Nuclear structure studies of highspin states with large arrays. Lecture 1: at the Joint ICTP-IAEA Workshop on Nuclear Data : Experiment, Theory and Evaluation Miramare, Trieste, Italy, August 2016 Paddy Regan Department of Physics, University of Surrey Guildford, GU2 7XH Å National Physical Laboratory, Teddington, UK p.regan@surrey.ac.uk; paddy.regan@npl.co.uk



Coexistence of collective and noncollective motion



Energy levels are determined by measuring gamma-rays decaying from excited states.

Many, many possible states can be populated...many different gamma-ray energies need to be measured at the same time (in coincidence).

(LN₂ cooled) germanium detectors have the combination of good energy resolution ($\Delta E \sim 2 \text{ keV} \otimes E_{\gamma}=1 \text{ MeV}$) and acceptable detection efficiency.

Various multi-detector 'arrays' of germanium detectors around the World. e.g., GAMMASPHERE, MINIBALL, GaSp YRASTBALL, JUROGAM, RISING, INGA, EXOGAM, AGATA, GRETINA.

Fusion-evaporation reactions best way to make the highest spins. Nuclear EM decay usually decay via 'near yrast' sequence (since decay prob ~ $E\gamma^{2L+1}$)

Compton Suppressed Arrays

For the last ~ 15 - 20 years, large arrays of Compton-suppressed Ge detectors such as EuroBall, JUROBALL, GASP, EXOGAM, TIGRESS, INGA, Gammasphere and others have been the tools of choice for nuclear spectroscopy.



EUROBALL



INGA



Gammasphere

Angular Momentum World of the Nucleus



Figure; originated from Prof. M.A.Riley (FSU)

Different nuclear reaction mechanisms?

- Heavy-ion fusion-evaporation reactions (makes mostly neutron-deficient residual nuclei).
- Spontaneous fission sources such as ²⁵²Cf (makes mostly neutron-rich residual nuclei).
- Deep-inelastic/multi-nucleon transfer and heavy-ion fusionfission reactions (makes near-stable/slightly neutron-rich residual nuclei).
- High-energy Projectile fragmentation / projectile fission at e.g., GSI, RIKEN, GANIL, MSU.
- Coulomb excitation, EM excitations via E2 (usually).
- Single particle transfer reactions (p,d)
- Beta decay ; alpha decay ; proton radioactivity
- Other probes (e,e'γ), (γ,γ'), (n,γ), (p,γ), (n,n'γ) etc.
 First four generally populate <u>'near-yrast'</u> states

 most useful to see 'higher' spins states and excitations.

Heavy-ion induced nuclear reactions on fixed targets can result in a range of different nuclear reactions taking place.



(2) the beam energy (higher or lower than the Coulomb repulsion between the two nuclei), and (3) the impact parameter, b.









Example: ⁹⁶Ru(⁴⁰Ca,xpyn)¹³⁶Gd-xp-yn





Hot, compound system recoils backwards at 0° in the lab frame. Example: ⁹⁶Ru(⁴⁰Ca,xpyn)¹³⁶Gd-xp-yn









Production of High-Spin States

$$E_{ex} = E_{cm} + Q_{fus} \tag{2.1.1}$$

 E_{cm} is the kinetic energy of the collision which is transferred to the compound system. It can be calculated by taking the kinetic energy of the beam, E_B and subtracting the kinetic energy of the recoiling compound system, E_R . Thus

$$E_{cm} = E_B - E_R \tag{2.1.2}$$

By conservation of momentum, for beam and target masses of M_B and M_T respectively, the velocity of the recoiling compound, V_R can be calculated using

$$M_B V_B = (M_T + M_B) V_R (2.1.3)$$

and by conservation of energy,

$$E_{cm} = E_B - \frac{1}{2} \left(M_T + M_B \right) V_R^2 \tag{2.1.4}$$

substituting in for V_R , and recalling that $E_B = \frac{1}{2}M_B V_B^2$, we obtain

$$E_{cm} = E_B \left(1 - \frac{M_B}{M_T + M_B} \right) \tag{2.1.5}$$



Doppler Shifts

germanium

θ

 $\Delta \theta \Delta \theta$

detector

Moving source - nucleus which emits gamma-ray ; Stationary observer - Ge detector.

$$E_S = E_0 \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \theta} \approx E_0 (1 + \beta \cos \theta)$$

The range in Doppler shifted energy across the finite opening angle of a detector ($\Delta \theta$) Causes a reduction in measured energy resolution due to Doppler Broadening.

This is made worse if there is also a spread in the recoil velocities (Δv) for the recoils.

$$\Delta E_s \approx E_o \cos\theta \frac{\Delta v}{c} - E_o \frac{v}{c} \sin\theta \Delta\theta$$

beam direction, velocity, v















Proton Number



Proton Number

<u>Do you evaporate protons or neutrons?</u>



<u>Do you evaporate protons or neutrons?</u>



<u>Near stable (compound) nuclei, Sp ~ Sn ~ 5-8 MeV.</u> Coulomb barrier means (HI,xn) favoured over (HI,xp)



Angular Momentum Input in HIFE Reactions?

$$\hbar l_{max} = \mu v R$$

$$\frac{1}{2}\mu v^2 = E_{cm} - V_c$$

Reduced mass of system, $\mu = m_b.m_T / (m_B+m_T)$

$$l_{max}^2 = \frac{2\mu R^2}{\hbar^2} \left(E_{cm} - V_c \right)$$

⁹⁸Mo + ${}^{12}C \rightarrow {}^{110}Cd$ fusion evaporation calculations using PACE4 S.F. Ashley, PhD thesis, University of Surrey (2007)



Increasing the beam energy increases the maximum input angular momentum,

but

Causes more nucleons to be evaporated (on average).

Also, increasing the beam energy increases the recoil velocity.

For the ¹¹⁰Cd compound nucleus: $S_n = 9.9 \text{ MeV}$ $S_p = 8.9 \text{ MeV}$

Coulomb barrier means neutron evaporation is much favoured.







Excitation function for various products of the reaction ${\rm ^{18}O+^{96}Zr}$

P.H. Regan et al., Phys. Rev. <u>C49</u> (1994) 1885



The transition probability for a state decaying from state J_i to state J_f , separated by energy E_{γ} , by a transition of multipole order L is given by [1, 7]

$$T_{fi}(\lambda L) = \frac{8\pi (L+1)}{\hbar L \left((2L+1)!! \right)^2} \left(\frac{E_{\gamma}}{\hbar c} \right)^{2L+1} B(\lambda L : J_i \to J_f)$$
(1.1.2)

where $B(\lambda L: J_i \to J_f)$ is called the *reduced matrix element*.

DCO and DCO Ratios

JULY 15, 1940

PHYSICAL REVIEW

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On Directional Correlation of Successive Quanta

DONALD R. HAMILTON* Harvard University, Cambridge, Massachusetts (Received May 6, 1940)

A theoretical investigation shows that there should be a correlation between the directions of propagation of the quanta emitted in two successive transitions of a single radiating system. This correlation is described by a function $W(\theta)$ which gives the relative probability that the second quantum will be emitted at an angle θ with the first; W is determined by the angular momenta of the three levels involved in the two transitions and by the multipole order of the radiation emitted in these transitions. The explicit

forms of W for all angular momenta and for dipole and quadrupole radiation are given; experimental determination of W in any given case should limit these factors to a small number of possibilities. This has particular interest as a means of investigating the nuclear energy levels involved in γ -radiation; here W should be observable by measuring the variation with θ of gamma-gamma coincidence counting rates.



Dropping a constant factor, our correlation function for calculation is '

$$\mathbf{W}(\boldsymbol{\theta}) = \mathbf{1} + (\mathbf{R}/\mathbf{Q}) \cos^2 \boldsymbol{\theta} + (\mathbf{S}/\mathbf{Q}) \cos^4 \boldsymbol{\theta}.$$

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TABLE 1. R/Q, for both transitions dipole.

	$\Delta J = -1$	$\Delta J = 0$	$\Delta J = 1$
$\Delta j = -1$	$\frac{1}{13}$	$\frac{-(2J-1)}{(14J+13)}$	$\frac{J(2J-1)}{(26J^2+67J+40)}$
$\Delta j = 0$	$\frac{-(2J+3)}{(14J+1)}$	$\frac{(2J-1)(2J+3)}{(12J^2+12J+1)}$	
$\Delta j = 1$	$\frac{(J+1)(2J+3)}{(26J^2-15J-1)}$		

TABLE II. R/Q, for first transition quadrupole, second dipole.

	$\Delta J = 1$	$\Delta J = 0$	$\Delta J = -1$
$\Delta j = 2$	$\frac{-3}{29}$	$\frac{3(2J+3)}{(26J-3)}$	$\frac{-3(J+1)(2J+3)}{(58J^2-23J+3)}$
$\Delta j = 1$	$\frac{3(J-5)}{(55J+61)}$	$\frac{-3(2J+3)(J-5)}{(58J^2+49J-15)}$	$\frac{3(2J+3)(J-5)}{(110J^2-49J+15)}$
$\Delta j = 0$	$\frac{(2J-3)(2J+5)}{(36J^2+92J+61)}$	$\frac{-(2J-3)(2J+5)}{5(4J^2+4J-1)}$	$\frac{(2J-3)(2J+5)}{(36J^2-20J+5)}$
$\Delta j = -1$	$\frac{3(2J-1)(J+6)}{(110J^2+269J+174)}$	$\frac{-3(2J-1)(J+6)}{(58J^2+67J-6)}$	$\frac{3(J+6)}{(55J-6)}$
$\Delta j = -2$	$\frac{-3J(2J-1)}{(58J^2+139J+84)}$	$\frac{3(2J-1)}{(26J+29)}$	$\frac{-3}{29}$

Dropping a constant factor, our correlation function for calculation is '

$\mathbf{W}(\boldsymbol{\theta}) = \mathbf{1} + (\mathbf{R}/\mathbf{O}) \cos^2 \boldsymbol{\theta} + (\mathbf{S}/\mathbf{O}) \cos^4 \boldsymbol{\theta}.$

DIRECTIONAL CORRELATION OF QUANTA

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 $\overline{(2J-1)(16J^3-42J^2+29J+3)}$

TABLE III. R/Q , for both transitions quadrupole.				
	$\Delta J = 0$	$\Delta J = -1$	$\Delta J = -2$	
$\Delta j = -2$	$\frac{-(2J-3)(2J+1)}{(2J+3)(6J+5)}$	$\frac{(J+3)}{(17J+15)}$	$\frac{1}{8}$	
$\Delta j = -1$	$\frac{(5J-2)(2J-3)(2J+5)}{(20J^3+52J^2+41J+6)}$	$\frac{-(17J^2+17J-30)}{(35J^2+35J+6)}$	$\frac{(J-2)}{(17J+2)}$	
$\Delta j = 0$	$\frac{-(2J-3)(4J^2+4J-7)(2J+5)}{(2J-1)(2J+3)(4J^2+4J-1)}$	$\frac{(5J+7)(2J-3)(2J+5)}{(20J^3+8J^2-3J+3)}$	$\frac{-(2J+1)(2J+5)}{(2J-1)(6J+1)}$	
$\Delta j = 1$		$\frac{-(2J+3)(17J^3+69J^2-77J-105)}{(70J^4-9J^3-73J^2-27J-9)}$	$\frac{(2J+3)(J^2+18J+5)}{(34J^3-57J^2+8J+3)}$	
A:2			$(J+1)(2J+3)(2J^2-9J+1)$	

 $\Delta j = 2$

	$\Delta J = 2$	$\Delta J = 1$
$\Delta j = -2$	$\frac{J(2J-1)(2J^2+13J+12)}{(2J+3)(16J^3+90J^2+161J+84)}$	$\frac{(2J-1)(J^2-16J-12)}{(34J^3+159J^2+224J+96)}$
$\Delta j = -1$		$\frac{-(2J-1)(17J^3-18J^2-164J-24)}{(70J^4+289J^3+374J^2+188J+24)}$

TABLE IV. S/Q, for both transitions quadrupole.

	$\Delta J = 0$	$\Delta J = -1$	$\Delta J = -2$
$\Delta j = -2$	$\frac{4(J-1)(2J-3)}{3(2J+3)(6J+5)}$	$\frac{-4(2J-3)}{3(17J+15)}$	$\frac{1}{24}$
$\Delta j = -1$	$\frac{-16(J-1)(2J-3)(2J+5)}{3(20J^3+52J^2+41J+6)}$	$\frac{16(2J-3)(2J+5)}{3(35J^2+35J+6)}$	$\frac{-4(2J+5)}{3(17J+2)}$
$\Delta \mathbf{j} = 0$	$\frac{16(J-1)(J+2)(2J-3)(2J+5)}{3(2J-1)(2J+3)(4J^2+4J-1)}$	$\tfrac{-16(J+2)(2J-3)(2J+5)}{3(20J^3+8J^2-3J+3)}$	$\frac{4(J+2)(2J+5)}{3(2J-1)(6J+1)}$
$\Delta j = 1$		$\frac{16(2J-3)(2J+3)(J+2)(2J+5)}{3(70J^4-9J^3-73J^2-27J-9)}$	$\frac{-4(2J+3)(J+2)(2J+5)}{3(34J^3-57J^2+8J+3)}$
$\Delta j = 2$			$\frac{(J\!+\!1)(2J\!+\!3)(J\!+\!2)(2J\!+\!5)}{3(2J\!-\!1)(16J^3\!-\!42J^2\!+\!29J\!+\!3)}$

	$\Delta J = 2$	$\Delta J = 1$	
$\Delta j = -2$	$\frac{J(2J-1)(J-1)(2J-3)}{2(2J+3)(16J+2)(16J+20)(2J+16J+24)}$	-4(2J-1)(J-1)(2J-3)	
	5(23+3)(103+903+1013+84)	$3(34J^{2}+139J^{2}+224J+90)$	
$\Delta i = -1$		16(2J-1)(J-1)(2J-3)(2J+5)	
		$3(70J^4 + 289J^3 + 374J^2 + 188J + 24)$	

JULY 15, 1940

PHYSICAL REVIEW

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On Directional Correlation of Successive Quanta

DONALD R. HAMILTON* Harvard University, Cambridge, Massachusetts (Received May 6, 1940)

First real 'evidence' of angular correlations between successive gamma rays;

Radioactive decays of

⁶⁰Co ($I^{\pi}=5^+$, $T_{1/2}=5.27$ yrs to ⁶⁰Ni) ⁴⁶Sc ($I^{\pi}=4^+$, $T_{1/2}=84$ days to 46Ti) ⁸⁸Y ($I^{\pi}=4^-$, $T^{1/2}=107$ days to ⁸⁸Sr) ¹³⁴Cs ($I^{\pi}=4^+$, $T_{1/2}=2.1$ yrs to ¹³⁴Ba).

(note, says ⁸⁶Y in paper, means ⁸⁸Y)

Angular Correlation of Successive Gamma-Ray Quanta

EDWARD L. BRADY AND MARTIN DEUTSCH Massachusetts Institute of Technology, Cambridge, Massachusetts September 10, 1947

THEORETICAL considerations^{1,2} predict a directional correlation of successive quanta of the form

$$W(\theta) = 1 + \sum_{i=1}^{l} A_i \cos^{2i\theta}$$

if 2*l* is the highest multipole order occurring. Attempts to demonstrate this effect experimentally have heretofore been inconclusive.³ We have studied coincidences between successive gamma-rays of Co⁵⁰, Sc⁴⁶, Y⁸⁶ (106 day), and Cs¹³⁴ at angles 180° and 90° between the counters, and found a pronounced anisotropy in the first two named and no correlation within the experimental error in the last two. Our results, together with the gamma-ray energies concerned, are shown in Table I. The quantity $(W(\pi)$

TABLE I. Anisotropy of gamma-ray coincidences.

Source	Coso	Sc48	Y86	Cs184
Gamma-rays	$\substack{0.21 \pm 0.025 \\ 1.1, 1.3}$	0.20±0.035 0.89, 1.12	-0.05 ± 0.03 0.91, 1.89	0.01 ±0.04 0.58, 0.78
Reference	4	5	6	7

 $-W(\pi/2))/W(\pi/2)$ should be equal to ΣA_i . Our results
JUNE 1, 1950

The probability, per unit solid angle, that two successive gamma-rays are emitted at an angle θ is proportional to

$$W(\theta) = 1 + \sum_{1}^{l} a_i \cos^{2i\theta}$$

where 2l is the order of the lowest multipole in the cascade. Thus, if both gamma-rays are quadrupoles $W(\theta) = 1 + a_1 \cos^2 \theta + a_2 \cos^4 \theta$. If one is a dipole $W(\theta)$ $=1+a_1\cos^2\theta$, etc. A further restriction on the number of terms in $W(\theta)$ is $a_i = 0$ for $i > J_2$. J_2 is the spin of the intermediate state in the cascade. Thus if J_2 is zero or $\frac{1}{2}$, the angular correlation will always be isotropic; if J_2 is 1 or $\frac{3}{2}$, the correlation will at most contain terms in $\cos^2\theta$. The coefficients a_1 and a_2 have been given by Hamilton² for all possible combinations of angular momenta. In Table I we have listed the values of these coefficients from Hamilton's paper for the values of Jwhich are of interest in connection with our experiments. Coefficients for octupole radiation should be very useful, but have not yet been published. If the transition involves mixed multipoles, e.g., electric quadrupole and magnetic dipole components, the situation becomes very complicated and the coefficients depend not only on the relative intensities of the two components but also on their relative phases.5

Angular Correlation of Successive Gamma-Rays*

E. L. BRADY† AND M. DEUTSCH Laboratory for Nuclear Science and Engineering, Department of Physics and Chemistry Department, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received February 6, 1950)

The angular correlation of successive gamma-rays emitted by six even-even nuclei has been investigated and found to be anisotropic in every case, and of the magnitude expected theoretically. Effects of external magnetic fields and of chemical binding on the correlations are found to be smaller than the experimental uncertainty for some exploratory experiments. Interpretation of the results in terms of the nuclear states involved is in general possible by use of additional evidence such as relative transition probabilities.

TABLE 1	Ι.	Coefficients	for	the	angula	r correlation	with	the	spin	of
		t	he	grou	ind stat	$e J_1 = 0.$				

J 2	J_1	Multipoles	<i>a</i> 1	<i>d</i> 2
1	0	Dipole-Dipole	1	0
1	1	Dipole-Dipole	-1/3	0
1	2	Dipole-Dipole	-1/3	0
1	1	Quadrupole-Dipole	-1/3	0
1	2	Quadrupole-Dipole	3/7	0
1	3	Quadrupole-Dipole	-3/29	0
2	3	Dipole-Ouadrupole	-3/29	0
2	2	Dipole-Ouadrupole	3/7	0
2	1	Dipole-Ouadrupole	-1/3	0
2	0	Ouadrupole-Ouadrupole	-3	4
2	1	Ouadrupole-Ouadrupole	5	-16/3
2	2	Ouadrupole-Ouadrupole	-15/13	16/13
2	3	Ouadrupole-Ouadrupole	0	-1/3
2	4	Quadrupole-Quadrupole	1/8	1/24



FIG. 4. Correlation of gamma-rays from Cs134 and Na24.



High-precision γ-ray spectroscopy of the cardiac PET imaging isotope ⁸²Rb and its impact on dosimetry

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FIG. 1. (a) Singles spectrum from the decay of ⁸²Rb. (b) Spectrum obtained by gating on the 776-keV transition in ⁸²Kr. In both panels, strong transitions belonging to the decay of ⁸²Rb are labeled by their energy in keV, while background lines are indicated with a *.



FIG. 6. Angular correlation analysis of confirmed and newly identified $0^+ \rightarrow 2^+ \rightarrow 0^+$ cascade transitions in ⁸²Kr. Solid circles give the measured intensity as a function of angle between detectors while solid lines are the theoretical predictions for a 0–2-0 sequence. Numbers in each panel give the cascade energies, in keV. In panel (c), the angular correlation of the 1180-776.5 cascade (stars) is given along with the theoretical predictions for a 2–2-0 sequence with $\delta = -0.52$ (dotted line).



OF GAMMA RAYS FROM ALIGNED NUCLEI*

T. YAMAZAKI

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$$W(\theta) = \sum_{k} A_k P_k \left(\cos \theta \right)$$

For $\Delta I=2$ EM transitions, the singles angular distribution is of the form:

$$W(\theta) = A_0 \left\{ 1 + A_2 P_2 \left(\cos \theta \right) + A_4 P_4 \left(\cos \theta \right) \right\}$$

$$P_2\left(\cos\theta\right) = \frac{1}{2}\left(3\cos^2\theta - 1\right)$$

p351

$P_4\left(\cos\theta\right) = \frac{1}{8}\left(35\cos^4\theta - 30\cos^2\theta + 3\right)$

E Der Mateosian and A.W. Sunyar, Atomic Data and Nuclear Data Tables
13 (1974) p407
K.S. Krane, R.M. Steffen and R.M. Wheeler, Nuclear Data Tables 11 (1973)



K.R. Pohl et al. Phys. Rev. C53 (1996) p2682

<u>Selection and identification of high-spins states.</u>

- Need a top quality gamma-ray spectrometer to measure full-energies of emitted gamma rays from (high-spin) excited nuclear states.
- Helpful to have some sort of channel selection device (e.g., recoil separator; fragment detector).
- Timing between reaction and detection of gamma ray(s) and also the time differences between individual gamma rays in a decay sequences can also be helpful in channel selection and decay scheme building.
- Use EM selection rules, transition rates and DCO/W(θ) etc. to assign spin and parities to excited states.



Recoil (Mass) Separators

Gas Filled

- Pros: High Efficiency
- Cons: No Mass Resolution
- Examples: RITU (Jyvaskyla), BGS (Berkeley)

Vacuum

- Pros: Mass Resolution
- Conn: Low Efficiency
- Examples: FMA (Argonne), RMS (Oak Ridge)

Using Fragment Mass Analyzer (FMA) for High Spin Studies



- Separates ions produced at the target position as a function of M/q at the focal plane.
- 8.9 meters long with a +/- 20% energy acceptance.
- Mass resolution is ~ 350:1.
- Multiple detector configurations at focal plane.

PHYSICAL REVIEW C, VOLUME 60, 064308

Near yrast study of the fpg shell nuclei ⁵⁸Ni, ⁶¹Cu, and ⁶¹Zn

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Can be used to select very weak channels (1 part in 10⁶ or less); Good example is SHE studies where most compound nuclei fission.



. Reiter et al., Phys. Rev. Lett. 82 (1999) 509

Recoil Decay Tagging (Isotopic Identification)



D. Seweryniak et al., PRL 86 (2001) 1458.

-

Can use 'fine structure' in radioactivity to select decays to specific states (i.e., different single particle configurations).





Some high-spin nuclear physics phenomena.

- What happens to the nucleus under rotational stress?
 - Observe sequences of 'rotational bands' in many nuclei.
 - Single particle effects/excitations can 'compete'
 - 'alignments' and 'backbending'.
 - Coriolis effects; moment of inertia differences; band crossings.
 - Other (stable) deformed nuclear shapes exist.
 - Static octupole (β_3) deformations; parity doublets.
 - Very stable, very elongated nuclear charge and mass distributions ('superdeformation').

Quasi-particle aligned angular momentum

Total <u>aligned</u> angular momentum (I_x) can be calculated using Pythagoras theorem:

$$I_x(I) = \sqrt{I(I+1) - K^2} \approx \sqrt{\left(I + \frac{1}{2}\right)^2 - K^2}$$

The rotational frequency can be derived (from the canonical relation) to give

$$\omega \approx \frac{E_{\gamma}}{\sqrt{(I + \frac{3}{2})^2 - K^2} - \sqrt{(I - \frac{1}{2})^2 - K^2}}$$

Many plots of I (= Ix) for even-even nuclei ground state sequences vs. ω show 'discontinuities' at the 'critical' frequency.



Rotation of nuclear core can cause 'breaking' of nuclear pairs close to the nuclear (Fermi) surface.

Biggest energy effect from Coriolis interaction is for orbitals with largest j_x values (i.e. high-J, low- Ω Nilsson orbitals).



Rotation of nuclear core can cause 'breaking' of nuclear pairs close to the nuclear (Fermi) surface.

Biggest energy effect from Coriolis interaction is for orbitals with largest j_x values (i.e. high-J, low- Ω Nilsson orbitals).



The experimental data can give a value for the total angular momentum of state, aligned parallel to the axis of rotation, Ix (from Pythagoras)

$$I_x(I) = \sqrt{I(I+1) - K^2} \approx \sqrt{\left(I + \frac{1}{2}\right)^2 - K^2}$$

Single particle contribution to 'aligned' angular momentum can be separated from the collective, rotational contribution to Ix.

$$i_x(\omega) = I_x(\omega) - I_{\text{ref}}(\omega)$$

The core, collective angular momentum component of the total Ix can be parameterised as a function of rotational frequency (ω) using the <u>'Harris parameters'</u> to fit the nuclear moment of inertia.

$$I_{\rm ref} = \left(\mathcal{I}_{(0)} + \mathcal{I}_{(2)}\omega^2\right)\omega$$



$$E_{rot}(I) = \frac{\hbar^2}{2\mathcal{I}^{(0)}(I)}I(I+1)$$

The kinematic moment of inertia is given by

$$\mathcal{I}^{(1)}(I) = \frac{I}{\omega}$$

while the dynamic moment is given by

$$\omega = \frac{dE(I)}{dI_x(I)} \approx \frac{E(I+1) - E(I-1)}{I_x(I+1) - I_x(I-1)}$$

$$I_x(I) = \sqrt{I(I+1) - K^2} \approx \sqrt{\left(I + \frac{1}{2}\right)^2 - K^2}$$

$$\omega \approx \frac{E_{\gamma}}{\sqrt{(I+\frac{3}{2})^2 - K^2} - \sqrt{(I-\frac{1}{2})^2 - K^2}}$$



P.H. Regan et al. / Nuclear Physics A586 (1995) 351-376



Binary-reaction spectroscopy of 99,100 Mo: Intruder alignment systematics in N=57 and N=58 isotones

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Collective Model B(E2), B(M1) values?

The reduced in-band transition probabilities¹ are given by,

$$B(E2; I_i K \to I_f K) = \frac{5}{16\pi} e^2 Q_o^2 | < I_i 2K0 | I_f K > |^2$$

$$B(M1; I_i K \to I_f K) = \frac{3}{4\pi} e^2 |\langle I_i 1 K 0 | I_f K \rangle |^2 (g_K - g_R)^2 K^2$$

where Q_o is the intrinsic quadrupole moment and g_K and g_R are the intrinsic and rotational gyromagnetic ratios respectively. The relevant Clebsch-Gordon coefficients² are given below.

$$E2(\Delta I = 2) = \left[\frac{3(I-K)(I-K-1)(I+K)(I+K-1)}{(2I-2)(2I-1)I(2I+1)}\right]^{1/2}$$
$$E2(\Delta I = 1) = -K \left[\frac{3(I-K)(I+K)}{(I-1)I(2I+1)(I+1)}\right]^{1/2}$$
(2.4.57)
$$M1(\Delta I = 1) = -\left[\frac{(I-K)(I+K)}{I(2I+1)}\right]^{1/2}$$

 $^1{\rm K.E.G.}$ Löbner in, The Electromagnetic Interaction in Nuclear Spectroscopy, W.D. Hamilton (Ed), North-Holland (1975) Chapter 5

²The Theory of Atomic Spectra, Condon and Shortley (1935) reprinted (1963) p76-77



P.H. Regan et al. / Nuclear Physics A586 (1995) 351-376

$$B(M1) = \frac{3}{8\pi} \frac{K^2}{I^2} \Big[(g^{(1)} - g_R) (\sqrt{I^2 - K^2} - i_x^{(1)}) - (g^{(2)} - g_R) i_x^{(2)} \Big]^2 \\ B(E2: I \rightarrow I - 2) = \frac{5}{16\pi} Q_0^2 \frac{3(I - K)(I - K - 1)(I + K)(I + K + 1)}{(2I - 2)(2I - 1)I(2I + 1)} \\ \int_{10^2} \frac{10^6 cd}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \\ = \frac{10^3}{10^4} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})}}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})}}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})}}{\sqrt{(h_{11/2})^2 x(g_{9/2})^2}} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})}}{\sqrt{(h_{11/2})^2 x(g_{9/2})}} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})}}{\sqrt{(h_{11/2})^2 x(g_{9/2})}} \frac{10^4}{\sqrt{(h_{11/2})^2 x(g_{9/2})}} \frac{\sqrt{(h_{11/2})^2 x(g_{9/2})}}{\sqrt{(h_{11/2})^2 x(g_{9/2})}} \frac{10^4}{\sqrt{(h_{11/2})^2 x($$

Octupole Collectivity



I. Ahmad & P. A. Butler, Annu. Rev. Nucl. Part. Sci. 43, 71 (1993).
 P. A. Butler and W. Nazarewicz, Rev. Mod. Phy. 68, 349 (1996).

$$B(E1:I_i \to I_f) = \frac{3D_0^2}{4\pi} < I_i 010 | I_f 0 >^2 = \frac{3D_0^2}{4\pi} \frac{(I-K)(I+K)}{I(2I+1)}$$





J. Smith et al., Phys. Rev. Lett. 75 (1995) 1050.

Where to find enhanced octupole collectivity

Long-range interactions between single particle states with $\Delta j = \Delta l = 3$;



Difficult regions to study experimentally

Problem?

 Standard fusion-evaporation reactions make (usually) neutron-deficient nuclei.

How do you make and study neutron-rich nuclei?

- (low-cross-section) fusion evap. reactions, e.g., ${}^{18}O + {}^{48}Ca \rightarrow 2p + {}^{64}Fe$
 - Limited compound systems using stable / beam target combinations.
 - Highly selective reactions (if good channel selection applied).
- Spontaneous fission sources (e.g., ²⁴⁴Cm)
 - Good for some regions of the nuclear chart, but little/no selectivity in the 'reaction' mechanism.
 - Can make quite high spins in each fragment ($10 \rightarrow 20\hbar$)
- Fusion fission reactions
 - e.g., ${}^{18}O + {}^{208}Pb \rightarrow {}^{226}Th^* \rightarrow f_1 + f_2 + xn$ (e.g., ${}^{112}_{44}Ru + {}^{112}_{46}Pd + 2n$)
 - Doesn't make very neutron-rich, little selectivity.
 - Medium spins (~10 ħ in each fragment) populated
- Heavy-ion deep-inelastic / multi-nucleon transfer reactions (e.g.,
 - e.g. $^{136}Xe + {}^{198}Pt \rightarrow {}^{136}Ba + {}^{194}Os + 2n$.
 - Populations Q-value dependent; medium spins accessed in products, make nuclei 'close' to the original (stable) beam and target species.
 - Selectivity can be a problem, large Doppler effects.
- Projectile fragmentation (or Projectile Fission)
 - (v. different energy regime)
 - Need a fragment separator.

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TOPICAL REVIEW

Spectroscopic studies with the use of deep-inelastic heavy-ion reactions

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Deep-Inelastic Reactions





$$L = \mu R^2 \omega + \Im_p \omega_p + \Im_t \omega_t,$$
$$\mu = \frac{A_p A_t}{A_p + A_t}.$$

J. Phys. G: Nucl. Part. Phys. 32 (2006) R151-R192

TOPICAL REVIEW

Spectroscopic studies with the use of deep-inelastic heavy-ion reactions

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Figure 2. Production cross sections for binary reaction products in 54 Fe + 106 Cd collisions. In the same experiment cross sections for fusion–evaporation products were determined. Reprinted with permission from [11] © 1994 American Physical Society.


Figure 3. Isobar-integrated production cross sections (a) and the average N/Z ratios (b) as a function of binary reaction product mass in ${}^{106}Cd + {}^{54}Fe$ collisions. Reprinted with permission from [11] © 1994 American Physical Society.

Both the target-like and beam-like fragments and the intermediated fusion-fission residues are usually **stopped** in a thick/backed target.

R155

For discrete gamma rays decaying from states with effective lifetimes of a few picoseconds, there is <u>no</u> <u>Doppler shift</u> effect as the sources are stopped in the target and have v/c=0.

Prompt decays from higher-spin / faster lifetime states (< 1ps) will be 'smeared' out by the Doppler broadening effect.

Backed/thick target experiments can not correct for Doppler shifts as the direction and velocity of the emitting fragment is not known. e.g., ⁸²Se + ¹⁹²Os at INFN-Legnaro.

Discrete gamma rays detected using GASP array.

Triples gamma-ray coincidences measured within ~ 50 ns timing window.

Discrete states to ~ 12ħ observed in BLF.

More like ~ 20 \hbar in some of the TLFs.



PHYSICAL REVIEW C 76, 054317 (2007)

Yrast studies of ^{80,82}Se using deep-inelastic reactions

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Vol. 36 (2005)

Counts per keV Channel

No 4

OBLATE COLLECTIVITY IN THE YRAST STRUCTURE OF ¹⁹⁴Pt*

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328.5

338.8

(a)

10000

5000 世 空

0

200

_____205(192Os Target) 5.0#

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255.0#

290#

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391

400 γ – Rav Energy (keV) ⋇

562.5

600.4

592.1#

600

. . . .

619.4#

482.8



States to spins of >20 ħ can be populated in DIC.



¹³⁶Xe beam on thick, backed ¹⁹²Os target at Argonne National Lab.

Gamma rays measured using GAMMASPHERE

Gamma rays decaying following isomeric states are all stopped in the target, no Doppler shifts.

Evidence for population of states with I>25 ħ.



Physics Letters B 720 (2013) 330-335

isomers and excitation modes in the gamma-soft nucleus ¹⁹²Os

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Conservation of linear angular momentum gives,

$$P_0 = P_p \cos \theta_p + P_t \cos \theta_t$$
$$0 = P_p \sin \theta_p - P_t \sin \theta_t$$



After some algebra manipulation, the relation of the recoil momenta to the initial beam momentum is given by,

$$P_{p,t} = P_0 \frac{\sin(\theta_t, \theta_p)}{\sin(\theta_p + \theta_t)}$$
(2.2)



Figure 2.3: Calculated velocities of the projectile and the target recoils for the particular case of a ¹³⁶Xe beam at 850 MeV in the laboratory frame impinging on a ¹⁹⁸Pt target. An elastic collision and simple two-body kinematics have been assumed.

¹³⁶Ba studied via deep-inelastic collisions: Identification of the $(\nu h_{11/2})_{10+}^{-2}$ isomer

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¹³⁶Xe beam on a thin ¹⁹⁸Pt target.

Residual reaction nuclei measured in 'binary' pairs using CHICO, a position sensitive gas detector.

Gamma rays from beam and target-like fragments measured in GAMMASPHERE.

Difference in time of flight between BLF and TLF hitting CHICO can be used to deduce which fragments is which (heavier one usually moved more slowly due to COLM).

Angle differences between CHICO and GAMMASPHERE can be used for Doppler Corrections.



¹³⁶Ba studied via deep-inelastic collisions: Identification of the $(\nu h_{11/2})_{10+}^{-2}$ isomer

J. Valiente-Dobón,^{1,*} P. H. Regan,^{1,2} C. Wheldon,^{1,3} C. Y. Wu,⁴ N. Yoshinaga,⁵ K. Higashiyama,⁵ J. F. Smith,⁶ D. Cline,⁴ R. S. Chakrawarthy,⁶ R. Chapman,⁷ M. Cromaz,⁸ P. Fallon,⁸ S. J. Freeman,⁶ A. Görgen,⁸ W. Gelletly,¹ A. Hayes,⁴ H. Hua,⁴ S. D. Langdown,^{1,2} I. Y. Lee,⁸ X. Liang,⁷ A. O. Macchiavelli,⁸ C. J. Pearson,¹ Zs. Podolyák,¹ G. Sletten,⁹ R. Teng,⁴ D. Ward,⁸ D. D. Warner,¹⁰ and A. D. Yamamoto^{1,2}



$$\cos \Theta = \sin \theta_R \sin \theta_\gamma (\cos \phi_R \cos \phi_\gamma + \sin \phi_R \sin \phi_\gamma) + \cos \theta_R \cos \theta_\gamma, \qquad (3)$$

where θ_R and ϕ_R are the scattering angles of the recoils (BLFs and TLFs), and θ_{γ} and ϕ_{γ} are the detection angles of the γ rays from GAMMASPHERE.

e.g., ¹³⁶Xe beam on thin ¹⁹⁸Pt target at 850 MeV (~20% above the coulomb barrier)

The maximum velocities of the binary partners were $\beta \approx 11\%$, as determined from the two-body kinematics of the reaction. Therefore, the prompt γ rays emitted in flight were heavily Doppler shifted. However, it was possible to correct the prompt γ -ray energies for the Doppler effect on an event-by-event basis using the interaction position of the recoils as measured by Chico. The interaction position determined the velocities β_{BLF} and β_{TLF} of the recoiling beam and target nuclei, respectively. By conservation of linear momentum [20] and assuming the limiting case of no particle evaporation,

$$P_{BLF,TLF} = \frac{P_0 \sin(\theta_{TLF,BLF})}{\sin(\theta_{BLF} + \theta_{TLF})},$$
(1)

where $P_{BLF} = m_{(136_{Xe})}\beta_{BLF}c$ and $P_{TLF} = m_{(198_{Pt})}\beta_{TLF}c$ are the momenta of the recoiling beam and target nuclei, respectively; θ_{BLF} and θ_{TLF} are the laboratory scattering angles of the recoiling beam and target nuclei, respectively, and P_0 is the momentum of the incident beam. Note that since it is not possible to determine the mass of the recoils with Chico, the momenta of the recoils are calculated assuming the beam ¹³⁶Xe and target ¹⁹⁸Pt masses.

The Doppler-shifted γ rays were corrected according to [22]

$$E_{S} = E_{0} \frac{\sqrt{1 - \beta^{2}}}{1 - \beta \cos \Theta}, \qquad (2)$$

where E_0 is the energy in the rest frame of the nucleus and Θ is the emission angle relative to the trajectory of the nucleus in the laboratory frame. The angle Θ was determined using the expression

Timing of fragments and measured reaction gamma rays correlated to beam pulse.







Use spectrometer to 'tag' on one of the reaction fragments for Doppler Correction. e.g., ⁸²Se (Z=34) beam on thin ¹⁷⁰Er (Z=68) target at INFN-Legnaro. PHYSICAL REVIEW C 81, 034310 (2010)

Spectroscopy of neutron-rich ^{168,170}Dy: Yrast band evolution close to the $N_p N_n$ valence maximum

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FIG. 3. Spectrum of γ -ray energies from targetlike fragments gated on the beamlike fragments ⁸⁴Kr (top) and on beamlike fragments ⁸⁴Kr plus a short time of flight (bottom). The transitions identified as the rotational band in ¹⁶⁸Dy are marked with solid lines.

Gate on ⁸⁴Kr (Z=36) fragment in PRISMA. Complementary fragment (assuming no neutron evaporation) for ⁸²Se+¹⁷⁰Er reaction for ⁸⁴Kr is ¹⁶⁸Dy (Z=66) (+2p transfer channel). Shortest time of flight in PRISMA associated with least neutron evaporation.

Measure BLFs directly in PRISMA spectrometer and gammas in CLARA gamma-ray array. Reverse correct for heavier TLF using 2-body kinematics.

