Partition Functions of N=4 Supersymmetric Yang-Mills

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I will discuss partition functions of topologically twisted N=4 supersymmetric Yang-Mills theories supported on a 4-manifold. These partition functions are mathematically generating functions of Euler numbers of moduli spaces of semi-stable sheaves. If the 4-manifold is a rational or ruled surface, the partition functions can be determined explicitly for arbitrary rank, and expressed in terms of indefinite theta functions. The latter functions are convergent sums over subsets of indefinite lattices, which do not exhibit the modular transformation properties familiar from theta functions associated to definite lattices. I will explain how a correspondence between the partition functions and D-instanton corrections to hypermultiplet moduli spaces in string theory, provides complementary terms to add to the partition function such that it transforms as a modular form. This is a confirmation of the physical S-duality.