# Some Aspects of Nuclear Isomers and Excited State Lifetimes

Lecture 2:

at the

Joint ICTP-IAEA Workshop on Nuclear Data : Experiment, Theory and Evaluation

Miramare, Trieste, Italy, August 2016

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# Outline

- What is an isomer ?
- Electromagnetic transition rates.
- Weisskopf Single-Particle Estimates
- Shell Structure in near spherical nuclei.
  - Odd-A singly magic nuclei (e.g., $^{205}Au_{126}$ ; $^{131}In_{82}$)
  - Why are E1s 'naturally' hindered ?
- Seniority isomers, $j^2$ & $j^n$ configurations ?
- Near Magic nuclei.
  - Limited valence space and core breaking.
- Deformed Nuclei.
  - the Nilsson Model, K-isomers.
- Measurements of excited state nuclear lifetimes
  - Electronic coincidences (Fast-timing with $LaBr_3(Ce)$
  - Doppler Shift methods (RDM, DSAM)
  - Transition quadrupole moments (Qo)

# Some good recent reviews; useful references and equations..

Phys. Scr. **T152** (2013) 014015 (20pp)

## Isomers, nuclear structure and spectroscopy

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Atomic Data and Nuclear Data Tables 103–104 (2015) 50–105

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Atomic Data and Nuclear Data Tables

journal homepage: www.elsevier.com/locate/adt

## Configurations and hindered decays of *K* isomers in deformed nuclei with A > 100

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**IOP** Publishing

Reports on Progress in Physics

)034-4885/79/7/076301

Rep. Prog. Phys. **79** (2016) 076301 (46pp)

## Review of metastable states in heavy nuclei

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Received 13 June 2015, revised 13 April 2016
Accepted for publication 18 April 2016
Published 31 May 2016

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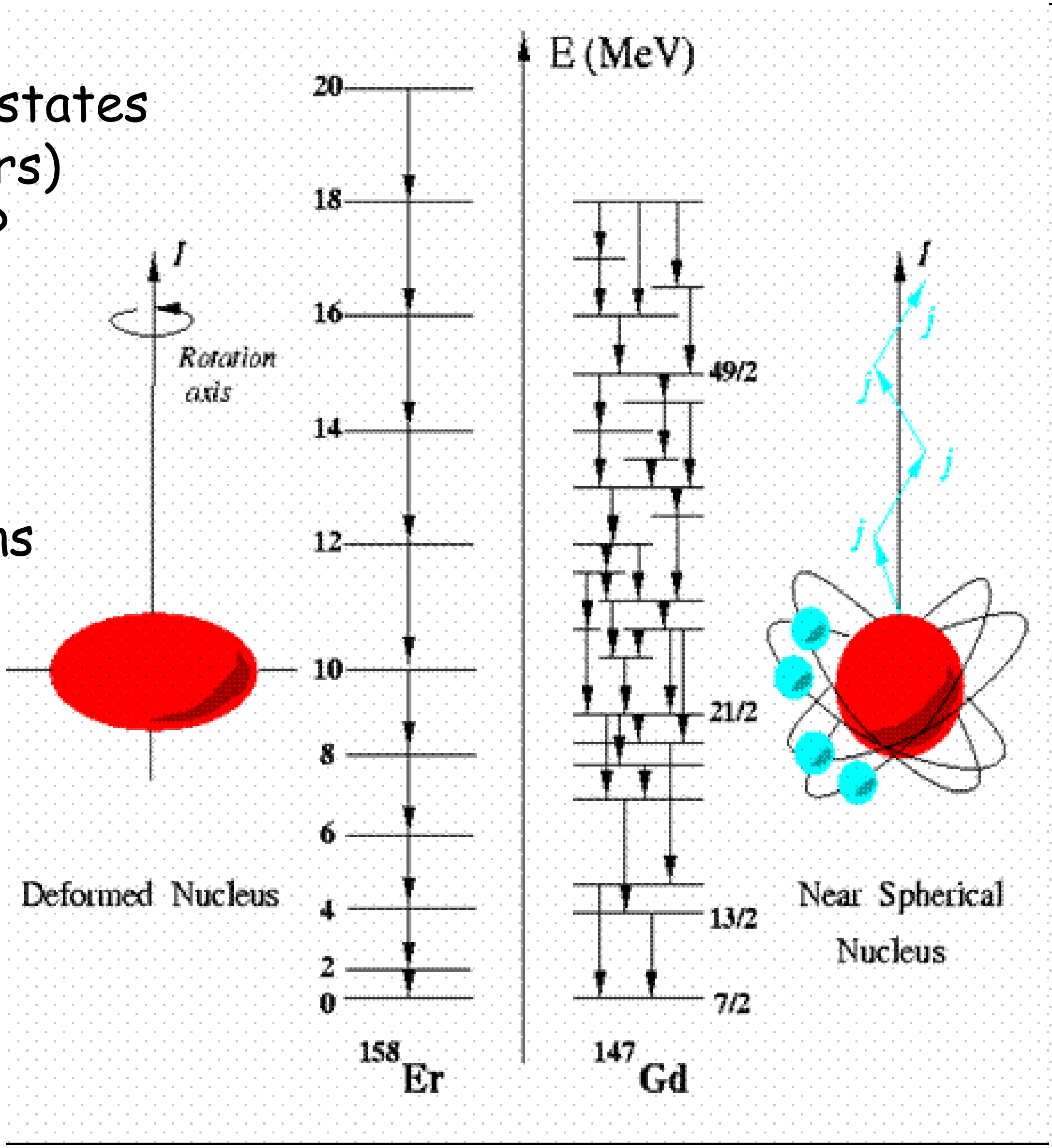
## Some nuclear observables.

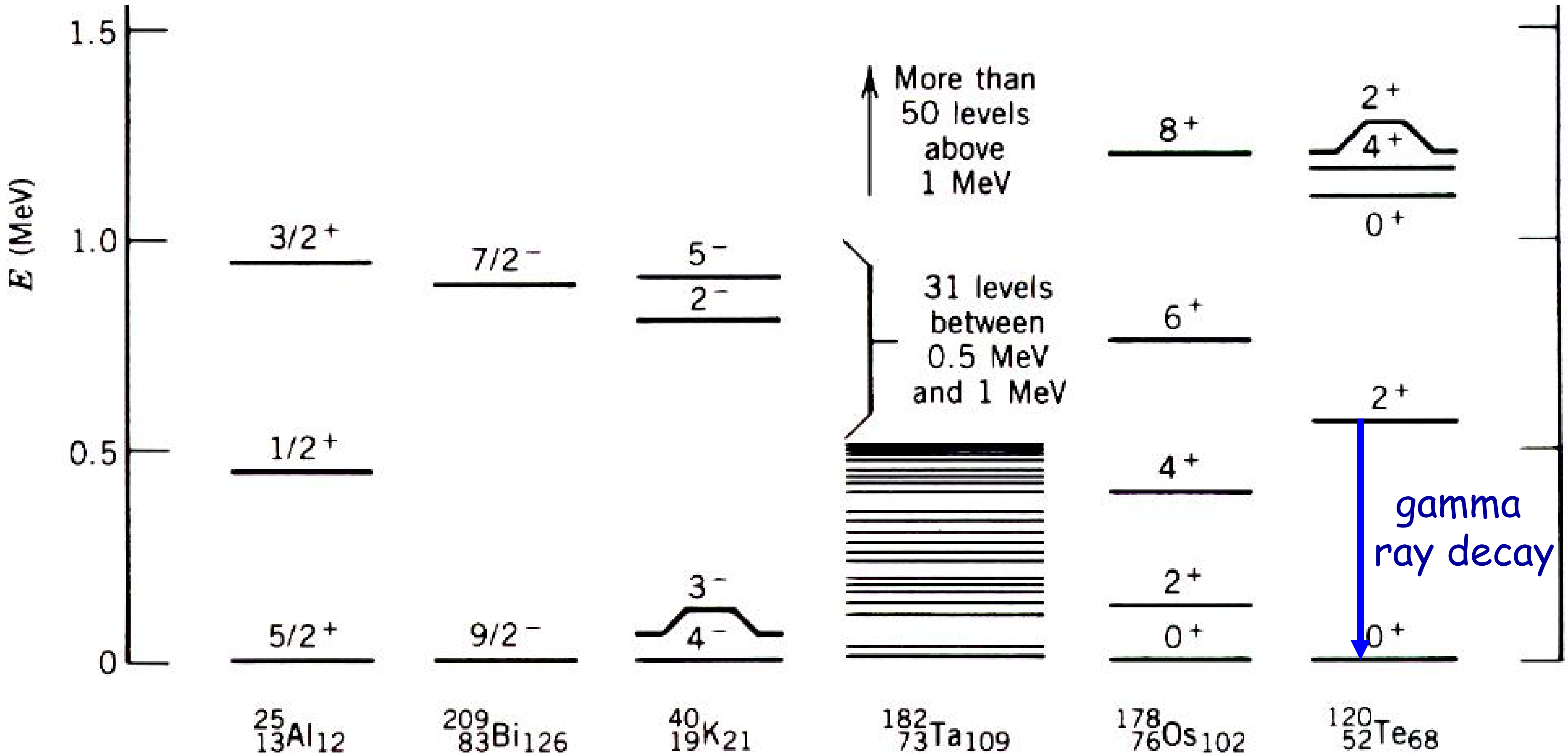
1) Masses and energy differences
2) Energy levels
3) Level spins and parities
4) EM transition rates between states
5) Magnetic properties (g-factors)
6) Electric quadrupole moments?

This is the essence of nuclear structure physics.

How do these change as functions of N, Z, I, Ex ?



# Measuring Excited Excited States –
# Nuclear Spectroscopy & Nuclear (Shell) Structure




- Nuclear states labelled by spin and parity quantum numbers and energy.
- Excited states (usually) decay by gamma rays (non-visible, high energy light).
- Measuring gamma rays gives the energy differences between quantum states.

The lifetime of an isomeric state is related to the total decay width, $\Gamma$, a linear sum of all partial decay widths ($\gamma$ ray, conversion electrons, $\alpha$ decay, $\beta$ decay, fission, etc.), through the uncertainty relationship (in convenient units):

$$\Gamma \times \tau = \hbar = 0.6582 \times 10^{-15}\ [\mathrm{eV} \cdot \mathrm{s}], \tag{3}$$

where $\tau$ is the level mean life, which is related to the half-life as $T_{1/2} = \ln 2 \times \tau$.

For an isomeric state with $N$ branches, predominantly $\gamma$ rays and internal conversion in the present cases, the partial $\gamma$-ray mean life of an individual transition $j$, $\tau_\gamma^j$, is given by:

$$\tau_\gamma^j = \tau^{exp} \times \frac{\sum_{k=1}^{N} I_\gamma^k \times (1 + \alpha_T^k)}{I_\gamma^j}, \tag{4}$$

Atomic Data and Nuclear Data Tables **78,** 1–128 (2001)
doi:10.1006/adnd.2001.0858, available online at http://www.idealibrary.com on IDEAL®

# TRANSITION PROBABILITY FROM THE GROUND TO THE FIRST-EXCITED $2^+$ STATE OF EVEN–EVEN NUCLIDES*

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Adopted values for the reduced electric quadrupole transition probability, $B(E2)\uparrow$, from the ground state to the first-excited $2^+$ state of even–even nuclides are given in Table I. Values of $\tau$, the mean life of the $2^+$ state; $E$, the energy; and $\beta$, the quadrupole deformation parameter, are also listed there. The ratio of $\beta$ to the value expected from the single-particle model is presented. The intrinsic quadrupole moment, $Q_0$, is deduced from the $B(E2)\uparrow$ value. The product $E \times B(E2)\uparrow$ is expressed as a percentage of the energy-weighted total and isoscalar $E2$ sum-rule strengths.

Table II presents the data on which Table I is based, namely the experimental results for $B(E2)\uparrow$ values with quoted uncertainties. Information is also given on the quantity measured and the method used. The literature has been covered to November 2000.

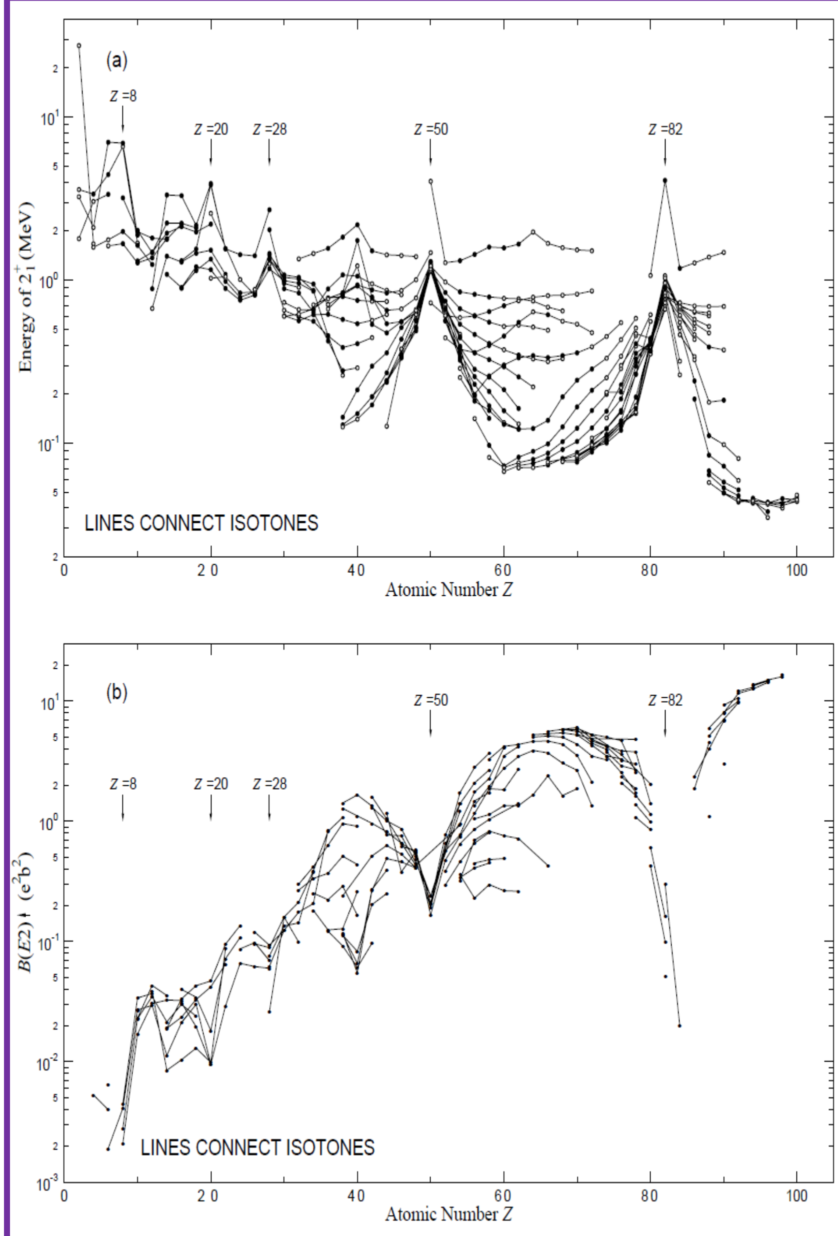
The adopted $B(E2)\uparrow$ values are compared in Table III with the values given by systematics and by various theoretical models. Predictions of unmeasured $B(E2)\uparrow$ values are also given in Table III. 

## EXPLANATION OF TABLES

### TABLE I. Adopted Values of $B(E2)\uparrow$ and Related Quantities

Throughout this table, italicized numbers refer to the uncertainties in the last digits of the quoted values.

| | |
|---|---|
| Nuclide | The even $Z$, even $N$ nuclide studied |
| $E$(level) | Energy of the first excited $2^+$ state in keV either from a compilation or from current literature |
| $B(E2)\uparrow$ | Reduced electric quadrupole transition rate for the ground state to $2^+$ state transition in units of $e^2b^2$ |
| $\tau$ | Mean lifetime of the state in ps<br>$\tau = 40.81 \times 10^{13} E^{-5} [B(E2)\uparrow/e^2b^2]^{-1}(1+\alpha)^{-1}$ (see Table II for the $\alpha$ values when $\alpha > 0.001$) |
| $\beta$ | Deformation parameter<br>$\beta = (4\pi/3ZR_0^2)[B(E2)\uparrow/e^2]^{1/2}$, where<br>$R_0^2 = (1.2 \times 10^{-13} A^{1/3}\ \mathrm{cm})^2$<br>$= 0.0144 A^{2/3}\ \mathrm{b}$ |
| $\beta_{(sp)}$ | $\beta$ from the single-particle model<br>$\beta_{(sp)} = 1.59/Z$ |
| $Q_0$ | Intrinsic quadrupole moment in b<br>$Q_0 = \left[\frac{16\pi}{5}\ \frac{B(E2)\uparrow}{e^2}\right]^{1/2}$ |



## What are isomers ?

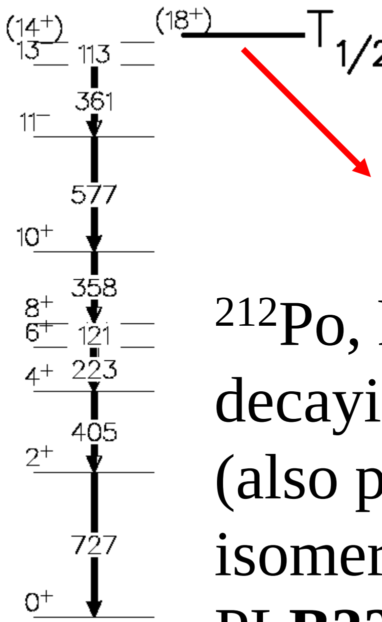
Metastable (long-lived) nuclear excited state.
'Long-lived' could mean:
~$10^{-19}$ seconds, shape isomers in $\alpha$-cluster resonances or
~$10^{15}$ years $^{180}$Ta $9^{-}\rightarrow1^{+}$ decay.

## Why/when do nuclear isomers occur ?

(i) large change in spin **('spin-trap')**
(ii) small transition energy between states **(seniority isomers)**
(iii) dramatic change in structure/shape (fission isomers) and/or underlying symmetry **(*K*-isomers)**

## What information do isomers gives you ?

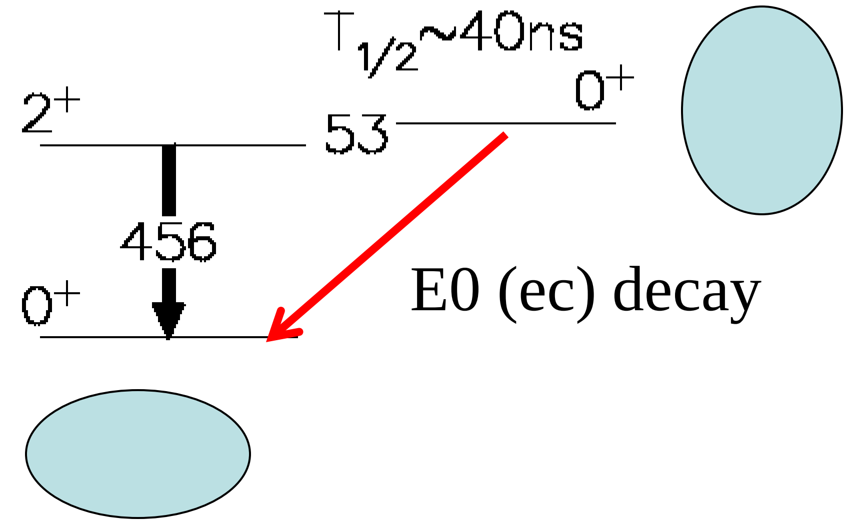
Isomers occur due to differences in ***single-particle structure***.
EM transitions are **hindered** between states with very different underlying structures.

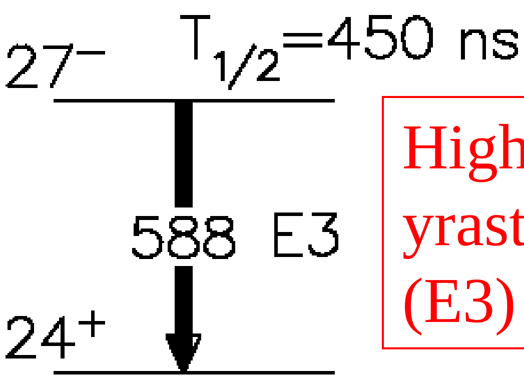



α–decay to states in $^{208}$Pb.

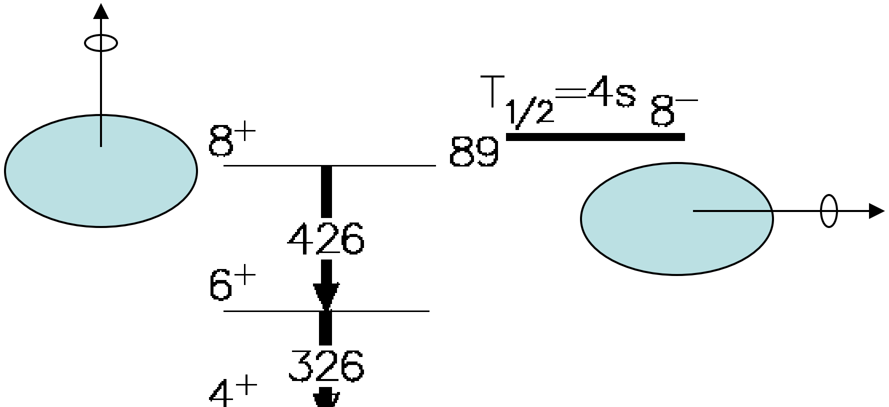
$^{212}$Po, high-spin α-decaying yrast trap. (also proton decaying isomers, e.g, $^{53}$Co PL**B33** (1970) 281ff.

$^{74}$Kr, shape isomer



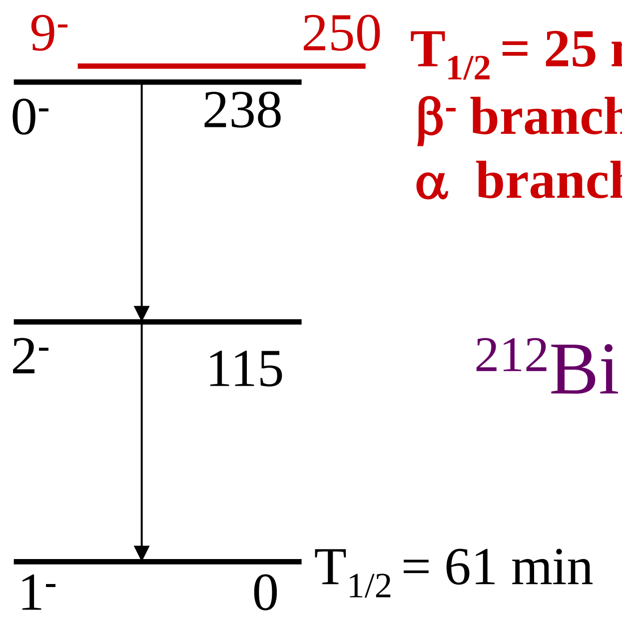

High-spin, yrast-trap (E3) in $^{212}$Fr



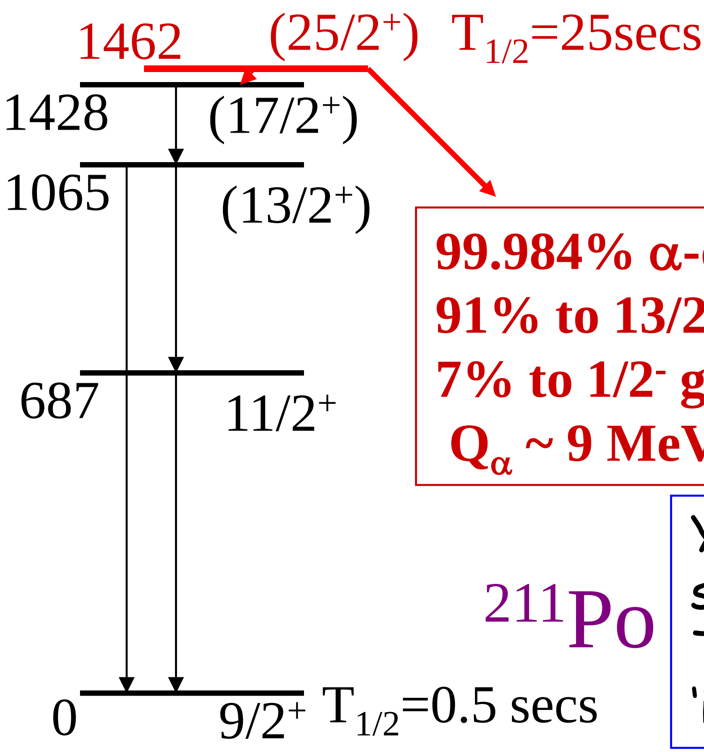

K-isomer in $^{178}$Hf

‘High-spin’ α and β-decaying isomers just above $^{208}$Pb, basically as a result of ‘yrast’ (spin) traps..

$^{212}$Bi, Z=83, N=129,
$9^-$ from $\nu g_{9/2} \times \pi h_{9/2}$




**$T_{1/2}$ = 25 min**
**$\beta^-$ branch =33%**
**α branch = 67%**




**99.984% α-decay branch,**
**91% to 13/2$^+$ isomer in $^{207}$Pb,**
**7% to 1/2$^-$ ground state in $^{207}$Pb,**
**$Q_\alpha$ ~ 9 MeV per decay**

Yrastness is what causes these isomers…they simply have ‘nowhere to go’ to (easily).
This yrastness is itself often caused by high-j ‘intruders’ in the nuclear single particle spectrum….

DWK, Rep. Prog. Phys. **79**, 076301_(2016)

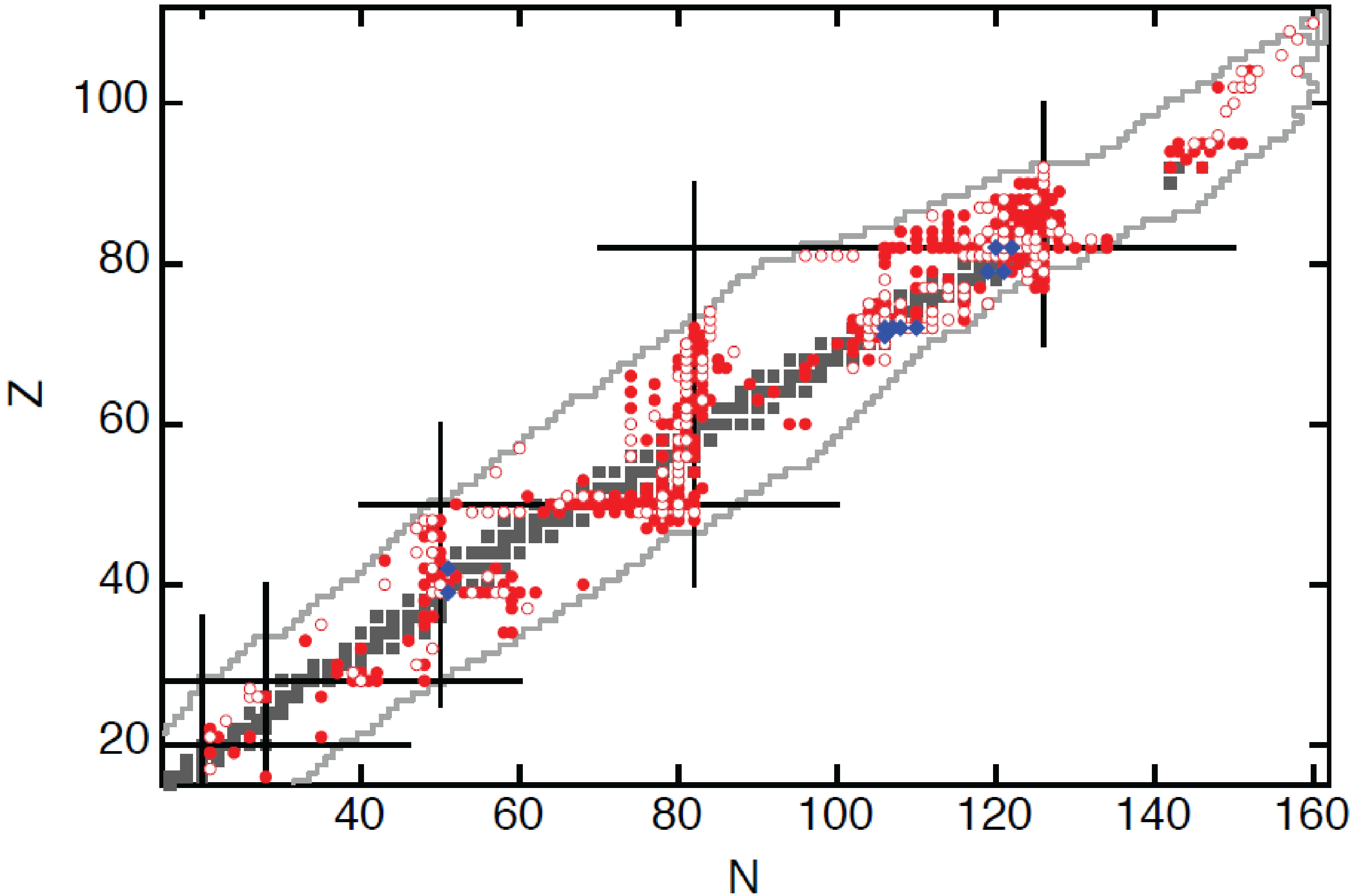
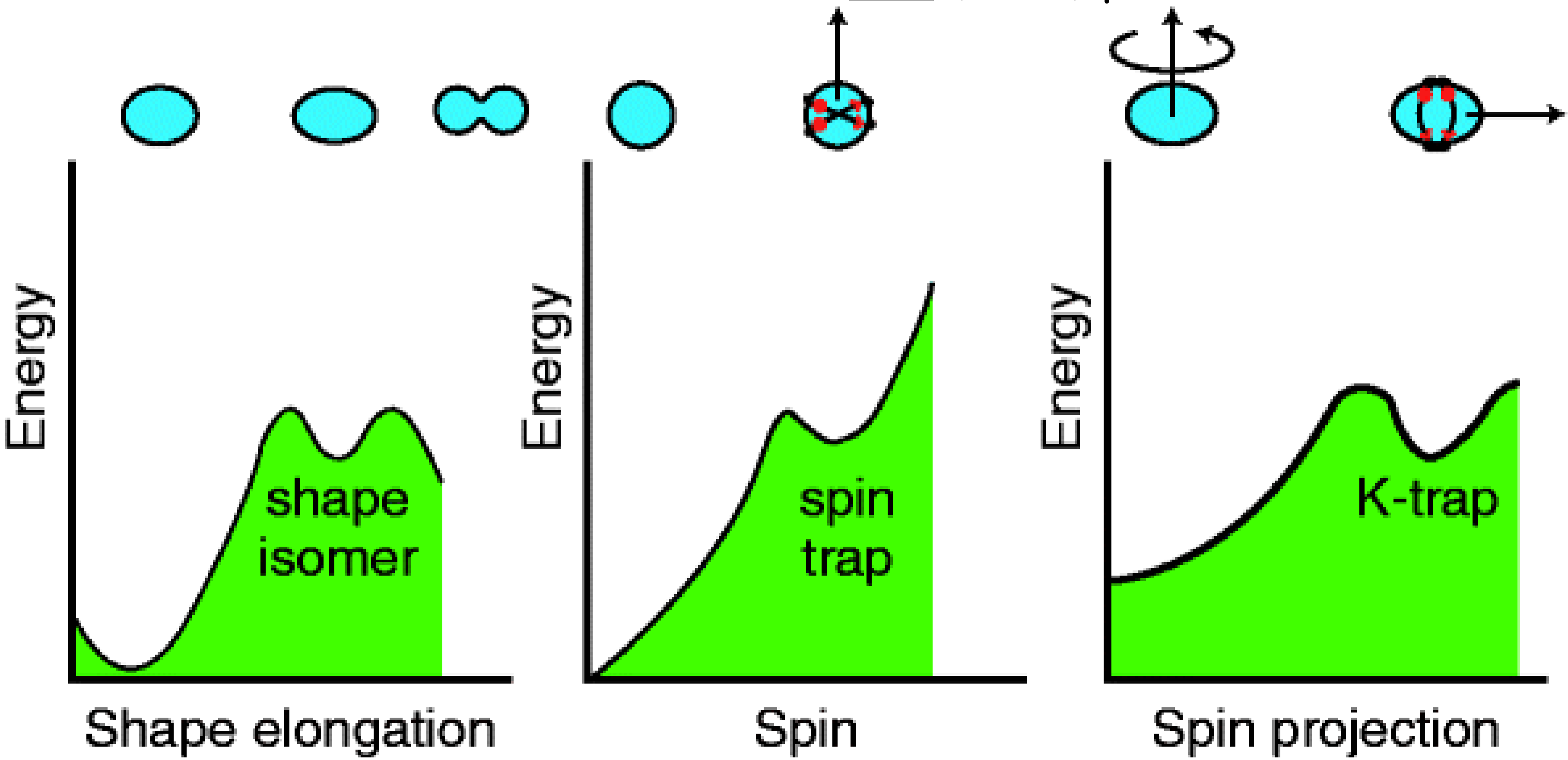



**Figure 1.** Nuclear chart illustrating the distribution of isomers with excitation energies greater than 600 keV, with data from Audi *et al.* [44]. The filled red circles correspond to 200 ns $< T_{1/2} <$ 100 $\mu$s, the open red circles correspond to 100 $\mu$s $< T_{1/2} <$ 1 hr, and the filled blue diamonds are for $T_{1/2} >$ 1 hr.

P.M.Walker and G.D.Dracoulis. Nature, **399** (1999) p35




**What about EM transition rates between low-energy states in nuclei ?**

# EM decay selection rules reminder.

Suppose we are concerned with a transition between the states $i$ and $f$ of characters (spin, parity) $(J_i, \pi_i)$ and $(J_f, \pi_f)$ respectively; defining a quantity $p$, which is 0 for even parity and +1 for odd parity, we see that the multipoles that can contribute are delimited by

$$J_i + J_f \geqq L \geqq |J_i - J_f|,$$

and by the further conditions:

$$p_i + p_f + L = \text{odd for magnetic multipoles}$$
$$p_i + p_f + L = \text{even for electric multipoles.}$$

TABLE I

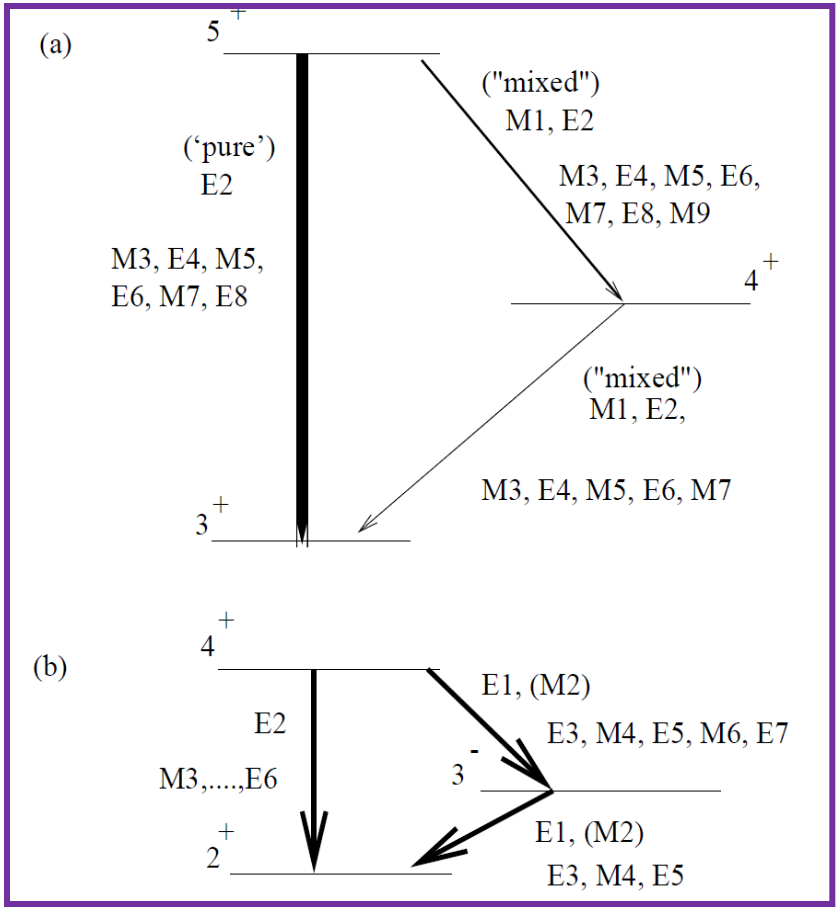
OBSERVED MULTIPOLE TRANSITIONS

| Multipole L | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Parity change | Yes | *E*1 ⟷ | *M*2 | *E*3 | *M*4 | *E*5 |
| | No | *M*1 ⟷ | *E*2 | *M*3 | *E*4 | |

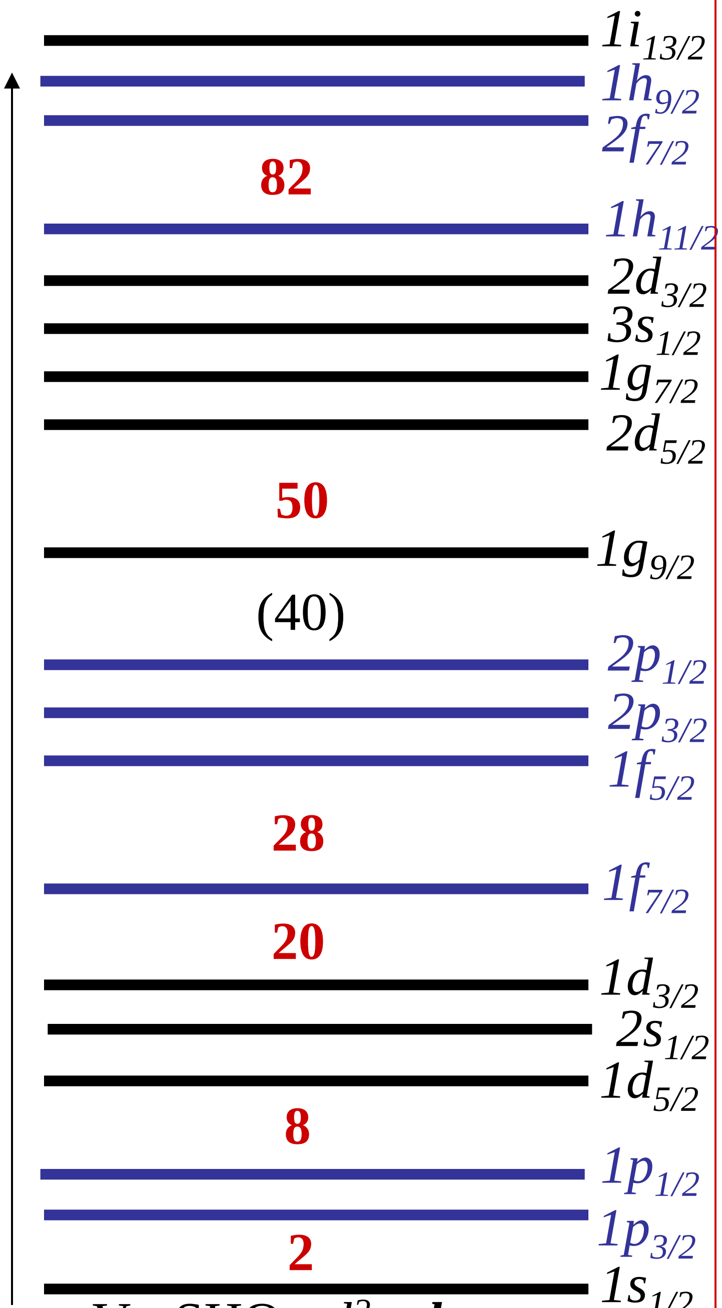
Since the quantity that enters in the vector potential of the $L$-multipole is:

$$j_L(kr) \cong \frac{(kr)^L}{(2L+1)!!}, \qquad 9.$$

we see that the lowest possible multipole transition is greatly favored for $kr \ll 1$. The range of energies for which $(kr) \ll 1$ is frequently called the "long wave-length" region. In fact, usually only the lowest possible multipole contributes, but sometimes also the next order will appear. It is this fact that makes the measurement of multipole order so useful a tool in the assignment of characters to levels. The most important examples of mixed multipoles are found for $M1+E2$ and $E1+M2$ transitions; these are indicated in the above table by connecting lines.




From M.Goldhaber & J.Weneser, Ann. Rev. Nucl. Sci. 5 (1955) p1-24




V= SHO + $l^2$.+ ***l.s.***

Independent particle model (from 1950s)

Protons & neutron (fermions) fill orbits by PEP.

Each $\underline{j}=\underline{l}+\underline{s}$ level has $2j+1$ projections of $m_j$

e.g., $g_{9/2}$ orbit can have

[(2 x 9/2)+1 ] = 10 protons
with *mj* from -9/2, -7/2,…..,,+7/2, +9/2.

Clustering of levels causes energy 'gaps'
leading to **MAGIC NUMBERS.**

Can 'approximate' the average / mean field experienced by each nucleon by e.g., as:

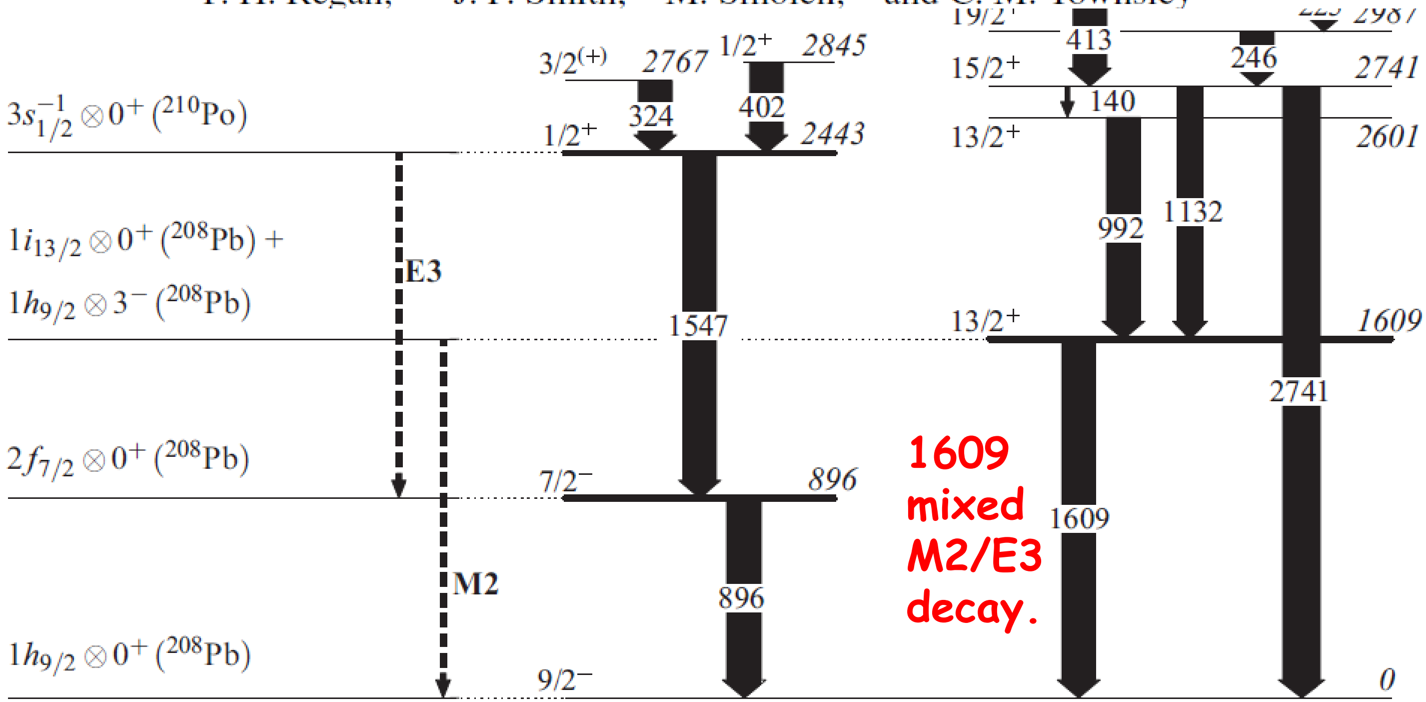
H = HO + a$\underline{\boldsymbol{l}}.\underline{\boldsymbol{l}}$ + b$\underline{\boldsymbol{l}}.\underline{\boldsymbol{s}}$

*Changes in the Hamiltonian alters the level ordering.*

PHYSICAL REVIEW C **93**, 014309 (2016)

# $E3$ and $M2$ transition strengths in $^{209}_{83}$Bi

O. J. Roberts,[1,2,*] C. R. Niţă,[1,3] A. M. Bruce,[1] N. Mărginean,[3] D. Bucurescu,[3] D. Deleanu,[3] D. Filipescu,[3] N. M. Florea,[3,4] I. Gheorghe,[3,5] D. Ghiţă,[3] T. Glodariu,[3] R. Lica,[3] R. Mărginean,[3] C. Mihai,[3] A. Negret,[3] T. Sava,[3] L. Stroe,[3] R. Şuvăilă,[3] S. Toma,[3] T. Alharbi,[6,7] T. Alexander,[6] S. Aydin,[8] B. A. Brown,[9] F. Browne,[1] R. J. Carroll,[6] K. Mulholland,[10] Zs. Podolyák P. H. Regan,[6,11] J. F. Smith,[10] M. Smolen,[10] and C. M. Townsley[6]




| $E_x$ (keV) | $J^\pi_i$ | $T_{\frac{1}{2}}$ (ns) | $E_\gamma$ (keV) | $J^\pi_f$ | Lλ | Measured transitions | $B(L\lambda)$ ($\times 10^3\ e^2$ fm$^6$ or $\mu_N^2$ fm$^2$) | SM ($\times 10^3\ e^2$ fm$^6$ or $\mu_N^2$ fm$^2$) | FFS ($\times 10^3\ e^2$ fm$^6$ or $\mu_N^2$ fm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1609 | $\frac{13}{2}{}^+_1$ | 0.120(15) | 1609 | $\frac{9}{2}{}^-_1$ | $E3$ | ($^{208}_{82}$Pb $3^- \otimes \pi 1h_{9/2}$)→ ($^{208}_{82}$Pb $0^+ \otimes \pi 1h_{9/2}$) | 12(2) | 0.52 | 9.8 [40] |
| | | | | | $M2$ | ($^{208}_{82}$Pb $0^+ \otimes \pi 1i_{13/2}$) → ($^{208}_{82}$Pb $0^+ \otimes \pi 1h_{9/2}$) | 0.038(5) | 0.43 | 0.033 [41] |
| 2443 | $\frac{1}{2}{}^+_1$ | 9.02(24) | 1547 | $\frac{7}{2}{}^-_1$ | $E3$ | ($^{210}_{84}$Po $0^+ \otimes \pi 3s_{1/2}$)→ ($^{208}_{82}$Pb $0^+ \otimes \pi 2f_{7/2}$) | 6.3(2) | 0 | |

# EM Transition Rates

Classically, the average power radiated by an EM multipole field is given by

$$P(\sigma L) = \frac{2(L+1)c}{\varepsilon_0 L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+2}[m(\sigma L)]^2$$

*m(σL)* is the time-varying electric or magnetic multipole moment.
ω is the (circular) frequency of the EM field

For a quantized (nuclear) system, the decay probability is determined by the MATRIX ELEMENT of the EM MULTIPOLE OPERATOR, where

$$m_{fi}(\sigma L) = \int \psi_f^* m(\sigma L)\psi_i dv$$ i..e, integrated over the nuclear volume.

We can then get the general expression for the probability per unit time for gamma-ray emission, $\lambda(\sigma L)$ , from:

$$\lambda(\sigma L) = \frac{1}{\tau} = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L+1)}{\varepsilon_0 \hbar L[(2L+1)!!]^2}\left(\frac{\omega}{c}\right)^{2L+1}[m_{fi}(\sigma l)]^2$$

(see Introductory Nuclear Physics, K.S. Krane (1988) p330).

# How is measuring the lifetime useful?

<u>Nuclear structure information.</u>
The '<u>**reduced matrix element**</u>', **B(λL)** tells us the overlap between the initial and final nuclear single-particle wavefunctions.

$$T_{fi}(\lambda L) = \frac{8\pi(L+1)}{\hbar L\left((2L+1)!!\right)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\lambda L : J_i \rightarrow J_f)$$

Transition probability (i.e., 1/mean lifetime as measured for state which decays by EM radiation)

(trivial) gamma-ray energy dependence of transition rate, goes as. *$E_\gamma^{2L+1}$ e.g., $E_\gamma^5$ for E2s for example.*

Nuclear EM transition rates between excited states are fundamental in nuclear structure research.

$$T_{fi} = \frac{1}{\tau} = \frac{8\pi(L+1)}{\hbar\left((2L+1)!!\right)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B\left(\lambda L: I_i \rightarrow I_f\right)$$

The extracted ***reduced matrix elements***, *B(λL)* give insights e.g.,

- Single particle / shell model-like: ~ 1 Wu (NOT for E1s)
- Deformed / collective: faster lifetimes, ~10s to 1000s of Wu (in e.g., superdeformed bands)
- Show underlying symmetries and related selection rules such as K-isomerism: MUCH slower decay rates ~ $10^{-3\rightarrow 9}$ Wu and slower).

| Multipolarity | Electric Transition Rate ($s^{-1}$) | Magnetic Transition Rate ($s^{-1}$) |
|---|---|---|
| 1 | $1.587 \times 10^{15}\ E_\gamma^3\ B(E1)$ | $1.779 \times 10^{13}\ E_\gamma^3\ B(M1)$ |
| 2 | $1.223 \times 10^{9}\ E_\gamma^5\ B(E2)$ | $1.371 \times 10^{7}\ E_\gamma^5\ B(M2)$ |
| 3 | $5.689 \times 10^{2}\ E_\gamma^7\ B(E3)$ | $6.387 \times 10^{0}\ E_\gamma^7\ B(M3)$ |
| 4 | $1.649 \times 10^{-4}\ E_\gamma^9\ B(E4)$ | $1.889 \times 10^{-6}\ E_\gamma^9\ B(M4)$ |
| 5 | $3.451 \times 10^{-11}\ E_\gamma^{11}\ B(E5)$ | $3.868 \times 10^{-13}\ E_\gamma^{11}\ B(M5)$ |

Table 2.2: Transition probabilities $T(s^{-1})$ expressed by $B(EL)$ in $(e^2(fm)^{2L})$ and $B(ML)$ in $(\frac{e\hbar}{2mc}(fm)^{2L-2})$. $E_\gamma$ is the $\gamma$-ray energy, in MeV. (Taken from ref [69]).

Transition rates get slower (i.e., longer lifetimes associated with) higher order multipole decays

# Weisskopf Single Particle Estimates:

- These are 'yardstick' estimates for the speed of EM transitions for a given electromagnetic multipole order.
- They depend on the size of the nucleus (i.e., A) and the energy of the transition / gamma-ray energy ($E_\gamma^{2L+1}$)
- They estimate the transition rate for spherically symmetric proton orbitals for nuclei of radius $r=r_0A^{1/3}$.

The ***half-life*** (in $10^{-9}$s), equivalent to 1 Wu is given by (DWK):

$$T_{1/2}(E1) = 6.76 \times A^{-2/3}E^{-3} \times 10^{-6}$$
$$T_{1/2}(M1) = 2.20 \times E^{-3} \times 10^{-5}$$
$$T_{1/2}(E2) = 9.52 \times A^{-4/3}E^{-5}$$
$$T_{1/2}(M2) = 3.10 \times A^{-2/3}E^{-5} \times 10^{1}$$
$$T_{1/2}(E3) = 2.04 \times A^{-2}E^{-7} \times 10^{7}$$

where the transition energy $E$ is in MeV and $A$ is the atomic mass number

Weisskopf, V.F., 1951.

Radiative transition probabilities in nuclei.

*Physical Review*, **83**(5), p.1073.



where $K$ is the low frequency dielectric constant, $K_0$ is the optical constant, $\rho$ the density, and $\chi$ the compressibility. In Table I are listed the values of $\partial \ln K/\partial p$ calculated from (4) and (1) next to the experimental values of $\partial \ln K/\partial p$. The calculated values of $\partial \ln K/\partial p$ differ from those of Rao by the term $a(K-K_0)/K$, which arises from the difference between (1a) and (2a).

Equation (4) is derived assuming that the inner field polarizing the dielectric is independent of pressure. Since the values of $-\partial \ln K/\partial p$ obtained from (4) do not account for all the change in the dielectric constant, it seems consistent to expect that the inner field is not constant but does decrease with increasing pressure. This conclusion agrees with the one reached in my original paper using the theories of Hojendahl and Mott and Littleton.

[1] D. A. A. S. Narayana Rao, Phys. Rev. 82, 118 (1951).
[2] S. Mayburg, Phys. Rev. 79, 375 (1950).

## Radiative Transition Probabilities in Nuclei

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*Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts*
(Received July 20, 1951)

CONSIDER a transition from nuclear state $a$ to nuclear state $b$ with emission of a quantum of multipole radiation of angular momentum $l$ ($2^l$-pole) and $z$ component $m$. The transition probability per unit time is given by[1]

$$T(l, m)=\frac{8\pi(l+1)}{l[(2l+1)!!]^2}\frac{\kappa^{2l+1}}{\hbar}|A(l, m)+A'(l, m)|^2, \quad (1)$$

where $\kappa=2\pi\nu/c$ is the wave number of the emitted radiation, and the quantities $A$, $A'$ are the multipole matrix elements caused by the electric currents and by the magnetization (spins), respectively. We find for electric radiation

$$A(l, m)=Q(l, m)=e\sum_{k=1}^{Z}\int r_k^l\, Y_{lm}^*(\theta_k, \phi_k)\, \varphi_b^*\varphi_a\, d\tau, \quad (2)$$

$$A'(l, m)=Q'(l, m)=-\frac{i\kappa}{l+1}\frac{e\hbar}{2Mc}\sum_{k=1}^{A}\mu_k \times\int r_k^l\, Y_{lm}^*(\theta_k, \phi_k)\, \mathrm{div}(\varphi_b^*\mathbf{r}_k\times\boldsymbol{\sigma}_k\varphi_a)\, d\tau, \quad (3)$$

where $\varphi_a$ and $\varphi_b$ are the wave functions of the nuclear states, $M$ is the mass of each nucleon, $\mathbf{r}_k=(r_k, \theta_k, \phi_k)$ is the position vector of the $k$th nucleon, $\boldsymbol{\sigma}_k$ is its Pauli spin vector, and $\mu_k$ is its magnetic moment in nuclear magnetons. The sum in (2) extends over the protons, the sum in (3) over both protons and neutrons. These expressions are approximations valid for $\kappa R\ll 1$, where $R$ is the nuclear radius.

The corresponding expressions for magnetic multipole radiation are

$$A(l, m)=M(l, m)=-\frac{1}{l+1}\frac{e\hbar}{Mc}\sum_{k=1}^{Z} \times\int r_k^l\, Y_{lm}^*(\theta_k, \phi_k)\, \mathrm{div}(\varphi_b^*\mathbf{L}_k\varphi_a)\, d\tau, \quad (4)$$

$$A'(l, m)=M'(l, m)=-\frac{e\hbar}{2Mc}\sum_{k=1}^{A}\mu_k \times\int r_k^l\, Y_{lm}^*(\theta_k, \phi_k)\, \mathrm{div}(\varphi_b^*\boldsymbol{\sigma}_k\varphi_a)\, d\tau, \quad (5)$$

where $\mathbf{L}_k=-i\mathbf{r}_k\times\boldsymbol{\nabla}_k$ is the orbital angular momentum operator (in units of $\hbar$) for the $k$th nucleon.

We can estimate these matrix elements by the following exceedingly crude method. We assume that the radiation is caused by a transition of one single proton which moves independently within the nucleus, its wave function being given by $u(r)Y_{lm}(\theta, \phi)$. In addition we also assume that the final state of the proton is an $S$ state.[2] We then obtain

$$Q(l, m)\sim[e/(4\pi)^{\frac{1}{2}}][3/(l+3)]R^l \quad (6)$$

where the integral $\int r^l u_b(r)u_a(r)r^2dr$ over the radial parts of the proton wave functions was set approximately equal to $3R^l/(l+3)$. The other matrix elements are estimated by replacing div by $R^{-1}$. We get the rough order-of-magnitude guess

$$M(l, m)\sim[e/(4\pi)^{\frac{1}{2}}][3/(l+3)][\hbar/Mc]R^{l-1}, \quad (7)$$

$$M'(l, m)\sim[e/(4\pi)^{\frac{1}{2}}][3/(l+3)]\mu_P[\hbar/Mc]R^{l-1}, \quad (8)$$

where $\mu_P$ is the magnetic moment of the proton ($=2.78$). $Q'(l, m)$ can be neglected compared to $Q(l, m)$. We therefore get a ratio of roughly

$$(1+\mu_P^2)(\hbar/McR)^2\sim 10(\hbar/McR)^2$$

between the transition probability of a magnetic multipole and an electric one of the same order. This ratio is energy-independent in contrast to widespread belief.

Inserting these estimates into (1) we get for the transition probability of an electric $2^l$-pole

$$T_E(l)\cong\frac{4.4(l+1)}{l[(2l+1)!!]^2}\left(\frac{3}{l+3}\right)^2\left(\frac{\hbar\omega}{197\text{ Mev}}\right)^{2l+1} \times(R\text{ in }10^{-13}\text{ cm})^{2l}\, 10^{21}\text{ sec}^{-1} \quad (9)$$

and for a magnetic $2^l$-pole

$$T_M(l)\cong\frac{1.9(l+1)}{l[(2l+1)!!]^2}\left(\frac{3}{l+3}\right)^2\left(\frac{\hbar\omega}{197\text{ Mev}}\right)^{2l+1} \times(R\text{ in }10^{-13}\text{ cm})^{2l-2}\, 10^{21}\text{ sec}^{-1}. \quad (10)$$

The assumptions made in deriving these estimates are extremely crude and they should be applied to actual transitions with the greatest reservations. They are based upon an extreme application of the independent-particle model of the nucleus and it was assumed that a proton is responsible for the transition. On the basis of our assumptions the electric multipole radiation with $l>1$ should be much weaker for transitions in which a single neutron changes its quantum state. No such differentiation is apparent in the data.

In spite of these difficulties it may be possible that the order of magnitude of the actual transition probabilities is correctly described by these formulas. We have published these exceedingly crude estimates only because of the rather unexpected agreement with the experimental material which was pointed out to us by many workers in this field.

The author wishes to express his appreciation especially to Dr. M. Goldhaber and Dr. J. M. Blatt for their great help in discussing the experimental material and in improving the theoretical reasoning.

[1] We use the notation $(2l+1)!!=1\cdot 3\cdot 5\cdots(2l+1)$.
[2] This latter assumption can be removed; the corrections consist in unimportant numerical factors.

## Nuclear Magnetic Resonance in Metals: Temperature Effects for $Na^{23}$

H. S. Gutowsky
*Noyes Chemical Laboratory, Department of Chemistry, University of Illinois, Urbana, Illinois**
(Received July 2, 1951)

KNIGHT reported[1] that nuclear magnetic resonance frequencies are higher in metals than in chemical compounds. It has been proposed[2] that such frequency shifts are primarily the result of the contribution of conduction electrons to the magnetic field at the nuclei in the metal. This note gives an account of some related preliminary results including temperature and chemical effects, and also detailed line shape studies. Our experiments have been at fixed frequency using equipment and procedures outlined previously.[3,4]

The effect of temperature on the $Na^{23}$ magnetic resonance shift in the metal, relative to a sodium chloride solution, is given in

| Multipolarity | Electric Transition Rate (s$^{-1}$) | Magnetic Transition Rate (s$^{-1}$) |
|---|---|---|
| 1 | $1.0 \times 10^{14} A^{2/3} E_\gamma^3$ | $3.1 \times 10^{13} E_\gamma^3$ |
| 2 | $7.3 \times 10^{7} A^{4/3} E_\gamma^5$ | $2.2 \times 10^{7} A^{2/3} E_\gamma^5$ |
| 3 | $3.4 \times 10^{1} A^{2} E_\gamma^7$ | $1.0 \times 10^{1} A^{4/3} E_\gamma^7$ |
| 4 | $1.1 \times 10^{-5} A^{8/3} E_\gamma^9$ | $3.3 \times 10^{-6} A^{2} E_\gamma^9$ |
| 5 | $2.4 \times 10^{-12} A^{10/3} E_\gamma^{11}$ | $7.4 \times 10^{-13} A^{8/3} E_\gamma^{11}$ |

Table 2.3: Single-particle Weisskopf transition probability estimates as a function of the atomic mass measured in atomic mass units $A$ and the $\gamma$-ray energy $E_\gamma$ in MeV.

$$B_{sp}(EL) = \frac{1.2^{2L}}{4\pi}\left(\frac{3}{L+3}\right)^2 A^{\frac{2L}{3}} e^2 fm^{2L} \tag{2.15}$$

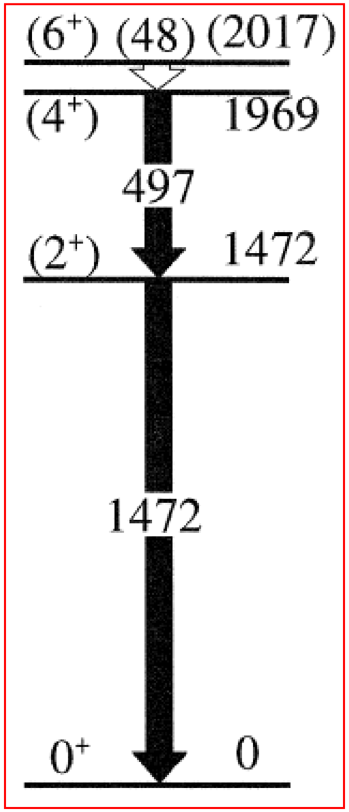
$$B_{sp}(ML) = \frac{10}{\pi} 1.2^{2L-2}\left(\frac{3}{L+3}\right)^2 A^{2L-2} 2\left(\frac{e\hbar}{2Mc}\right)^2 fm^{2L-2} \tag{2.16}$$

# EM Selection Rules and their Effects on Decays

- Allowed decays have:

$$\left|I_i - I_f\right| \leq \lambda \leq \left|I_i - I_f\right|$$

e.g., decays from $I^{\pi} = 6^{+}$ to $I^{\pi} = 4^{+}$ are allowed to proceed with photons carrying angular momentum of $\lambda = 2,3,4,5,6,7,8,9$ and $10\,\hbar$.



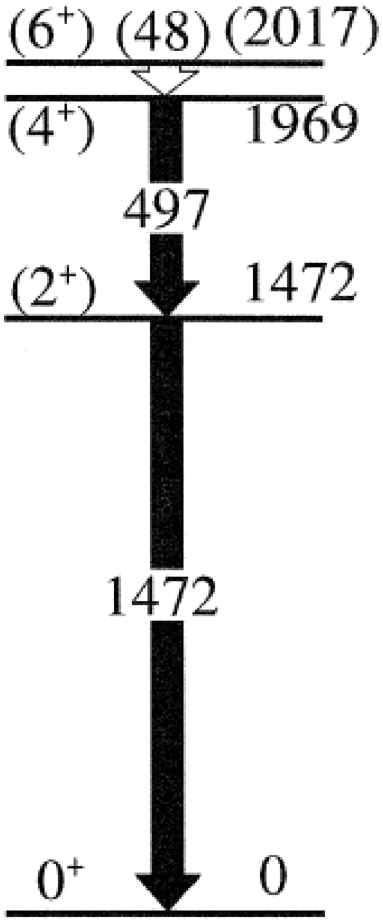

e.g., $^{102}Sn_{52}$

Why do we only observe the E2 decays ?

Are the other multipolarity decays allowed / present ?

Need also to conserve parity between intial and final states, thus, here the transition can not change the parity.

This adds a further restriction : Allowed decays now restricted to E2, E4, E6, E8 and E10 ; and M3, M5, M7, M9




$^{102}$Sn

| $E_\gamma$ | E2 (1Wu) | M3 (1Wu) | E4 (1Wu) |
|---|---|---|---|
| 48 ($6^+ \to 4^+$) | 112μs | 782,822 s | 2.5E+14s |
| 555 ($6^+ \to 2^+$) | | | 66,912s |

| | | | |
|---|---|---|---|
| 497 ($4^+ \to 2^+$) | 0.9ns | 61ms | 180,692s |
| 1969 ($4^+ \to 0^+$) | | | 751ms |

Conclusion, in general see a cascade of (stretched) E2 decays in near-magic even-even nuclei.

# Weisskopf single-particle estimates

*$\tau_{sp}$ for 1 Wu at A~100 and $E_\gamma$ = 200 keV*

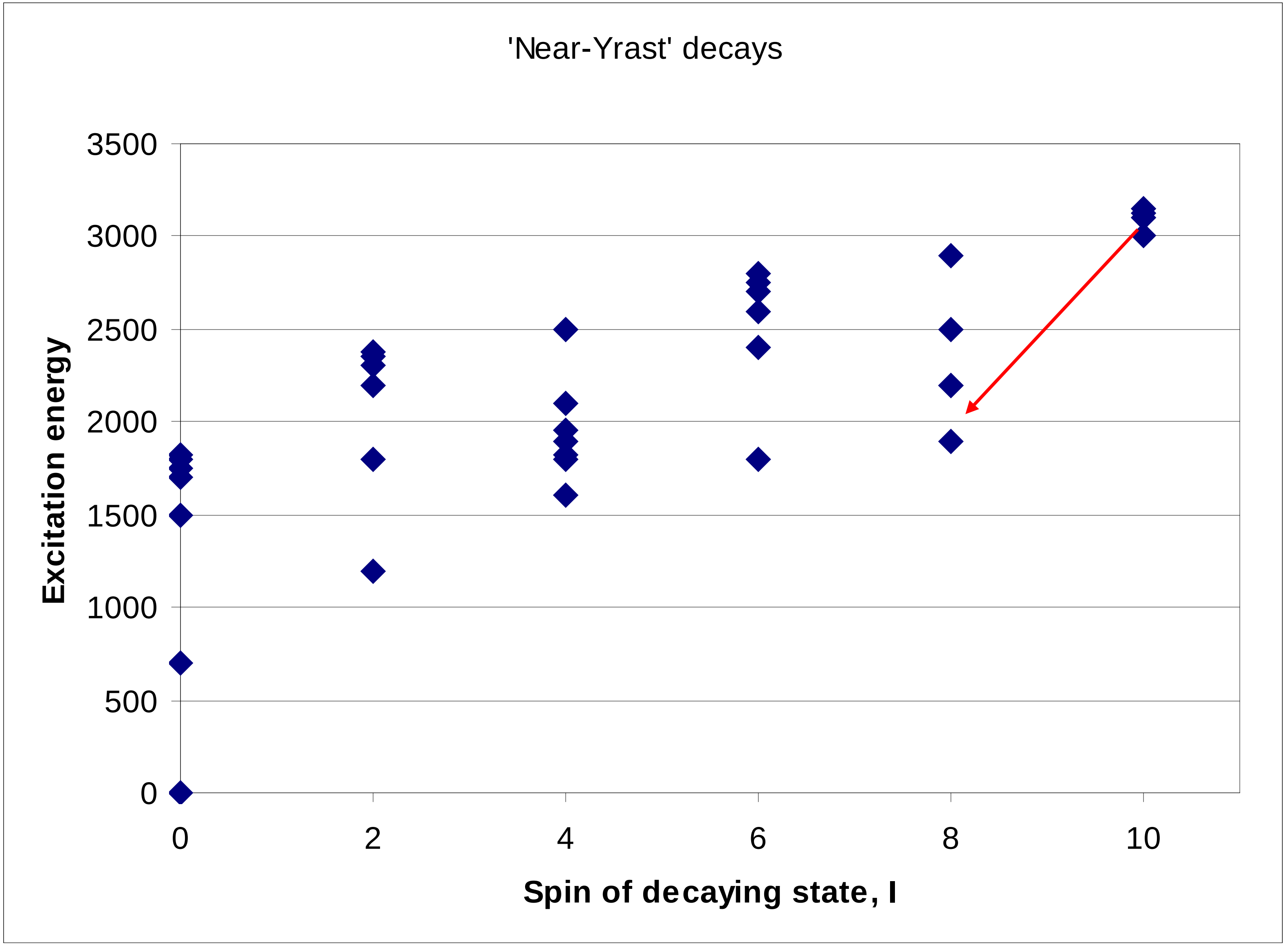
| | | |
|---|---|---|
| M1<br>2.2 ps | M2<br>4.1 ms | M3<br>36 s |
| E1<br>5.8 fs | E2<br>92 ns | E3<br>0.2 s |

*The lowest order multipole decays are highly favoured.*

*BUT need to conserve angular momentum so need at minimum $\lambda = I_i - I_f$ is needed for the transition to take place.*

*Note, for low $E_\gamma$ and high - $\lambda$, internal conversion also competes/dominates.*

The EM transition rate depends on $E_\gamma^{2\lambda+1}$ ; the highest energy transitions for the lowest $\lambda$ are (usually) favoured.
This results in the preferential population of yrast and near-yrast states.



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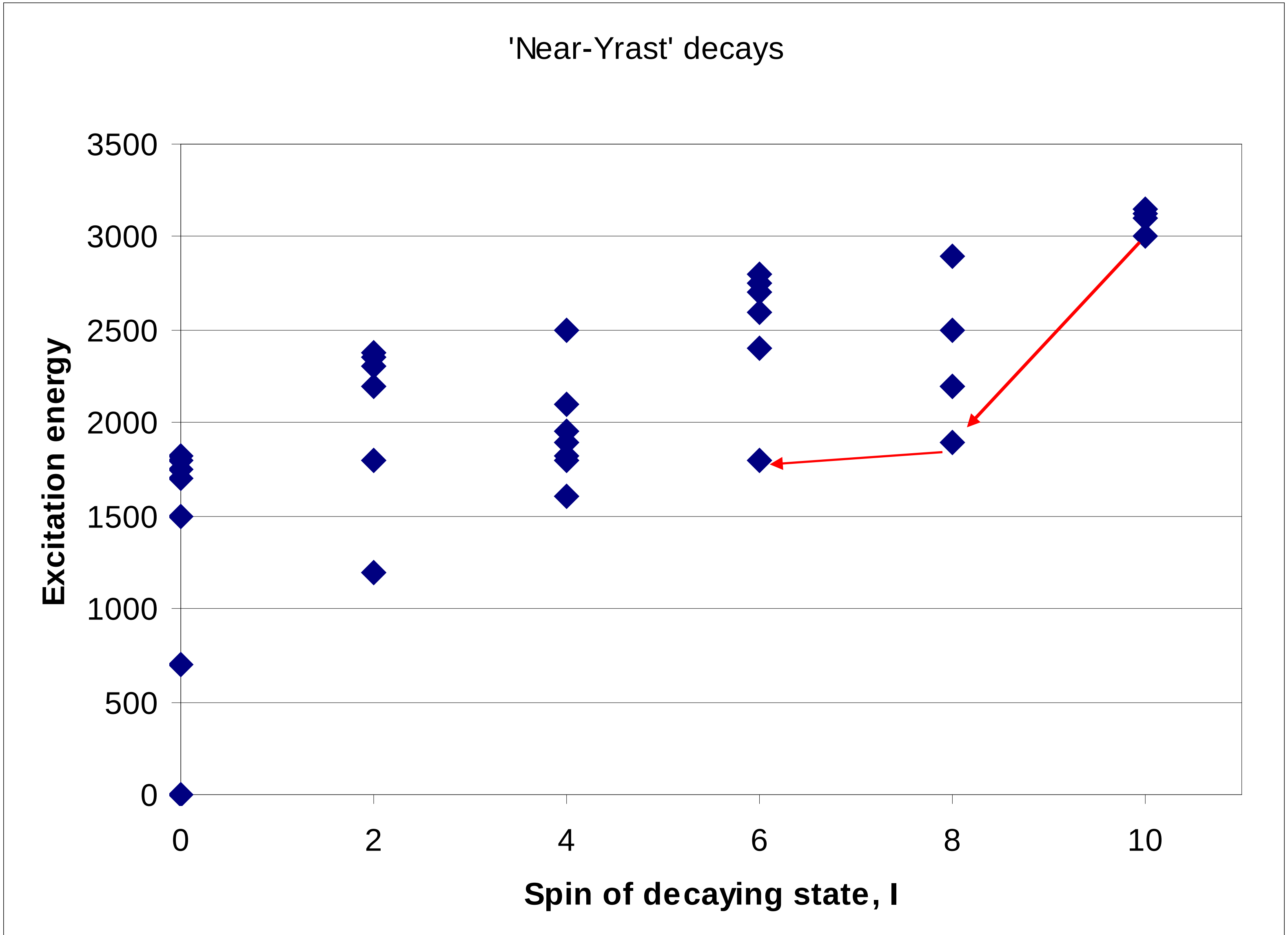

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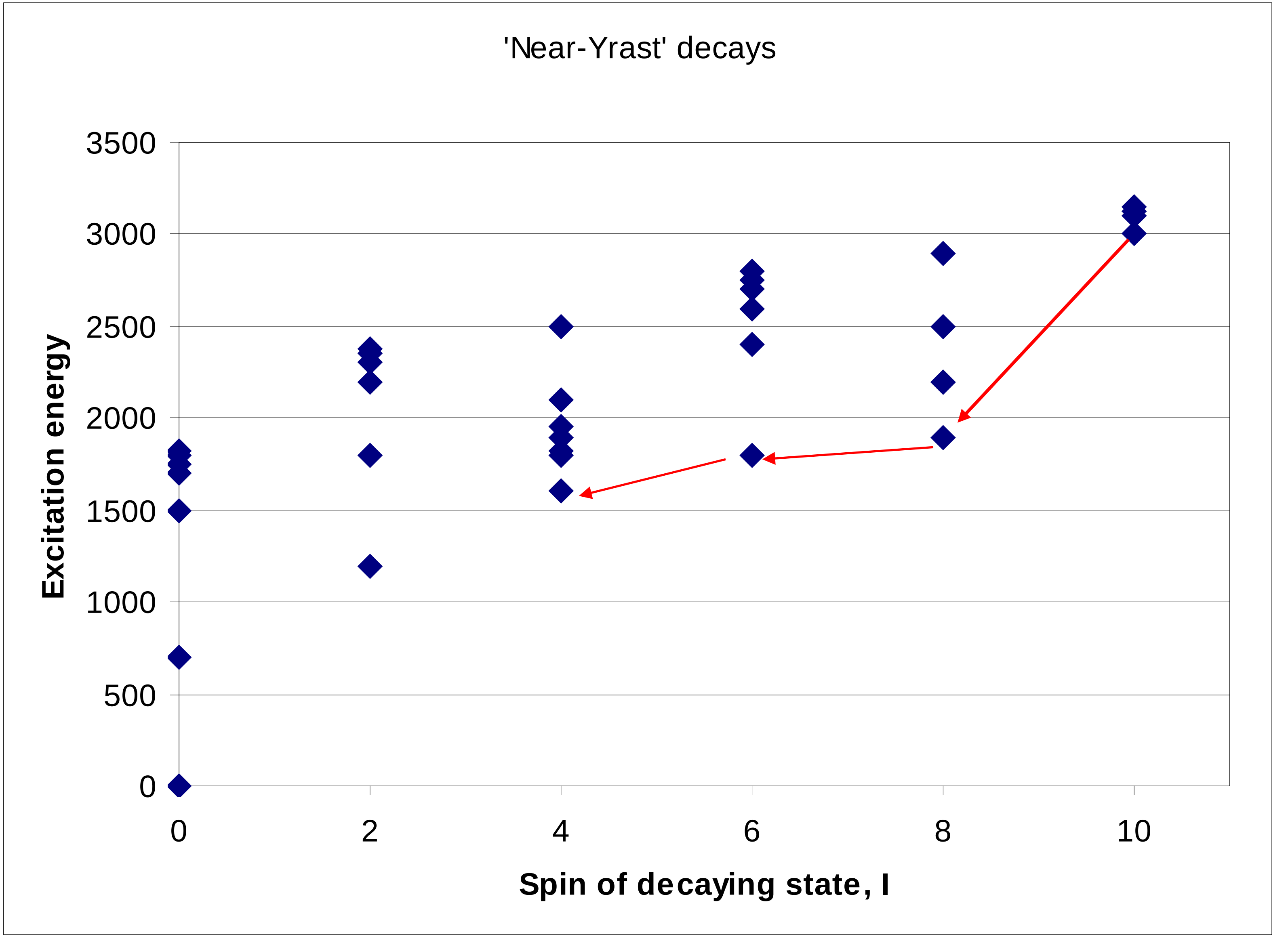

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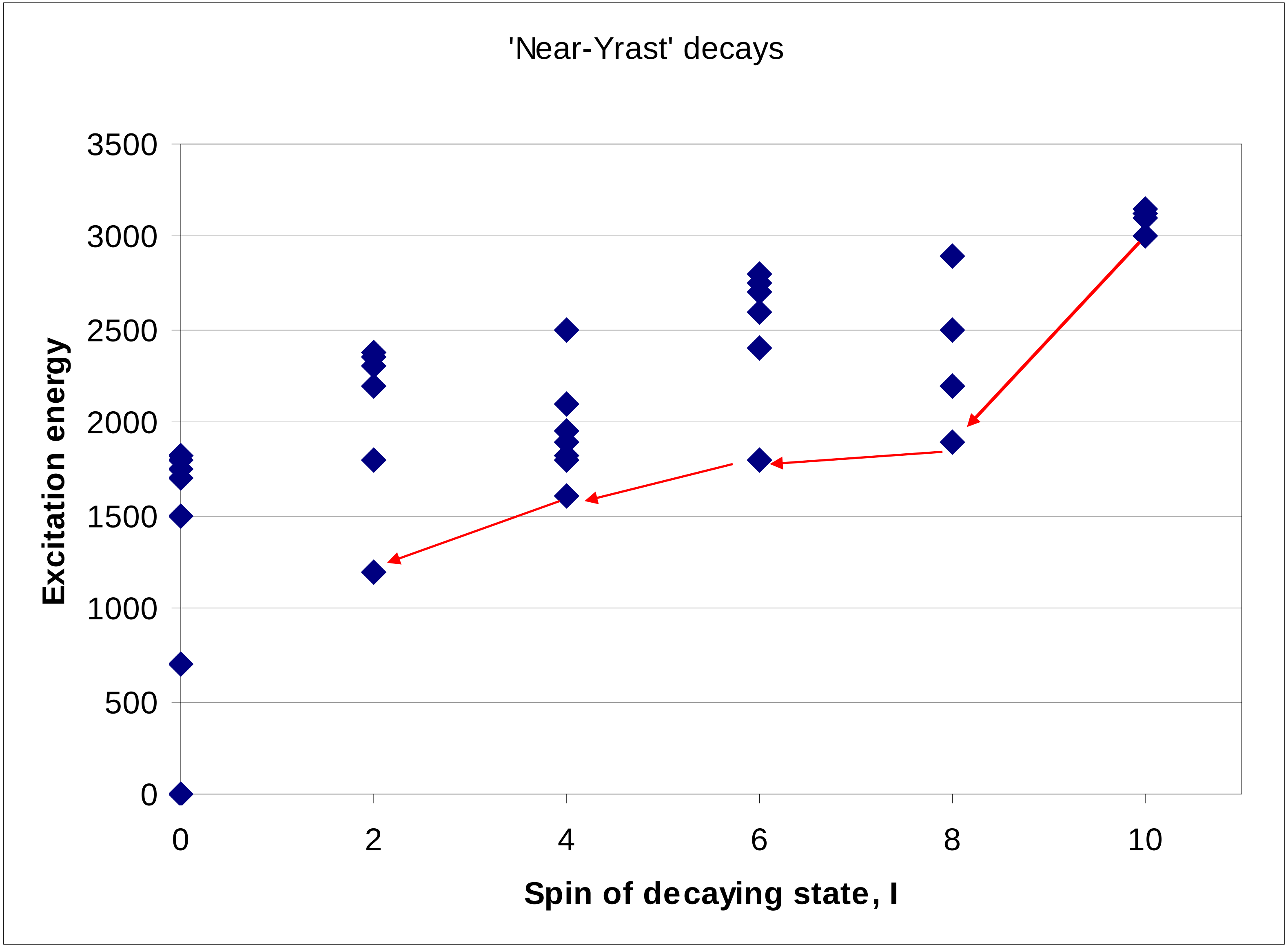

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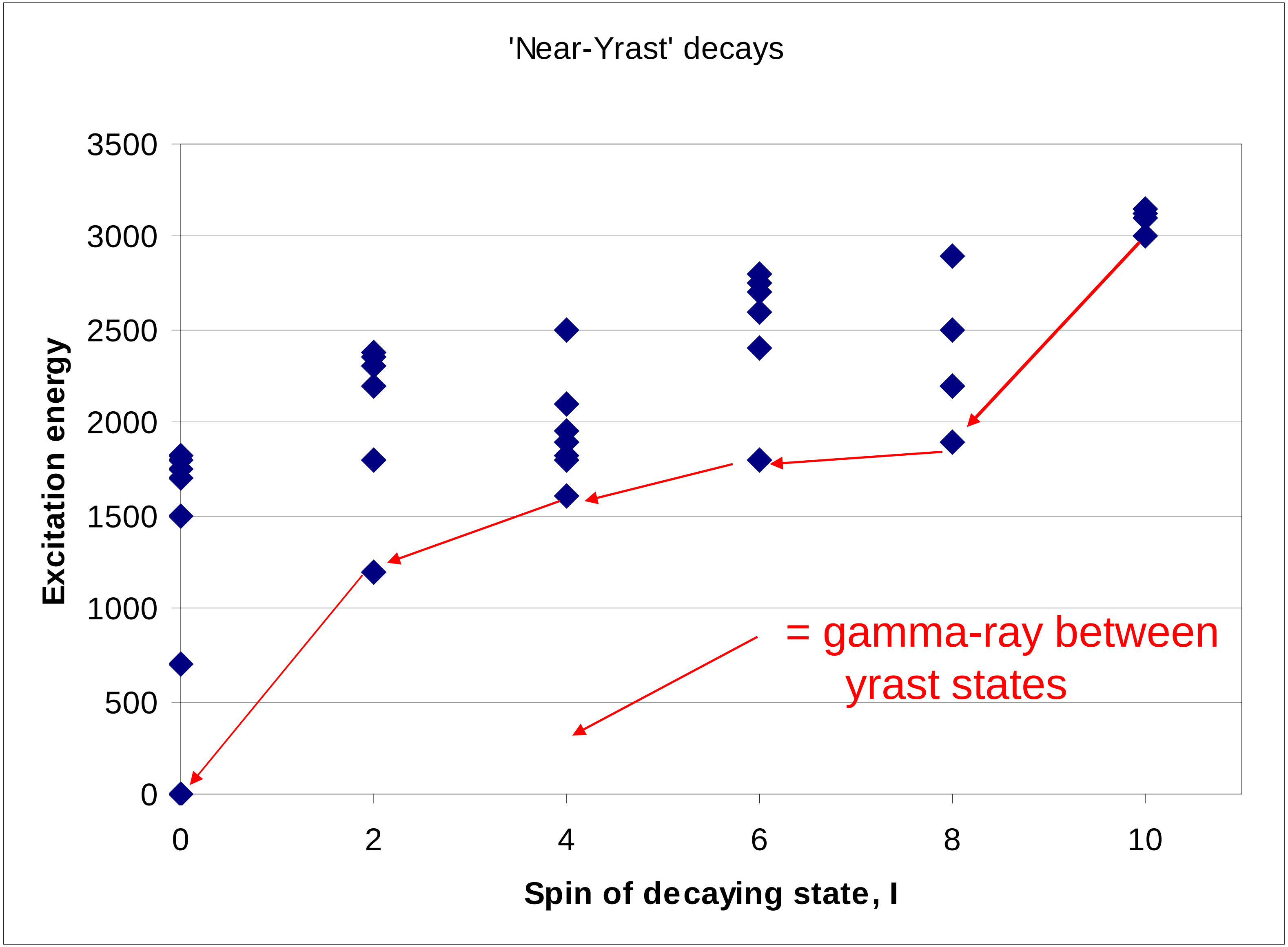
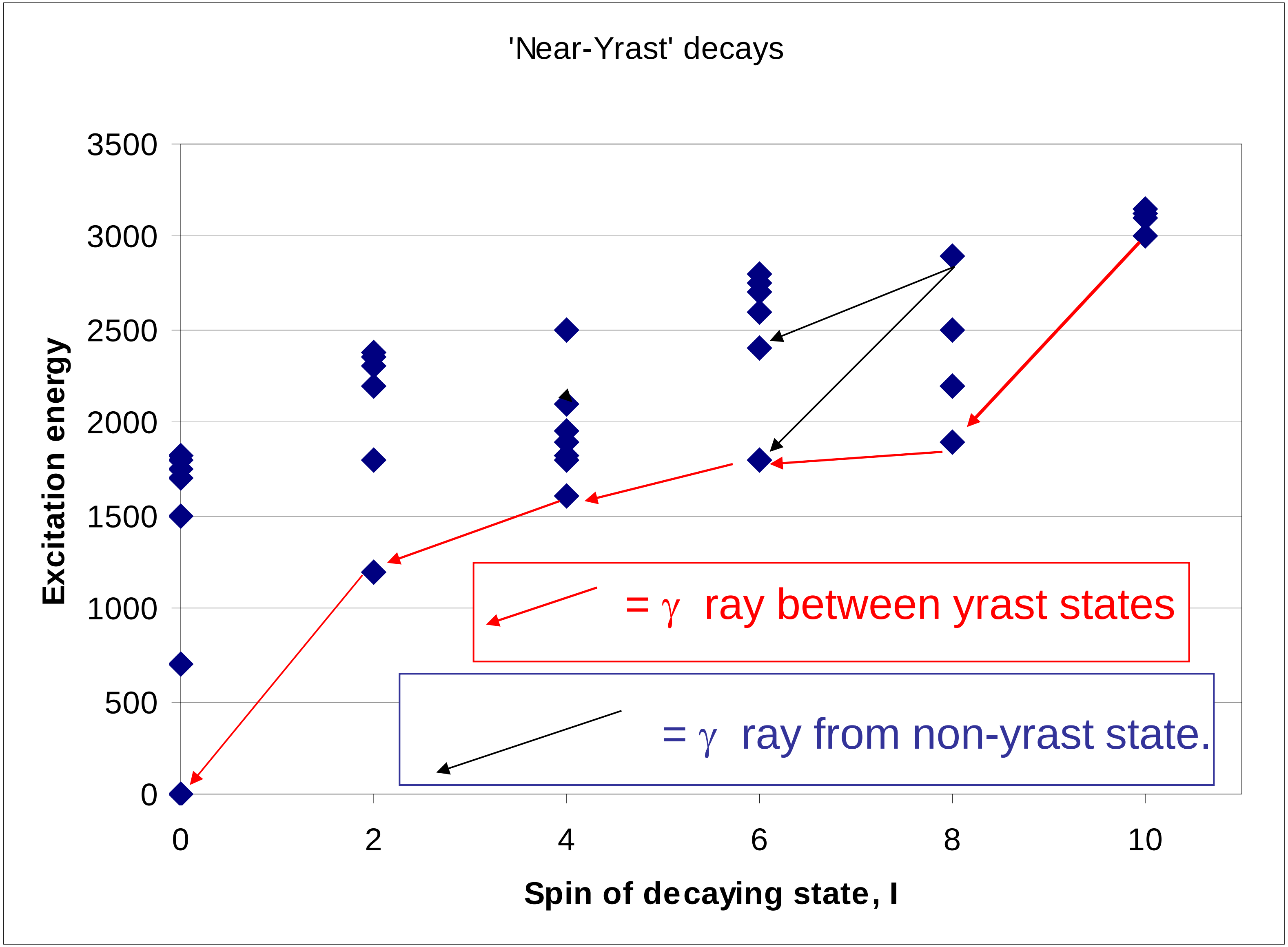


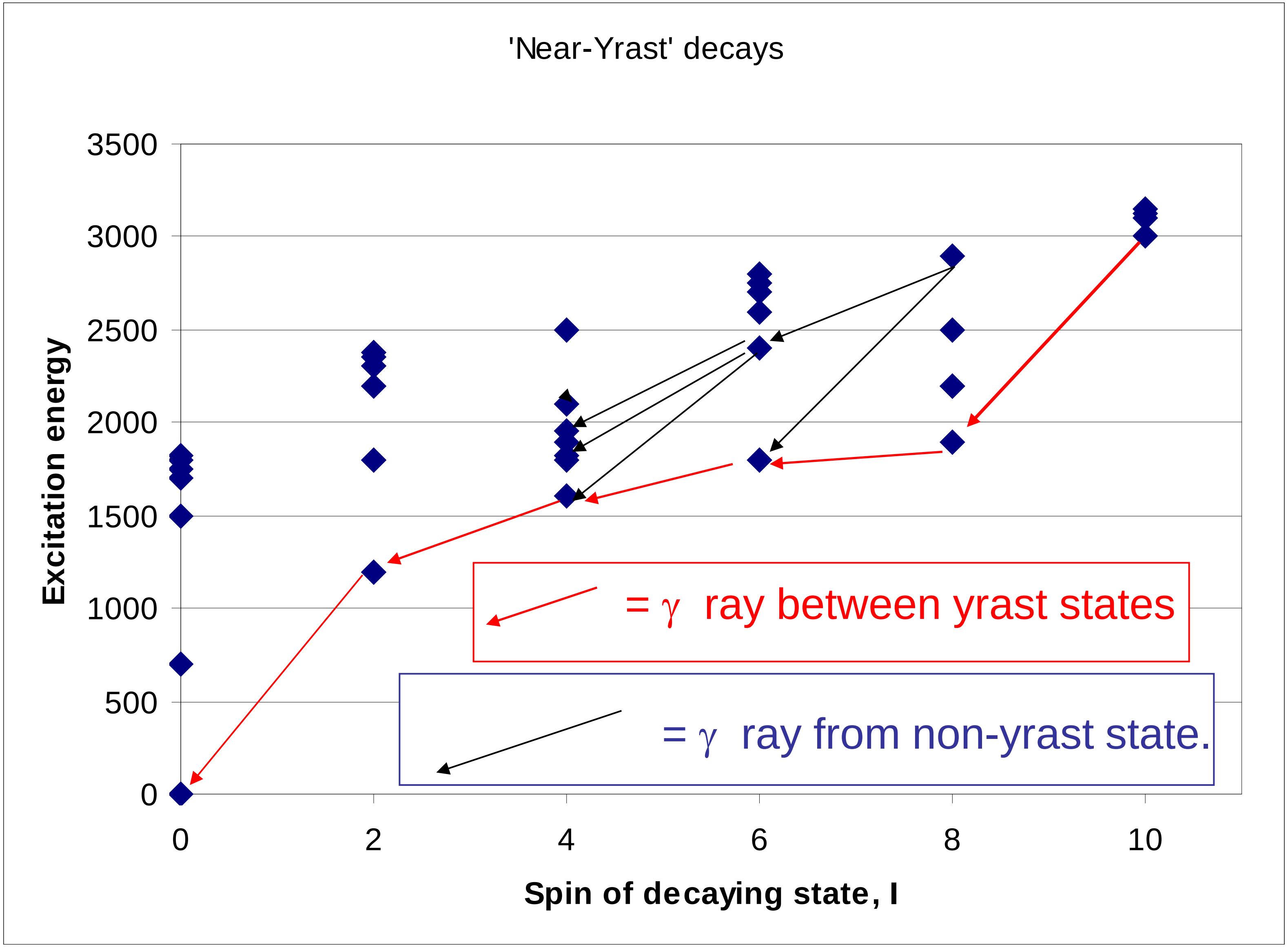
The EM transition rate depends on $E\gamma^{2\lambda+1}$, (for E2 decays $E_\gamma^5$)
Thus, the highest energy transitions for the lowest $\lambda$ are usually favoured.
Non-yrast states decay to yrast ones (unless very different $\phi$, K-isomers)



The EM transition rate depends on $E\gamma^{2\lambda+1}$, (for E2 decays $E_\gamma^5$)

Thus, the highest energy transitions for the lowest $\lambda$ are usually favoured.

Non-yrast states decay to yrast ones (unless very different $\phi$, K-isomers etc.)



The EM transition rate depends on $E\gamma^{2\lambda+1}$, (for E2 decays $E_\gamma^5$)
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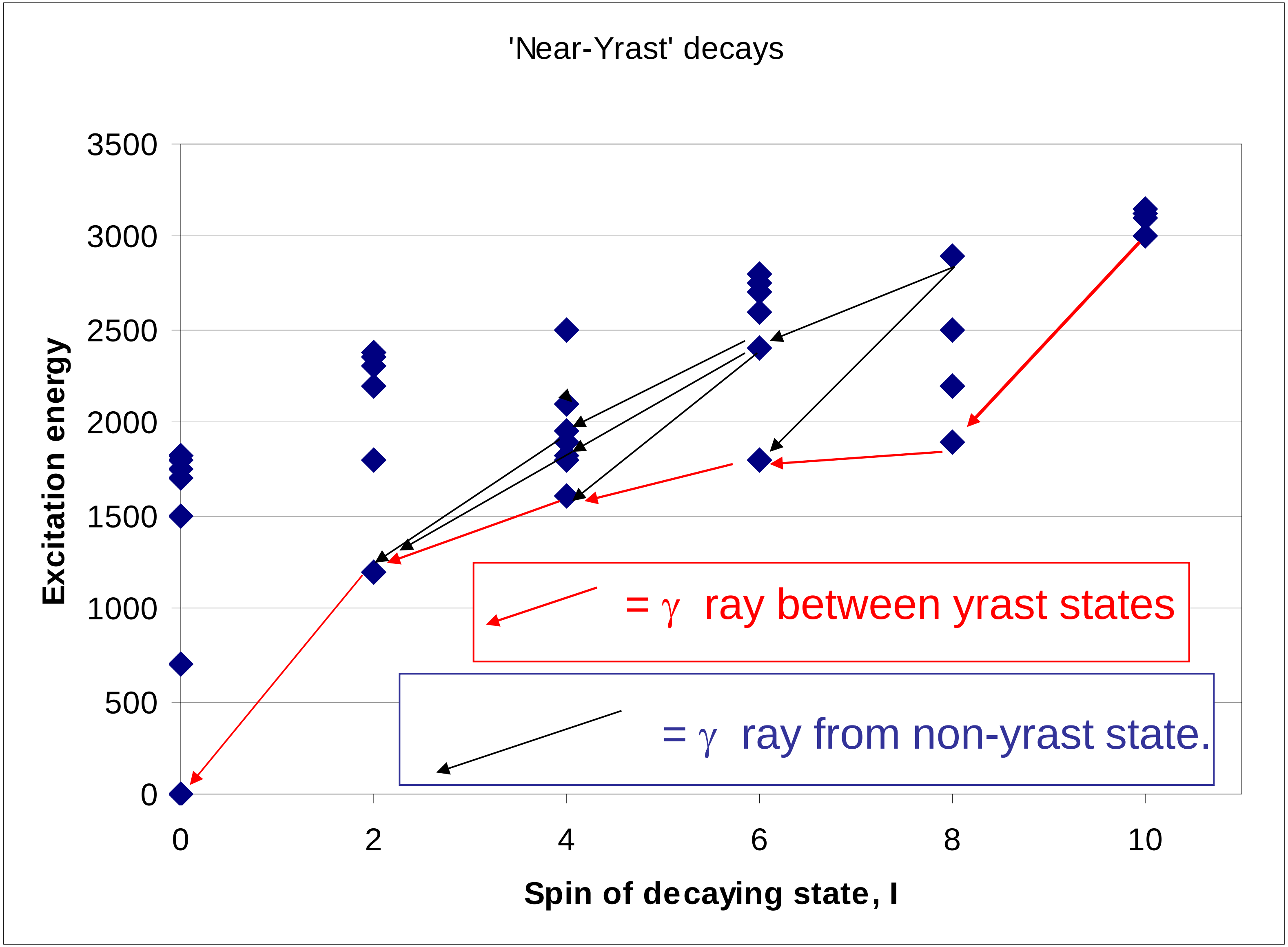
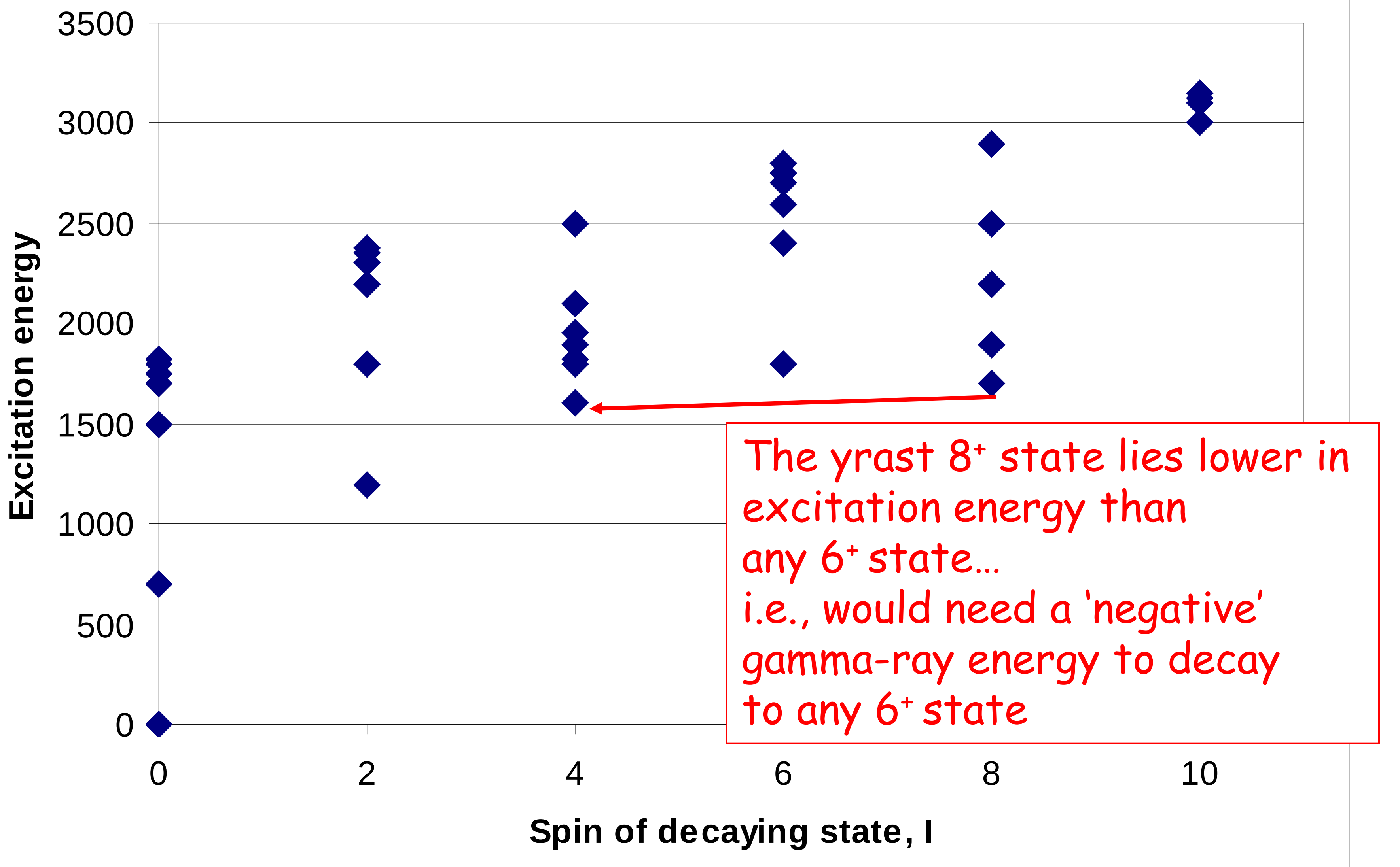
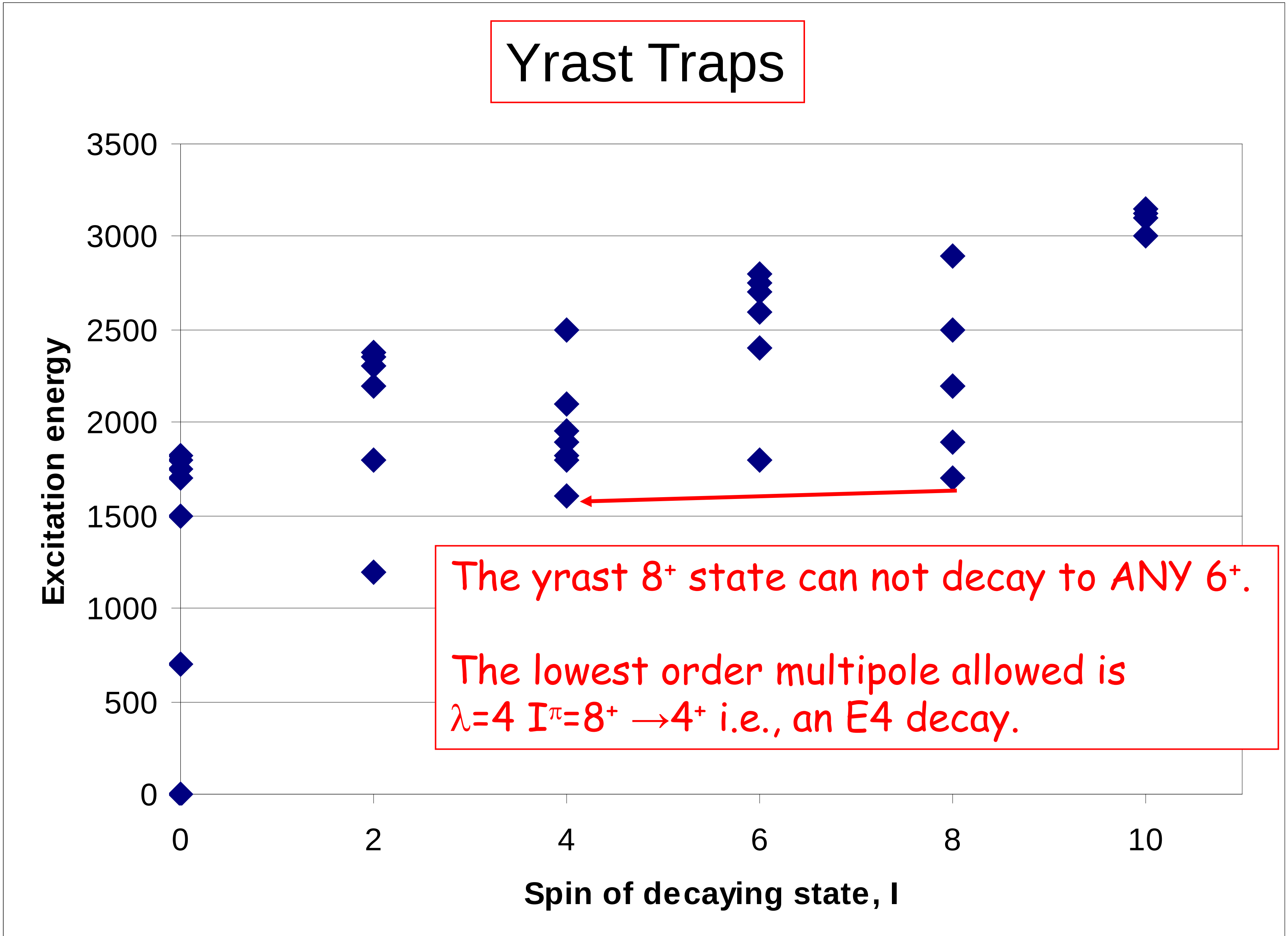
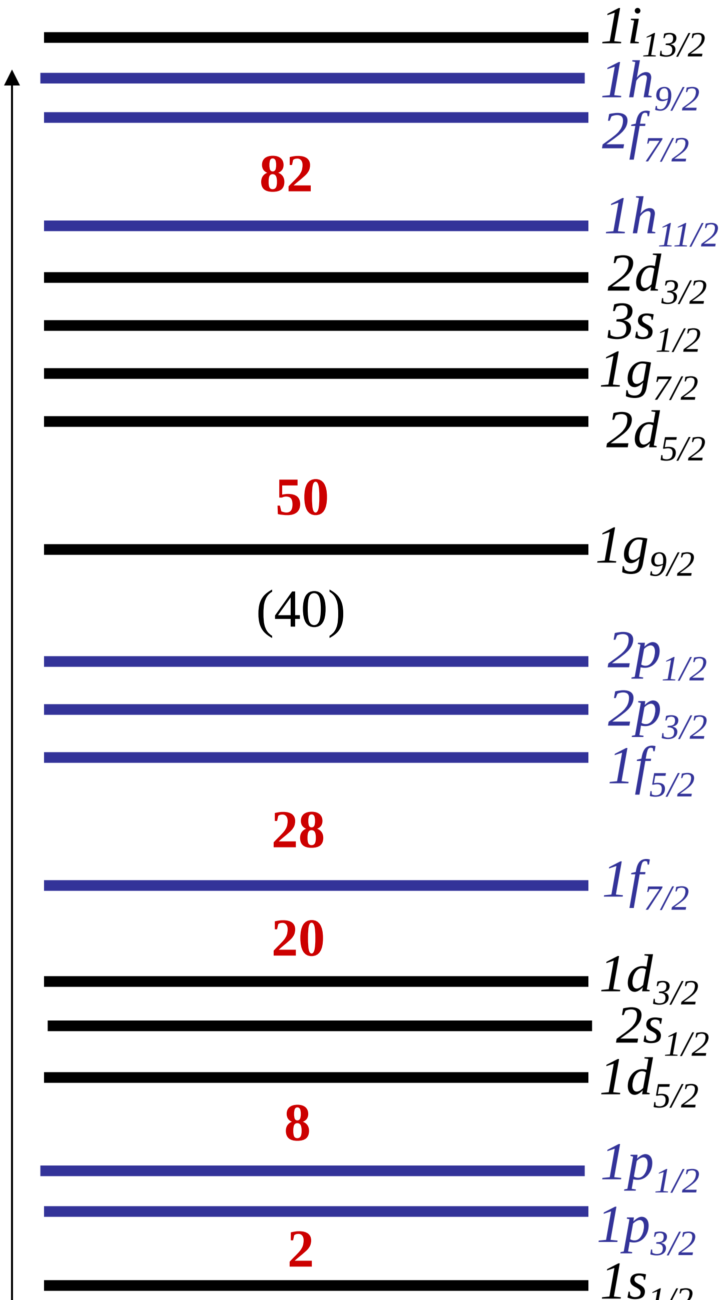


# Yrast Traps


3500
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
Excitation energy
Spin of decaying state, I
The yrast 8+ state lies lower in
excitation energy than
any 6+ state…
i.e., would need a 'negative'
gamma-ray energy to decay
to any 6+ state


Yrast Traps
3500
3000
2500
2000
1500
1000
500
0
Excitation energy
0
2
4
6
8
10
Spin of decaying state, I
The yrast $8^+$ state can not decay to ANY $6^+$.
The lowest order multipole allowed is
$\lambda=4$ $I^\pi=8^+ \rightarrow 4^+$ i.e., an E4 decay.

‘single-particle’-like transitions.




V= SHO + $l^2$.+ ***l.s.***

Basic, independent particle model (with very simple residual interactions added, such as δ - (contact) interaction) predicts large host of ***isomers in the vicinity of closed shells / magic numbers***.

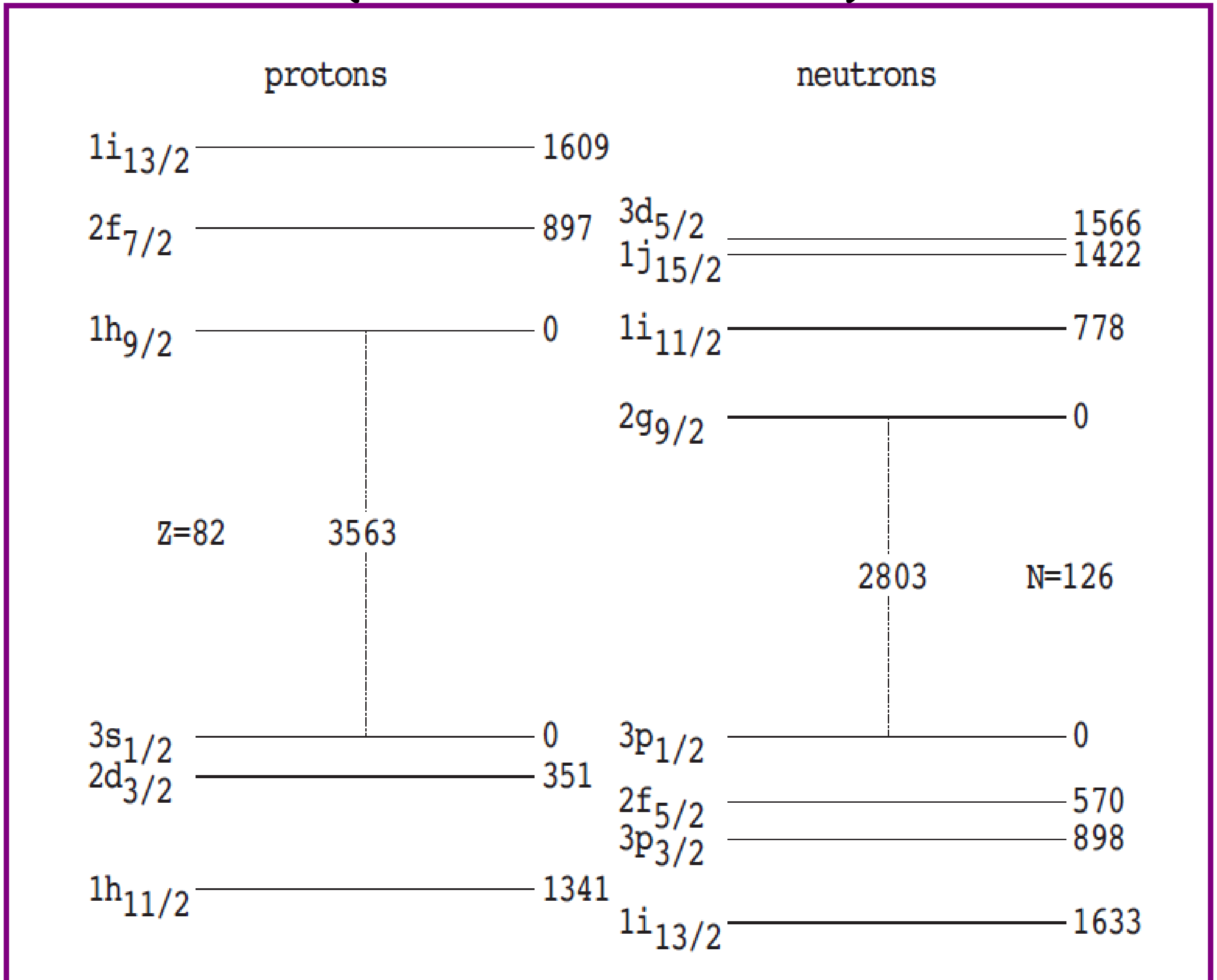
Two categories

**1)** **Spin-trap isomers** - from particularly favoured coupling of (often high-j intruder) particles gives rise to high-spin state at low excitation energy. This state ‘has nowhere to decay to’ unless decays by high multipolarity (thus slow) transition.

$$|J_i+J_f| > \Delta J > |J_i-J_f|$$

2) **Seniority isomers** – δ-interaction can demonstrate with geometric picture how (single) jn multiplet looks like $j^2$ multiplet. Small energy difference between $J_{max}$ and ($J_{max}$-2) states cause ‘seniority isomers’.

# Relative energies of orbits close to $^{208}$Pb (from DWK2016)



Physics Letters B 672 (2009) 116–119

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# Proton–hole excitation in the closed shell nucleus $^{205}$Au

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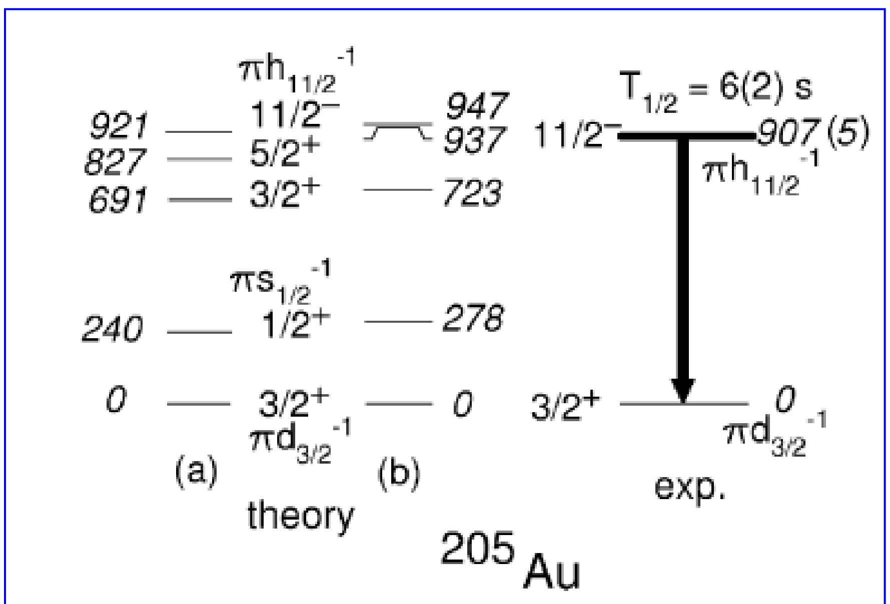
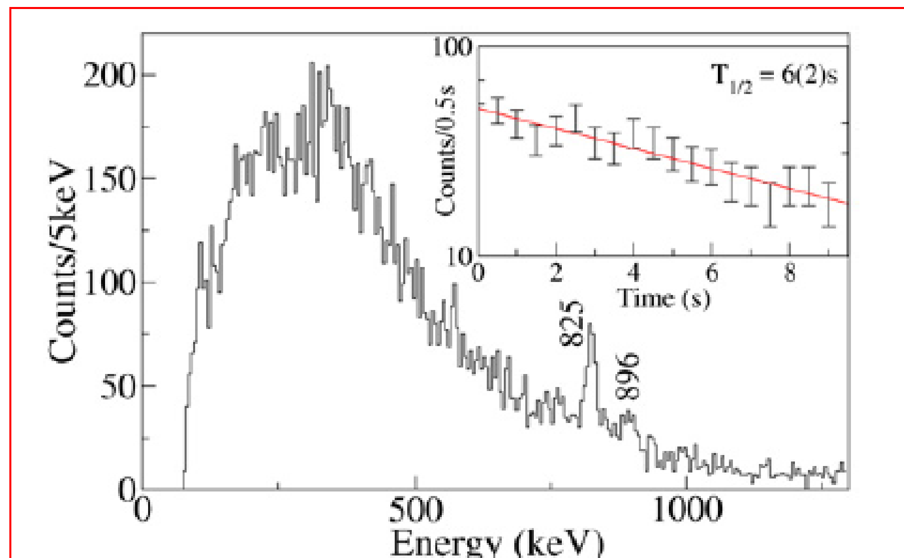
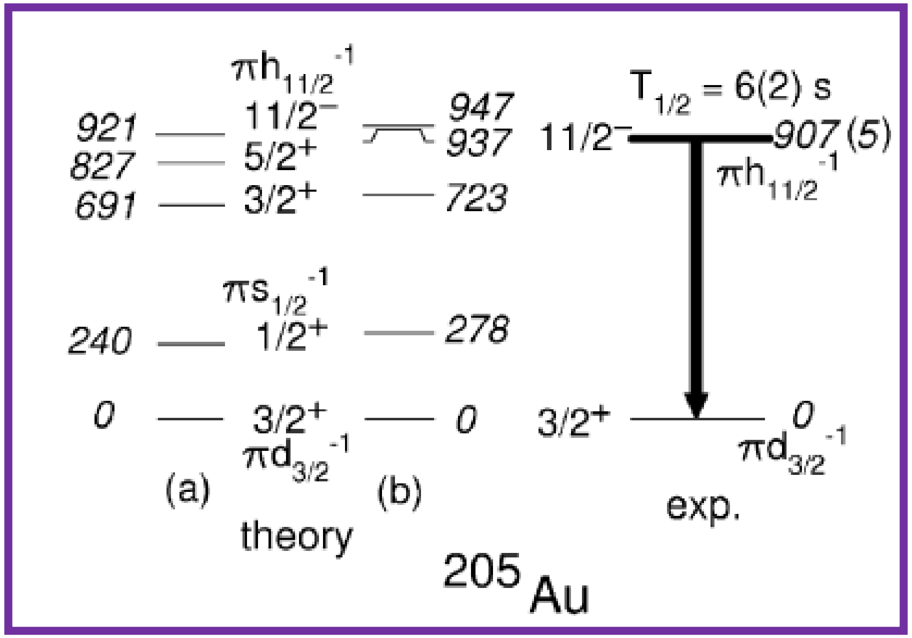
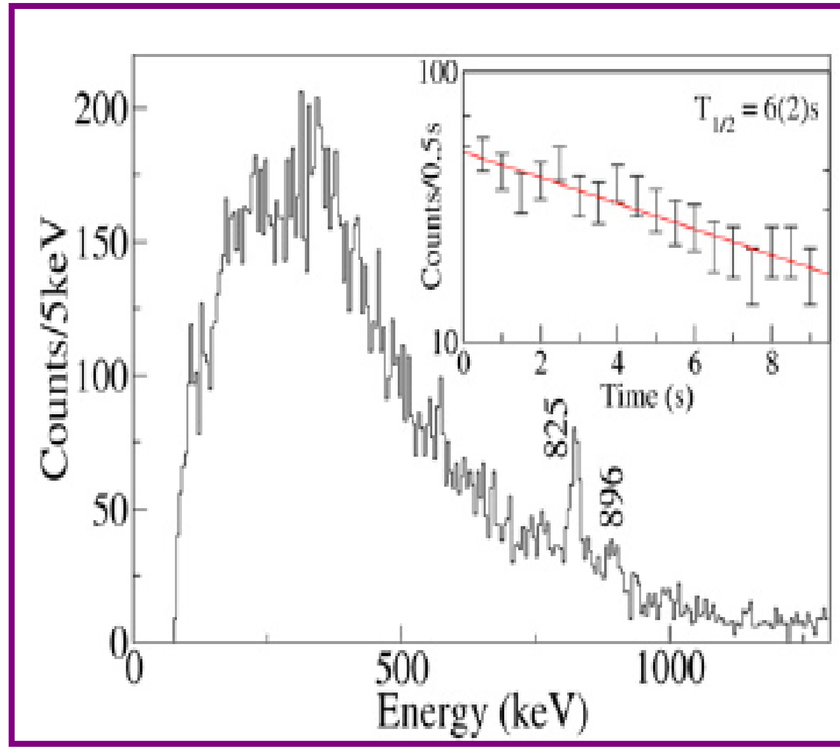


Zs. Podolyak et al., Phys. Lett. **B672** (2009) 116



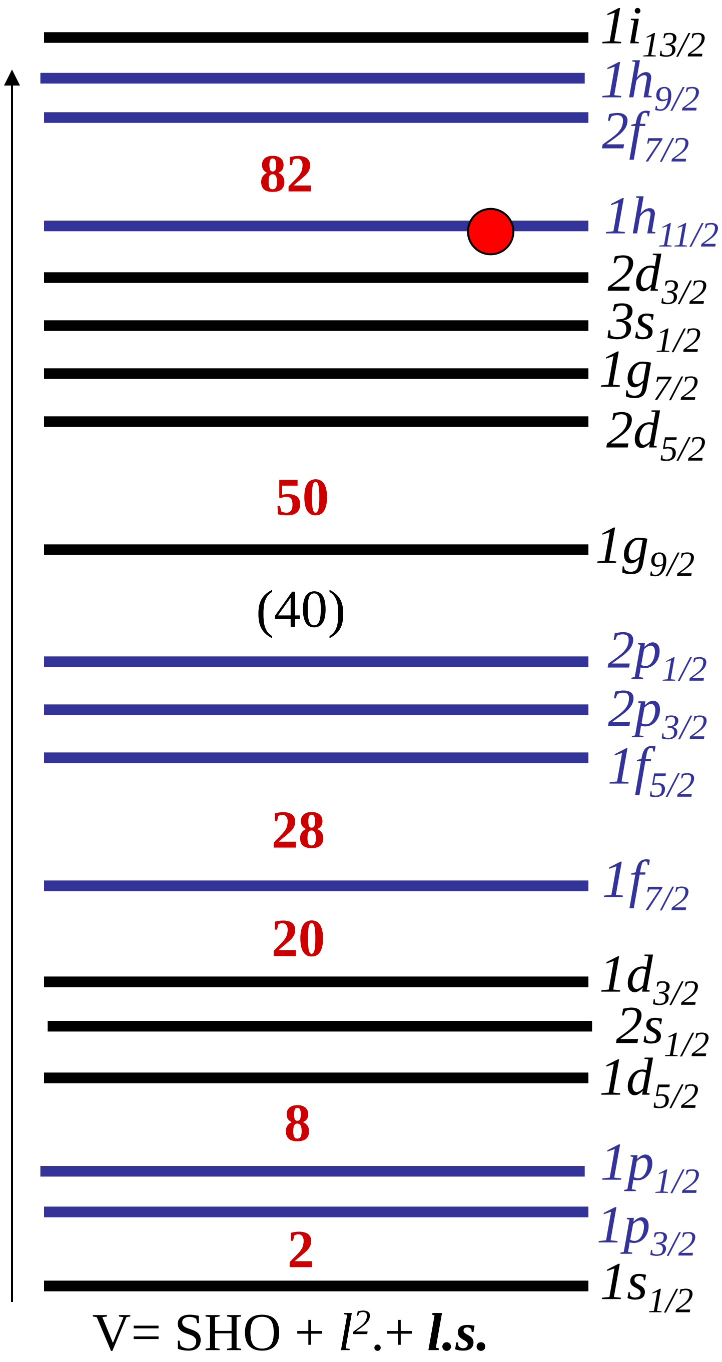

N=126 ; Z=79. Odd, single proton transition; $h_{11/2} \rightarrow d_{3/2}$ state (holes in Z=82 shell).

Angular momentum selection rule says lowest multipole decay allowed is
**$\lambda$ = ( 11/2 - 3/2 ) = $\Delta$ L = 4**

Change of parity means lowest must transition be M4.

1Wu 907 keV M4 in $^{205}$Au has $T_{1/2}$= 8 secs; corresponding to a near 'pure' single-particle (proton) transition from ($h_{11/2}$) $11/2^-$ state to ($d_{3/2}$) $3/2^+$ state.

(Decay here is observed following INTERNAL CONVERSION).
These competing decays to gamma emission are often observed in isomeric decays




## Why are E1 transitions usually isomeric?

E1 single particle decays need to proceed between orbitals which have Δ L=1 and change parity, e.g.,

$f_{7/2}$ and $d_{5/2}$

or $g_{9/2}$ and $f_{7/2}$

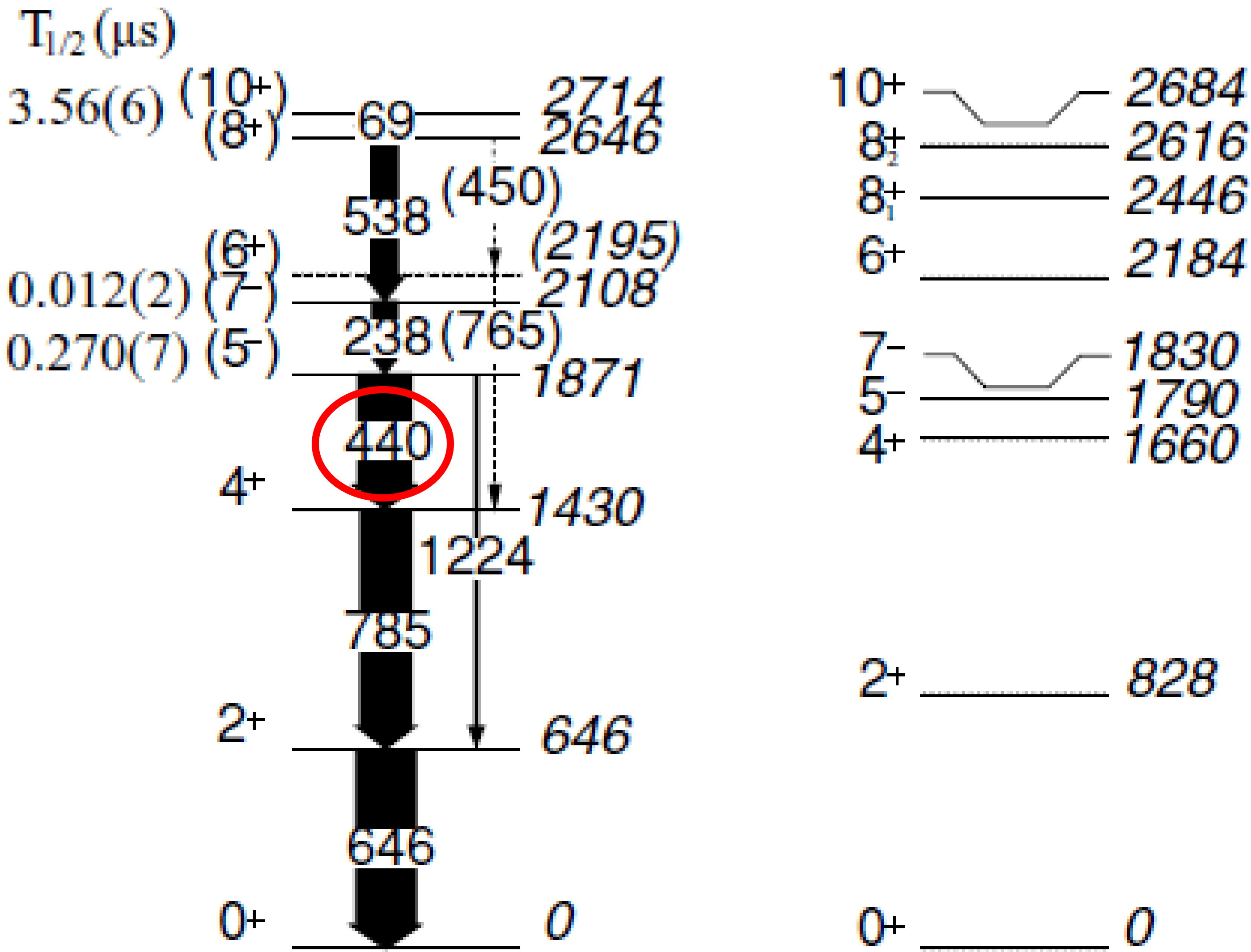
or $h_{11/2}$ and $g_{9/2}$

or $p_{3/2}$ and $d_{5/2}$

What about typical 2-particle configs. e.g.,

$I^{\pi}=5^{-}$ from $(h_{11/2})^{-1} \times (s_{1/2})^{-1}$

$I^{\pi}=4^{+}$ from $(d_{3/2})^{-1} \times (s_{1/2})^{-1}$




e.g., $^{128}Cd_{80}$, isomeric 440 keV E1 decay.

A 1 Wu 440 keV E1 should have $T_{1/2}$ ~ $4 \times 10^{-15}$s; actually has 270 ns (i..e hindered by ~$10^{8}$ )

L. Cáceres,[1,2,*] M. Górska,[1]

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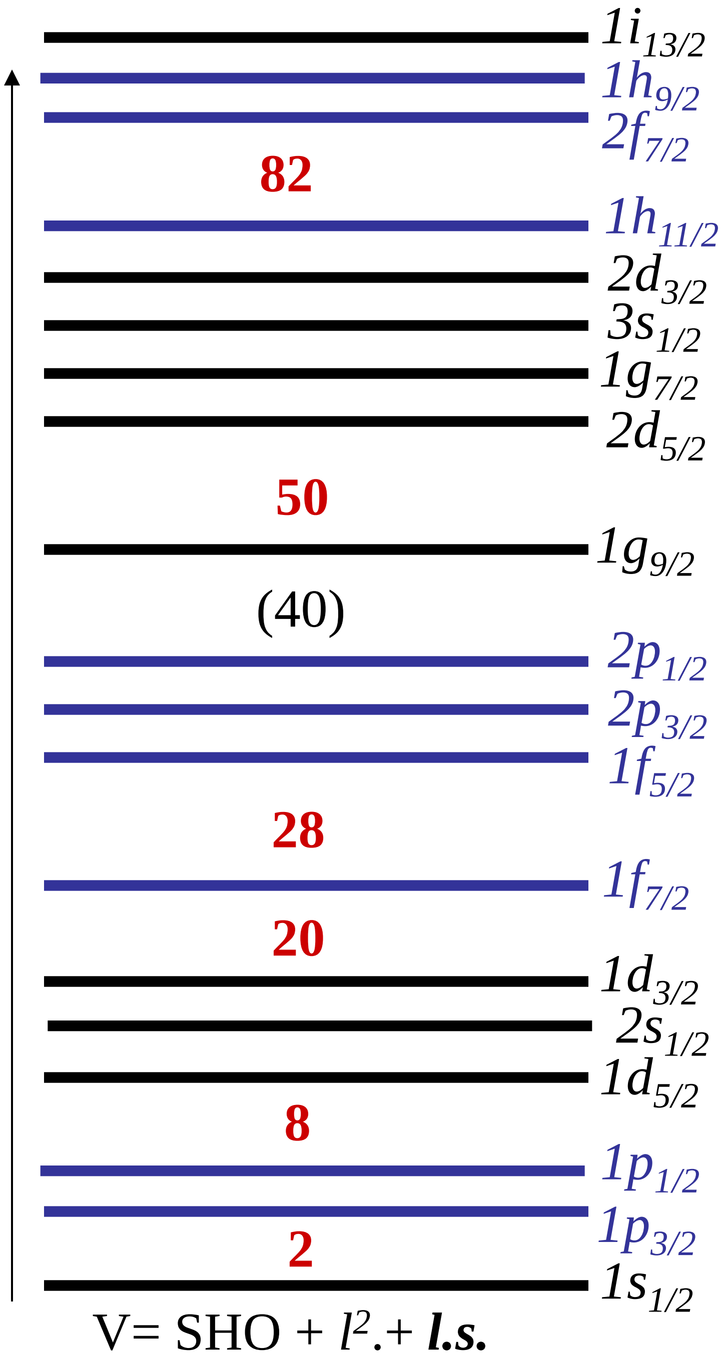



# Why are E1 s isomeric?

E1s often are observed with decay probabilities of $10^{-5} \rightarrow 10^{-9}$ Wu

E1 single particle decays need to proceed between orbitals which have $\Delta L = 1$ and change parity, e.g.,

$f_{7/2}$ and $d_{5/2}$

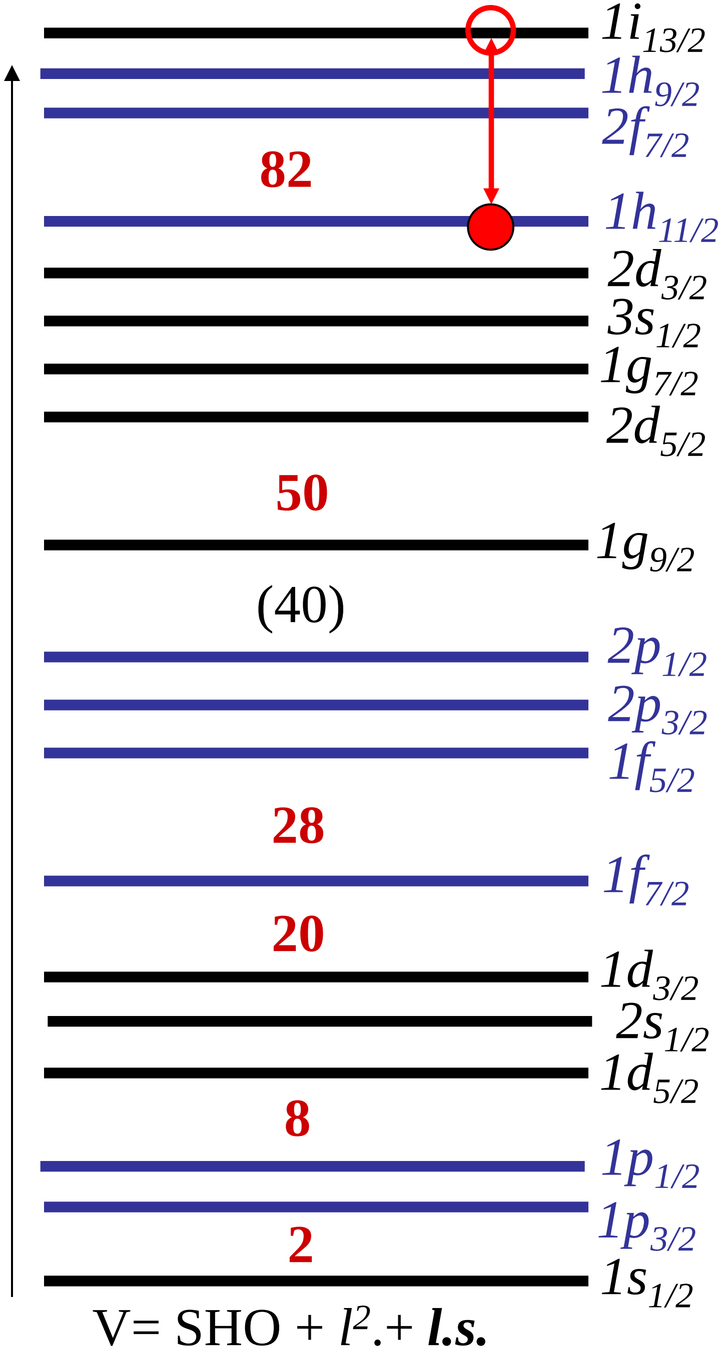
or $g_{9/2}$ and $f_{7/2}$

or $h_{11/2}$ and $g_{9/2}$

or $i_{13/2}$ and $h_{11/2}$

or $p_{3/2}$ and $d_{5/2}$

BUT these orbitals are along way from each other in terms of energy / other orbitals between them in the (spherical) mean-field single-particle spectrum.




# Why are E1 s isomeric?

E1 single particle decays need to proceed between orbitals which have $\Delta$ L=1 and change parity, e.g.,

What about typical 2-particle configs. e.g.,

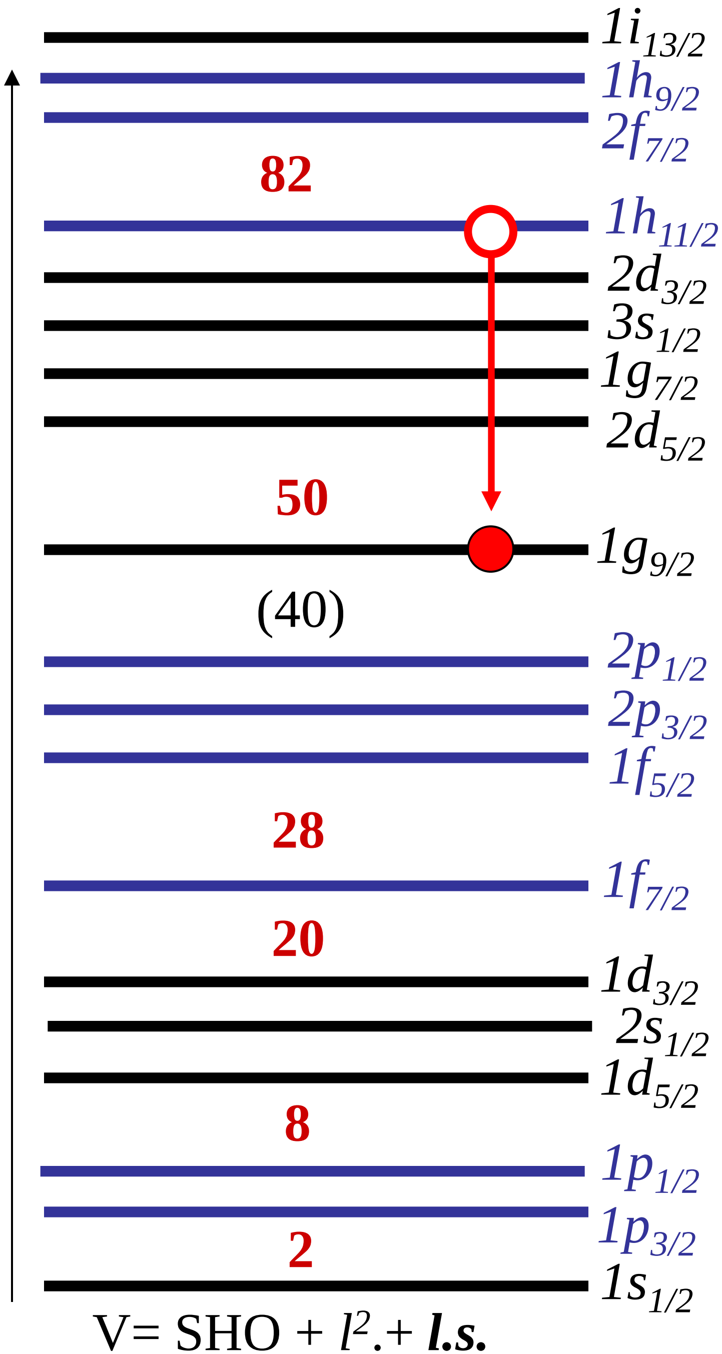
$I^{\pi}=5^{-}$ from mostly $(h_{11/2})^{-1} \times (s_{1/2})^{-1}$

$I^{\pi}=4^{+}$ from mostly $(d_{3/2})^{-1} \times (s_{1/2})^{-1}$

No E1 'allowed' between such orbitals.

E1 occur due to (very) small fractions of the wavefunction from orbitals orginating in higher shells.

Small overlap wavefunction in multipole matrix element allows 'slow' E1s to proceed.




## Why are E1 s isomeric?

E1s often observed with decay probabilities Of $10^{-5} \rightarrow 10^{-8}$ Wu

E1 single particle decays need to proceed between orbitals which have $\Delta$ L=1 and change parity, e.g.,

$f_{7/2}$ and $d_{5/2}$

or $g_{9/2}$ and $f_{7/2}$

or $h_{11/2}$ and $g_{9/2}$

or $p_{3/2}$ and $d_{5/2}$

BUT these orbitals are along way from each other in terms of energy in the mean-field single particle spectrum.

# More complex nuclei.

Simple signatures of nuclear structure such as $E(2^+)$ and $R(4/2)$ can help show us which regions of the nuclear chart are best explained by:

- Spherical 'single-particle' excitations

or

- Quadrupole deformed regions (Nilsson model)

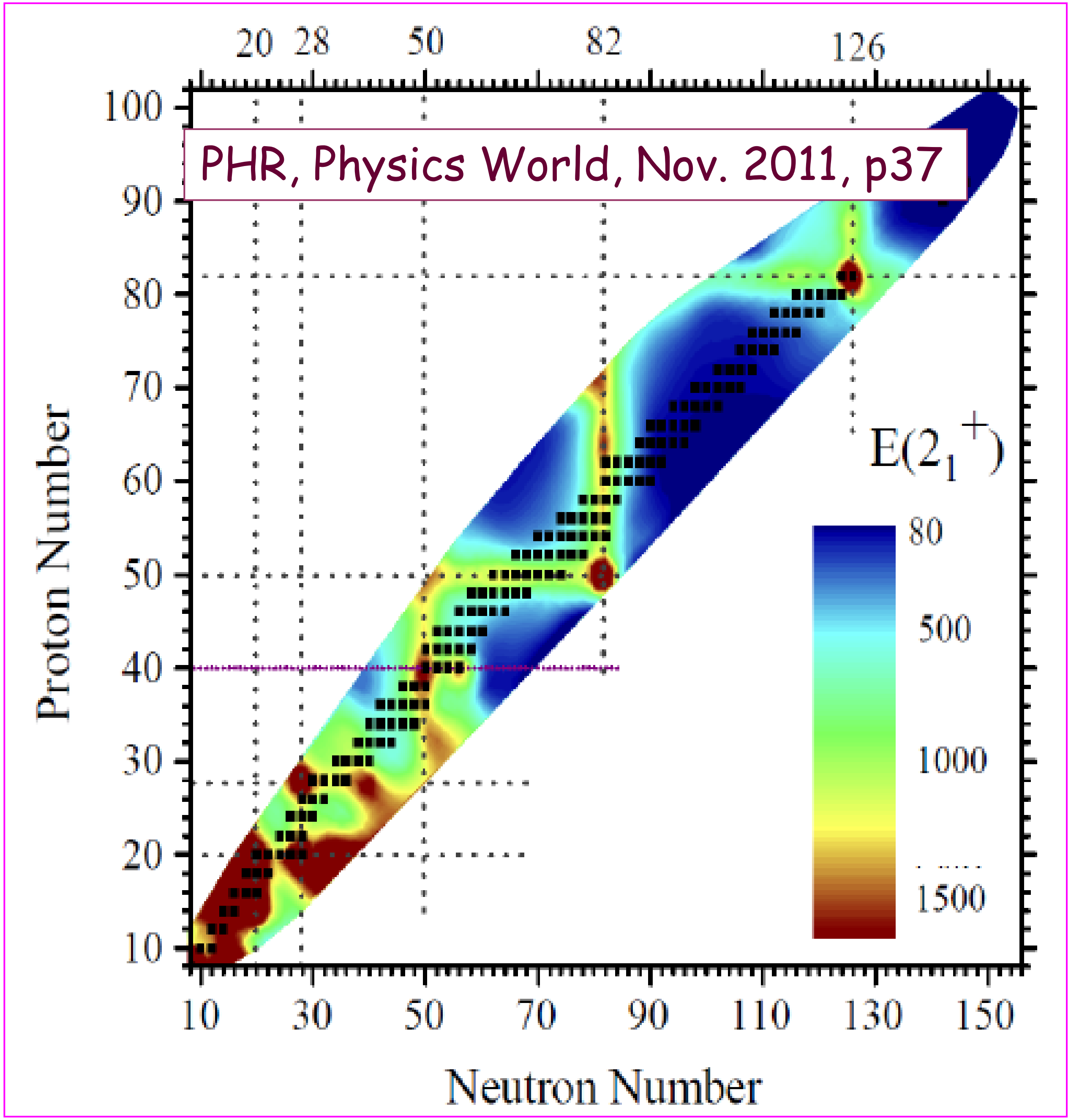
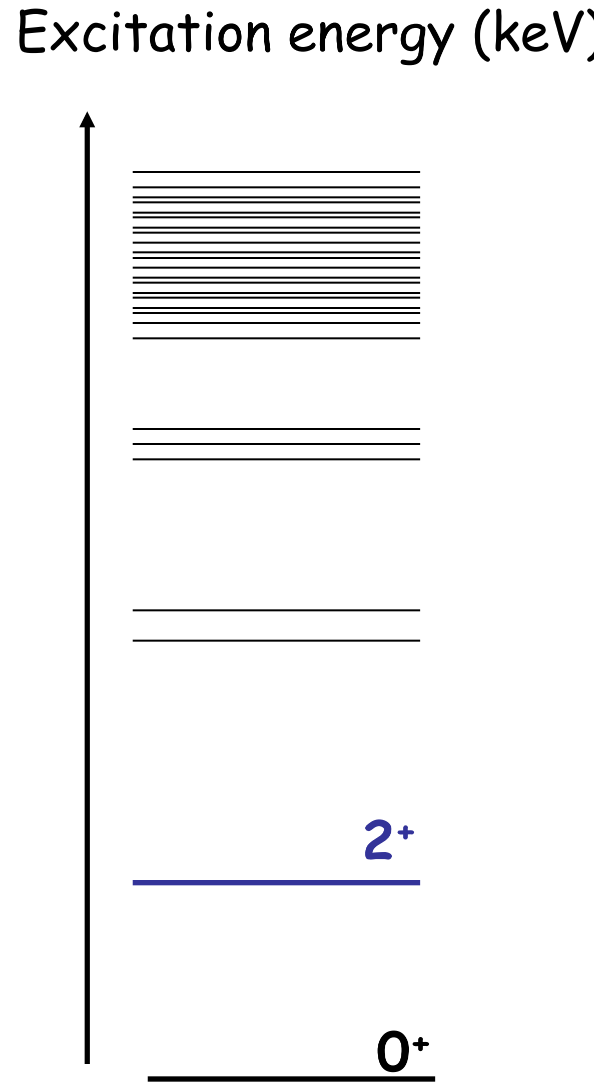
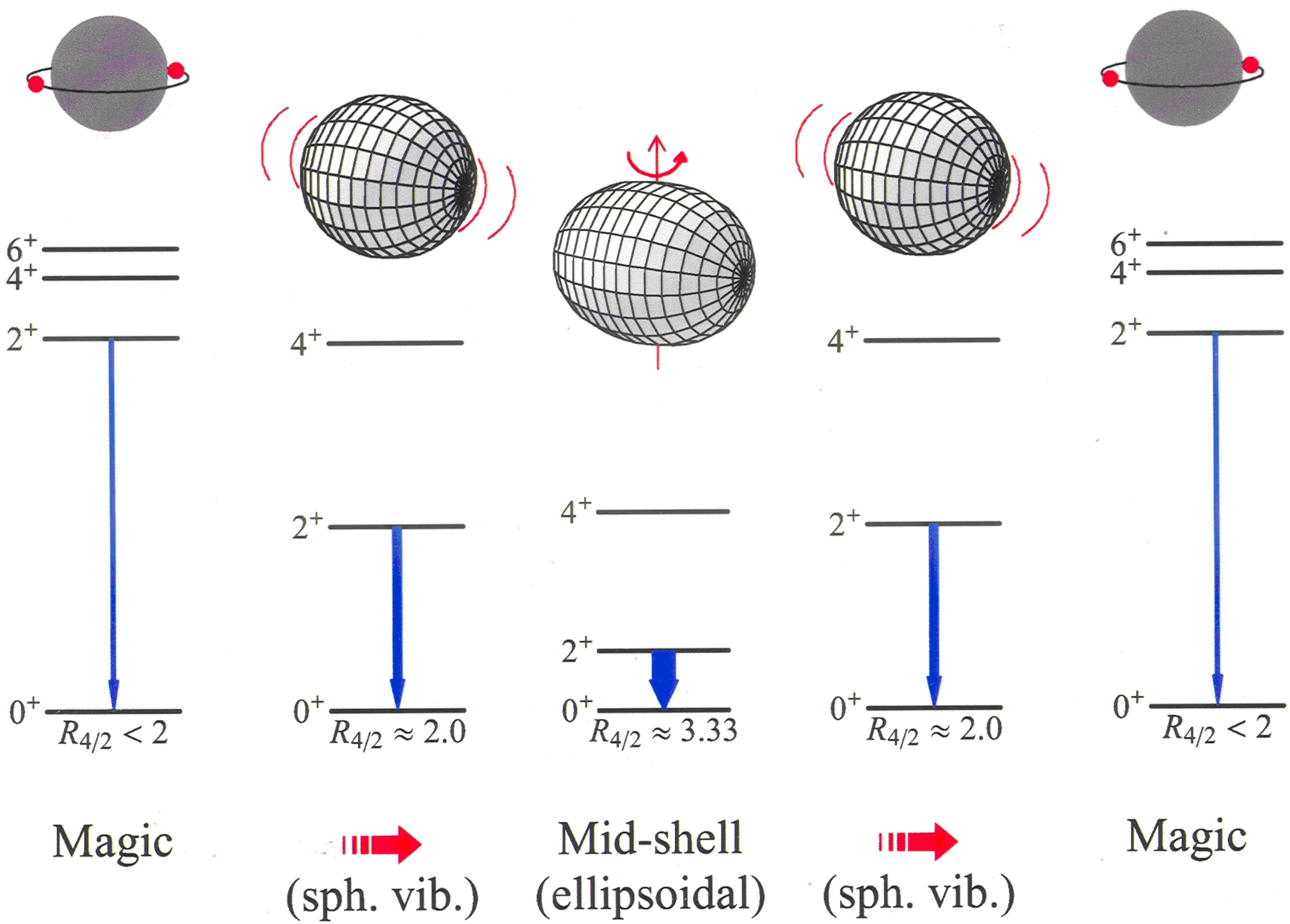


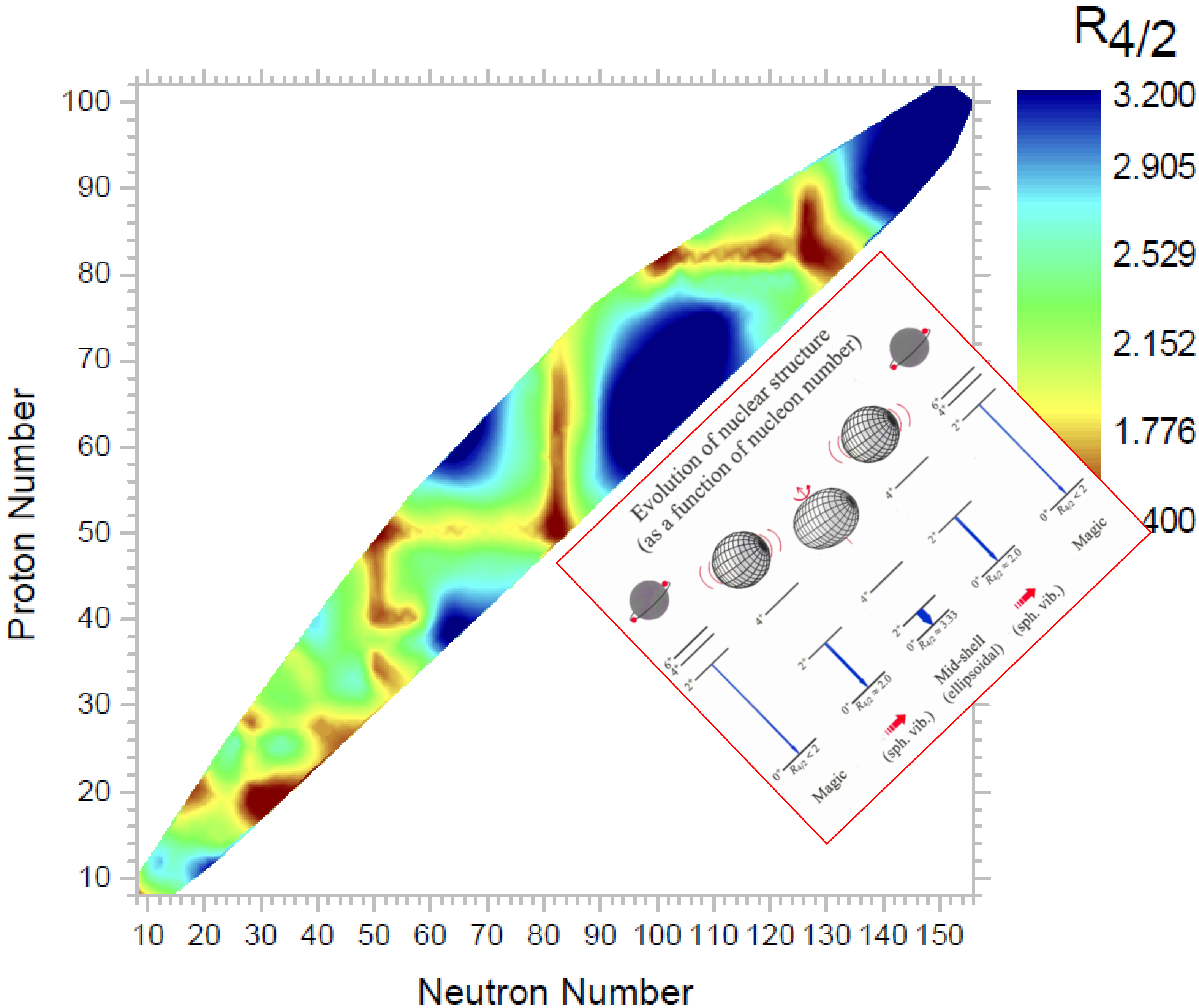

Figure courtesy Burcu Cakirli (Istanbul U.)




Ground state Configuration. Spin/parity $I^{\pi}=0^+$ ; $E_x$ = 0 keV

# Evolution of nuclear structure
# (as a function of nucleon number)





$R_{4/2}$
3.200
2.905
2.529
2.152
1.776
400
Proton Number
Neutron Number
10
20
30
40
50
60
70
80
90
100
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
Evolution of nuclear structure
(as a function of nucleon number)
Magic
(sph. vib.)
Mid-shell
(ellipsoidal)
(sph. vib.)
Magic

PHYSICAL REVIEW C **69**, 034317 (2004)

**Transition from the seniority regime to collective motion**

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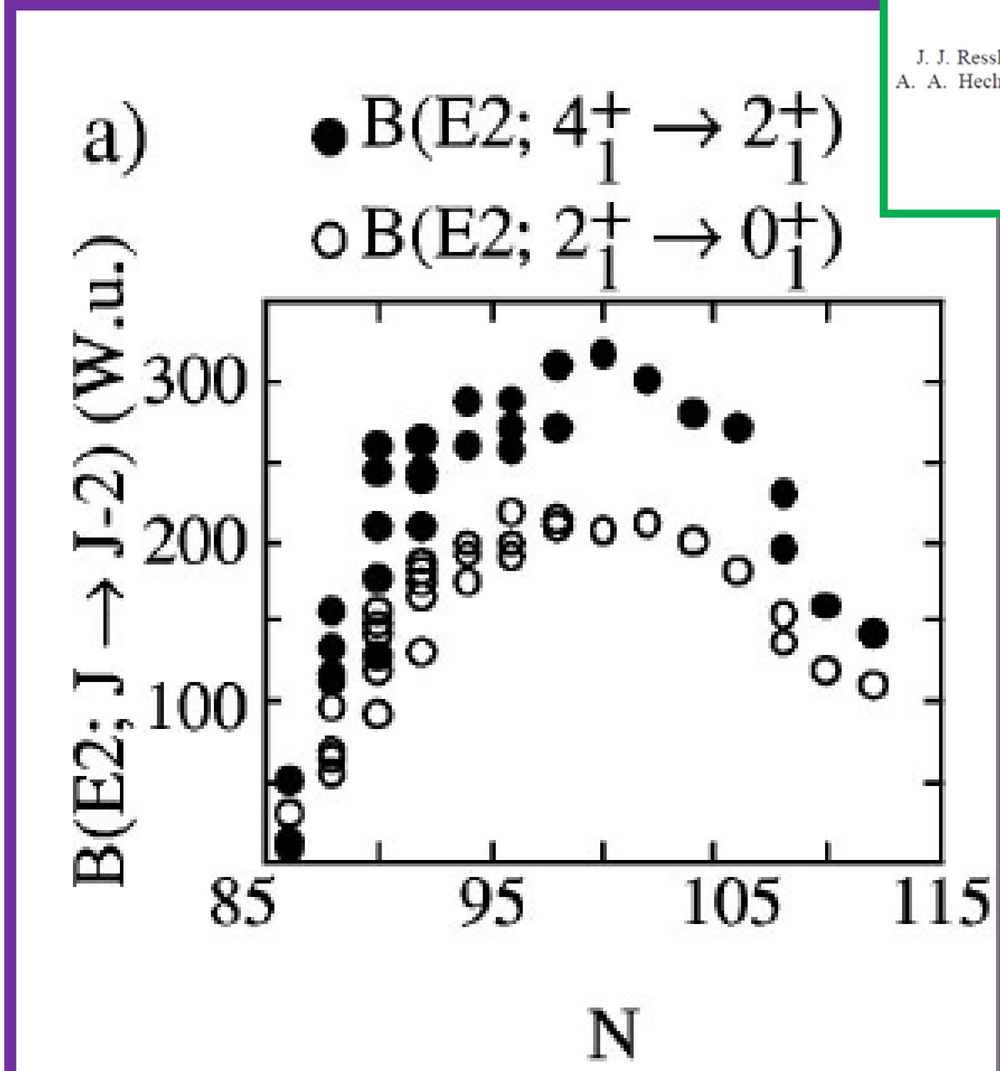
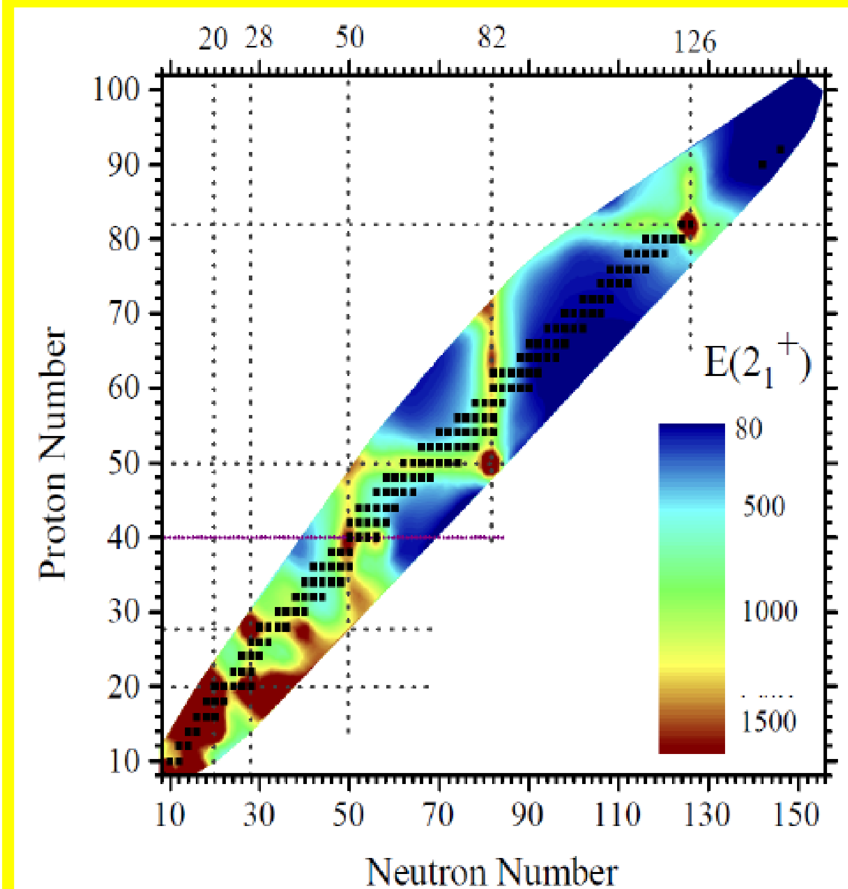
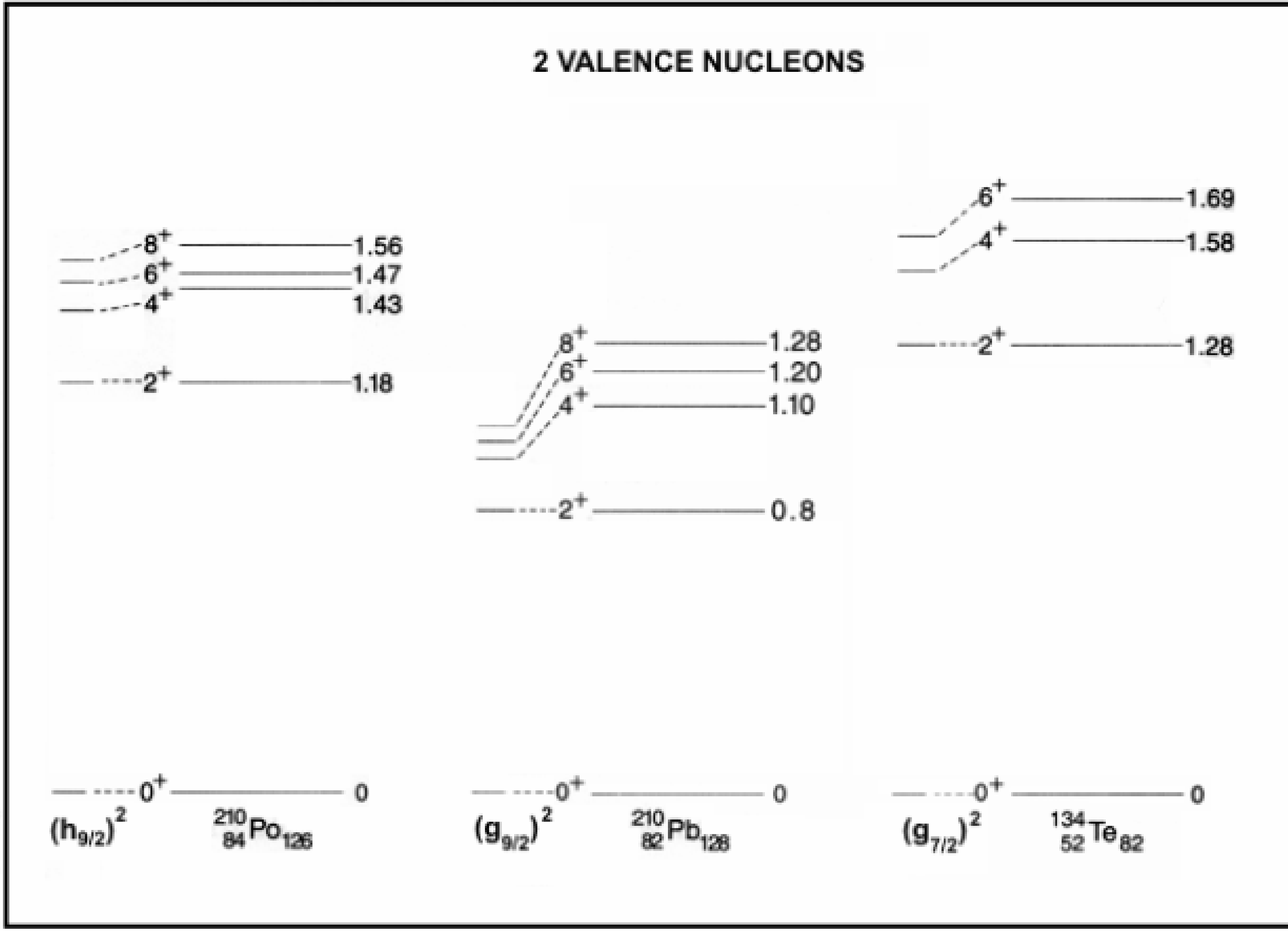
(Received 17 September 2003; published 15 March 2004)




B(E2) values for low-lying even-even nuclei with Z =62 (Sm) – 74 (W).
Very 'collective' transitions (>100 Wu) with maximum B(E2) at mid-shell.
This correlates with the lowest $E(2^+)$ excitation energy values.

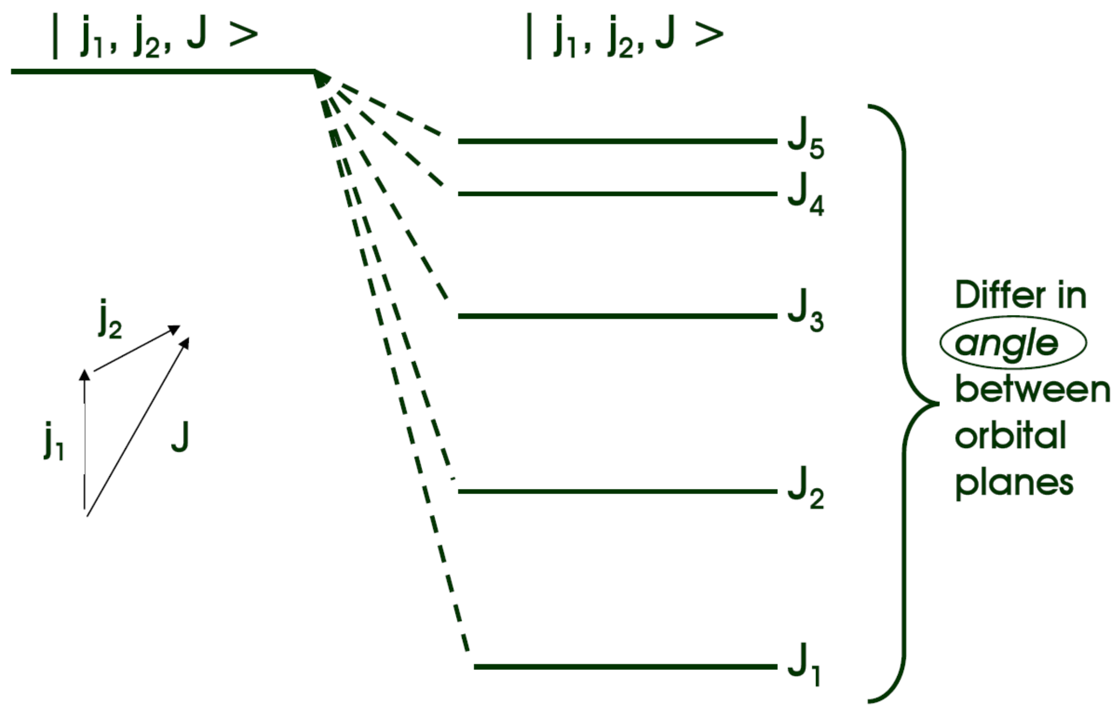



$j^2$ ('seniority) configurations observed in doubly-magic + 2-nucleon nuclei.

# Residual Interactions—Diagonal Effects

Consider 2 particles, in orbits $j_1$, $j_2$ coupled to spin $J_i$, and interacting with a residual interaction, $V_{12}$.

## 2 Identical Nucleons

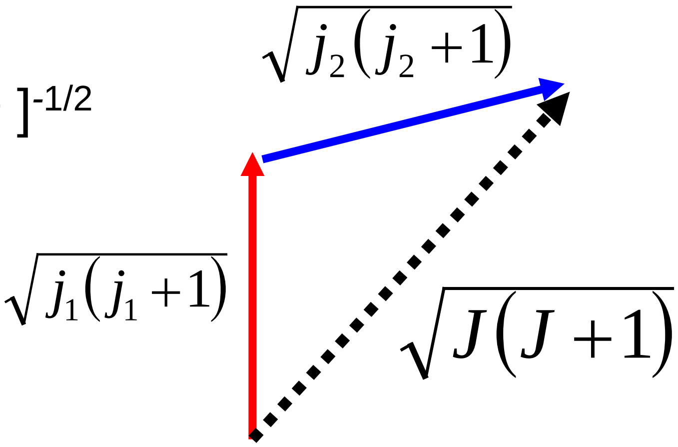



NO RESIDUAL INTERACTION

RESIDUAL INTERACTION

# Geometric Interpretation of the δ residual interaction for $j^2$ configuration coupled to Spin J

Use the cosine rule and recall that the magnitude of the spin vector of spin $j$ = $[\,j\,(j+1)\,]^{-1/2}$




$$J^2 = j_1^2 + j_2^2 - 2\,j_1 j_2 \cos(\theta)$$

*therefore*

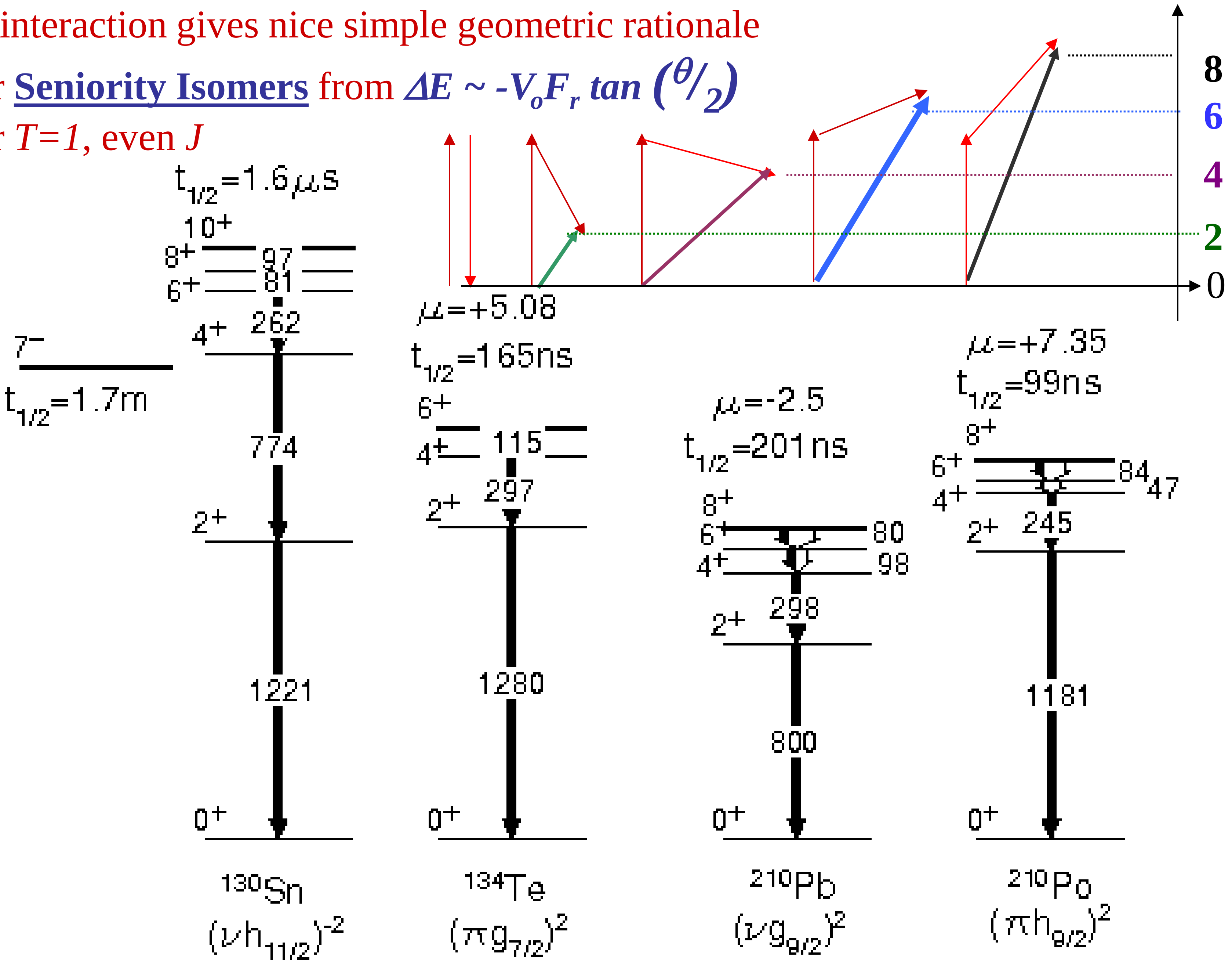
$$J(J+1) = j_1(j_1+1) + j_2(j_2+1) - \sqrt{j_1(j_1+1)}\sqrt{j_2(j_2+1)}\cos(\theta)$$

$$\therefore \text{for } j_1 = j_2 = j \quad \cos^{-1}\left[\frac{J(J+1) - 2j(j+1)}{j(j+1)}\right]$$

δ–interaction gives nice simple geometric rationale

for **Seniority Isomers** from $\Delta E \sim -V_o F_r \tan(\theta/2)$

for $T=1$, even $J$




See e.g., Nuclear structure from a simple perspective, R.F. Casten Chap 4.)

PHYSICAL REVIEW C **69**, 034317 (2004)

**Transition from the seniority regime to collective motion**

J. J. Ressler,[1] R. F. Casten,[1] N. V. Zamfir,[1] C. W. Beausang,[1] R B. Cakirli,[1,2] H. Ai,[1] H. Amro,[1] M. A. Caprio,[1] A. A. Hecht,[1] A. Heinz,[1] S. D. Langdown,[1,3] E. A. McCutchan,[1] D. A. Meyer,[1] C. Plettner,[1] P. H. Regan,[1,3] M. J. S. Sciacchitano,[1] and A. D. Yamamoto[1,3]
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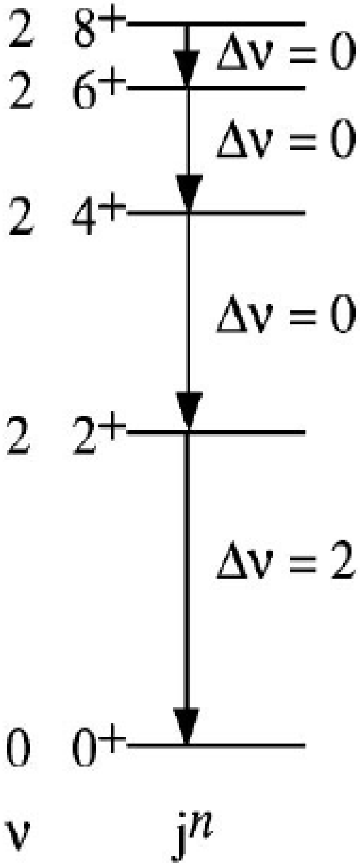
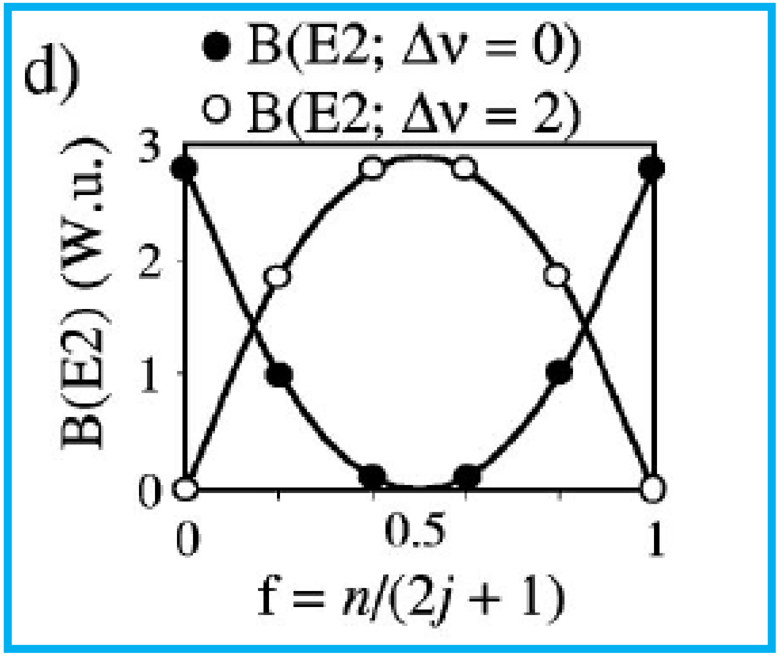
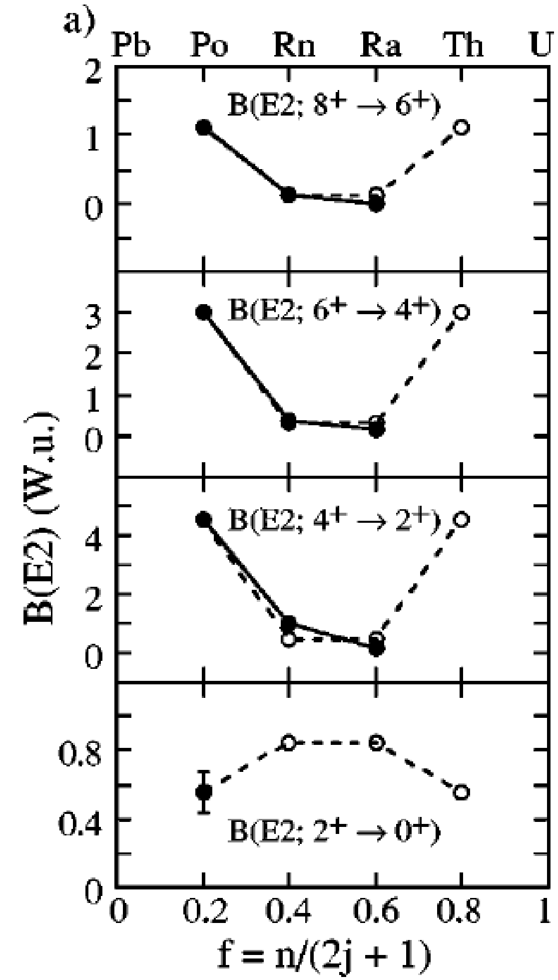
(Received 17 September 2003; published 15 March 2004)

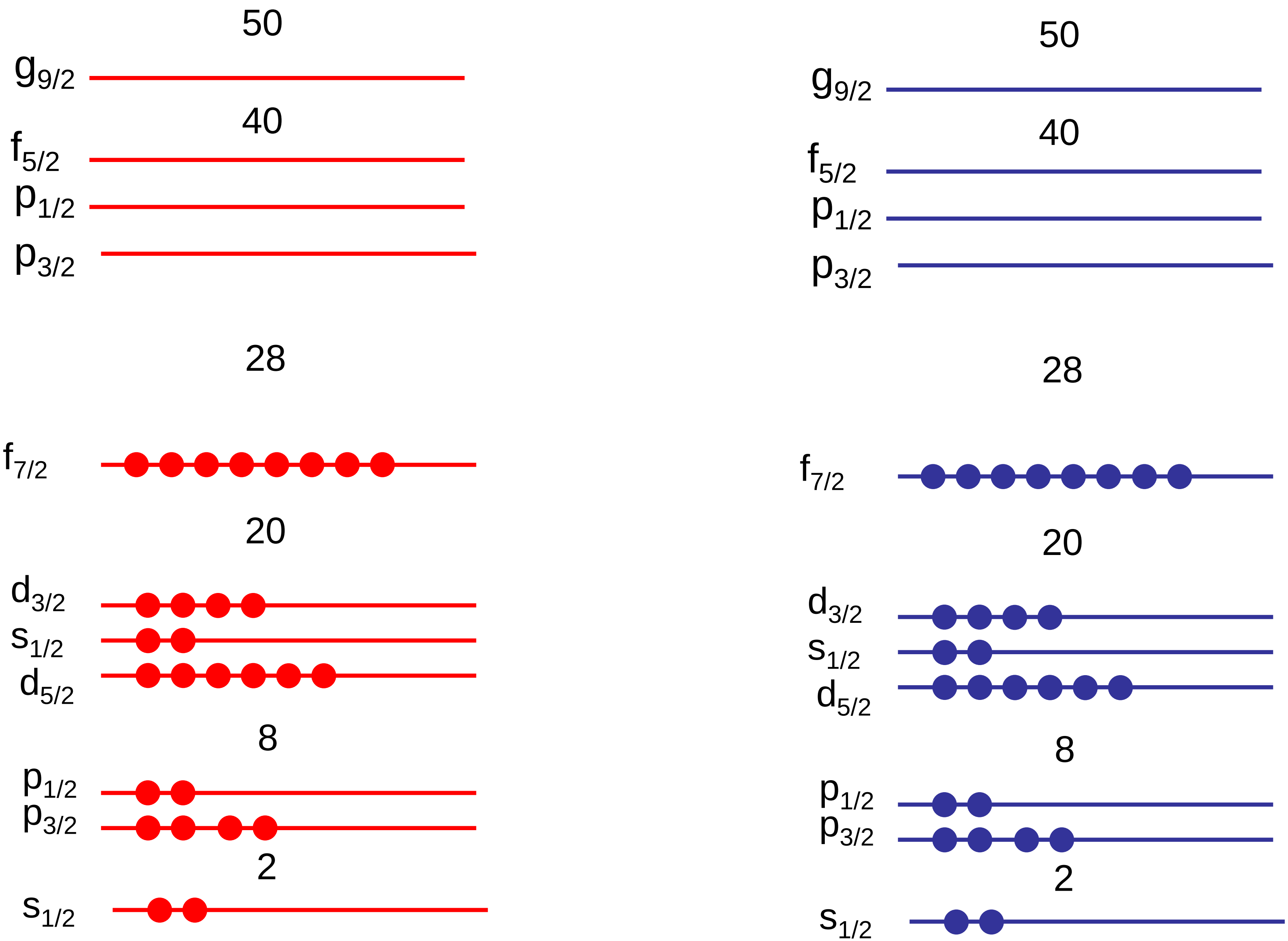


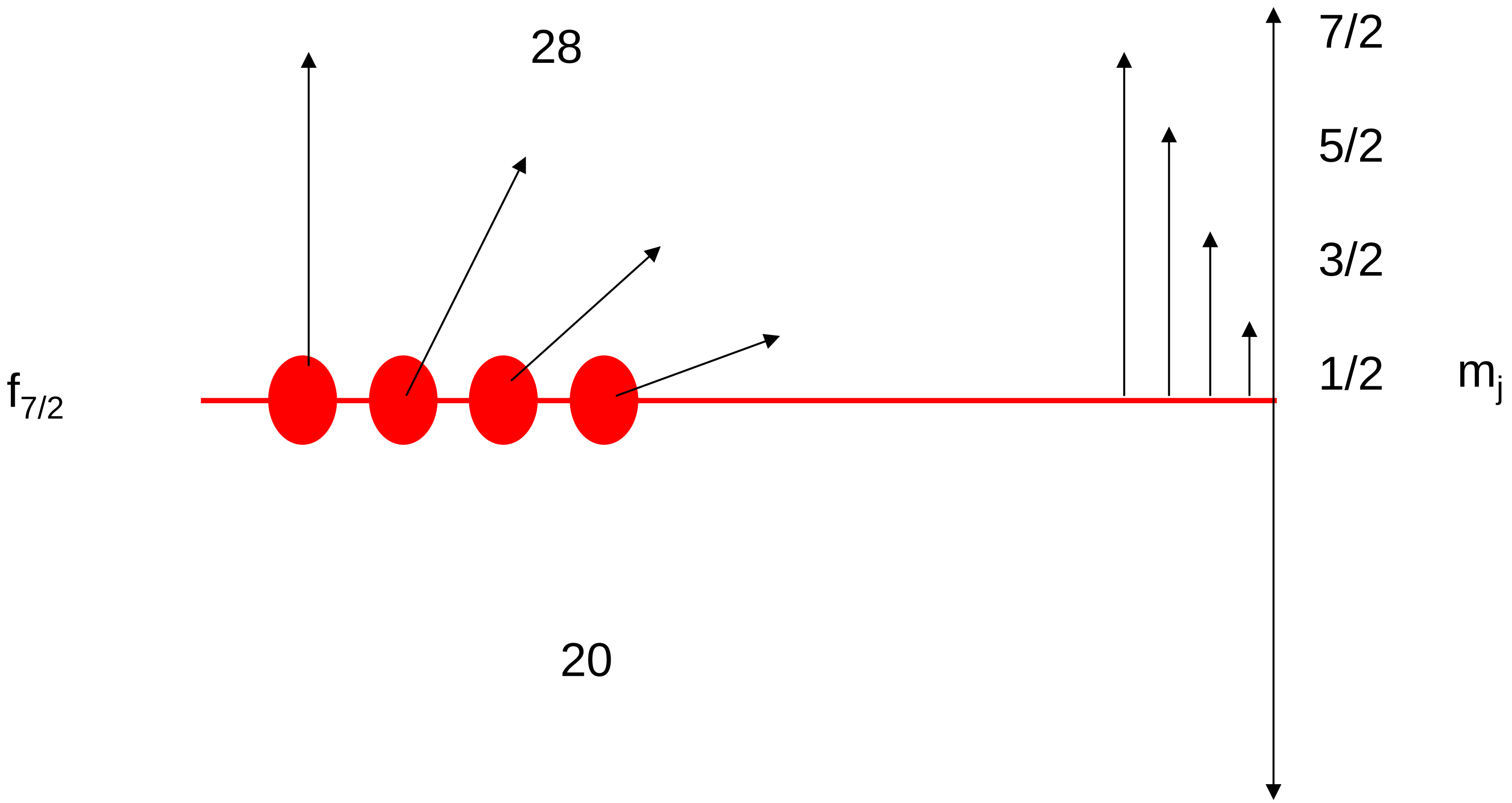




Fractional filling of single-j shell (e.g., $\pi h_{9/2}$) gives parabolic evolution of B(E2) values for **(a)** **Seniority Conserving** ($J_{max} \rightarrow J-2$) and **(b) Seniority Breaking** ($2^+ \rightarrow 0^+$) transitions.

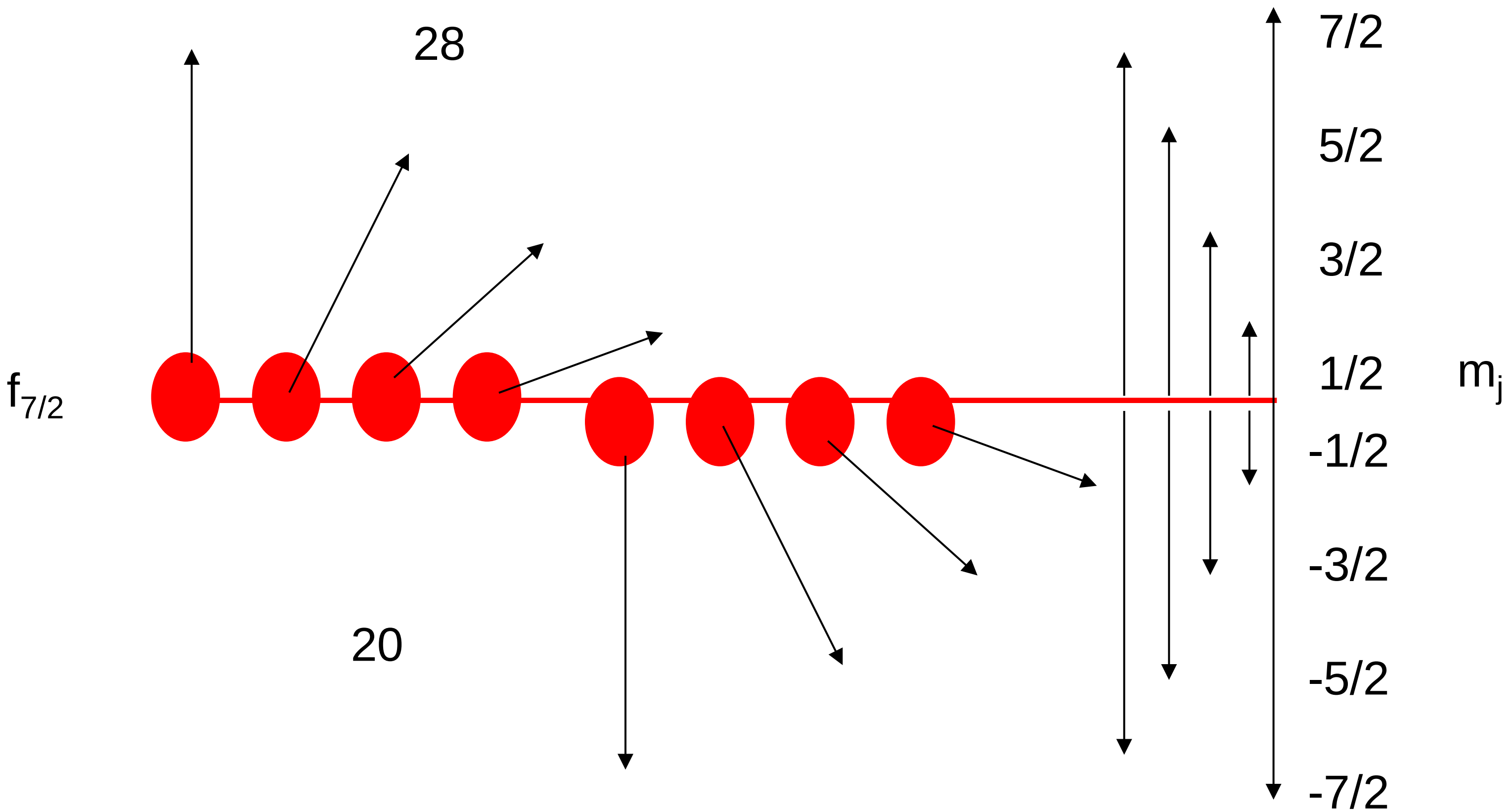
# Exhausting the spin ?



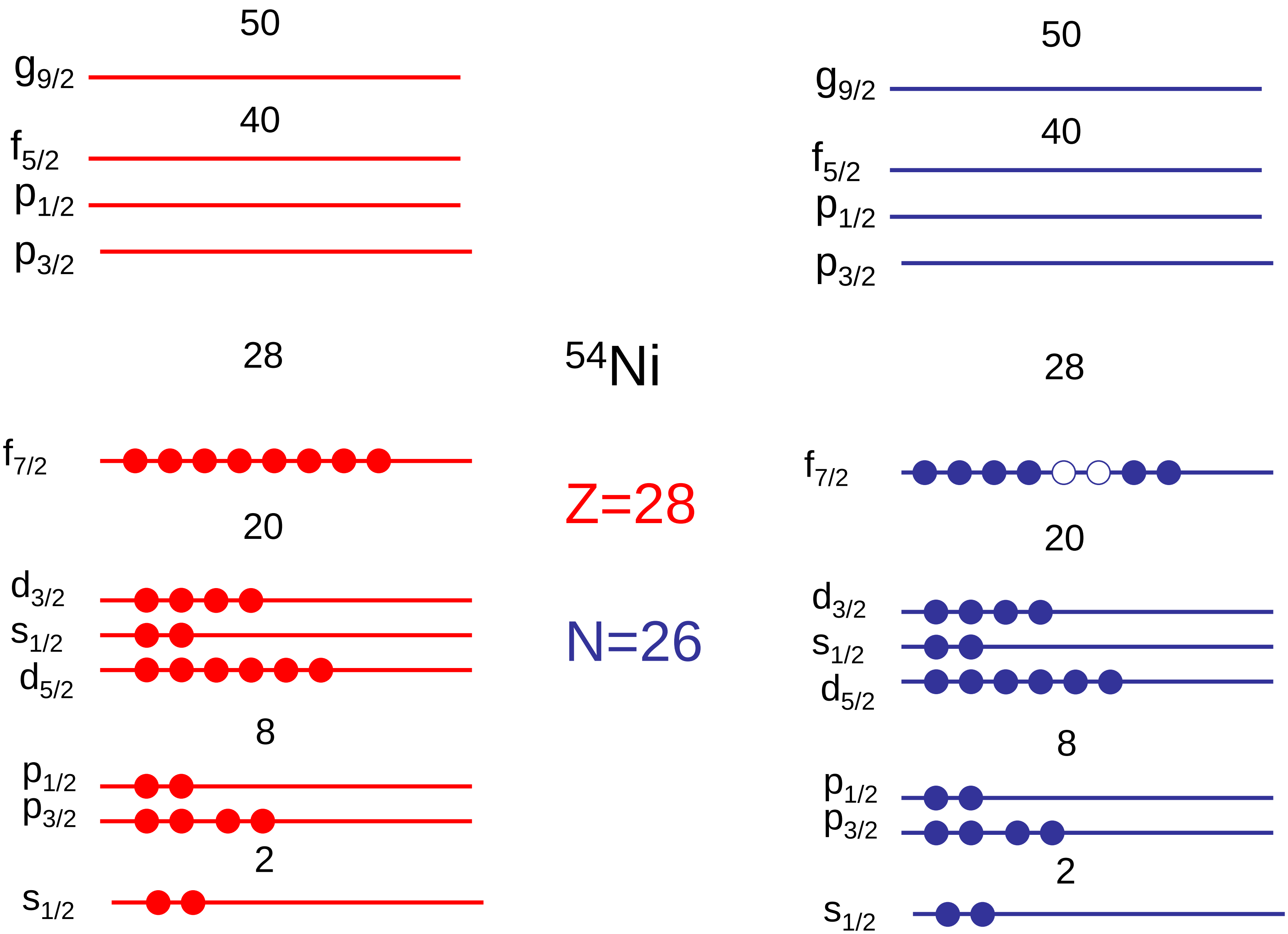

Doubly magic, closed shell nucleus, $^{56}$Ni, $I^{\pi}=0^{+}$




i.e., max spin in $f_{7/2}$ orbits can be generated from 4 occupied states, = $M_j$ = 7/2 + 5/2 + 3/2 + 1/2 = **8 ħ**




Fully filled (closed) $f_{7/2}$ shell would have Mj = J = 0




2 neutron ‘holes’ in $f_{7/2}$ shell $I^{\pi}=0^+$, $2^+$, $4^+$ & $6^+$ : $(\nu f_{7/2})^{-2}$

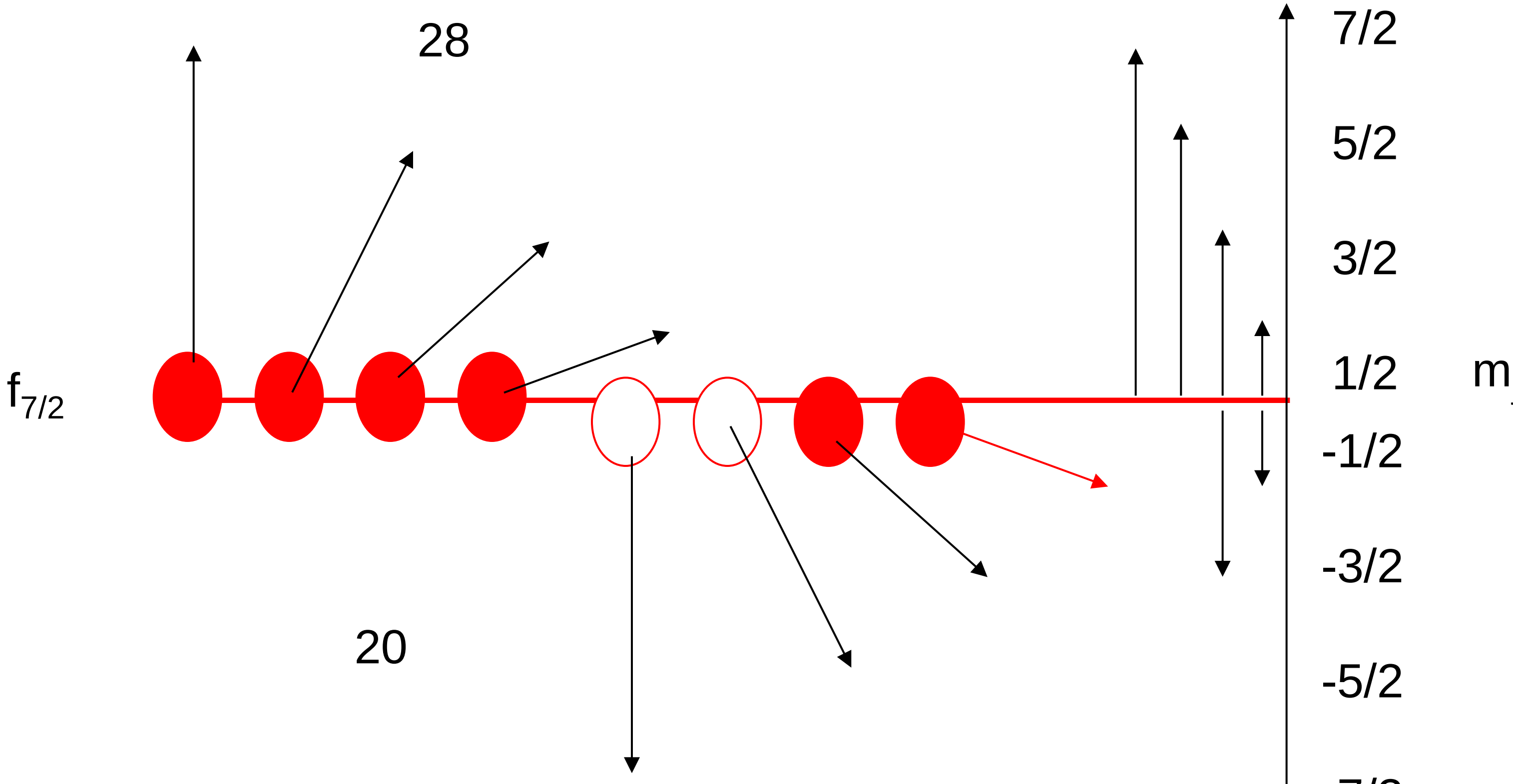
# Spins for Identical Nucleons in Equivalent Orbits
# The "m-scheme" – Pauli Principle
## e.g., $|f^2_{7/2}\rangle$

*m scheme for the configuration* $|(7/2)^2 J\rangle$*

| $j_1 = 7/2$ $m_1$ | $j_2 = 7/2$ $m_2$ | $M$ | $J$ |
|---|---|---|---|
| 7/2 | 5/2 | 6 | |
| 7/2 | 3/2 | 5 | |
| 7/2 | 1/2 | 4 | |
| 7/2 | –1/2 | 3 | 6 |
| 7/2 | –3/2 | 2 | |
| 7/2 | –5/2 | 1 | |
| 7/2 | –7/2 | 0 | |
| 5/2 | 3/2 | 4 | |
| 5/2 | 1/2 | 3 | |
| 5/2 | –1/2 | 2 | 4 |
| 5/2 | –3/2 | 1 | |
| 5/2 | –5/2 | 0 | |
| 3/2 | 1/2 | 2 | |
| 3/2 | –1/2 | 1 | 2 |
| 3/2 | –3/2 | 0 | |
| 1/2 | –1/2 | 0 | 0 |

* Only positive total $M$ values are shown. The table is symmetric for $M < 0$.

# running out of spin…




Maximum spin available for 'core' excitations in $^{54}$Ni (and its 'mirror' nucleus $^{54}$Fe (Z=26, N=28) from 2 holes in $f_{7/2}$ orbits is $I^{\pi}=6^{+}$

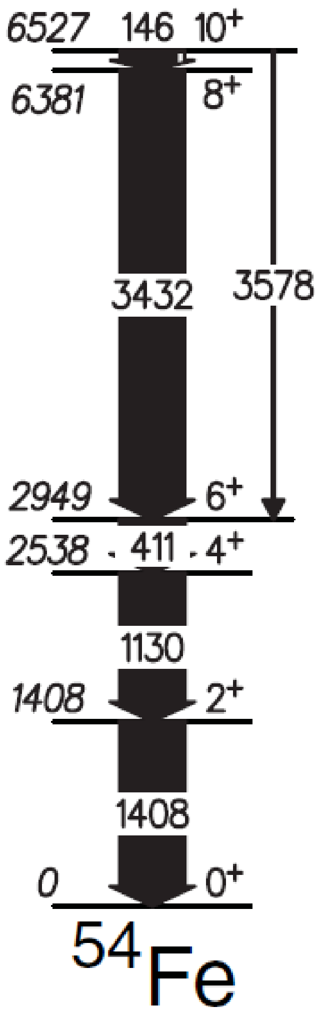
# What happens next?

Q. How do you generate higher angular momentum states when the maximum spin that valence space is used up (i.e. $j^2$ coupled to $J_{max} = (j-1)$) ?

A. Break the valence core and excite nucleons across magic number gaps. This costs energy (can be ~3-4 MeV), but result in spin increase of 4 ħ.

# Competing E4 and E2 transitions with core breaking?

We can have cases where low-energy (~100 keV) E2 decays competing with high-energy (~4 MeV) E4 transitions across magic shell closures, e.g. $^{54}Fe_{28}$.

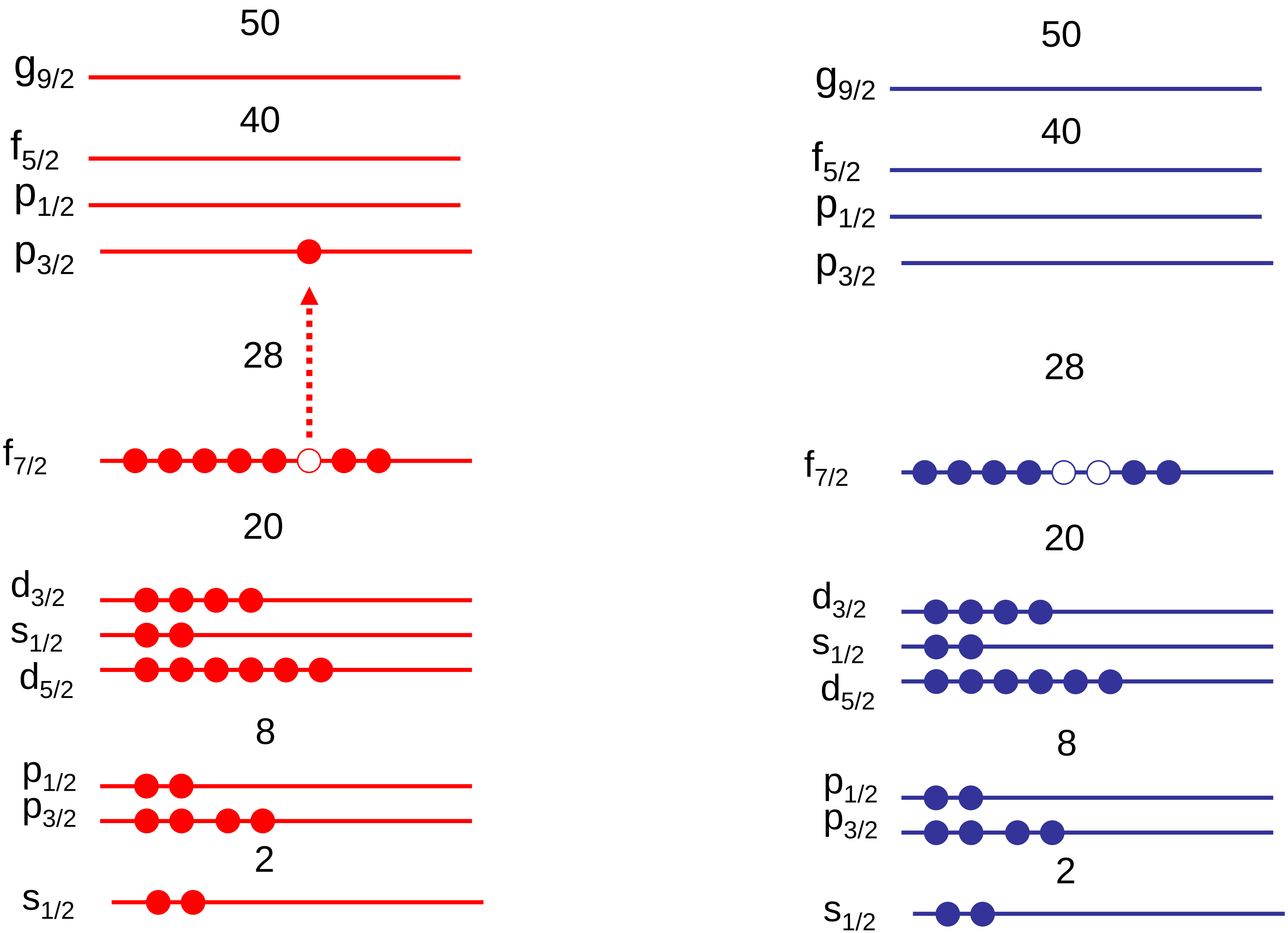



Z=26; N=28 case.

- 2 proton holes in $f_{7/2}$ shell.
- Maximum spin in simple valence space is $I^{\pi}=6^{+}$.
- i.e., $(\pi f_{7/2})^{-2}$ configuration coupled to $I^{\pi}= 6^{+}$

Additional spin requires exciting (pairs) of nucleons across the N or Z=28 shell closures into the $f_{5/2}$ shell.

| $E_{\gamma}$ | E2 (1Wu) | M3 (1Wu) | E4 (1Wu) |
|---|---|---|---|
| 146 keV ($10^{+}\rightarrow 8^{+}$) | 1.01 ms | 613 s | 21 E+6s |
| 3578 keV ($10^{+}\rightarrow 6^{+}$) | | | 6.5 ms |

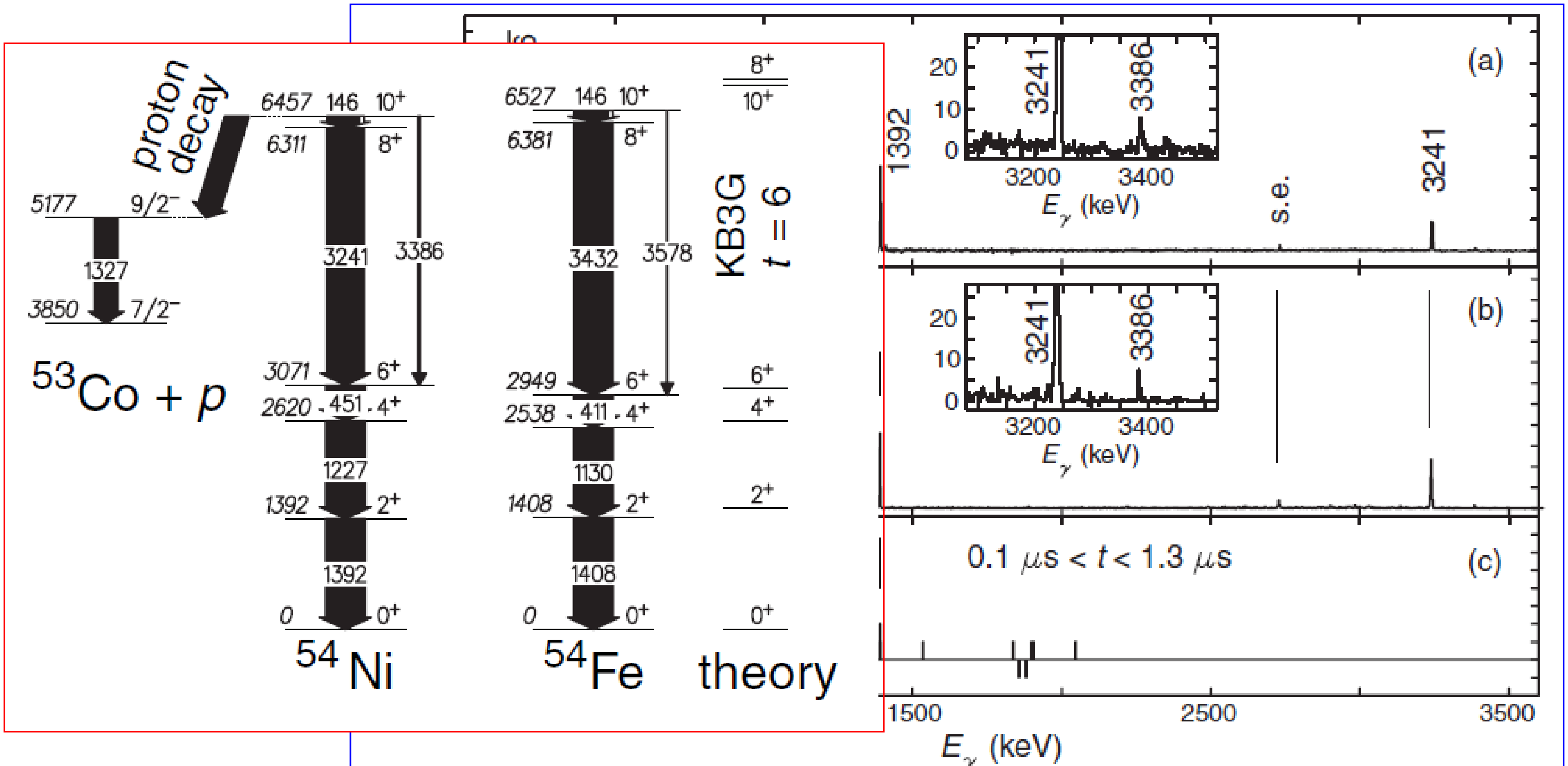


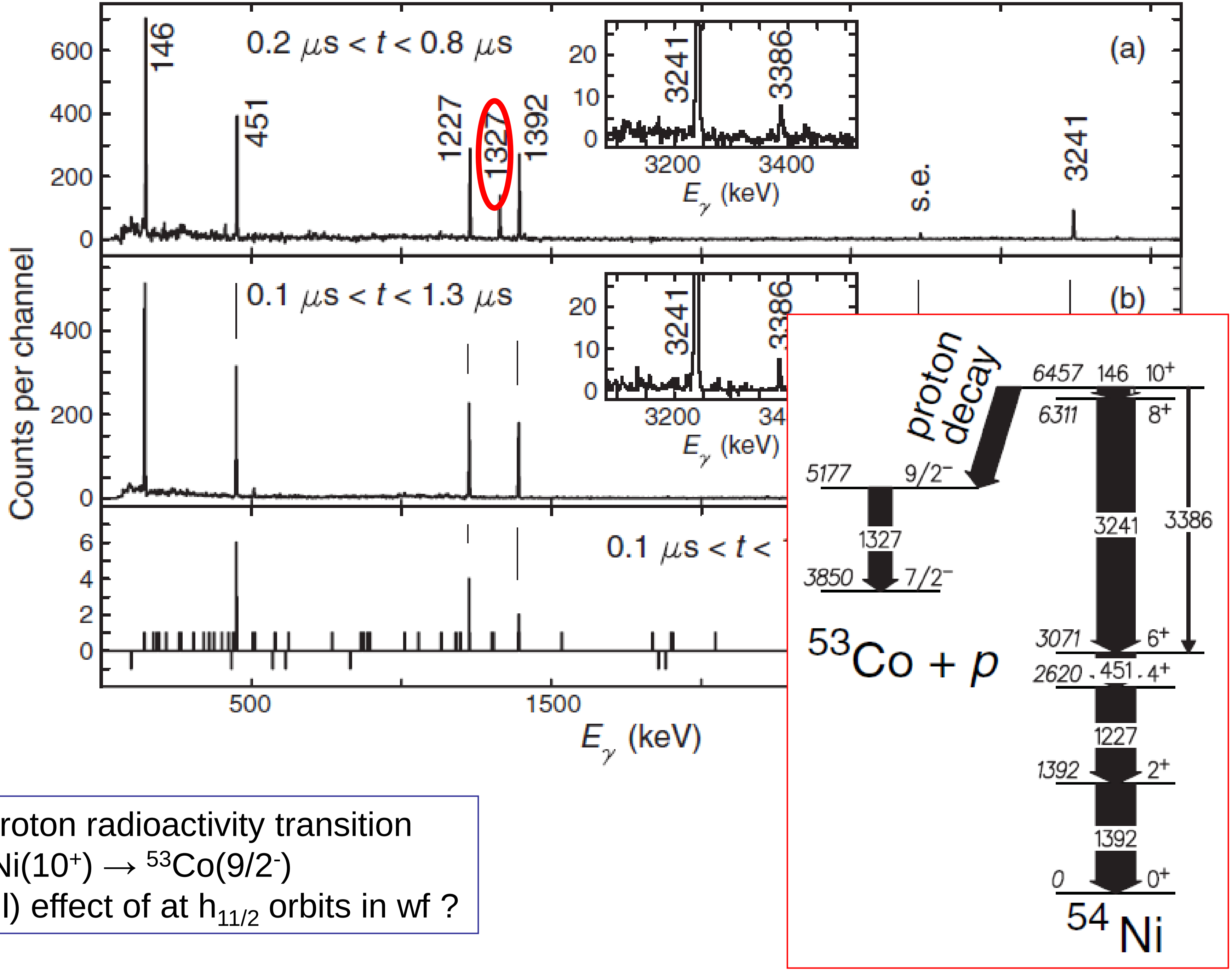

break core $I^{\pi}=10^{+}$ from $(\nu f_{7/2})^{-2}_{6+} \times (\pi f_{7/2} \times \pi p_{1/2},p_{3/2})_{4+}$

PHYSICAL REVIEW C **78**, 021301(R) (2008)

# Isospin symmetry and proton decay: Identification of the $10^+$ isomer in $^{54}Ni$

D. Rudolph,[1] R. Hoischen,[1,2] M. Hellström,[1] S. Pietri,[3] Zs. Podolyák,[3] P. H. Regan,[3] A. B. Garnsworthy,[3,4] S. J. Steer,[3] F. Becker,[2,*] P. Bednarczyk,[2,5] L. Cáceres,[2,6] P. Doornenbal,[2,7,†] J. Gerl,[2] M. Górska,[2] J. Grębosz,[2,5] I. Kojouharov,[2] N. Kurz,[2] W. Prokopowicz,[2,5] H. Schaffner,[2] H. J. Wollersheim,[2] L.-L. Andersson,[1] L. Atanasova,[8] D. L. Balabanski,[8,9] M. A. Bentley,[10] A. Blazhev,[7] C. Brandau,[2,3] J. R. Brown,[10] C. Fahlander,[1] E. K. Johansson,[1] A. Jungclaus,[6] and S. M. Lenzi[11]

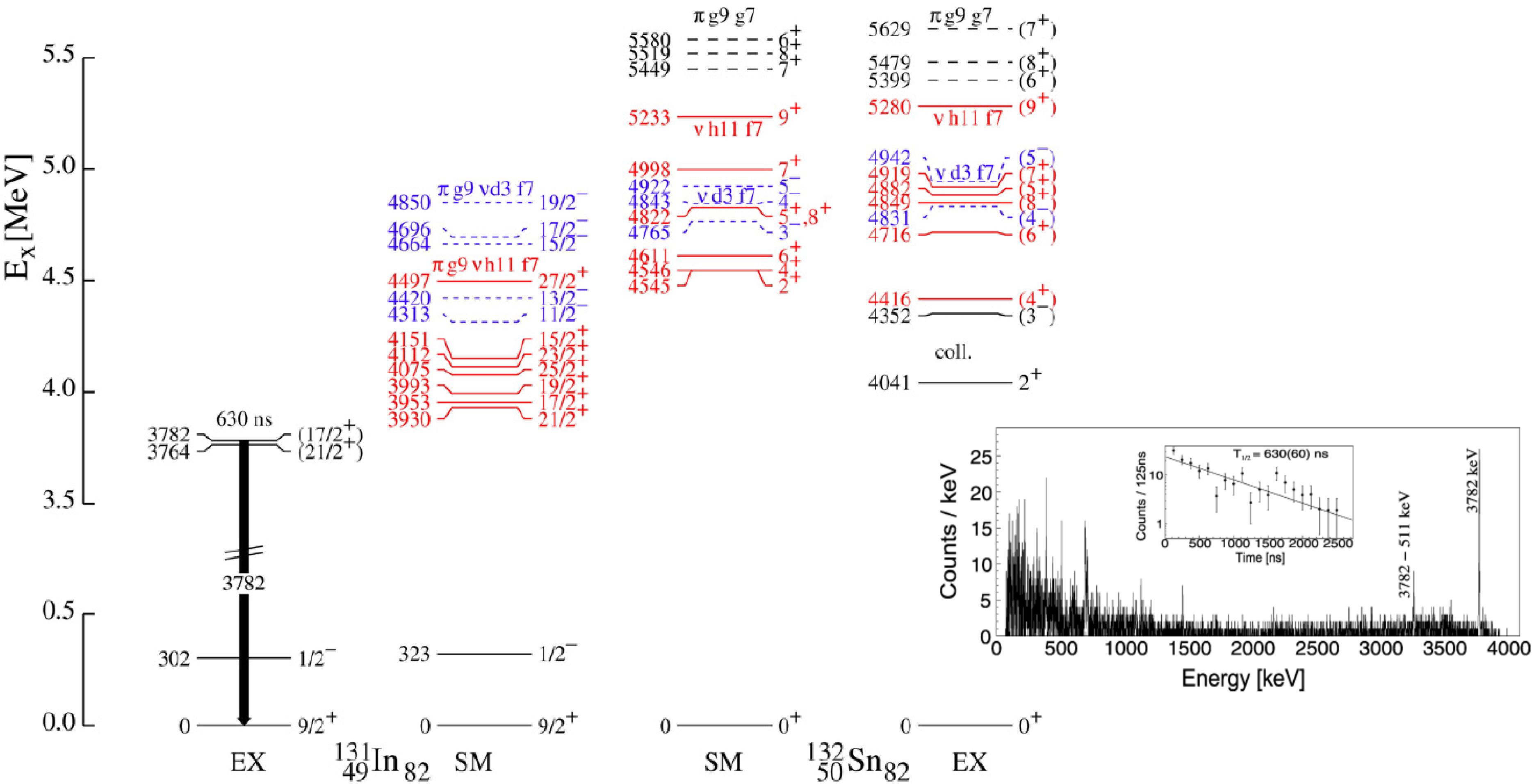





L=5 proton radioactivity transition
for $^{54}$Ni(10$^{+}$) → $^{53}$Co(9/2$^{-}$)
(Small) effect of at $h_{11/2}$ orbits in wf ?

Other examples of 'core' breaking isomeric decay:
Signature is a ~ 4 MeV decay from isomeric state.
see e.g. around $^{132}Sn_{82}$ doubly-magic closed core.

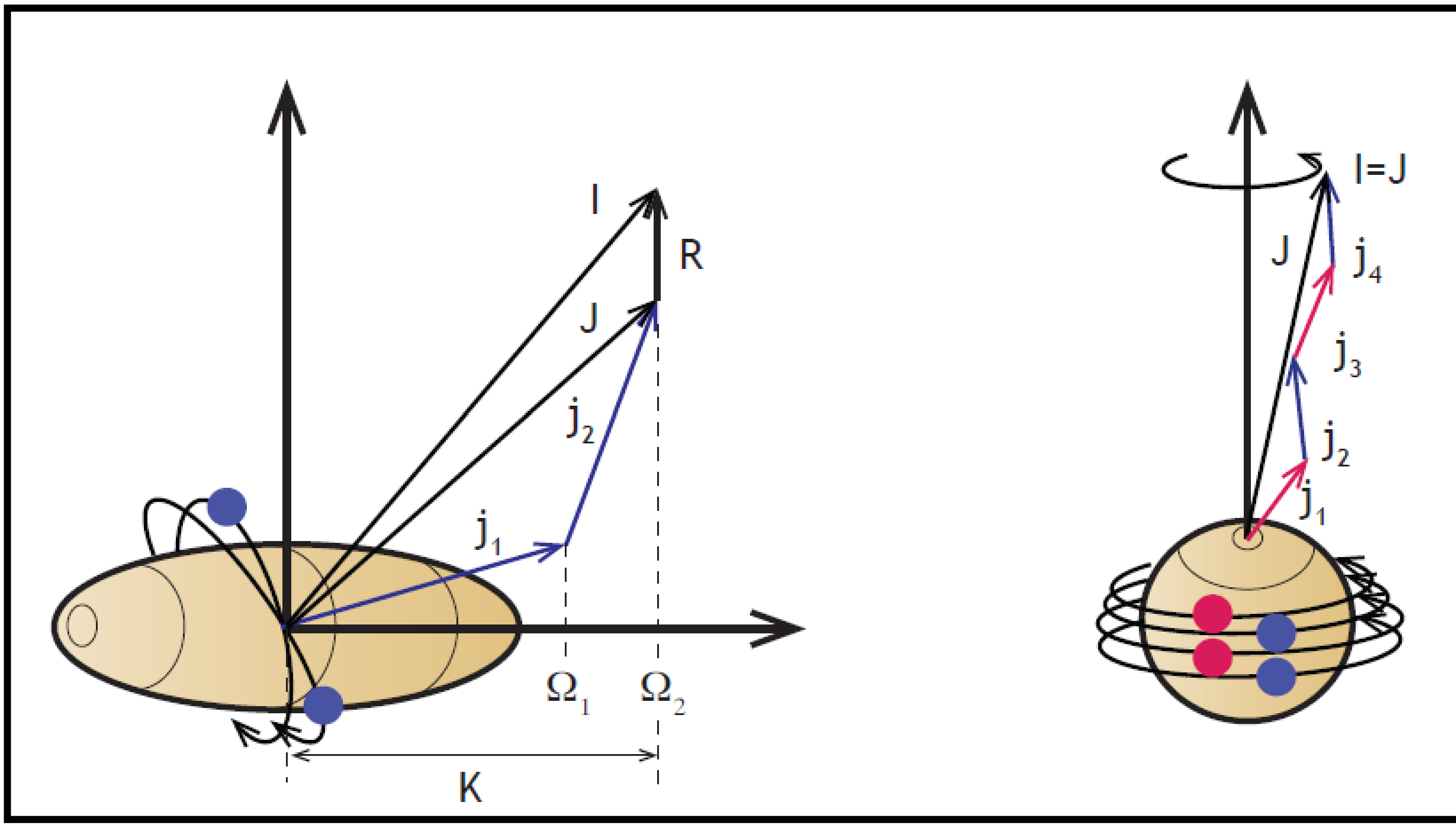
$17/2^+ \rightarrow 9/2^+$ (E4) in $^{131}$In; $T_{1/2}$=630 ns, B(E4)=1.48(14) Wu.




M. Górska et al. / Physics Letters B 672 (2009) 313–316

# What is the nuclear structure at higher spins ?

# Angular momentum coupling for multi-unpaired nucleons?

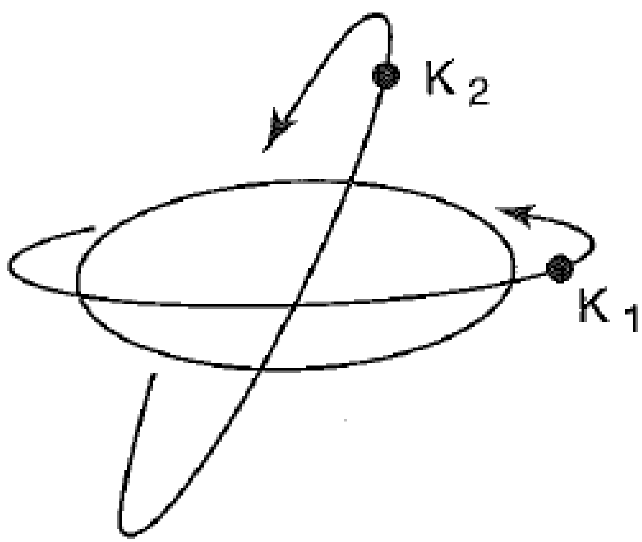


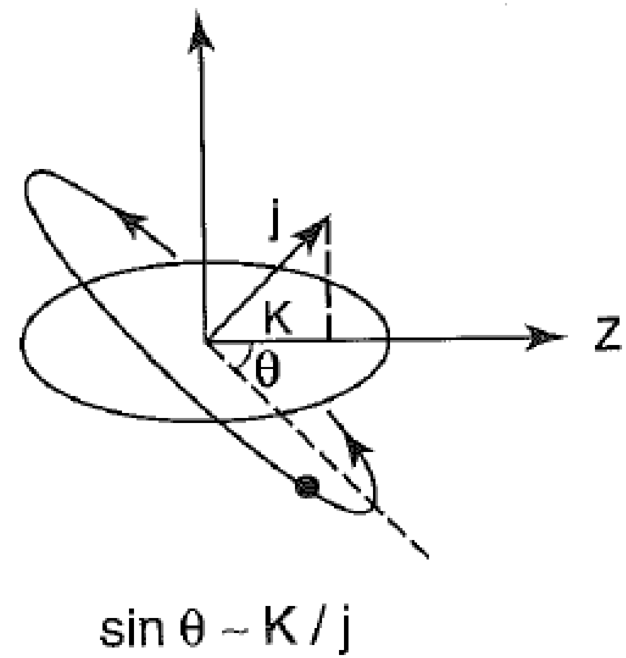

From DWK 2016

# Unpaired Particles in Deformed Nuclei:

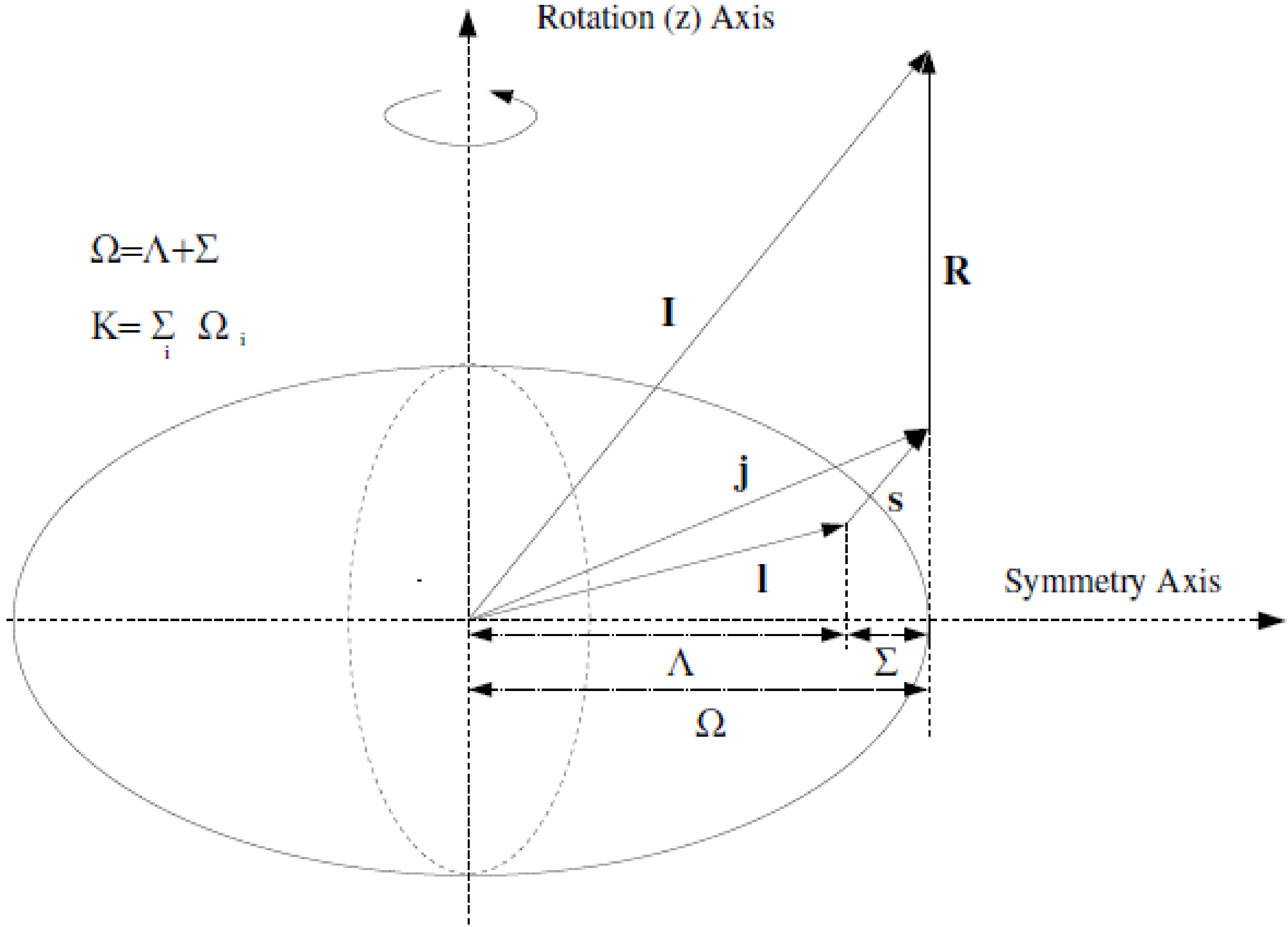
# The Nilsson Model

# Deformed Shell Model: The Nilsson Model




$$H = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\left[\omega_x^2(x^2+y^2)+\omega_z^2 z^2\right] + C\mathbf{l}\bullet\mathbf{s} + D\mathbf{l}^2$$

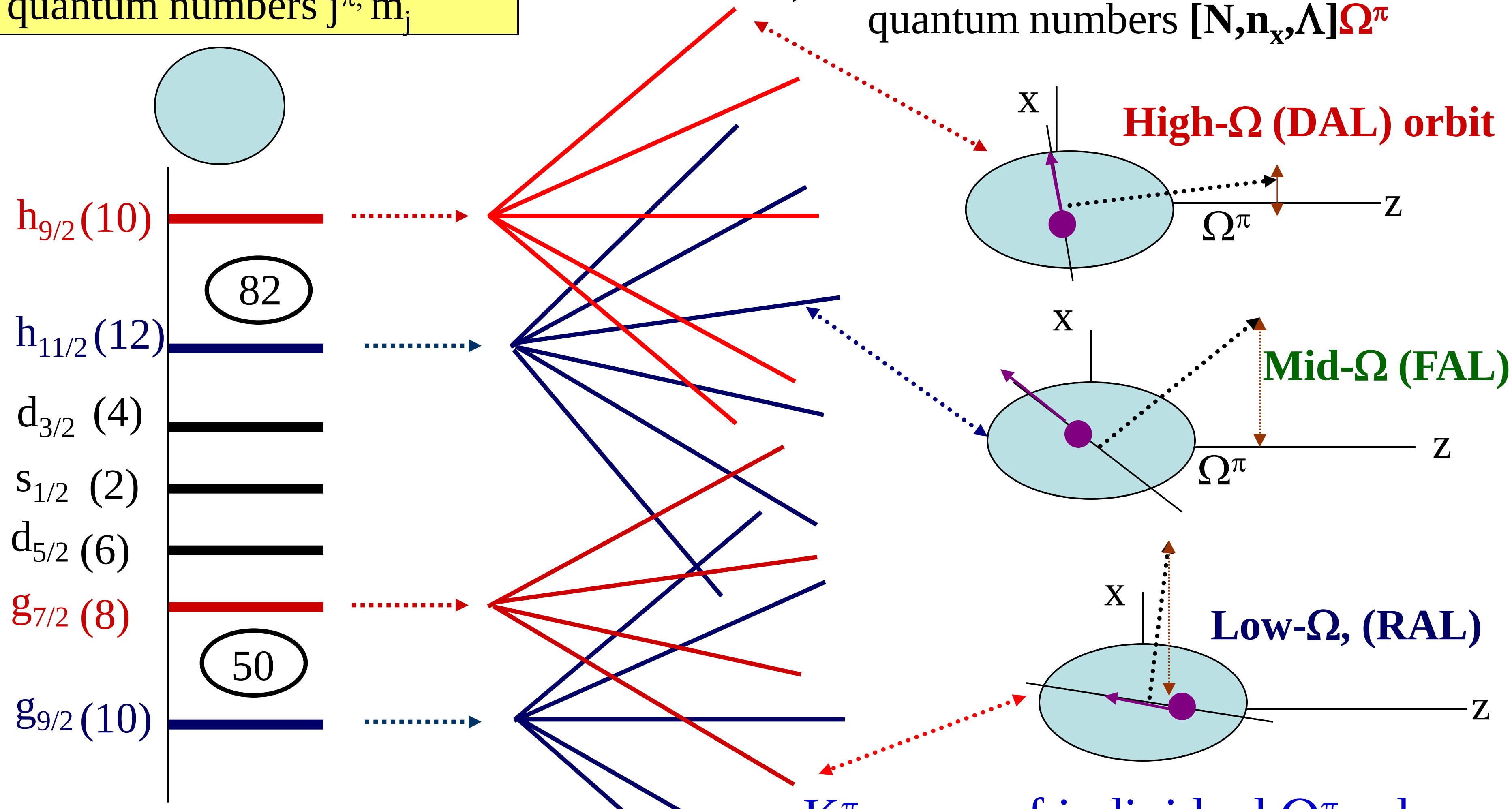

Rotation (z) Axis
Ω=Λ+Σ
K= Σ_i Ω_i
I
R
j
s
l
Symmetry Axis
Λ
Σ
Ω

# Effect of Nuclear Deformation on K-isomers

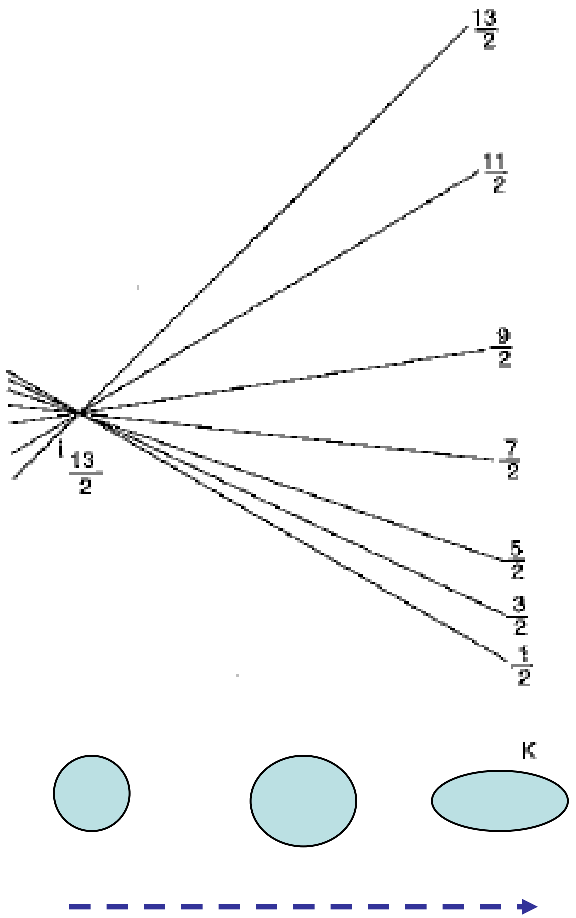
**Spherical**, harmon. oscilator
$H = \hbar\omega + a\,\boldsymbol{l.l} + b\,\boldsymbol{l.s}$,
quantum numbers $j^{\pi}$, $m_j$

**Nilsson scheme: Quadrupole deformed**
3-D HO. where $\hbar\omega \rightarrow \hbar\omega_x + \hbar\omega_y + \hbar\omega_z$
axial symmetry means $\omega_x = \omega_y$
quantum numbers $[N, n_x, \Lambda]\Omega^{\pi}$

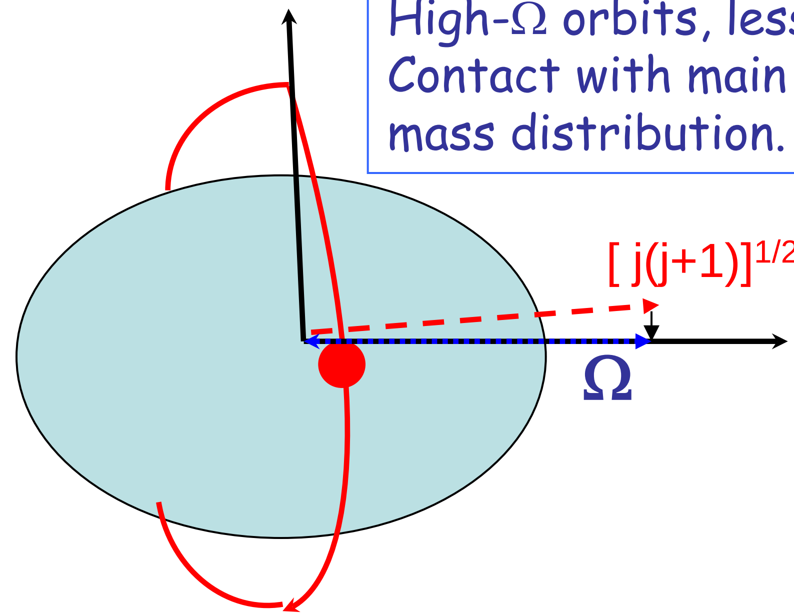
**> prolate $\beta_2$**



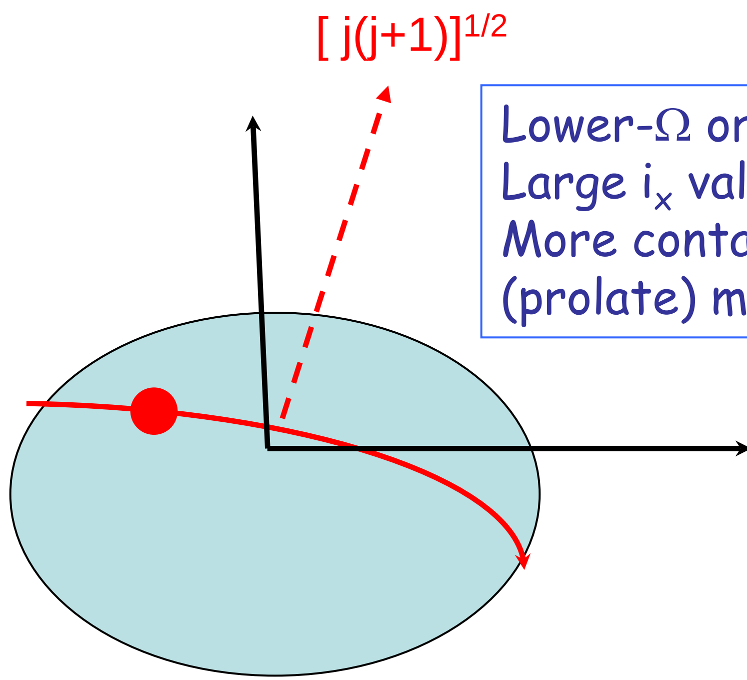

$K^{\pi}$= sum of individual $\Omega^{\pi}$ values.




Increasing (prolate) deformation, bigger splitting.




High-Ω orbits, less Contact with main mass distribution.




Lower-Ω orbits, have Large $i_x$ values and More contact with main (prolate) mass distribution.

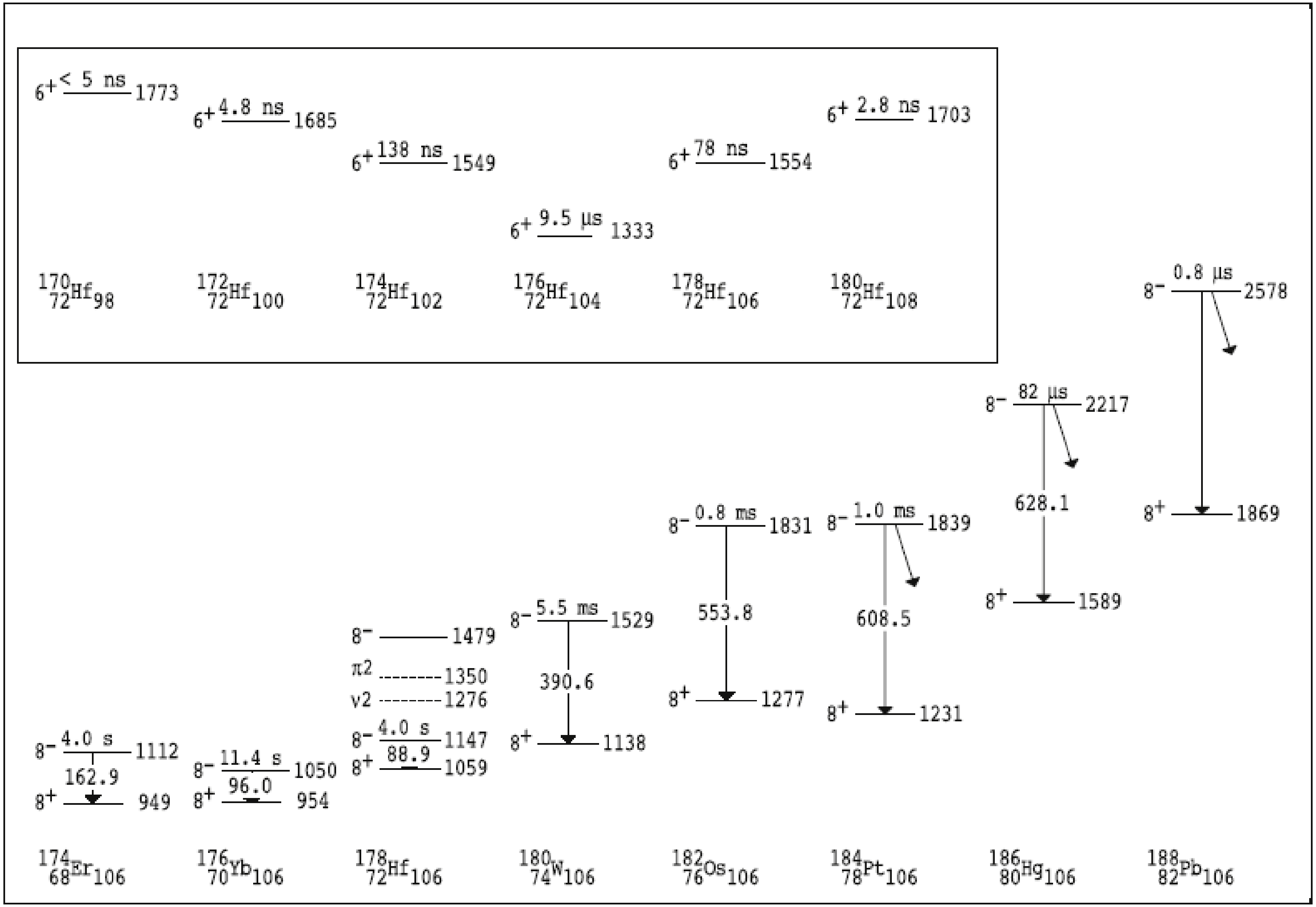
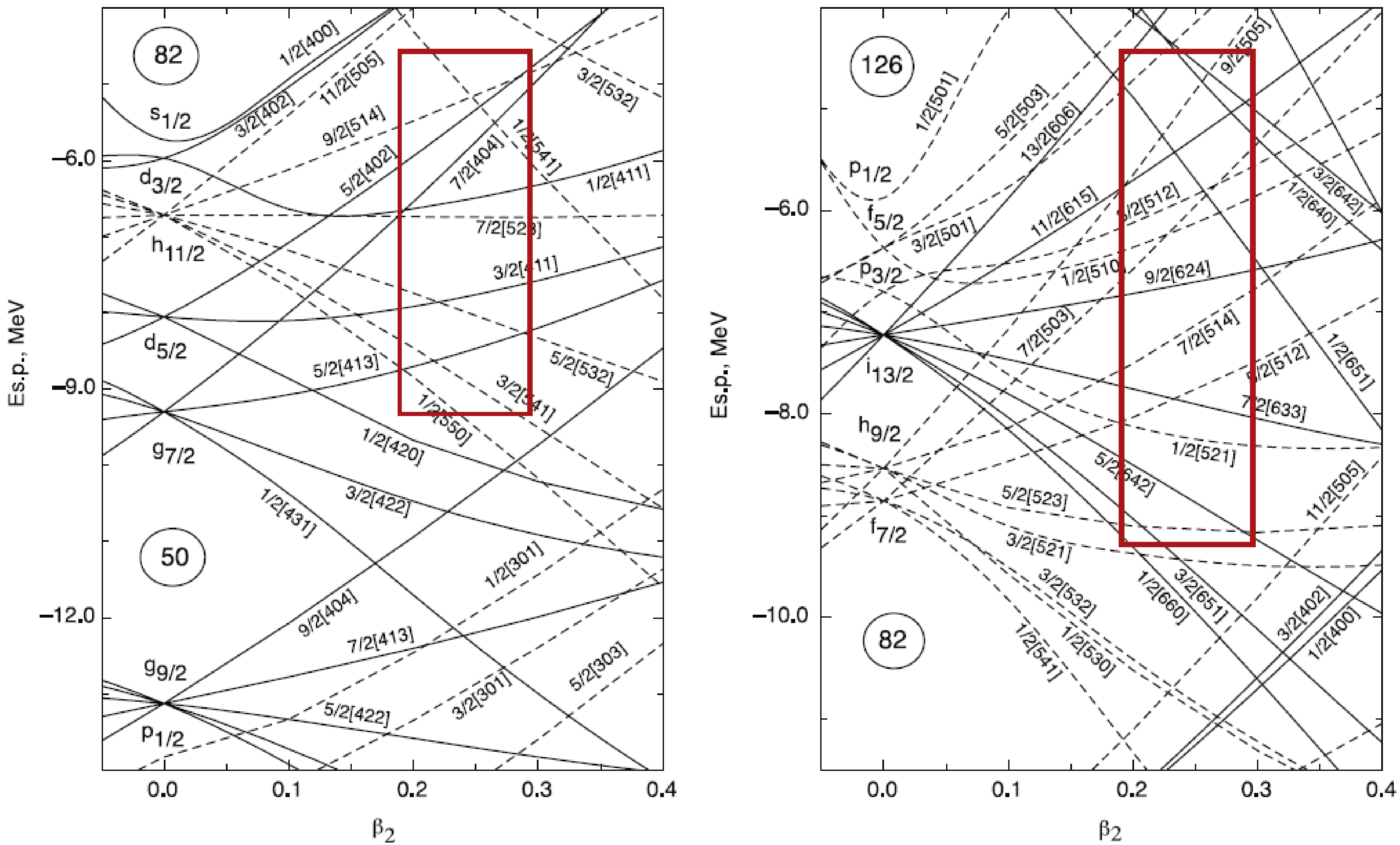


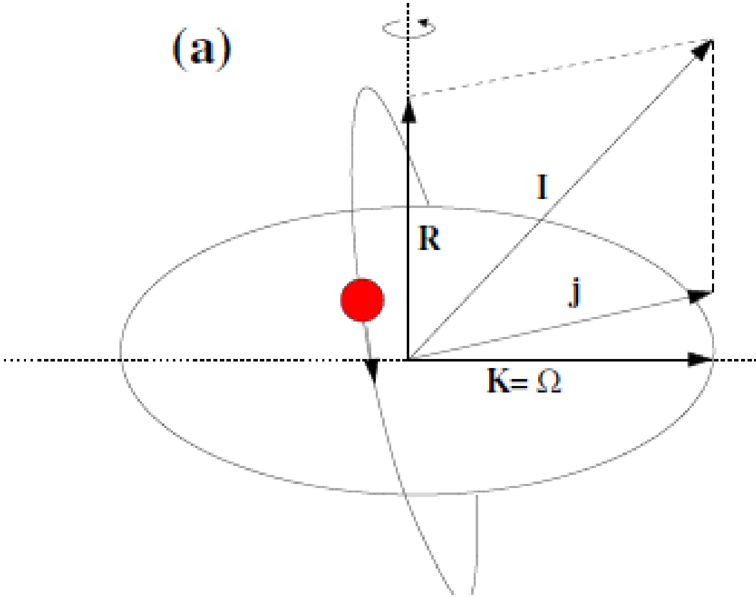

**Fig. 4.** Systematics of $K^\pi = 6^+$ isomers in the $Z = 72$ (Hf) isotopes and the $K^\pi = 8^-$ isomers in the $N = 106$ isotones.

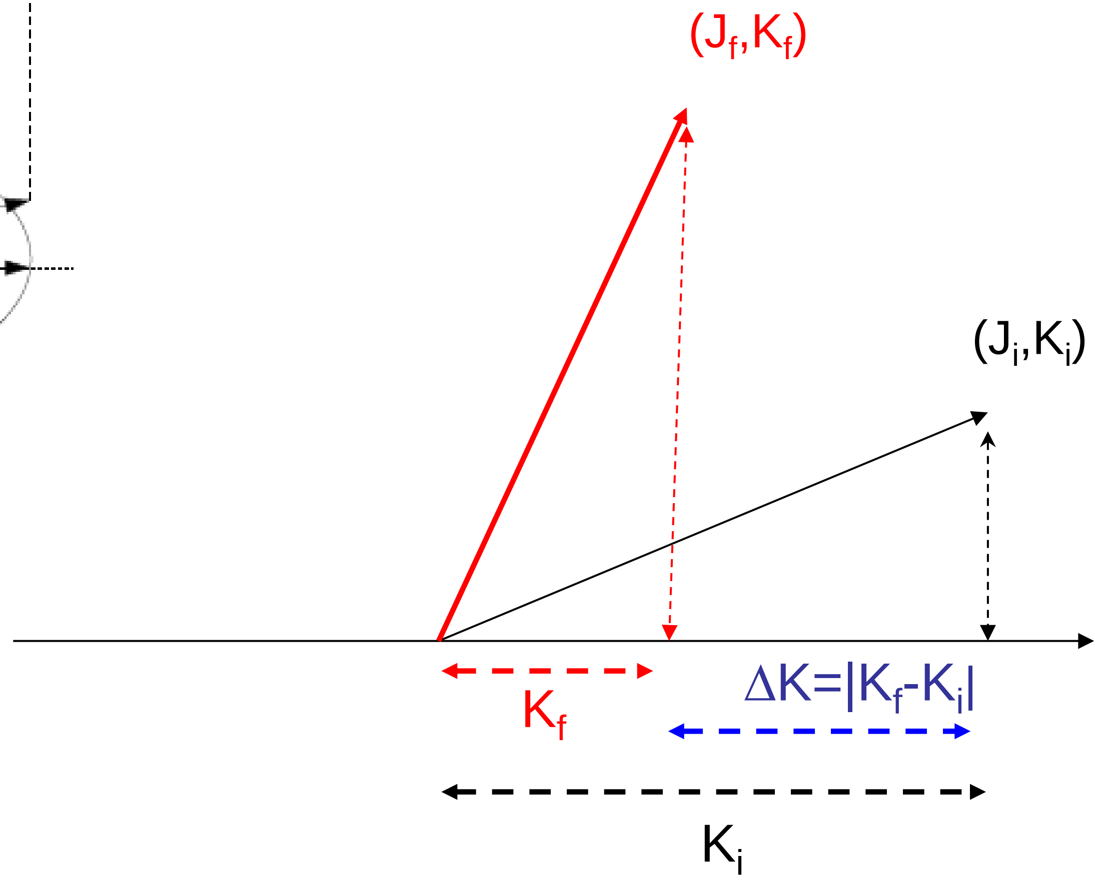



Nilsson levels for protons (left) and neutrons (right) in the $A \sim 170$–190 region. Boxes indicate the main orbitals of interest

From F.G.Kondev et al., ADNDT 103-104 (2015) p50-105

# K isomers




$$|K_i - K_f| = \Delta K \leq \lambda$$

$$\nu = \Delta K - \lambda$$

$$f_\nu = \left[\frac{T^{\gamma}_{1/2}}{T^{W}_{1/2}}\right]^{1/\nu}$$ = reduced hindrance for a K-isomeric decay transition.

# K-isomers in deformed nuclei

In the strong-coupling limit, for orbitals where $\Omega$ is large, unpaired particles can sum their angular momentum projections on the nuclear axis if symmetry to give rise to 'high-K' states, such that the total spin/parity of the high-K Multi-particle state is give by:

$$J^{\pi} = K^{\pi} = \sum_i \Omega_i^{\Pi(\pi_i)}$$

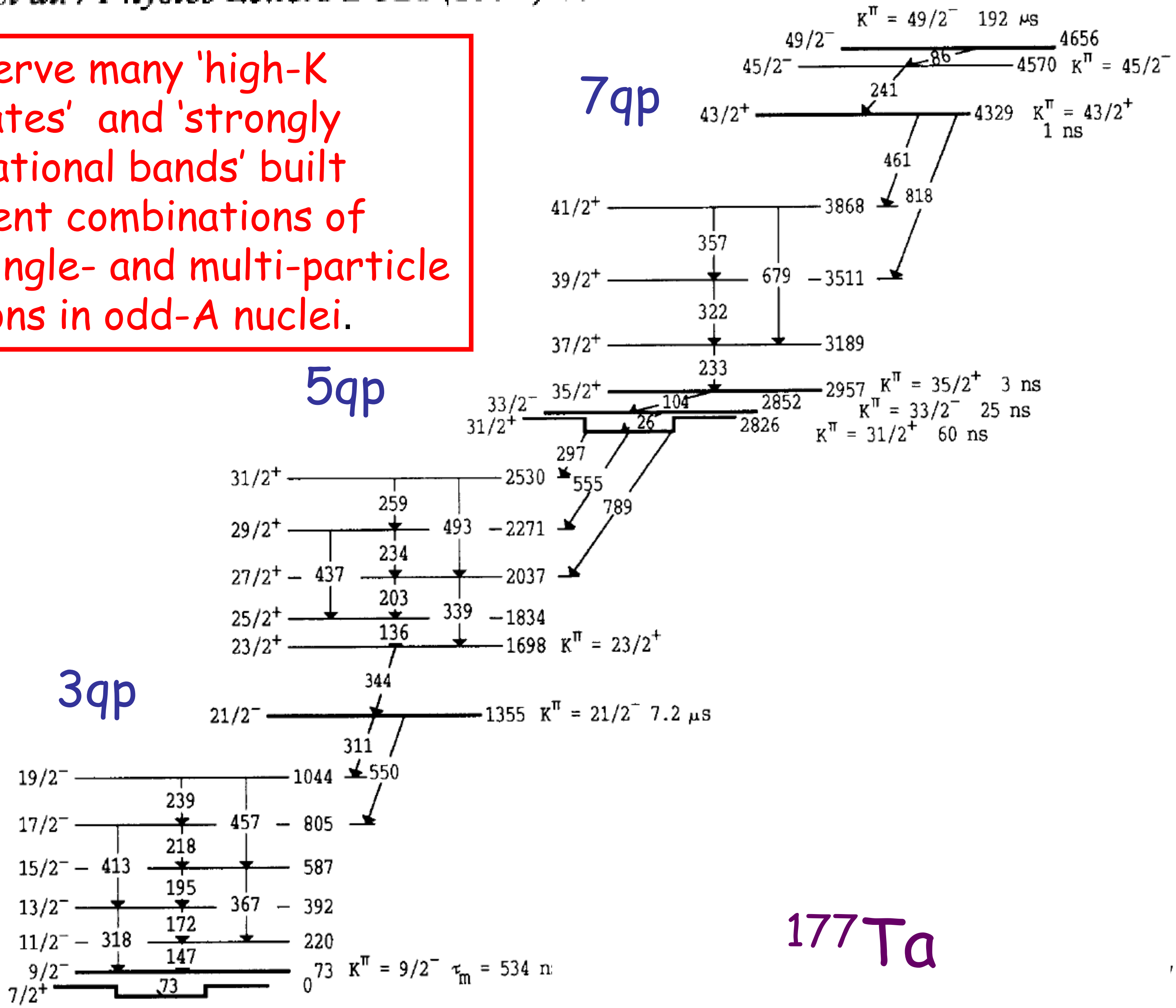
These, high-K multi-quasi-particle states are expected to occur at excitations energies of:

$$E^* \approx \sum_k \sqrt{(\epsilon_k - \epsilon_F)^2 + \Delta^2}$$

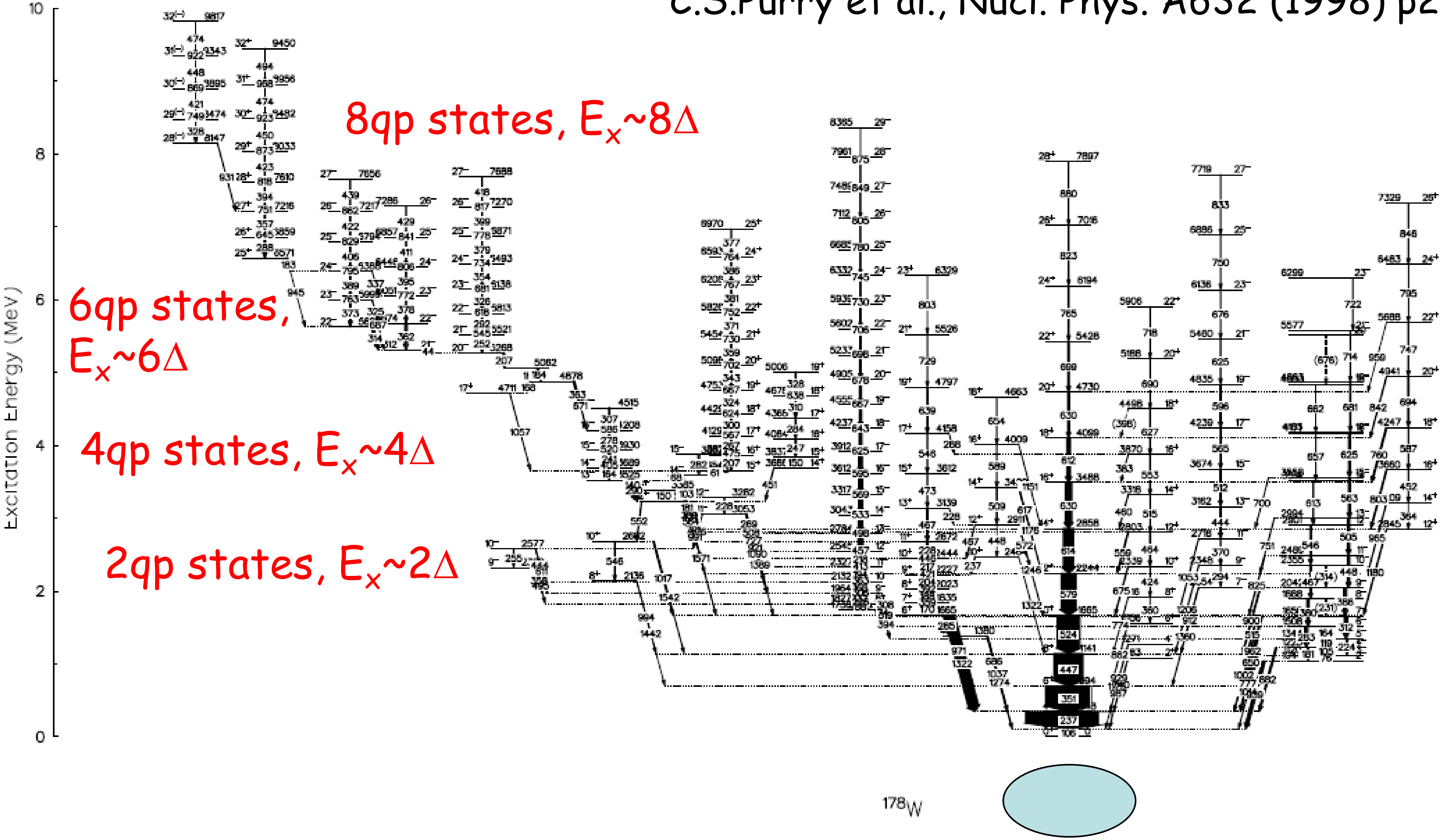
where $\varepsilon_k$ is the single-particle energy; $\varepsilon_F$ is the Fermi energy and $\Delta$ is the pair gap (which can be obtained from odd-even mass differences)

M. Dasgupta et al. / Physics Letters B 328 (1994) 16–21

We can observe many 'high-K isomeric states' and 'strongly coupled rotational bands' built upon different combinations of deformed single- and multi-particle configurations in odd-A nuclei.



C.S.Purry et al., Nucl. Phys. A632 (1998) p229




$^{178}$W: different and discrete 0, 2, 4, 6 and 8 quasi-particle band structures are all observed:

These are built on different underlying single-particle (Nilsson) orbital configurations.

# ‘Forbiddenness’ in K isomers

We can use single particle (‘Weisskopf’) estimates for transitions rates for a given multipolarity. ($E_g$ (keV) , $T_{1/2}$(s), Firestone and Shirley, *Table of Isotopes* (1996).

Weisskopf Estimates for $T^{1/2}$ $A = 180,\ E_\gamma = 500$ keV

$$E1 \to T_W^{1/2} = 6.76\times10^{-6} E_\gamma^{-3} A^{-2/3} \to 1.6\times10^{-15}\,s$$

$$M1 \to T_W^{1/2} = 2.20\times10^{-5} E_\gamma^{-3} \to 1.8\times10^{-13}\,s$$

$$E2 \to T_W^{1/2} = 9.52\times10^{6} E_\gamma^{-5} A^{-4/3} \to 3.0\times10^{-10}\,s$$

$$M2 \to T_W^{1/2} = 3.10\times10^{7} E_\gamma^{-5} A^{-2/3} \to 3.1\times10^{-8}\,s$$

*Hindrance (F)* (removing dependence on multipolarity and $E_\gamma$) is defined by

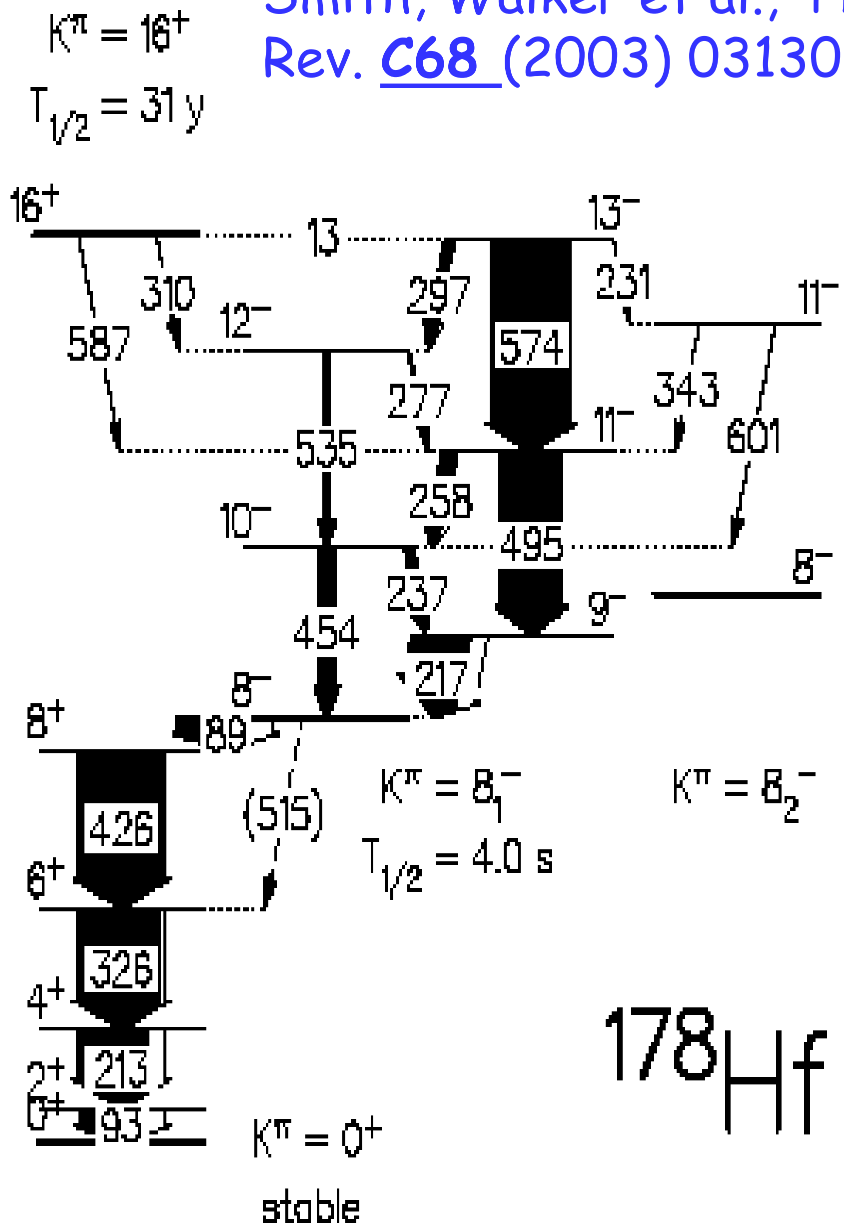
$$F = \left(\frac{T^\gamma_{1/2}}{T^W_{1/2}}\right) = \text{ratio of expt. and Weisskopf trans. rates}$$

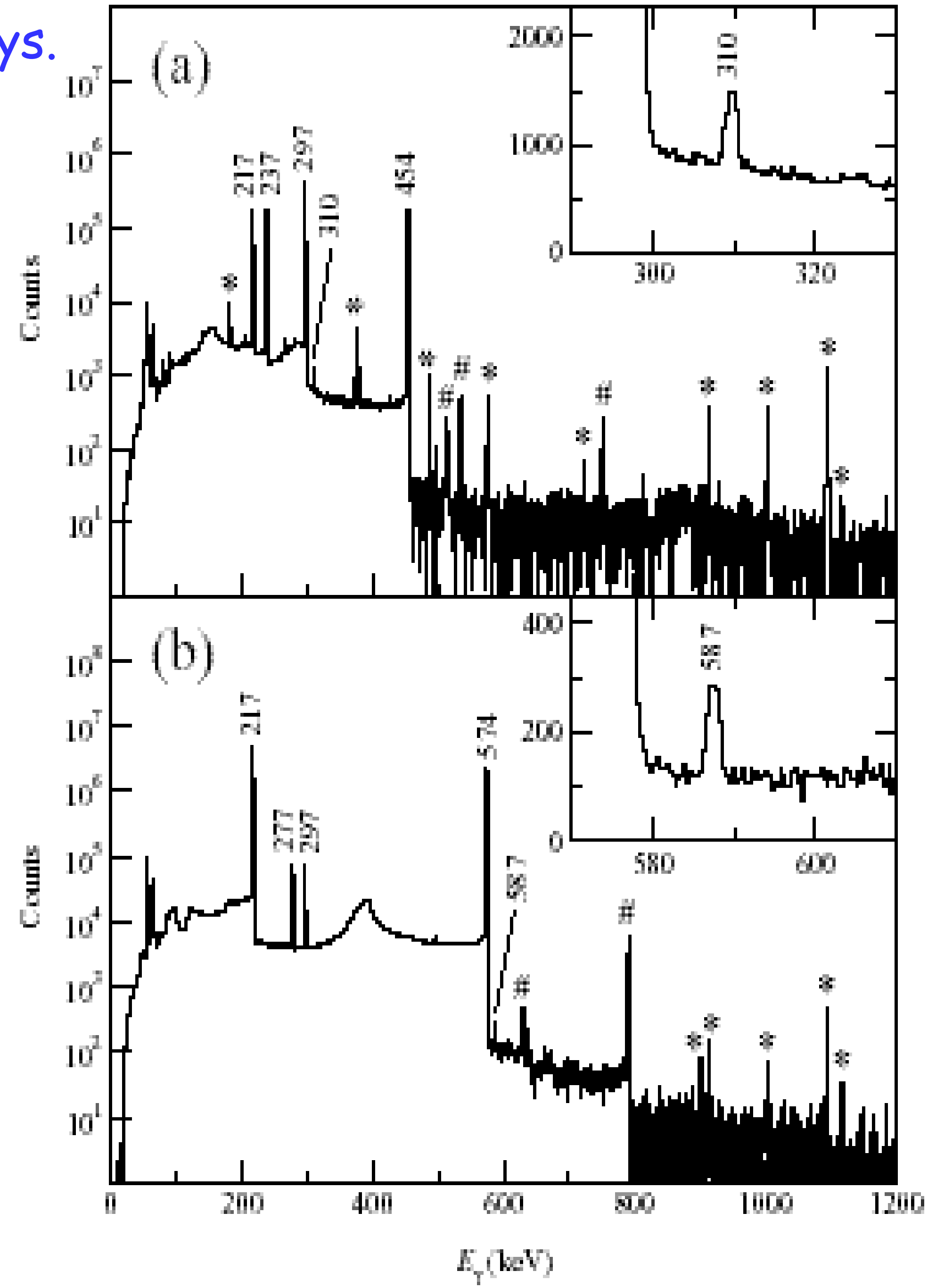
***Reduced Hindrance ( $f_\nu$ )*** gives an estimate for the ‘goodness’ of K- quantum number and validity of K-selection rule ( = a measure of axial symmetry).

$$f_\nu = F^{1/\nu} = \left(\frac{T^\gamma_{1/2}}{T^W_{1/2}}\right)^{1/\nu}, \quad \nu = \Delta K - \lambda$$

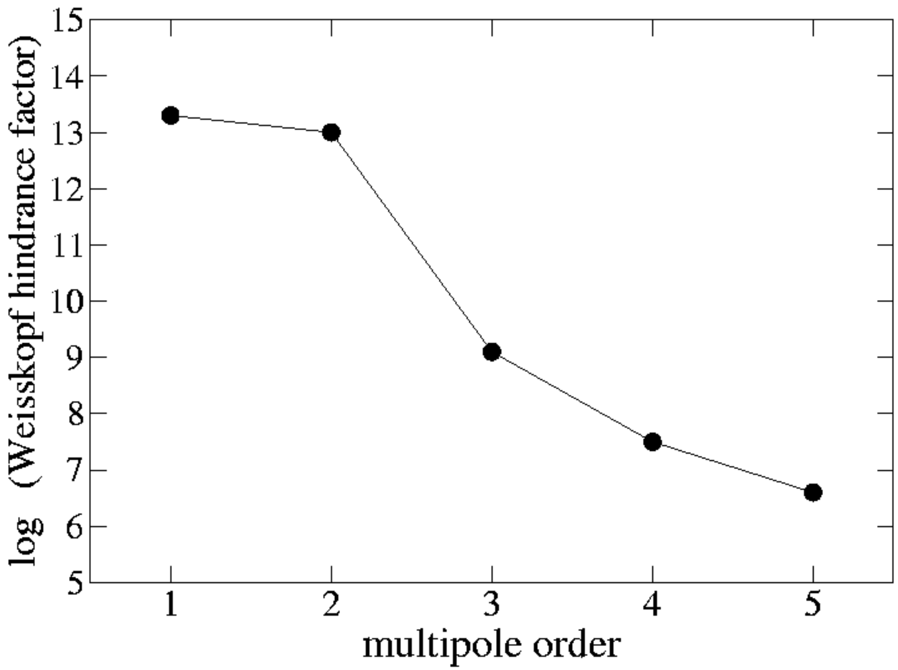
$f_\nu \sim 100$ typical value for ‘good’ K isomer (see Lobner Phys. Lett. **B26** (1968) p279)

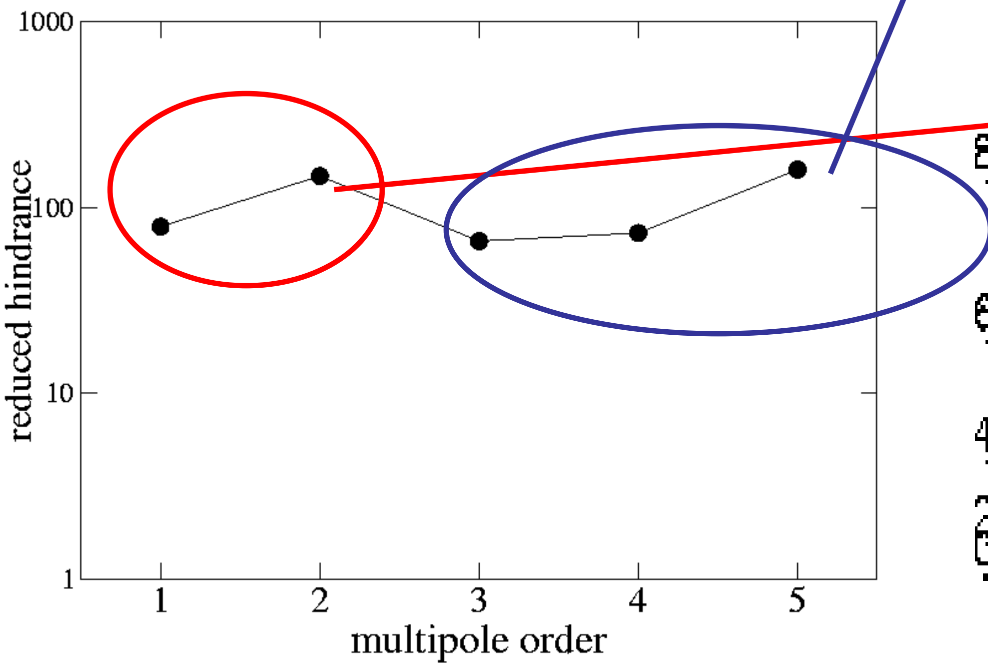
Smith, Walker et al., Phys. Rev. C68 (2003) 031302

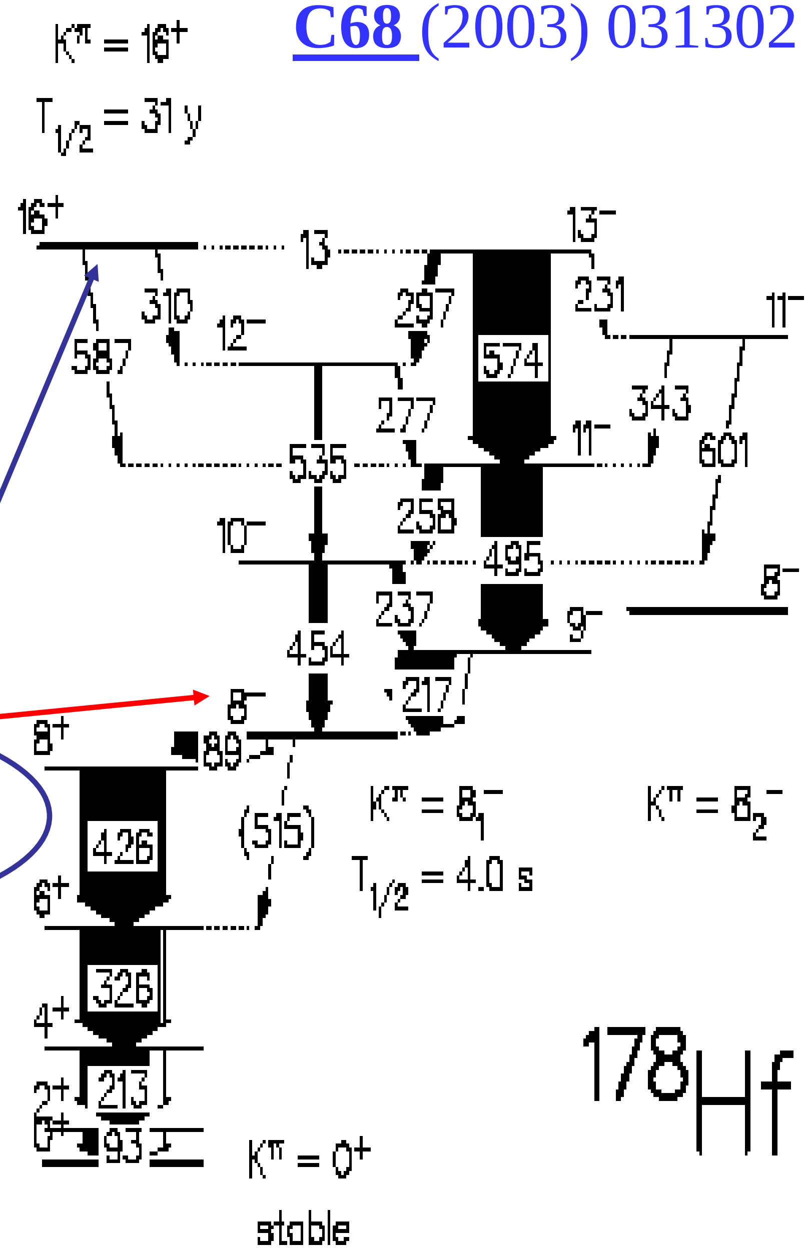


Smith, Walker et al., Phys. Rev. **C68** (2003) 031302



# Measurements of EM Transition Rates

Rep. Prog. Phys., Vol. 42, 1979.

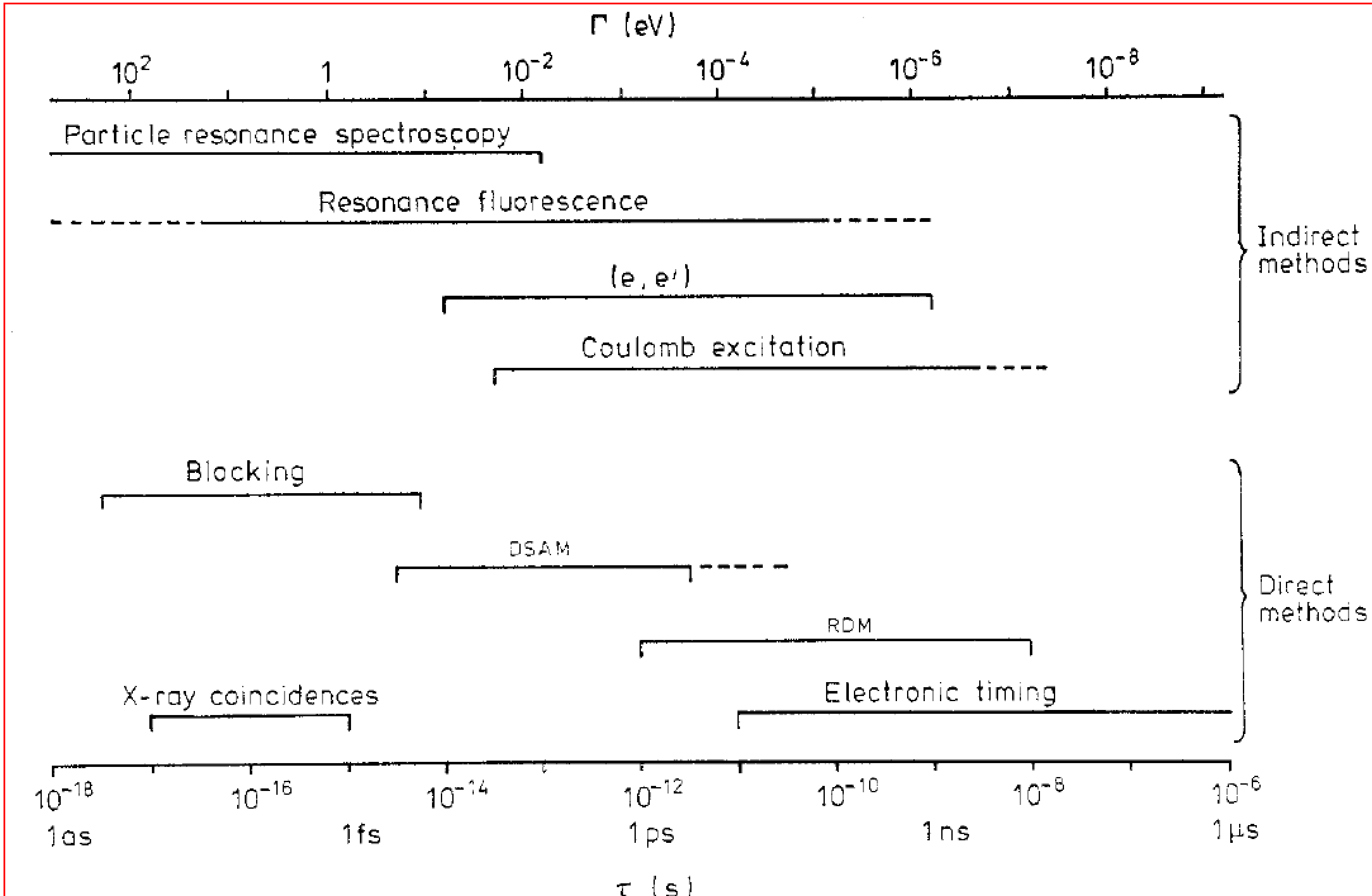
# The measurement of the lifetimes of excited nuclear states

PJ NOLAN† and JF SHARPEY-SCHAFER‡

$$\Gamma\tau = \hbar.$$

$$\Gamma \propto |\langle\psi_f|\hat{O}_{decay}|\psi_i\rangle|^2$$

$$\Gamma_{total} = \sum_j \Gamma_j.$$



# THE MEASUREMENT OF SHORT NUCLEAR LIFETIMES[1]

By A. Z. Schwarzschild and E. K. Warburton
*Brookhaven National Laboratory, Upton, New York*[2]

Annual Review of Nuclear Science (1968) **18** p265-290



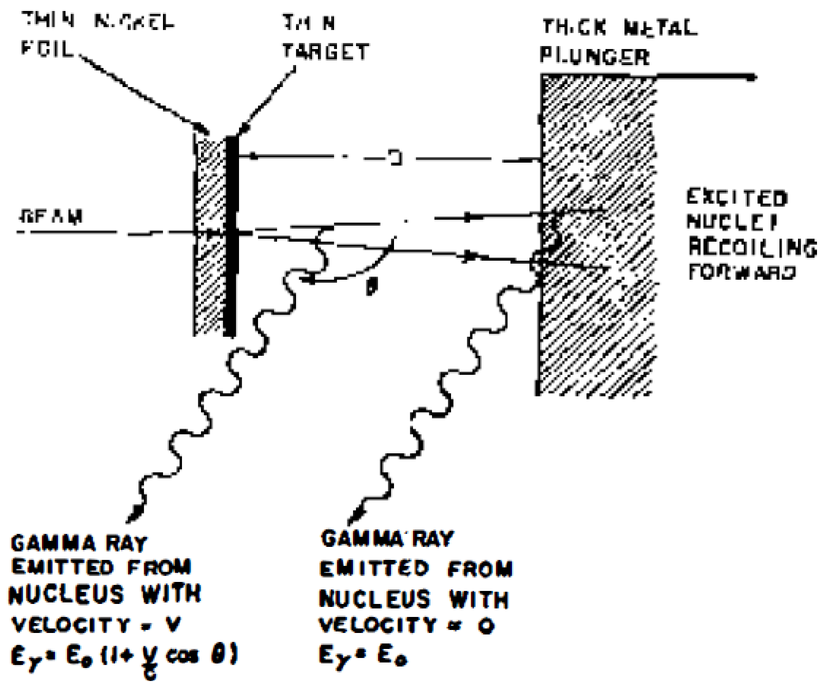
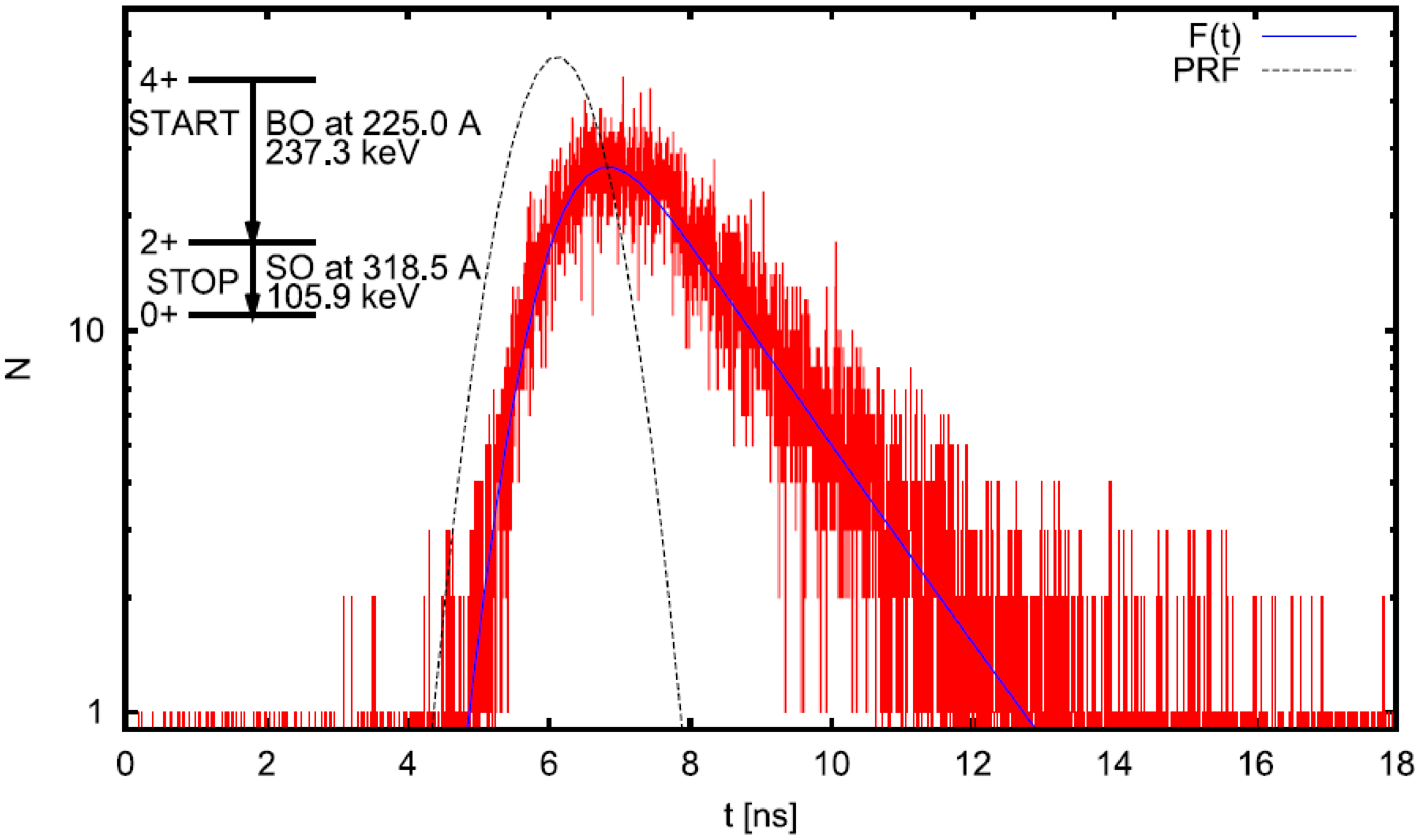



Fig. 5. Recoil method of measuring lifetimes of excited states.

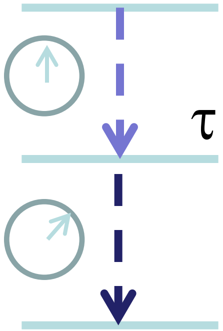
# Fast-timing Techniques

*M. Rudigier et al. / Nuclear Physics A 847 (2010) 89–100*




Gaussian-exponential convolution to account for timing resolution

# Some quick revision on extracting (nuclear excited state) lifetimes.

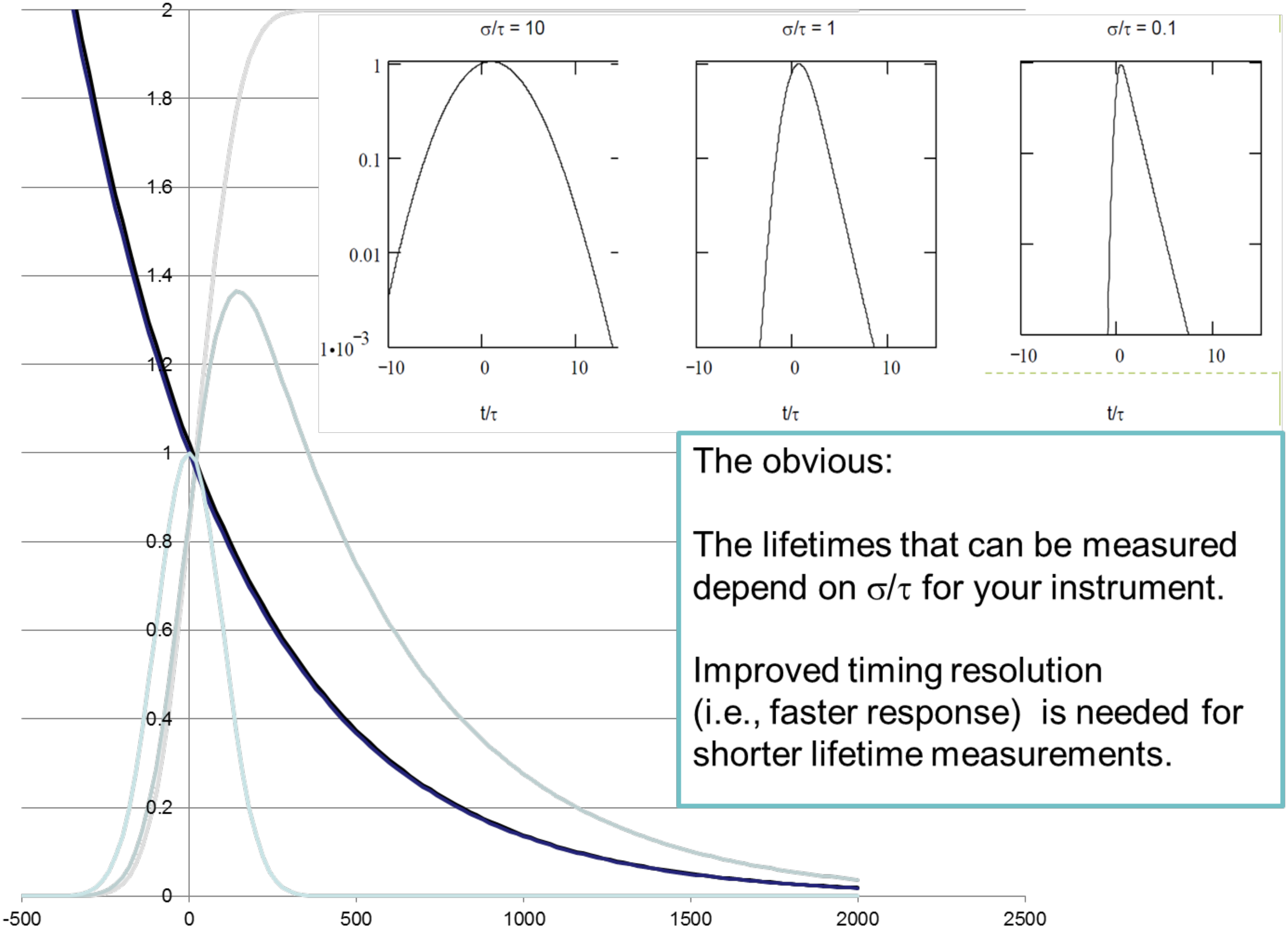



- Assuming no background contribution, the experimentally measured, 'delayed' time distribution for a $\gamma$–$\gamma$–$\Delta$t measurement is given by:

$$D(t) = n\lambda \int_{-\infty}^{t} P(t' - t_0) e^{-\lambda(t - t')} dt' \quad \text{with} \quad \lambda = 1/\tau,$$

- *P(t'-t₀)* is the (Gaussian) prompt response function and $\lambda = 1/\tau$, where $\tau$ is the mean lifetime of the intermediate state.

See e.g., Z. Bay, Phys. Rev. **77** (1950) p419;
T.D. Newton, Phys. Rev. **78** (1950) p490;
J.M.Regis et al., EPJ Web of Conf. **93** (2015) 01014

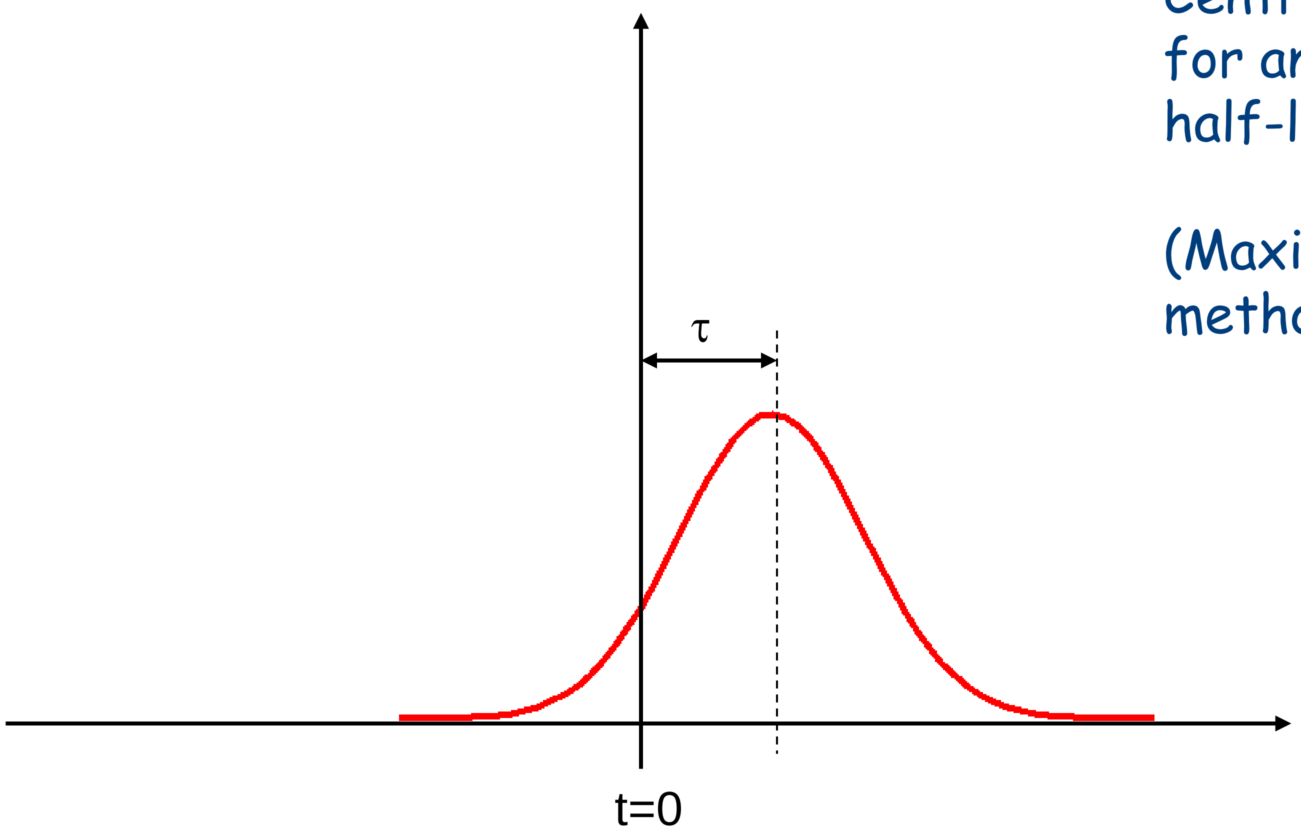



The obvious:

The lifetimes that can be measured depend on σ/τ for your instrument.

Improved timing resolution (i.e., faster response) is needed for shorter lifetime measurements.

# Fast-timing Techniques

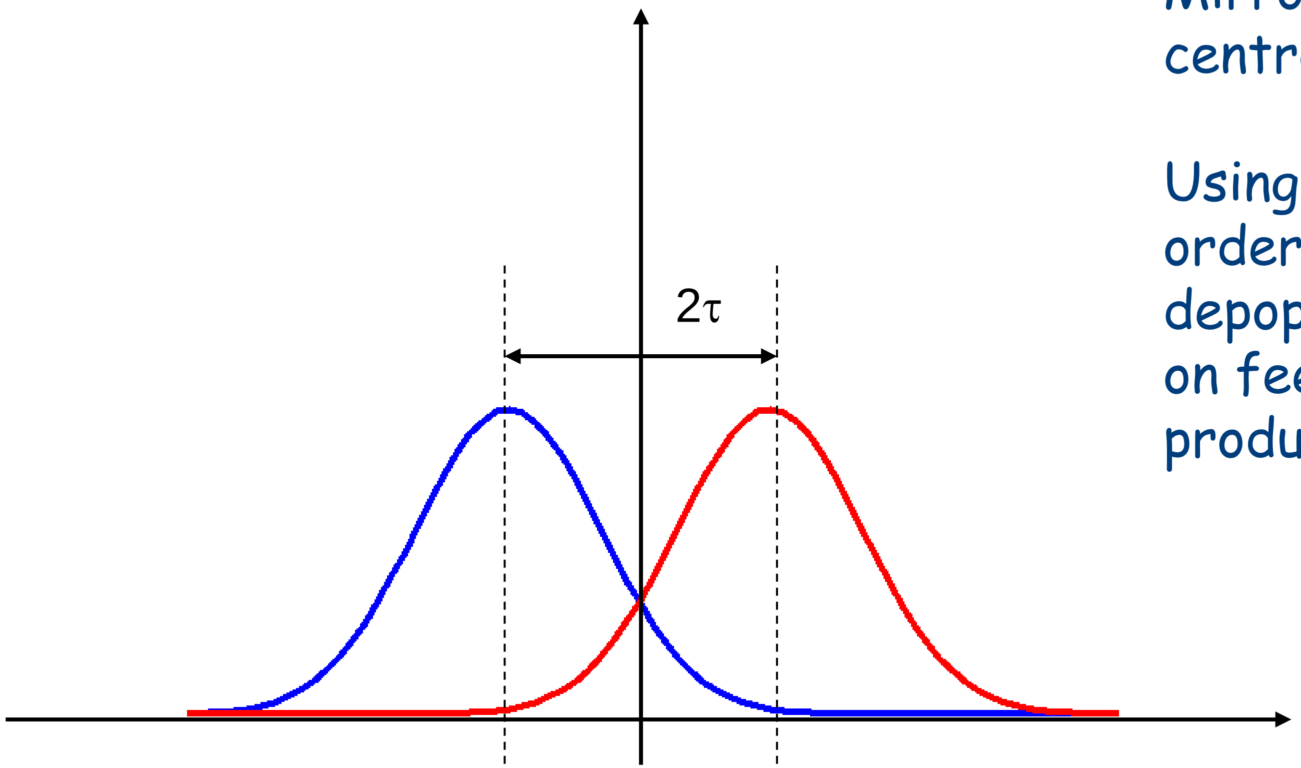



Centroid shift method for an analysis of short half-lives

(Maximum likelihood method)

Difference between the centroid of observed time spectrum and the prompt response give lifetime, $\tau$

# Fast-timing Techniques

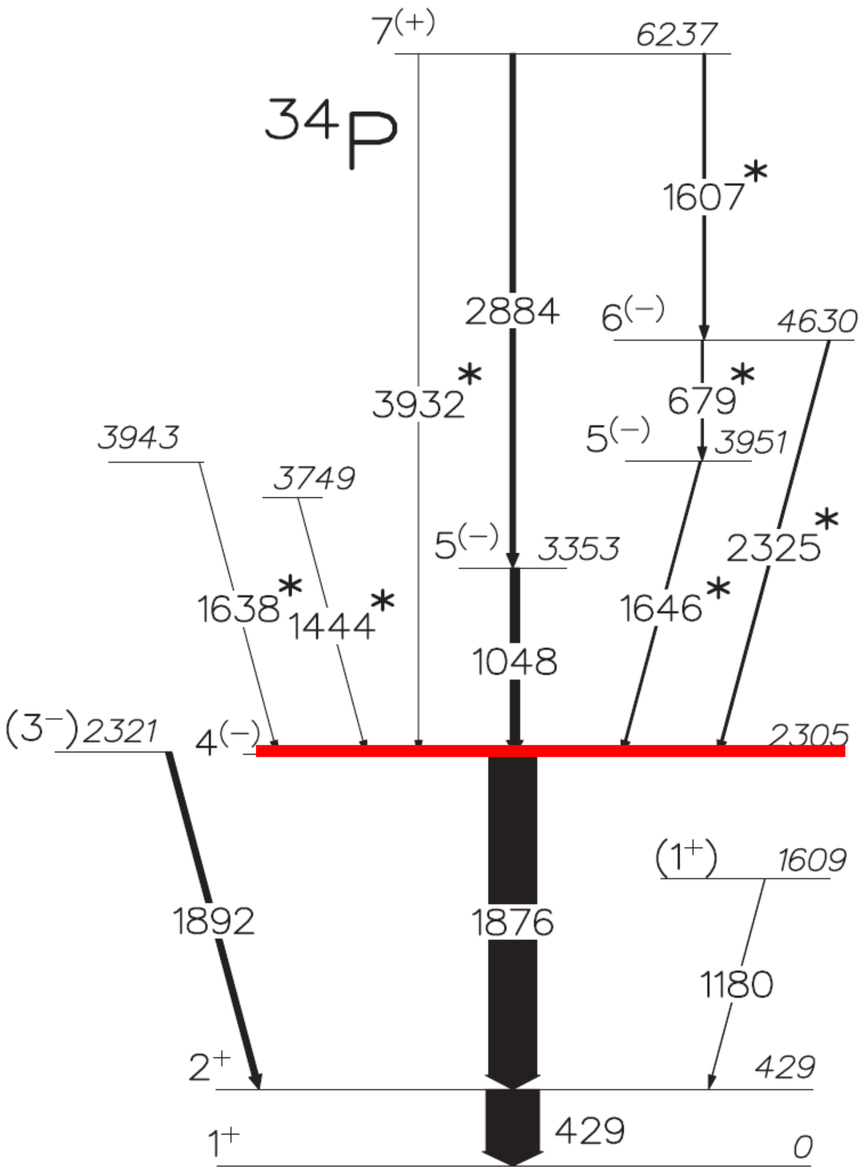



Mirror-symmetric centroid shift method.

Using reversed gate order (e.g. start TAC on depopulating gamma, stop on feeding gamma) produces opposite shift

Removes the need to know where the prompt distribution is and other problems to do with the prompt response of the detectors

# An example, 'fast-timing' and id of M2 decay in $^{34}$P.




P. C. BENDER *et al.* PHYSICAL REVIEW C 80, 014302 (2009)

R. CHAKRABARTI *et al.* PHYSICAL REVIEW C 80, 034326 (2009)

- Study of $^{34}$P identified low-lying $I^{\pi}=4^{-}$ state at E=2305 keV.
- $I^{\pi}=4^{-} \rightarrow 2^{+}$ transition can proceed by M2 and/or E3.
- *Aim of experiment was to* measure precision lifetime for 2305 keV state and obtain B(M2) and B(E3) values.
- Previous studies limit half-life to 0.3 ns < $t_{1/2}$ < 2.5ns

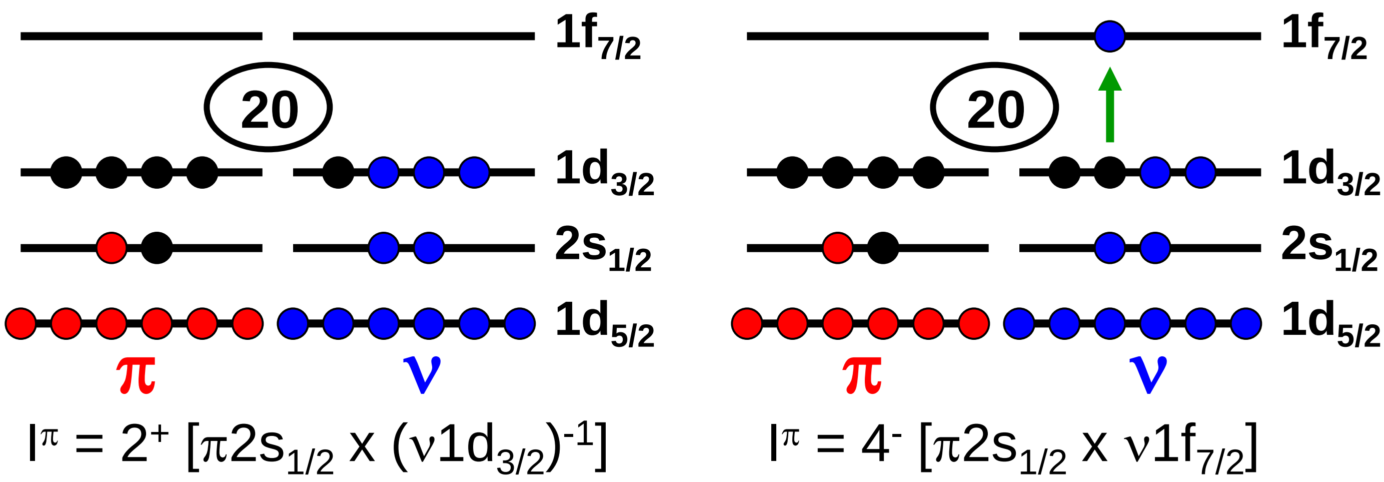
P.J.R.Mason et al., Phys. Rev. C85 (2012) 064303.

# Physics….which orbitals are involved?

- Theoretical (shell model) predictions suggest $2^+$ state based primarily on $[\pi 2s_{1/2} \times (\nu 1d_{3/2})^{-1}]$ configuration and $4^-$ state based primarily on $[\pi 2s_{1/2} \times \nu 1f_{7/2}]$ configuration.

- Thus expect transition to go mainly via $f_{7/2} \rightarrow d_{3/2}$, M2 transition.

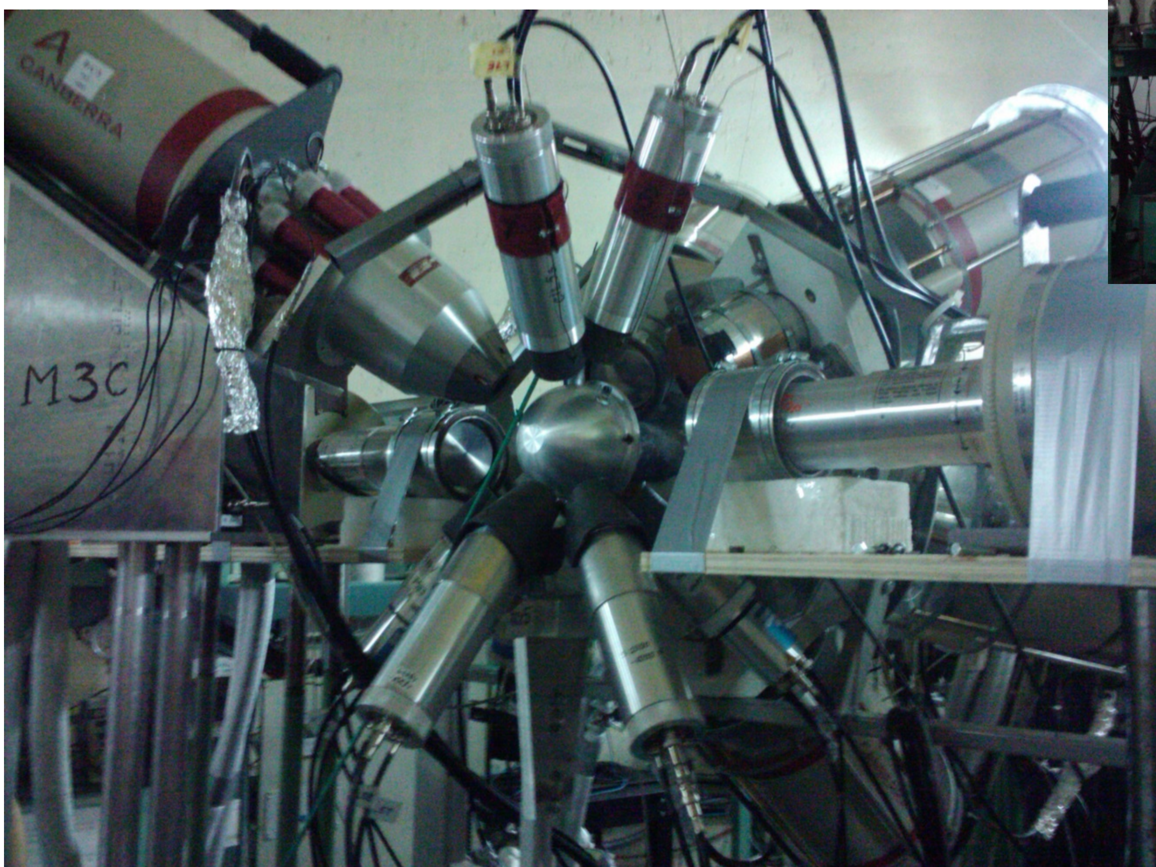
- Different admixtures in $2^+$ and $4^-$ states may also allow some E3 components (e.g., from, $f_{7/2} \rightarrow s_{1/2}$) in the decay.



# Experiment to Measure Yrast $4^-$ Lifetime in $^{34}P$

$^{18}O(^{18}O,pn)^{34}P$ fusion-evaporation at 36 MeV $\sigma$ ~ 5 – 10 mb

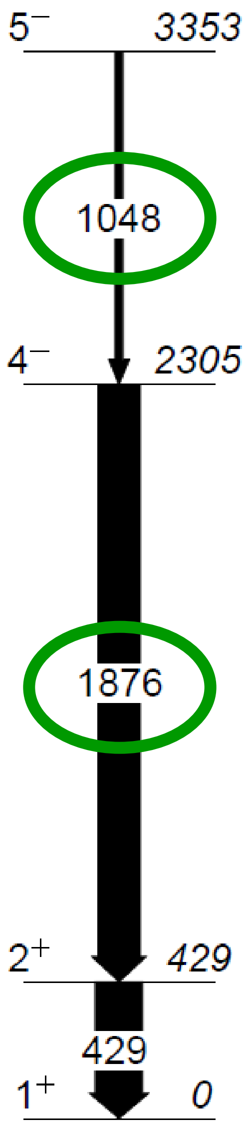
50mg/cm$^2$ $Ta_2{}^{18}O$ enriched foil; $^{18}O$ Beam from Bucharest Tandem (~20pnA)




Array 8 HPGe (unsuppressed) and 7 $LaBr_3$:Ce detectors

-3 (2"x2") cylindrical
-2 (1"x1.5") conical
-2 (1.5"x1.5") cylindrical

# Ge-Gated Time differences




PHYSICAL REVIEW C **85**, 064303 (2012)

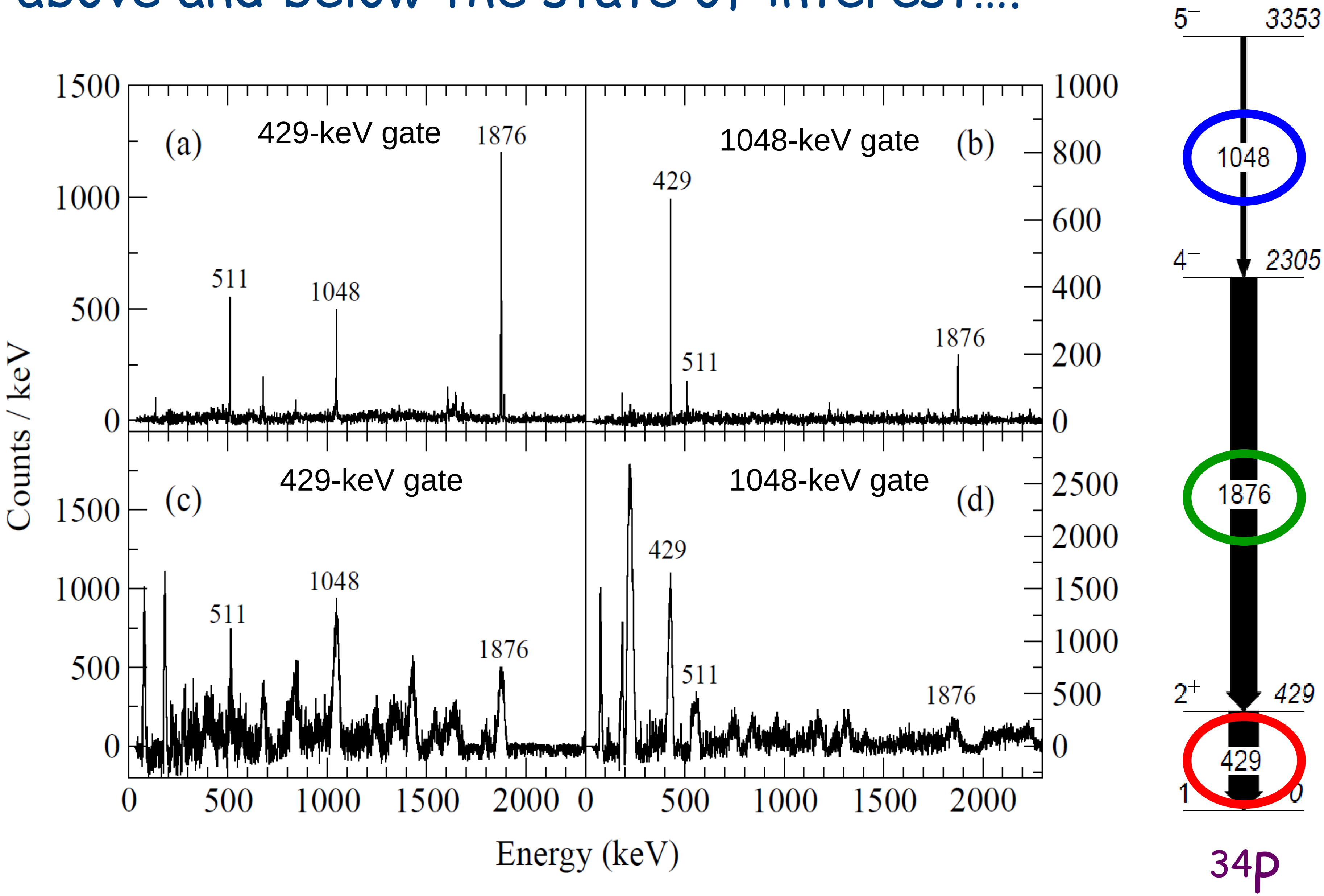
**Half-life of the $I^{\pi} = 4^{-}$ intruder state in $^{34}$P: $M2$ transition strengths approaching the island of inversion**

P. J. R. Mason,[1] T. Alharbi,[1,2] P. H. Regan,[1] N. Mărginean,[3] Zs. Podolyàk,[1] E. C. Simpson,[1] N. Alkhomashi,[4] P. C. Bender,[5] M. Bowry,[1] M. Bostan,[6] D. Bucurescu,[3] A. M. Bruce,[7] G. Căta-Danil,[3] I. Căta-Danil,[3] R. Chakrabarti,[8] D. Deleanu,[3] P. Detistov,[9] M. N. Erduran,[10] D. Filipescu,[3] U. Garg,[11] T. Glodariu,[3] D. Ghiţă,[3] S. S. Ghugre,[8] A. Kusoglu,[6] R. Mărginean,[3] C. Mihai,[3] M. Nakhostin,[1] A. Negret,[3] S. Pascu,[3] C. Rodríguez Triguero,[7] T. Sava,[3] A. K. Sinha,[8] L. Stroe,[3] G. Suliman,[3] and N. V. Zamfir[3]

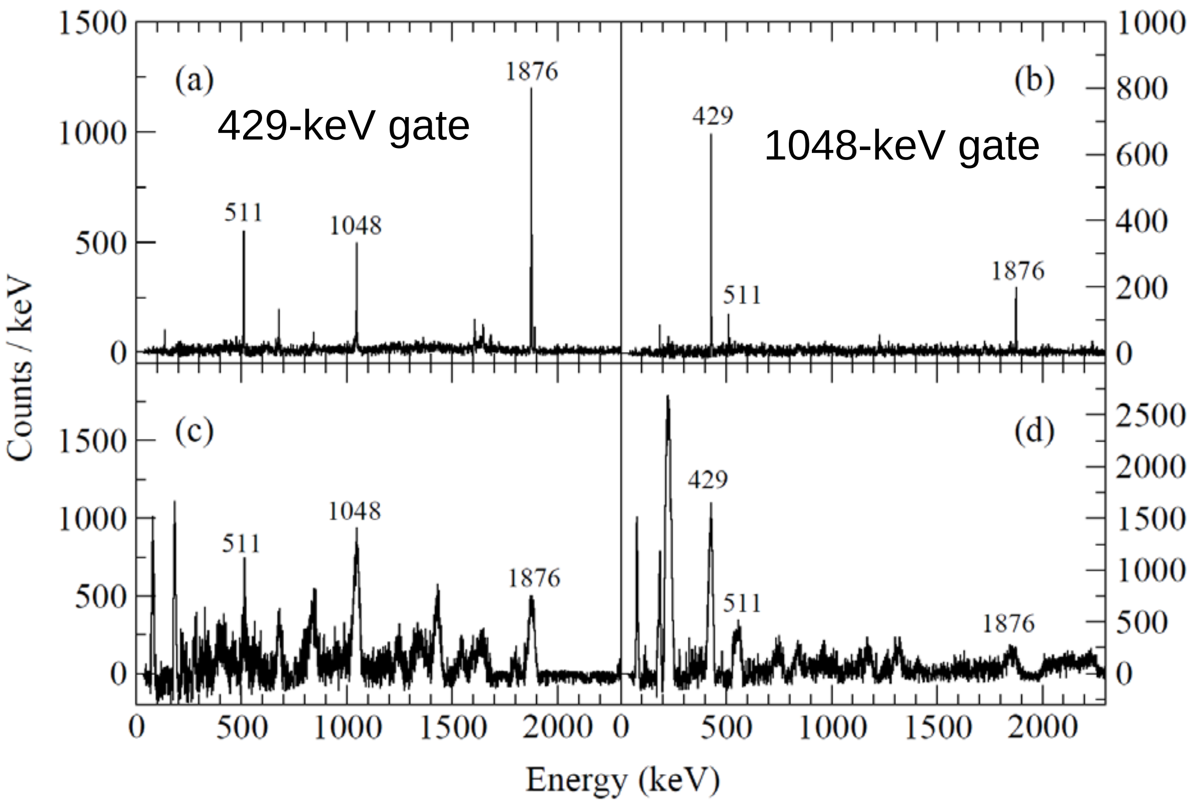
Gates in $LaBr_3$ detectors to observe time difference and obtain lifetime for state

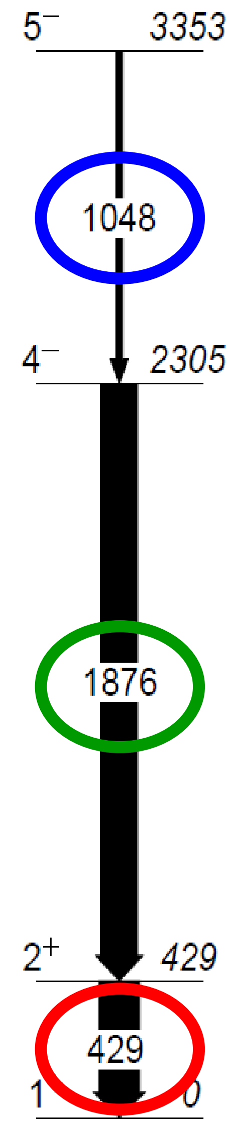
Ideally, we want to measure the time difference between transitions directly feeding and depopulating the state of interest ($4^{-}$)

# Gamma-ray energy coincidences 'locate' transitions above and below the state of interest....



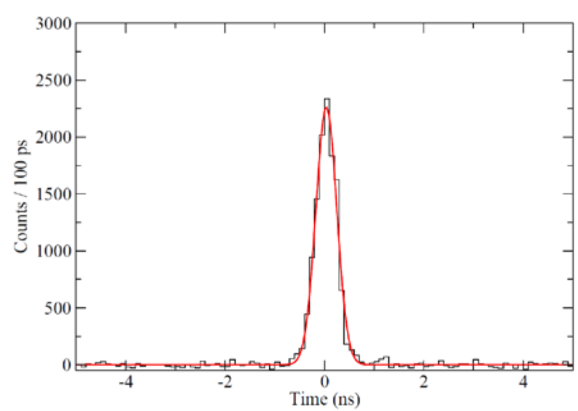
# LaBr$_3$ - LaBr$_3$ Energy-gated time




The 1876-429-keV time difference in $^{34}P$ should show prompt distribution as half-life of $2^+$ is much shorter than prompt timing response.

Measured FWHM = 470(10) ps



# Successful nanosecond lifetime measurement in $^{34}$P (June 2010)

PHYSICAL REVIEW C **85**, 064303 (2012)

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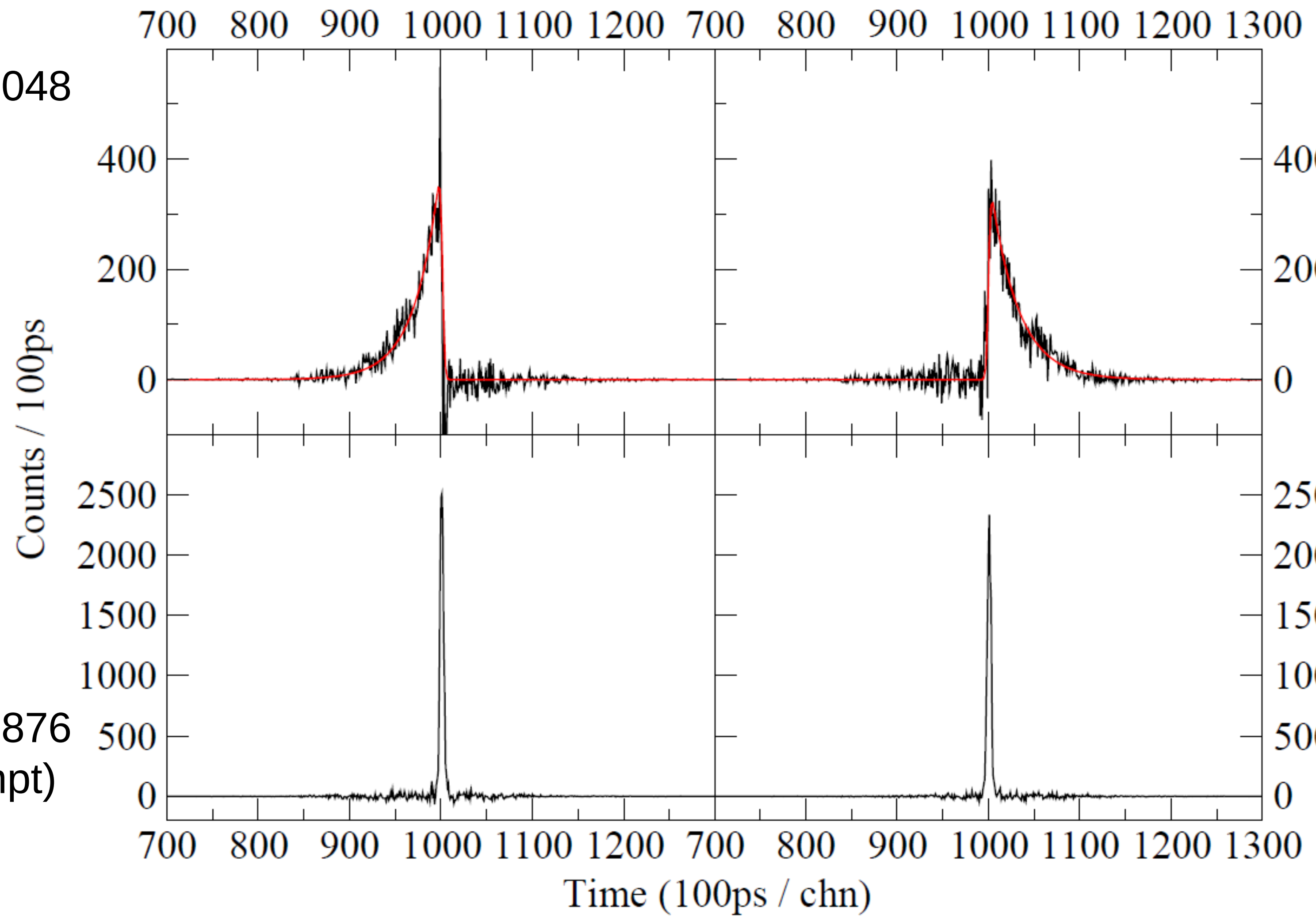
# Result: $T_{1/2}$ ($I^{\pi}=4^{-}$) in $^{34}P$= 2.0(1) ns

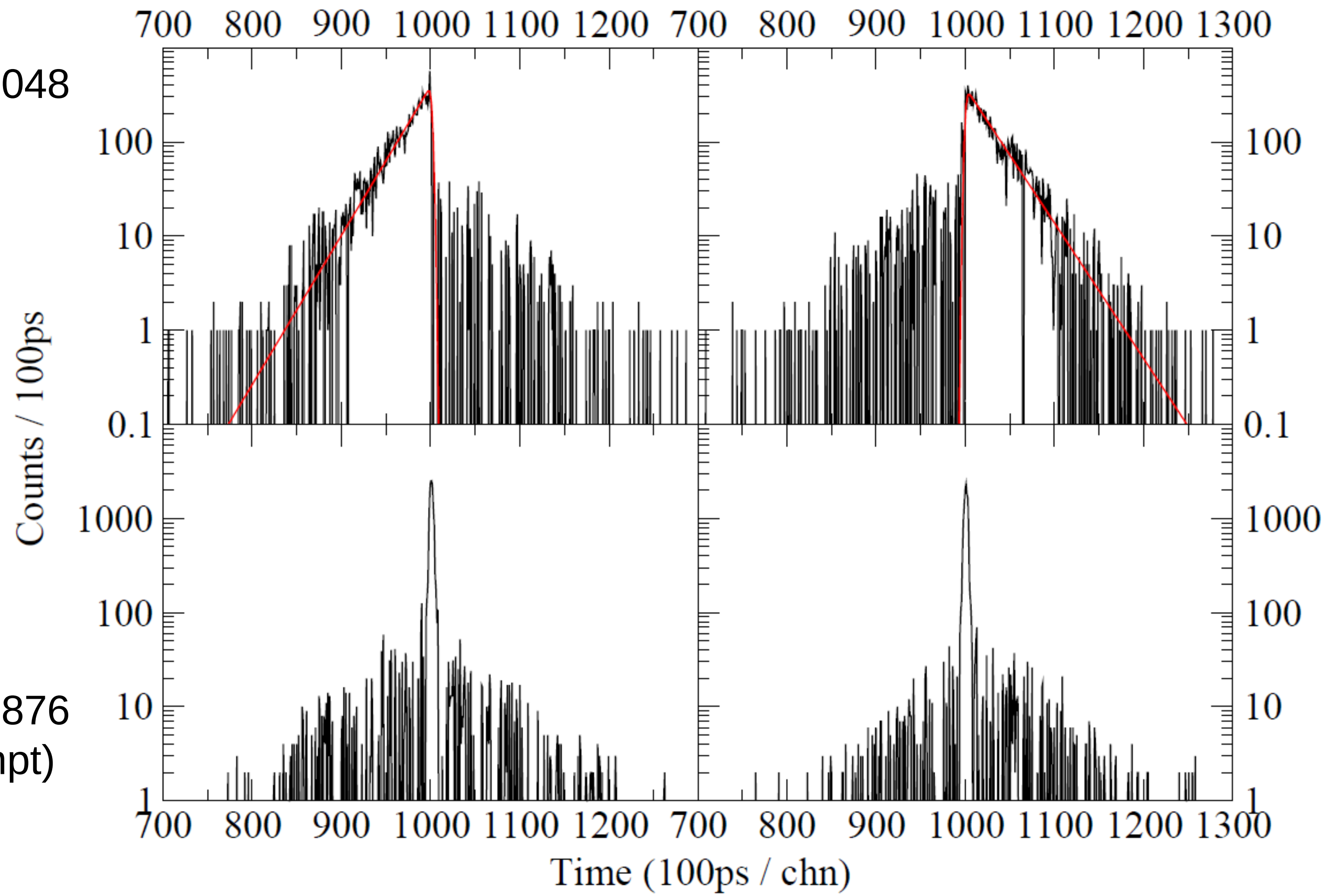
429 / 1048




429 / 1876
(~prompt)

# $T_{1/2}$ = 2.0(1)ns = 0.064(3) Wu for 1876 M2 in $^{34}$P.

429 / 1048




429 / 1876
(~prompt)

What about ‘faster’ transitions..
i.e. $< \sim 10$ ps ?

# Deconvolution and lineshapes

- *If the instrument time response function R(t) is Gaussian of width* $\sigma$*,*

$$R(t) = A_1 \exp\left(-\frac{t^2}{2\sigma^2}\right)$$

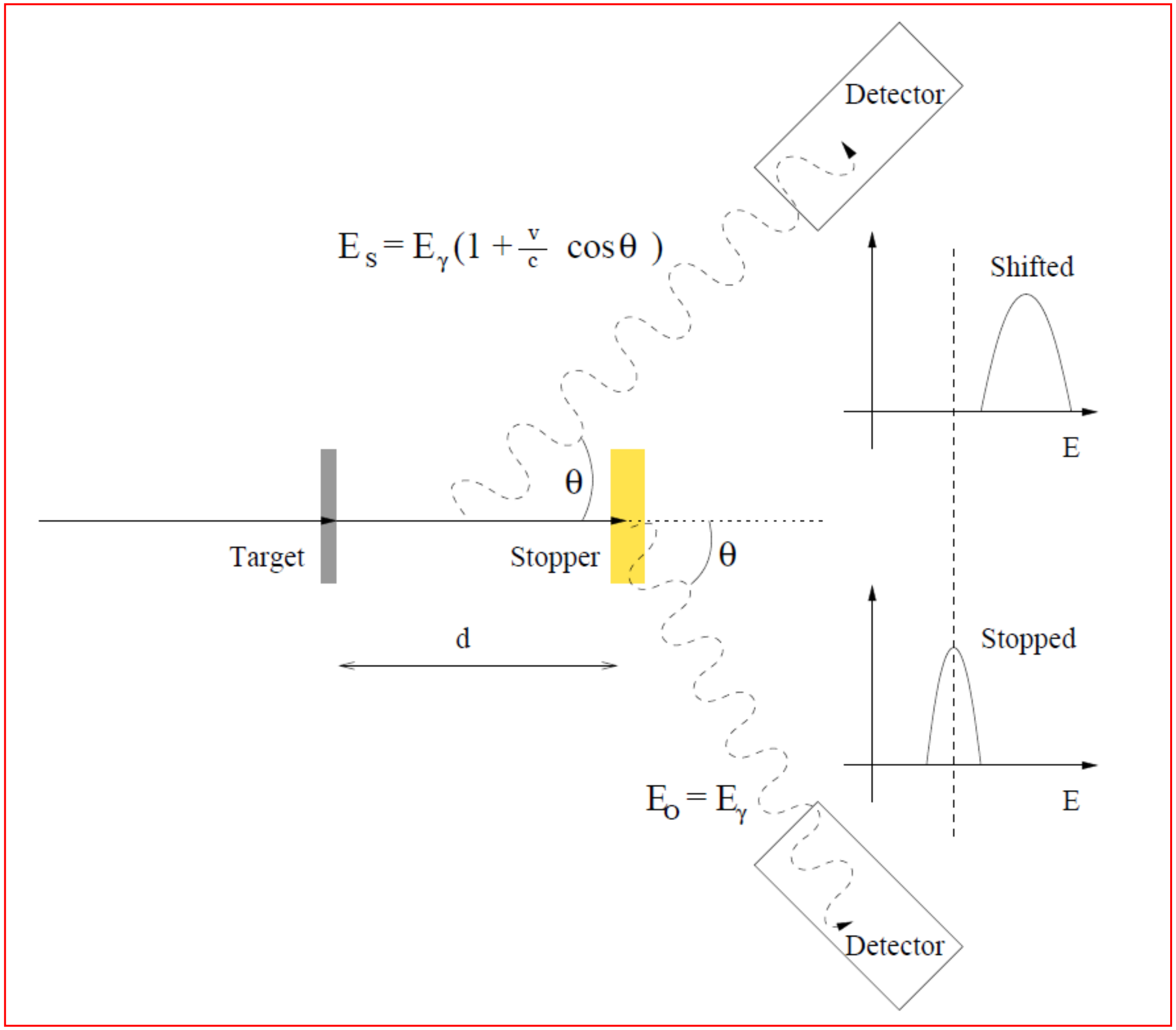
- *If the intermediate state decays with a mean lifetime* $\tau$*, then*

$$S(t) = A_2 \exp\left(-\frac{t}{\tau}\right)$$

- *The deconvolution integral for a single state lifetime is given by (ignoring the normalisation coefficients).*

$$I(t) = \exp\left(\frac{\sigma^2}{2\tau^2} - \frac{t}{\tau}\right)\left(1 - \mathrm{erf}\left(\frac{\sigma^2 - \tau t}{\sqrt{2}\sigma\tau}\right)\right)$$

*1-erf(x) is the complementary error function of the variable, x.*




$$
\begin{aligned}
E_0 &= E_0 \\
E_S &= E_0 \left( \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} \right)
\end{aligned}
$$

$$E_S \approx E_0(1+\beta\cos\theta)$$

$$
\begin{aligned}
I_0 &= N_0 \exp\left(\frac{-d}{v\tau}\right) \\
I_S &= N_0 \left(1 - \exp\left(\frac{-d}{v\tau}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
R &= \frac{I_0}{I_0 + I_S} = \exp\left(\frac{-d}{v\tau}\right) \\
\tau &= \frac{-d}{v\ln(R)}
\end{aligned}
$$

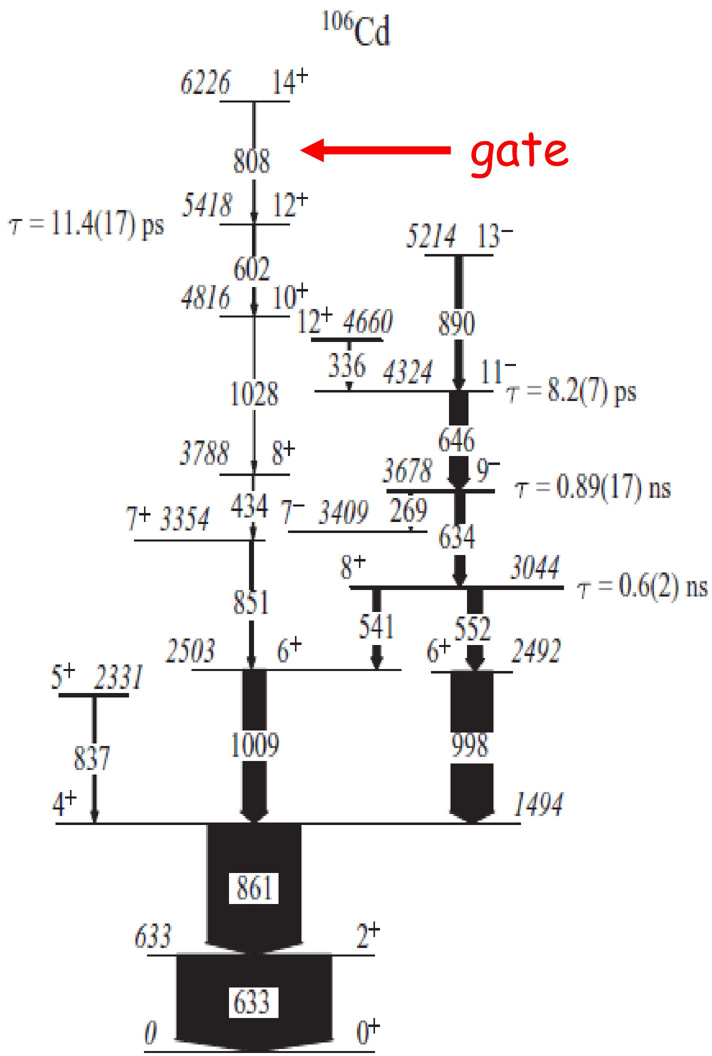
$$B(E2) = \frac{1}{1.225 \times 10^9 E_\gamma^5 \tau (1+\alpha)},$$

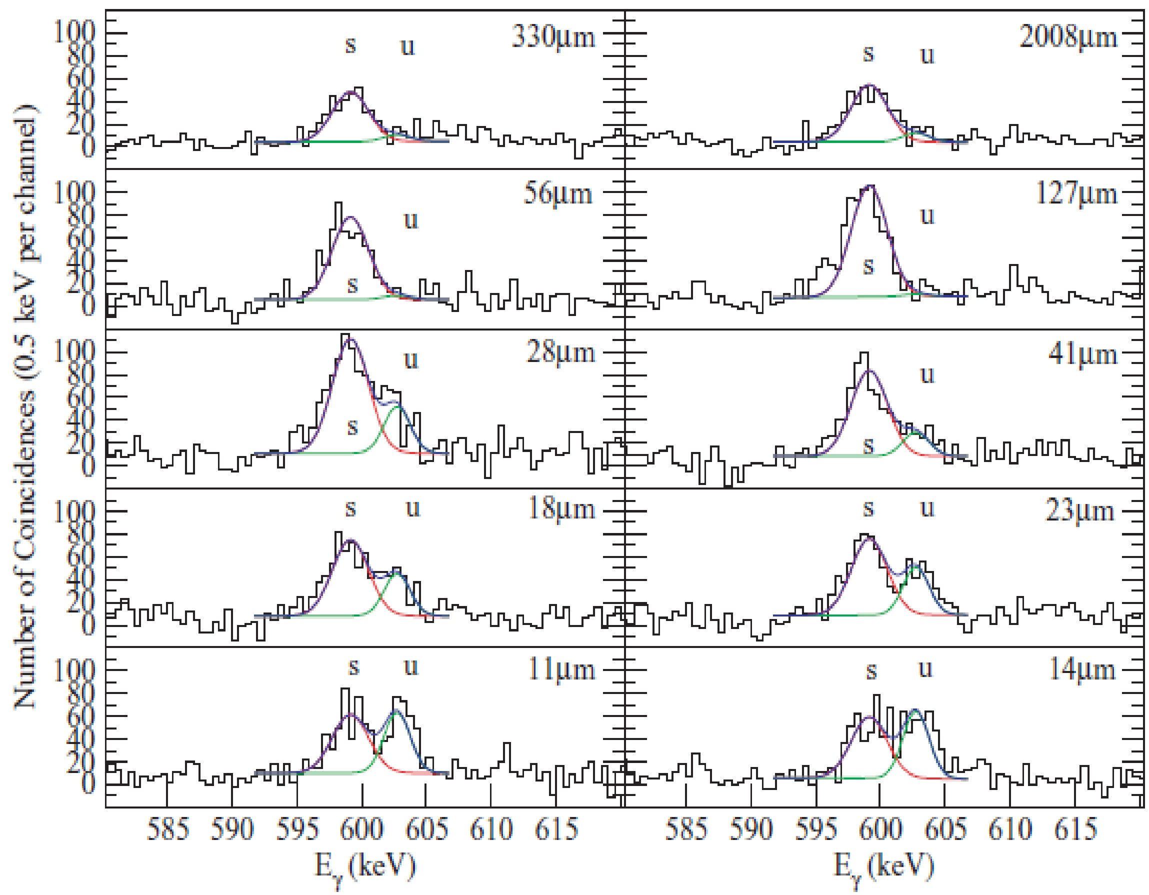
$$
\begin{aligned}
B(E2) &= \frac{5}{16\pi} e^2 Q_t^2 |\langle J_i K 20 | J_f K \rangle|^2, \\
Q_t &\approx \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_2 (1 + 0.16\beta_2),
\end{aligned}
$$

PHYSICAL REVIEW C **76**, 064302 (2007)

# Intrinsic state lifetimes in $^{103}$Pd and $^{106,107}$Cd

S. F. Ashley,[1,2,*] P. H. Regan,[1] K. Andgren,[1,3] E. A. McCutchan,[2] N. V. Zamfir,[2,4,5] L. Amon,[2,6] R. B. Cakirli,[2,6] R. F. Casten,[2] R. M. Clark,[7] W. Gelletly,[1] G. Gürdal,[2,5] K. L. Keyes,[8] D. A. Meyer,[2,9] M. N. Erduran,[6] A. Papenberg,[8] N. Pietralla,[10,11] C. Plettner,[2] G. Rainovski,[10,12] R. V. Ribas,[13] N. J. Thomas,[1,2] J. Vinson,[2] D. D. Warner,[14] V. Werner,[2] E. Williams,[2] H. L. Liu,[15] and F. R. Xu[15]



# Collective Model B(E2), B(M1) values.

The reduced in-band transition probabilities[1] are given by,

$$B(E2; I_i K \rightarrow I_f K) = \frac{5}{16\pi} e^2 Q_o^2 | < I_i 2 K 0 | I_f K > |^2$$
$$B(E2; I_i K \rightarrow I_f K) = \frac{5}{16\pi} e^2 Q_o^2 | < I_i 1 K 0 | I_f K > |^2 \qquad (2.4.56)$$
$$B(M1; I_i K \rightarrow I_f K) = \frac{3}{4\pi} e^2 | < I_i 1 K 0 | I_f K > |^2 (g_K - g_R)^2 K^2$$

where $Q_o$ is the intrinsic quadrupole moment and $g_K$ and $g_R$ are the intrinsic and rotational gyromagnetic ratios respectively. The relevant Clebsch-Gordon coefficients[2] are given below.

$$E2(\Delta I = 2) = \left[ \frac{3(I-K)(I-K-1)(I+K)(I+K-1)}{(2I-2)(2I-1)I(2I+1)} \right]^{1/2}$$
$$E2(\Delta I = 1) = -K \left[ \frac{3(I-K)(I+K)}{(I-1)I(2I+1)(I+1)} \right]^{1/2} \qquad (2.4.57)$$
$$M1(\Delta I = 1) = - \left[ \frac{(I-K)(I+K)}{I(2I+1)} \right]^{1/2}$$

[1] K.E.G. Löbner in, The Electromagnetic Interaction in Nuclear Spectroscopy, W.D. Hamilton (Ed), North-Holland (1975) Chapter 5

[2] The Theory of Atomic Spectra, Condon and Shortley (1935) reprinted (1963) p76-77

PHYSICAL REVIEW C **76**, 064302 (2007)

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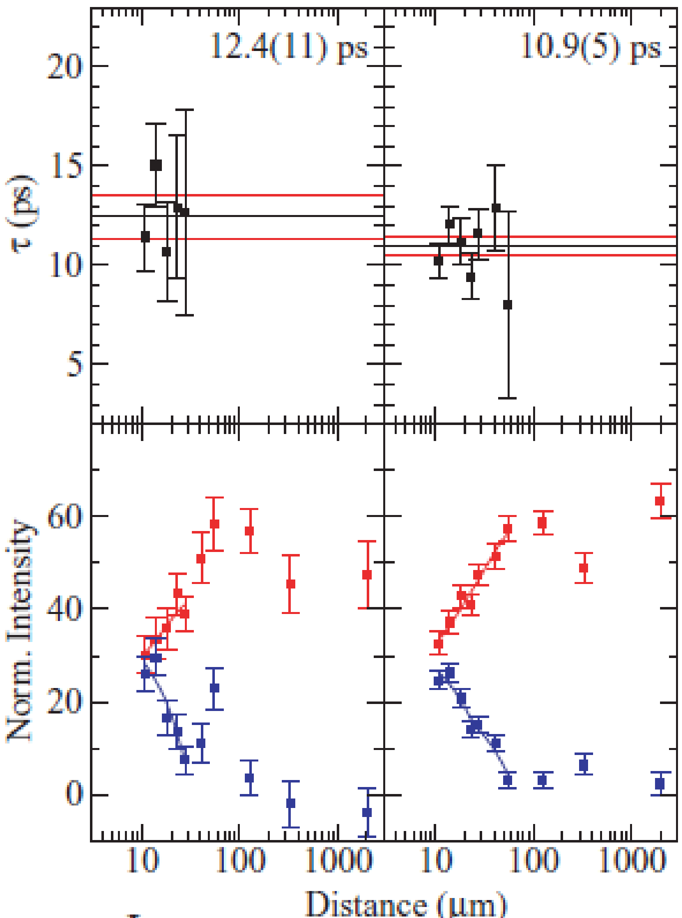


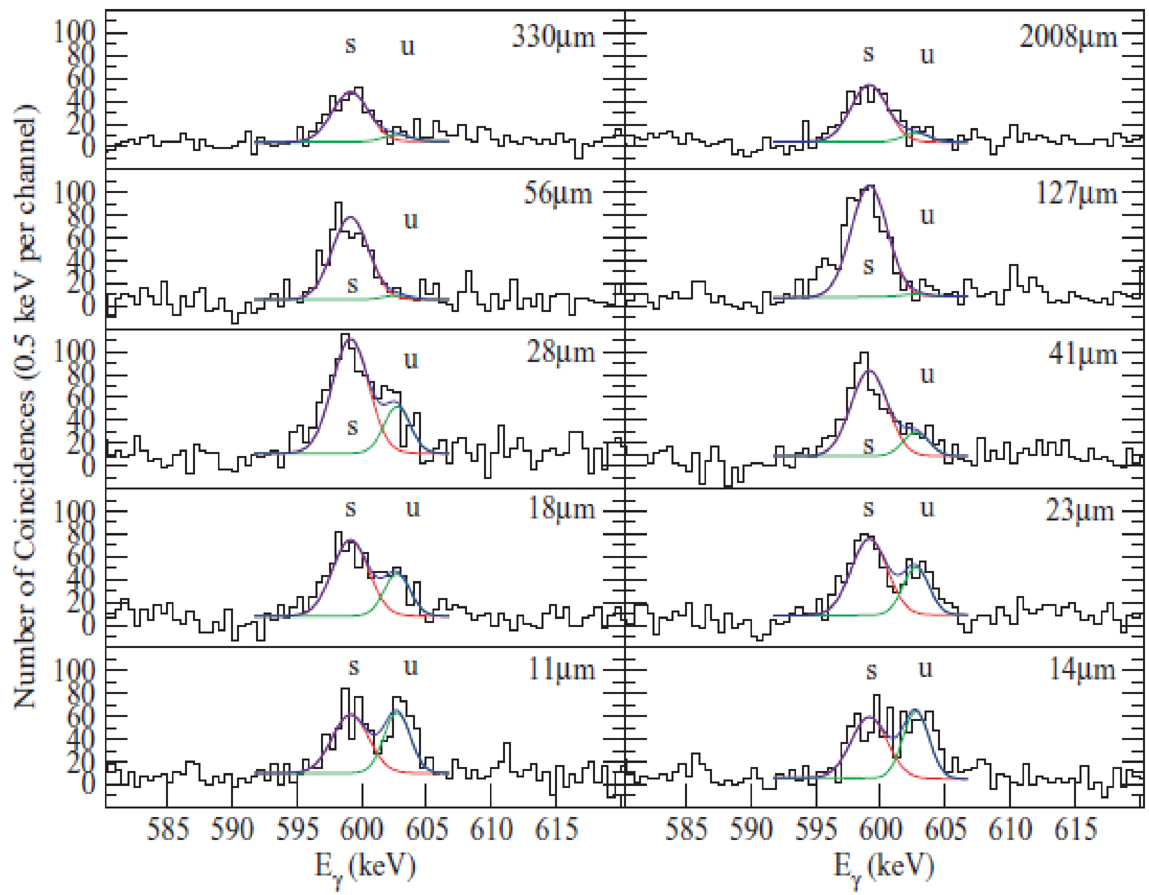


$$\tau = \frac{I_U}{v \frac{dI_s}{dx}}$$

# Collective (Quadrupole) Nuclear Rotations and Vibrations

- What are the (idealised) excitation energy signatures for quadrupole collective motion (in even-even nuclei) ?
  - (extreme) theoretical limits

Perfect, quadrupole (ellipsoidal), axially symmetric quantum rotor with a constant moment of inertia (I) has rotational energies given by (from $E_{class}(rotor) = L^2/2I$)

$$E_J = \frac{\hbar^2}{2\ell} J(J+1), \qquad \frac{E(4^+)}{E(2^+)} = \frac{4(5) = 20}{2(3) = 6} = 3.33$$

# Collective (Quadrupole) Nuclear Rotations and Vibrations

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  - (extreme) theoretical limits

Perfect, quadrupole (ellipsoidal), axially symmetric quantum rotor with a constant moment of inertia (I) has rotational energies given by (from $E_{class}$(rotor) = ½ $L^2/2I$)

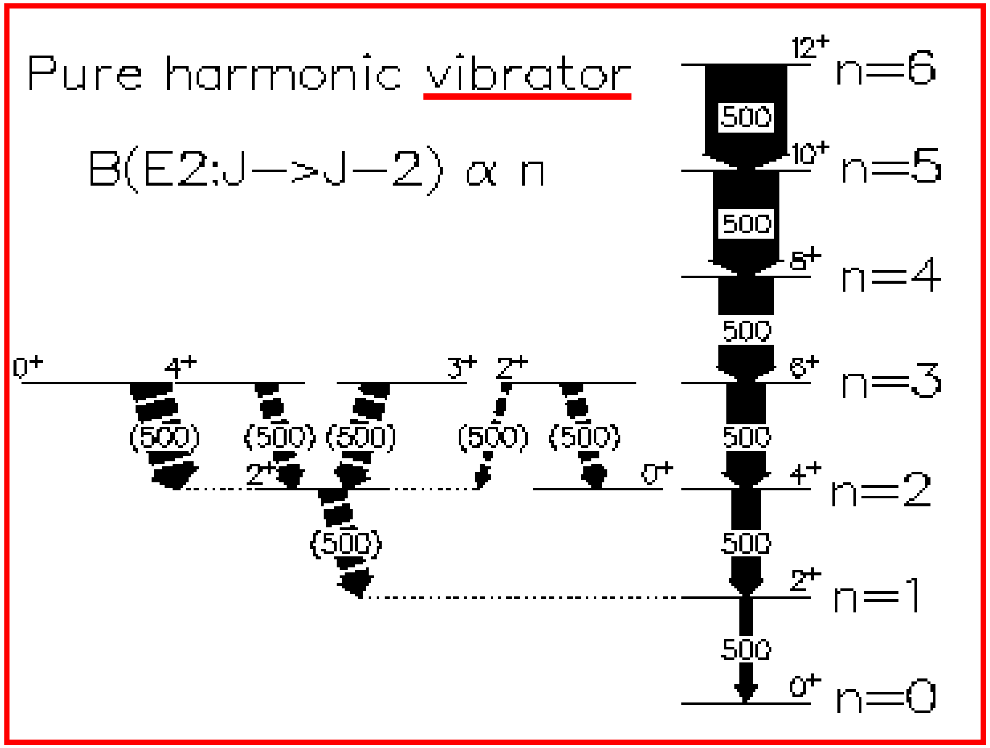
$$E_J = \frac{\hbar^2}{2\ell} J(J+1), \qquad \frac{E(4^+)}{E(2^+)} = \frac{4(5)=20}{2(3)=6} = 3.33$$

Perfect, quadrupole vibrator has energies given by the solution to the harmonic oscillator potential ($E_{classical}={}^1/_2 k\Delta x^2 + p^2/2m$ ).

$$E_N = \hbar\omega N \qquad \frac{E(4^+)}{E(2^+)} = \frac{2}{1} = 2.00$$

# Other Signatures of (perfect) vibrators and rotors

Decay lifetimes give B(E2) values.
Also selection rules important
(eg. $\Delta n=1$).

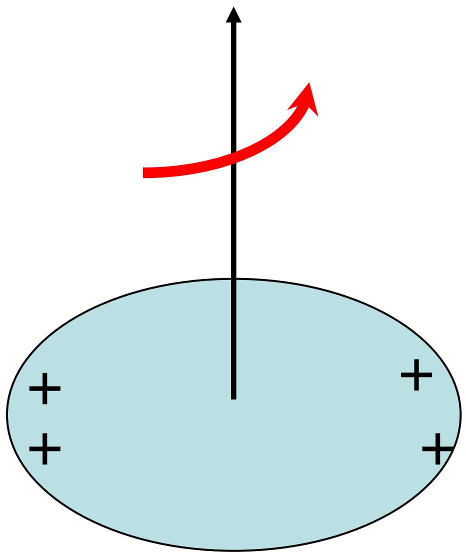



$E_\gamma=\hbar\omega$ ; $\Delta E\gamma$ $(J\rightarrow J-2)=0$

For ('real') examples, see

J. Kern et al., Nucl. Phys. **A593** (1995) 21

# Other Signatures of (perfect) vibrators and rotors

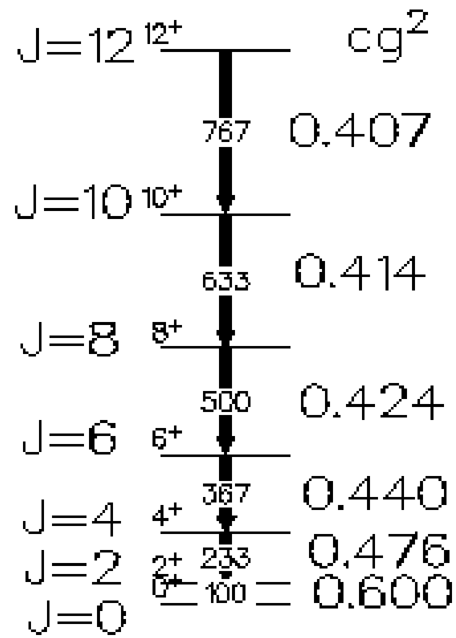


$$E_x=(\hbar^2/_{2I})J(J+1)$$

Perfect rotor

$$B(E2) = kQ^2 \langle J_i K 2 0 | j_f K \rangle^2$$

$$B(E2) \;\alpha\; \frac{3J(J-1)(J+1)}{(2J-2)(2J-1)(2J+1)} \quad \sim 3/8$$



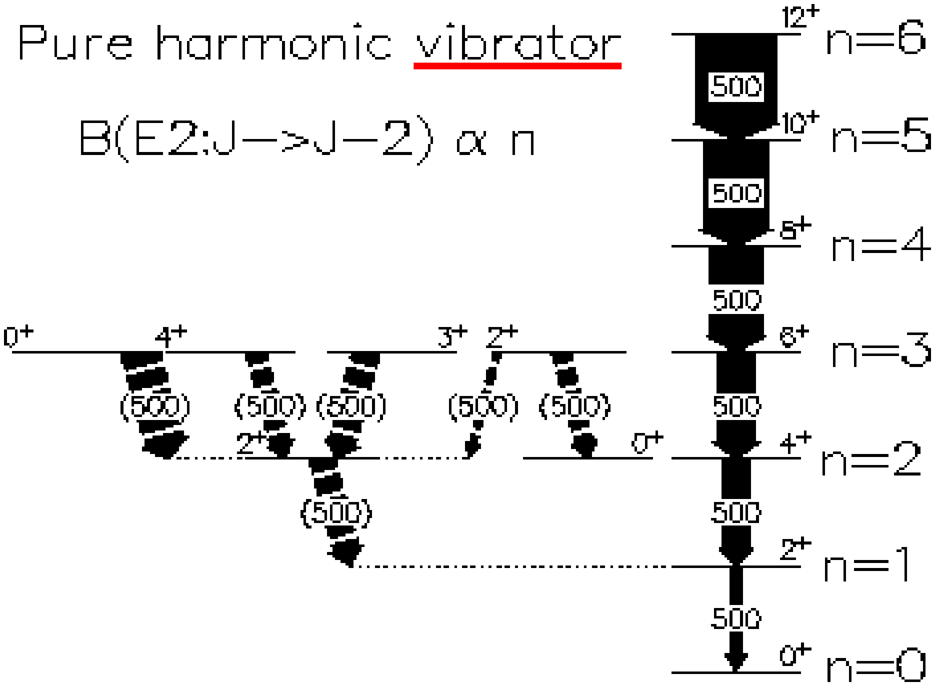

$E_x=(\hbar^2/_{2I})J(J+1)$ , i.e., $E\gamma\ (J \rightarrow J-2)= (\hbar^2/_{2I})[J(J+1) - (J-2)(J-3)] = (\hbar^2/_{2I})(6J-6)$;

# Other Signatures of (perfect) vibrators and rotors

Decay lifetimes give B(E2) values.
Also selection rules important
(eg. $\Delta n=1$).

Pure harmonic vibrator

$B(E2{:}J\rightarrow J-2)\ \alpha\ n$



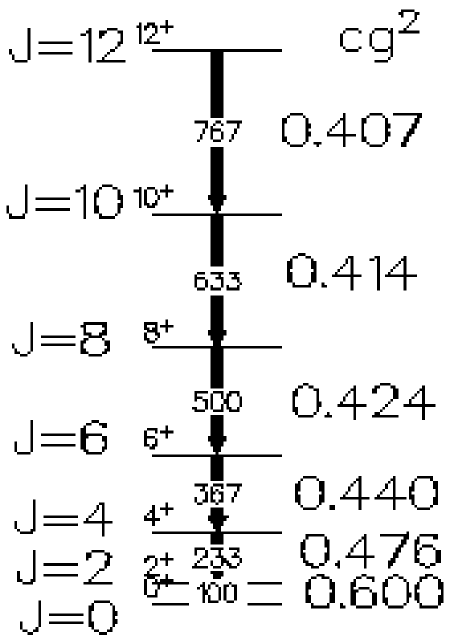

$E_\gamma=\hbar\omega$ ; $\Delta E\gamma\ (J\rightarrow J\text{-}2)=0$

For ('real') examples, see
J. Kern et al., Nucl. Phys. **A593** (1995) 21

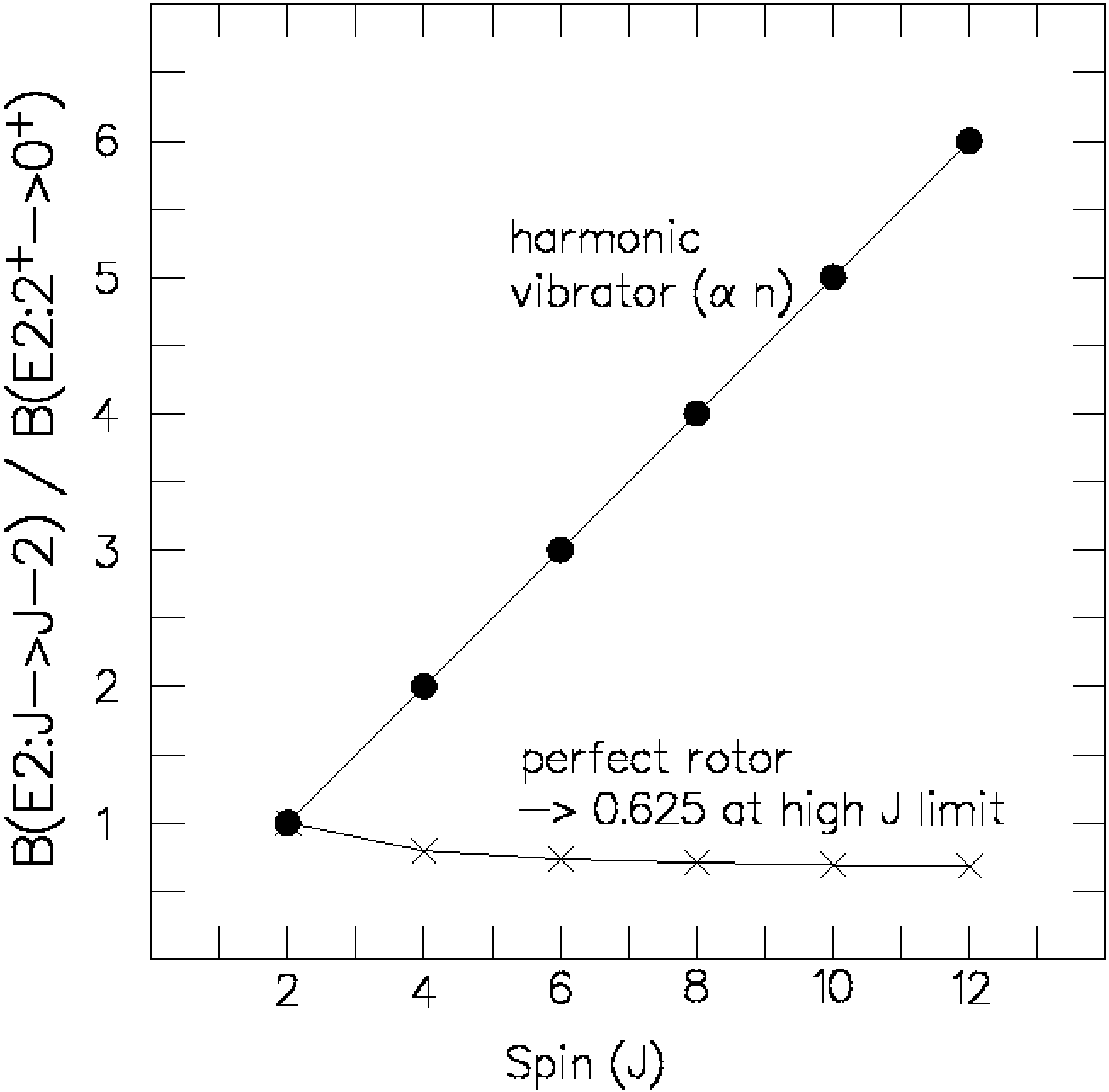
Perfect rotor

$B(E2) = kQ^2\langle J_iK20|j_fK\rangle^2$

$B(E2)\ \alpha\ \dfrac{3J(J-1)(J+1)}{(2J-2)(2J-1)(2J+1)}$ ~3/8

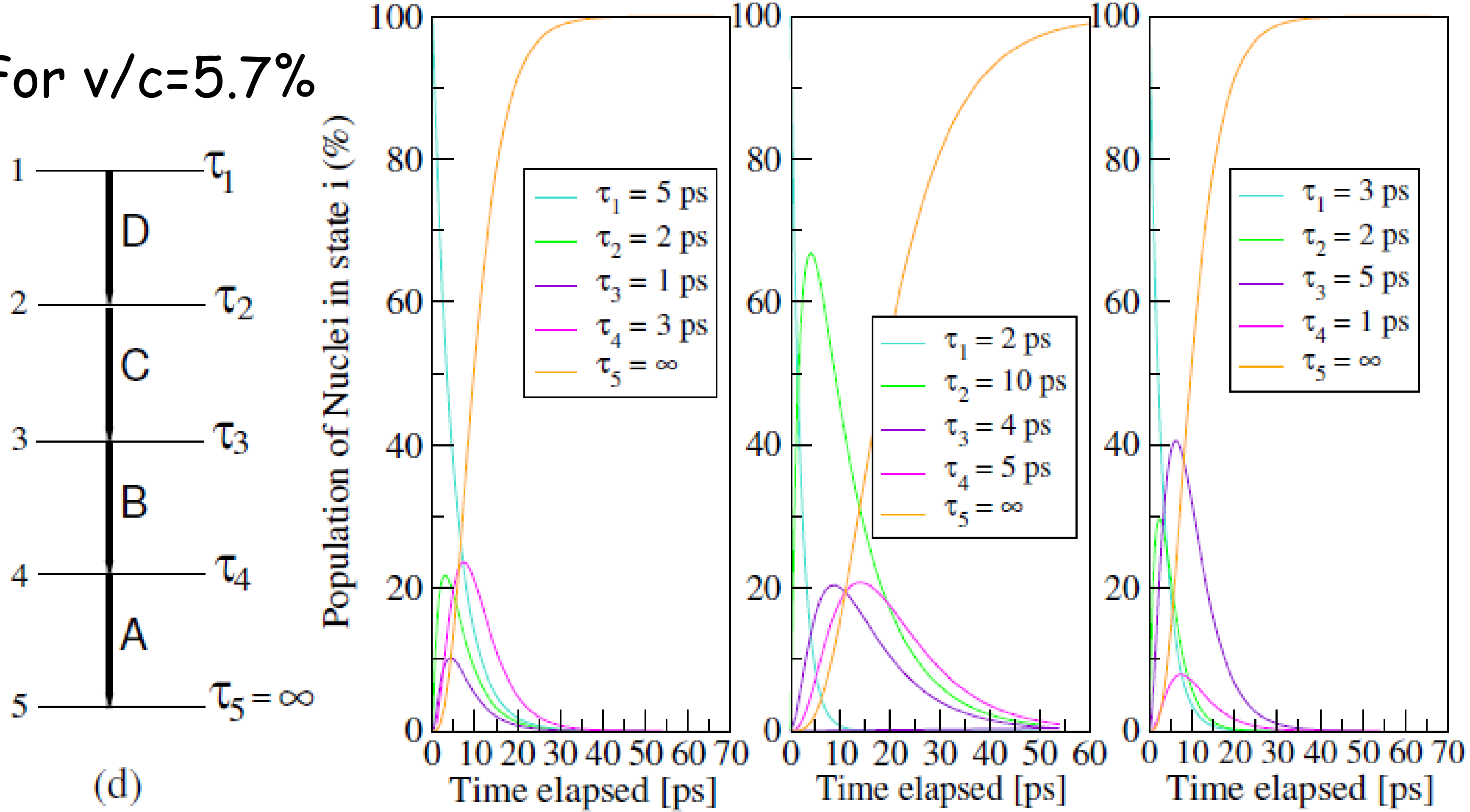



$E_x=(\hbar^2/_{2I})J(J+1)$ , i.e., $E\gamma\ (J\rightarrow J\text{-}2)=$
$(\hbar^2/_{2I})[J(J+1) - (J\text{-}2)(J\text{-}3)] =$
$(\hbar^2/_{2I})(6J\text{-}6)$; $\Delta E\gamma=(\hbar^2/_{2I})*12=\text{const}$.


B(E2:J−>J−2) / B(E2:2+−>0+)
1
2
3
4
5
6
harmonic
vibrator (α n)
perfect rotor
−> 0.625 at high J limit
2
4
6
8
10
12
Spin (J)

ASIDE: Multistep cascades, need to account for decay lifetimes of states feeding the state of interest…..need to account for the Bateman Equations.

For v/c=5.7%

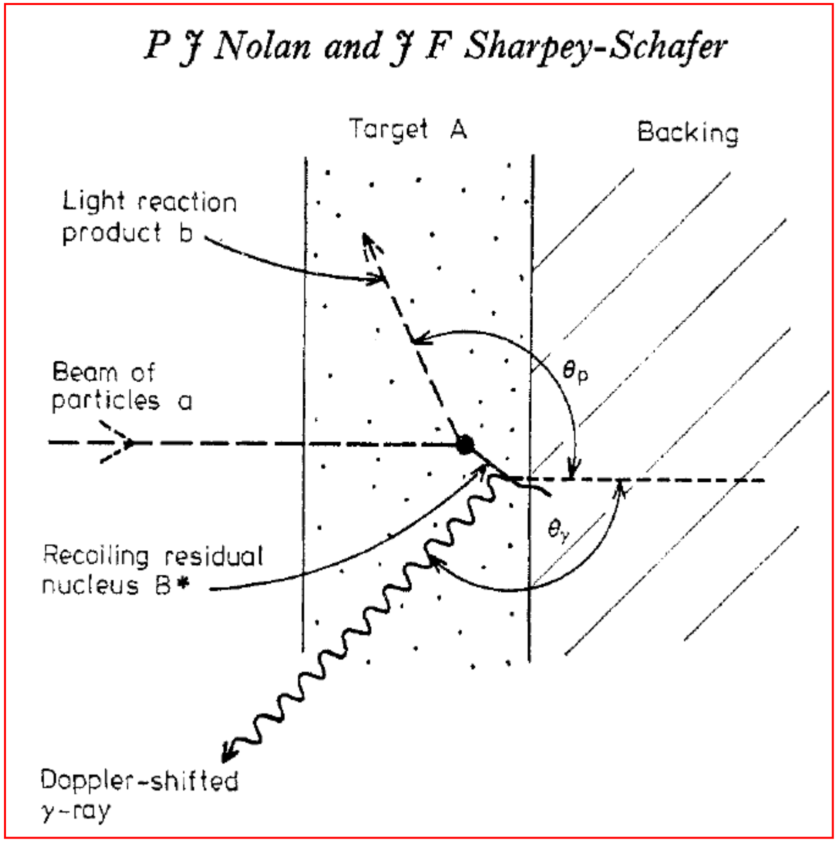



(a) Four-step decay (b)

This can be accounted for by using the 'differential decay curve method' by gating on the Doppler shifted component of the direct feeding gamma-ray to the state of interest, see G. Bohm et al., Nucl. Inst. Meth. Phys. Res. **A329** (1993) 248.

If the lifetime to be measured is so short that all of the states decay in flight, the RDM reaches a limit.

To measure even shorter half-lives (<1ps).
In this case, make the 'gap' distance zero !! i.e., have nucleus slow to do stop in a backing.

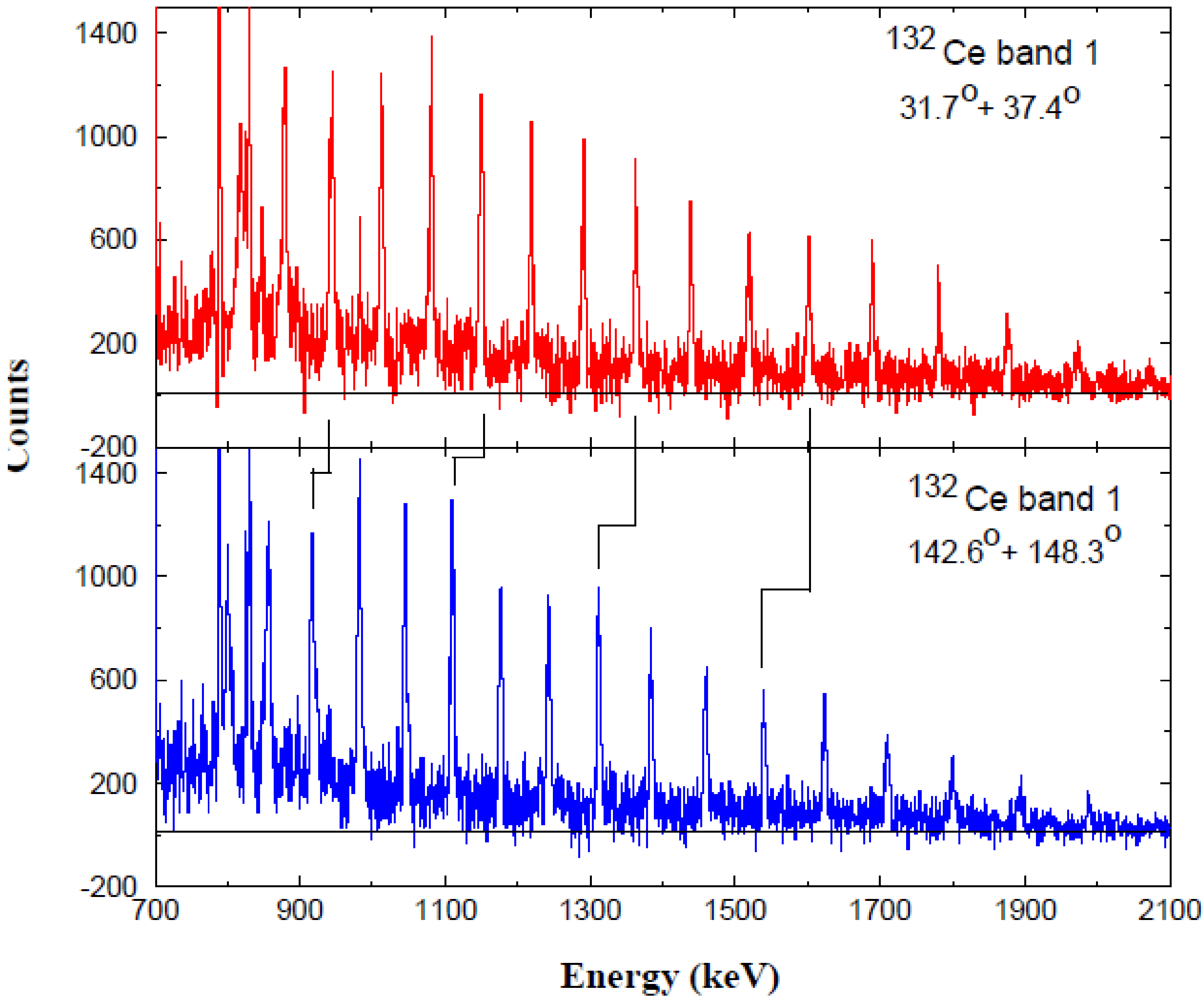



We can use the quantity
$F(\tau) = (v_s / v_{max})$.

$E_s(v,\theta) = E_0(1+{}^{v}/_{c} \cos(\theta))$ (for $v/c < 0.05$)

Measuring the centroid energy of the Doppler shifted line gives the **average** value for the quantity $E_s$ (and this v) when transition was emitted.

The ratio of vs divided by the maximum possible recoil velocity (at t=0) is the quantity, $F(\tau)$ = fractional Doppler shift.



132 Ce band 1
31.7° + 37.4°
132 Ce band 1
142.6° + 148.3°
Counts
Energy (keV)
1400
1000
600
200
-200
700
900
1100
1300
1500
1700
1900
2100

In the rotational model,

$$\frac{1}{\tau} = 1.223 E_\gamma^5 \frac{5}{16} Q_o^2 | < J_i K 20 | J_f K > |^2$$

where the CG coefficient is given by,

$$< J_i K 20 | J_f K > = \sqrt{\frac{3(J-K)(J-K-1)(J+K)(J+K-1)}{(2J-2)(2J-1)J(2J+1)}}$$

Thus, measuring $\tau$ and knowing the transition energy, we can obtain a value for $Q_0$

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R^2 \beta_2 \left(1 + \frac{1}{8}\sqrt{\frac{5}{\pi}}\beta_2 \ldots\right)$$

VOLUME 59, NUMBER 19 PHYSICAL REVIEW LETTERS 9 NOVEMBER 1987

# Intrinsic Quadrupole Moment of the Superdeformed Band in $^{152}$Dy

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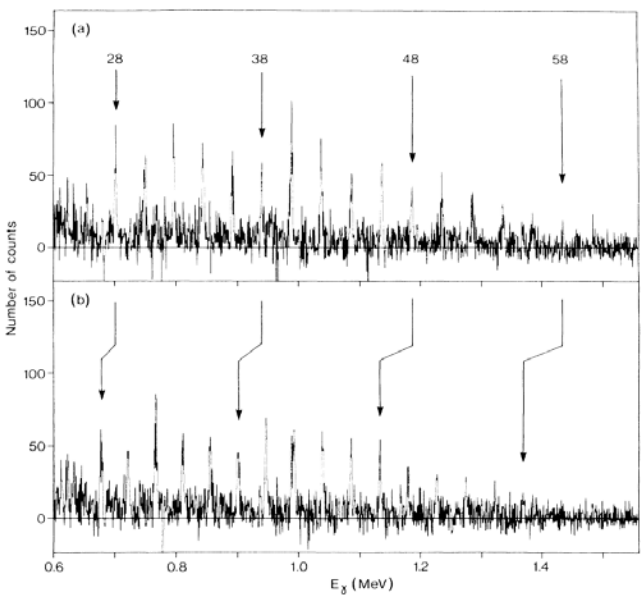
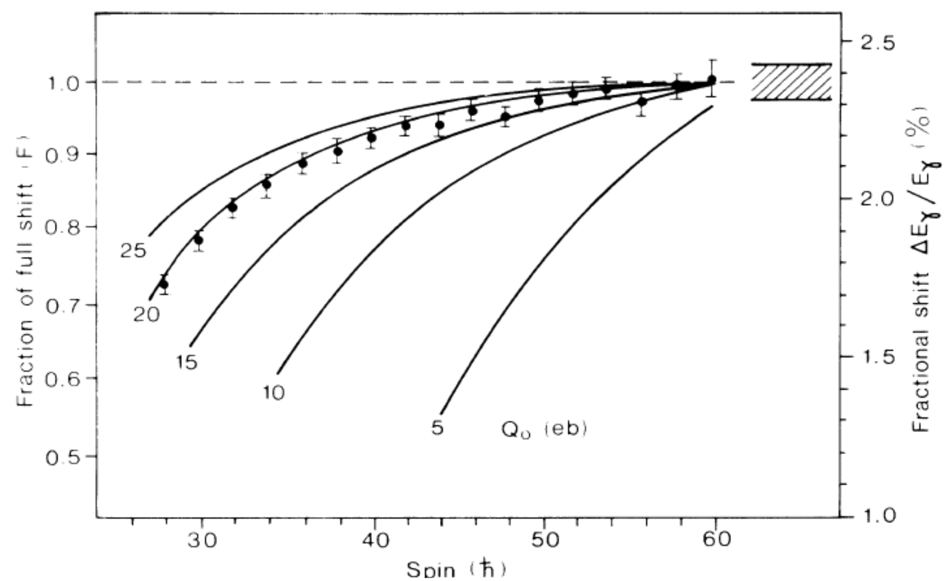
(Received 9 February 1987; revised manuscript received 3 June 1987)




If we can assume a constant quadrupole moment for a rotational band (Qo), and we know the transition energies for the band, correcting for the feeding using the Bateman equations, we can construct 'theoretical' F(τ) curves for bands of fixed $Q_0$ values