible with $\mathcal{W}$-conformal symmetry:

$$\eta^{ij} W_{ijk} = 0 \quad \Longrightarrow \quad A^{(3)}(p^2) = 0 \quad \Longrightarrow \quad \langle W_{ijk}(p) W_{lmn}(-p)\rangle = 0$$

3. Give up $\mathcal{W}$-conformal symmetry:

$$A^{(3)}(p^2) = \frac{c^{(3)}}{p^2}$$

Weyl and $\mathcal{W}$-Weyl anomalies from OPE

OPE of holomorphic currents:

$$T(z)T(w) \sim \frac{c}{(z-w)^4} + \dots$$

$$W(z)W(w) \sim \frac{c^{(3)}}{(z-w)^6} + \dots$$

$$\vdots$$

$$W^{(s)}(z)W^{(s)}(w) \sim \frac{c^{(s)}}{(z-w)^{2s}} + \dots$$

Each spin gives one $\mathcal{W}_s$-Weyl anomaly

Generating function of Weyl anomaly

- Without higher-spin fields, 2D effective action is uniquely given by Polyakov action

$$W_{2\mathrm{D}}[g] = \int R \frac{1}{\Box} R$$

- Analogue of Polyakov action for other cases is not known
  - 4D CFT ? *Deser Schwimmer '93; Deser '96,'99*
  - 2D CFT with higher-spin symmetry ?

Computing effective action from CFT is one-loop

Bndy Weyl anomaly from bulk

Henningson Skenderis '98

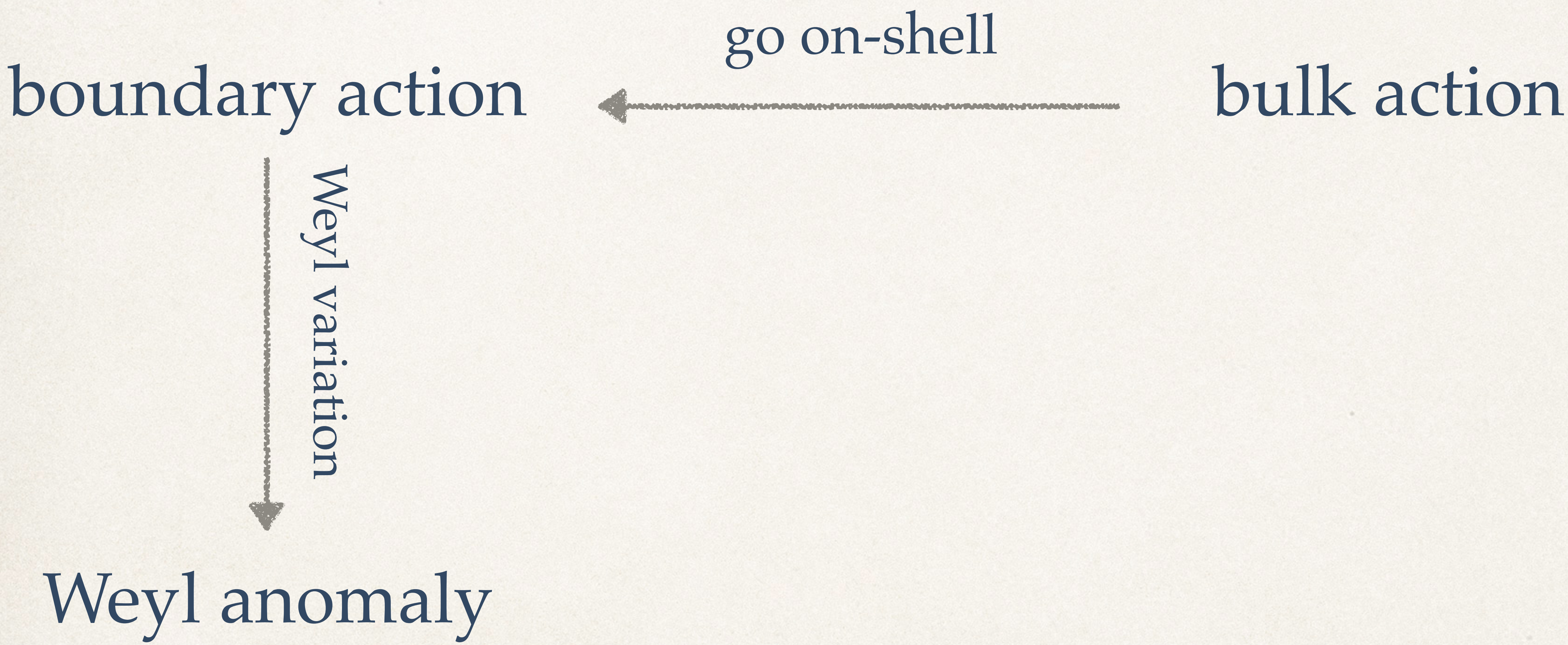
boundary action ← go on-shell ← bulk action

↓ Weyl variation

Weyl anomaly

Bndy Weyl anomaly from bulk

Henningson Skenderis '98



Bulk computation of boundary Weyl anomaly is Classical

Bndy Weyl anomaly from bulk original procedure

Henningson Skenderis '98

go on-shell

boundary action ← bulk action

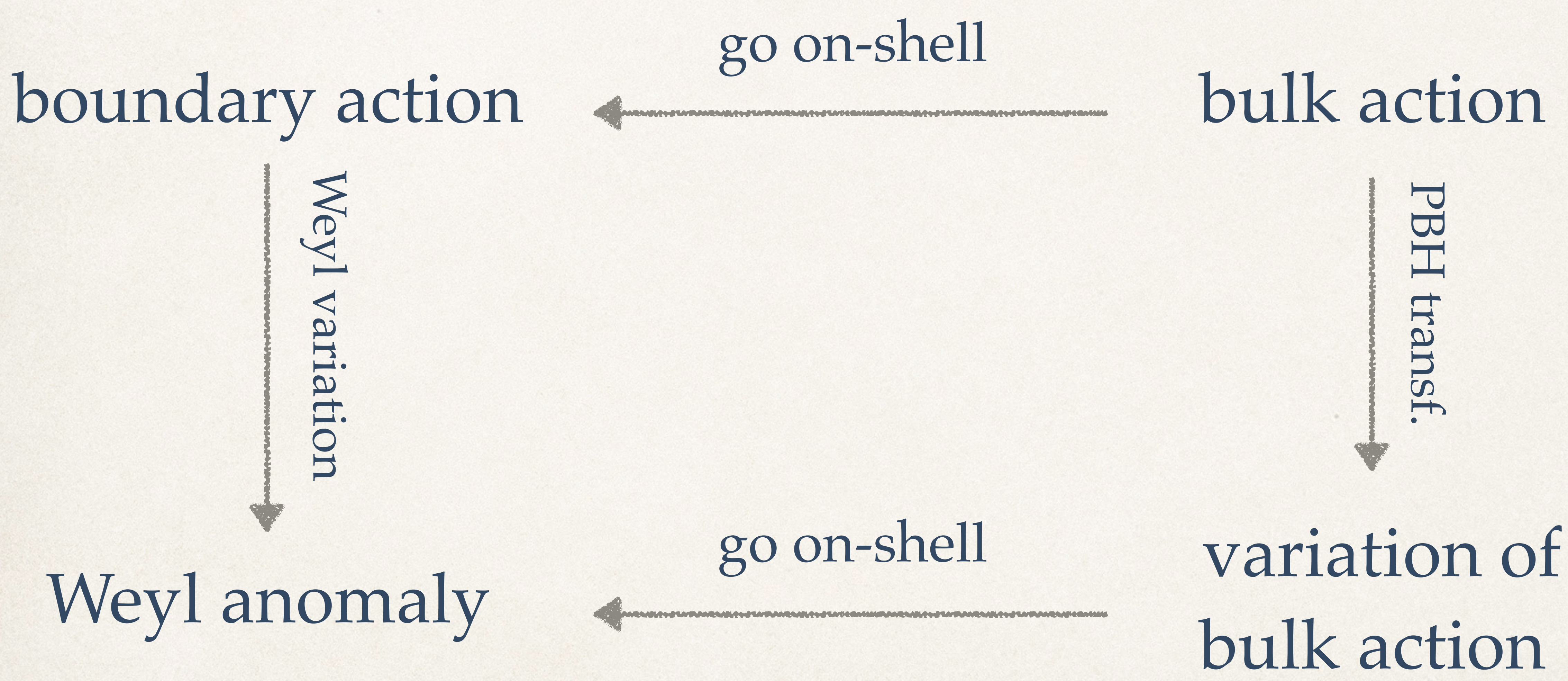
↓ Weyl variation

Weyl anomaly

Weyl variation of on-shell bulk action gives Weyl anomaly

Bndy Weyl anomaly from bulk
PBH procedure

Imbimbo Schwimmer Theisen Yankielowicz '99



PBH procedure for pure gravity

Step-1: Fefferman-Graham gauge of bulk metric (asympt. AdS_{2n+1})

$$ds^2 = \frac{d\rho^2}{\rho^2} + \frac{1}{\rho^2} g_{ij}(\rho, x) dx^i dx^j$$

FG expansion : $g_{ij}(\rho, x) = \overset{(0)}{g}_{ij}(x) + \rho^2 \overset{(2)}{g}_{ij}(x) + \rho^4 \overset{(4)}{g}_{ij}(x) + \ldots$

Step-2: PBH transformation $\equiv$ Bulk diffeo ξ^μ preserving FG gauge

$=$ Boundary Weyl

Step-3: Weyl anomaly from PBH transformation on bulk action

$$\langle T^i_i \rangle = \left(\delta_\xi S^{\text{bulk}}\right)|_{\text{on-shell}} = -2 \int_{\partial\mathcal{M}} d^d x \, \sigma(x) \left[\rho\sqrt{G}\mathcal{L}(G)\right]|_{\rho=0,\text{on-shell}}$$

Step-4: rewrite in terms of boundary data (i.e. go on-shell)

Bndy Weyl anomaly from bulk
PBH procedure

Imbimbo Schwimmer Theisen Yankielowicz '99

PBH procedure: a easy way to compute Weyl anomaly from bulk

What is higher-spin theory ?

- gravity theory coupled to higher-spin gauge symmetry
- dual CFT with higher-spin currents

Why higher-spin ?

- Higher-spin gravity (Vasiliev's theory) is an interesting extension of Einstein gravity
- Holography with higher-spin symmetry is different from traditional Gauge/Gravity duality
- Can use higher-spin symmetry to study string theory

Problem: there is no covariant metric-like formulation of higher-spin theory

We want metric-like formulation of higher-spin theory

pure gravity
(metric formulation)

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- Diffeomorphism invariance
- Riemannian geometry

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higher-spin
(metric-like formulation)

- Diffeo is coupled to higher-spin gauge transformation
- What is higher-spin geometry?

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pure gravity
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?

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We want metric-like formulation of higher-spin theory

pure gravity
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↓ ?

higher-spin
(metric-like formulation)

- Diffeo is coupled to higher-spin gauge transformation
- What is higher-spin geometry?

Campoleoni Fredenhagen Pfenninger Theisen '12

Chern-Simons formulation of higher-spin theory

pure gravity
(metric formulation)

pure gravity
sl(2) Chern-Simons

higher-spin
sl(N) Chern-Simons

3D higher Spin theory in AdS_3 — Action

Action:

$$S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}] \qquad \text{with} \qquad S_{\rm CS}[A] = \frac{k}{4\pi}\int_{\mathcal{M}} \mathrm{Tr}[AdA + \frac{2}{3}A^3]$$

Lorentzian: $A, \tilde{A} \in \mathfrak{sl}(N, \mathbb{R})$

Euclidean: $A, \tilde{A} \in \mathfrak{sl}(N, \mathbb{C})$ and $\tilde{A} = -A^\dagger$

Translation to metric-like formalism

1. Dreibein and spin connection

$$e = \frac{A - \tilde{A}}{2} \qquad \omega = \frac{A + \tilde{A}}{2}$$

2. metric and higher-spin fields

$$G_{\mu\nu} = \mathrm{Tr}[e_\mu e_\nu] \qquad \varphi_{\mu\nu\rho} = \mathrm{Tr}[e_{\{\mu} e_\nu e_{\rho\}}] \qquad \ldots$$

3D higher Spin theory in AdS_3 — Spectrum

Spectrum

1. Choose an $\mathfrak{sl}(2)$ subalgebra that corresponds to spin-2:

$$\text{spin-2}: \qquad \{L_1, \quad L_0, \quad L_{-1}\}$$

2. Decompose $\mathfrak{sl}(N)$ in terms of irreps of the gravitonal $\mathfrak{sl}(2)$

$$\text{spin-s} \ : \qquad \{W_m^{(s)}\} \qquad m = -s+1, \ldots, s-1$$

Principal embedding: 1 spin-s field for each $s = 2, ..., N$

$$L_1 \quad L_0 \quad L_{-1}$$

$$W_2 \quad W_1 \quad W_0 \quad W_{-1} \quad W_{-2}$$

3D higher Spin theory in AdS_3 — Spectrum

Spectrum

1. Choose an $\mathfrak{sl}(2)$ subalgebra that corresponds to spin-2:

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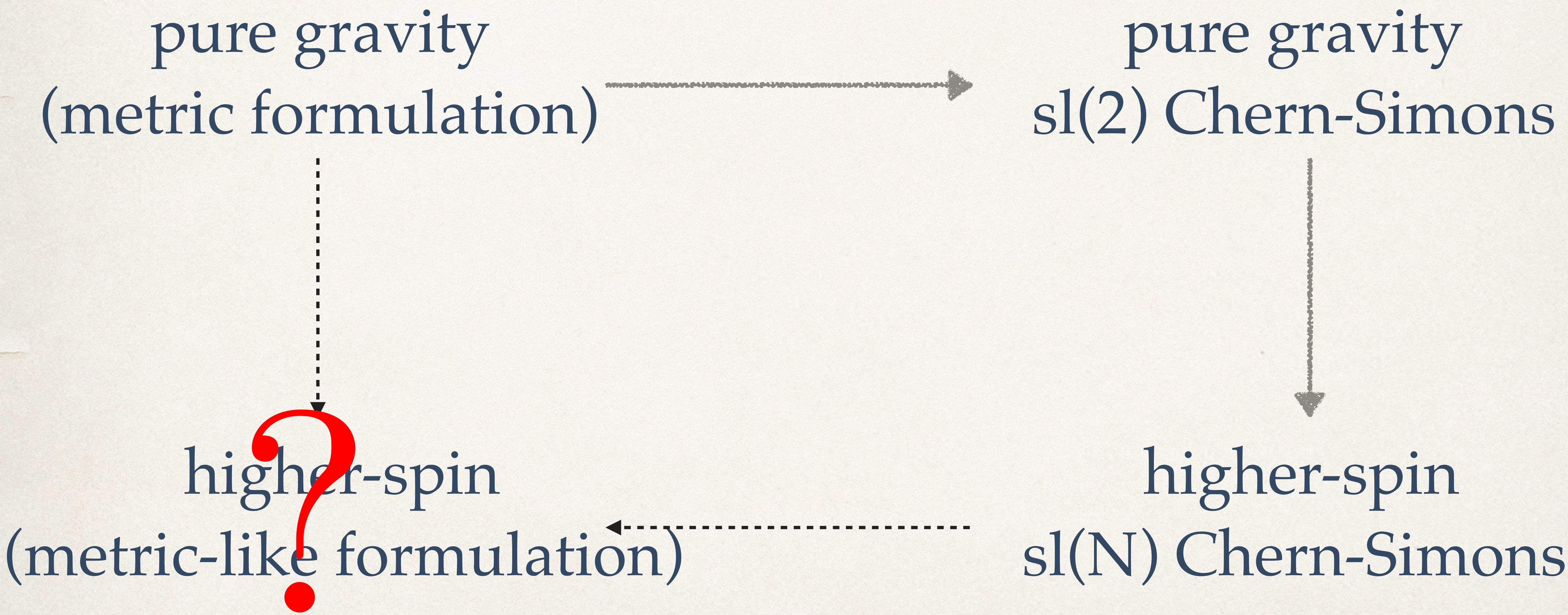
Principal embedding: 1 spin-s field for each $s = 2, ..., N$

$$L_1 \quad L_0 \quad L_{-1}$$

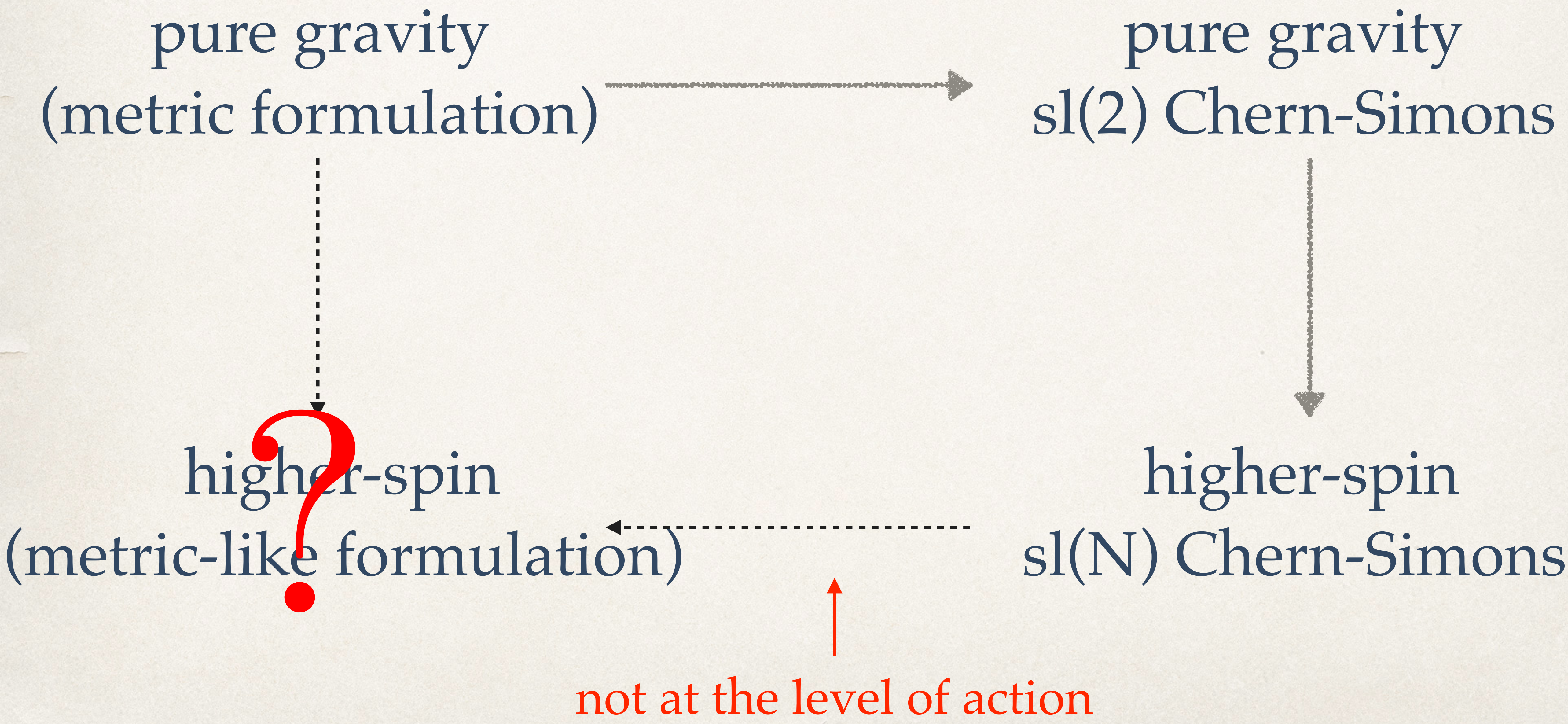
$$W_2 \quad W_1 \quad W_0 \quad W_{-1} \quad W_{-2}$$

lowest weight/zero/highest weight modes

From Chern-Simons to metric-like formulation



From Chern-Simons to metric-like formulation



From Chern-Simons to metric-like formulation

Li Theisen '15

PBH procedure in pure gravity (metric formulation) → PBH procedure in sl(2) Chern-Simons

↓

PBH procedure in sl(N) Chern-Simons

From Chern-Simons to metric-like formulation

Li Theisen '15

PBH procedure in pure gravity (metric formulation)

→

PBH procedure in sl(2) Chern-Simons

↓

PBH procedure in sl(N) Chern-Simons

↓

Weyl and W-Weyl anomalies in sl(N) Chern-Simons

PBH procedure for $\mathfrak{sl}(2)$

Step-1: Fefferman-Graham gauge of $\mathfrak{sl}(2)$ (i.e. pure gravity)

$$\begin{cases} A(\rho, x) &= \rho^{L_0}\; a(x)\; \rho^{-L_0} - \frac{d\rho}{\rho} L_0 \\ \tilde{A}(\rho, x) &= \rho^{-L_0}\; \tilde{a}(x)\; \rho^{L_0} + \frac{d\rho}{\rho} L_0 \end{cases}$$

with $\quad \mathrm{Tr}[L_0(a - \tilde{a})] = 0$

Step-2: PBH transformation for $\mathfrak{sl}(2)$: $\quad U(\rho, x) = \rho^{L_0}\; u(x)\; \rho^{-L_0}$

with $\quad u_2 = \sigma_2 L_0$

Step-3: Weyl anomalies from PBH transformation on bulk action

$$\text{Weyl Anomaly} = c\,\mathrm{Tr}\left[L_0\left(\partial a_{\bar{z}} - \bar{\partial} a_z\right)\right]$$

Step-4: rewrite in terms of boundary data

PBH procedure for $\mathfrak{sl}(N)$

Step-1: Fefferman-Graham gauge of $\mathfrak{sl}(N)$

$$\begin{cases} A(\rho,x) & = \rho^{L_0}\ a(x)\ \rho^{-L_0} - \frac{d\rho}{\rho} L_0 \\ \tilde{A}(\rho,x) & = \rho^{-L_0}\ \tilde{a}(x)\ \rho^{L_0} + \frac{d\rho}{\rho} L_0 \end{cases}$$

with $\quad \mathrm{Tr}[L_0(a-\tilde{a})]=0 \quad$ and $\quad \mathrm{Tr}[W_0(a-\tilde{a})]=0 \quad \dots$

Step-2: PBH transformation for $\mathfrak{sl}(N)$: $\quad U(\rho,x)=\rho^{L_0}\ u(x)\ \rho^{-L_0}$

with $\quad u_2=\sigma_2 L_0 \qquad$ and $\qquad u_3=\sigma_3 W_0 \qquad \dots$

Step-3: Weyl anomalies from PBH transformation on bulk action

$$\text{Weyl Anomaly} = c\,\mathrm{Tr}\big[L_0\left(\partial a_{\bar{z}} - \bar{\partial} a_z\right)\big]$$

$$\mathcal{W}_3\text{-Weyl Anomaly} = c\,\mathrm{Tr}\big[W_0\left(\partial a_{\bar{z}} - \bar{\partial} a_z\right)\big] \quad \dots$$

Step-4: rewrite in terms of boundary data

From Chern-Simons to metric-like formulation

Li Theisen '15

PBH procedure in pure gravity (metric formulation)

→

PBH procedure in sl(2) Chern-Simons

↓

PBH procedure in sl(N) Chern-Simons

↓

Weyl and W-Weyl anomalies in sl(N) Chern-Simons

←

Weyl and W-Weyl anomalies higher-spin (metric-like formulation)

Weyl anomaly and $\mathcal{W}$-Weyl anomaly in conformal gauge

Conformal gauge

- No source turned on; $N-1$ conformal mode: Toda fields $\{\Phi_s\}$
- Effective action is local (Toda action)
- Boundary metric and spin-3 field ($\Psi_L \equiv \frac{1}{2}(\Phi_1+\Phi_2) \quad \Psi_W \equiv \frac{1}{2}(\Phi_1-\Phi_2)$)

$$g = e^{\Psi_L}\cosh(\Psi_W)\,dzd\bar{z} \qquad \text{and} \qquad \varphi = 0$$

Weyl anomaly

$$\mathcal{A}_2 = \frac{c}{6\sqrt{g}}\,\partial\bar{\partial}\Psi_L$$

$\mathcal{W}$-Weyl anomaly

$$\mathcal{A}_3 = \frac{c}{18\sqrt{g}}\,\partial\bar{\partial}\Psi_W$$

Weyl anomaly and $\mathcal{W}$-Weyl anomaly in lightcone gauge

Light-cone gauge

- Turn on (chiral) sources μ_2, μ_3 $\qquad (\bar{\mu}_s = 0)$

$$T_{zz} = \frac{\delta}{\delta\mu_2} \qquad \text{and} \qquad W_{zzz} = \frac{\delta}{\delta\mu_3}$$

- Boundary metric and spin-3 field

$$g = (dz + \mu_2 d\bar{z})\, d\bar{z} \qquad \text{and} \qquad \varphi = \mu_3\, d\bar{z}^3$$

Weyl anomaly

$$\mathcal{A}_2 = \frac{c}{3}\, \partial^2 \mu_2$$

$\mathcal{W}$-Weyl anomaly

$$\mathcal{A}_3 = \frac{c}{18}\, \partial(\partial^2 - T)\mu_3$$

	Weyl anomaly	W-Weyl anomaly
pure gravity *metric*	$-\frac{c}{12}R[g]$	
pure gravity *sl(2)*	$c\,\mathrm{Tr}\left[L_0\left(\partial a_{\bar z}-\bar\partial a_z\right)\right]$	
higher-spin *sl(N)*	$c\,\mathrm{Tr}\left[L_0\left(\partial a_{\bar z}-\bar\partial a_z\right)\right]$	$c\,\mathrm{Tr}\left[W_0\left(\partial a_{\bar z}-\bar\partial a_z\right)\right]$
higher-spin *metric-like*	conformal $\frac{c}{6\sqrt{g}}\,\partial\bar\partial\Psi_L$ light-cone $\frac{c}{3}\,\partial^2\mu_2$	conformal $\frac{c}{18\sqrt{g}}\,\partial\bar\partial\Psi_W$ light-cone $\frac{c}{18}\,\partial(\partial^2-T)\mu_3$
higher-spin *metric-like* *covariant*	?	?

Summary

bulk computation of boundary conformal anomalies in 2D CFT with higher-spin symmetries

- Weyl anomaly and W-Weyl anomaly
- adapt PBH procedure to sl(N) Chern-Simons theory
- conformal gauge and light-cone gauge

Future

- translate anomalies into (covariant expression of) metric and higher-spin fields
- effective action in terms of metric and higher-spin fields (generalization of Polyakov action to higher-spin)
- 4d?

Thank You !