

Plasma Confinement

Plasma Physics

A collection of charged particles whose dynamics is essentially controlled by collective interactions.

E.M. fields \rightarrow Particle Motion \rightarrow E.M. Fields

Full ionization is not essential

Collective interactions for Coulomb forces is Large

$$V \sim \frac{1}{r}, \tau \sim \text{Volume} \sim r^3$$



$\tau V \sim r^2$ increases with the size

Plasmas Can be of several types.

Temperature (T) and density (n) are the major determinants. [We shall restrict to only neutral plasma]

1) Statistics	Classical	Quantum
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$$T \ll E_{\text{fermi}} \approx \frac{(n h^3)^{2/3}}{2m}$$

2) Two-particle Collision Physics	Classical	Quantum
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$$\lambda_{d.b} \ll n^{-1/3}$$

(3) Non-Relativistic	$T \ll mc^2$
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4) Nearly an ideal gas, i.e., essentially freely moving particles bumping into each other occasionally (A bump for a coulomb plasma is a 90° collision)

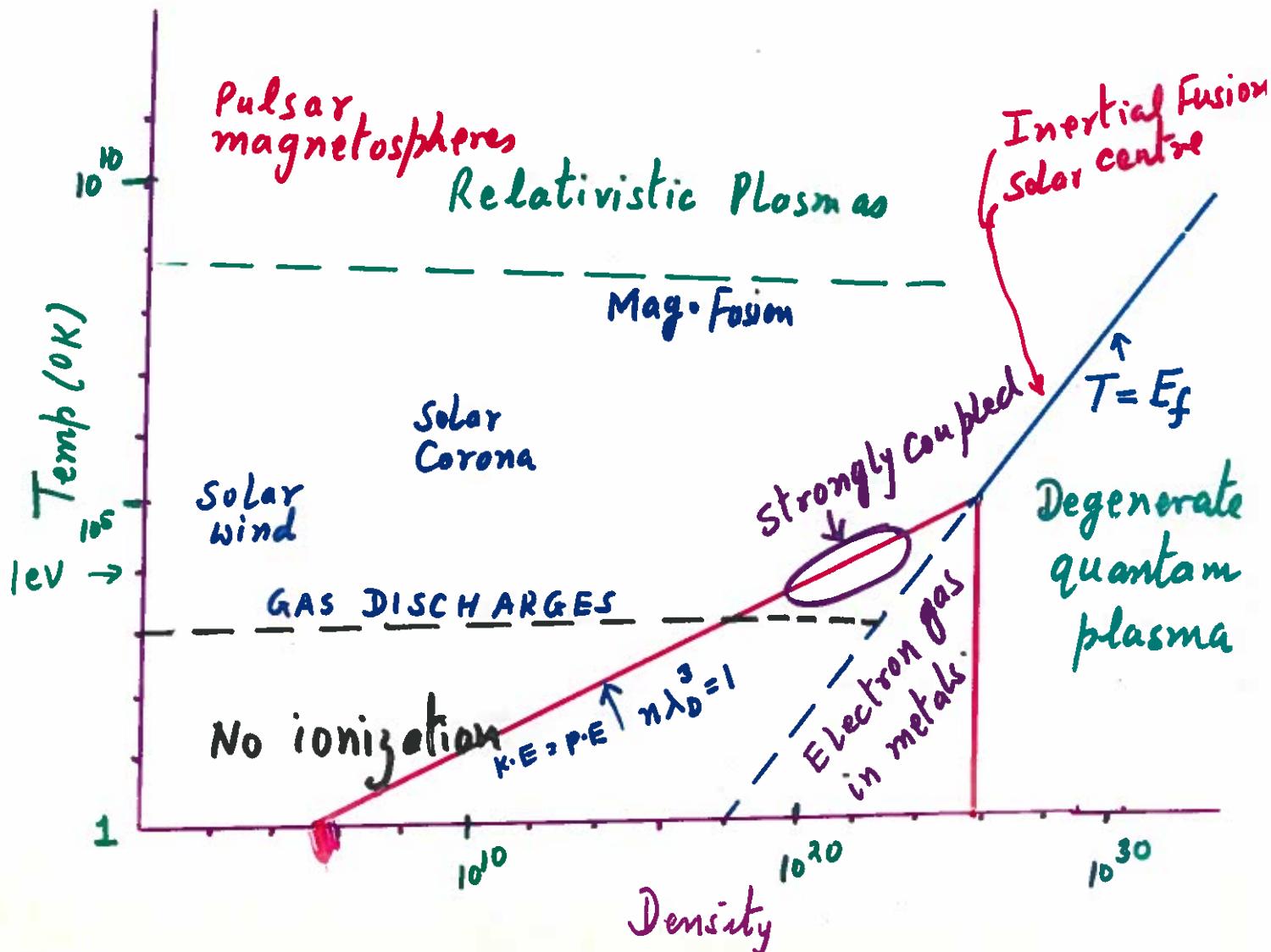
K-Energy > P-Energy

$$T \gg \frac{e^2}{r} \sim e^2 n^{1/3} \Rightarrow n \lambda_D^3 \gg 1$$

Debye length

5) Magnetized Plasma : Embedded in a strong external magnetic field

$$\Omega_{i,e} = \frac{qB}{m_{i,e}c} > \nu_c, \dots$$



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Basic Fusion Considerations

The idea is to exploit some exothermic nuclear reaction to produce useful energy.

D-T reaction will be the first to be tried because the cross sections are sufficiently large at comparatively low temps $\sim 10 \text{ keV}$.

At 10 keV \rightarrow D, T will be fully ionized

\Rightarrow hence plasma physics;
in fact lots of it !!

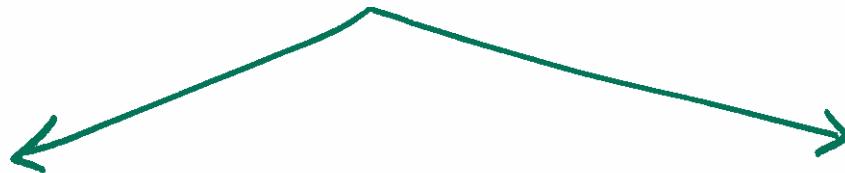
To really produce energy, we must have requiring

i) Large number of nuclear reactions leading to

a) high densities, n

b) Large times of interaction t

leading to the famous Lawson Criterion
for (D-T), $nt \sim 10^{14}$ at $T \sim 10 \text{ keV}$



Confinement

Heating

Broad Scope, and draws on almost all branches of classical physics, and stimulates many

Magnetic Confinement

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Confinement is studied under three different headings.

- 1) Equilibrium : Timeindependent solution of the dynamical equations [Maxwell's Equations + appropriate plasma equations] →

$$S_0 = f_0 \text{, } B_0 \text{, } n_0 \text{, } T_0$$

the distribution function the magnetic field density Temp.

⇒ The attempt is to determine the geometry of the mag. field which will keep the plasma particles together.

- 2) stability : Will a perturbation on S_0 grow or damp in time?

$$S = S_0 + \delta(t), \quad \delta(t) \sim e^{-\gamma t + i\omega t}$$

- 3) Transport : - These, in general, are the loss processes which take place in a stable confined plasma.
Collisional Transport, for example.

Equilibrium

Single Particle Confinement, Plasma Confinement.

→ $\frac{2}{3}$, straight uniform field.

Seal the third direction

open Ended devices



Mirror Machines

$$B_z = B_{z0} (1 + f(z))$$

$f(z)$ is +ve definite

Confinement is due to the existence of an adiabatic invariant, $\mu = \frac{1}{2} \frac{m v_{\perp}^2}{B}$, the magnetic moment. The required conditions are

$$\frac{1}{B} \left| \frac{\partial B}{\partial x} \right| < a_i = \frac{v_{\perp}}{\Omega} \quad \frac{1}{B} \left| \frac{\partial B}{\partial t} \right| < \Omega$$

Larmor radius

$L \gg a_i$ • Why strong mag. fields

$$\text{energy } E = \frac{1}{2} m v_{\perp}^2 + \frac{1}{2} m v_{\parallel}^2 = \frac{1}{2} m v_{\parallel}^2 + \mu B$$

$$\Rightarrow \frac{1}{2} m v_{\parallel}^2 = E - \mu B$$

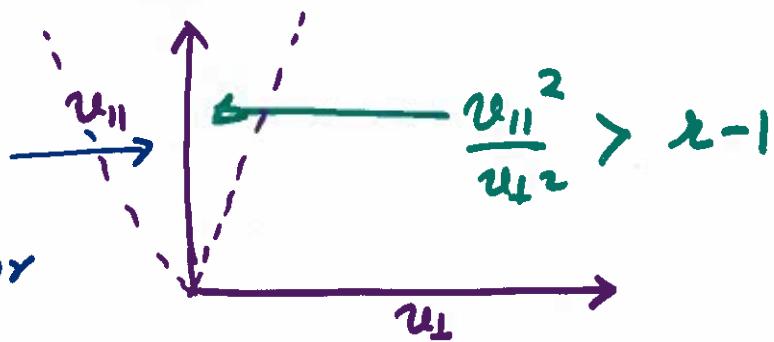
Reflection occurs if $E < \mu B_{\max}$

No Confinement

$$\epsilon > \mu B_{\max}$$

Loss Cone

$$\frac{B_{\max}}{B_{\min}} = r, \text{ the mirror ratio}$$



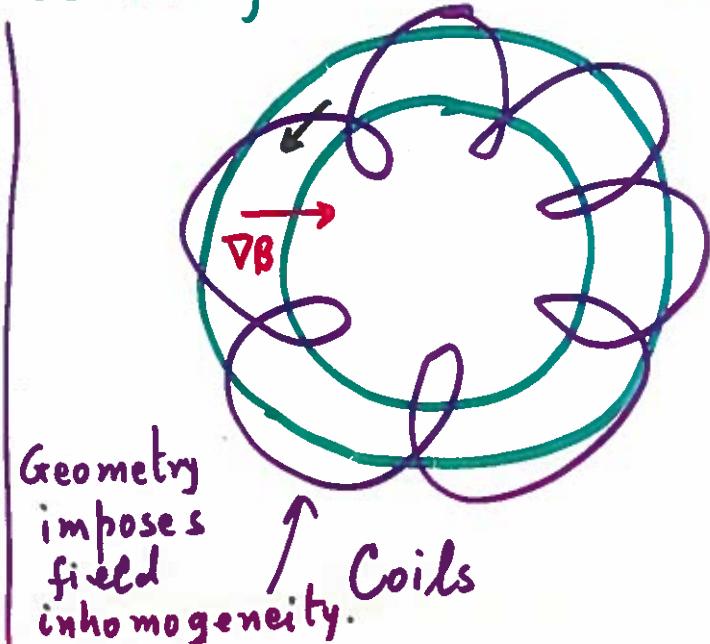
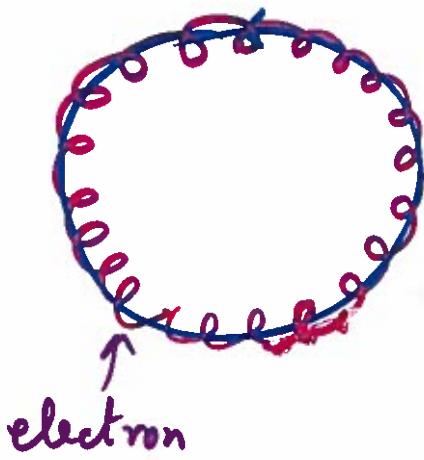
The ends are leaky ; single particle confinement is not perfect.

Keep the system running by injecting more and more particles.

Plasma Confinement not possible due to 'Loss-Cone'-instability precisely at $\omega \sim \omega_{ci}$, destruction of μ .
 $b(t) \sim (\sin \omega t) e^{rt}$.

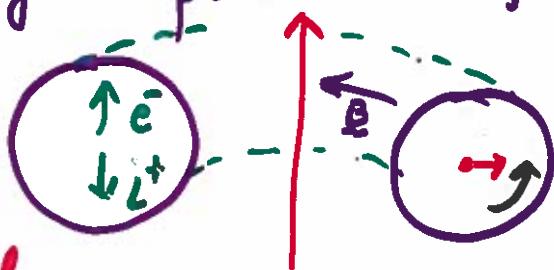
\Rightarrow A very active field of research, Tandem mirrors, end plugging, thermal barriers

Alternative : Close the field line on itself.



Particles in an inhomogeneous mag. field suffer a
charge dependent drift $v_0 \propto v (\nabla B \times \vec{B})$

Toroidal section

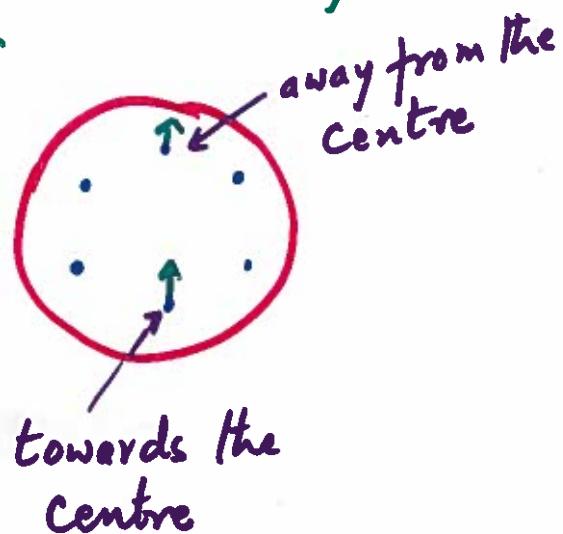
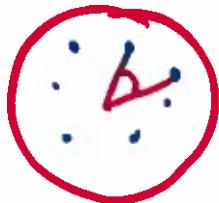


symmetry axis
of the torus
or the donut

- ↑ poloidal direction
- ← toroidal direction
- radial direction

A purely toroidal field is not capable of confining a plasma. The introduction of a helical twist to the field line.

$\Delta = i =$ rotational transform



If $v_{\parallel}(\text{along } B) \gg v_0$, then on the average the charge centres do not drift, and an equilibrium can be there.

$v_0 \rightarrow$ charge separation
electric field \tilde{E} ,

then

$$\frac{e \tilde{E} \times \tilde{B}}{\tilde{B}^2} \text{ motion}$$

takes the particles to the walls.

The whole process takes microseconds for a typical set of parameters.

Rotational Transform

i produced by external currents (superimposed helical windings) \rightarrow stellarators .

i produced by inducing current in the plasma
 \rightarrow [Tokamak
Heated plasma] (stabilized pinch)

$i = \tilde{B}_p$, the poloidal magnetic field.

Stellarators are non axisymmetric, difficult to make, even harder to theorize :- attempted in the beginning, and given up. [There is a bit of a resurgence in Stellarator research now; the principal reasons are:

- ① Heating and B_p are separately controlled.
- (3) Lack of equilibrium plasma current may stabilize some troublesome instabilities in a Tokamak.

But the principal approach is still the good old Tokamak : Axisymmetric, essentially circular cross section (though a D-shape or a bean-shape^{machine} may be better for high power reactors), primary heating due to the induced current. etc.

Tokamaks

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For no charge separation, $\nabla \cdot \underline{J} = 0$.

For a toroidal field $\underline{J}_\perp = \alpha (\underline{b} \times \nabla b)$, $\nabla \cdot \underline{J}_\perp \neq 0$
 $\therefore \nabla \cdot \underline{J}_{\parallel} \neq 0 \Rightarrow$ Must have J_{\parallel} , the current along the field line (i , or B_p).

Experimentally, they claim to have three kinds of mag. fields.

- 1) Large Toroidal Field B_T (external coils)
- 2) B_p introduced by plasma current controlled
- 3) A vertical field (from feed-back external coils). But really

$$\underline{B} = \underline{B}_T + \underline{B}_p$$

poloidal direction is, of course, not the azimuthal direction. The vertical field is just the externally applied part of the toroidal field.

For a closed system, Magneto hydrodynamics is a good leading-order description.

Confinement (again)

In simple terms Confinement means the creation and maintenance of a pressure gradient.

The Leading order description [Equilibrium]

$$\tilde{J} \times \tilde{B} = c \nabla p \quad \text{Force balance}$$

$$\nabla \times \tilde{B} = \frac{4\pi}{c} \tilde{J}$$

$$\nabla \cdot \tilde{B} = 0$$

We wish now to determine \tilde{B} such that $\nabla p \neq 0$ solution is possible.

Scenario [Finite Volume enclosed by a Wall
 $p=0$ at the wall

+ Currents and Conductors outside the plasma
(These provide the external boundary condition
on \tilde{B})

Not all fields are Confining. For example

i) Vacuum Field with $\nabla \times \tilde{B} = 0$ inside the plasma

This field has the lowest free energy, but unfortunately, the only solution is

$$\nabla p = 0 \quad \text{everywhere.}$$

a) Force-free field

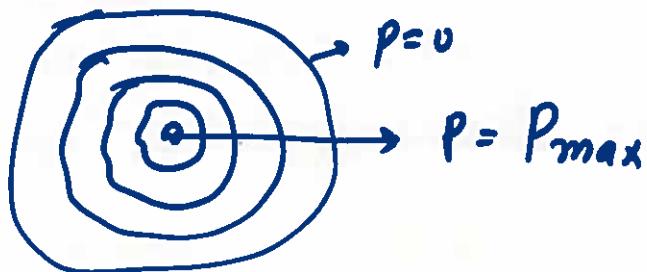
$$\nabla \times \tilde{B} = \lambda \tilde{B}, \quad \tilde{J}_\perp = 0 \Rightarrow \nabla p = 0$$

$J_{\parallel} \neq 0$, sharp boundary pinches.

We are interested in having $\nabla p \neq 0$ (To be exact $\nabla p = 0$ on a set of measure zero).

$p = p(\tilde{x})$ be a smooth function

We want the existence of isobaric (constant pressure) surfaces which can have a general shape, but which must be nested.



i.e., they can neither intersect nor go to infinity.

Consequences of our equations:

$$\nabla \cdot \tilde{J} = 0$$

Charge neutrality

$$\tilde{B} \cdot \nabla p = 0, \quad \tilde{J} \cdot \nabla p = 0$$

$p = p_0$ surface is a magnetic surface, i.e., the magnetic field has no \perp component to the surface.

It also is a current surface, there is no current \perp to the surface

Poincaré's Theorem

Theorem:- A surface smoothly covered by a nonvanishing finite vector field is topologically a torus.

We found that an isobaric surface is a magnetic surface, i.e., it is traced out by a magnetic field line. Since $\underline{B} \neq 0$, nor ∞ , the magnetic surfaces (current surfaces, isobaric surfaces) must be toroidal!

Isobaric surfaces ^{are} nested toroids. The innermost toroid^{^1} is degenerate, and is called the magnetic axis. $\nabla p \neq 0$, everywhere except on the magnetic axis.

A very important concept is of Surface Quantities

$$\underline{B} \cdot \nabla K = 0$$

K is a surface quantity, and can be used to label the surface, and effectively serve as the co-ordinate \perp to the surface (radial co-ordinate).

A certain poloidal flux ψ is the preferred ^{variable}.

Question

Can we always obtain solutions to the equilibrium equations with $\nabla \rho \neq 0$.

Grad 'No' not, in general, . Solutions can be guaranteed only if there is axisymmetry ($\frac{\partial \rho}{\partial \theta} = 0$) or some other symmetry.

The nice good magnetic surfaces are fragile, and can be messed up by field errors and instabilities.

Fusion :- One requires approximate and not mathematically rigorous surfaces

The existence of Isobaric surfaces traced out by a line of \underline{B} is at the heart of the magnetic confinement.

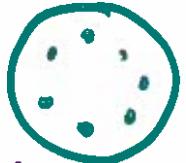
Destruction of Surfaces = Loss of Confinement

Magnetic Surfaces

In general each surface ($\phi = \phi_0$) will be ergodically traversed by a single field line, but

There is one very important exception

When the field line closes on itself



$$q = \frac{2\pi}{\lambda} = \frac{n}{m} \text{ is a rational number}$$

Although such a line does not trace out a surface, it is still called a magnetic surface, but a Rational Surface

Flux or
surface
quantities

- α = Toroidal flux function
- ψ = poloidal flux function
- χ → A poloidal angle like variable
- ϑ → toroidal angle

$$q = \frac{d\alpha}{d\psi} \simeq \frac{r B_T}{R B_\vartheta} \sim \text{Safety parameter.}$$

What do rational surfaces do?

Profound Effects on the Stability of the plasma.

Instabilities can be menacing around these surfaces.

A Simple Calculation

$$\tilde{B} \cdot \nabla p = 0 \quad [\tilde{\nabla} \times \tilde{B} = C \nabla p]$$

$$\tilde{B} = \tilde{B}_0 + \tilde{b}, \quad p = p_0 + \tilde{p}$$

Equilibrium $\tilde{B}_0 \cdot \nabla p_0 = 0 \Rightarrow p_0 = p_0(\psi)$

1st order Equation $\tilde{b} \cdot \nabla p_0 = - \tilde{B}_0 \cdot \nabla \tilde{p}$

$$\nabla p_0 = \frac{\partial p_0}{\partial \psi} \nabla \psi$$

$$\tilde{B}_0 \cdot \nabla = (\tilde{B}_0 \cdot \nabla x) \left(\frac{\partial}{\partial x} + q \frac{\partial}{\partial \phi} \right)$$

$$\Rightarrow \left(\frac{\partial}{\partial x} + q \frac{\partial}{\partial \phi} \right) \tilde{p} = - \left(\frac{\tilde{b} \cdot \nabla \psi}{\tilde{B}_0 \cdot \nabla x} \right) \frac{dp_0}{d\psi}$$

$$\tilde{p}(b) = \sum_{n,m} e^{imx - i\ell\phi} \tilde{p}_{n,m}(b, m, n)$$

and using properties of \tilde{B}_0 , etc

$$\tilde{p}_{n,m} \approx - \frac{(b_\psi)_{n,m} dp_0/d\psi}{m - nq}$$

At the rational surface $q = m/n$, $p_{nm} \rightarrow \infty$

unless, of course, $dp_0/d\psi \rightarrow 0$, Poor Confinement.

This immense magnification of perturbations at the rational surfaces makes one wonder about the fragility of the equilibrium.

Region of poor confinement

$\psi = \psi_s$ be the rational surface

$$q(\psi_s) = m/n$$

$$q(\psi) = q(\psi_s) + (\psi - \psi_s) q'$$

$$m - nq = -nq'(\psi - \psi_s) = -nq' \Delta \psi$$

$$(\Delta \psi)_{m,n} \approx \frac{b_{m,n}}{n} \quad (p = p_0)$$

Singular region becomes small with n .

Further $b_{m,n} \sim O(n^{-1}, m^{-1}, \dots)$ from field errors etc.

This is indeed the saving grace. Large rationals have tiny singular regions about them. Thus the equilibrium is not sufficiently destroyed.

'KAM' theorem

Vol. of singular region $\rightarrow 0$ as $b_{m,n} \rightarrow 0$

- (1) $q = 43/19$ is perhaps not a very bad surface
- (2) $q = 1.01$, mathematically fine, but physically bad

Surfaces

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Bad regions are localized in the near vicinity of low-order rational surfaces.

Significance of q and q'

A perturbation for which

$$\mathbf{B} \cdot \nabla f = 0$$

$$\mathbf{B} \cdot \nabla \sim \frac{\partial}{\partial x} + q \frac{\partial}{\partial \phi} \sim m - lq \propto k_{\parallel}$$

or equivalently $k_{\parallel} = 0$ is called a flute mode \longrightarrow is the most dangerous perturbation.

If $q = \text{Const}$, then a plasma can support flute modes, and is likely to be very unstable

If $q' \neq 0$, then only flute-like modes which have $\mathbf{B} \cdot \nabla f \neq 0$ in some small region, can be present.

Thus shear ($q' \neq 0$) does take the bite away from the flute modes. Still, though not as bad the most dangerous instabilities tend to ^{be} flute-like ($k_{\parallel} \approx 0$).

Rational surface : The field line bites its own tail , and hence it does not trace out a surface ~~at~~ at all .

Inaccessible region $\rightarrow \cdot \cdot \cdot$
 $\cdot \cdot \cdot$

Irrational Surface :- A genuine surface , where no region on the surface is inaccessible along the field line .

Electrons (current flows only on the surface) on an irrational surface can rush to any part of the surface to wipe out a charge disparity

On the rational surface, however, electrons can not neutralize a charge build up in the inaccessible regions . There is no restoration to charge neutrality \longrightarrow perturbations can grow .

For Equilibrium , we want electrons not to go across the surfaces (otherwise , surfaces don't have much meaning) However , on the surface we want them to be infinitely mobile to wipe out perturbations .

Particle Motions

$$\hat{b} = \frac{\underline{B}}{B}, \quad \underline{v} = \hat{b} v_{||} + \underline{v}_{\perp}$$

$$f = \frac{v_{\perp}}{v_{\parallel}} \sim \frac{v_{T\perp}}{v_{T\parallel}}, \quad v_{T\perp} = (2T/m)^{1/2}$$

A strongly magnetized plasma (magnetically confined)

$$\frac{f}{L} \ll 1. \quad L \text{ is a typical equilibrium length}$$

$$L^{-1} \approx \frac{1}{n(x)} \frac{dn(x)}{dx}$$

To a leading order approximation, gyration of the particles ~~is~~ neglected

Guiding centre plasma (g.c.)

$$\underline{v}_{g.c.} = \hat{b} v_{||} + \underline{v}_0 + \underline{v}_E + \dots$$

$$\underline{v}_E = c \frac{\underline{E} \times \underline{B}}{B^2}$$

$$\underline{v}_0 = \frac{1}{\Omega} \hat{b} \times \left[v_{||}^2 (\hat{b} \cdot \nabla) \hat{b} + \frac{v_{\perp}^2}{2B} \hat{b} \times \nabla B \right]$$

\downarrow curvature drift \downarrow grad B drift

What are these drifts and how big are they?

$v_{\parallel} \sim v_{th}$, mag. field does nothing to the parallel motion

$$\frac{v_D}{\sim} \simeq \frac{1}{\Omega} \frac{v_{th}^2}{L} \simeq \frac{1}{\Omega} \frac{\delta}{L} v_{th} \ll 1$$

$$\frac{\delta}{L} \sim \delta \quad \downarrow \delta \omega_T, \quad \omega_T = \frac{v_{th}}{L}$$

To the leading order, the particles follow the field lines.

How do most of the wave motions scale in time $\frac{d}{dt} \sim \omega$

$$\omega = \Omega \sim \delta^{-1} \omega_T \quad \begin{array}{l} \text{Label} \\ \text{High frequency} \end{array} \quad \begin{array}{l} \text{Free Energy} \\ \frac{\partial f}{\partial \dot{x}^2} > 0 \end{array}$$

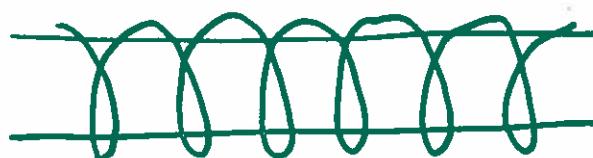
$$\omega \sim \omega_T \sim \delta^{-1} \Omega \quad \begin{array}{l} \text{MHD} \\ \text{kink-type} \end{array} \quad \begin{array}{l} \text{magnetic} \\ \text{free energy} \end{array}$$

$$\omega \sim \delta \omega_T \sim \delta^2 \Omega \quad \begin{array}{l} \text{Drift type} \\ \text{fast} \\ \text{motions} \end{array} \quad \begin{array}{l} \text{Expansion} \\ \text{free energy} \end{array}$$

$$\omega \sim \delta^2 \omega_T \sim \delta^3 \Omega \quad \begin{array}{l} \text{Transport} \end{array} \quad \begin{array}{l} \text{Inevitable.} \end{array}$$

Stability (M.H.D)

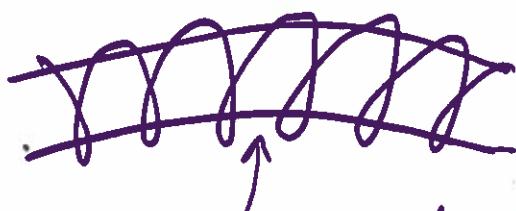
Worst modes for which $k_{\parallel} \approx 0$.



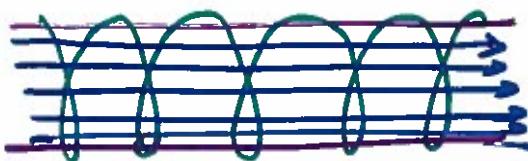
$$\beta + \frac{B^2}{8\pi} = \text{const.} \quad [\text{Consequence of our equations}]$$

Only B_p , $B_T = 0$

A typical instability is a Kink-Instability



B becomes large \Rightarrow plasma must move onto lower B ; thus the kink grows.



\rightarrow strong axial or Toroidal field

Tension in strong B_T hinders the formation of kinks (or other horrible things)

$B_p \rightarrow$ Equilibrium, $B_T \rightarrow$ stability. But a plasma prone to kinks dies in microseconds. MHD instabilities have to be eliminated.

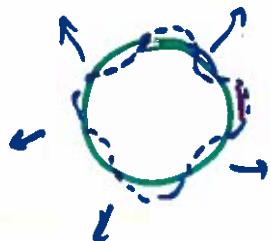
Stability Criterion

MHD instabilities are suppressed if

$$\eta \simeq \left(\frac{RB_p}{rB_T} \right)^{-1} = \frac{rB_T}{RB_p} > 1 \text{ everywhere}$$

and if

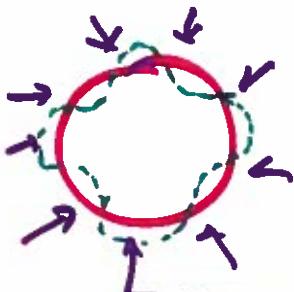
the average curvature of the field is good,
i.e. field increases away from the plasma.



B increasing
outwards

$$\rho + \frac{B^2}{8\pi} = \text{const}$$

pushes the bulge backwards for
stability



B increasing inwards

β low, plasma is pushed into the
bulge.

A Tokamak, in general, does have average
good curvature

$$B_T \sim \frac{1}{R} \sim \frac{1}{R_0 + r \cos\theta}$$



bad curvature

good curvature

No M·H·D Instabilities can be tolerated.
For geometrical reasons

$q \sim 3-4$ at the edge of the plasma so that at the centre it remains above 1. (It actually does not).

If q is to be large for stability, then for a given B_T (money, technology etc), the plasma current or B_p has to be limited

$$B_p < \left(\frac{I}{Rq}\right) B_T$$

Further, the particle temp. in a Tokamak is basically due to the ohmic dissipation; thus the plasma pressure p also has to be limited :

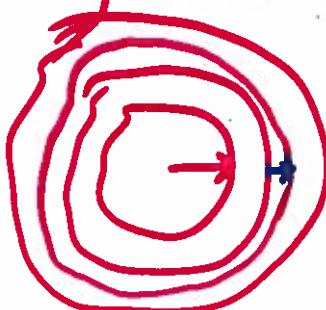
$$\beta = \frac{p}{B^2/8\pi} \ll 1 \text{ for a Tokamak}$$

[II stability regime etc . . .]

Q1

The next important class of instabilities are the resistive instabilities. [Collisional as well as collisionless]

These again grow by exploiting the free energy available in plasma current



$$b_y = 0$$

B = on the surface

Non-zero b_y can connect two neighbouring surfaces, and the particles can move along the field line to go to another surface \rightarrow another and be eventually lost. The process can be quite catastrophic.

Formation of islands, major disruptions.

These instabilities are called tearing modes, for they tear the magnetic surfaces, and create ergodic volumes (no confinement).

Instabilities

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Because of a whole range of them, one cannot design a machine to be totally stable.

Macro Instabilities

fast, destroy the whole plasma
must be stabilized
(fairly ^{well} understood)

Generally driven by pressure gradients, current, bad curvature etc.

Microinstabilities

Comparatively slow, affect local regions of plasma, and have to be lived with not so well understood.

In fact, the same source of free energy drives these, although detailed mechanisms are vastly different

Transport

(24)

Neutral gas $D \sim \frac{(\Delta x)^2}{\tau} \sim v \lambda_{mfp}^2 \sim \frac{v u_H^2}{\nu}$
uniform

Across a mag. field $D_c \sim \frac{\rho_L^2}{\tau} \sim \frac{\nu u_H^2 m_e^2}{e^2 g^2} \sim \frac{\nu u_H^2}{B^2}$

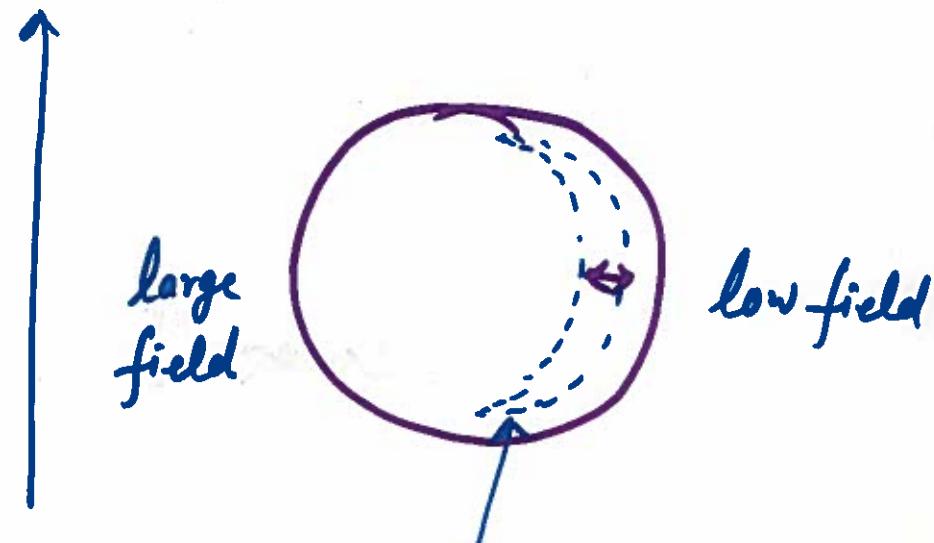
low collisionality, high B would restrict D to a low enough value that we could have fusion before diffusion.

Unfortunately, life turns out to be not so simple.

a)

$$n_{tr} \sim (2\epsilon)^{1/2} n_0$$

$$\epsilon = \gamma/R$$



$$\frac{D_{tr}}{D_c} \sim (2\epsilon) \frac{\lambda_b^2}{\epsilon} \sim 10^2 D_c$$

Trapped particles
Neoclassical transport

$$D_{obs} \sim 10^2 D_{N.C.} !!$$

It is generally believed that this anomalously high rate of energy loss of electrons is due to microinstabilities. The belief is strengthened by the fact that experimental observations seem to confirm high levels of density and temp. fluctuations consistent with theoretical expectations from microinstability theory.

This is ^{one} of the most important theoretical questions of fusion physics.

Scaling laws are experimentally established and also by theoretical nonlinear instability studies.

$$\tau_c^E \sim \frac{a^2}{D} \rightarrow \begin{array}{l} \text{larger size} \\ \text{smaller } D \end{array}$$

$$T_{\text{obs}} \sim a^2 D_{\text{obs}}^{-1} \sim n Z_{\text{eff}} R^{1.63} a^{0.98}] \text{No depend on } B$$

empirical

$$T_{\text{th}} \sim a^2 D_{\text{thc}}^{-1} \sim n Z_{\text{eff}} R^{1.85} a^{1.2} (I)^{\frac{1}{3}}$$

(typical)

$\boxed{\tau_c^E \sim 15-20 \text{ msec}}$