Chapter VI
Rectifying and amplifying stresses

I) Linear medium: dipole conservation

1. Mean-stress theorem
2. Dipole conservation in linear media
3. Relating the macroscopic stress to macroscopic work
4. Relating the macroscopic force dipole to macroscopic work

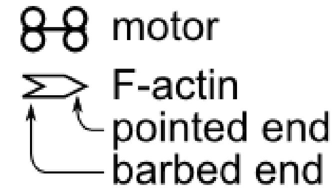
II) Nonlinear (bucklable) medium: amplification

1. Radial force balance
2. Double-check: the linear case
3. Force amplification

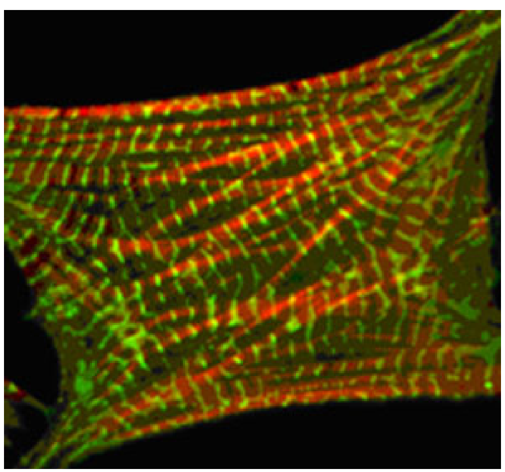
III) Discussion

Reference:
Ronceray & ML, *Soft Matter* **11**, 1597 (2015)
Ronceray, Broedersz & ML, *Proc. Natl. Acad. Sci. U.S.A.* **113**, 2827 (2016)

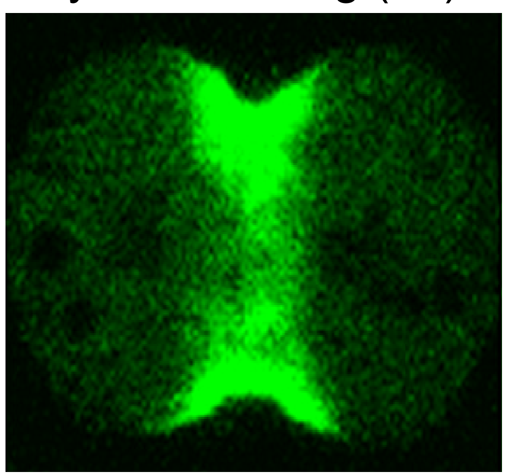
Actomyosin is an active, contractile material



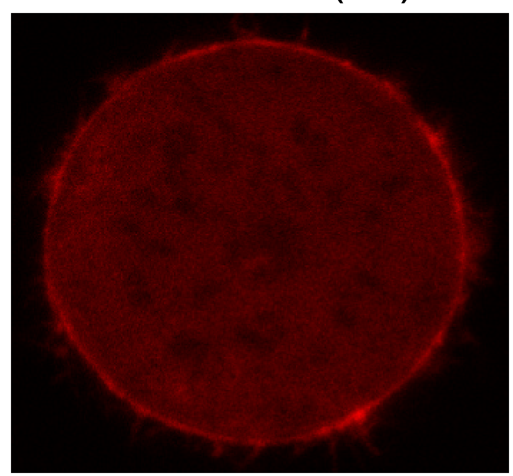
striated muscle



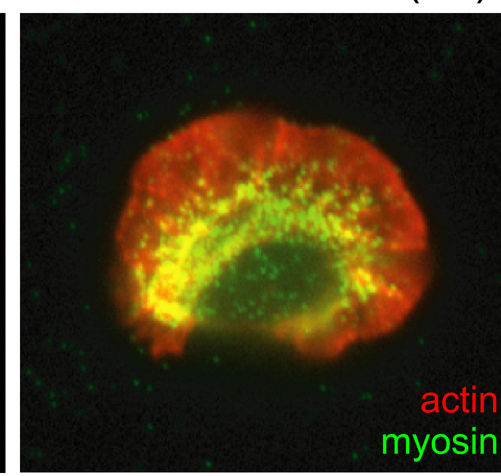
cytokinetic ring (1d)



cell cortex (2d)



lamellar network (3d)

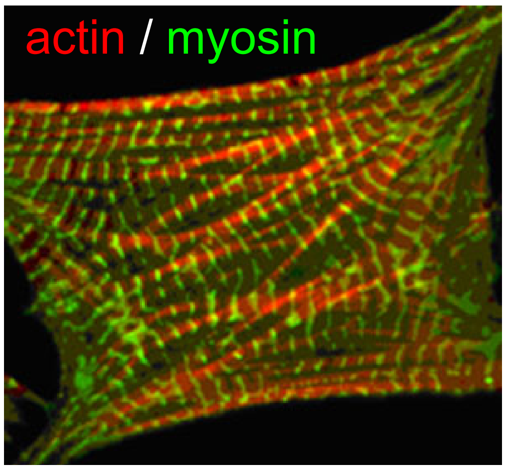


ordered

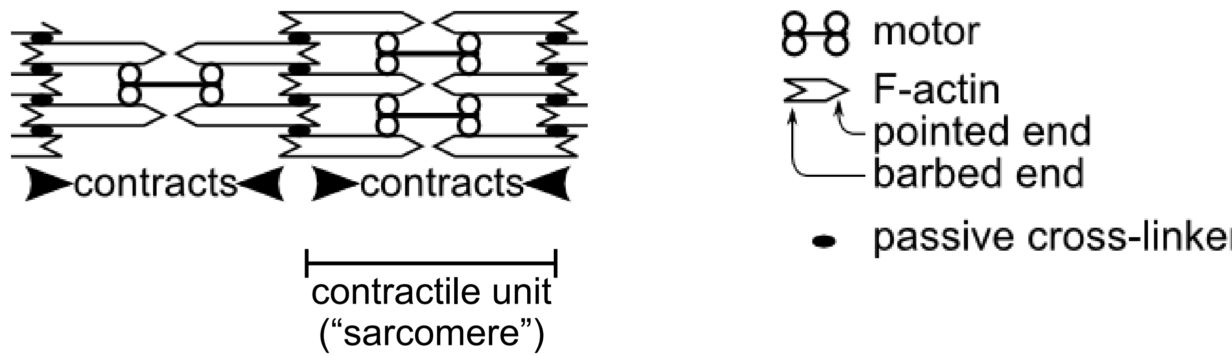
disordered

Lappalainen lab website ; Yumura *J. Cell. Biol.* 2001 ; Clark *et al. Biophys. J.* 2013 ; Barnhart *et al. PLoS Biol.* 2011

F-actin ordering accounts for striated muscle contractility



Contraction mechanism:



Universal contractility paradigm?
Myosin generates contractile forces,
which the actin scaffold transmits over long distances.

Myosin motors have no intrinsic propensity for contraction

Why is disordered actomyosin contractile rather than extensile?

Linear networks cannot favor contraction over extension

motor force dipole: $\mathcal{D} = \sum_i \boldsymbol{f}_i \cdot \boldsymbol{r}_i$

$$\sigma_{\text{far field}} = -\rho \mathcal{D}_{\text{local}} + \Sigma$$

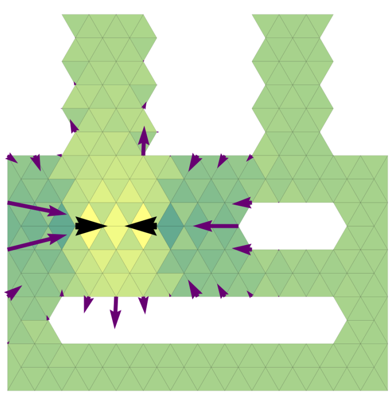
active stress

motor density

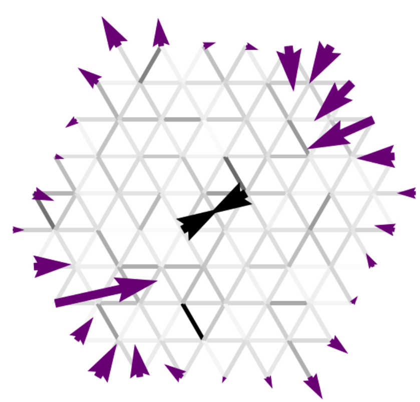
$\Sigma = \int \mathrm{Tr}(\sigma)\, \mathrm{d}V$

integrated “pressure”
Vanishes for homogeneous linear materials.

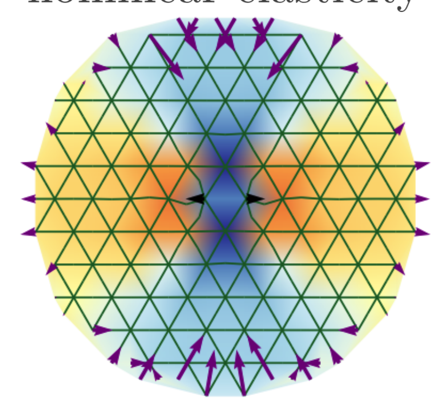
$\Sigma = 0$
even in strange geometries



$\langle \Sigma \rangle = 0$
even in disordered materials

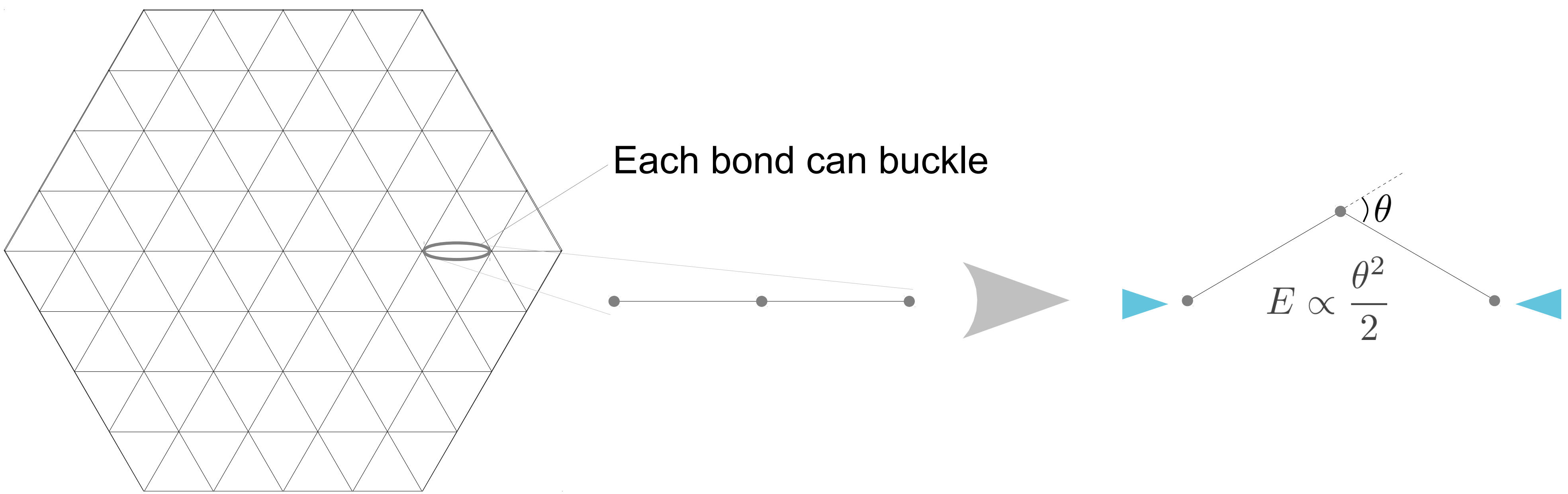


$\Sigma \neq 0$
in the presence of nonlinear elasticity

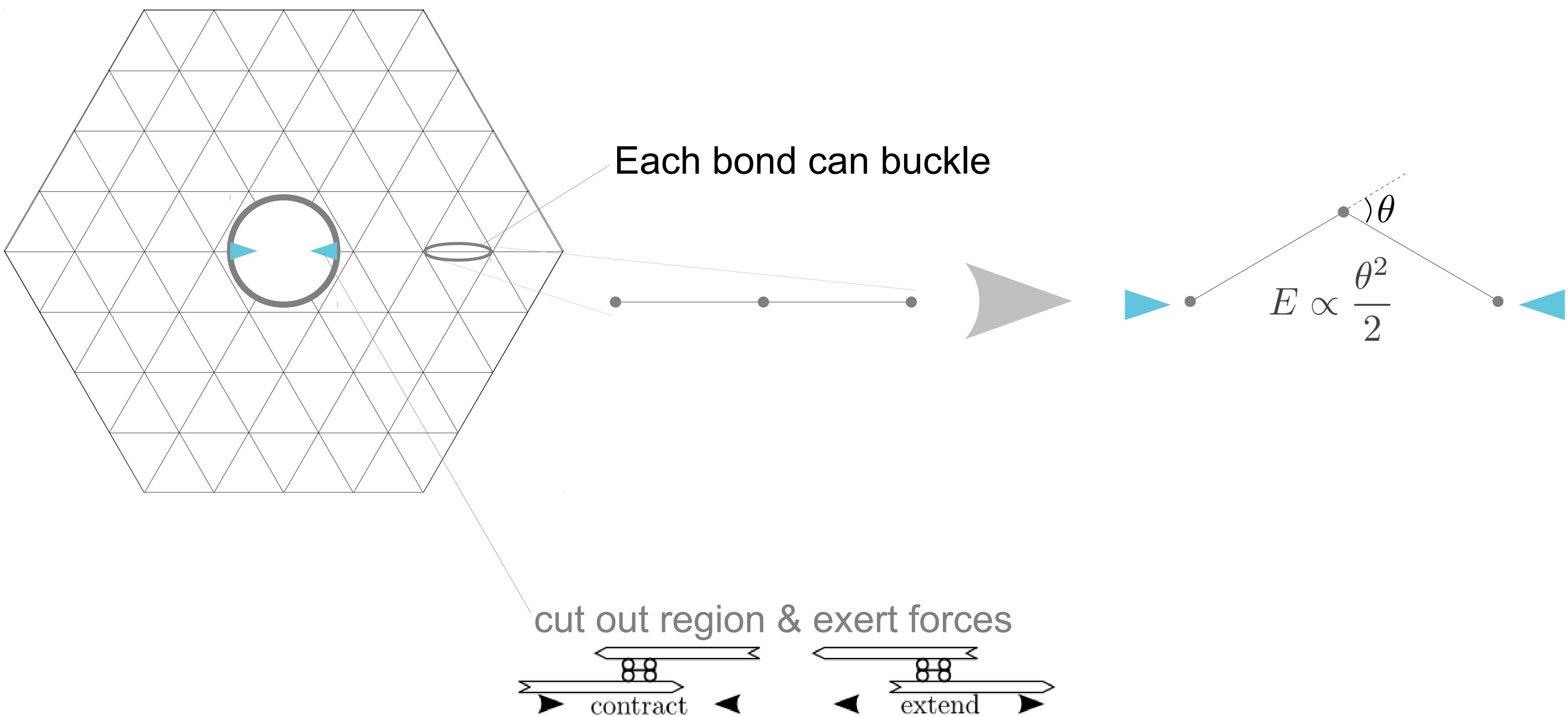


Ronceray & ML *Soft Matter 2015*

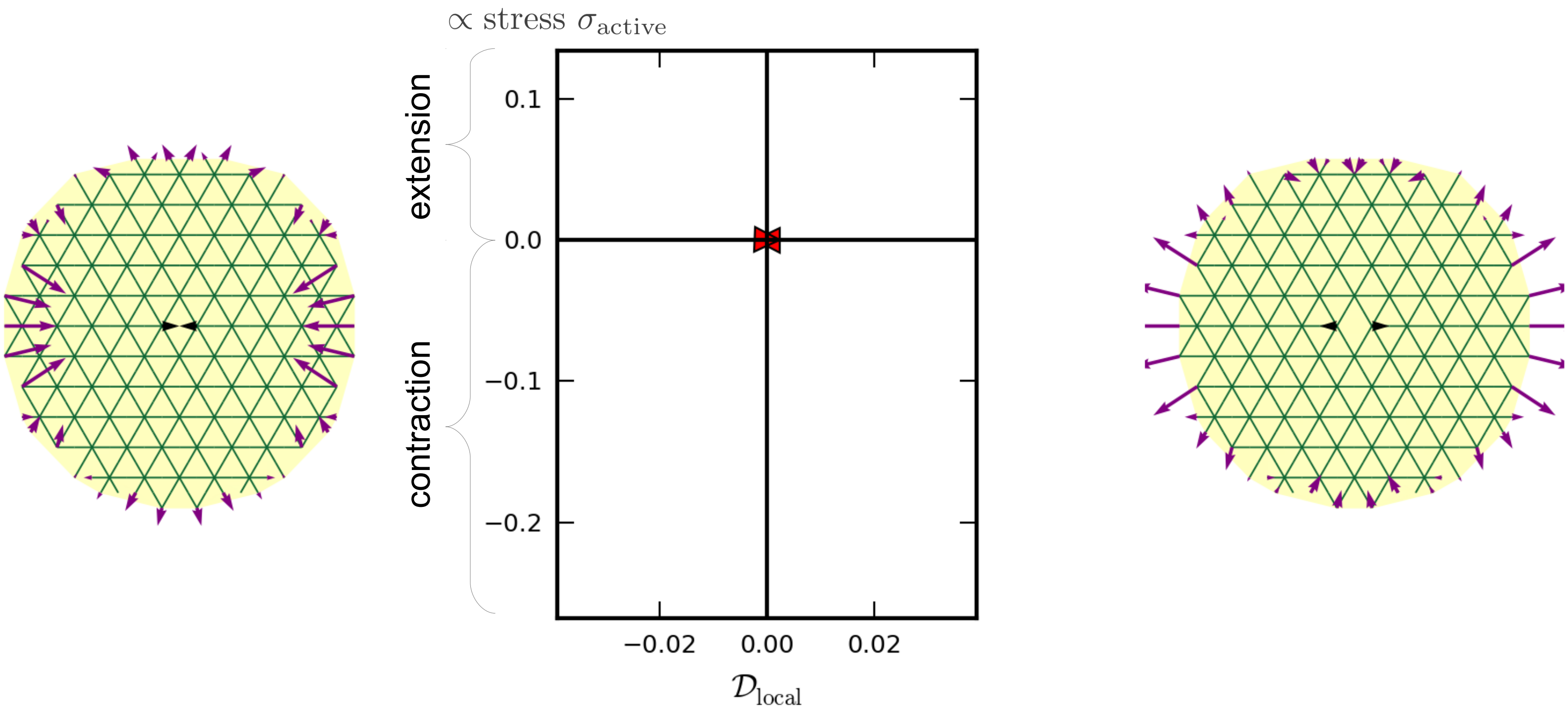
We consider a network with filaments susceptible to buckling



We consider a network with filaments susceptible to buckling

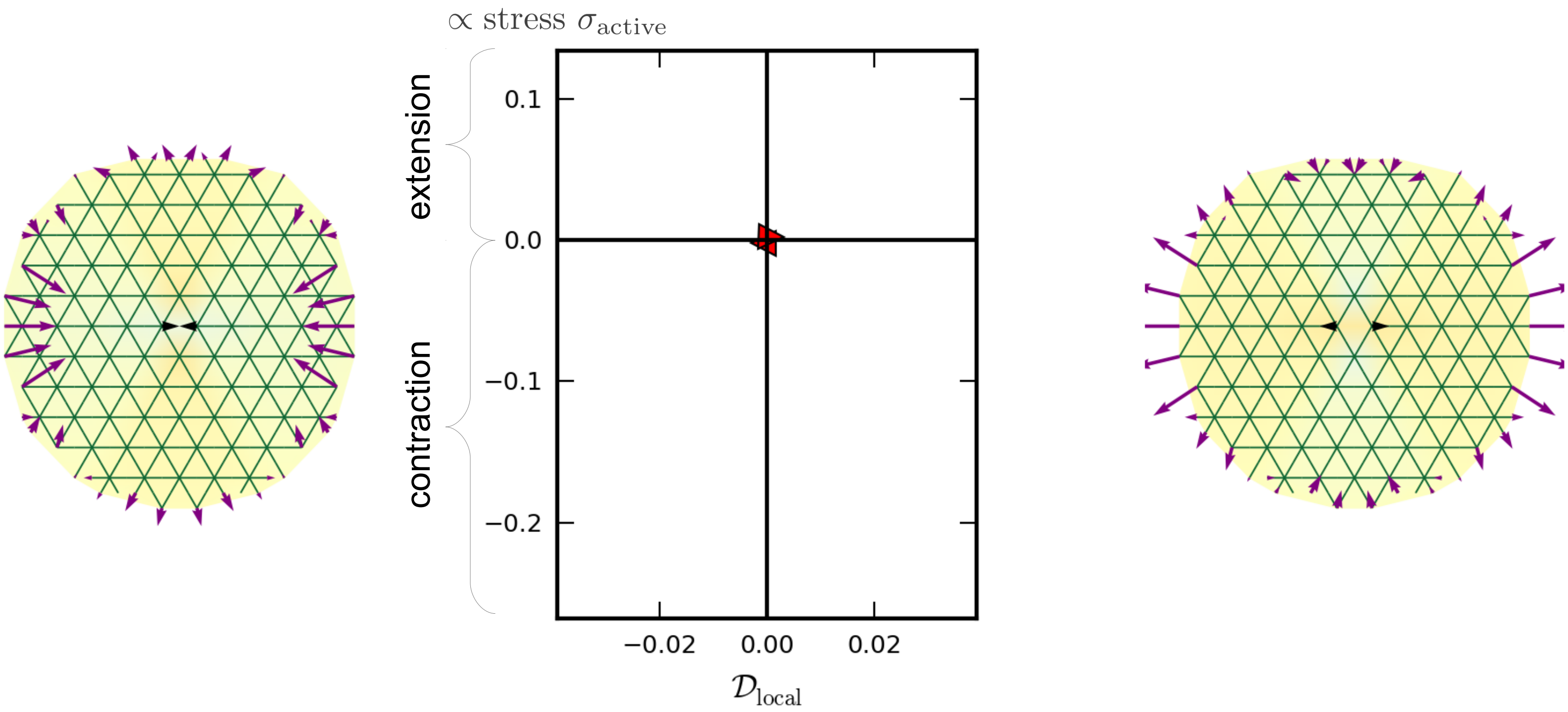


Forces below the buckling threshold yield linear force transmission



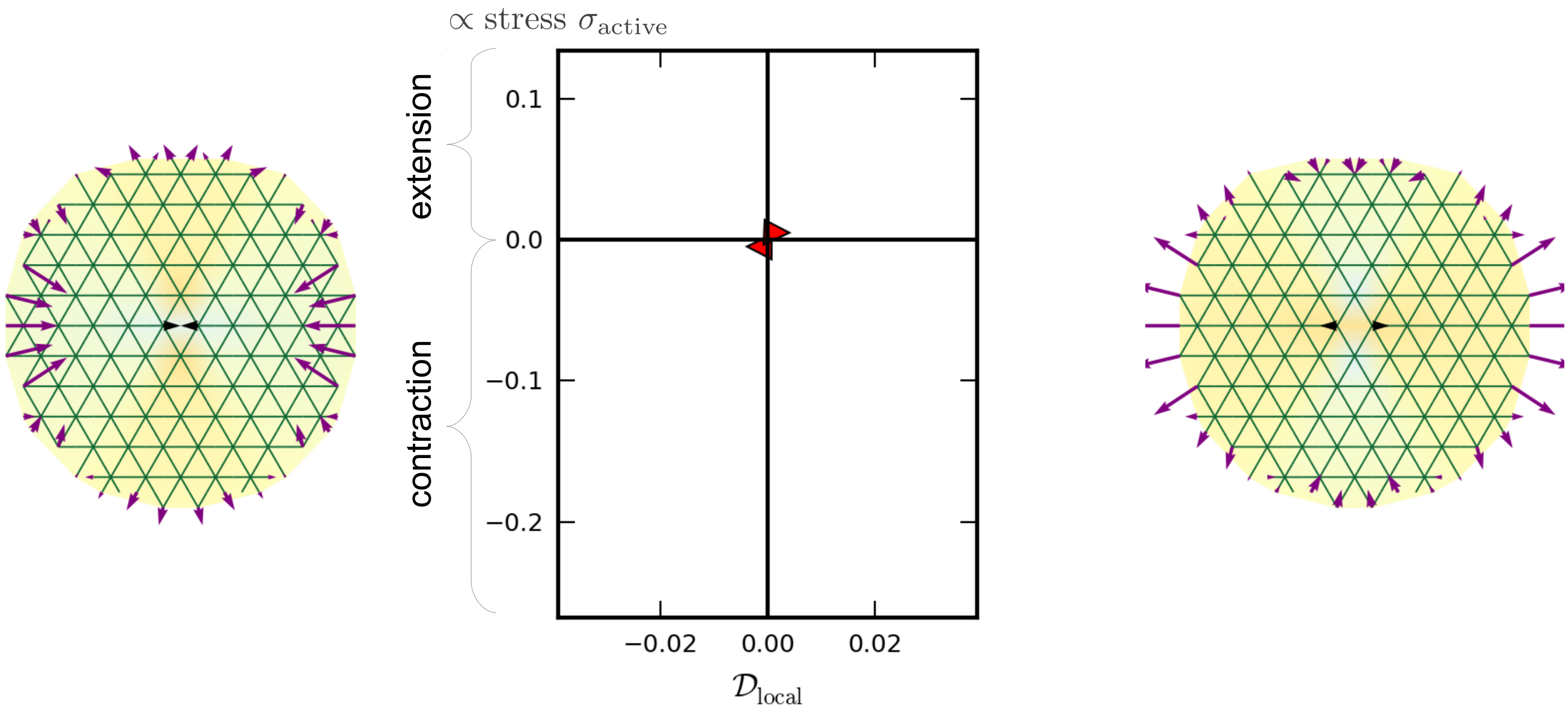
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



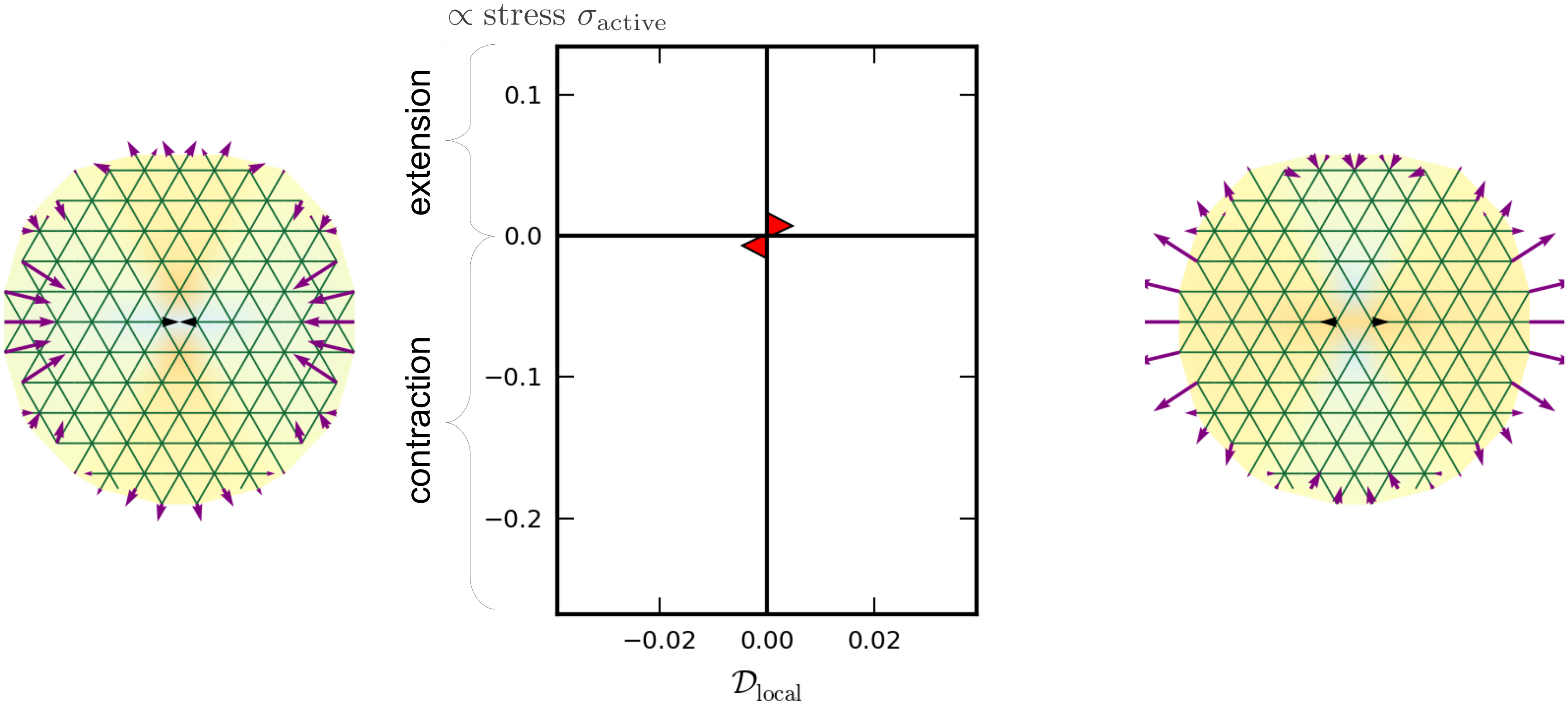
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



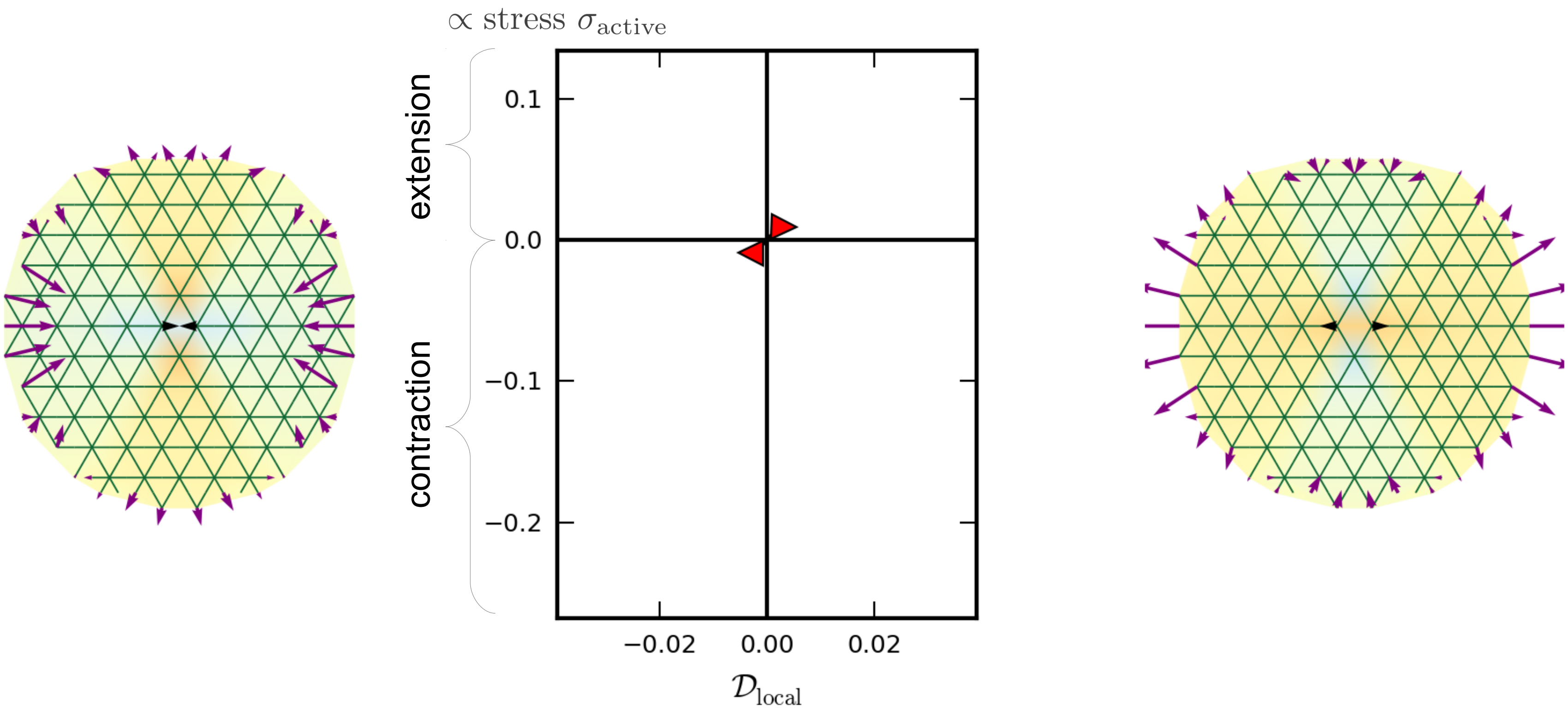
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



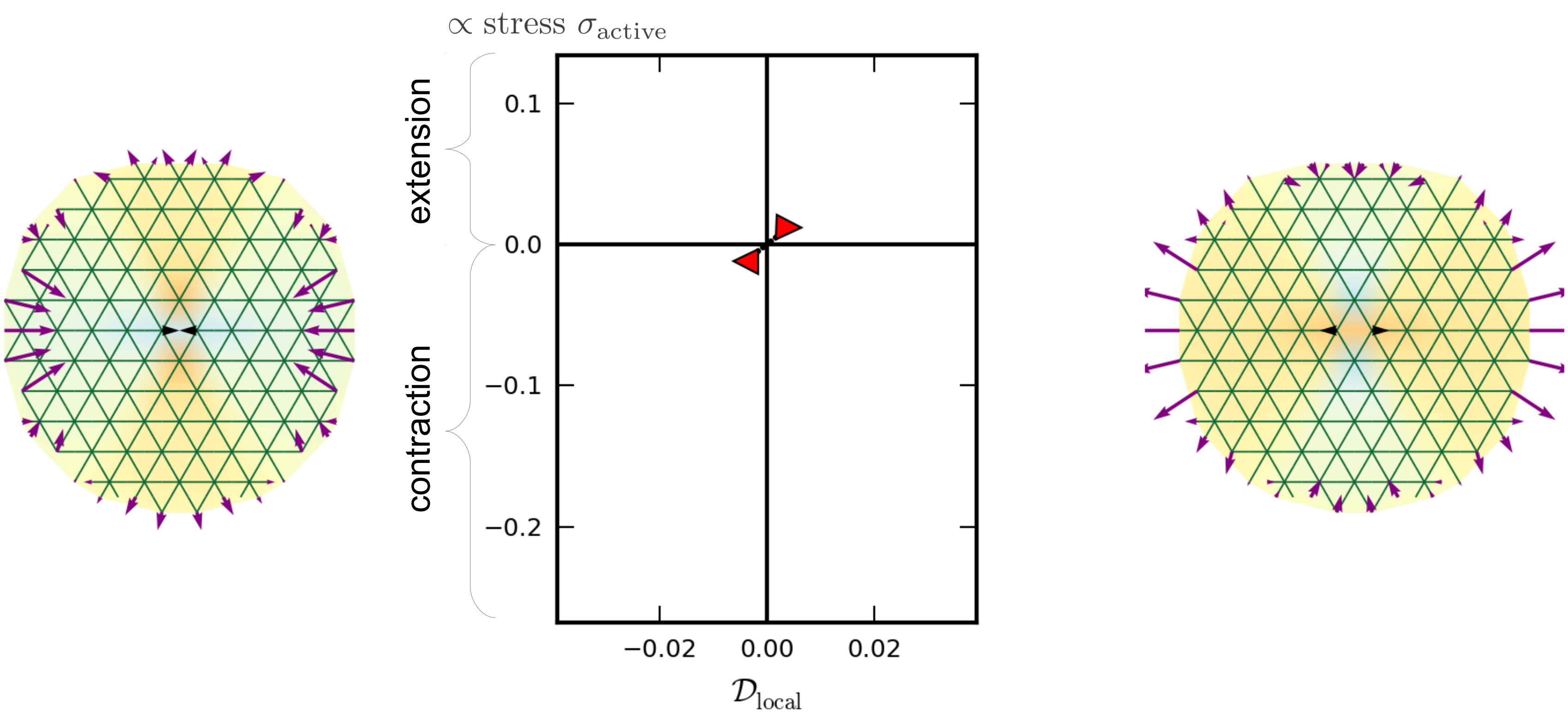
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



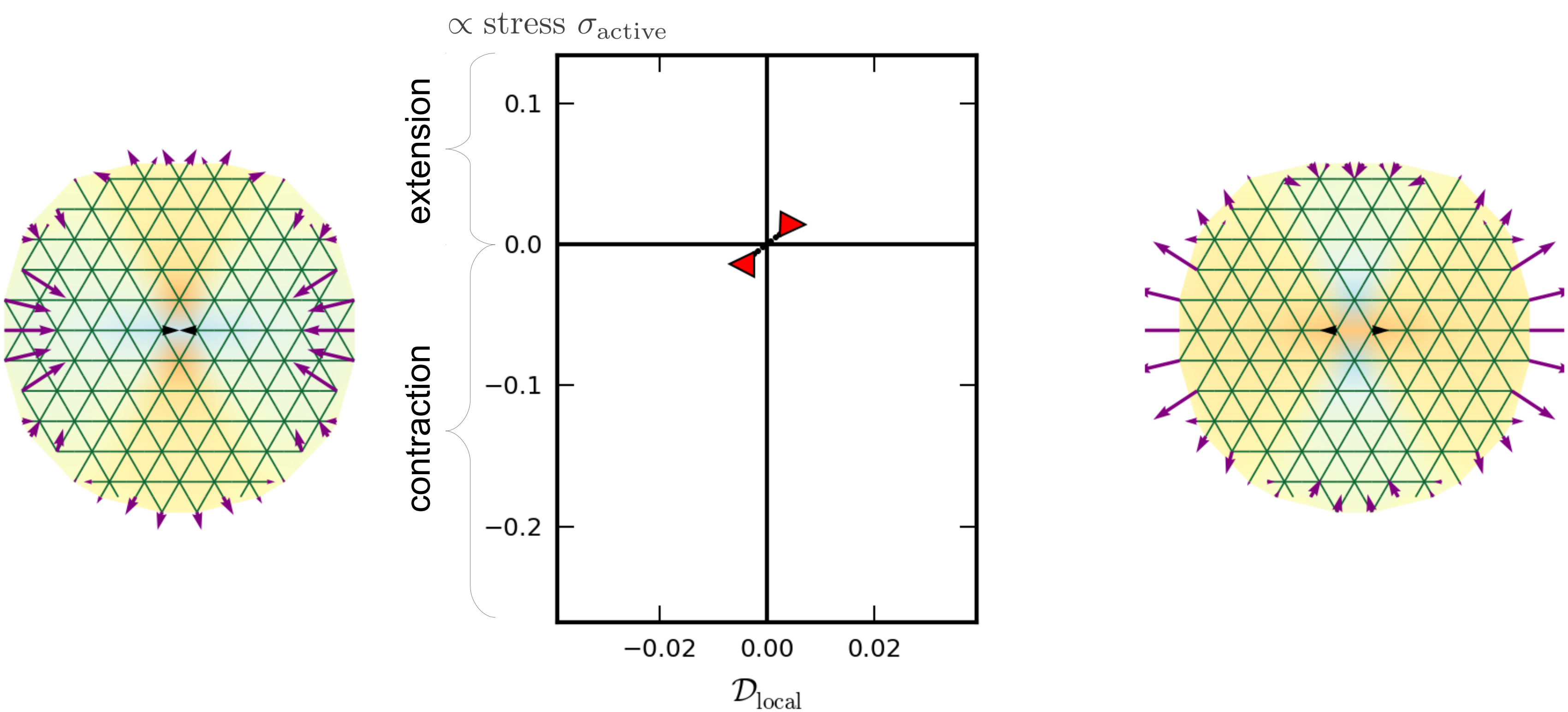
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



legend:
compression
extension

Forces below the buckling threshold yield linear force transmission

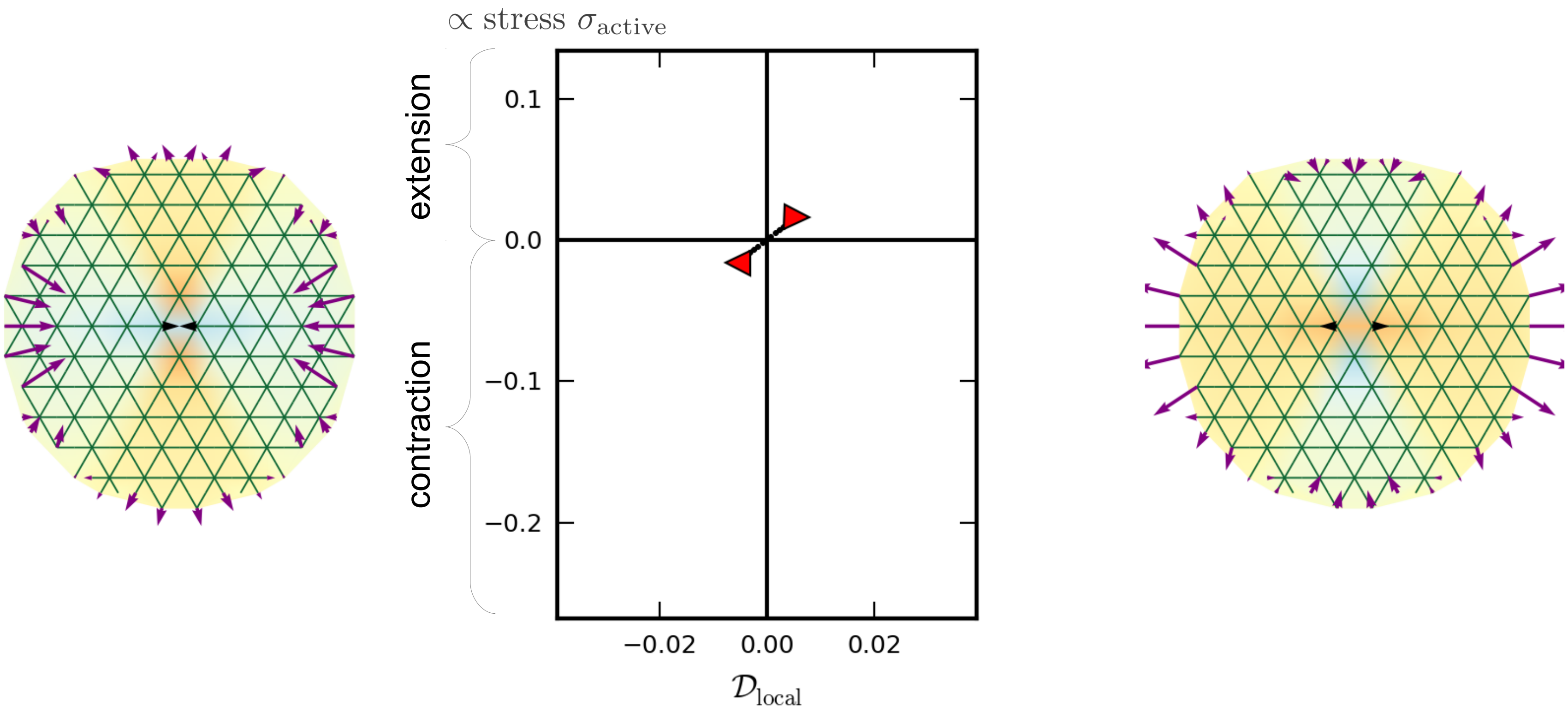


legend:

compression

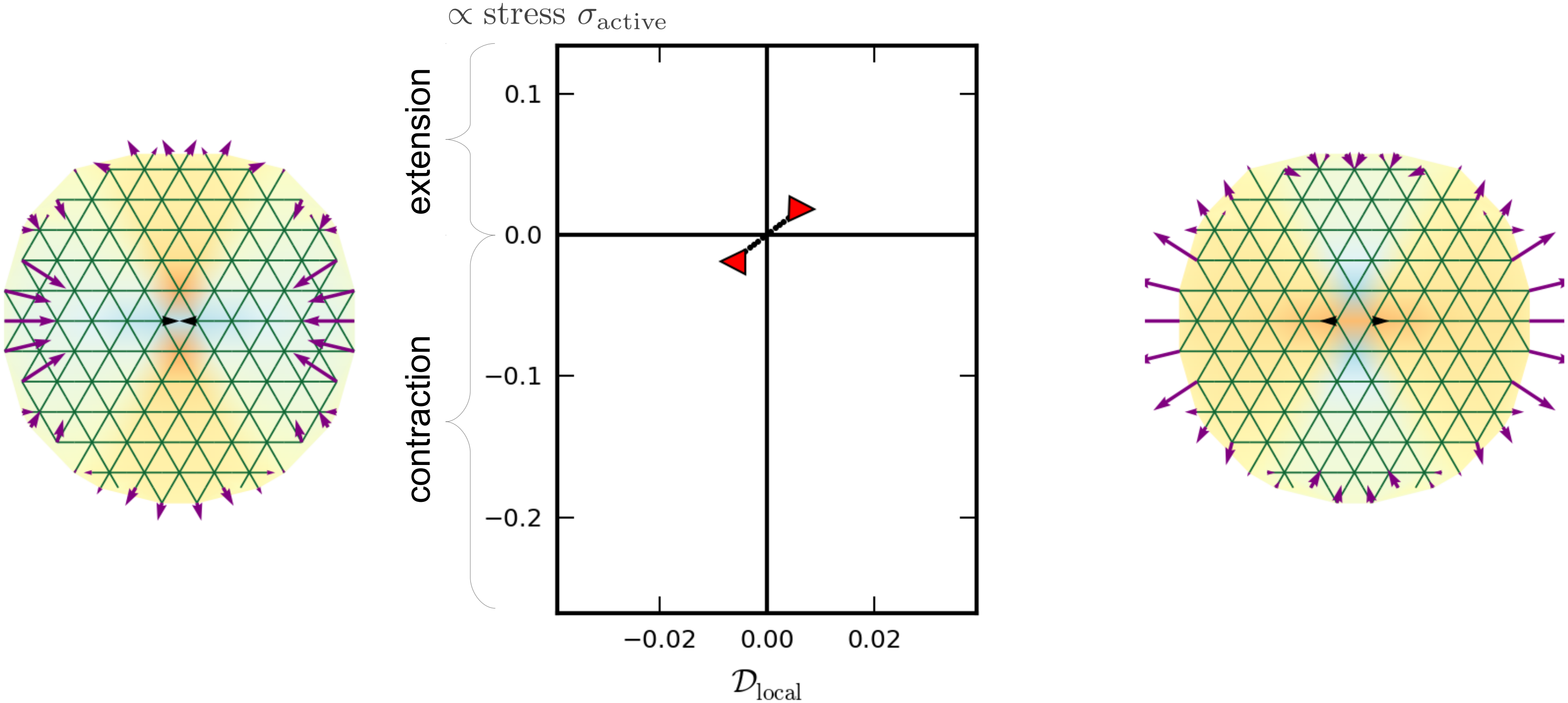
extension

Forces below the buckling threshold yield linear force transmission



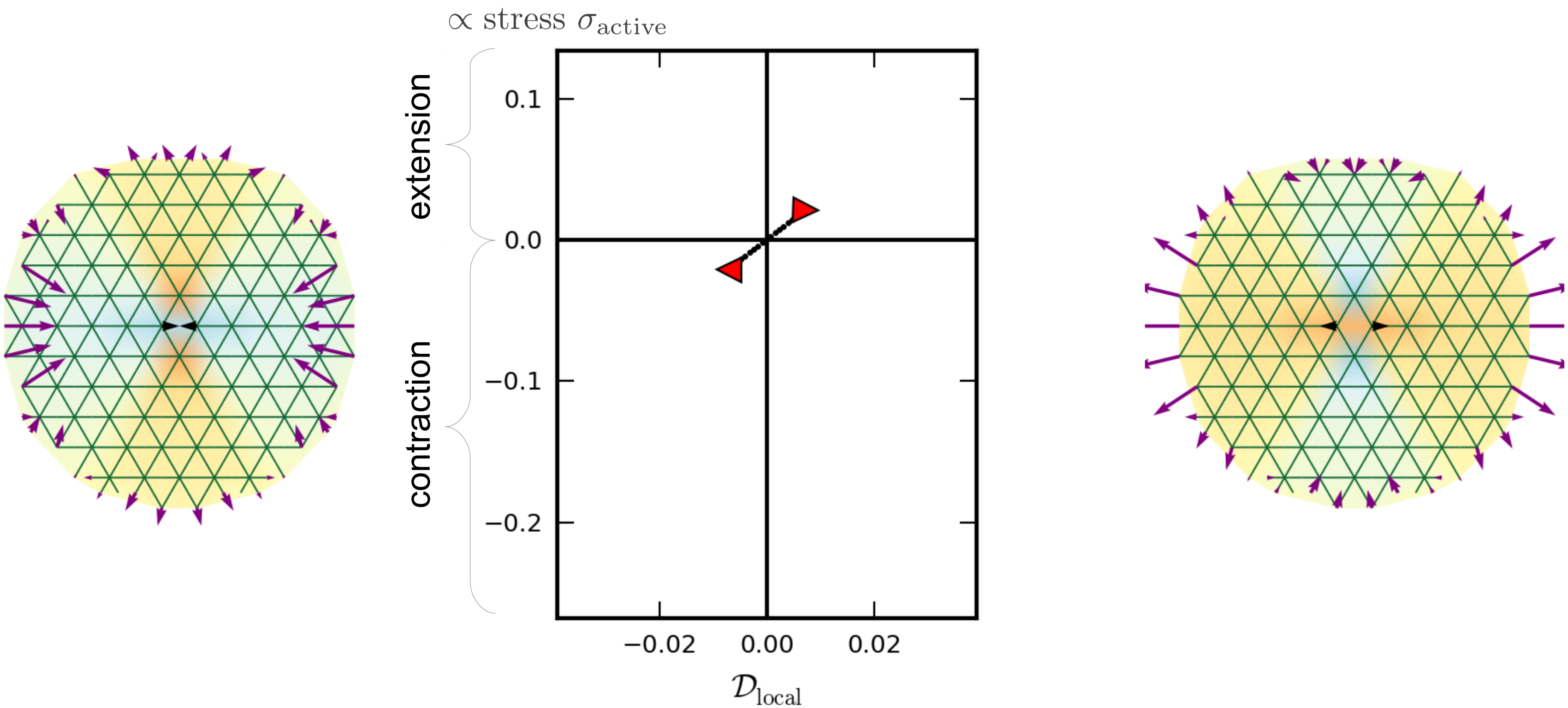
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



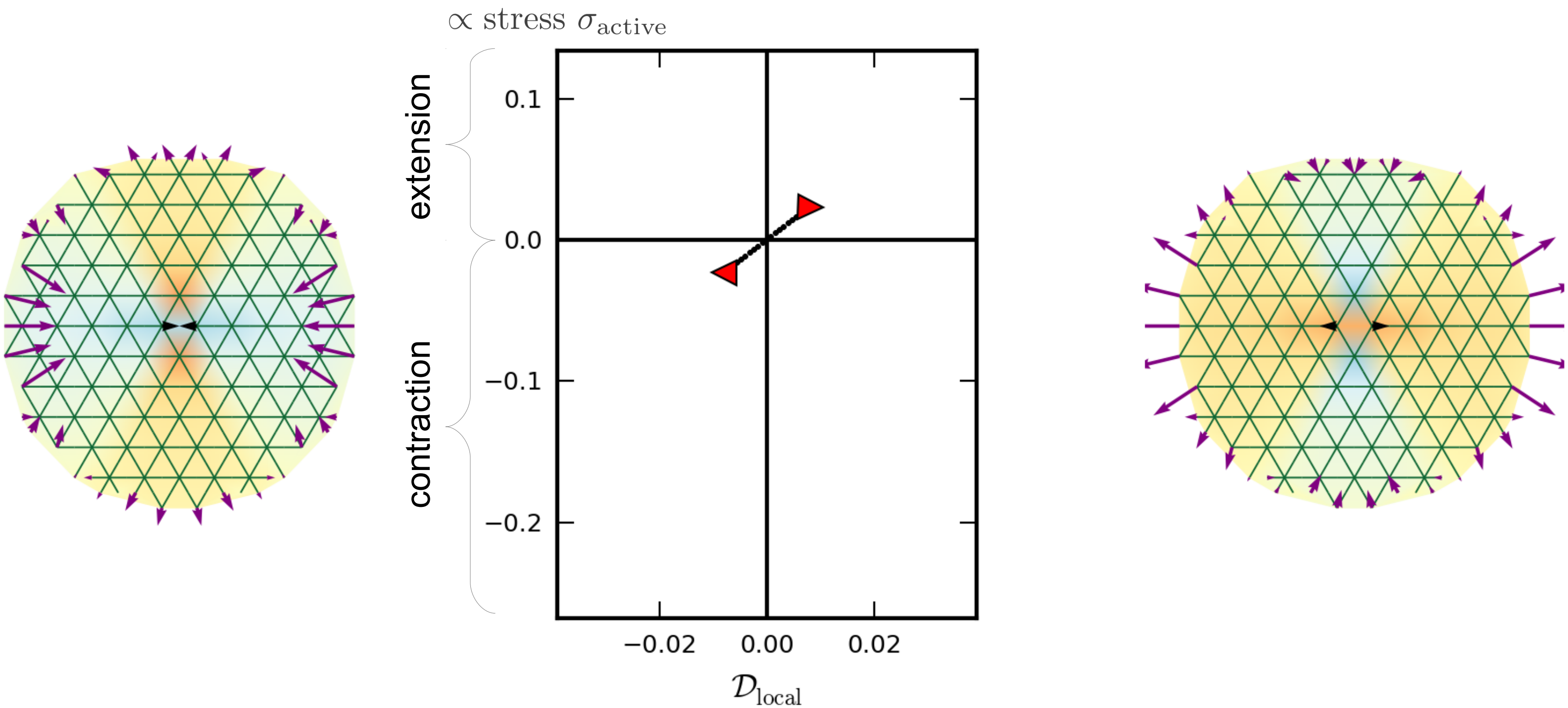
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



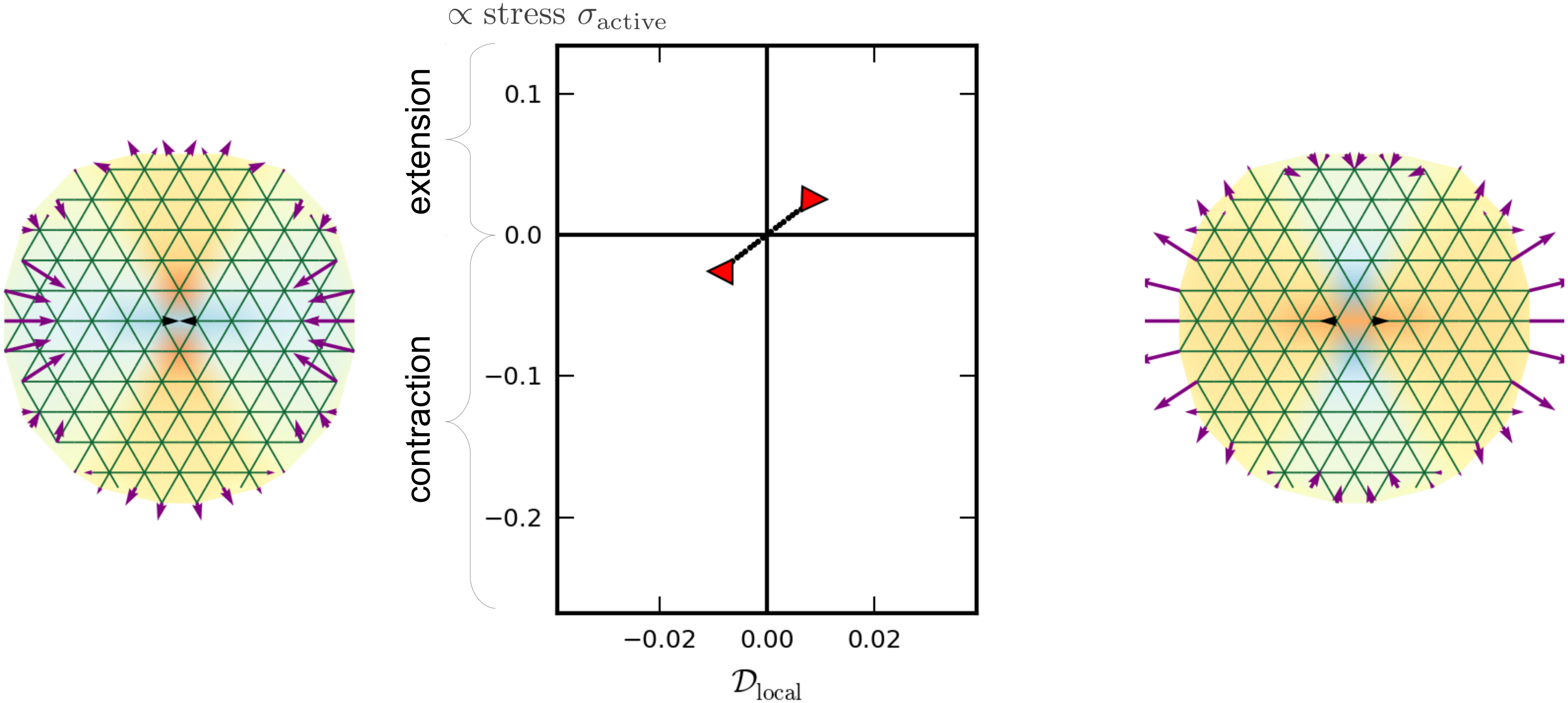
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



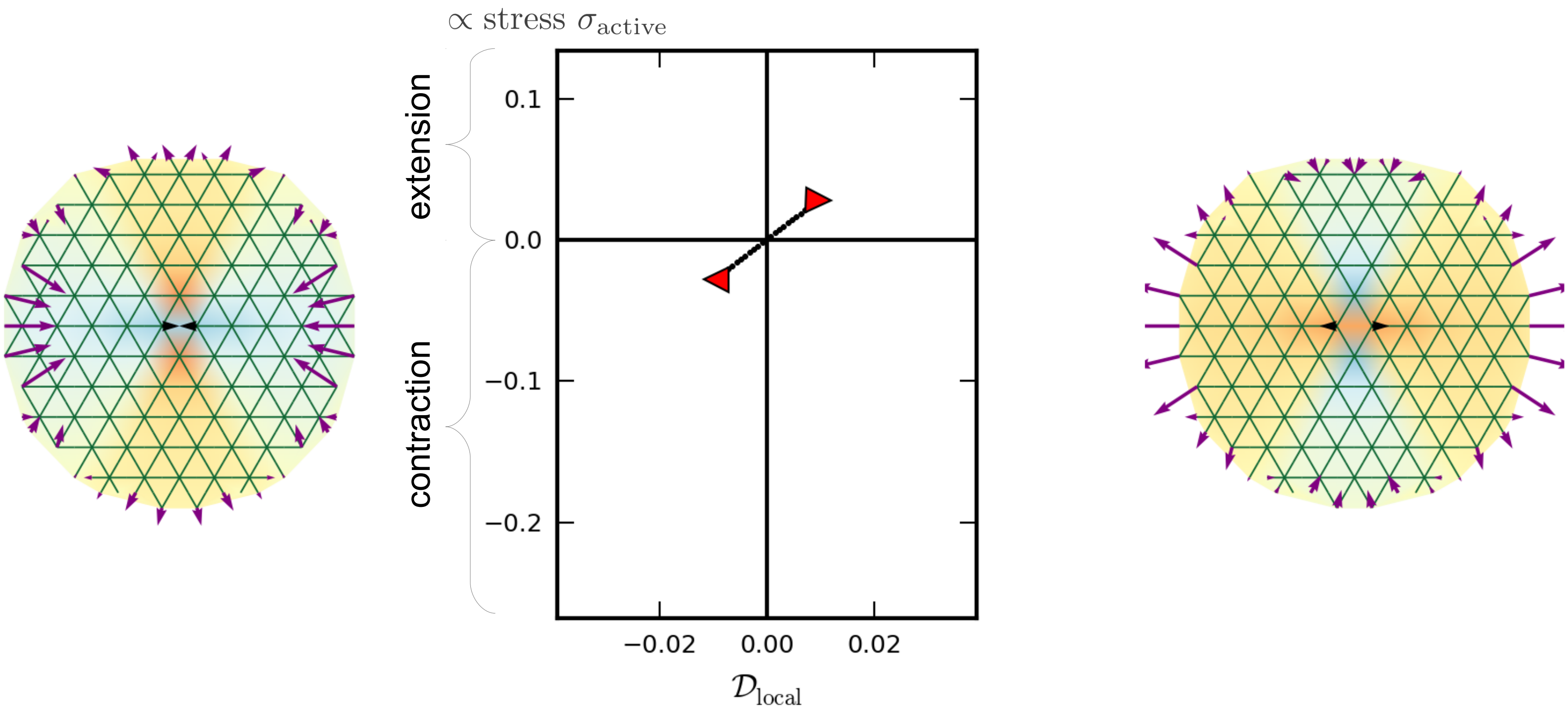
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



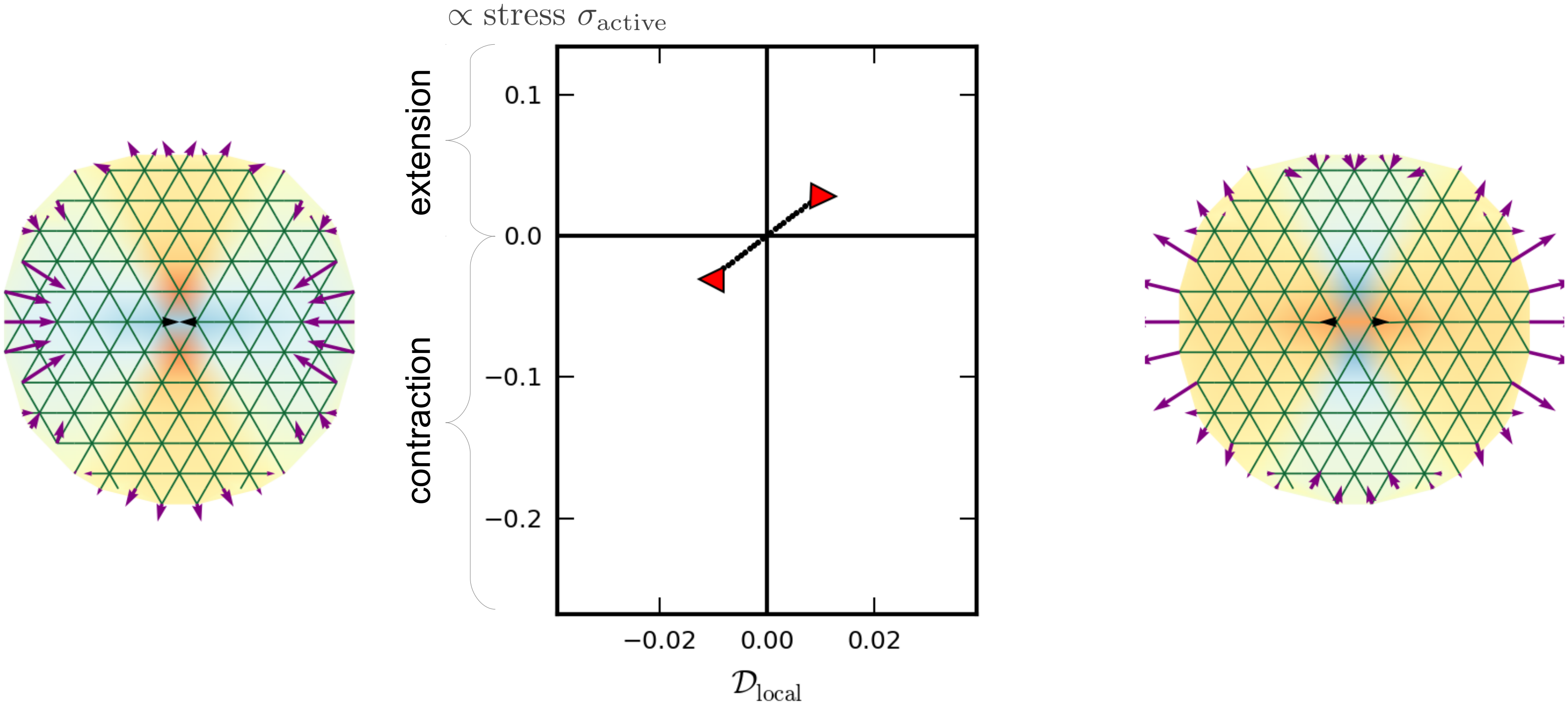
legend:
compression
extension

Forces below the buckling threshold yield linear force transmission



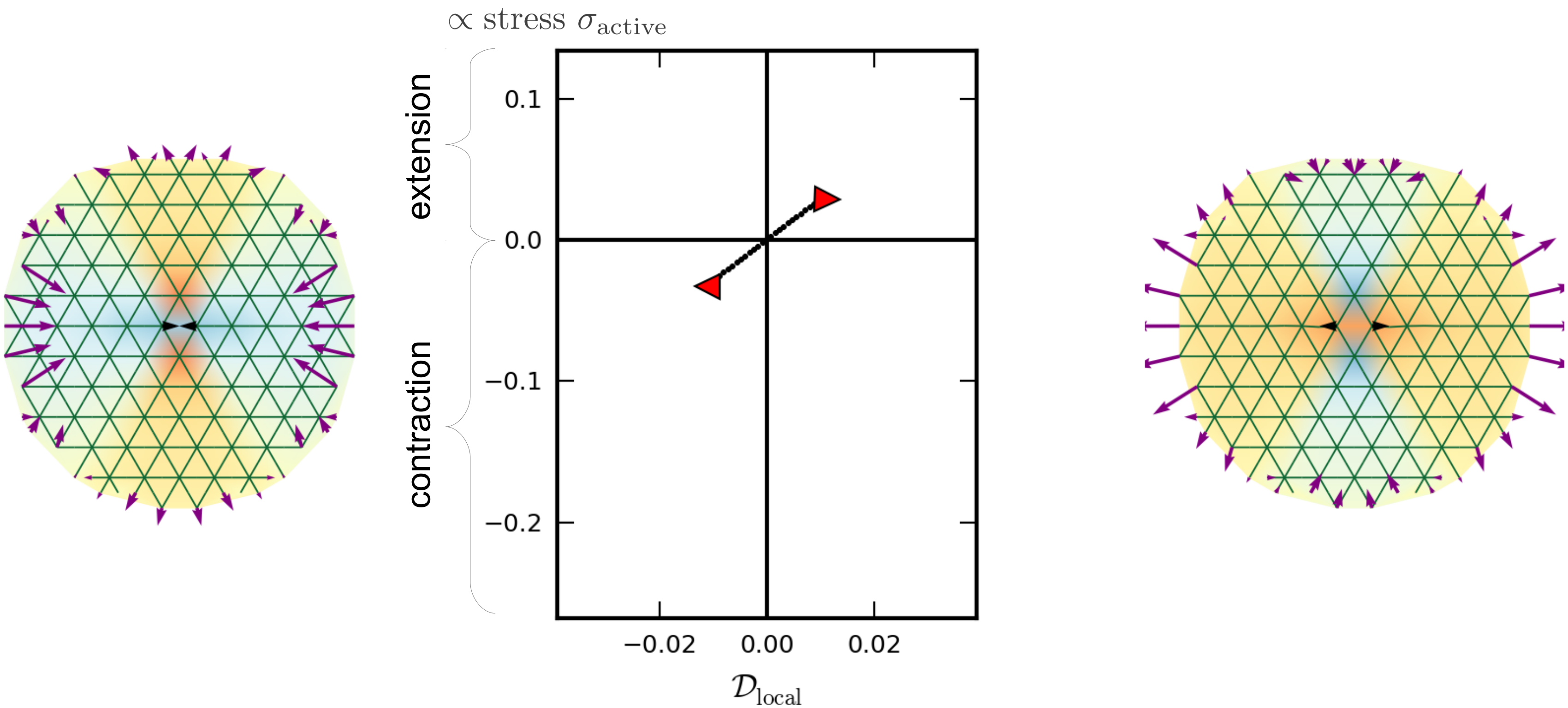
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



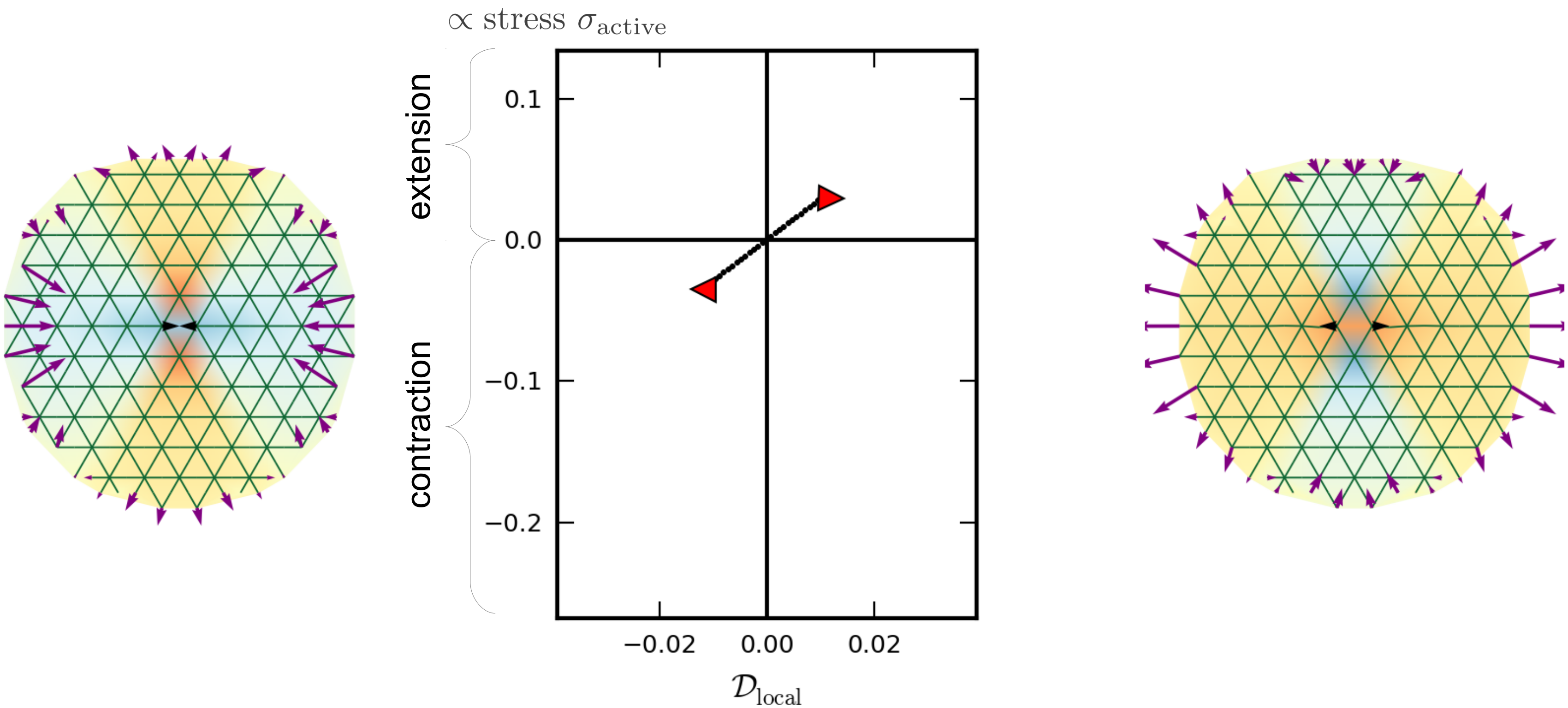
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



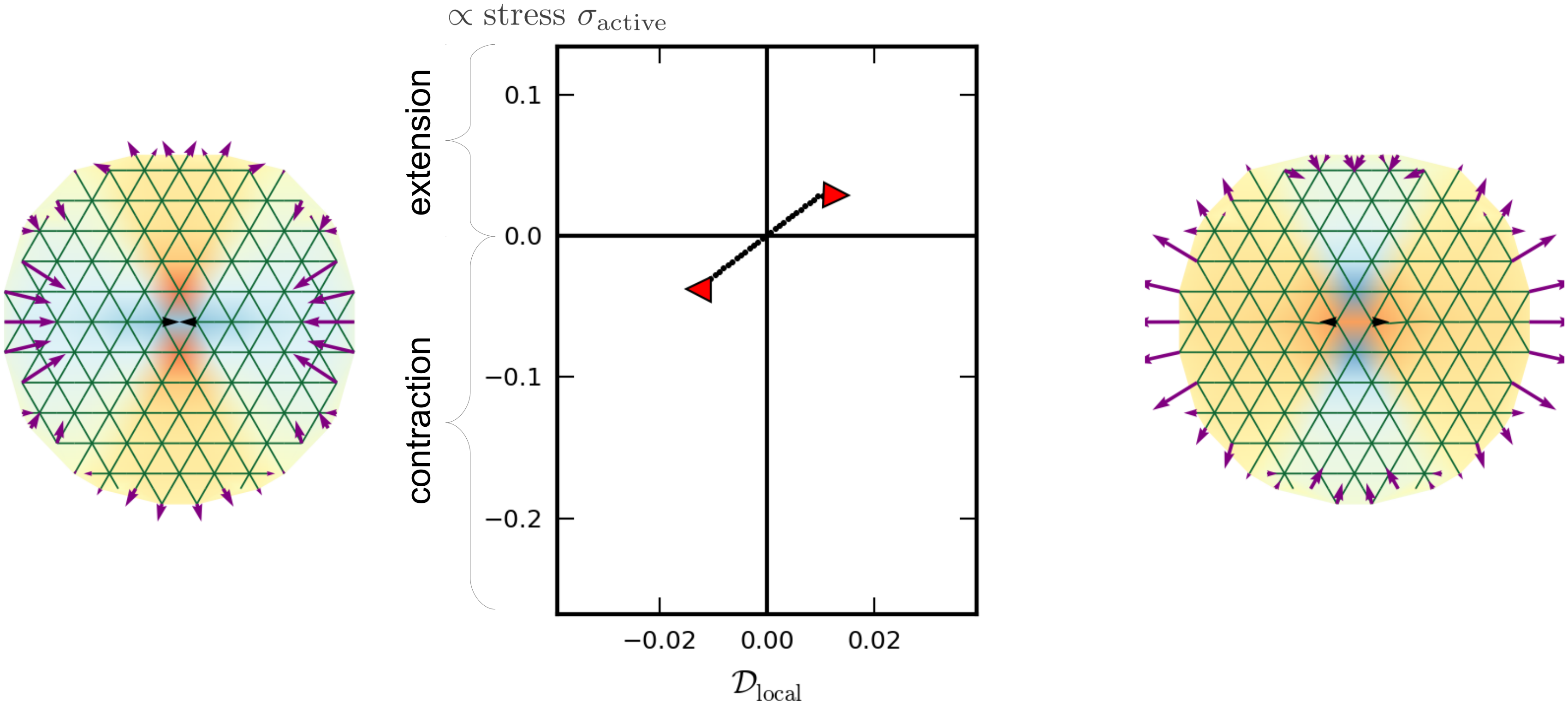
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



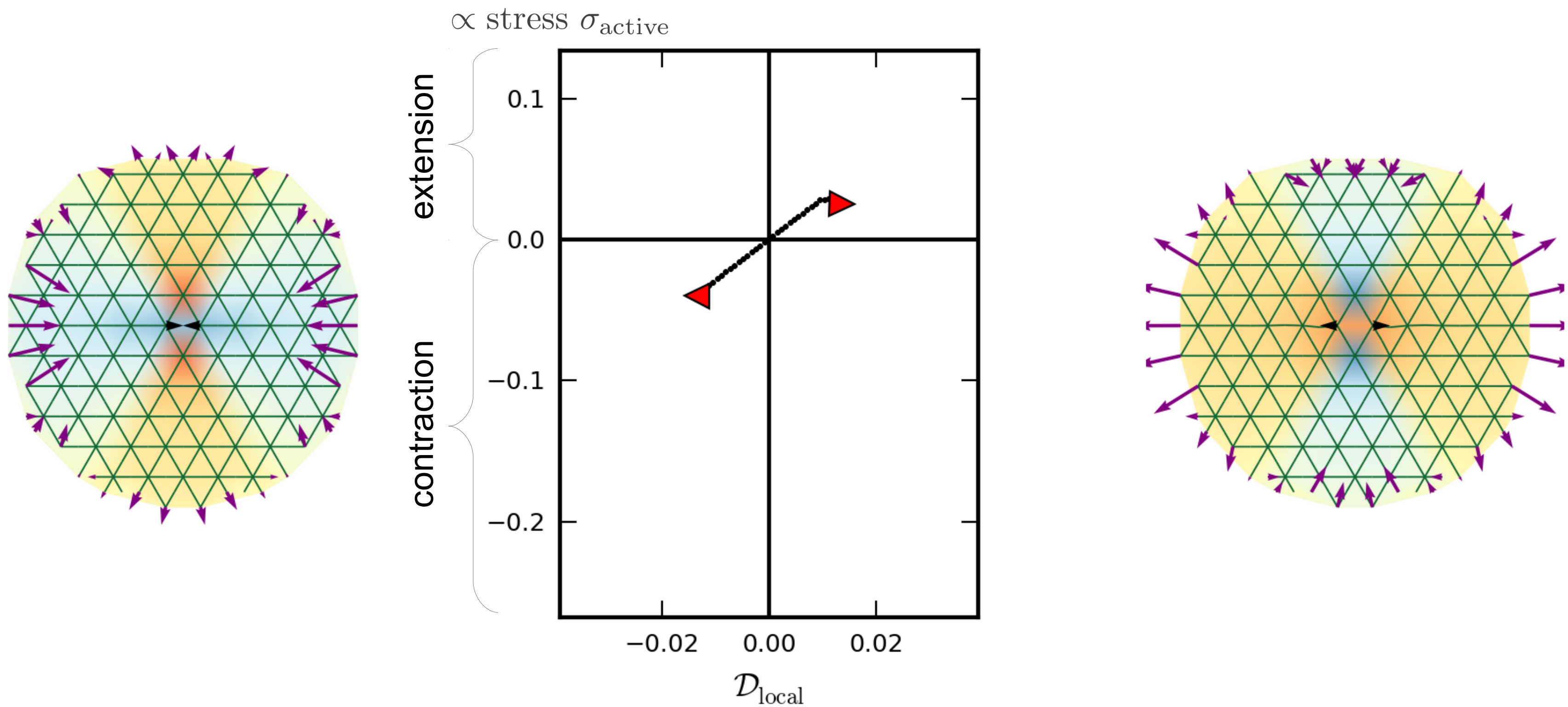
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



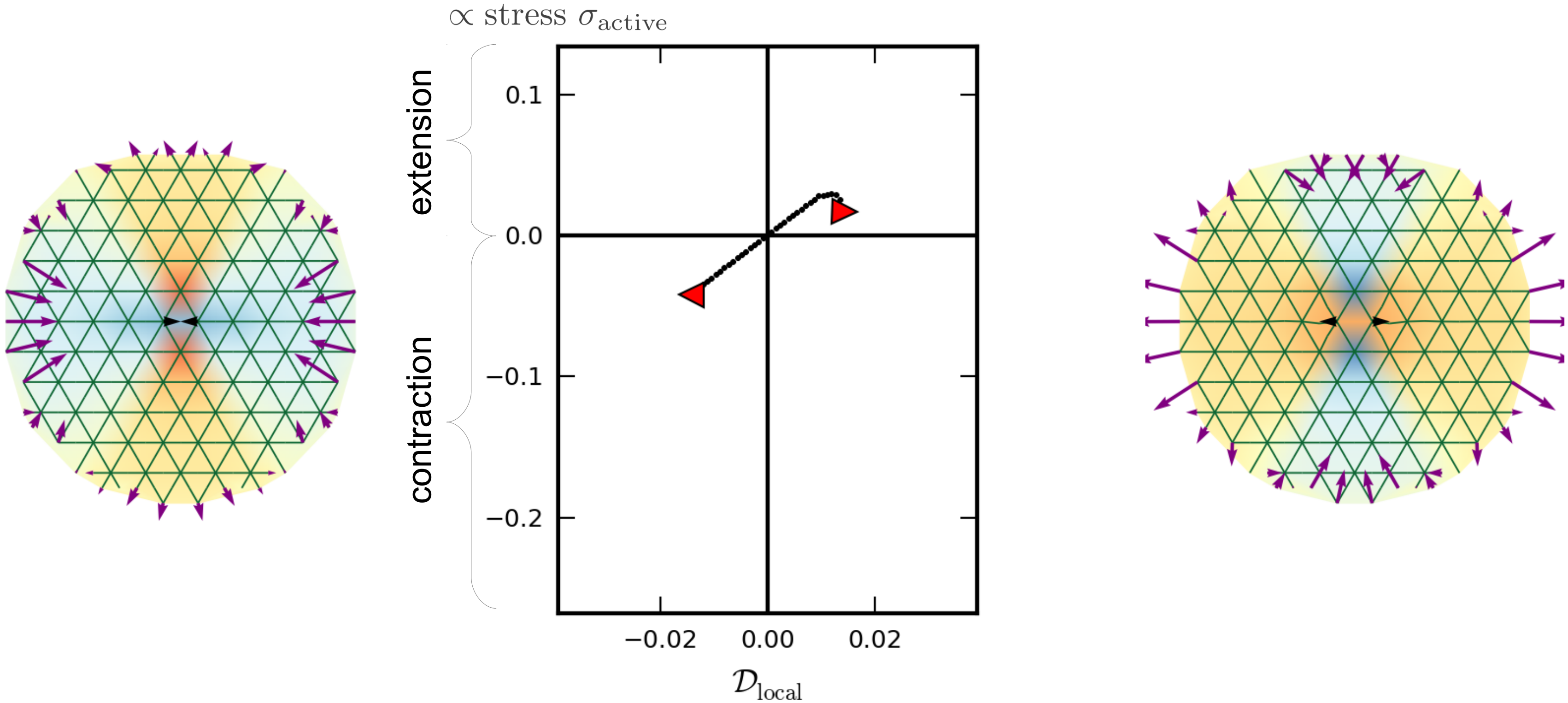
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



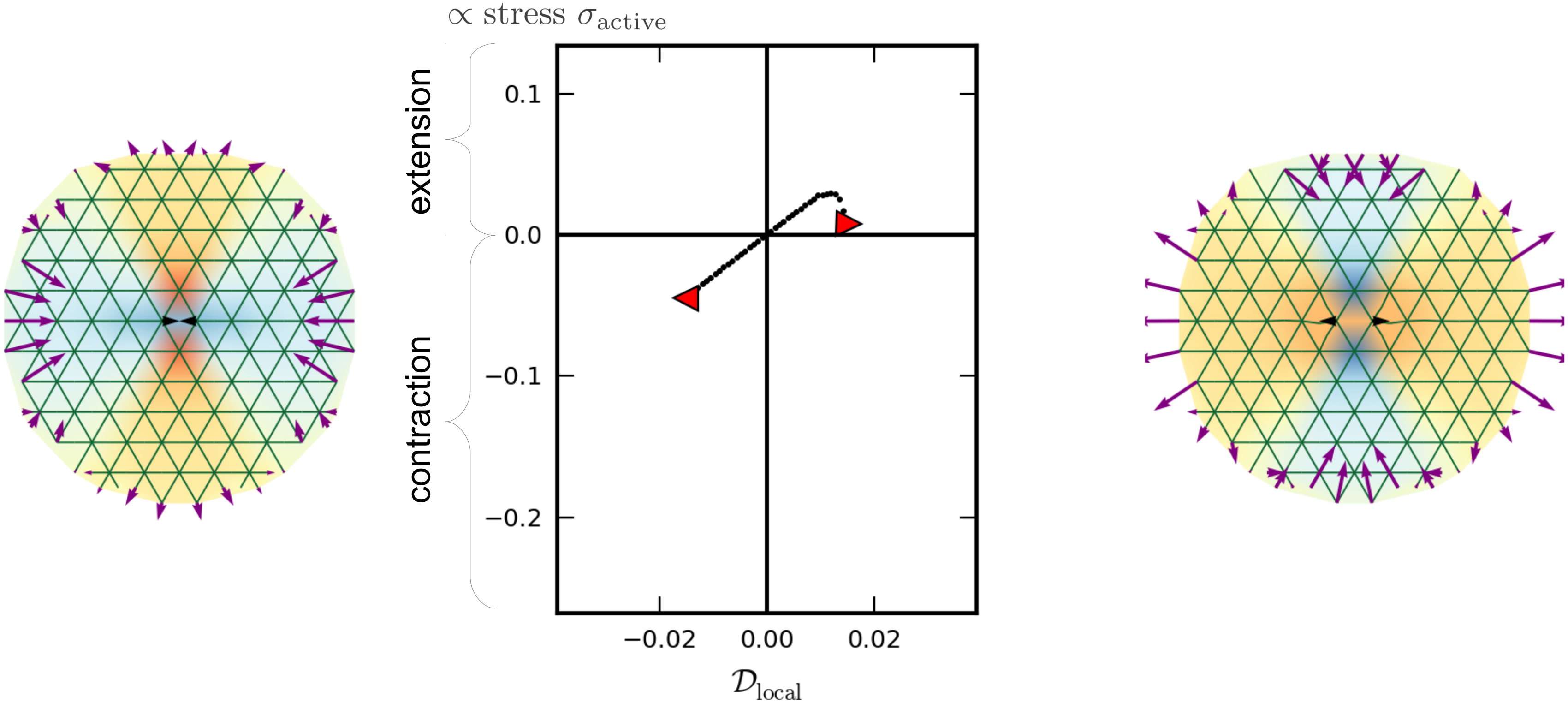
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



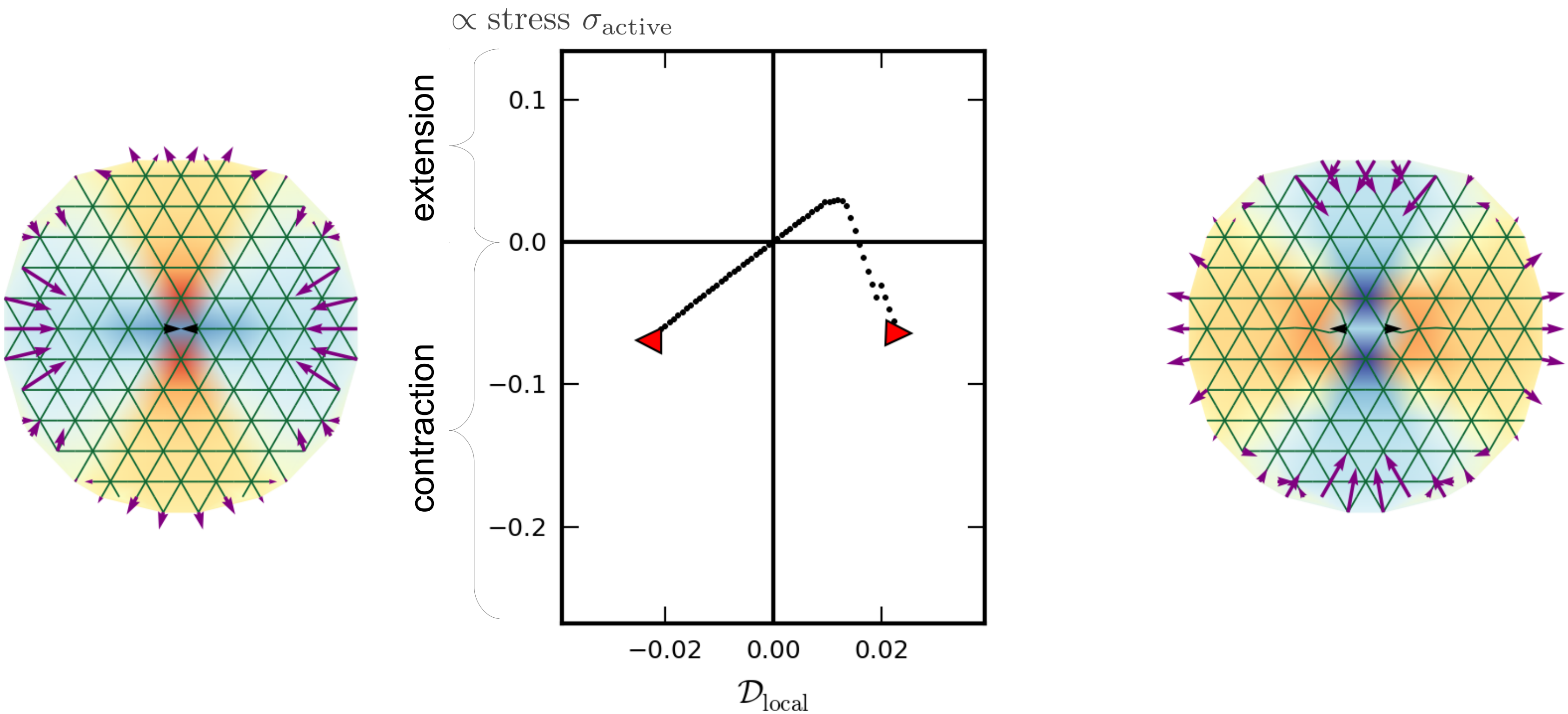
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



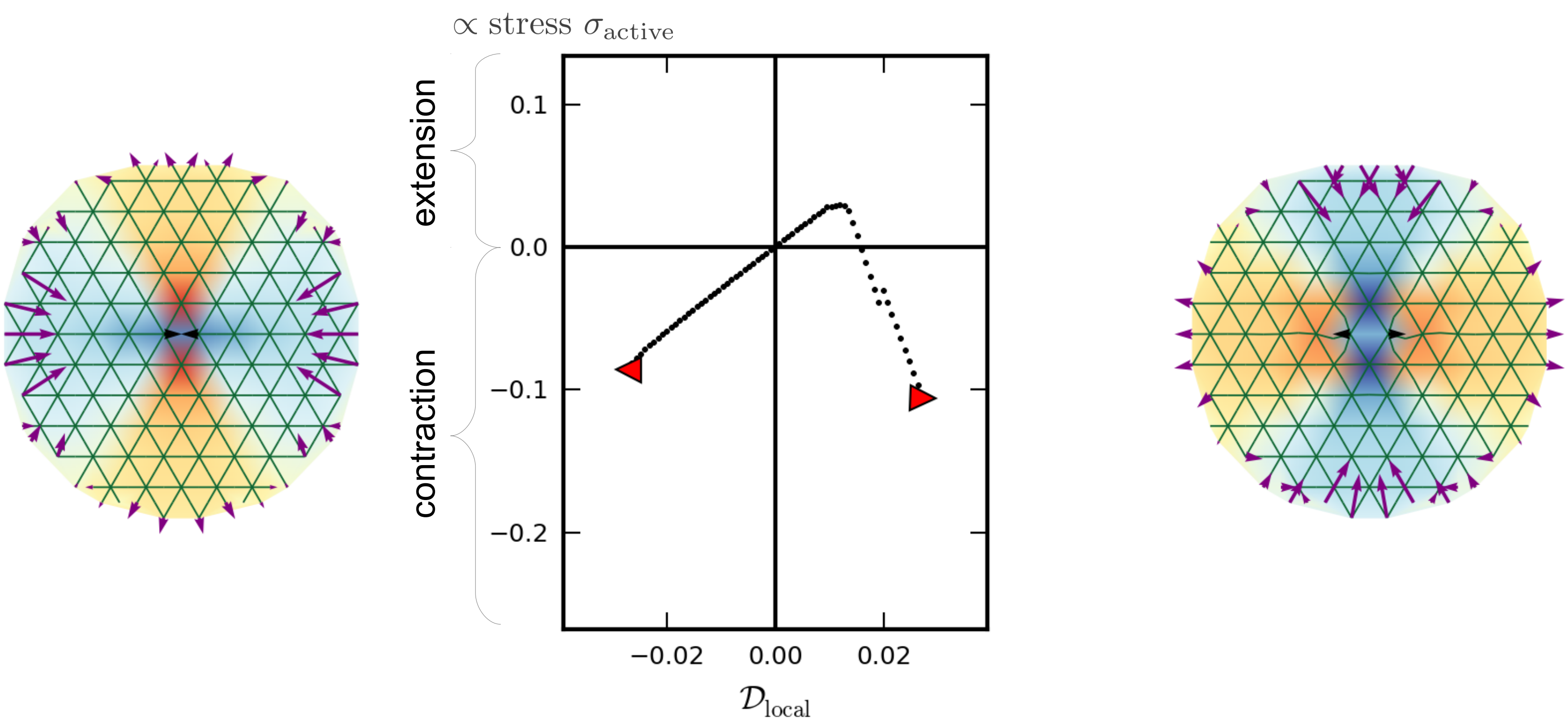
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



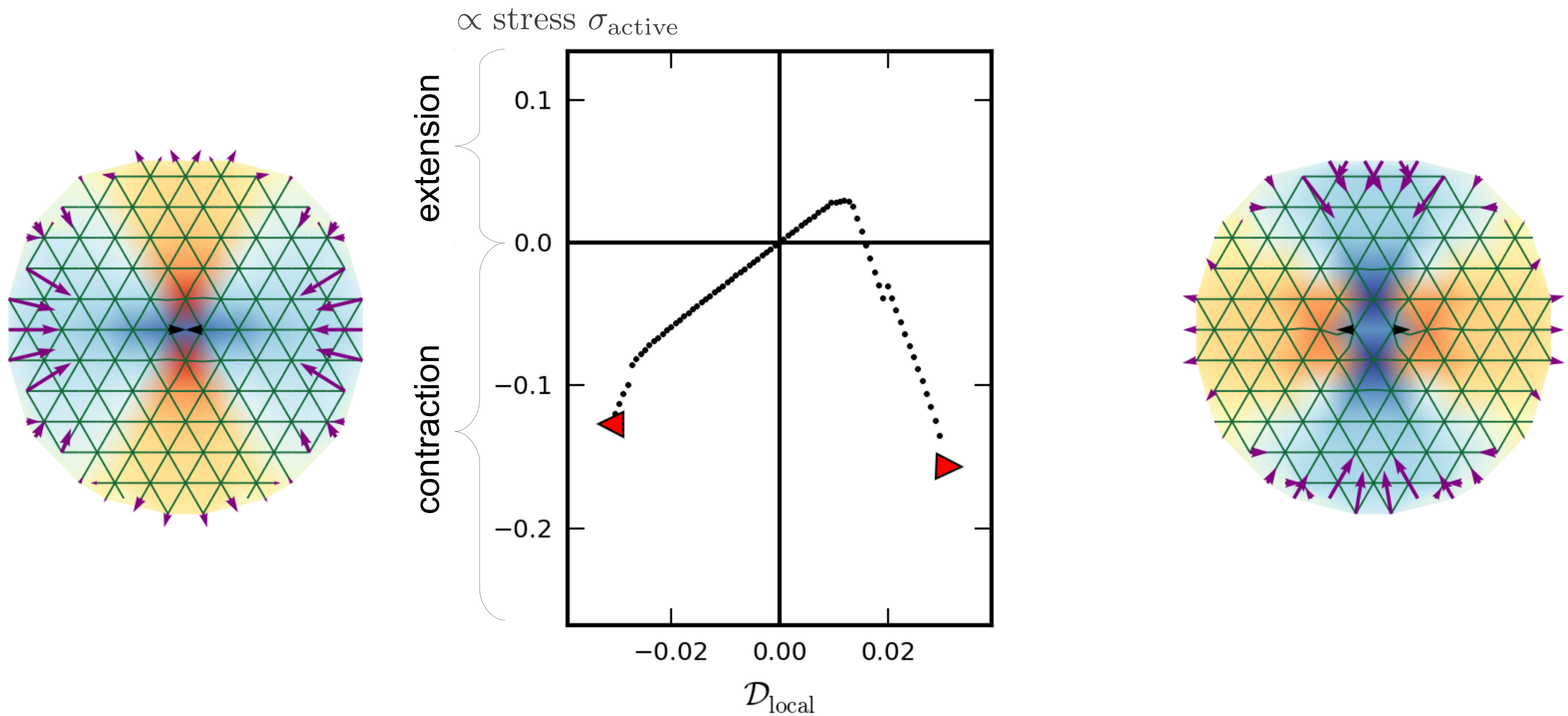
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction



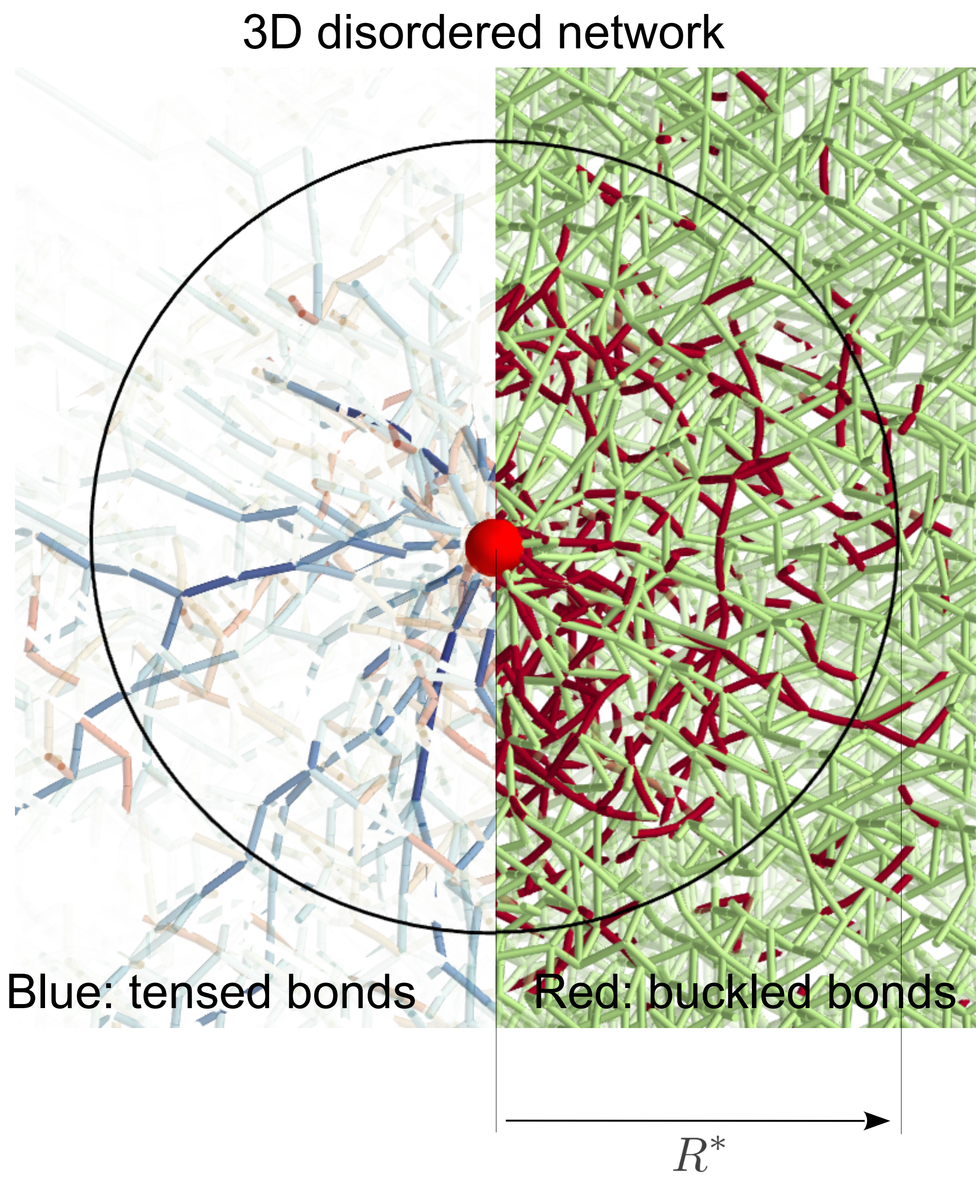
legend:
compression
extension

Buckling rectifies forces of any sign towards contraction

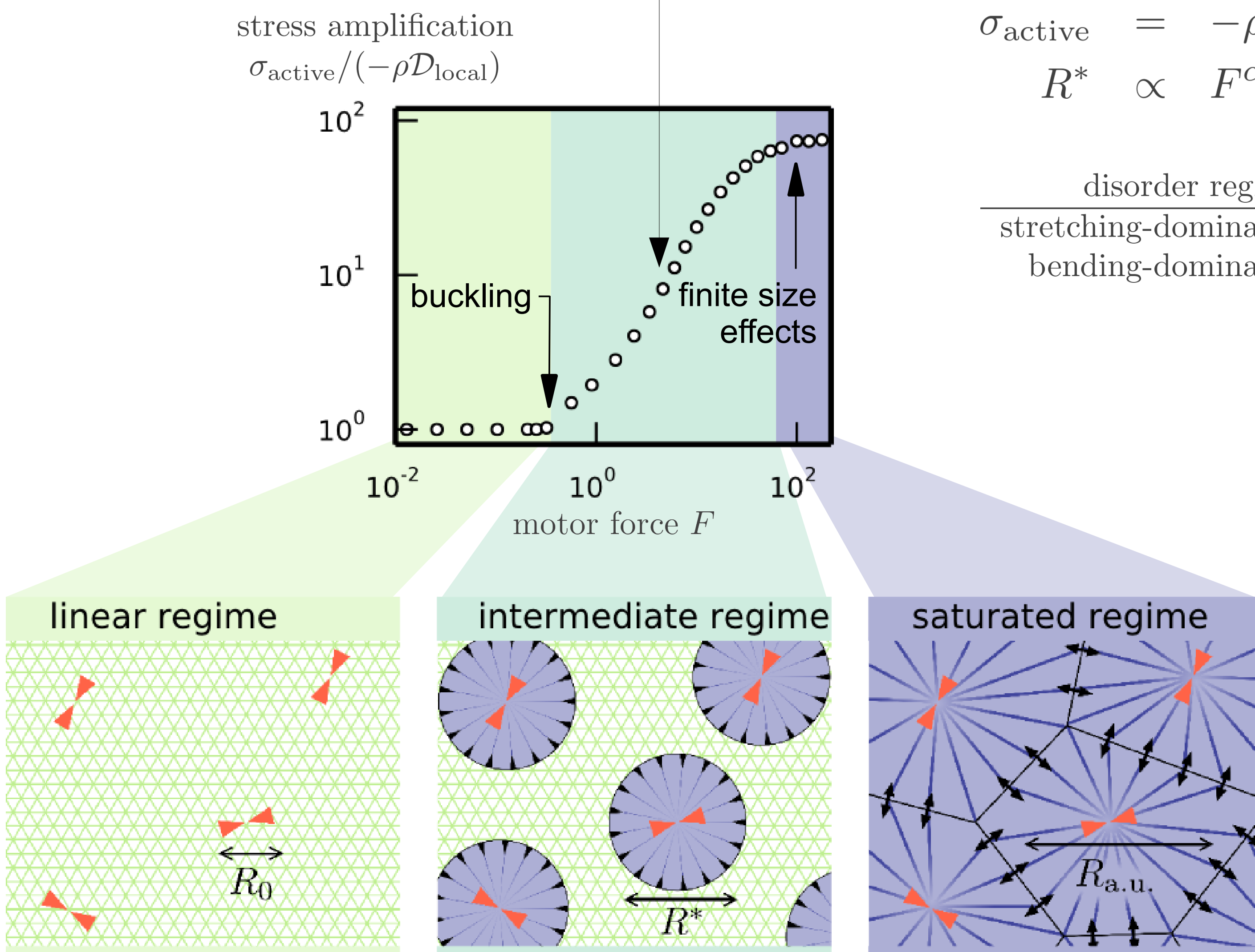


legend:
compression
extension

Stress amplification is governed by an emergent buckling length scale



Large-scale buckling generates large stress amplification



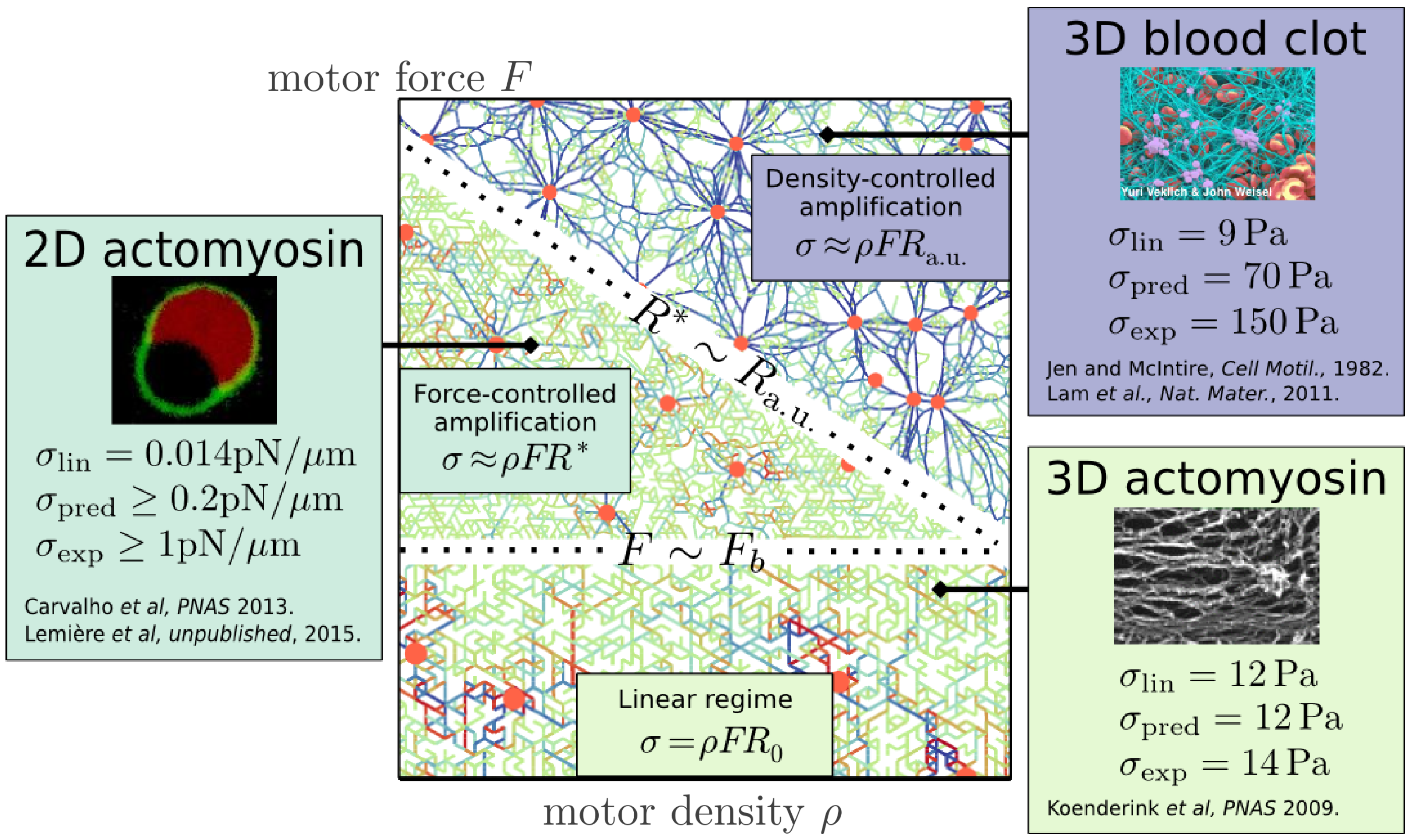
buckling zone controlled

$$\sigma_{\mathrm{active}} = -\rho\mathcal{D}_{\mathrm{eff}} \approx \rho F R^*$$

$$R^* \propto F^{\alpha}$$

disorder regime	2D	3D
stretching-dominated	$\alpha = 1$	$\alpha = 1/2$
bending-dominated	$\alpha \simeq 0.4$	$\alpha \simeq 0.25$

Experiments support all predicted amplification regimes



Ronceray, Broedersz, ML *Proc. Natl. Acad. Sci. U.S.A.* 2016