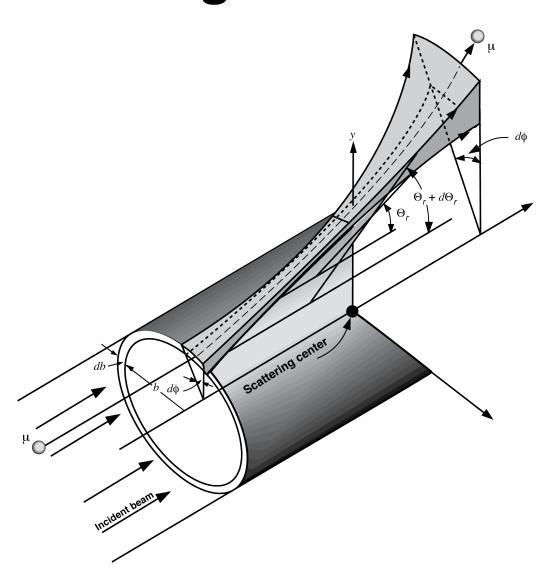
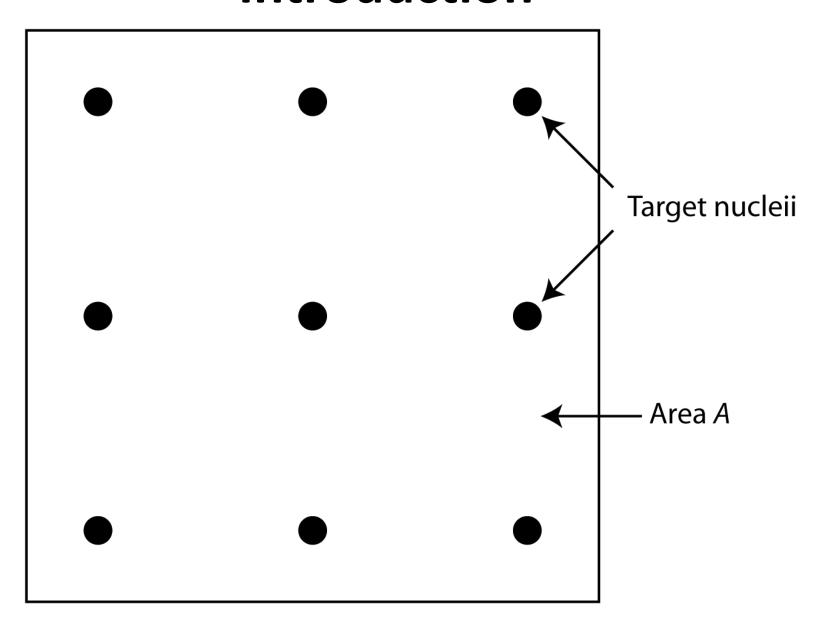
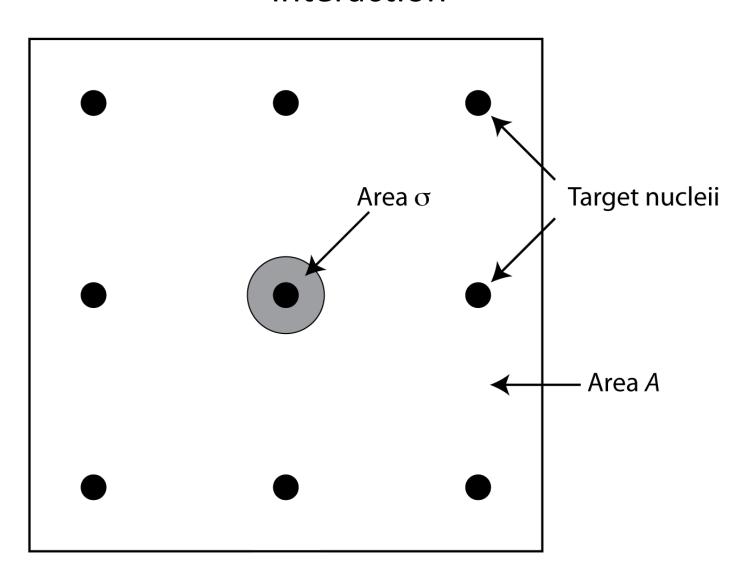
Part II Introduction to Reaction and Scattering Cross-Sections



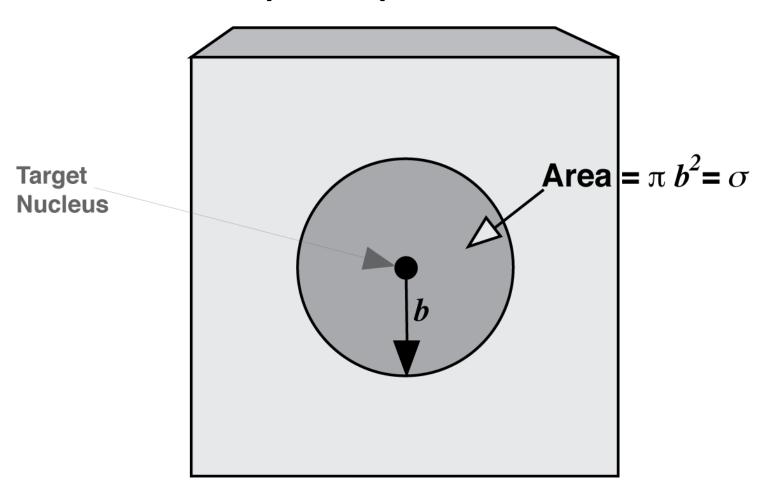
1. Reaction Cross-Section Introduction



The cross-section, σ , is the area that one atom presents to a beam of incident particles for initiating a specific "interaction"



The impact parameter, b



$$\sigma \left[\text{cm}^2 \right] = \pi \left(b \left[\text{cm} \right] \right)^2$$
$$\sigma = \pi b^2$$

Transmutation reactions

	Be6 5.0E-21 s	Be7 3/- 53.28 d	Be8 ~7E-17 s	Be9 ^{3/-}	Be10 1.6E6 a
	2 p, α	$\frac{\epsilon}{\gamma}$ 477.6 $\sigma_{\rm p}$ 3.9E4, 2E4 σ_{α} .14, .06	2 α . 0461	σ ₇ 8 mb, 4 mb	β⁻ .556 noγ Ĝ _γ <1 mb
	6.01972	E .862	8.0053051	9.0121822	E .5561
	Li5 3/7 ~3E-22 s	Li6 ¹⁺ 7.5*	Li7 3/- 92.5*	Li8 ²⁺ 0.84 s	Li9 3/- 177 ms
	ρ, α	σ _α 941, 423 σ̂ _γ 39 mb, 17 mb	σ _γ .045, 020	β ⁻ 13 (2α) 1.57	β- 13.5, 11.0, ··· (n) .3, ··· (2α) .7, ···
L.	5.01254	6.015122	7.016003	E 16.004	E 13.606
He3 1/+ 0,000138	He4 99.999862	He5 3/- 7.6E-22 s	He6 807 ms	He7 (3/)- 3E-21 s	He8 119 ms
σ _p 5.33E3, 2.40E3		n, α	β- 3.510 noγ	n .	β ⁻ 10.,··· γ 980.7 (n) .61-3.0
σ_{γ}^{ν} .05 mb 3.01602930	σ ₇ 0 4.00260323	5.01222	E 3.507	7.02803	tω E 10.65
H2 1+	H3 1/+				· 50
0.015	12.3 a				
0.015	β ⁻ .0186 noγ				6
	1107				
σy .52 mb, .23 mb	$\overline{\sigma}_{\gamma}$ < 6 μ b				

$${}_{3}^{6}\text{Li} + {}_{0}^{1}n \rightarrow {}_{2}^{4}\text{He} + {}_{1}^{3}\text{H}$$

$${}_{3}^{6}\text{Li} (n,\alpha) {}_{1}^{3}\text{H}$$

Li transmutation via thermal *n* capture

$$\sigma_{\alpha} = 941 \text{ [barns]} = 941 \cdot 10^{-24} \text{ [cm}^2\text{]}$$

$$b_{\alpha} = \sqrt{\frac{\sigma_{\alpha}}{\pi}} = 1.73 \cdot 10^{-11} \text{ [cm]} = 1.73 \cdot 10^{-13} \text{ [m]} = 173 \text{ [fm]}$$

How large is 173 fm?

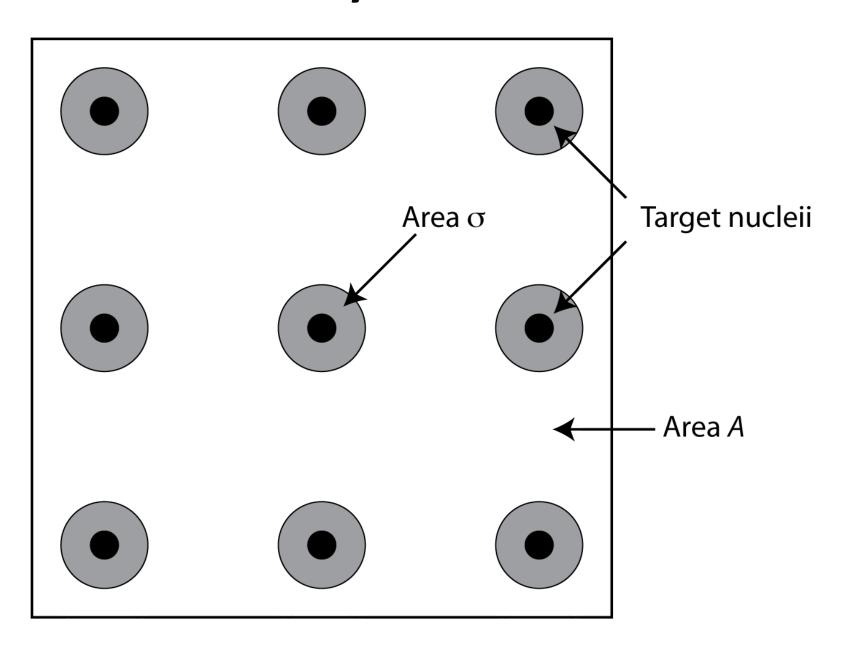
$$R = 1.25 \cdot 10^{-5} M_T^{1/3} [\text{Å}]$$

$$M_T(^6\text{Li}) = 6.01522 \text{ [amu]}$$

$$R = 2.27332 \cdot 10^{-5} \, \text{[Å]}$$

$$= 2.27332 \cdot 10^{-5} \, [\text{Å}] = 2.27 \, [\text{fm}]$$

Reaction Cross-Section(s). Relationship to the Probability of Interaction



Fluence

$$\Phi \left[\frac{\text{interactions}}{\text{cm}^2} \right] = \Phi_0 \left[\frac{\text{projectile particles}}{\text{cm}^2} \right] \hat{\rho} \left[\frac{\text{target atoms}}{\text{cm}^2} \right] \quad \sigma \left[\frac{\text{cm}^2}{\text{target atom}} \right]$$

$$\Phi = \Phi_0 \hat{\rho} \sigma$$

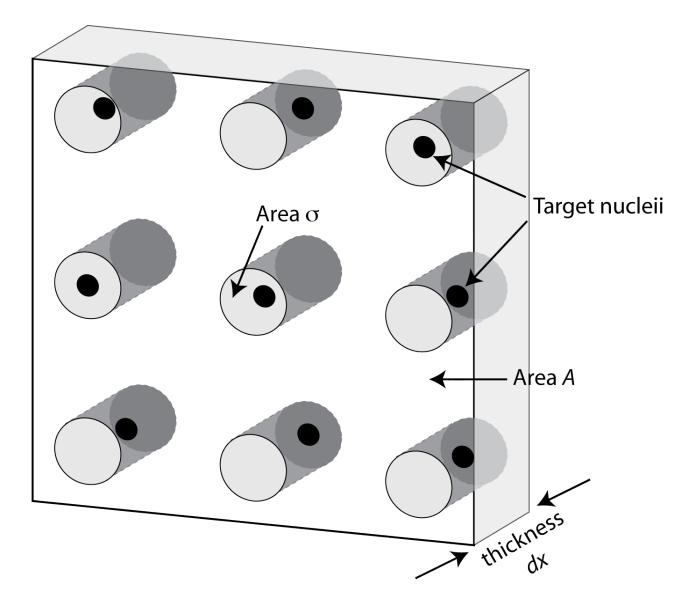
Probability vs. Cross-Section

Probability of Interaction =
$$\frac{\Phi\left[\frac{\text{interactions}}{\text{cm}^{2}}\right]}{\Phi_{0}\left[\frac{\text{projectile particles}}{\text{cm}^{2}}\right]}$$

$$= \hat{\rho}\left[\frac{\text{target atoms}}{\text{cm}^{2}}\right] \quad \sigma\left[\frac{\text{cm}^{2}}{\text{target atom}}\right]$$

$$P = \frac{\Phi}{\Phi_{0}} = \hat{\rho} \sigma$$

3-D



$$\Phi\left[\frac{\text{interactions}}{\text{cm}^{2}}\right] = \Phi_{0}\left[\frac{\text{projectile particles}}{\text{cm}^{2}}\right]\rho\left[\frac{\text{target atoms}}{\text{cm}^{3}}\right] \cdot dx \text{ [cm] } \sigma\left[\frac{\text{cm}^{2}}{\text{target atom}}\right]$$

$$\Phi = \Phi_{0}\left(\rho \cdot dx\right)\sigma$$

3-D

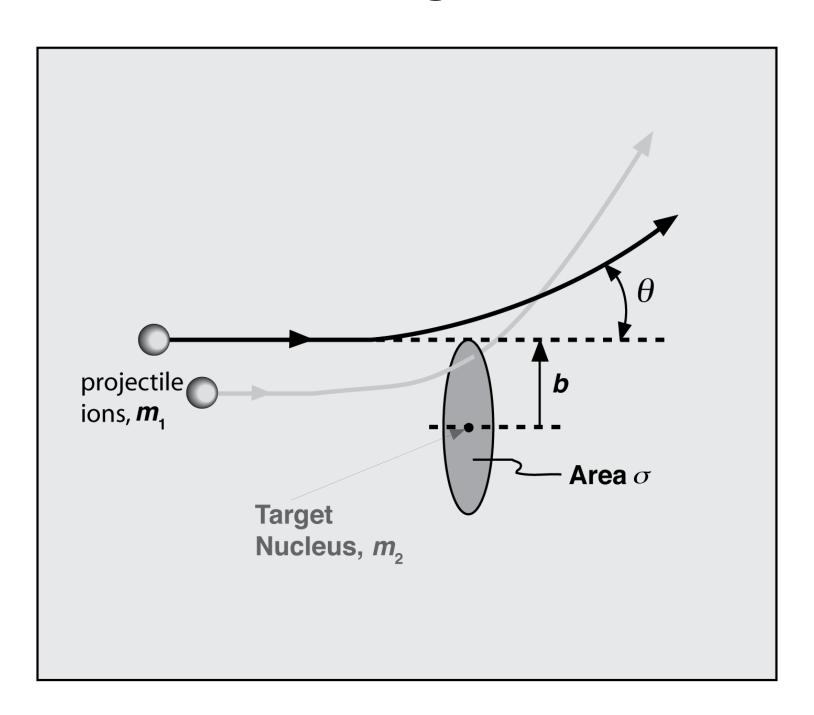
Probabilty of Interaction

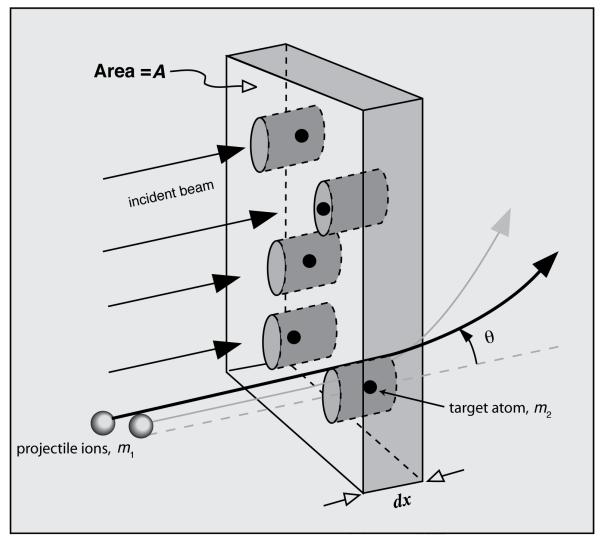
$$=\frac{\Phi\left[\frac{\text{interactions}}{\text{cm}^2}\right]}{\Phi_0\left[\frac{\text{projectile particles}}{\text{cm}^2}\right]}$$

$$= \rho \left[\frac{\text{target atoms}}{\text{cm}^3} \right] \cdot dx \text{ [cm]} \quad \sigma \left[\frac{\text{cm}^2}{\text{target atom}} \right]$$

$$P = \frac{\Phi}{\Phi_0} = (\rho \cdot dx) \, \sigma$$

The Total Scattering Cross-Section, σ





$$\Phi\begin{bmatrix} \frac{\text{projectile ions}}{\text{scattered to}} \\ \frac{\theta \le \theta \le \pi}{\text{cm}^2} \end{bmatrix} = \Phi_0 \begin{bmatrix} \frac{\text{projectile ions}}{\text{cm}^2} \end{bmatrix} \rho \begin{bmatrix} \frac{\text{target nuclei}}{\text{cm}^3} \end{bmatrix} \cdot dx \text{ [cm]} \quad \sigma \begin{bmatrix} \frac{\text{cm}^2}{\text{target nucleus}} \end{bmatrix}$$

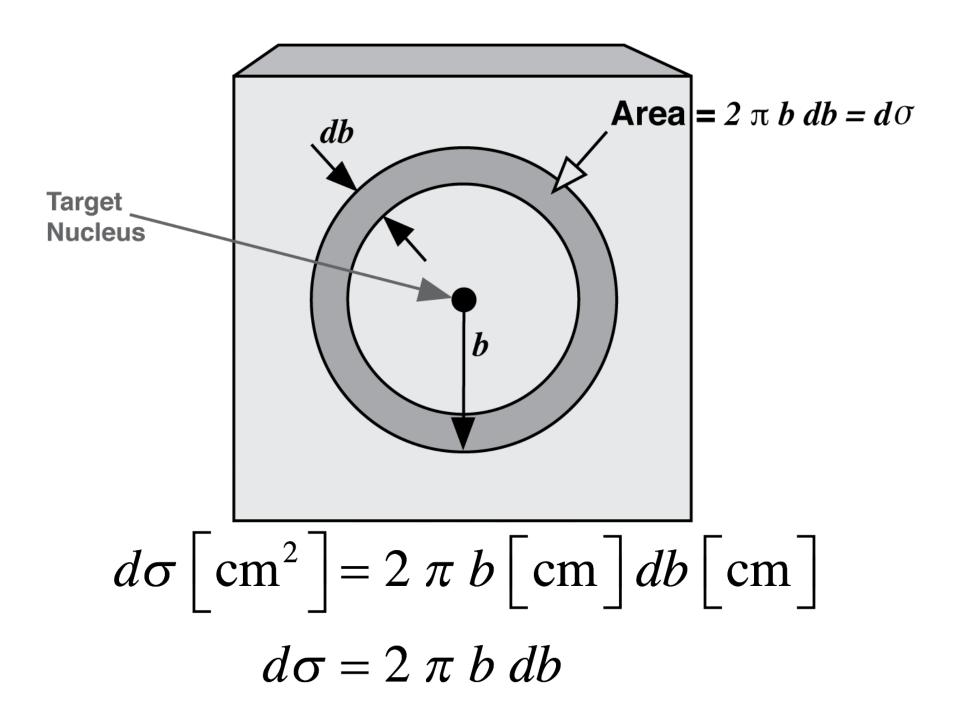
$$\Phi = \Phi_0 \left(\rho \cdot dx \right) \sigma$$

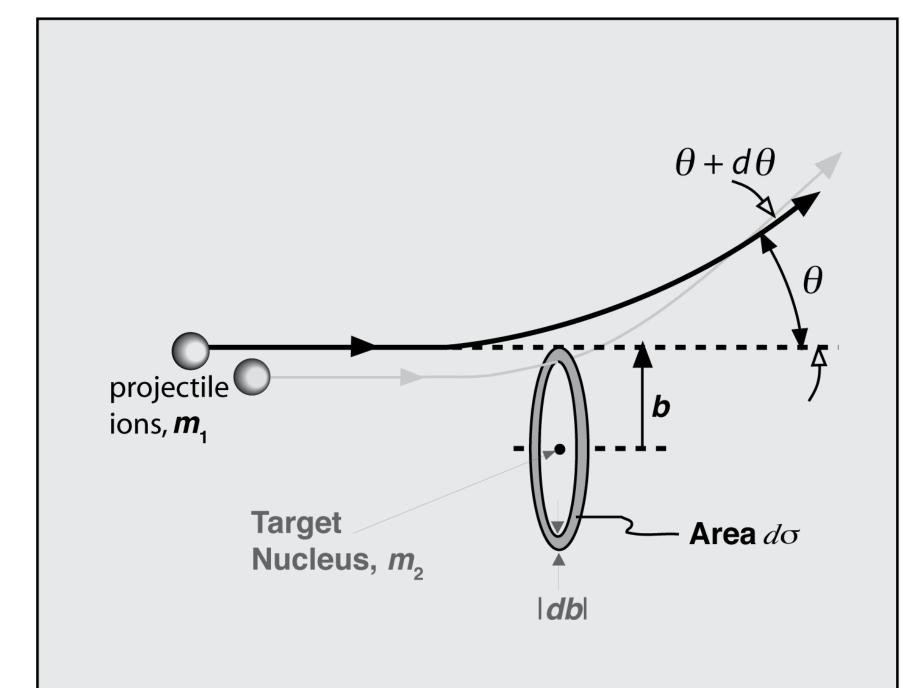
Probabilty of Scattering to $\theta \leq \theta \leq \pi$

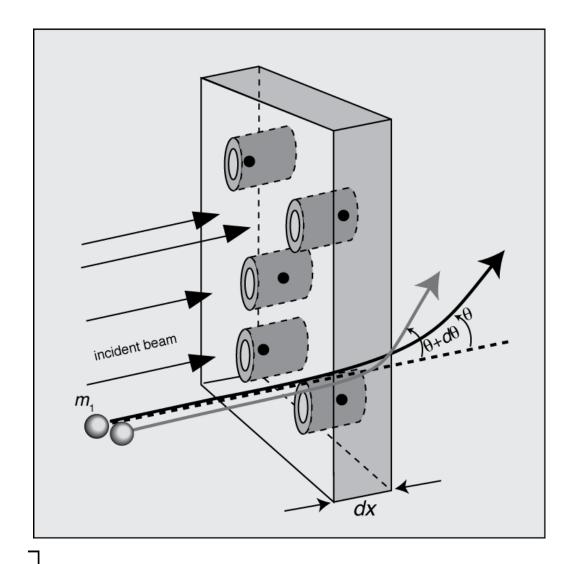
$$\Phi \begin{bmatrix}
\frac{\text{projectile ions}}{\text{scattered to}} \\
\frac{\theta \le \theta \le \pi}{\text{cm}^2}
\end{bmatrix} = \rho \begin{bmatrix}
\frac{\text{target nuclei}}{\text{cm}^3}
\end{bmatrix} \cdot dx \text{ [cm]} \quad \sigma \begin{bmatrix}
\frac{\text{cm}^2}{\text{target nucleus}}
\end{bmatrix}$$

$$P = \frac{\Phi}{\Phi_0} = (\rho \cdot dx) \,\sigma$$

The Differential Scattering Cross-Section, $d\sigma/d\Omega$







$$d\Phi \begin{vmatrix} \text{projectile ions} \\ \frac{\theta \le \theta \le \theta + d\theta}{\text{cm}^2} \end{vmatrix} = \Phi_0 \left[\frac{\text{projectile ions}}{\text{cm}^2} \right] \rho \left[\frac{\text{target nuclei}}{\text{cm}^3} \right] \cdot dx \text{ [cm]} \quad d\sigma \left[\frac{\text{cm}^2}{\text{target nucleus}} \right]$$

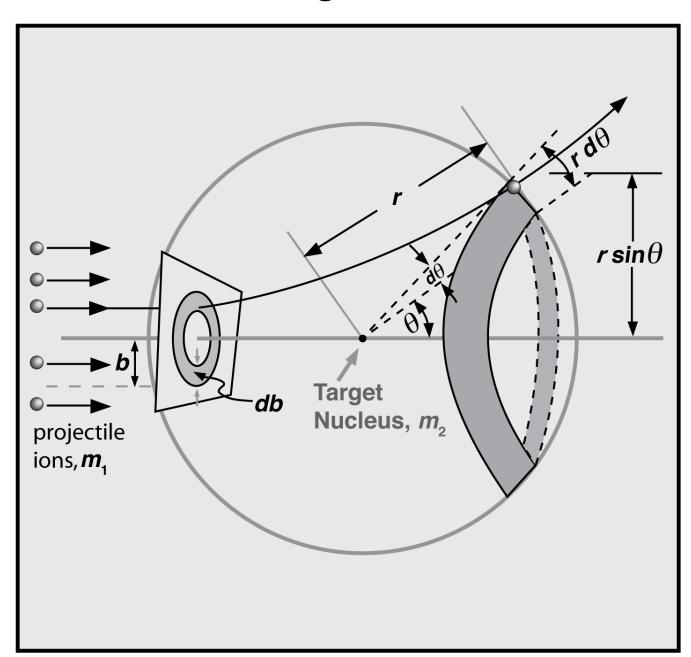
$$d\Phi = \Phi_0 \left(\rho \cdot dx \right) d\sigma$$

Differential Probabilty of Scattering to $\theta \le \theta \le \theta + d\theta$

$$d\Phi = \frac{d\Phi \left[\begin{array}{c} \frac{\text{projectile ions}}{\text{scattered to}} \\ \frac{\theta \le \theta \le \theta + d\theta}{\text{cm}^2} \end{array}\right]}{\Phi_0 \left[\begin{array}{c} \frac{\text{projectile ions}}{\text{2}} \end{array}\right]} = \rho \left[\begin{array}{c} \frac{\text{target nuclei}}{\text{cm}^3} \end{array}\right] \cdot dx \text{ [cm]} \quad d\sigma \left[\begin{array}{c} \frac{\text{cm}^2}{\text{target nucleus}} \end{array}\right]$$

$$P(\theta, \theta + d\theta) = \frac{d\Phi}{\Phi_0} = (\rho \cdot dx) d\sigma$$

Solid angle, $d\Omega$, associated with scattering in 3-D, to angles between θ and θ + $d\theta$



$$\frac{d\Omega}{4\pi} = \frac{2\pi (r \sin \theta)(r d\theta)}{4\pi r^2}$$
$$d\Omega = 2\pi \sin \theta d\theta$$

Fractional ion fluence scattered through angle θ and within solid angle Ω

$$d\Phi\begin{bmatrix} \frac{\text{projectile ions}}{\text{scattered to}} \\ \frac{\theta \leq \theta \leq \theta + d\theta}{\text{cm}^2} \end{bmatrix} = \rho \begin{bmatrix} \frac{\text{target nuclei}}{\text{cm}^3} \end{bmatrix} \cdot dx \text{ [cm]} \int_{\Omega} \frac{d\sigma}{d\Omega} \begin{bmatrix} \frac{\text{cm}^2}{\text{target nucleus} \cdot \text{steradian}} \end{bmatrix} d\Omega \begin{bmatrix} \text{steradians} \end{bmatrix}$$

$$P(\Omega(\theta)) = \frac{d\Phi}{\Phi_0} = (\rho \cdot dx) \int_{\Omega} \frac{d\sigma}{d\Omega} d\Omega$$

Rutherford backscattering (RBS) detector:

Fractional ion fluence scattered through angle θ and within solid angle Ω

$$P(\Omega(\theta)) = \frac{d\Phi}{\Phi_0} = (\rho \cdot dx) \frac{d\sigma}{d\Omega} \Omega$$

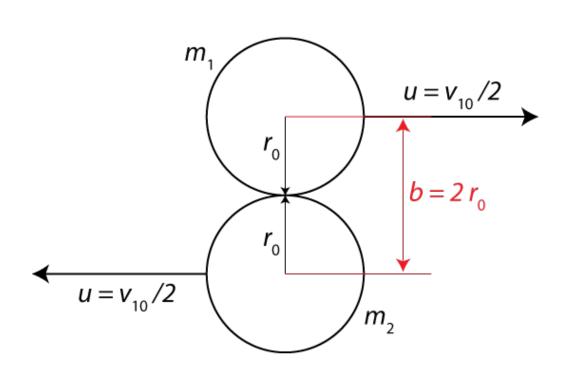
The Differential Energy Transfer Cross-Section, $d\sigma(E,T)/dT$

$$d\sigma(b)/db \rightarrow d\sigma(E,\theta)/d\Omega \rightarrow d\sigma(E,T)/dT$$

State (without proof) an important relationship:

$$\frac{d\sigma(E,T)}{dT} = \frac{4\pi}{\Lambda E} \frac{d\sigma(E,\theta(T))}{d\Omega}$$

Cross-Sections for a Hard-Sphere Interaction Potential



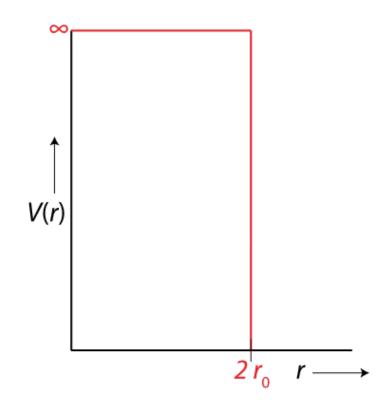


Figure 13.

$$\sigma = \pi b^2$$
 \rightarrow $d\sigma = 2 \pi b db$ \rightarrow

$$\frac{d\sigma(b)}{db} = 2 \pi b$$

Hard sphere – scattering is isotropic in the *CM* system

$$\frac{d\sigma(E,\theta(T))}{\sigma(E)} = \frac{d\Omega(\theta)}{4 \pi} = \frac{2 \pi \sin\theta d\theta}{4 \pi}$$

Hard-sphere Differential energy transfer cross-section

$$\frac{d\sigma(E,T)}{dT} = \frac{4 \pi}{\Lambda E} \frac{d\sigma(E,\theta(T))}{d\Omega}$$
$$= \frac{4 \pi}{\Lambda E} \frac{\sigma(E)}{4 \pi}$$

$$\frac{d\sigma(E,T)}{dT} = \frac{\sigma(E)}{\Lambda E}$$

Cross-Sections for a *Rutherford*Interaction Potential

$$V(r) = \frac{K}{r}$$

$$K = (Z_1 e) (Z_2 e)$$

$$= Z_1 Z_2 e^2$$

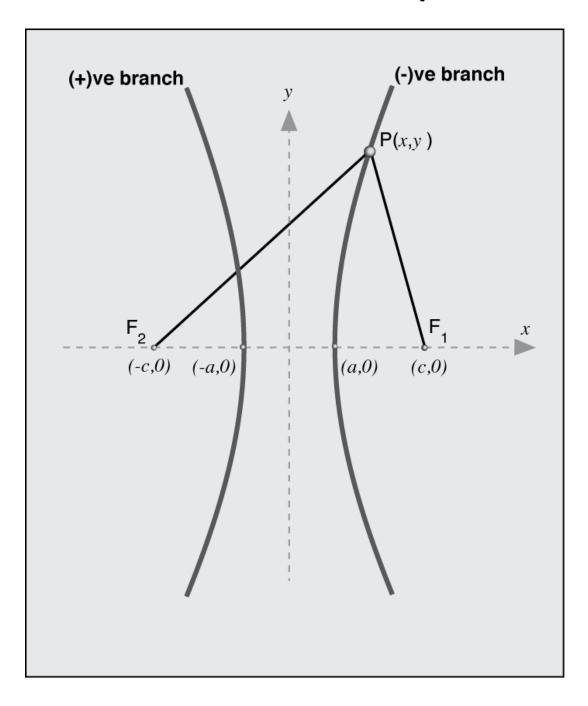
Cross-Sections for a *Rutherford*Interaction Potential

$$d\sigma(b)/db \rightarrow d\sigma(E,\theta)/d\Omega \rightarrow d\sigma(E,T)/dT$$

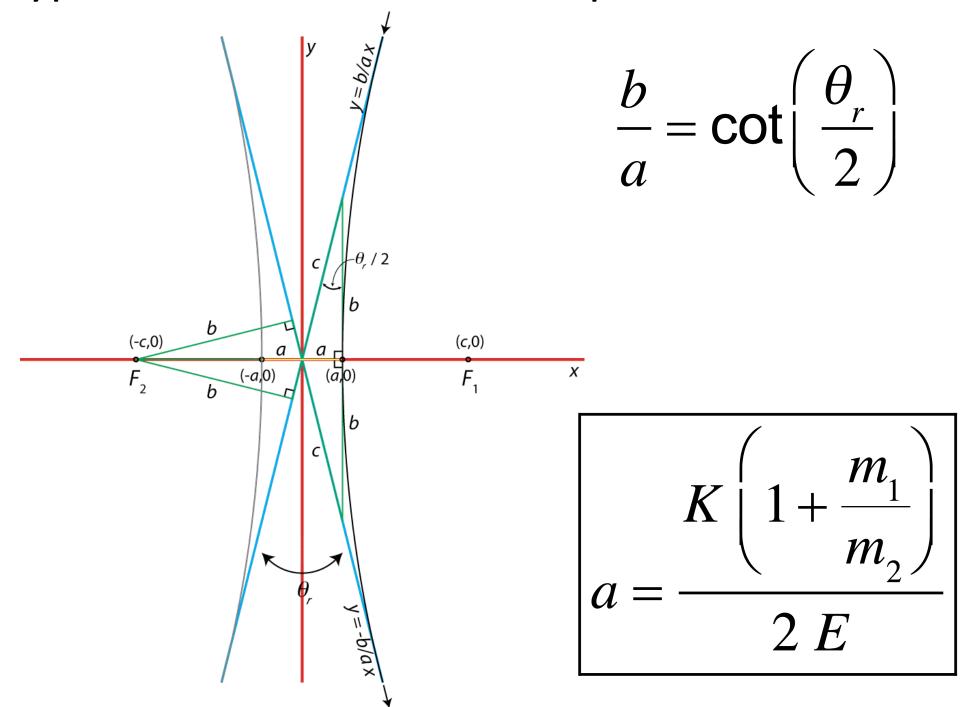
$$\left| \frac{d\sigma(b)}{db} = 2 \pi b \right|$$

Next, we require an equation that provides a one-to-one mapping of

Hyperbolic solution to the equation of motion



Hyperbolic solution to the equation of motion



Rutherford cross-section for angular scattering

$$\frac{d\sigma(E,\theta)}{d\Omega} = \left(\frac{K\left(1 + \frac{m_1}{m_2}\right)}{4E}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

Rutherford cross-section for energy transfer

$$d\sigma(E,\theta)/d\Omega \rightarrow d\sigma(E,T)/dT$$

$$\left| \frac{d\sigma(E,T)}{dT} = \frac{\pi K^2 x}{E T^2} \right|$$

where
$$x = m_1/m_2$$
 and $K = Z_1 Z_2 e^2$

Rutherford scattering: Differential probability of energy transfer between T and T+dT

$$P(T,T+dT) = (\rho dx) \frac{d\sigma(E,T)}{dT} dT$$
$$= (\rho dx) \frac{\pi K^2 x}{E T^2} dT$$

$$\sigma(E) = \int_{T=0}^{T=\Lambda E} d\sigma(E,T)$$

$$= \int_{T=0}^{T=\Lambda E} \frac{\pi K^2 x}{E} \frac{dT}{T^2}$$

$$= \frac{\pi K^2 x}{E} \int_{T=0}^{T=\Lambda E} \frac{dT}{T^2}$$

$$= \frac{\pi K^2 x}{E} \int_{T=0}^{T=\Lambda E} \frac{dT}{T^2}$$

$$= -\frac{\pi K^2 x}{E} \frac{1}{T} \Big|_{T=0}^{T=\Lambda E}$$

$$= \frac{\pi K^2 x}{E} \left(\frac{1}{0} - \frac{1}{\Lambda E}\right)$$

The solution: The Screened Coulomb Potential

$$V(r) = \frac{K}{r} e^{-\frac{r}{\hat{a}}}$$