Problems on "Combinatorial deformation theory: from combinatorics to models and back" lectured by Jeffery Brock.

Throughout we use the following notations:

- $S = S_{g,n}$ denotes a compact surface with g genus and n boundary components.
- $C(S)$ denotes the curve graph and $d_{C(S)}$ denotes the graph metric on $C(S)$. If $x, y \in C(S)$ then $i(x,y)$ denotes the minimum geometric intersection number between x and y up to isotopy.
- $P(S)$ denotes the pants graph and $d_{P(S)}$ denotes the graph metric on $P(S)$.
- $T(S)$ denotes Teichumüller space and $T(S)_{wp}$ denotes Teichumüller space equipped with Weil–Petersson metric $d_{T(S)_{wp}}$.

$< May/23/2016 >$

1. Let a, b, c be positive real numbers. Construct $S_{0,3}$ such that the length of its three boundary components are a, b, and c.
2. Show $C(S)$ is connected.
3. Let $x, y \in C(S)$. Show $d_{C(S)}(x,y) \leq 2 \cdot i(x,y) + 1$. Furthermore, show $d_{C(S)}(x,y) \leq 2 \cdot \log_2 i(x,y) + 2$.
4. Show $C(S)$ has infinite diameter.

$< May/24/2016 >$

1. Discuss questions from $< May/23/2016 >$.
2. Prove that there exists a constant L depending only on S so that for each X in $T(S)$ there exists a pants decomposition $P \in P(S)$ so that $\max_{a \in P} l_X(a) < L$ where $l_X(a)$ is the distance of a measured by the hyperbolic metric on X.
3. Let $P \in P(S)$ and $V_k(P) = \{X \in T(S) | \max_{a \in P} l_X(a) < k\}$. Let L be the constant given by the above. Show the diameter of $V_L(P)$ is uniformly bounded in $T(S)_{wp}$ for all $P \in P(S)$.
4. Show $P(S_{1,1})$ is quasi-isometric to $T(S_{1,1})_{wp}$. Hint. Use the above two results.

$< May/26/2016 >$

1. Discuss questions from $< May/23/2016 >$ and $< May/24/2016 >$.
2. Let M_ϕ be the mapping torus with monodromy ϕ. Show that if M_ϕ is hyperbolic then ϕ is pseudo–Anosov.
3. Let A be a multicurve in S and $P_A(S)$ be the subgraph of $P(S)$ where the vertices are pants decompositions which contain A. Let $S = S_{2,0}$ and A be a multicurve such that $|A| = 2$. Is $P_A(S)$ geodesically convex in $P(S)$? What about when $|A| = 1$?
4. Let $S = S_{1,1}$ and ϕ be pseudo–Anosov. By using Brock–Bromberg inequality
$$Vol(M_\phi) \leq 3\sqrt{\frac{\pi}{2}(2g - 2 + n)}||\phi||_{wp}$$
where $||\phi||_{wp} = \inf_{X \in T(S)} d_{T(S)_{wp}}(x, \phi(x))$, obtain a lower bound on Weil-Petersson length of the imaginary axis in terms of the volume of the ideal hyperbolic octahedron.