6d strings and exceptional instantons

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Geometric correspondence of gauge theories, ICTP
Sep 15, 2016

Talk based on:

Hee-Cheol Kim, SK, Jaemo Park,
"6d strings from new chiral gauge theories"
1608.03919.

Hee-Cheol Kim, Joonho Kim, SK, Jaemo Park,
work in progress.

See also the following recent papers, which partly overlap with ours:

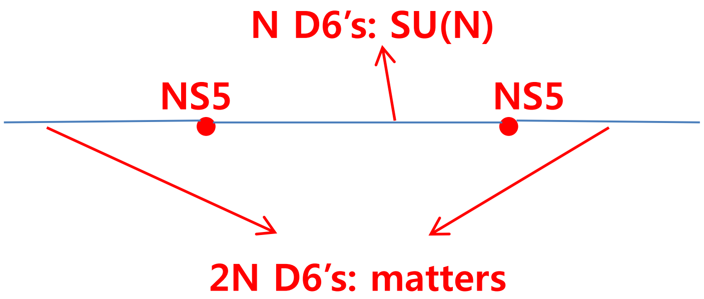
Shimizu, Tachikawa,
"Anomaly of strings of 6d N=(1,0) theories"
1608.05894.

Del Zotto, Lockhart,
"On exceptional instanton strings"
1609.00310.

6d SCFTs

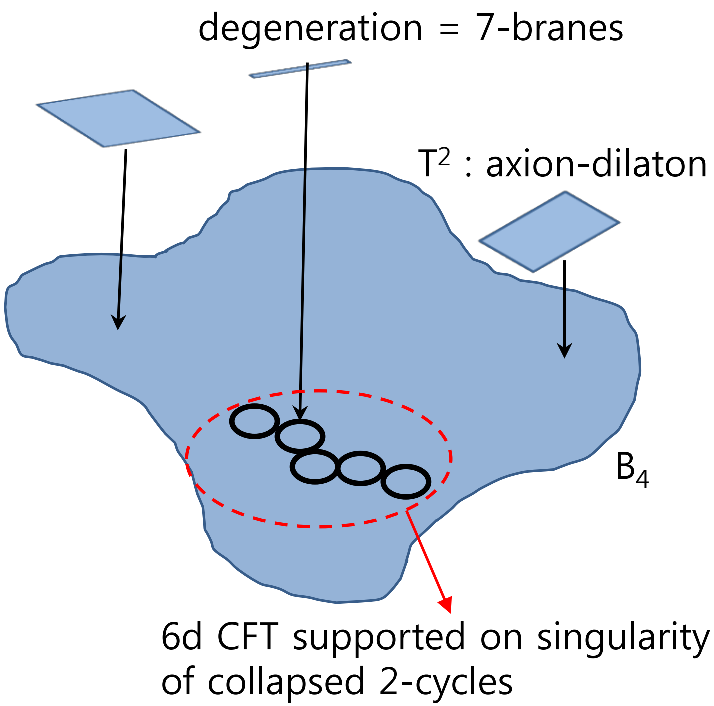
- From branes (e.g. IIA): NS5 (tensor), D6 (vectors), + D6, D8 (hypers). For instance,

  [Hanany, Zaffaroni] [Brunner, Karch] (1997)



- Type IIB on ADE singularities: N=(2,0) SCFTs [Witten] (1995)
- N=(1,0) SCFTs: F-theory on $R^{5,1}$ x (elliptic CY_3)

  [Morrison, Vafa] [Witten] (1996)



- A "classification" by going to tensor branch

$$B_{\mu\nu} \text{ with } H = dB = \star dB \ , \quad \Psi^A \ , \quad \boxed{\Phi}$$ → VEV

[Morrison, Taylor] (2012) [Heckman, Morrison, Vafa] (2013)

[Heckman, Morrison, Rudelius, Vafa] (2015)

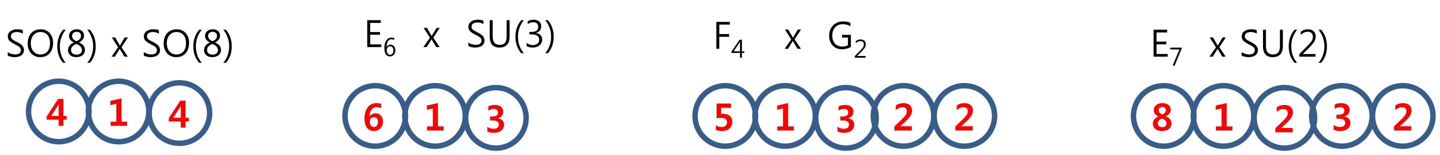
The “atoms”

- Building blocks of 6d SCFTs [Morrison,Vafa] [Witten] (1996) [Morrison,Taylor] (2012)
- SCFTs with lower dimensional tensor branch:

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

base	3, 2	3, 2, 2	2, 3, 2
gauge symmetry	$G_2 \times SU(2)$	$G_2 \times Sp(1) \times \{0\}$	$SU(2) \times SO(7) \times SU(2)$
matters	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1})+\frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2})$

- To construct more complicated 6d SCFTs, [Heckman,Morrison,Vafa] [H,M, Rudelius,V]
- make quivers of these atoms: glue two CFTs using n=1 SCFT, gauging subgroups of E_8.



- “unHiggs” to bigger gauge groups w/ more hypermultiplet matters

Self-dual strings

- Tensor branch: "self-dual strings" charged under 2-form
- D3-branes wrapping 2-cycles
- Like W-bosons/monopoles/dyons in 4d gauge theories in Coulomb branch

- Half-BPS in 6d (1,0) theories: 2d N=(0,4) CFTs on the strings

- To each 6d "atoms" correspond a family of 2d "atoms" for N=(0,4) SCFTs

- Effective description in tensor branch (when there is a gauge symmetry):
- 6d SYM + hypermultiplet matters, coupled to Abelian tensor multiplets

$$S^{\rm bos}_{\rm v+t} = \int \left[\frac{1}{2} d\Phi \wedge \star d\Phi + \frac{1}{2} H \wedge \star H\right] + \sqrt{c}\int \left[-\Phi {\rm tr}(F \wedge \star F) + B \wedge {\rm tr}(F \wedge F)\right]$$

$$H \equiv dB + \sqrt{c}\,{\rm tr}\left(AdA - \frac{2i}{3}A^3\right)$$

- Self-dual strings = instanton string solitons in 6d SYM

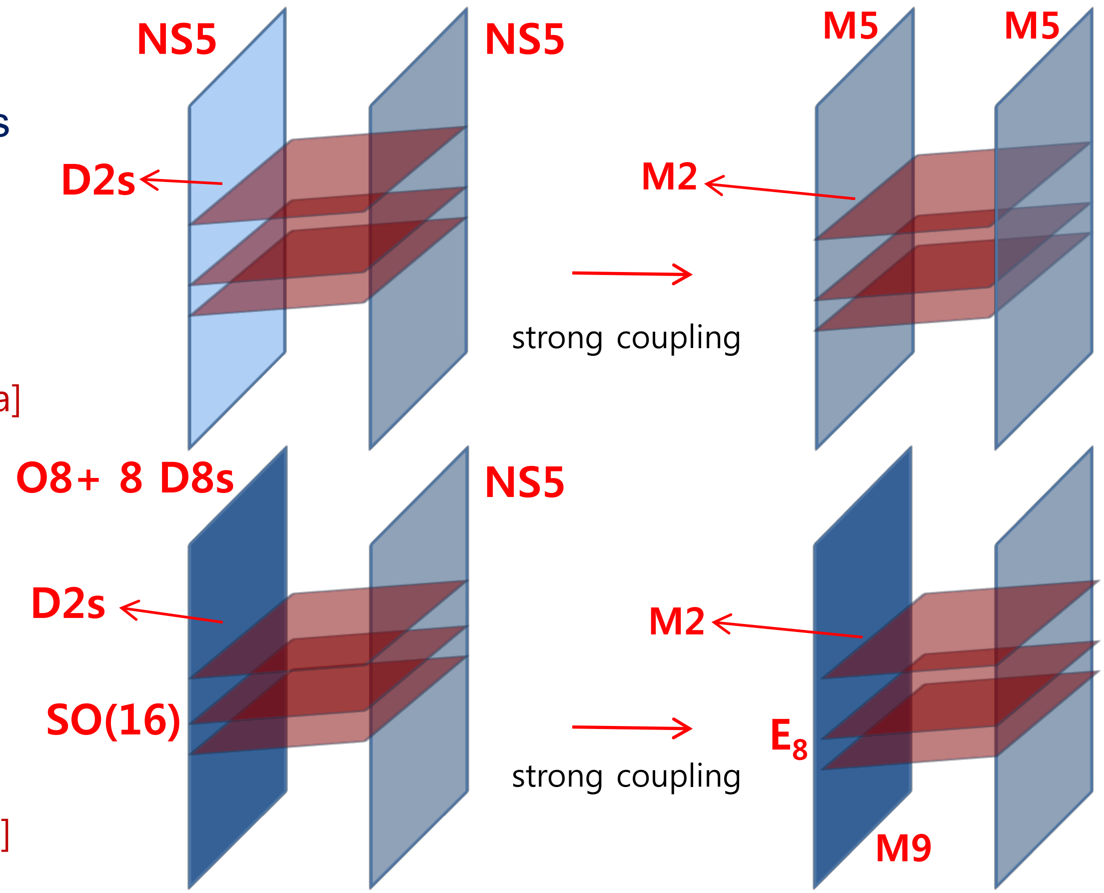
Self-dual strings & gauge theories

- Some 2d CFTs come from UV gauge theories. (GLSM) [Witten] (1993)
- "top-down" construction:

- D-brane construction of 6d strings
- Open strings are light 2d d.o.f.

n = 2: "M-strings":
[Haghighat, Iqbal, Kozcaz, Lockhart, Vafa]

n = 1: "E-strings":
[J. Kim, SK, K. Lee, Park, Vafa]



n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

- (p,q) 7-branes in generic F-theory setting (not just D7s): not just fundamental strings

Bottom-up: soliton strings in 6d SYM

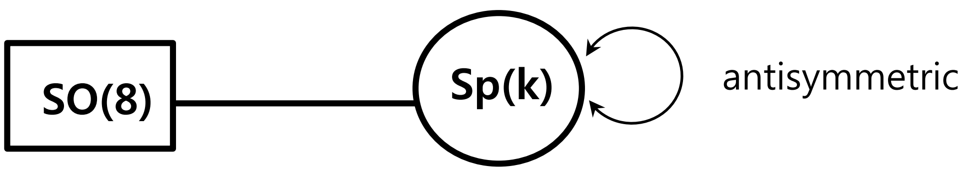
- For other SCFTs, we have Yang-Mills intuitions: self-dual strings = instanton strings

$$F_{\mu\nu} = \star_4 F_{\mu\nu} \qquad k \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\,(F\wedge F) \in \mathbb{Z} \qquad S \leftarrow \int B \wedge \mathrm{tr}(F\wedge F)$$

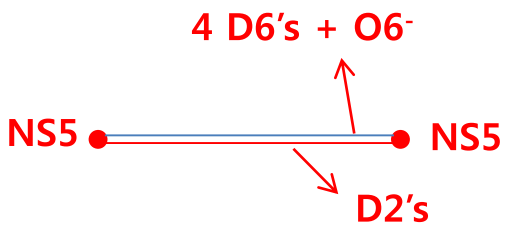
- Classical gauge group (ABCD): ADHM construction suggests 2d gauge theories.
- Most gauge groups are exceptional.

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

- Apparently simple cases: n=3,4 w/ SU(3), SO(8)
- n=4: [Haghighat, Klemm, Lockhart, Vafa]
- SO(8) ADHM construction
- Good QFT: e.g. Sp(k) anomaly-free

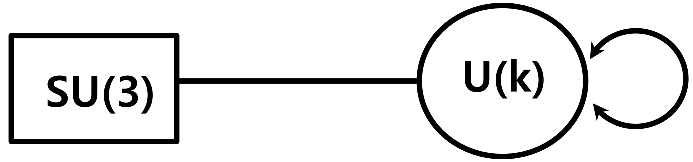


- Also constructed from top-down: D-brane realization

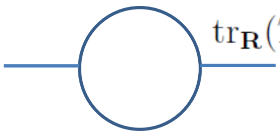


Strings of SU(3) SCFT

- Naively, one may also try a guess w/ SU(3) ADHM ("bottom-up" approach)
- SU(3) ADHM for k instantons:



- U(k) anomaly doesn't cancel:



$\mathrm{tr}_{\mathbf{R}}(T^a T^b) = D_{\mathbf{R}} \delta^{ab}$

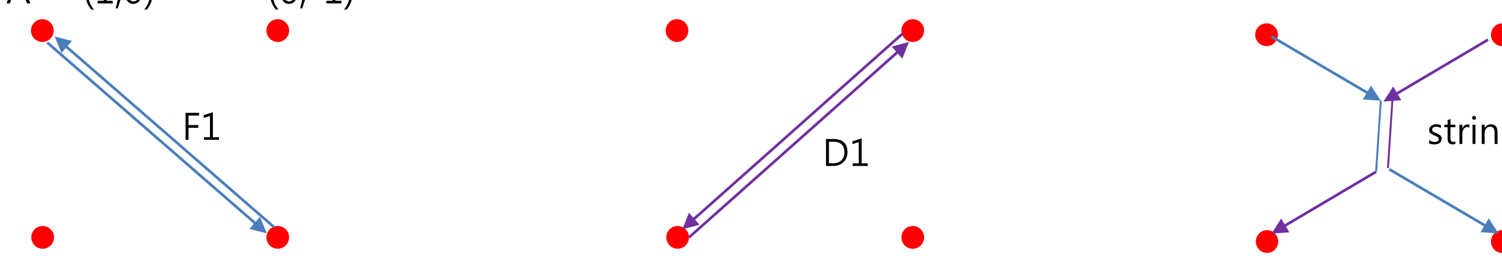
$D_{\mathbf{k}} = 1$

$D_{\mathbf{adj}} = 2k$

$\sim\ 2 \cdot 3 \cdot 1 + 2 \cdot 2k - 2 \cdot 2k = 6 \neq 0$

fields	$U(k)$	$SU(3)$	$SU(2)_F$	$SU(2)_1$	$SU(2)_2$
$\lambda_{\dot\alpha A-}$	**adj**	**1**	**1**	**2**	**2**
$q_{\dot\alpha}(\to \psi_{A+})$	**k**	$\bar{\mathbf{3}}$	**1**	**2**	**1**
$a_{\alpha\dot\beta}(\to \chi_{\alpha A+})$	**adj**	**1**	**2**	**2**	**1**

- This failure is natural. (Here, SU(3) doesn't come from open strings ending on 3 D-branes.)
- SU(3) is "nonperturbative" or "exceptional" [Grassi, Halverson, Shaneson]



Exceptional instanton strings

- So including SU(3), the followings are realized "non-perturbatively."

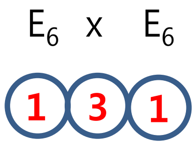
n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

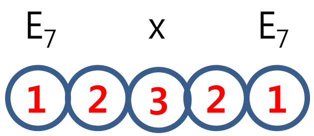
- SU(3) SCFT is a building block of all the "exotic atoms"

base	$3,2$	$3,2,2$	$2,3,2$
gauge symmetry	$G_2 \times SU(2)$	$G_2 \times Sp(1) \times \{0\}$	$SU(2) \times SO(7) \times SU(2)$
matters	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1})+\frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2})$

- Related to G_2 instantons, & instantons in "SO(7) + spinors" by Higgsings.

- Builds new SCFTs, "conformal matters" (later)





- Strategy for the SU(3) strings:
- Employ bottom-up approach.
- Cure the pathology of naïve SU(3) quiver.

The cure for SU(3)

- **Result:** can't make N=(0,4) gauge theory. Can have one by sacrificing some SUSY in UV.

- Add the following N=(0,2) superfields to the anomalous SU(3) ADHM :

fields	$U(k)$	$SU(3)$	$SU(2)_F$	$SU(2)_1$	$SU(2)_2$
$\lambda_{\dot\alpha A-}$	**adj**	**1**	**1**	**2**	**2**
$q_{\dot\alpha}(\rightarrow \psi_{A+})$	**k**	$\bar{\mathbf{3}}$	**1**	**2**	**1**
$a_{\alpha\dot\beta}(\rightarrow \chi_{\alpha A+})$	**adj**	**1**	**2**	**2**	**1**

$$
\begin{aligned}
(\phi,\chi) &: \text{ chiral multiplet in } (\bar{\mathbf{k}},\bar{\mathbf{3}}) \\
(b,\xi)+(\tilde b,\tilde\xi) &: \text{ two chiral multiplet in } (\overline{\mathbf{anti}},\mathbf{1}) \\
(\hat\lambda,\hat G) &: \text{ complex Fermi multiplet in } (\mathbf{sym},\mathbf{1}) \\
(\check\lambda,\check G) &: \text{ complex Fermi multiplet in } (\mathbf{sym},\mathbf{1}) \\
(\zeta,G_\zeta) &: \text{ complex Fermi multiplet in } (\bar{\mathbf{k}},\mathbf{1})\ . \\
(\tilde\phi,\tilde\chi) &: \text{ chiral multiplet in } (\mathbf{k},\mathbf{1}) \\
(\eta,G_\eta) &: \text{ complex Fermi multiplet in } (\bar{\mathbf{k}},\mathbf{1})
\end{aligned}
$$

- anomaly: SU(k) from ADHM ~ $2\cdot 3\cdot 1 + 2\cdot 2k - 2\cdot 2k = 6 \neq 0$

  from others ~ $+3\cdot 1 + 2(k-2) - (k+2) - (k+2) - 1 = -6$

  $D_{\rm sym} = k+2$

  $D_{\rm anti} = k-2$

  U(1) $+3\cdot 2\cdot 1^2\cdot k + 3\cdot 1^2 \cdot k + 2\cdot 2^2\cdot \frac{k^2-k}{2} - 2^2\cdot\frac{k^2+k}{2} - 2^2\cdot\frac{k^2+k}{2} - 1^2\cdot k = 0$

- Can turn on superpotentials to get the correct SU(3) instanton moduli space: but preserving only N=(0,1) SUSY [H.-C.Kim, SK, J. Park]

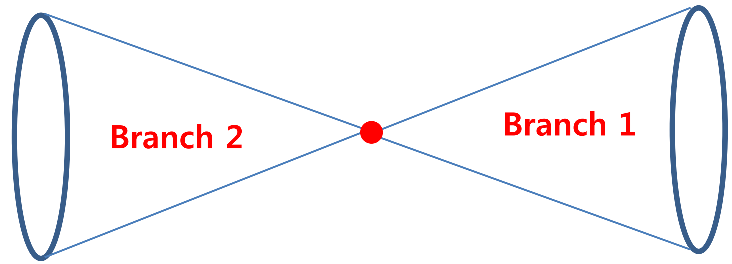
The moduli space & UV completion

- Classical moduli space: vanishing $V(\phi_{\text{ADHM}}, \phi_{\text{others}}) = V_1(\phi_{ADHM}) + V_2(\phi_{\text{others}}, \phi_{\text{ADHM}})$

1) branch 1: extra fields = 0. ADHM fields satisfy

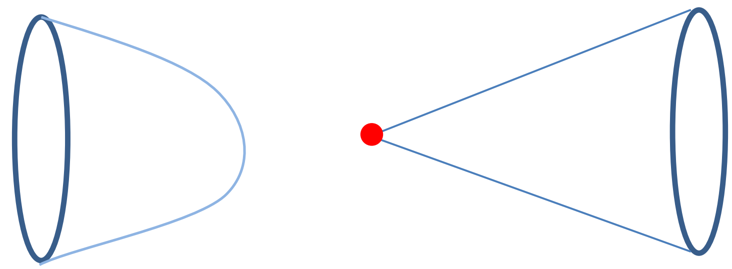
$$D^I \equiv q_{\dot{\alpha}}(\tau^I)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{q}^{\dot{\beta}} + (\tau^I)^{\dot{\alpha}}{}_{\dot{\beta}}[a_{\alpha\dot{\alpha}}, a^{\alpha\dot{\beta}}] = 0$$

- SU(3) instanton moduli space: hyper-Kahler quotient, N=(0,4) SUSY enhancement
- 1-loop correction doesn't spoil this zero potential condition



2) branch 2: We find another branch. (k=1)

- Classical: meets 1st branch at small instanton singularity



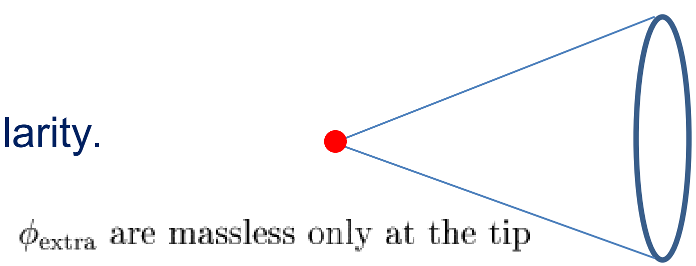
- Quantum: 1-loop correction only at 2nd branch

  conjecture: detached from the 1st branch (IR decoupling)

  [Melnikov, Quigley, Sethi, Stern] (2012)

- Non-linear sigma model in IR: small instanton singularity.

  Extra light d.o.f. at small instanton singularity. UV completion.



Other observables

- elliptic genus:

$$H_\pm \equiv \frac{H \pm P}{2} \qquad H_- \sim \{Q, \overline{Q}\}$$

$$Z_k(\tau, \epsilon_{1,2}, m_a) = \mathrm{Tr}\left[(-1)^F e^{2\pi i \tau H_+} e^{2\pi i \bar{\tau} H_-} e^{2\pi i \epsilon_1 (J_1 + J_R)} e^{2\pi i \epsilon_2 (J_2 + J_R)} \cdot \prod_{a \in \text{flavor}} e^{2\pi i m_a F_a}\right]$$

- Easy to compute w/ a UV gauge theory: contour integral

  [Benini,Eager,Hori,Tachikawa] (2013)

- Our U(k) gauge theory: [Flume, Poghossian] [Bruzzo, Fucito, Morales, Tanzini] (2002)

$$Z_k^{SU(3)} = (-1)^{\frac{k^2-k}{2}} \eta^{6k} \sum_{\vec{Y};|\vec{Y}|=k} \prod_{i=1}^{3} \prod_{s \in Y_i} \frac{\theta_1(2u(s))\theta_1(2\epsilon_+ - 2u(s))\theta_1(\epsilon_+ + u(s))}{\prod_{j=1}^{3} \theta_1(E_{ij})\theta_1(E_{ij} - 2\epsilon_+)\theta_1(\epsilon_+ - u(s) - v_j)}$$

$$\times \prod_{i \leq j}^{3} \prod_{s_{i,j} \in Y_{i,j}; s_i < s_j} \frac{\theta_1(u(s_i) + u(s_j))\theta_1(2\epsilon_+ - u(s_i) - u(s_j))}{\theta_1(\epsilon_{1,2} - u(s_i) - u(s_j))}$$

$$E_{ij} = v_i - v_j - \epsilon_1 h_i(s) + \epsilon_2(v_j(s) + 1)$$

$$u(s) = v_i - \epsilon_+ - (m-1)\epsilon_1 - (n-1)\epsilon_2$$

- 1d limit, replacing all θ_1 functions to sine functions, agrees with Nekrasov's SU(3) instanton partition function: We found an alternative "ADHM-like" formalism
- Novel results in 2d: For simplicity, let us consider single string k=1

$$Z_1^{SU(3)}(v, \epsilon_{1,2}) = -\frac{\eta^2}{\theta_1(\epsilon_{1,2})} \sum_{i=1}^{3} \frac{\eta^4 \theta_1(4\epsilon_+ - 2v_i)\theta_1(v_i)}{\prod_{j(\neq i)} \theta_1(v_{ij})\theta_1(2\epsilon_+ - v_{ij})\theta_1(2\epsilon_+ + v_j)}$$

Tests

- k=1 (tests also done at k=2,3): computation from topological strings [Haghighat, Klemm, Lockart, Vafa]

$$\log Z(\tau, \epsilon_+, \epsilon_+, \mu) = \sum_{g\geq 0, n\geq 0} (\epsilon_1\epsilon_2)^{g-1}(\epsilon_1+\epsilon_2)^n F_{g,n}(\tau,\mu)$$

$$F_{0,0} = -\left[\frac{\theta_1(2v_1)\theta_1(v_1)}{\theta_1(v_{12})^2\theta_1(v_{13})^2\theta_1(v_2)\theta_1(v_3)} + (1,2,3\to 2,3,1) + (1,2,3\to 3,1,2)\right]$$
$$= e^{-\pi i\tau + 2\pi i v_{12} + 2\pi i v_{23}} \sum_{d_0,d_1,d_2=0}^{\infty} N_{d_0,d_1,d_2}\left(\frac{e^{2\pi i\tau}}{e^{2\pi i v_{12}}e^{2\pi i v_{23}}}\right)^{d_0} e^{2\pi d_1 v_{12}} e^{2\pi d_2 v_{23}}$$

$d_1 \backslash d_2$	0	1	2	3	4	5
0	1	3	5	7	9	11
1	3	4	8	12	16	20
2	5	8	9	15	21	27
3	7	12	15	16	24	32
4	9	16	21	24	25	35
5	11	20	27	32	35	36

Table 1: q^0

$d_1 \backslash d_2$	0	1	2	3	4	5
0	3	4	8	12	16	20
1	4	16	36	60	84	108
2	8	36	56	96	144	192
3	12	60	96	120	180	252
4	16	84	144	180	208	288
5	20	108	192	252	288	320

Table 2: q^1

$d_1 \backslash d_2$	0	1	2	3	4	5
0	5	8	9	15	21	27
1	8	36	56	96	144	192
2	9	56	149	288	465	651
3	15	96	288	456	735	1080
4	21	144	465	735	954	1371
5	27	192	651	1080	1371	**1632**

Table 3: q^2

$d_1 \backslash d_2$	0	1	2	3	4	5
0	7	12	15	16	24	32
1	12	60	96	120	180	252
2	15	96	288	456	735	1080
3	16	120	456	1012	1788	**2796**
4	24	180	735	1788	**2823**	**4356**
5	32	252	1080	**2796**	**4356**	**5760**

Table 4: q^3

black numbers: computed from top. strings

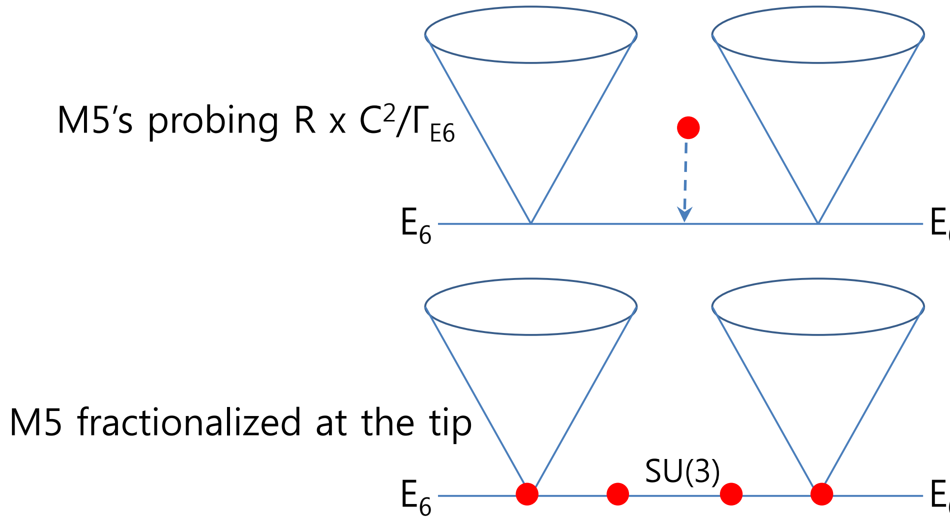
red: our prediction

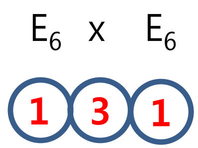
complete agreement

E_6 x E_6 conformal matter

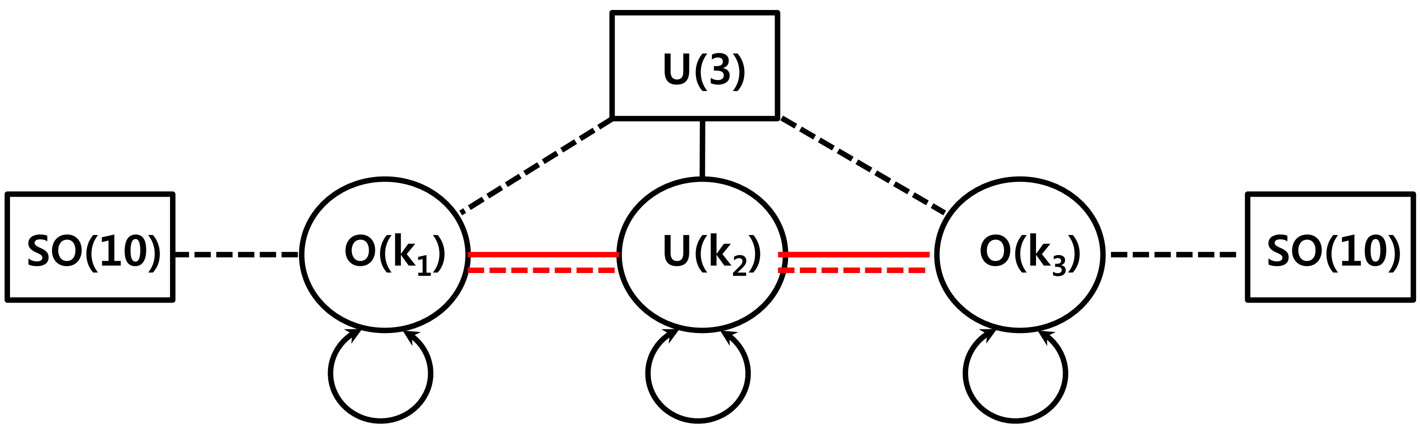
- 2d quivers for strings w/ higher dim'l 6d tensor branches
- E_6 x E_6 conformal matter:

  [Del Zotto, Heckman, Tomasiello, Vafa]





- 3 types of string charges: connect 3 theories w/ bifundamental matters



\- SO(10) x U(1) x SO(10) enhances to E_6 x E_6: partly checked from elliptic genus

More tests: anomaly inflows from 6d

- 2d anomalies of global symmetries
- Computable from 6d gauge anomaly cancelation w/ 2d defects (anomaly inflow)
- Results from the inflow mechanism [H.-C. Kim, SK, J. Park] (see also [Shimizu, Tachikawa])

$$I_4^{\rm inflow} = I_4^{(1)} + I_4^{(2)} = \Omega^{ij} k_i \left[I_j + \frac{1}{2} k_j \chi(T_4) \right] \qquad I_4^{2d} = -I_4^{\rm inflow}$$

- Agree with the anomalies calculated from our 2d gauge theories

- k instanton strings for G=SU(3): both calculations yield

$$I_4^{2d} = -\frac{3k}{4}{\rm Tr}F_G^2 - 3kc_2(R) - \frac{k}{4}p_1(T_6) - \frac{3k^2}{2}\chi_4(T_4)$$

- E_6 x E_6 conformal matter: both calculations yield

$$\begin{aligned} I_4^{2d} = \; & k_1(k_2+k_3)\chi_4(T_4) - k_1\left(\frac{3}{4}{\rm Tr}F_{SU(3)}^2 + 3c_2(R) + \frac{p_1(T_6)}{4} + \frac{3}{2}k_1\chi_4(T_4)\right) \\ & + k_2\left(\frac{1}{4}{\rm Tr}F_{E_6^L}^2 + \frac{1}{4}{\rm Tr}F_{SU(3)}^2 - c_2(R) + \frac{p_1(T_6)}{4} - \frac{k_2}{2}\chi_4(T_4)\right) \\ & + k_3\left(\frac{1}{4}{\rm Tr}F_{E_6^R}^2 + \frac{1}{4}{\rm Tr}F_{SU(3)}^2 - c_2(R) + \frac{p_1(T_6)}{4} - \frac{k_2}{2}\chi_4(T_4)\right) \; . \end{aligned}$$

UnHiggsing to exceptional instantons

- 6d Higgsings are reflected in 2d QFT as massive deformations

- Allowed unHiggsing sequences: all exceptional ~ "terminates after finite sequence"

$$(SU(3)) \leftarrow (G_2, n_{\mathbf{7}}=1) \leftarrow (SO(7), n_{\mathbf{7}}=0, n_{\mathbf{8}}=2) \leftarrow (SO(8), n_{\mathbf{8}_v}=n_{\mathbf{8}_s}=n_{\mathbf{8}_c}=1)$$

$$\leftarrow \begin{cases} (SO(N), n_{\mathbf{N}}=N-7, n_{\mathbf{S}}=\cdots)_{N=9,\cdots,12} \\ (F_4, n_{\mathbf{26}}=2) \leftarrow (E_6, n_{\mathbf{27}}=3) \leftarrow (E_7, n_{\frac{1}{2}\mathbf{56}}=5) \leftarrow (E_8, n_{\text{inst}}=9) \end{cases}$$

- G_2 & SO(7) instantons w/ 6d matters [Hee-Cheol Kim, Joonho Kim, SK, Jaemo Park]

- An (anomaly-free) quiver for SO(7) instantons: only SU(4) ⊂ SO(7) is manifest in UV

Standard SU(4) ADHM

$$A_\mu, \lambda_0, \lambda \;:\; \mathcal{N}=(0,4)\ U(k) \text{ vector multiplet}$$
$$q_i, \tilde{q}^i \;:\; (\mathbf{k}, \bar{\mathbf{4}}) + (\bar{\mathbf{k}}, \mathbf{4})$$
$$a, \tilde{a} \;:\; (\mathbf{adj}, \mathbf{1})$$

Extra chiral multiplet to make it a novel "SO(7) ADHM"

$$\phi_i \;:\; (\bar{\mathbf{k}}, \bar{\mathbf{4}})$$
$$b, \tilde{b} \;:\; (\overline{\mathbf{anti}}, \mathbf{1})$$
$$\hat{\lambda} \;:\; (\mathbf{sym}, \mathbf{1})$$
$$\check{\lambda} \;:\; (\mathbf{sym}, \mathbf{1})$$

Extra 2d field induced by 6d hypers in 8

$$\Psi_i \;:\; (\mathbf{k}, \mathbf{1})$$
$$\tilde{\Psi}_i \;:\; (\bar{\mathbf{k}}, \mathbf{1}) \quad (i=1,2)$$

$$\mathbf{8} \to \mathbf{4} + \bar{\mathbf{4}}$$

- Can Higgs SO(7) to G_2 with one 7. Further Higgsing to our alternative SU(3) ADHM.

Applications

[H.-C. Kim, J. Kim, SK, J. Park]

- **Application 1**: Reduce to 1d. ADHM-like QM for exceptional instantons

- G_2 instantons, & SO(7) w/ matters in spinor rep.
- 1d Witten indices: e.g. one G_2 instanton

$$\oint \frac{d\phi}{2\pi i} \frac{2\sinh\epsilon_+ \cdot 2\sinh\phi \cdot 2\sinh(\epsilon_+ - \phi) \cdot 2\sinh\frac{\epsilon_+ + \phi}{2}}{2\sinh\frac{\epsilon_+ \pm (u - v_{1,2,3})}{2} \cdot 2\sinh\frac{\epsilon_+ - \phi - v_{1,2,3}}{2}} \cdot \frac{1}{2\sinh\frac{\epsilon_+ \pm \phi}{2}}$$

$$\sum_{i=1}^{3} \frac{2\sinh(2\epsilon_+ - v_i) \cdot 2\sinh\frac{v_i}{2}}{\prod_{j(\neq i)} 2\sinh\frac{v_{ij}}{2} \cdot 2\sinh\frac{2\epsilon_+ - v_{ij}}{2} \cdot 2\sinh\frac{2\epsilon_+ + v_j}{2}} \cdot \frac{1}{2\sinh\frac{v_i}{2} \cdot 2\sinh\frac{2\epsilon_+ - v_i}{2}}$$

One (k=1) G_2 instanton partition function from 1d gauge theory

$$= \frac{t^{\frac{3}{2}}(1+t)(1+t\chi_{\mathbf{7}}^{G_2}(v)+t^2)}{\prod_{i<j}(1-te^{-v_{ij}})(1-te^{v_{ij}})} = t^{\frac{3}{2}} \sum_{n=0}^{\infty} \chi_{(0,n)}^{G_2}(v) t^n$$

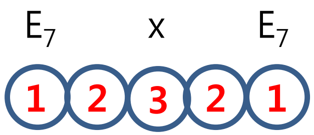
[Cremonesi, Ferlito, Hanany, Mekareeya] (2014)

- A strength of our approach: can add the effects of hypermultiplet matters in 7

- **Application 2**: 6d self-dual strings of "exotic atoms"

base	3, 2	3, 2, 2	2, 3, 2
gauge symmetry	$G_2 \times SU(2)$	$G_2 \times Sp(1) \times \{0\}$	$SU(2) \times SO(7) \times SU(2)$
matters	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1}) + \frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2})$

- They all contain the base '3' : either G_2 with one 7 or SO(7) with two 8's
- With these atoms, one can study the strings of E_7 x E_7 conformal matter



Concluding remarks

- 6d CFTs are hard. Even the 2d QFTs on solitons are hard for many 6d theories.

- We are getting solid clues on 2d gauge theories on self-dual strings:
- related to exceptional instantons' ADHM-like descriptions
- Using tensor branch observables for CFT physics at symmetric phase? (e.g. $S^5 \times S^1$ index)

- Extension to other exceptional instantons…? (reduced UV symmetry, …)
- For an exceptional group G_r of rank r, we are seeking for ADHM-like UV gauge theories, which exhibit only SU(r+1) subgroup as its UV symmetry.
- $SU(3) \subset G_2$ (already discovered), $SU(8) \subset E_7$, $SU(9) \subset E_8$ (trying similar constructions).
- Cancelation of 2d gauge anomalies, 2d global anomalies, Witten index/elliptic genus…

- Our 2d CFTs = 4d Argyres-Douglas theories on S^2: see also [Del Zotto, Lockhart] (2016)
- More insight on the self-dual strings from AD theories? [Maruyoshi, Song] (2016)