Overview of Indian-Ocean dynamics

Targeted Training Activity on:

Towards Improved Monsoon Simulations

International Centre for Theoretical Physics

Trieste, Italy

June 13–17, 2016

Dynamics of wind-driven circulations in the IO

Targeted Training Activity on:

Towards Improved Monsoon Simulations

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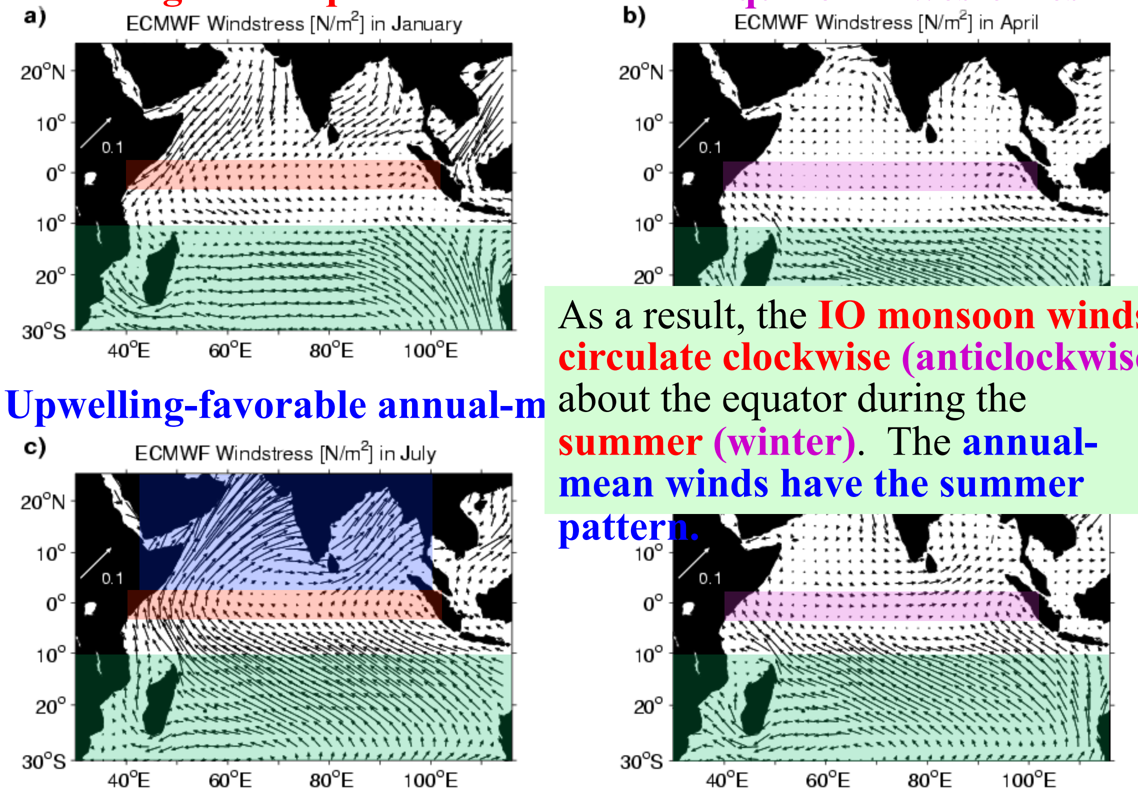
Climatological wind forcing

Reversing cross-equatorial winds

Equatorial westerlies



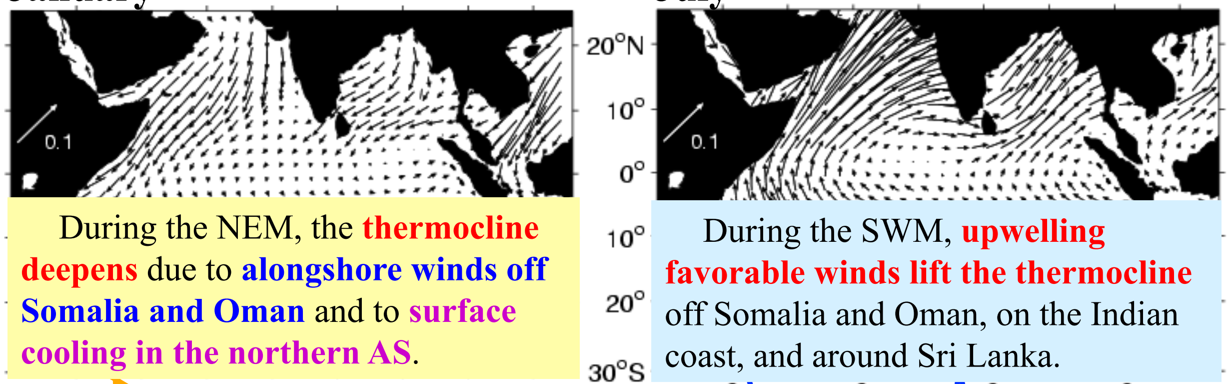
Upwelling-favorable annual-m

As a result, the **IO monsoon winds circulate clockwise (anticlockwise)** about the equator during the **summer (winter)**. The **annual-mean winds have the summer pattern.**

Relatively steady southeast tradewinds

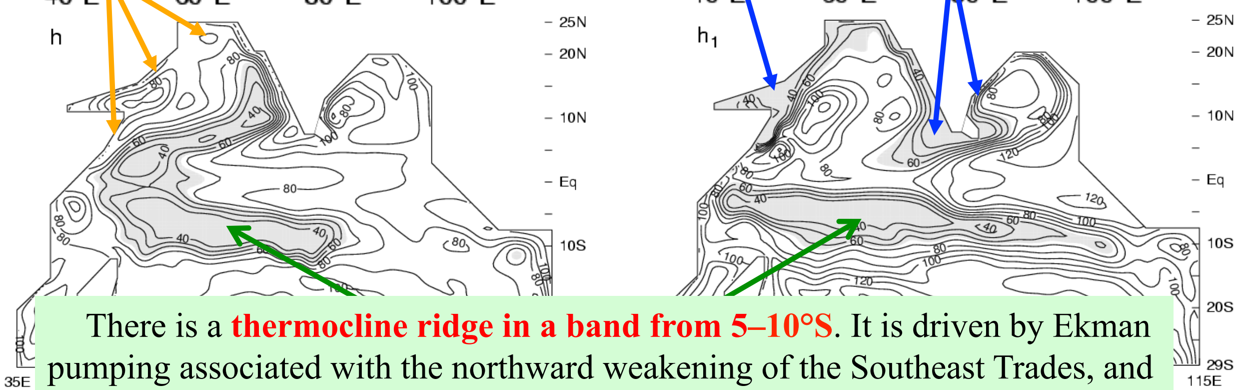
Wind-forced thermocline response

January **July**



During the NEM, the **thermocline deepens** due to **alongshore winds off Somalia and Oman** and to **surface cooling in the northern AS**.

During the SWM, **upwelling favorable winds lift the thermocline** off Somalia and Oman, on the Indian coast, and around Sri Lanka.

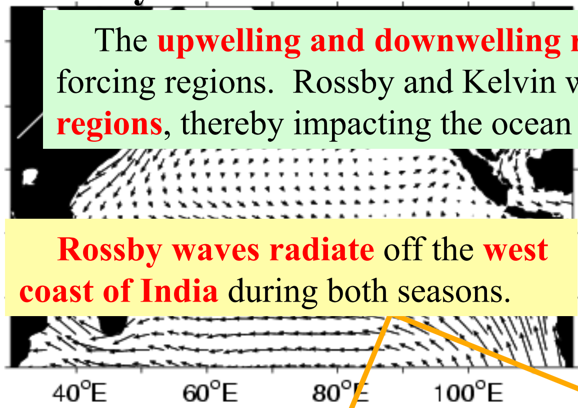


There is a **thermocline ridge in a band from 5–10°S**. It is driven by Ekman pumping associated with the northward weakening of the Southeast Trades, and is **stronger in northern summer** when the Trades are stronger.

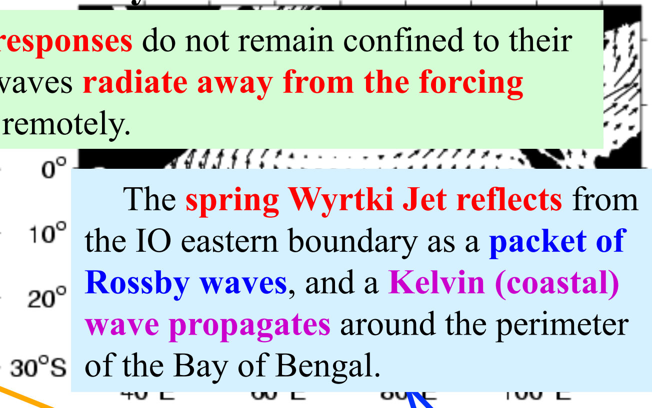
(1993)

Wave radiation

January



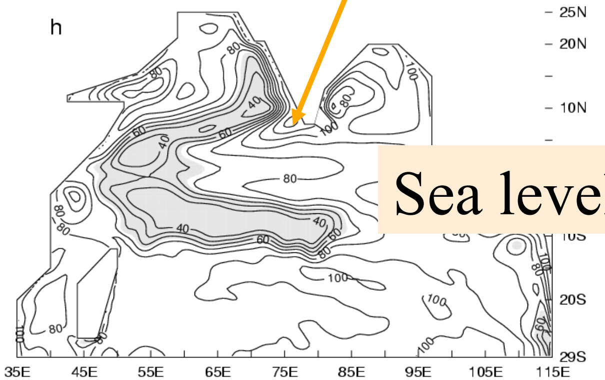
July

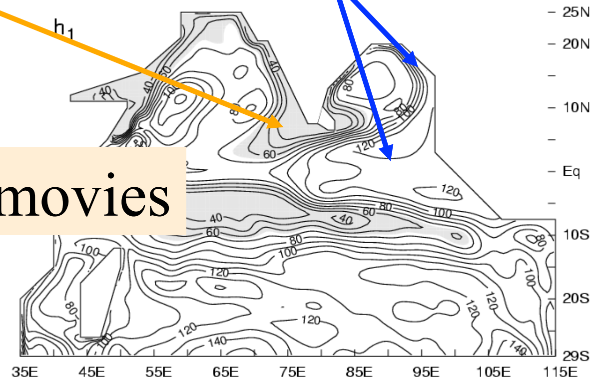


The **upwelling and downwelling responses** do not remain confined to their forcing regions. Rossby and Kelvin waves **radiate away from the forcing regions**, thereby impacting the ocean remotely.

Rossby waves radiate off the **west coast of India** during both seasons.

The **spring Wyrtki Jet reflects** from the IO eastern boundary as a **packet of Rossby waves**, and a **Kelvin (coastal) wave propagates** around the perimeter of the Bay of Bengal.





Sea level movies

McCreary, Kundu, and Molinari (1993)

Goal

To provide an **introduction** to the dynamics of **wind-driven circulations in the North Indian Ocean (NIO)**, both for its **mean state and climatic variability**.

Approach

Our approach is to **split the complete IO response into smaller pieces** in the **interior, coastal, and equatorial oceans**. We then **obtain simple solutions** that illustrate the **dominant processes** at work in each region. These simpler solutions **provide the "language"** needed to discuss more complex problems.

Questions

What types of ocean models are useful for studying NIO phenomena?

model hierarchy: OGCMs, LCS model, layer models

What types of ocean waves impact NIO phenomena?

gravity & Rossby waves; Kelvin waves; (shelf waves)

How does the wind drive ocean circulations?

Ekman flow; Ekman pumping; excitation of Rossby and Kelvin waves; adjustment to Sverdrup balance

How do wind-driven dynamics differ in the interior, equatorial, and coastal oceans?

They are very similar, differing primarily in the types of waves that are generated.

References: ITCP

This lecture, **TTAlecture.pptx**, and other files can be obtained from the ITCP website at:

http://indico.ictp.it/event/7666/

Today's talk **focuses on physical concepts**, rather than mathematical solutions. There are **additional slides** at the end of **TTAlecture.pptx** that **derive some of the solutions** discussed today, as well as other **supportive material**.

References: NIOSS (2010)

The content & organization of this lecture were inspired by a set of lectures I gave at a **summer school held at NIO in 2010 (NIOSS)**. An overview of NIOSS can be found at the web site:

http://www.nio.org/index/option/com_newsdisplay/task/view/tid/4/sid/23/nid/255

References: NIOSS (2010)

All of the movies that I show were prepared during NIOSS, and they are stored in the **Tutorials folders** at the website

http://www.nio.org/index/option/com_eventdisplay/task/view/tid/4/sid/114/eid/143

They can also be obtained from the ICTP web site. I recommend that you **download all of these movies** onto your own computer.

A **description of each movie** is given in **ExperimentList.docx**, located in the **Tutorials folders** at the NIOSS website, as well as at the ICTP website.

References: INCOIS winter school (2015)

This lecture is a 2-hour version of a set of lectures I gave during a **winter school held at INCOIS in 2016**. The complete set of lectures can be downloaded from the web site:

http://www.incois.gov.in/portal/ITCOocean/course_materials.jsp

Organization

1) Hierarchy of ocean models

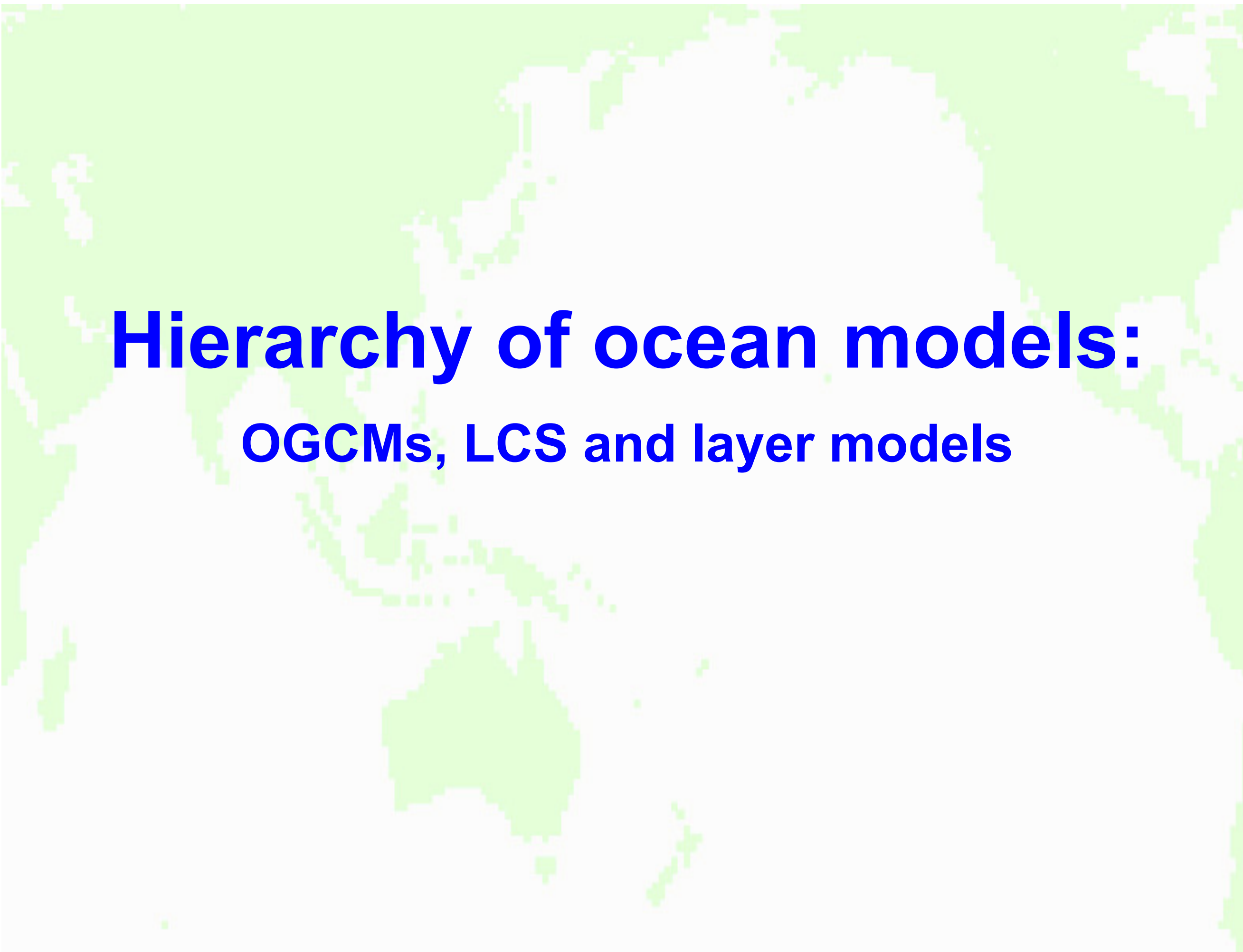
2) Midlatitude-ocean waves

3) Interior ocean

4) Equatorial ocean
   - 5a) Equatorial waves
   - 5b) Wind-forced solutions

6) Coastal Ocean

7) Summary



Hierarchy of ocean models:

OGCMs, LCS and layer models

Simpler ocean models

OGCMs are remarkably **good at simulating oceanic phenomena**.

At the same time, it is often **difficult to isolate basic processes** at work in OGCM solutions. Instead, **basic processes are illustrated better in simpler systems**. The simpler systems provide a **language for discussing phenomena and processes** in the more complicated ones. Moreover, **OGCM & simple solutions are often quite similar** to each other and to observations.

Here, I introduce equations for the **linear, continuously stratified (LCS) and 1½-layer models**, which are simpler equation sets that allow for **analytic solutions**. Most of our **understanding of ocean dynamics arises from analytic solutions** to these simpler equation sets.

OGCM equations

The following set of equations are the equations of motion that are solved in many OGCMs.

$$u_t + uu_x + vu_y + wu_z - fv + \frac{1}{\bar{\rho}}p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + uv_x + vv_y + wv_z + fu + \frac{1}{\bar{\rho}}p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

Many OGCMs adopt the **hydrostatic approximation**. In most physical situations, it is an **EXCELLENT** one.

$$T_t + uT_x + vT_y + wT_z = (\kappa_T T_z)_z + \nu_h \nabla^2 T,$$

$$S_t + uS_x + vS_y + wS_z = (\kappa_S S_z)_z + \nu_h \nabla^2 S,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0,$$

$$\rho = \rho\,(S, T, p)$$

LCS model: mode equations

The LCS model **linearizes and simplifies the OGCM equations** until it is possible to express the ***u*, *v*, and *p* fields as the expansions**

$$u = \sum_{n=0}^{\infty} u_n \psi_n, \quad v = \sum_{n=0}^{\infty} v_n \psi_n, \quad \frac{p}{\bar{\rho}} = \sum_{n=0}^{\infty} p_n \psi_n,$$

wl The resulting equations for u_n, v_n, and p_n are ,

$$\left(\partial_t + \frac{A_\nu}{c_n^2}\right) u_n - f v_n + p_{nx} = \frac{\tau^x}{\mathcal{H}_n} + \nu_h \nabla^2 u_n,$$

$$\left(\partial_t + \frac{A_\nu}{c_n^2}\right) v_n + f u_n + p_{ny} = \frac{\tau^y}{\mathcal{H}_n} + \nu_h \nabla^2 v_n,$$

$$\left(\partial_t + \frac{A_\kappa}{c_n^2}\right) \frac{p_n}{c_n^2} + u_{nx} + v_{ny} = \nu_h \nabla^2 \frac{p_n}{c_n^2}$$

$$\mathcal{H}_n^{-1} = \frac{\int_{-D}^{0} Z(z)\, dz}{\int_{-D}^{0} \psi_n^2\, dz}$$

See additional slides and HIGNotes.pdf for a derivation of (A).

Thus, the ocean's response is separated into a **superposition of independent responses associated with each mode**. They **differ only in the values of** , the **Kelvin-wave speed** for each mode.

LCS model: baroclinic and barotropic modes

The functions **are the baroclinic and barotropic modes** of the ocean. They require that the **ocean bottom is flat** at and that **depends only on z**. They are then solutions to

$$\left(\partial_z \frac{1}{N_b^2}\partial_z\right)\psi_n = \left(\frac{1}{N_b^2}\psi_{nz}\right)_z = -\frac{1}{c_n^2}\psi_n \tag{1}$$

subject to boundary conditions and normalization

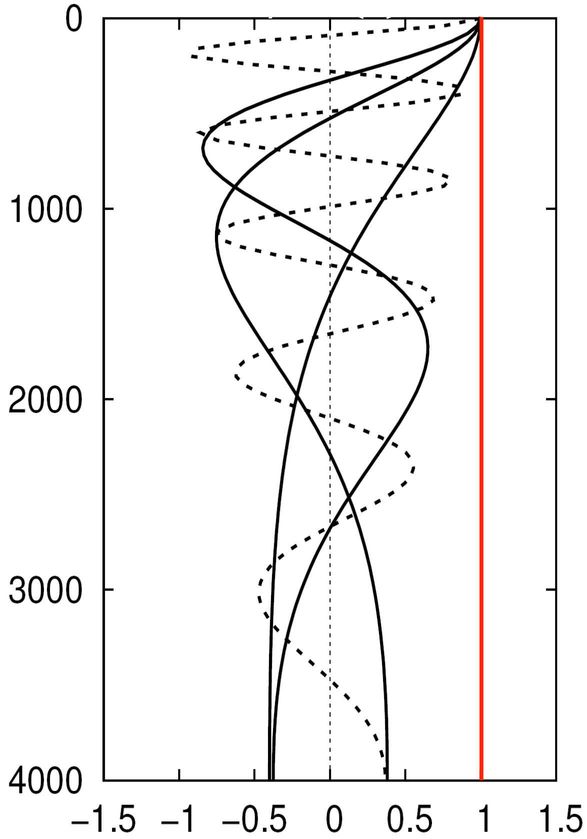
$$\psi_{nz}(-D) = \psi_{nz}(0) = 0, \qquad \psi_n(0) = 1 \tag{2}$$

Integrating (1) over the water column gives

$$-\int_{-D}^{0}\left(\frac{1}{N_b^2}\psi_{nz}\right)_z dz = -\frac{1}{N_b^2}\psi_{nz}\Big|_{-D}^{0} = \boxed{0 = -\frac{1}{c_n^2}\int_{-D}^{0}\psi_n\, dz} \tag{3}$$

Constraint (3) can be satisfied in two ways. Either $c_0 = \infty$ **in which case $\psi_0(z) = 1$ (barotropic mode)** or c_n **is finite and its value is set so that the integral of ψ_n vanishes (baroclinic modes)**.

LCS model: baroclinic and barotropic modes



When $N_b{}^2$ **decreases with depth** like

$$N_b^2 = -g\rho_{bz}/\rho_0 = ge^{z/b}$$

and c_n **is finite**, solutions to (1) are similar, except their **wavelength increases and amplitude decreases with depth**.

The **values of** c_n are different from, but are **similar to, those for constant density**.

When c_n **is infinite**, the solution to (1) that satisfies boundary conditions (2) is

$$\psi_0\left(z\right) = 1$$

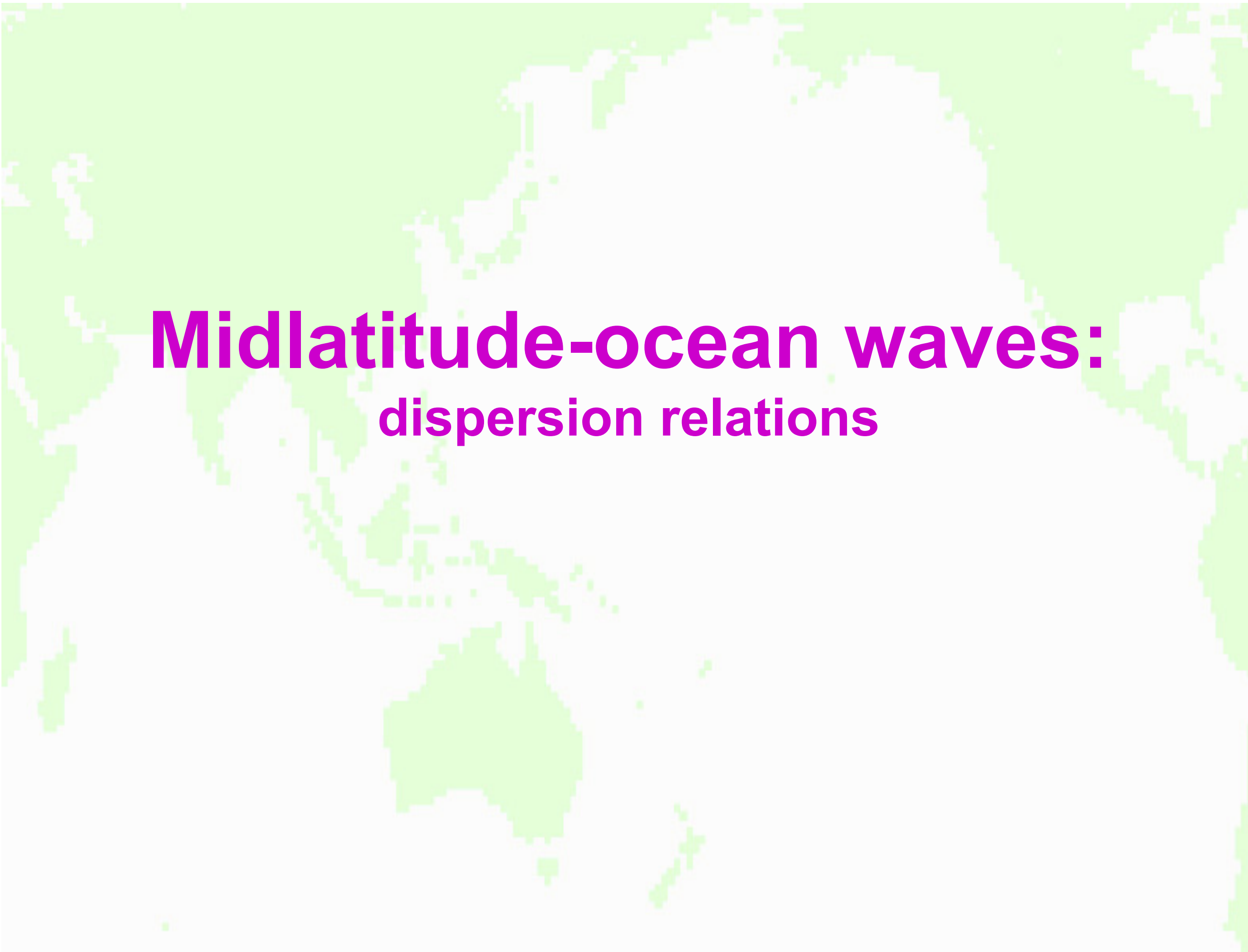
the **barotropic mode of the system**.

1½-layer model

If a particular phenomenon is **surface trapped**, it is often useful to study it with an **upper-layer model** that focuses on the surface flow. Such a model is the **1½-layer, reduced-gravity model**. In its **linear form**, its equations are

$$u_{nt} - f v_n + p_{nx} = \frac{\tau^x}{\mathcal{H}_n} - \frac{A_\nu}{c_n^2} u_n + \nu_h \nabla^2 u_n,$$
$$v_{nt} + f u_n + p_{ny} = \frac{\tau^y}{\mathcal{H}_n} - \frac{A_\nu}{c_n^2} v_n + \nu_h \nabla^2 v_n,$$
$$p_{nt} + c_n^2 \left(u_{nx} + v_{ny}\right) = -\frac{A_\kappa}{c_n^2} p_n + \nu_h \nabla^2 p_n$$

Most of the NIOSS movies are numerical **solutions to the LCS model for a single (*n* = 1) baroclinic mode** or, equivalently, **to a 1½-layer model**. A few movies are LCS solutions that are a **sum of a number of baroclinic modes**.



Midlatitude-ocean waves:

dispersion relations

v_n equation

To focus on the free waves, we neglect **forcing, damping**, and **friction** terms in the equations for a mode of the LCS model (or 1½-layer model) to get

$$\left(\partial_t + \right. \qquad u_{nt} - f v_n + p_{nx} = 0, \qquad \cancel{\nabla}^2 u_n,$$

$$\left(\partial_t + \right. \qquad v_{nt} + f u_n + p_{ny} = 0, \qquad \cancel{\nabla}^2 v_n,$$

$$\frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0.$$

Waves associated with a **superposition of vertical modes**

$$u = \sum_{n=0}^{N} u_n \psi_n(z), \quad v = \sum_{n=0}^{N} u_n \psi_n(z), \quad p = \sum_{n=0}^{N} \bar{\rho} p_n \psi_n(z).$$

propagate both horizontally and *vertically*.

v_n equation

Solving the unforced, inviscid equations for a **single equation in v_n**, and for convenience **dropping subscripts n** gives

$$v_{xxt} + v_{yyt} - \frac{1}{c^2} v_{ttt} - \frac{f^2}{c^2} v_t + \beta v_x = 0. \qquad (1)$$

Solutions See additional slides for a derivation of (1). use ***f* is a function of *y*** and the equation **includes *y* derivatives** (v_{yyt} term). There are, however, useful analytic solutions to **approximate versions of (1)**.

Dispersion relation of free waves

The simplest approximation (**mid-latitude *β*-plane approximation**) simply **"pretends" that *f* and *β* are both constant**. Then, solutions to (1) have the form of **plane waves**,

$$\exp\left(ikx + i\ell y - i\sigma t\right).$$

Then, we can set , , and in (1), resulting in the dispersion relation,

$$v_{xxt} \quad \sigma\left(k^2 + \ell^2 - \frac{\sigma^2}{c^2} + \frac{f^2}{c^2}\right) = -k\beta. \quad = 0.$$

The **dispersion relation provides a "biography"** for a model. It **describes everything about the waves** it supports.

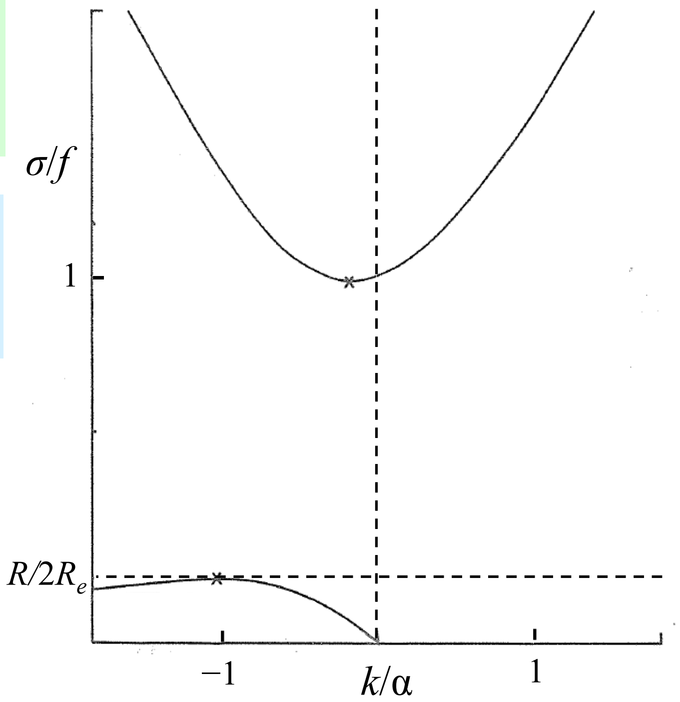
Gravity and Rossby waves

For convenience, the figure plots curves **when $\ell = 0$**.

When $\ell \neq 0$, the disp. rel. is a **circle for each σ**. So, the two curves become **circular bowls**.

The **top bowl** (**bottom bowl**) describes the **gravity waves** (**Rossby waves**) of the system.

$$\left(k + \frac{\beta}{2\sigma}\right)^2 + \ell^2 = \frac{\sigma^2 - f^2}{c^2} + \frac{\beta^2}{4\sigma^2}$$



Kelvin waves

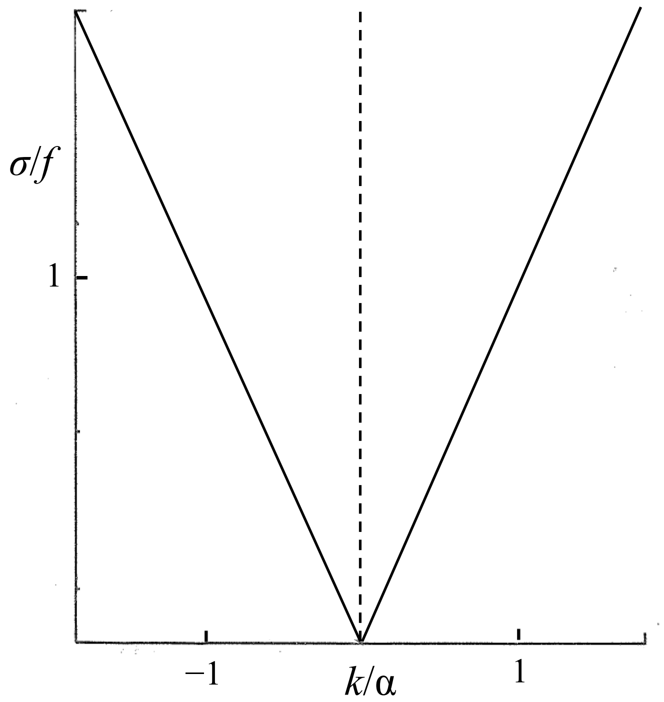
To derive the dispersion relation for GWs and RWs, we solved for a single equation in v. So, we **missed a wave with $v = 0$**, the **coastal Kelvin wave**.

See the additional slides for a derivation of the Kelvin wave.

Kelvin waves along zonal boundaries. KWs also exist along meridional boundaries.

The coastal KW propagates **along coasts at speed c** with the **coast to its right (in the NH)**, and **decays offshore with the scale $c/f = R$**, the Rossby radius of deformation.

$$\sigma = \pm ck$$

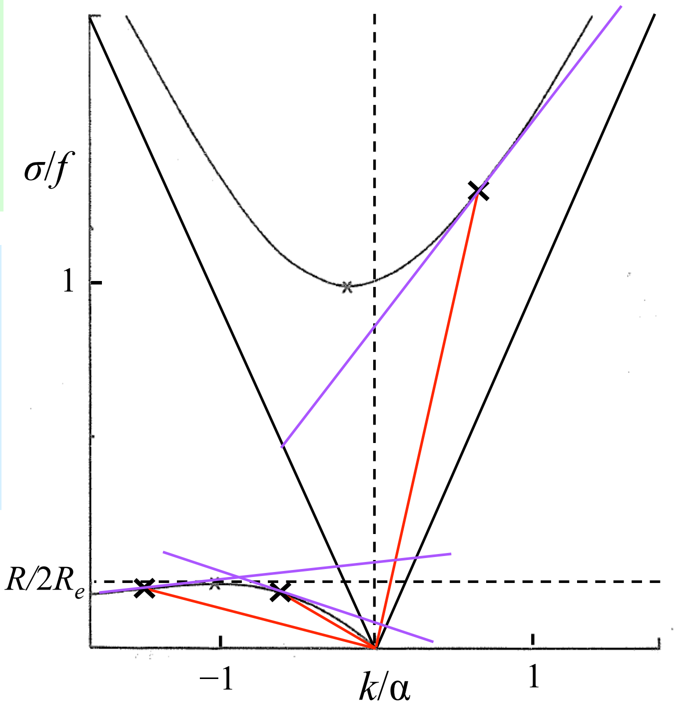


Phase and group speed

The figure shows the wave types that we have discussed.

$$\left(k + \frac{\beta}{2\sigma}\right)^2 + \ell^2 = \frac{\beta^2}{4\sigma^2} - \frac{f^2}{c^2}, \qquad \sigma = \pm ck$$

The **phase speed** of a wave with wavenumber k and frequency σ is the **slope of the line that extends from (0,0) to (σ,k)**.



The **group speed** of a wave with wavenumber k and frequency σ is the **slope of the line parallel to the dispersion curve at the point (σ,k)**.

Movies A1, A3, A2

Interior ocean:

Ekman pumping & adjustment to Sverdrup balance

Interior-ocean equations

Equations of motion for the , , and fields of a single baroclinic mode are

$$-fv + g'h = \tau^x/H,$$

$$fu + g'h_y = \tau^y/H,$$

$$h_t + H\left(u_x + v_y\right) = -\kappa\left(h - H\right)$$

This approximation is useful because it **filters out the gravity-wave response**. Thus, it only describes the **slowly varying parts** of the response, that is, its **directly forced and Rossby wave (if $\beta \neq 0$) parts**.

horizontal **viscosity terms are assumed small** dropped in the interior ocean, and are only retained to represent western boundary currents.

Ekman pumping (β=0) with κ = 0 forced by τ^x

Suppose the ocean is forced by a **zonal-wind stress** **switched on at** , that **is constant ( )**, and that there is **no damping** (= 0). Then, the solution is

$$h = H - w_{ek} t = H + \frac{\tau^x_y}{f} t$$

so that ***h* thickens (thins) continuously where** **> 0 (< 0)**.

With *h* known, the zonal and meridional velocities are

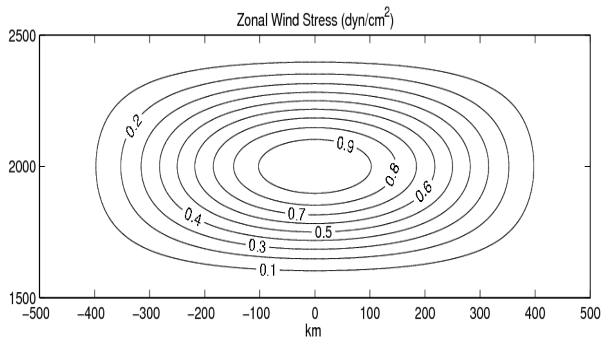
See additional slides for a derivation of this solution.

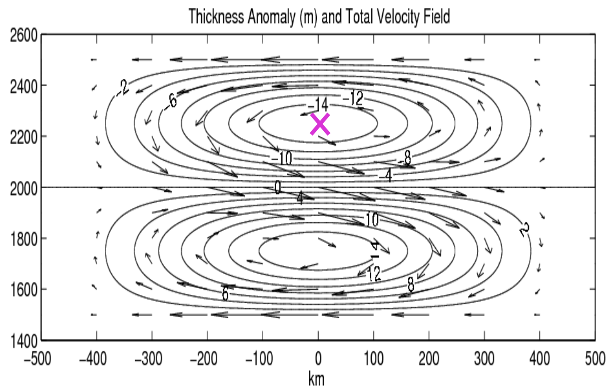
$$v = \frac{1}{f} g' h_x - \frac{\tau^x}{fH}, \qquad u = -\frac{1}{f} g' h_y$$

a superposition of **Ekman and geostrophic currents**.

Note that, although the **geostrophic flow grows linearly in time**, the **Ekman flow switches on instantly at *t* = 0 and thereafter remains constant**. It switches on instantly because the **interior-ocean equations filter out gravity (inertial) waves**.

Ekman pumping (β=0) with κ = 0 forced by τ^x





For this wind, **north of 2000 km and *h* thins**, and the opposite change happens south of 2000 km. The **constant Ekman drift shifts water continually from the northern to the southern half** of the domain. **Counter-rotating geostrophic gyres spin-up** in response to *h*.

How long does it take for the **layer bottom to upwell to the surface**?

$$H = -\frac{\min\left(\tau_y^x\right)}{f}t = \frac{\pi}{2\Delta y}\frac{\tau_o}{f}t \quad \Rightarrow \quad t = \frac{2\Delta y f}{\pi \tau_o}H$$

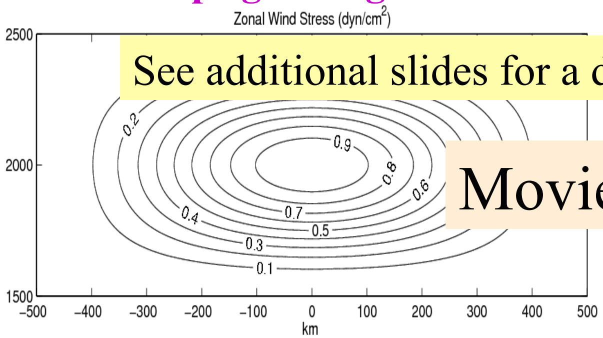
which for the above wind, H = 100 m, and $f = 10^{-4}$ s^{-1} is ***t* = 368 days**.

Ekman pumping ($\beta=0$) with $\kappa \neq 0$ forced by τ^x

Suppose the ocean is **forced by a zonal wind** (i.e.,) and there **is damping** ($\kappa \neq 0$). Then, the solution is

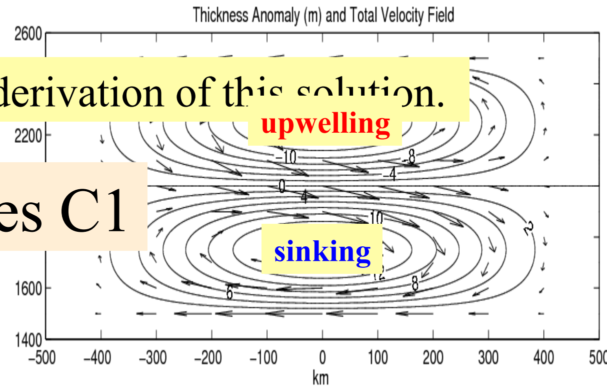
$$h = H - \frac{1 - e^{-\kappa t}}{\kappa} w_{ek} = H + \frac{1 - e^{-\kappa t}}{\kappa} \frac{\tau^x_y}{f}$$

so that ***h* stops growing**.



See additional slides for a derivation of this solution.

Movies C1



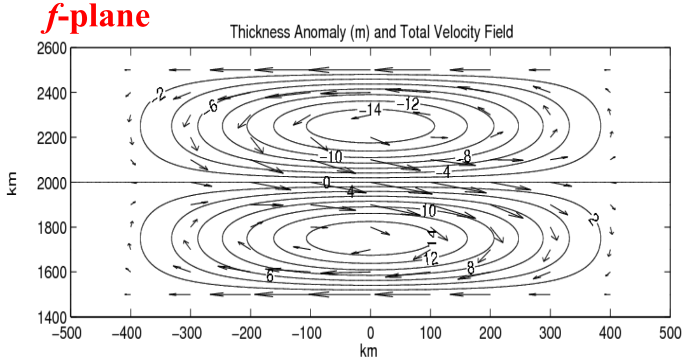
In steady state, **Ekman drift still flows from the northern to the southern half** of the domain. Water **entrains into the layer in the north to provide a source** for the Ekman drift and **detrains from the layer in the south to provide a sink**, forming an **overturning cell**.

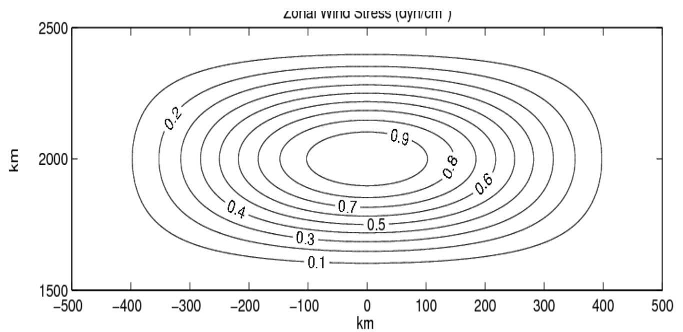
Adjustment to Sverdrup balance ($\beta \neq 0$) forced by τ^x

Consider the response to a switched-on patch when .

The **initial response** is the same as on the ***f*-plane**. **Ekman flow switches on instantly** because gravity waves are filtered out of the system, and **wind curl drives Ekman pumping**.

***f*-plane**



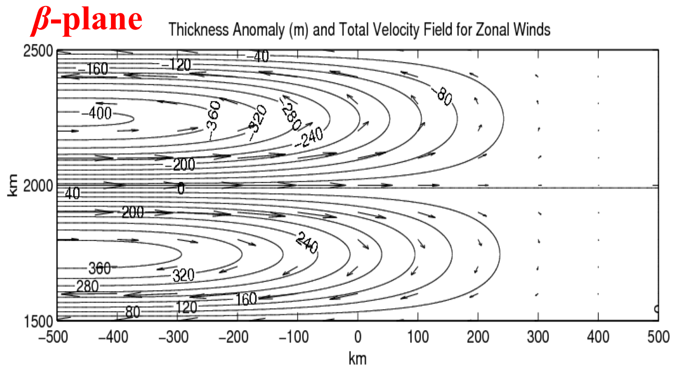


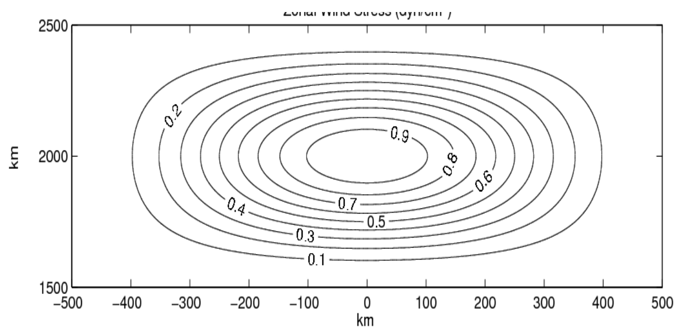
Adjustment to Sverdrup balance ($\beta \neq 0$) forced by τ^x

Consider the response to a switched-on patch when .

Subsequently, **westward radiation of Rossby waves** extends the response west of the forcing region, and **adjusts the circulation to Sverdrup balance**.

See additional slides for a derivation of this solution.

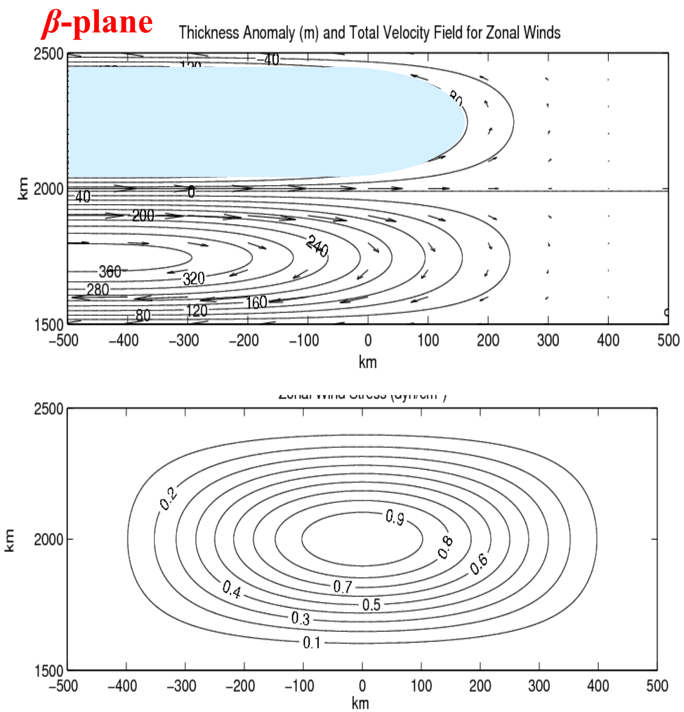




Adjustment to Sverdrup balance ($\beta \neq 0$) forced by τ^x

Consider the response to a switched-on   patch when   .

At any longitude, **Ekman pumping continues until the passage of Rossby waves**. Because they propagate slowly, the Ekman pumping can be **large enough for the bottom of the layer to rise to the surface (light blue area)**. In that case, the solution breaks down, and **there must be upwelling from deep ocean**.

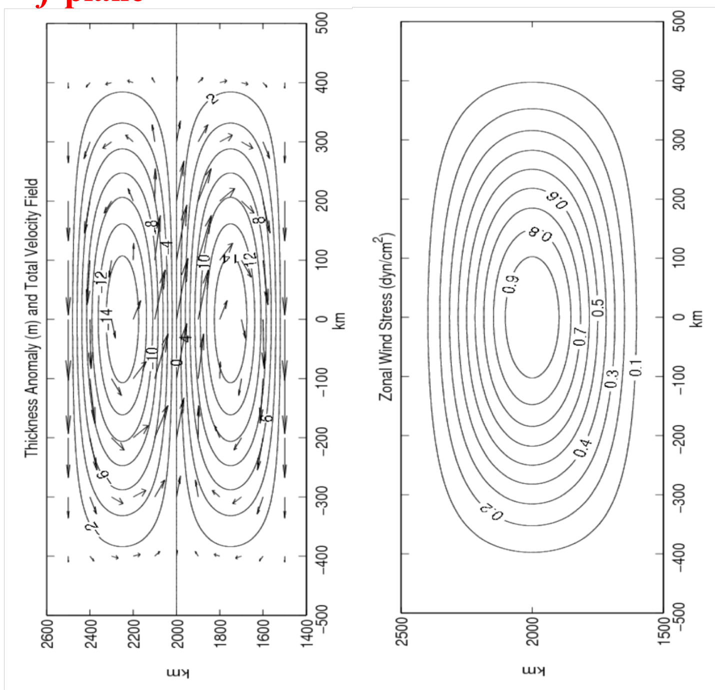


Adjustment to Sverdrup balance ($\beta \neq 0$) forced by τ^y

Consider the response to a switched-on **patch** when .

The **initial response** is the same as on the ***f*-plane**. **Ekman flow switches on instantly** because gravity waves are filtered out of the system, and **wind curl drives Ekman pumping**.

***f*-plane**



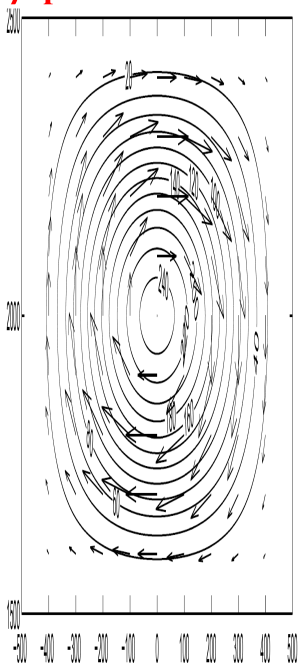
Adjustment to Sverdrup balance ($\beta \neq 0$) forced by τ^y

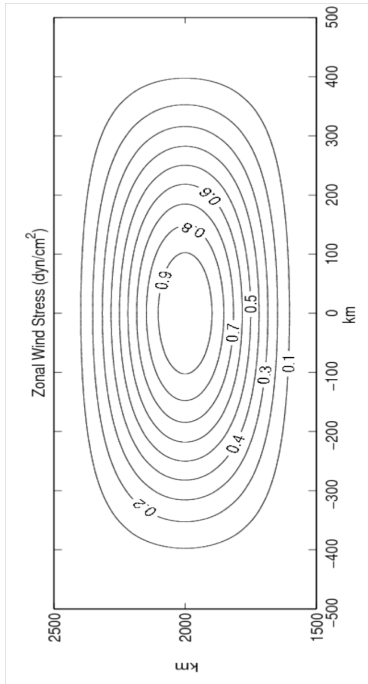
Consider the response to a switched-on patch when .

Subsequently, the **Rossby waves radiate westward** across the wind patch, but after their passage the adjusted **response remains confined to the forcing region**.

Movies C3 & C2a

***β*-plane**





Equatorial ocean:

equatorially trapped waves & wind-forced solutions

Questions

What forcing mechanisms drive equatorial currents?

zonal and meridional wind stress

What are equatorial waves?

equatorial gravity, Rossby, and Kelvin waves;
mixed Rossby/gravity (Yanai) wave

How do they differ from midlatitude waves?

dynamically very similar; extra Yanai wave;
discreteness

What are the key differences between 2-d and 3-d theories of equatorial circulation?

Yoshida Jet; establishment of p_x to balance τ^x

How do equatorial waves reflect from basin boundaries?

Kelvin- and Rossby-wave reflections

Equatorial-ocean equations

Equations for the , , and for a single baroclinic mode are

$$\left(\partial_t + \frac{A}{c_n^2}\right) u_n - f v_n + p_{nx} = \tau^x / \mathcal{H}_n + \nu_h \nabla^2 u_n,$$
$$\left(\partial_t + \frac{A}{c_n^2}\right) v_n + f u_n + p_{ny} = \tau^y / \mathcal{H}_n + \nu_h \nabla^2 v_n, \quad (1)$$
$$\left(\partial_t + \frac{A}{c_n^2}\right) \frac{p_n}{c_n^2} + u_{nx} + v_{ny} = 0,$$

Because *f* vanishes at the equator, **no terms can be dropped that allow for mathematically simple solutions** near the equator.

A useful assumption, though, is to set $f = \beta y$, known as the **equatorial β-plane approximation**. As a result, one can look for solutions as **expansions in Hermite functions**.

Equatorial ocean:

equatorially trapped waves

Equatorial gravity and Rossby waves

We look for unforced (free-wave) solutions to (1) of the form,

, **without damping ($A = 0$)**, and, for convenience, we **drop the subscript *n***. The **resulting *v* equation is**

$$\sigma k^2 v - \frac{\sigma^3}{c^2} v - \sigma \alpha_o^2 \left(\partial_{\eta\eta} - \eta^2 \right) v + k\beta v = 0 \qquad (2)$$

The mathematical c

See additional interior-ocean slides for a derivation.

ersion relation from (2) is that, because ***f*** **v** **tor**, it is **not possible to set $\phi_\ell(y) = \exp(i\ell y)$**, like we did for the interior ocean. Rather, **$\phi_\ell(y)$ is the set of solutions (eigenfunctions) that satisfy**

$$\left(\partial_{\eta\eta} - \eta^2 \right) \phi_\ell = \lambda_\ell \phi_\ell = -(2\ell + 1)\phi_\ell, \qquad (2\ )$$

and **vanish as $\eta \to \pm\infty$**, **where $\ell = 0, 1, 2, \ldots$**. They are referred to as **Hermite functions**.

$^{-1}$s^{-1}, its value is **$R_1 = 331$ km.**

Equatorial gravity and Rossby waves

The solutions to (2) can be represented as expansions in Hermite functions

$$v = \sum_{\ell=0}^{\infty} v_\ell(x)\phi_\ell(\eta)e^{ikx-i\sigma t}, \qquad (3)$$

where is a wave amplitude. **Each term in expansion (3) is an individual equatorial wave**.

Inserting term in (3) into (2) and using (2) gives

$$\sigma k^2 v_\ell - \frac{\sigma^3}{c^2}v_\ell + \sigma\alpha_o^2\left(2\ell+1\right)v_\ell + k\beta v_\ell = 0, \qquad (2)$$

which provides the **dispersion relation**

$$\sigma\left(k^2 + \alpha_\ell^2 - \frac{\sigma^2}{c^2}\right) + k\beta = 0, \qquad \alpha_\ell^2 = \alpha_o^2(2\ell+1),$$

for equatorial, Rossby and gravity waves.

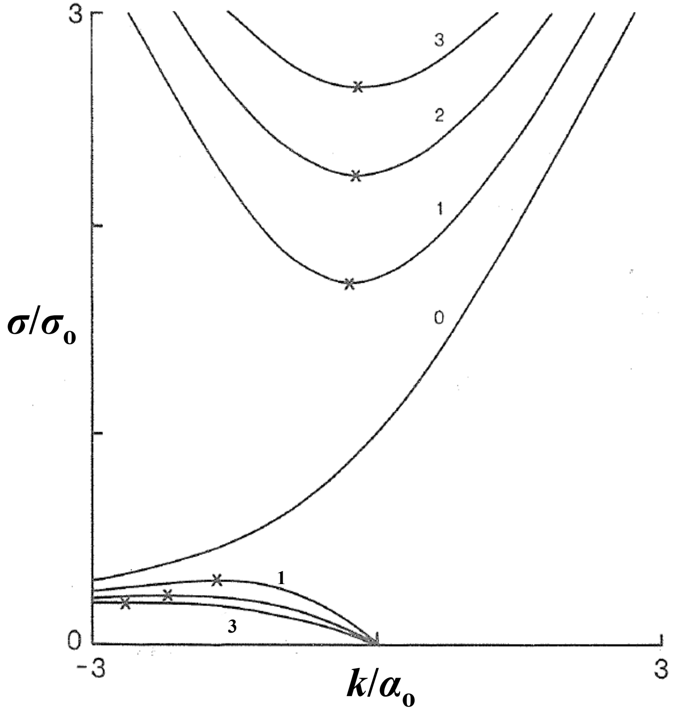
Equatorial gravity and Rossby waves

For each ℓ > 1, there is a **gravity wave** (large σ) and a **Rossby wave** (small σ). The plot shows waves for ℓ = 1, 2, and 3.
continuously for midlatitude

For ℓ = 0, there is a new type of wave, the **mixed Rossby-gravity (Yanai) wave**, which behaves like a Rossby (gravity) wave for k positive (negative).

See additional slides for a deriv. of the Yanai wave relation. disperson curves.

$$\sigma\left(k^2 + \alpha^2 - \frac{\sigma^2}{c^2}\right) + k\beta = 0$$



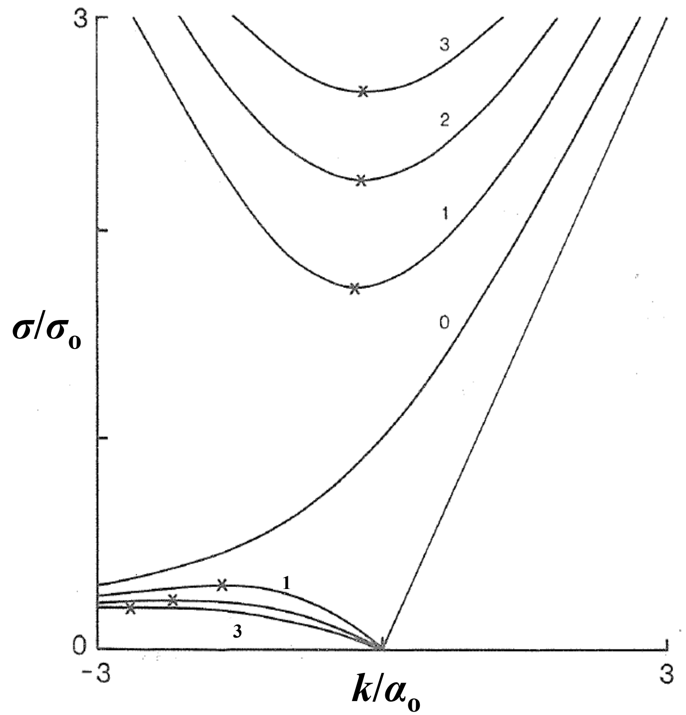
Theoretical equatorial waves

To derive the dispersion relation for GWs and RWs, we solved for a single equation in *v*. So, we **missed a wave with** $v=0$, the **coastal Kelvin wave** with the **dispersion relation** $\sigma=kc$.

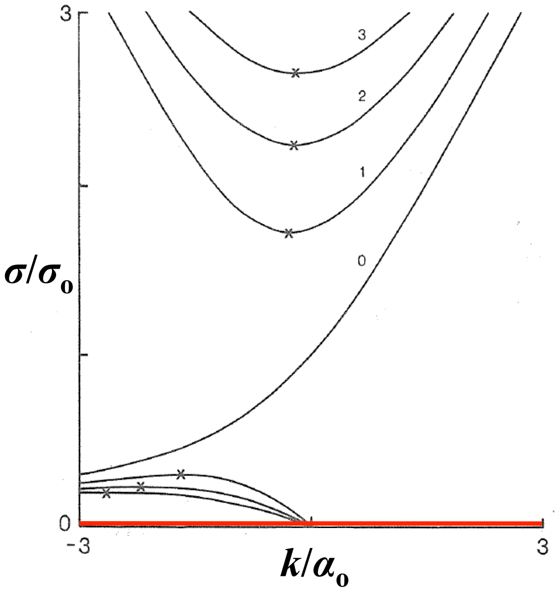
$$\sigma\left(k^2+\alpha_\ell^2-\frac{\sigma^2}{c^2}\right)+k\beta=0, \qquad \sigma=kc$$

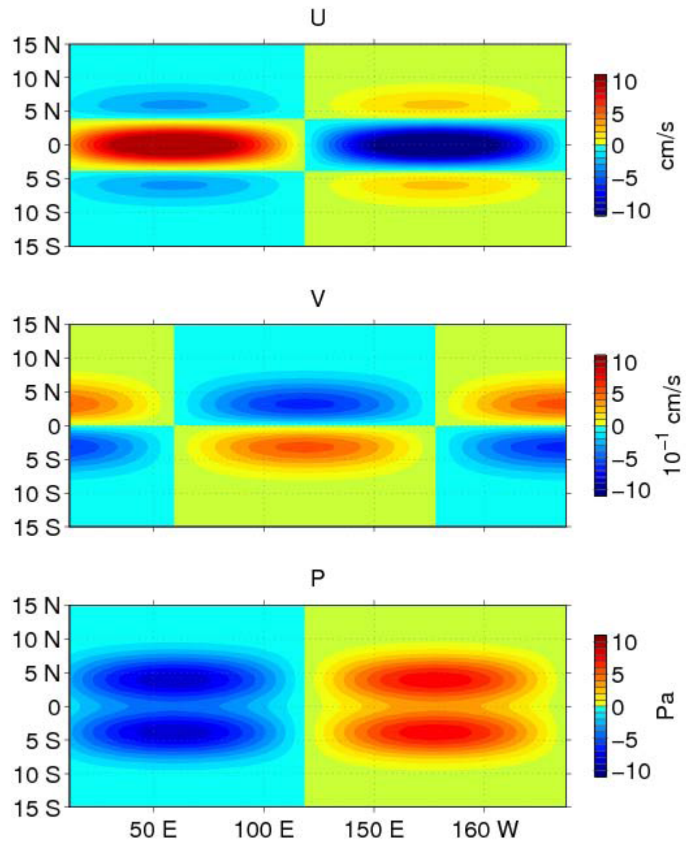
See additional slides for a deriv. of the equatorial KW.

and a **Rossby wave** (small σ). The plot indicates waves only for ℓ = 1, 2, and 3. In addition, there is the **Yanai wave** for $\ell=0$, and the **equat. Kelvin wave** with $v=0$.



Structure of equatorial Rossby waves

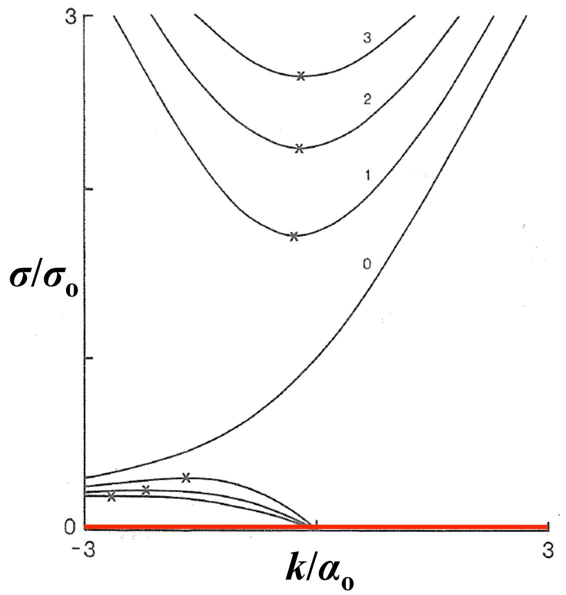


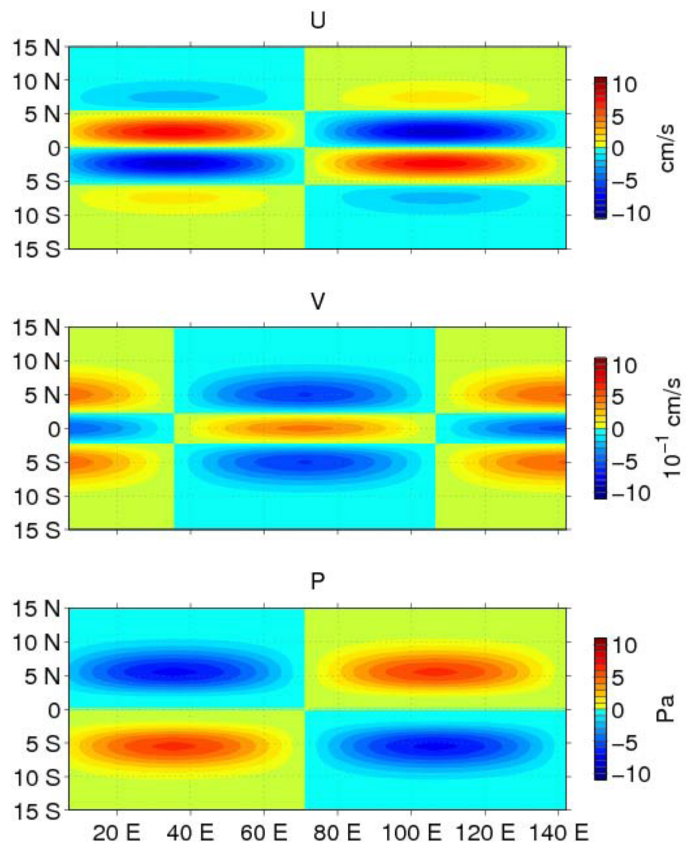


The u, v & p fields for an $\ell = 1$ **Rossby wave** when c_n **= 250 cm/s** and **P = 360 days**. For this P, σ/σ_o **= .03** and λ **= 240º**.

Courtesy of Francois Ascani

Structure of equatorial Rossby waves





The u, v, and p fields for an ℓ = 2 **Rossby wave** when c_n = **250 cm/s** and **P = 360 days**. For this P, σ/σ_o = **.03** and λ = **140°**.

Courtesy of Francois Ascani

Observed equatorial waves

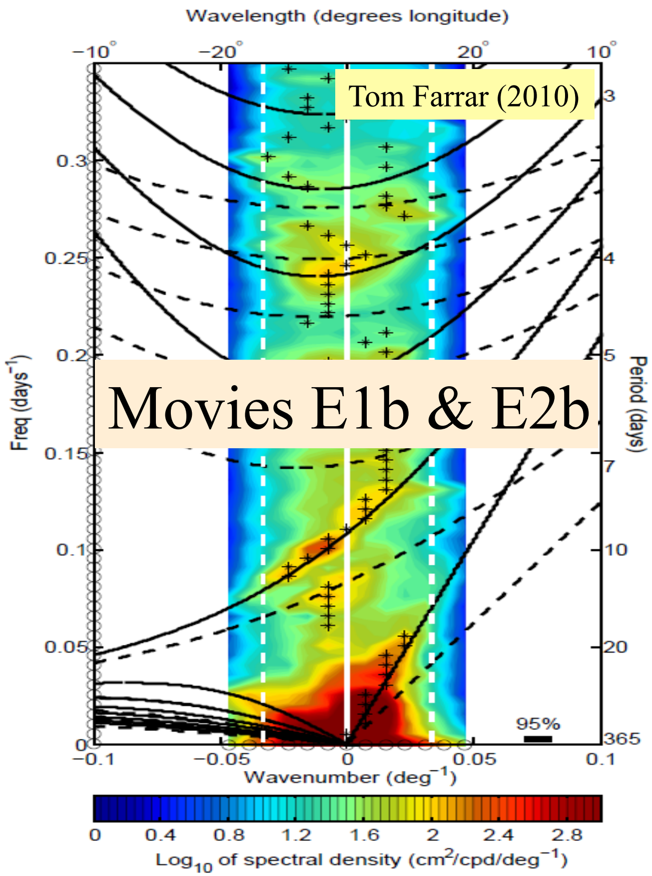
A lot of mathematics led to this set of dispersion curves. **Do any of these waves actually exist?!**

The first equatorially trapped waves to be discovered were **gravity waves** with periods of O(10 days) (Wunsch and Gill, 1976; *Deep-Sea Res.*).

The **equatorial Kelvin wave** was **discovered after it was predicted** (Knox and Halpern, 1982, *JMR*).

The **mixed Rossby-gravity (Yanai) wave** was first **observed in the atmosphere by Yanai**. In the ocean, it was (probably) **first detected in the Indian Ocean by Reverdin and Luyten (1986)** using altimeter data.

Who first detected an **equatorial Rossby wave**?



Equatorial ocean:

wind-forced solutions

x-independent (2-d) Yoshida Jet

Kozo Yoshida wrote down the first solution for an ***x*-independent (2d) equatorial current driven by zonal winds**. The (more complete) theoretical solution developed somewhat later (Dennis Moore) has come to be called the **"Yoshida Jet"** (Jim O'Brien).

The basic dynamics of the Yoshida Jet can be understood from the zonal-momentum equation. Neglecting the (the flow is assumed to be -independent) and **mixing** terms in the zonal momentum equation gives

$$\left(\partial_t + \frac{A}{c_n^2}\right) u_{nt} - f v_n = \tau^x / \mathcal{H}_n. + \nu_h \nabla^2 u_n.$$

Offshore, Ekman balance ($f v_n = \tau^x / \mathcal{H}_n$) holds, whereas **at the equator u_n continues to accelerate ($u_{nt} = \tau^x / \mathcal{H}_n$)**. The switch from one dynamical regime to the other occurs at $y \approx \alpha_{on}^{-1/2} = (\beta/c_n)^{-1/2}$.

Bounded (3-d) Yoshida Jet

In reality and models, equatorial zonal flows (Yoshida Jets) **don't continue to accelerate**. Why not?

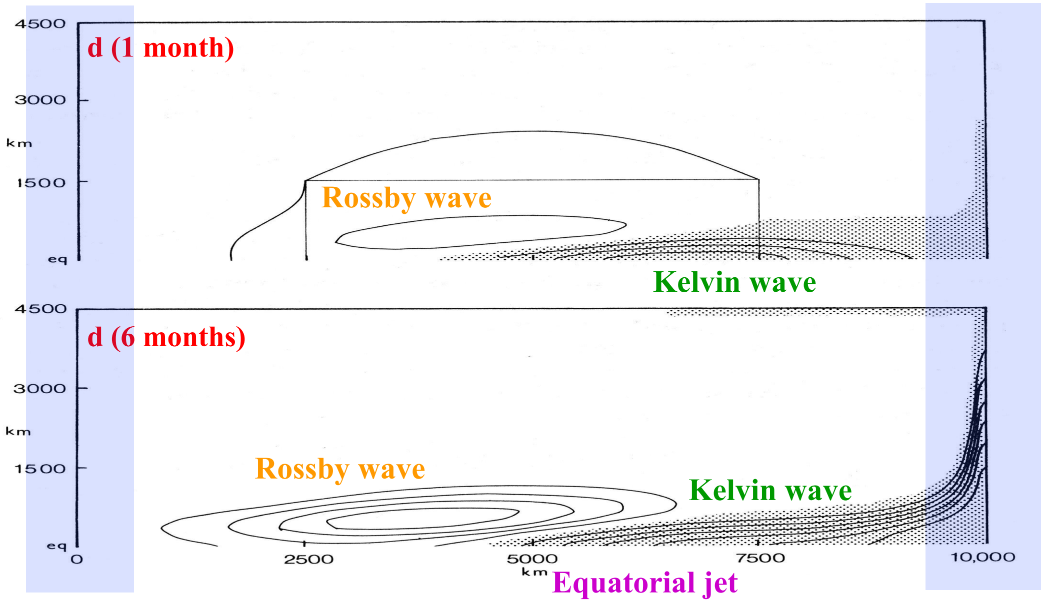
Because in the real world either the **wind forcing or the ocean basin is zonally bounded**, which **introduces *x*-dependence** into the solution. (An exception is the Southern Ocean, but we will not consider that case here.)

For convenience, we can still **drop the mixing terms** in the zonal momentum equation, and at the equator the **Coriolis term vanishes**. The boundaries, however, introduce *x*-dependence so we **cannot neglect the p_{nx} term**

$$\left(\partial_t + \frac{A}{c_n^2}\right) u \quad u_{nt} + p_{nx} = \tau^x/\mathcal{H}_n \quad + \nu_h \nabla^2 u_n.$$

In this case, the system **stops accelerating** by adjusting to a state where the **pressure gradient balances the wind**. It does so by **radiating equatorial Kelvin and Rossby waves**.

Bounded (3-d) Yoshida Jet

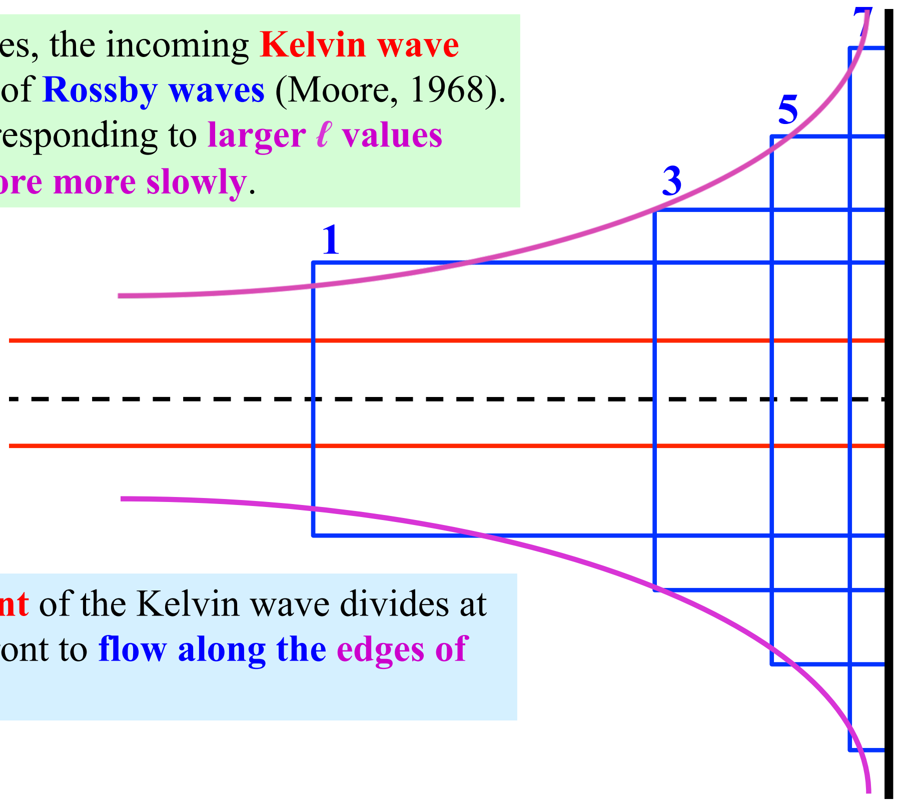


In response to forcing by a **patch of easterly wind**, an **accelerating Yoshida Jet initially develops** in the forcing region. Subsequently, **KWs and RWs radiate** from the forcing region. They **generate a steady, eastward, equatorial current** both east and west of the forcing region: the **bounded YJ**.

Eastern-boundary reflections

What happens when **basin boundaries are included**?

At low frequencies, the incoming **Kelvin wave** reflects as a packet of **Rossby waves** (Moore, 1968). with the waves corresponding to **larger ℓ values propagating offshore more slowly**.



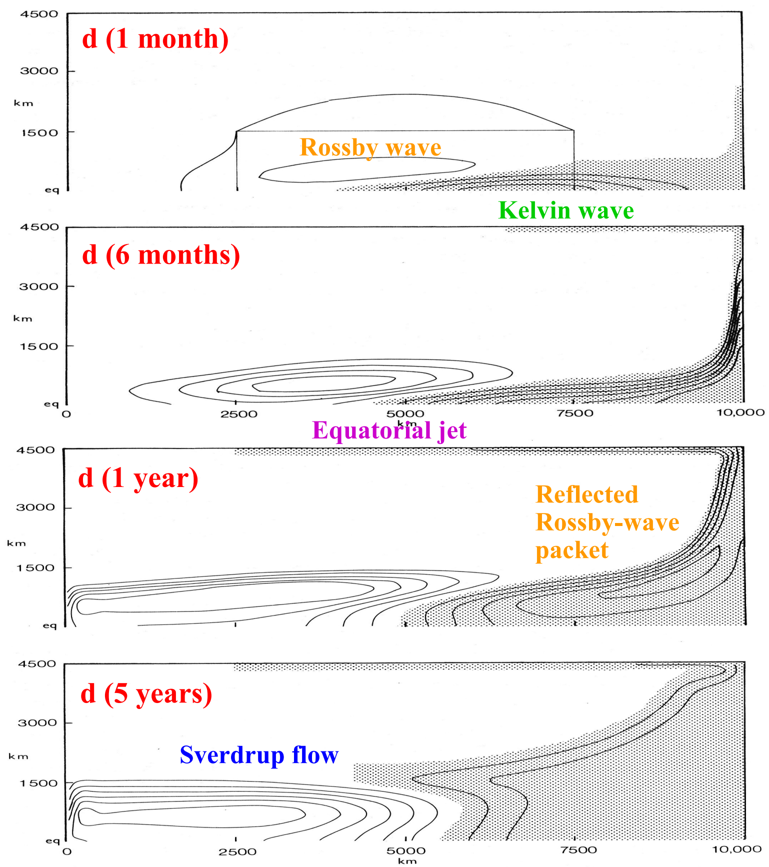
The **zonal current** of the Kelvin wave divides at the Rossby-wave front to **flow along the edges of the wave packet**.

Adjustment to steady state

In response to forcing by a **patch of wind in the interior ocean**, **KWs reflect from the eastern boundary** as a packet of RWs creating a characteristic **wedge-shaped pattern**. In addition, wind-generated **RWs reflect from the western boundary** to return to the interior ocean.

After multiple reflections, the solution eventually **adjusts to a steady state of Sverdrup balance**.

Movies F1b & F2

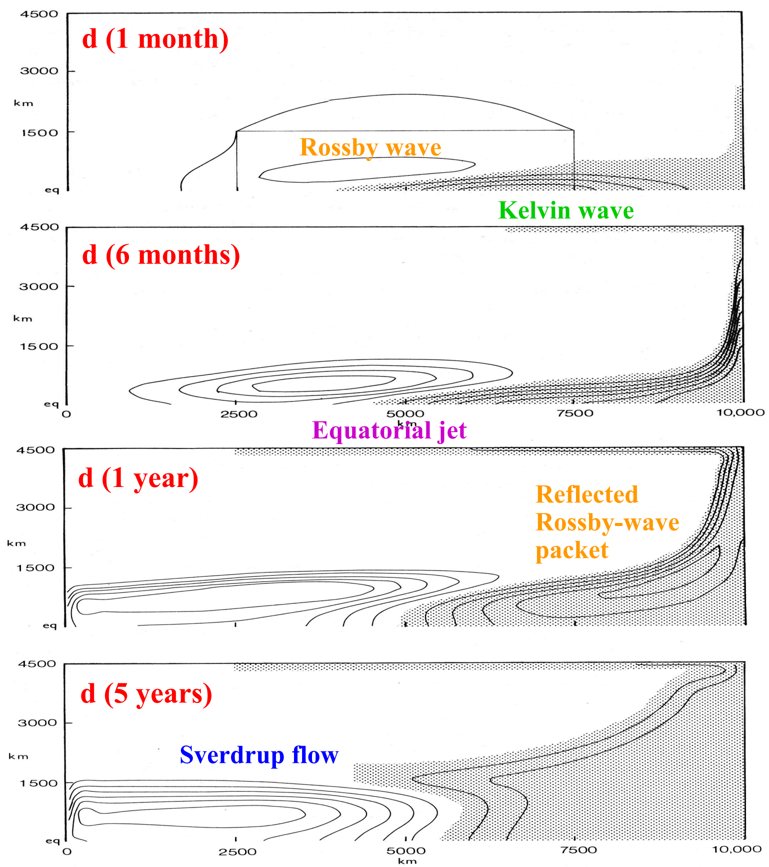


Multi-mode response to switched-on τ^x

How does the LCS model adjust when **many baroclinic modes are included**?

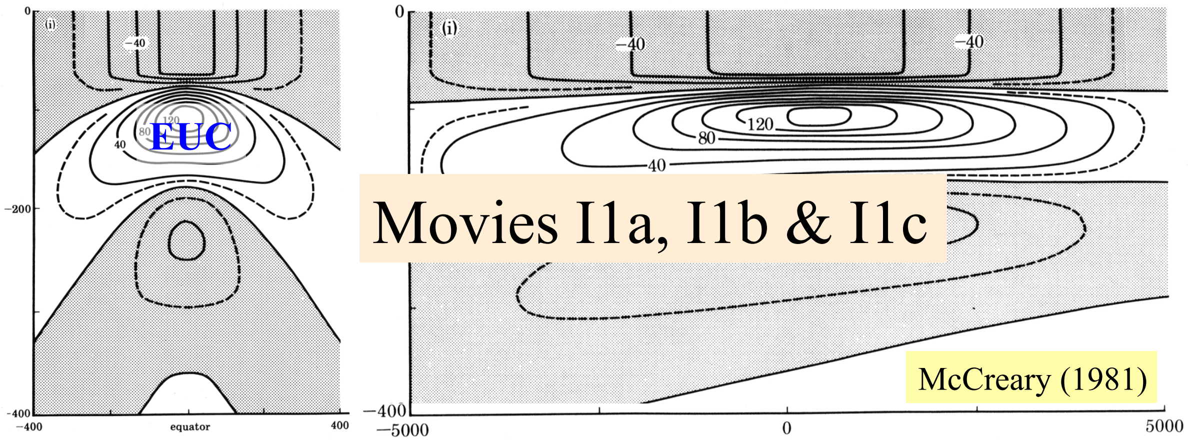
With damping, the ***n* > 1 responses are increasingly damped for larger *n*,** since $\nu = A/c_n^2$. In that case, **waves that radiate from the forcing region are increasingly weakened** for larger *n*.

propagation speeds of eq. waves are slower ($\propto c_n$ and $c_n < c_1$.



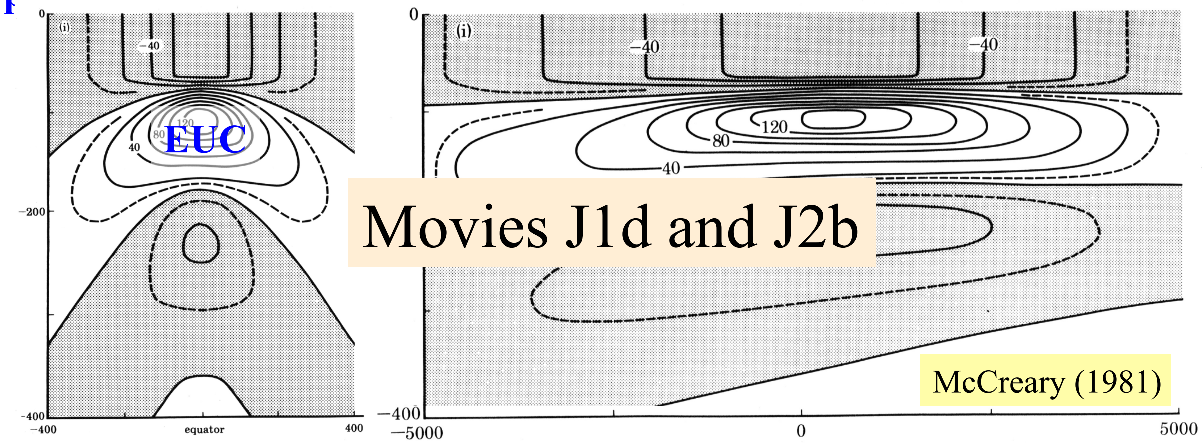
Multi-mode response to switched-on τ^x

With damping (vertical mixing), the LCS model produces a **realistic steady flow field** near the equator **with an EUC**.



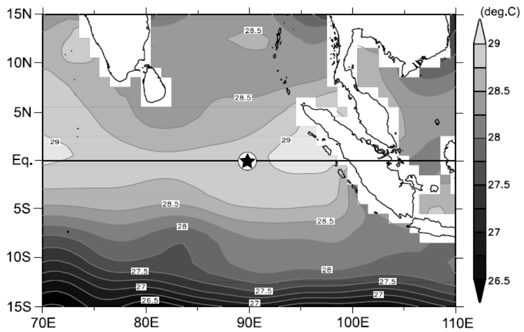
Multi-mode response to periodic τ^x

In the IO, the **steady component** of equatorial τ^x **is weak**. Instead, τ^x tends to oscillate at **annual, semiannual, and other (e.g., intraseasonal) periods**



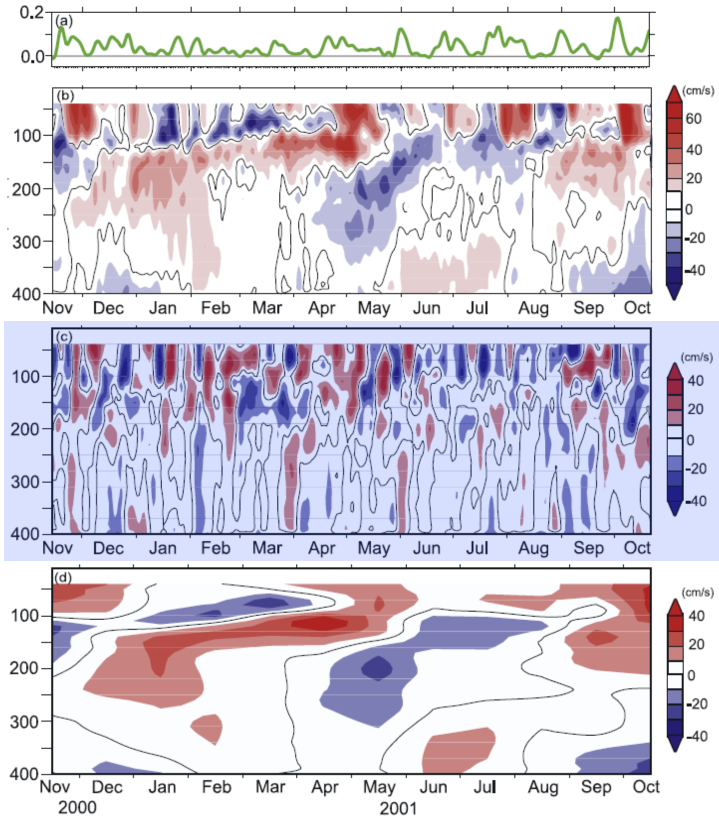
In response to periodic forcing, **equatorial waves** from a number of baroclinic modes superpose to **form beams that propagate vertically** as well as horizontally. **Kelvin** (**Rossby**) beams extend downward and **eastward** (**westward**) from the forcing region. Phase propagates **upwards** (**downwards**) across **downward-extending** (**upward-extending**) beams.

Upward phase propagation in the EEIO



The ***u* field (b & d)** shows a **strong semiannual cycle**.

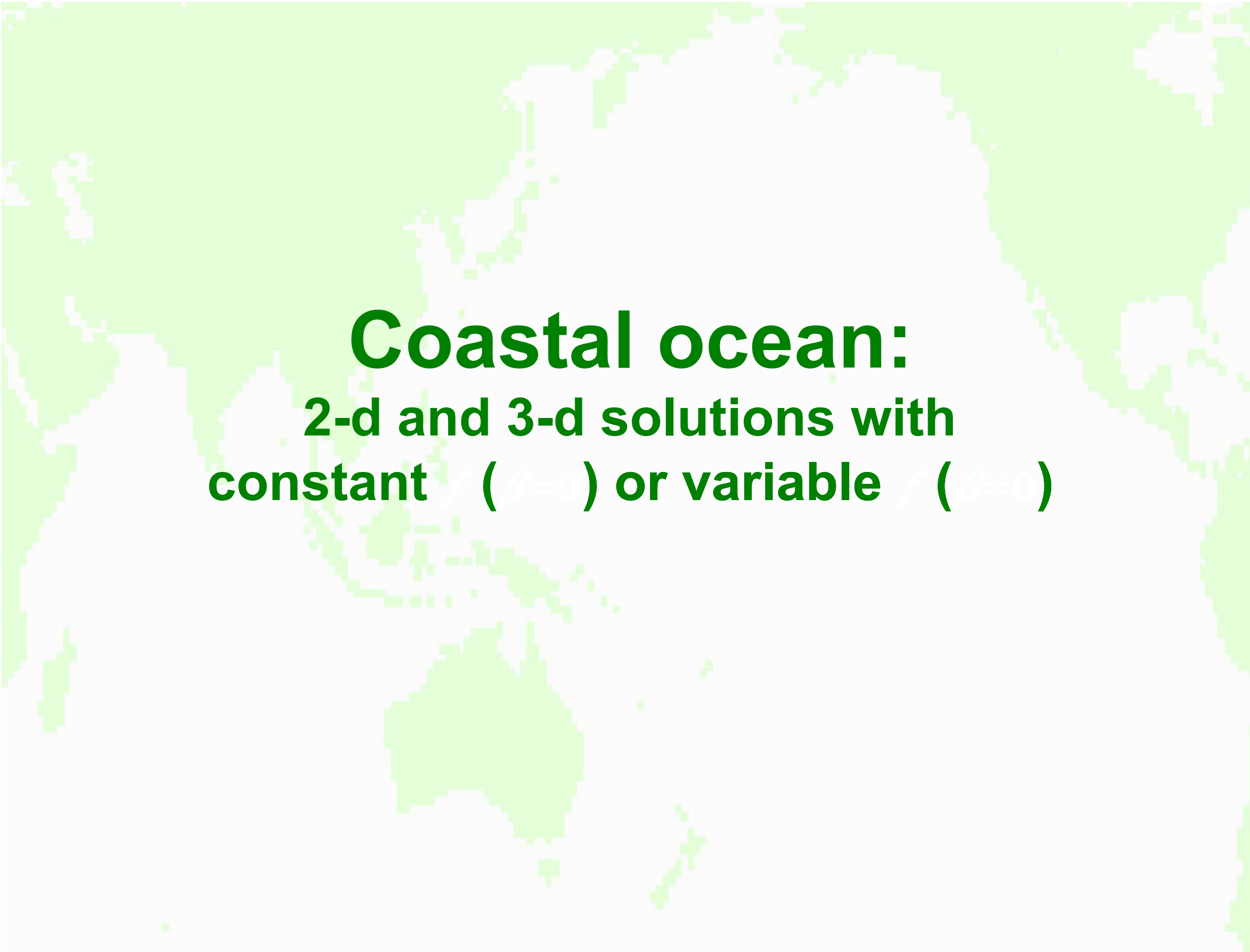
Above 200 m, the **phase of *u* propagates upwards**, indicating that it is **remotely forced (wave) signal**!



Masumoto et al. (2005)

Coastal ocean:

2-d and 3-d solutions with constant f ($\beta=0$) or variable f ($\beta\neq0$)



Questions

How does wind drive coastal currents?

across-shore Ekman flow driven by alongshore winds

What waves are generated at coasts?

Kelvin and Rossby waves; (shelf waves)

What are the key differences between 2-d and 3-d theories of coastal circulation?

wave radiation; establishment of p_y to balance τ^y

Why do eastern-boundary currents exist at all?

vertical mixing; (shelf trapping)

Coastal-ocean equations

A useful set of equations for the coastal ocean is

$$-fv + g'h_x = 0,$$

$$v_t + fu + g'h_y = \tau^y/H - \lambda v,$$

$$h_t + H\left(u_x + v_y\right) = -\kappa\left(h - H\right)$$

As for the interior-ocean equations, this approximation is useful because it **filters out gravity waves**. Thus, it only describes the **slowly varying response**, that is, its **directly forced & Rossby-wave (if $\beta \neq 0$) parts**.

mixing terms. In this way, the **alongshore flow is in geostrophic balance**, a property consistent with observations.

Forcing by a band of alongshore wind τ^y

All the coastal solutions discussed below are forced by a **band of alongshore winds** of the form,

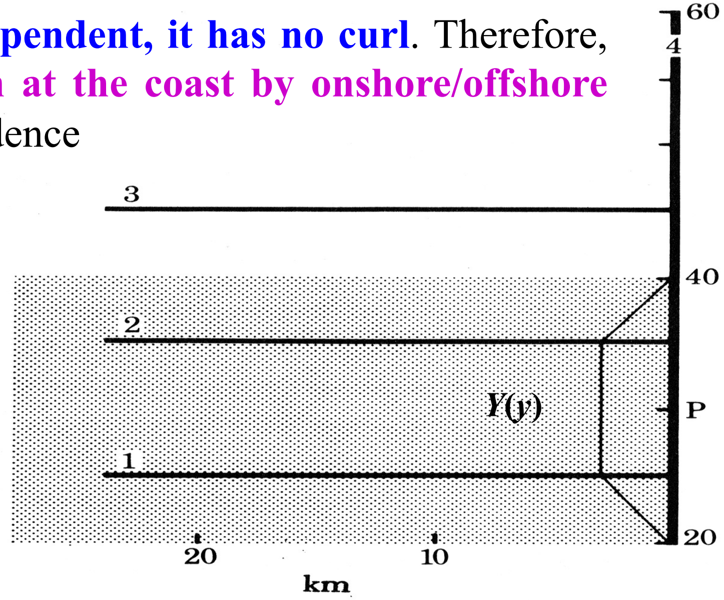
$$\tau^y(x, y, t) = \tau_o Y(y) T(t).$$

Since this **wind field is *x*-independent, it has no curl**. Therefore, the response is entirely **driven at the coast by onshore/offshore Ekman drift**. The time dependence is usually **switched-on**

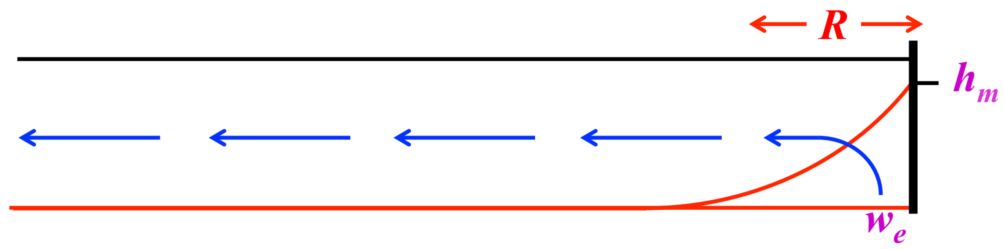
$$T(t) = \theta(t)\,,$$

except for a few solutions when it is **periodic**

$$T(t) = e^{-i\sigma t}$$



2-d response to switched-on $\tau \uparrow y$



Consider the **2-dimensional (x, h)** coastal response of a **1½-layer model** when the wind is **independent of y**.

If the alongshore winds are directed **southward, they force offshore Ekman drift**. Since there can be no flow through the coast, the **thermocline must rise** to conserve mass. It rises until it **intersects the surface mixed layer**, and then **subsurface water entrains (upwells)** into surface layer.

The offshore decay scale of the circulation is the **Rossby radius of deformation, R**. There is a geostrophic **coastal current v in the direction of the wind**.

2-d response to switched-on τ^y

The solution to the 2-d coastal equations **with** is

$$h = H + \frac{\tau^y}{Rf} t e^{x/R}$$

For **southward winds ( ), *h* thins at the coast**, and the coastal response **weakens exponentially offshore with width scale *R***.

How long d See additional slides for a derivation of (3). **e coast?** For the parameter choices

$$H = 100 \text{ m}, \quad f = 10^{-4} \text{ s}^{-1}, \quad R = 25 \text{ km}, \quad \tau^y = 1 \text{ dyn/cm}^2$$

the time is **29 days**.

3-d response to switched-on τ^y (β=0)

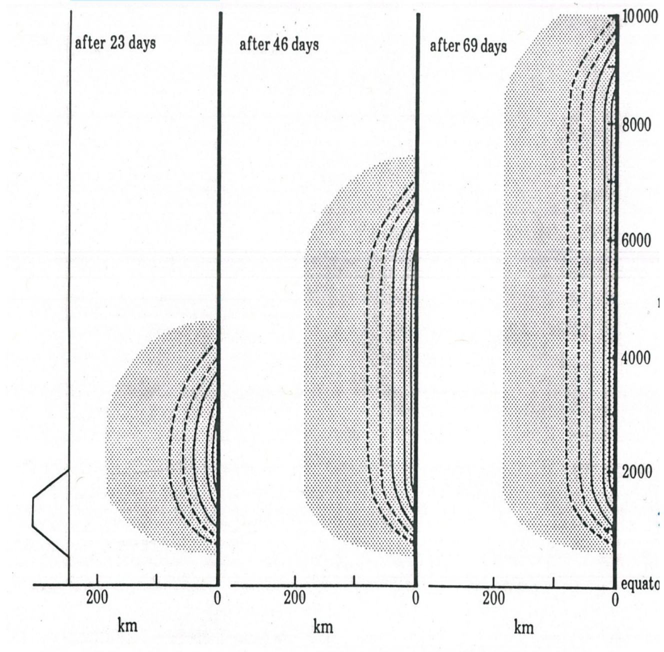
Two-dimensional coastal upwelling is altered dramatically when 3-d processes are included. Specifically, the **propagation of Kelvin waves** along the coast **stops the rise of *h***.

See additional slides for a derivation of this response.

upwelling, **coastal Kelvin waves extend the response north** of the forcing region. The **pycnocline tilts** in the latitude band of the wind, creating a **pressure force that balances** τ^y and stops the interface from rising further.

Movies H1a and H1b

***f*-plane**



3-d response to switched-on τ^y ($\beta \neq 0$)

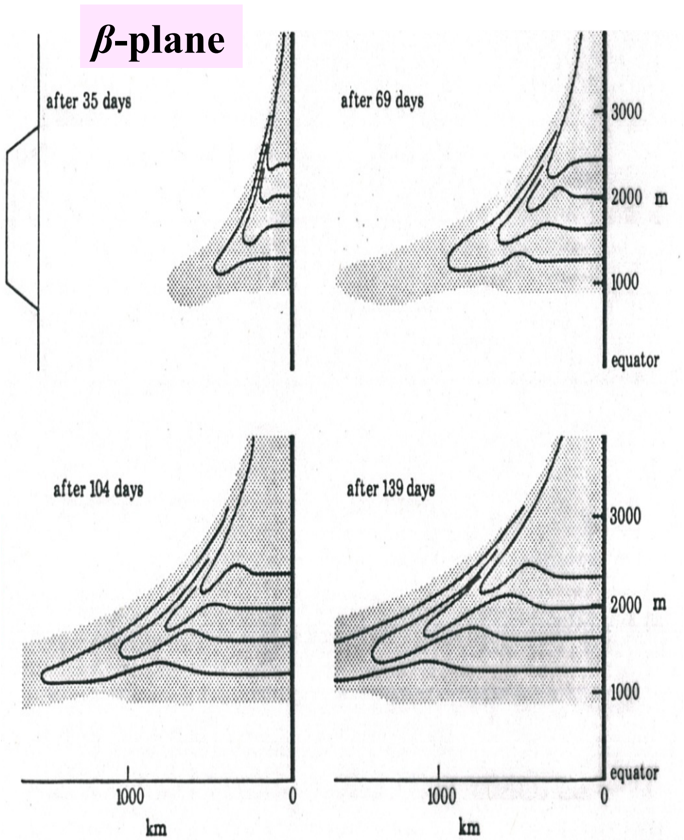
When $\beta \neq 0$, **Rossby waves carry the coastal response offshore**, leaving behind a state of rest in which p_y **balances** τ^y **everywhere**.

The RW speed is

$$c_r = -\beta \frac{c^2}{f^2}$$

So, **RWs propagate faster closer to the equator ($c_r \sim f$).**

Movie H1c

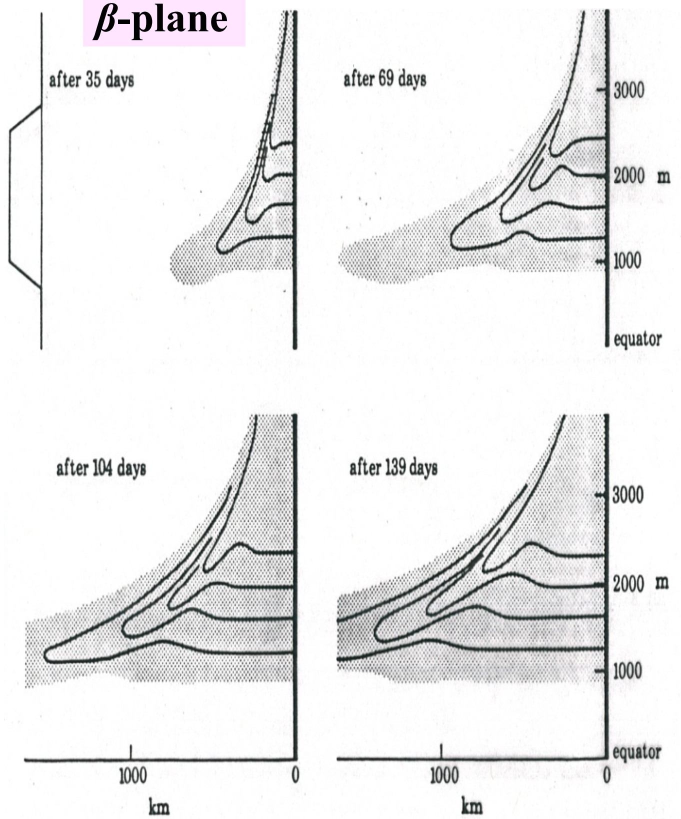


Multi-mode response to switched-on τ^y ($\beta \neq 0$)

A **fundamental question** of coastal dynamics is: Since Rossby waves propagate offshore, **why do eastern-boundary currents exist at all?**

A possible answer is that **many baroclinic modes contribute** to the coastal response, and that the **RWs associated with them are damped** before they can propagate offshore.

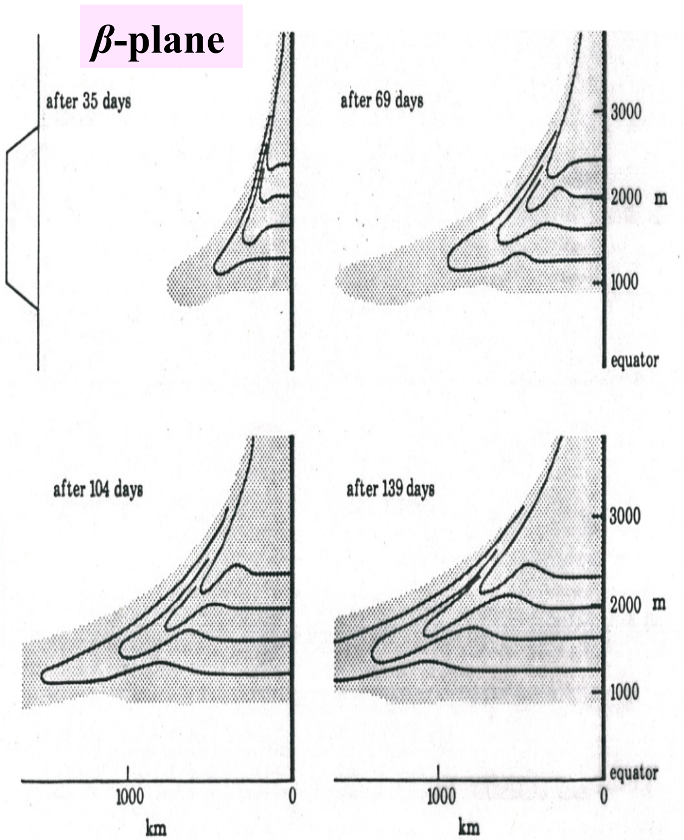
β-plane



Multi-mode response to switched-on τ^y ($\beta \neq 0$)

The plot shows the response of the ***n* = 1 mode without damping**. But, it also illustrates the ***n* > 1 responses**: the difference is that currents **propagate offshore more slowly**, since the RW propagation speed is $\propto c_n^{-2}$ **and** $c_n < c_1$.

With damping, the responses of the ***n* > 1 modes are increasingly damped** since $v = A/c_n^2$. In that case, the **Kelvin and Rossby waves that radiate from the forcing region are weakened** for larger n. For sufficiently large n, then, the response is **confined to the forcing region**.



Multi-mode response to switched-on $\tau \uparrow y$ $(\beta \neq 0)$

McCreary (1981) obtained a **steady-state, coastal solution to the LCS model with damping**.

The model **allows Rossby waves to propagate offshore**. A **steady coastal circulation remains**, however, because they are **damped by vertical diffusion**.

There is **upwelling in the band of wind forcing**. There is a **surface current** in the direction of the wind, and a **subsurface CUC**.

Movies I2c and I3c

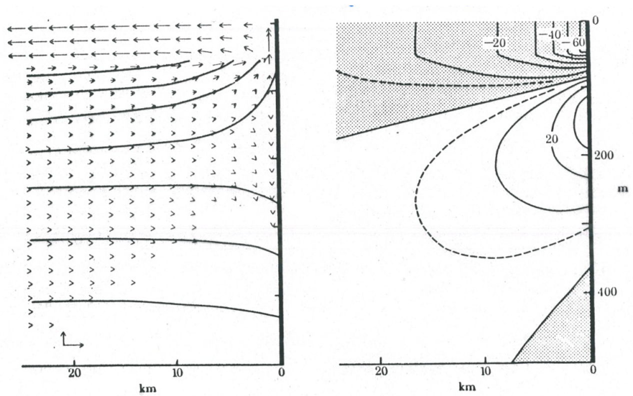
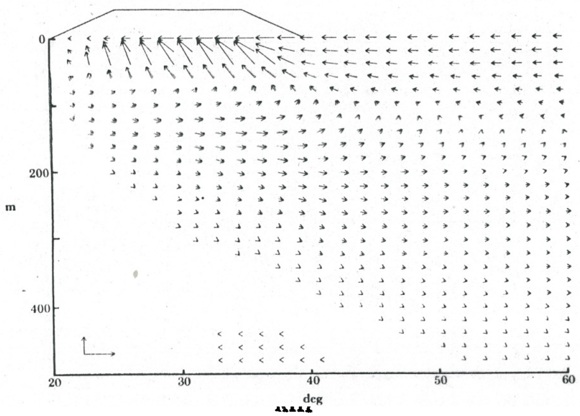
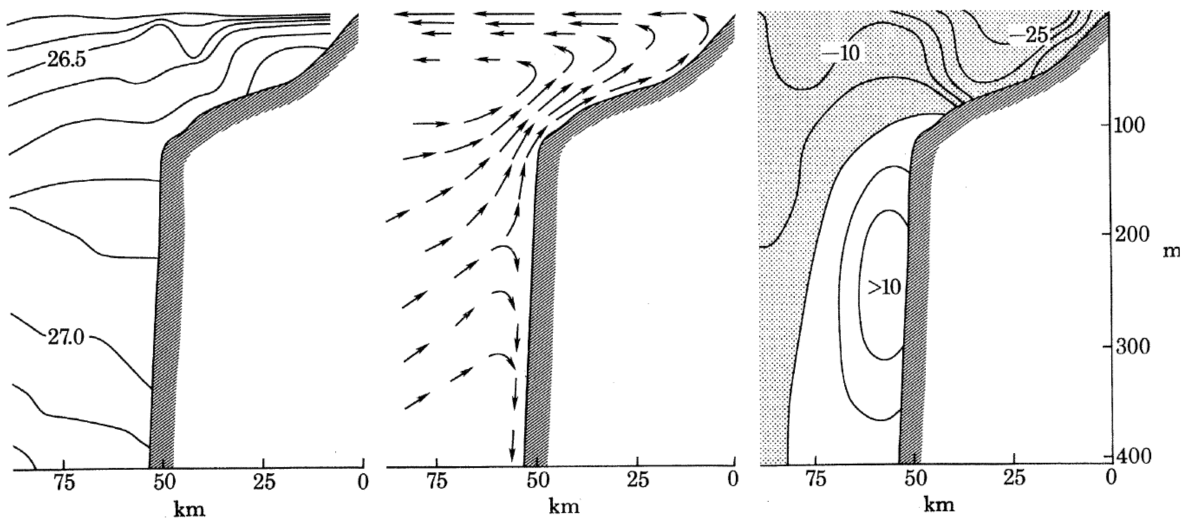


FIGURE 4*b*. As for figure 4*a*, but along section 2.



Observed eastern-coastal circulation



Mittelstadt et al. (1975), Huyer (1976)

The agreement with the McCreary (1981) model is striking. **Do eastern-boundary coastal in the real ocean exist due to diffusive damping?**

a **poleward undercurrent**, and **coastal upwelling**.

Multi-mode response to periodic τ^y ($\beta \neq 0$)

For a **switched-on** τ^y, **Kelvin waves** (**Rossby waves**) radiate poleward (offshore), leaving behind a **steady-state coastal circulation**.

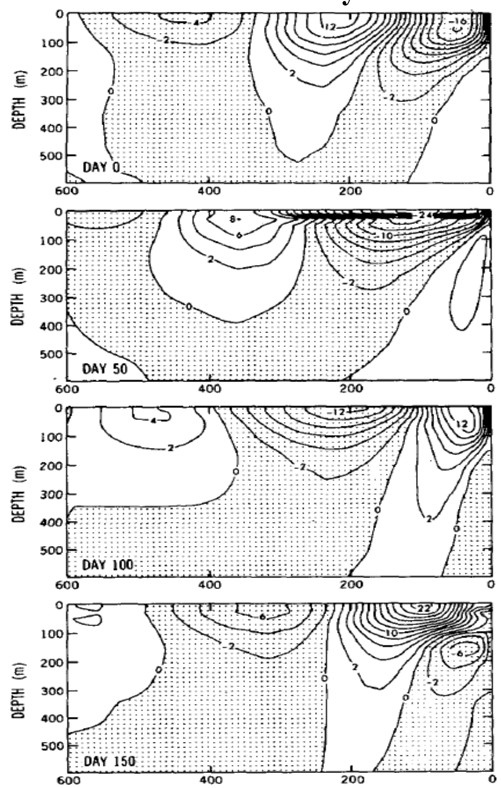
For a **periodic** τ^y, coastal **KWs and RWs** are **continually generated**.

Kelvin-wave packets associated with a number of baroclinic modes **form a "beam"** that **carries energy downward**. There is **upward phase propagation** across the beam.

Se… des for a… ion of Kelvin waves.

Movies K1e & K1d

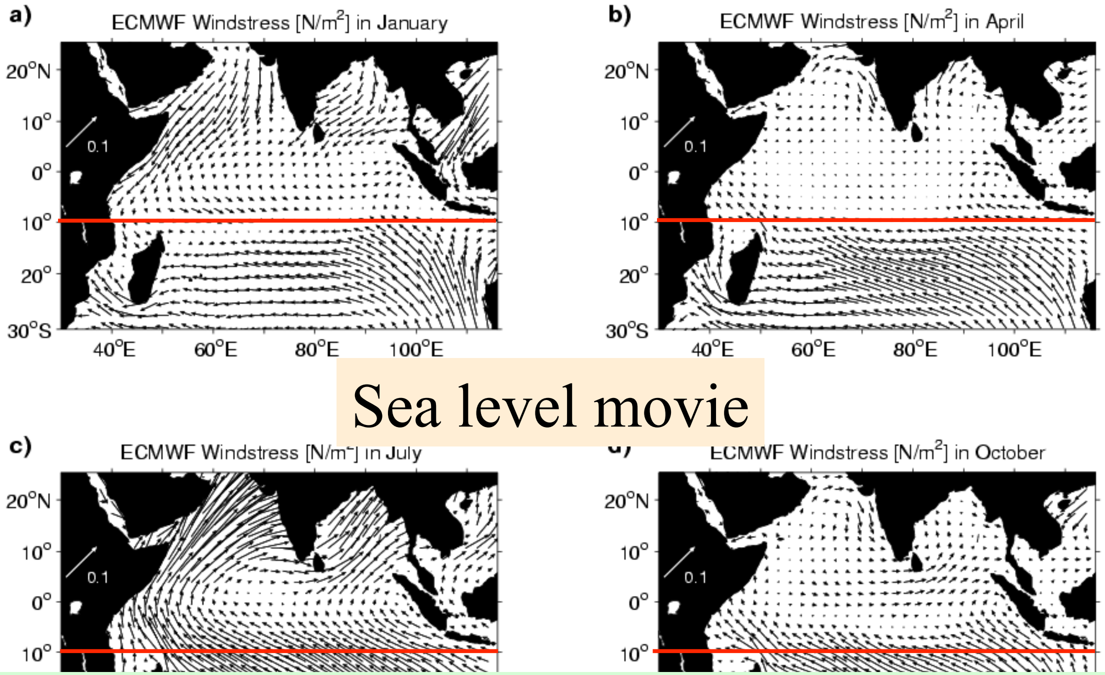
σ = 2π/200 days



Philander and Yoon (1982)

Summary

Indian Ocean phonomena



Sea level movie

The **winds in the North Indian Ocean** (north of ~10°S), **are highly variable** because of the monsoon. As a result, there are no **steady currents**, and the **propagation of remotely-forced waves** around the basin **is apparent**.

Additional slides

Hierarchy of ocean models:

derivation of LCS model equations

See **HIGNotes.pdf** for a detailed discussion.

LCS model

$$u_t - fv + \frac{1}{\bar{\rho}}p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}}p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$T_t + uT_x + vT_y + wT_z = (\kappa_T T_z)_z + \nu_h \nabla^2 T,$$

$$S_t + uS_x + vS_y + wS_z = (\kappa_S S_z)_z + \nu_h \nabla^2 S,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0,$$

$$\rho = \rho\,(S, T, p)$$

Impose ... $v_z)_z$.
Dropping ... the Vaisala frequency, ... **a very thin bound**... ally unimportan... s **GOOD**. and can on... fully with observations.

Linearize the equation of state to

Then, **set κ_T** ... **btain a single densi** ... **density effects in th** ... **κ_S deletes double diffu** ... enomena considered i ... **OOD**.

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}} p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$\rho_t + u\rho_x + v\rho_y + w\rho_z = (\kappa \rho_z)_z + \nu_h \nabla^2 \rho$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0,$$

$$\rho = \rho(S, T, p)$$

LCS model

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}} p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$\rho_t - w \frac{\bar{\rho}}{g} N_b^2 = (\kappa \rho_z)_z + \nu_h \nabla^2 \rho$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$$

The derivative ρ_{bz} is related to a **fundamental ocean frequency, the Vaisala frequency,** the square of which is

$$N_b^2(z) = -\frac{g}{\bar{\rho}} \rho_{bz}$$

Replace ρ_{bz} with N_b^2.

LCS model

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}} p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$\rho_t - w \frac{\bar{\rho}}{g} N_b^2 = (\kappa \rho)_{zz} + \nu_h \nabla^2 \rho$$

$$u_x + v_y + w_z = 0$$

Modify the form of **vertical diffusion from $(\kappa\rho_z)_z$ to $(\kappa\rho)_{zz}$**. This assumption is essential to allow the **expansion of solutions into vertical (barotropic and baroclinic) modes**. Since the precise form of vertical diffusion is not known, it is **OKAY**.

LCS model

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = \tau^x Z(z) + (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}} p_y = \tau^y Z(z) + (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$\rho_t - w \frac{\bar{\rho}}{g} N_b^2 = (\kappa \rho)_{zz} + \nu_h \nabla^2 \rho$$

$$u_x + v_y + w_z = 0$$

Wind stress enters the ocean in a surface mixed layer. To simulate this process in a simple way, we **introduce wind as a "body force" with the vertical profile *Z*(*z*)**. The body force differs from an actual mixed layer in that its **profile is uniform in space and constant in time**. This representation is **CONVENIENT** and **SENSIBLE**.

LCS model

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = \tau^x Z(z) + (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}} p_y = \tau^y Z(z) + (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$-\left(\partial_z \frac{1}{N_b^2} \partial_z\right) \frac{p_t}{\bar{\rho}} + u_x + v_y = -\left(\partial_z \frac{1}{N_b^2} \partial_z\right) \left[\left(\kappa \frac{p_z}{\bar{\rho}}\right)_z + \nu_h \nabla^2 \frac{p}{\bar{\rho}}\right]$$

$$w = -\frac{1}{\bar{\rho} N_b^2} \left[p_{zt} - (\kappa p_z)_{zz} - \nu_h \nabla^2 p_z\right]$$

$$\rho = -\frac{1}{g} p_z$$

Wind stress enters the ocean in a surface mixed layer. To simulate

Rewrite equations (1) − (3). First, **solve (1) for ρ** and **(2) for w in terms p_z**. Then, **insert both expressions into (3)**.

mixed layer in that its **profile is uniform in space and constant in time**. This representation is **CONVENIENT** and **SENSIBLE**.

LCS model

$$u_t - fv + \frac{1}{\bar\rho}p_x = \tau^x Z(z) + A_\nu\left(\partial_z \frac{1}{N_b^2}\partial_z\right)u + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar\rho}p_y = \tau^y Z(z) + A_\nu\left(\partial_z \frac{1}{N_b^2}\partial_z\right)v + \nu_h \nabla^2 v,$$

$$-\left(\partial_z \frac{1}{N_b^2}\partial_z\right)\frac{p_t}{\bar\rho} + u_x + v_y = -A_\kappa\left(\partial_z \frac{1}{N_b^2}\partial_z\right)^2\frac{p}{\bar\rho} - \nu_h\left(\partial_z \frac{1}{N_b^2}\partial_z\right)\nabla^2\frac{p}{\bar\rho}$$

$$w = -\frac{1}{\bar\rho N_b^2}\left[p_{zt} - (\kappa p_z)_{zz} - \nu_h \nabla^2 p_z\right]$$

$$\rho = -\frac{1}{g}p_z$$

Finally, assume that $\nu = A_\nu/N_b^2, \quad \kappa = A_\kappa/N_b^2$

In which case **all the *z*-operators have the same form**, a property **necessary to represent solutions as expansions in vertical modes**.

Mid-latitude ocean waves:

derivation of vln equation

Derivation of v_n equation

$$u_{nt} - f v_n + p_{nx} = 0$$

$$v_{nt} + f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0$$

$$u_{ntx} - f v_{nx} + p_{nxx} = 0$$

(−1) $$\frac{p_{ntt}}{c_n^2} + u_{nxt} + v_{nyt} = 0$$

$$p_{nxx} - \frac{p_{ntt}}{c_n^2} = v_{nyt} + f v_{nx}$$

$$\left(\partial_{xx} - \frac{1}{c^2}\partial_{tt}\right) p_n = v_{nyt} + f v_{nx}$$

Derivation of v_n equation

$$u_{nt} - f v_n + p_{nx} = 0$$

$$v_{nt} + f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0$$

$$\left(\partial_{xx} - \frac{1}{c^2}\partial_{tt}\right) p_n = v_{nyt} + f v_{nx}$$

$(-1/c_n{}^2)$ $\quad u_{ntt} - f v_{nt} + p_{nxt} = 0$

$$\frac{p_{ntx}}{c_n^2} + u_{nxx} + v_{nyx} = 0$$

$$u_{nxx} - \frac{u_{ntt}}{c^2} = -\frac{f}{c^2} v_{nt} - v_{nyx}$$

$$\left(\partial_{xx} - \frac{1}{c^2}\partial_{tt}\right) u_n = -\frac{f}{c^2} v_{nt} - v_{nyx}$$

Derivation of v_n equation

$$u_{nt} - f v_n + p_{nx} = 0$$

$$v_{nt} + f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0$$

$$\left(\partial_{xx} - \frac{1}{c^2}\partial_{tt}\right) p_n = v_{nyt} + f v_{nx}$$

$$\left(\partial_{xx} - \frac{1}{c^2}\partial_{tt}\right) u_n = -\frac{f}{c^2} v_{nt} - v_{nyx}$$

$$\left(\partial_{xx} - \frac{1}{c^2}\partial_{tt}\right) v_{nt} + f\left(-\frac{f}{c^2} v_{nt} - v_{nyx}\right) + \left(v_{nyt} + f v_{nx}\right)_y = 0$$

$$\left(\partial_{xx} - \frac{1}{c^2}\partial_{tt}\right) v_{nt} - \frac{f^2}{c^2} v_{nt} - \cancel{f v_{nyx}} + v_{nyyt} + \cancel{f v_{nxy}} + \beta v_x = 0$$

$$v_{xxt} + v_{yyt} - \frac{1}{c^2} v_{ttt} - \frac{f^2}{c^2} v_t + \beta v_x = 0$$

Mid-latitude ocean waves:

derivation of coastal Kelvin wave

Derivation of KW solution

$$u_{nt} + p_{nx} = 0$$

$$f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c^2} + u_{nx} = 0$$

$$(-c^2) \quad u_{ntx} + p_{nxx} = 0$$

$$p_{ntt} + c^2 u_{nxt} = 0$$

$$p_{ntt} - c^2 p_{nxx} = 0$$

Derivation of KW solution

$$u_{nt} + p_{nx} = 0$$

$$fu_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c^2} + u_{nx} = 0$$

$$p_{ntt} - c^2 p_{nxx} = 0$$

$$fu_{nt} + fp_{nx} = 0$$

$$(-1) \quad fu_{nt} + p_{nyt} = 0$$

$$fp_{nx} = p_{nyt}$$

Derivation of KW solution

$$u_{nt} + p_{nx} = 0$$

$$fu_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c^2} + u_{nx} = 0$$

$$p_{ntt} - c^2 p_{nxx} = 0$$

$$fp_{nx} = p_{nyt}$$

Look for solutions proportional to **exp (*ikx –iσt*)**. Set ***∂t* = *−iσ*** and ***∂x* = *ik***.

$$p_{ntt} - c^2 p_{nxx} = 0 \quad \Rightarrow \quad \sigma^2 = c^2 k^2 \quad \Rightarrow \quad \sigma = \pm ck$$

$$fp_{nx} = p_{nyt} \quad \Rightarrow \quad kfp_n = -\sigma p_{ny} \quad \Rightarrow \quad fp_n = \mp c p_{ny}$$

$$p_n = p_o \exp\left(\mp \alpha y\right) \exp\left[ik\left(x \mp ct\right)\right], \qquad \alpha = \frac{f}{c}$$

Interior ocean:

Ekman drift and inertial oscillations

Ekman drift and inertial oscillations (β = 0)

The most fundamental forced motion in the ocean is Ekman drift. In an inviscid, single-mode (or 1½-layer) model, **Ekman drift** occurs at an angle of **90° to the right (left) of the wind** in the **northern (southern) hemisphere**.

To illustrate this response as simply as possible, we assume that the **ocean is unbounded**, ***f* is constant**, and the forcing is by a **spatially uniform τ^x**. Then, the equation (1) simplifies to

$$v_{xxt} + v_{yyt} - \frac{1}{c^2}v_{ttt} \quad v_{tt} + f^2 v = -fF \quad F_{yx} - \frac{1}{c^2}G_{tt} + G_{xx} \qquad (2)$$

Why is it **"okay" to consider spatially uniform winds**? Because the typical **scale of wind the wind forcing** (~500−1000 km) is **much greater than the Rossby radius of deformation** (R ~ 25−50 km).

Ekman drift and inertial oscillations ($\beta = 0$)

Suppose the **wind switches on at $t = 0$**. We split the solution into a **time-independent, particular solution**

$$\cancel{v_{ptt}} + f^2 v_p = -fF \quad \Rightarrow \quad v_p = -\frac{F}{f}$$

and a **homogeneous solution that satisfies (2) with $F = 0$**

$$-v_{htt} + f^2 v_h = -\cancel{fF} \quad \Rightarrow \quad v_h = A \sin ft + B \cos ft$$

The **total solution** is then

$$v = -\frac{F}{f} + A \sin ft + B \cos ft$$

and ***A* and *B* are determined by applying initial conditions**.

Ekman drift and inertial oscillations ($\beta = 0$)

Assume that the ocean is at rest before the wind switches, so that appropriate **initial conditions are $u = v = 0$ at $t = 0$**.

We use the v momentum equation to **write the boundary condition for u in terms of v**. We have

$$v_t + fu = 0 \quad \Rightarrow \quad \boxed{v_t = 0 \quad @ \quad t = 0}$$

Applying the initial conditions gives

$$v(0) = -\frac{F}{f} + B = 0 \quad \Rightarrow \quad B = \frac{F}{f}$$

$$v_t(0) = fA = 0 \quad \Rightarrow \quad A = 0$$

so that

$$\boxed{v = -\frac{F}{f}(1 - \cos ft)}$$

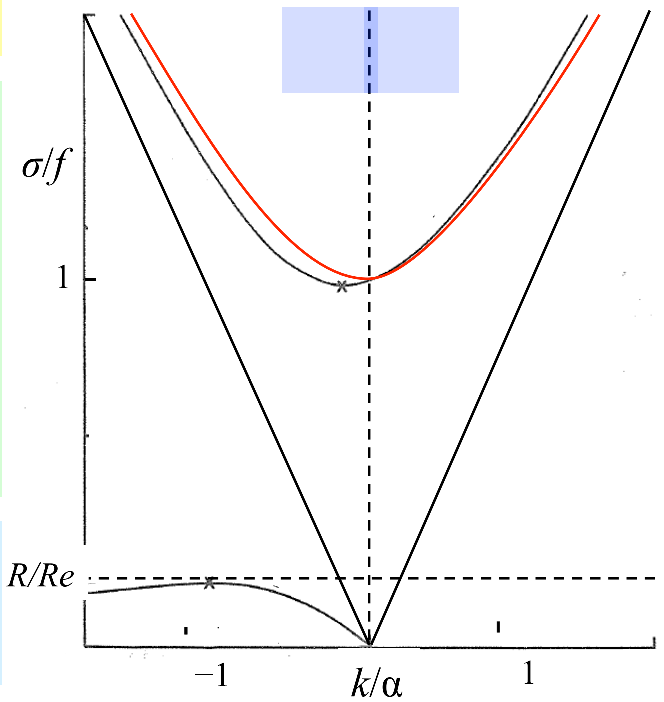
Ekman drift and inertial oscillations (β = 0)

The **steady-state solution is Ekman drift**, but **GWs at $\sigma = f$ are also generated** to satisfy the initial conditions.

Because **β = 0** and there are **no coasts, only GWs are possible.** Because the **wind is spatially uniform**, only GWs with **$k = \ell = 0$** can be excited. According to the disp. rel., the **waves with zero wavenumber are inertial waves with $\sigma = f$.**

If the **wind is not spatially uniform**, GWs with **$k > 0$ and $\sigma > f$** can are also excited.

$$v = -\frac{F}{f}(1 - \cos ft)$$



Ekman drift and inertial oscillations ($\beta = 0$)

To summarize, the solutions for ***u* and *v* when *f* is constant** are

$$u = -\frac{v_t}{f} = \frac{F}{f}\sin ft, \qquad v = -\frac{F}{f} + \frac{F}{f}\cos ft$$

a **steady, southward, Ekman drift** plus an **inertial oscillation** in which the **velocity vector rotates clockwise at a single frequency *f***.

Ekman drift and inertial oscillations ($\beta \neq 0$)

To summarize, the solutions for **u and v when f is constant** are

$$u = -\frac{v_t}{f} = \frac{F}{f}\sin ft, \qquad v = -\frac{F}{f} + \frac{F}{f}\cos ft$$

a **steady, southward, Ekman drift** plus an **inertial oscillation** in which the **velocity vector rotates clockwise at a single frequency f**.

Q: How does this simple response change when $\beta \neq 0$?

A: **Frequency f** and hence the clockwise **rotation of the velocity vector differ at each latitude**. Very quickly, **convergences (divergences) develop between different latitudes**, requiring water to downwell (upwell). This process **excites gravity waves with $\ell \neq 0$**, and is known as **β-dispersion**.

Movies B

Interior ocean:

derivation of Ekman pumping ($\beta = 0$)

Ekman pumping

Written **in terms of a 1½-layer model**, the interior-ocean equations are

$$-fv + g'h_x = \frac{\tau^x}{H}, \qquad fu + g'h_y = \frac{\tau^y}{H},$$

$$h_t + H\left(u_x + v_y\right) = -\kappa\left(h - H\right)$$

where

$$p_n \to g'\left(h - H\right), \quad c_n^2 \to g'H, \quad \mathcal{H}_n \to H, \quad \kappa = A/c_n^2$$

and the damping corresponds to **entrainment into or detrainment from the layer**.

Solving for a **single equation in *h*** gives

$$h_t - \beta\frac{g'H}{f^2}h_x + \ldots \equiv -w_{ek}$$

$$h_t + \kappa\left(h - H\right) = -w_{ek}$$

where ***w**$_{ek}$* **is the Ekman-pumping velocity**, the **rate at which wind curl raises or lowers subsurface isopycnals**.

Ekman pumping

When there is no damping ( ), the solution is

$$h = H - w_{ek} t = H + \frac{\tau_y^x}{f} t$$

so that ***h* grows continuously in time**. With damping (, it is

$$h = H - \frac{1 - e^{-\kappa t}}{\kappa} w_{ek} = H + \frac{1 - e^{-\kappa t}}{\kappa} \frac{\tau_y^x}{f}$$

so that ***h* stops growing**.

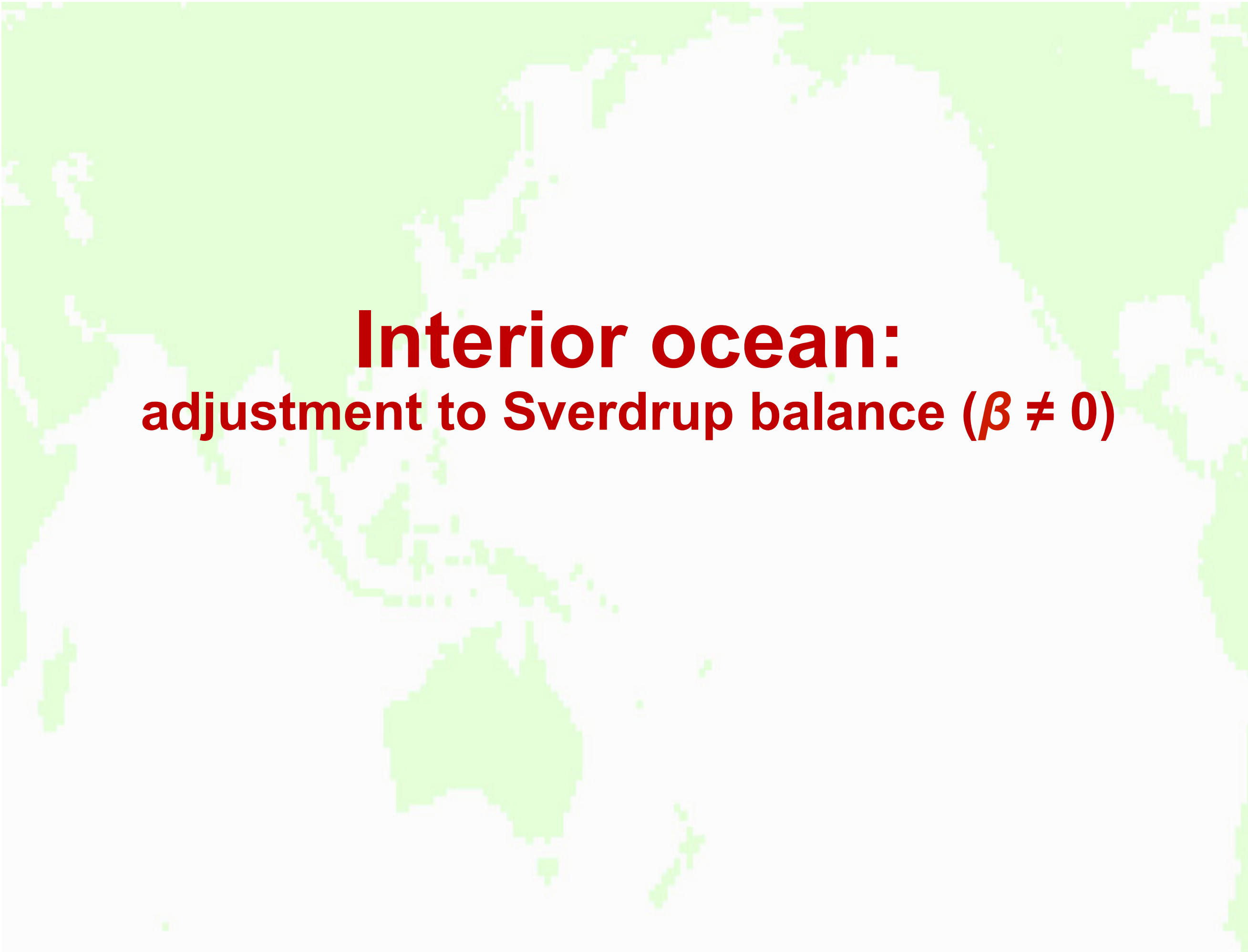
With h known, the zonal and meridional velocities are

$$v = \frac{1}{f} g' h_x - \frac{\tau^x}{fH}, \qquad u = -\frac{1}{f} g' h_y$$

a superposition of **Ekman and geostrophic currents**.

Interior ocean:

adjustment to Sverdrup balance ($\beta \neq 0$)



Adjustment when $\beta \neq 0$ and $\kappa = 0$

Suppose the model ocean allows ***f* to vary ($\beta \neq 0$)** and there is **no damping ($\kappa = 0$)**. Then, *h* satisfies

$$h_t - \beta \frac{g'H}{f^2} \boxed{h_t - c_r h_x = -w_{ek}, \qquad c_r = \beta \frac{g'H}{f^2}} \Bigg] \equiv -w_{ek}$$

We obtain the solution by splitting it into **steady-state (particular, forced)** and **transient (homogenous, wave)** parts

$$h_p - H = \boxed{\frac{1}{c_r} \int_{\infty}^{x} w_{ek}\, dx' \equiv \chi(x, y)}, \qquad h_h = \Lambda(x + c_r t, y)$$

where $\Lambda(x,y)$ is an as yet unspecified function.

To satisfy the **initial condition that $h = H$ at $t = 0$**, we must choose **$\Lambda(x,y) = -\chi(x,y)$**, so that

$$\boxed{h = h_p + h_h = H + \chi(x, y) - \chi(x + c_r t, y)}$$

Initial adjustment

To determine the **response a short time after the wind switches on**, we **expand the Rossby-wave term in a Taylor series about t = 0** to get

$$
\begin{aligned}
\lim_{t\to 0} h &= H + \chi(x,y) - \lim_{t\to 0}\chi(x + c_r t, y) \\
&= H + \chi(x,y) - \left[\chi(x,y) + c_r \chi'(x,y)\, t + \cdots\right] \\
&= H - c_r \chi'(x,y)\, t + \cdots = H - w_{ek} t + \cdots
\end{aligned}
$$

Thus, **at small times, the response is just Ekman pumping!**

The response does not change from Ekman pumping until the **Rossby waves have time enough to propagate significantly westward**.

Final adjustment

At longer times the solution for all the fields is

$$h = H + \frac{1}{c_r}\int_{\infty}^{x} \operatorname{curl}\left(\frac{\tau}{f}\right) dx' + \chi\left(x + c_r t, y\right)$$

$$v = \frac{g'}{f} h_x - \frac{\tau^x}{fH} = \frac{\operatorname{curl}\tau}{\beta H} + \frac{g'}{f}\chi_x\left(x + c_r t, y\right)$$

$$u = -\frac{g'}{f} h_y = -\frac{1}{H}\int_{\infty}^{x}\left(\frac{\operatorname{curl}\tau}{\beta}\right)_y dx' + \frac{g'}{f}\chi_y\left(x + c_r t, y\right)$$

where

$$\operatorname{curl}\tau = \tau^y_x - \tau^x_y, \qquad \operatorname{curl}\frac{\tau}{f} = \left(\frac{\tau^y}{f}\right)_x - \left(\frac{\tau^x}{f}\right)_y$$

A **packet of Rossby waves** propagates westward.

After their passage, the solution adjusts to a **steady-state Sverdrup balance**.

Interior ocean:

western boundary currents

Western-boundary currents

When **long-wavelength Rossby waves** (LWRWs) propagate to a western-ocean boundary, **zonal flow** associated with them is **channeled into a western-boundary current** (WBC).

Without momentum mixing, the LWRWs reflect as a packet of **short-wavelength Rossby waves** (SWRWs) that **continuously thins**.

Movie MassSource(300days).fli

Western-boundary currents

When **long-wavelength Rossby waves** (LWRWs) propagate to a western-ocean boundary, **zonal flow** associated with them is **channeled into a western-boundary current** (WBC).

Without momentum mixing, the LWRWs reflect as a packet of **short-wavelength Rossby waves** (SWRWs) that **continuously thins**.

Movie MassSource(300days).fli

With momentum mixing, the **WBC thinning stops** (or never appears at all) and its offshore structure **adjusts to steady-state profile**.

Movies D

Western-boundary currents

To find the structure of the western-boundary current, **neglect time-dependent and vertical-mixing terms** and **forcing terms** in the equations of motion, and for convenience **drop subscripts *n***.

$$-fv + p_x = -\nu u + \nu_h \nabla^2 u,$$

$$fu + p_y = -\nu v + \nu_h \nabla^2 v,$$

$$u_x + v_y = 0$$

Solving for a single equation in *v* then gives

$$-\beta v_x = \nu \nabla^2 v + \nu_h \nabla^4 v$$

Western-boundary currents

$$-\beta v_x = \nu v_{xx} + \nu_h v_{xxxx} \qquad (1)$$

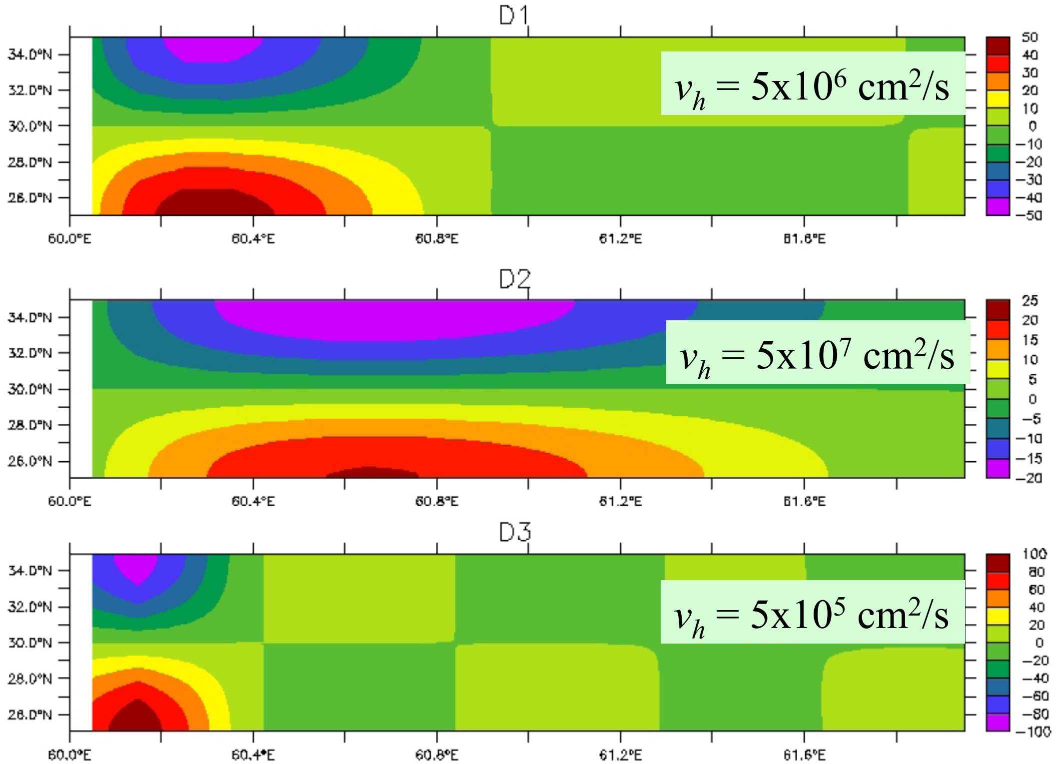
Adopting that **boundary-layer assumption that $L_y^2 \gg L_x^2$**, we **drop all *y*-derivative terms** from the right-hand side of (1).

With only Laplacian mixing ($v = 0$), then the solution to (1) is

$$v = v_0 \exp\left(-\frac{x}{r_m}\right) \sin\left(\frac{\sqrt{3}}{2}\frac{x}{r_m}\right), \qquad r_m = \left(\frac{\nu_h}{\beta}\right)^{\frac{1}{3}}$$

a **Munk layer**. This layer oscillates, as well as decays, offshore.

Western-boundary currents



In Solutions D1 − D3, the **WBCs are Munk layers** that decay and oscillate offshore.

Coastal ocean:

derivation of 2d coastal response

2-d response to switched-on τ↑y

It is easy to solve the coastal equations for the initial rise of the thermocline. At that time, the **response is inviscid**, and the coastal equations **written in terms of a 2-d, 1½-layer model** are

$$-fv + g'h_x = 0,$$

$$\frac{g'}{f}h_{xt} + fu = \frac{\tau^y}{H},$$

$$h_t + Hu_x = 0$$

Solving for a **single equation in *h*** gives

$$h_t - R^2 h_{xxt} = -\frac{\tau^y_x}{f} = 0, \qquad (1)$$

where $R^2 = g'H/f^2$ is the square of Rossby radius of deformation. The **forcing term vanishes** because **τ^y is independent of x**.

2-d response to switched-on τ^y

The general solution to (1) is

$$h = H \; \boxed{h = H + A\left(t\right) e^{x/R}} \; e^{-x/R}$$

The **coast is at $x = 0$** and the **ocean lies in the region $x < 0$**, so we have to **drop the B term** to ensure the solution is bounded as $x \to -\infty$.

To evaluate A, we **impose the boundary cond. that $u = 0$ at $x = 0$**. Using the v-momentum equation to **write u in terms of h** gives

$$fu = -v_t + \frac{\tau^y}{H} = -\frac{g'}{f} h_{xt} + \frac{\tau^y}{H} = 0 \quad @ \quad x = 0$$

and then

$$-\frac{g'}{f}\frac{A_t}{R} + \frac{\tau^y}{H} = 0 \quad \Rightarrow \quad \boxed{A = \frac{\tau^y}{H}\frac{f}{g'}Rt = \frac{\tau^y}{H}\frac{f^2}{g'}R\frac{t}{f} = \frac{\tau^y}{Rf}t}$$

2-d response to switched-on τ^y

The solution is then

$$h = H + \frac{\tau^y}{Rf} t e^{x/R}$$

For **southward winds ($\tau^y < 0$), *h* thins at the coast**, and the coastal response **weakens exponentially offshore with width scale *R***.

There is a **meridional geostrophic current associated with *h***,

$$v = \frac{g'}{f} h_x = \frac{g' \tau^y}{R^2 f^2} t e^{x/R} = \frac{\tau^y}{H} t e^{x/R}$$

a **coastally trapped jet flowing in the direction of the wind**.

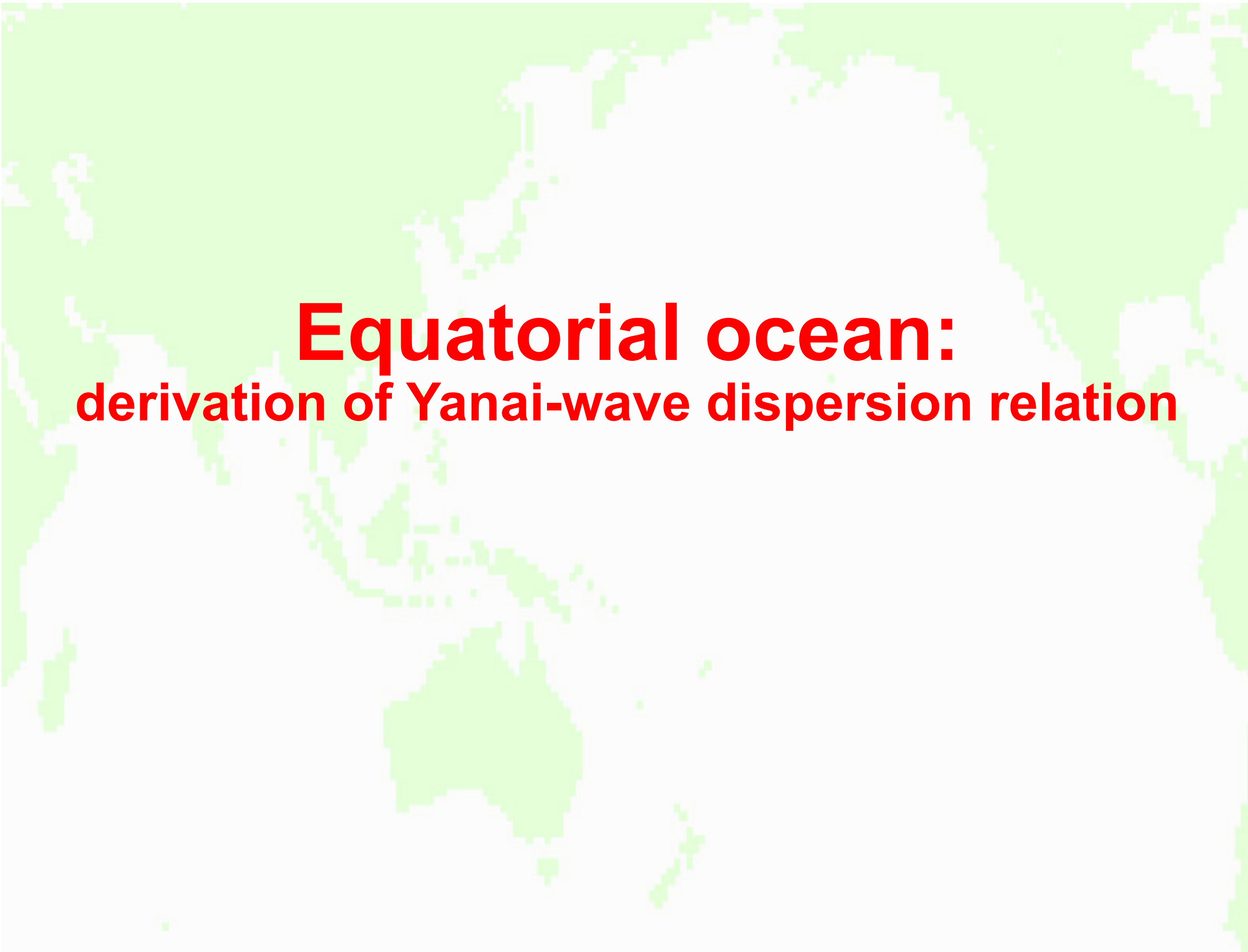
How long does it take for *h* to thin to the surface at the coast? For the parameter choices

$$H = 100 \text{ m}, \quad f = 10^{-4} \text{ s}^{-1}, \quad R = 25 \text{ km}, \quad \tau^y = 1 \text{ dyn/cm}^2$$

the time is **29 days**.

Equatorial ocean:

derivation of Yanai-wave dispersion relation



Mixed Rossby-gravity (Yanai) wave

The curious form of the Yanai-wave dispersion curve happens because it **factors into two parts when $\ell = 0$**. We have

$$\sigma \left(k^2 + \alpha_0^2 - \frac{\sigma^2}{c^2} \right) + k\beta = 0$$

$$\sigma \left(k^2 + \frac{\beta}{c} - \frac{\sigma^2}{c^2} \right) + k\beta = 0$$

$$\sigma \left(k^2 - \frac{\sigma^2}{c^2} \right) + \beta \left(k + \frac{\sigma}{c} \right) = 0$$

$$\left(k - \frac{\sigma}{c} + \frac{\beta}{\sigma} \right) \left(k + \frac{\sigma}{c} \right) = 0$$

Mixed Rossby-gravity (Yanai) wave

The curious form of the Yanai-wave dispersion curve happens because it **factors into two parts when $\ell = 0$**. We have

$$\left(k - \frac{\sigma}{c} + \frac{\beta}{\sigma}\right)\left(k + \frac{\sigma}{c}\right) = 0$$

The second factor describes a wave that **travels westward at the speed of a Kelvin wave**. It can be shown that this wave **blows up at ±∞, and so it must be discarded**.

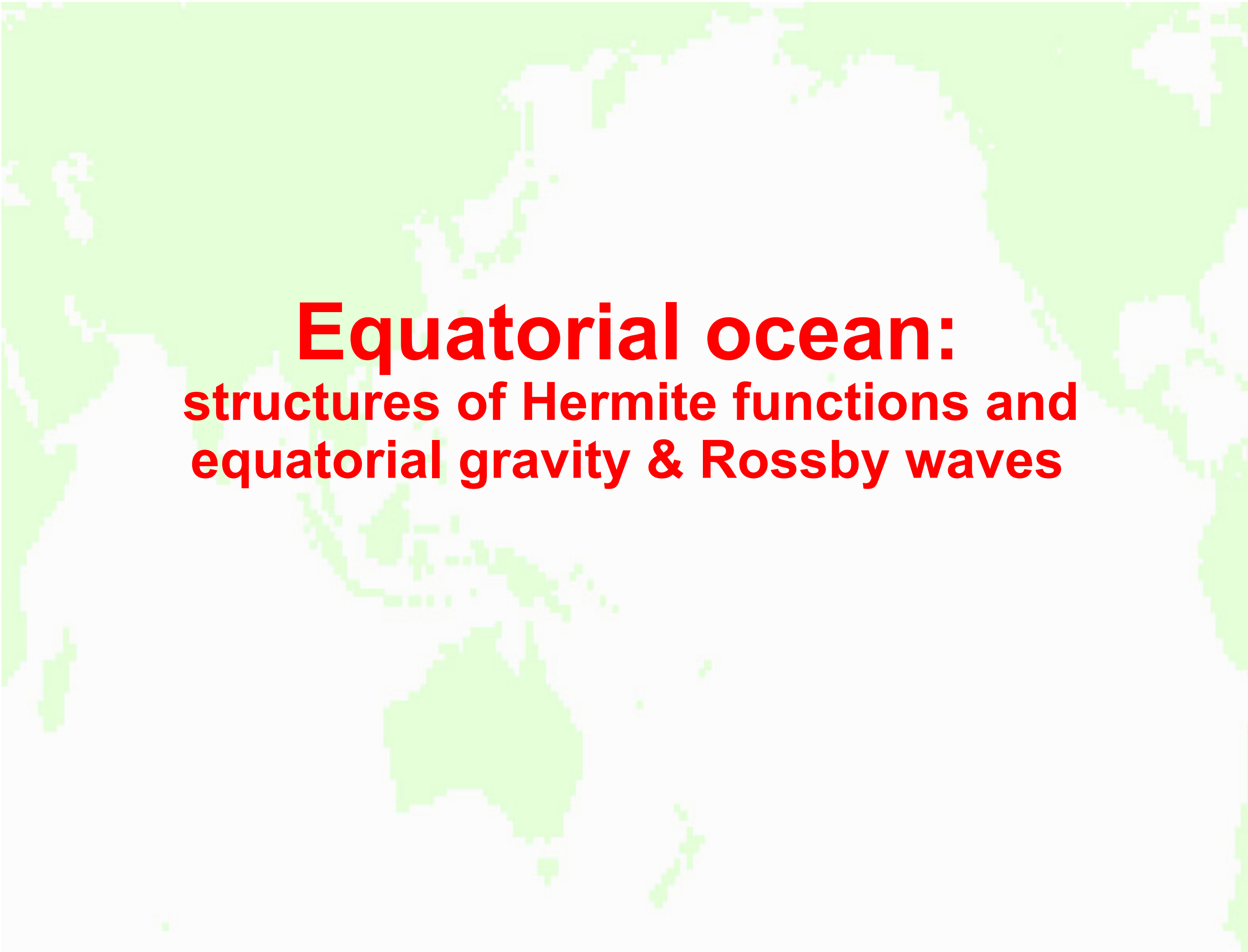
The single dispersion relation for the Yanai wave is then

$$\boxed{k - \frac{\sigma}{c} + \frac{\beta}{\sigma} = 0}$$

For **small and large values of σ**, the relation simplifies to,

$$\lim_{\sigma \to 0} k = -\frac{\beta}{\sigma}, \qquad \lim_{\sigma \to \infty} k = \frac{\sigma}{c}$$

the same **properties for Rossby and gravity waves**, respectively.



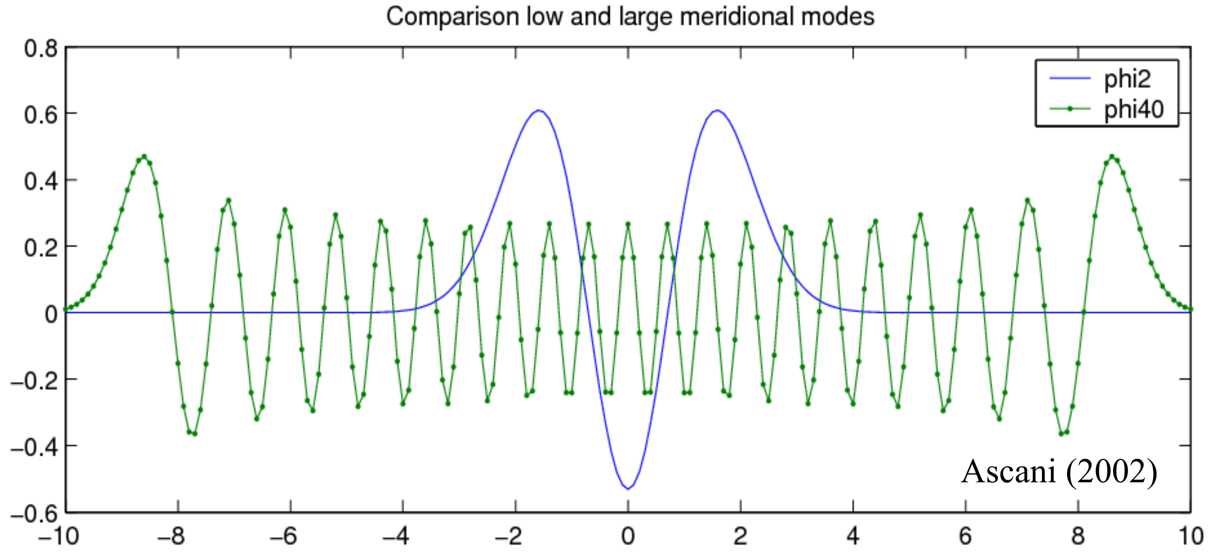
Equatorial ocean:

structures of Hermite functions and equatorial gravity & Rossby waves

Hermite functions

The figure plots the **first six Hermite functions ϕ_ℓ ($\ell = 0$–5)**. The

For large ℓ, the Hermite functions resemble **cosine or sine curves near the equator**. They begin to **decay at latitudes higher than the "turning latitude."** So, the **Hermite functions are equatorially trapped**.



Equatorial gravity and Rossby waves

The ***v_ℓ*, *u_ℓ*, and *p_ℓ* fields** for equatorially trapped **Rossby and gravity waves** are

$$v_\ell = \mathcal{V}_\ell \phi_\ell \exp\left(ik_j^\ell x - i\sigma t\right),$$

$$u_\ell = -ic\alpha_0 \mathcal{V}_\ell \left(\sqrt{\frac{\ell+1}{2}} \frac{\phi_{\ell+1}}{ck_j^\ell - \sigma} - \sqrt{\frac{\ell}{2}} \frac{\phi_{\ell-1}}{ck_j^\ell + \sigma} \phi_{\ell-1}\right) \exp\left(ik_j^\ell x - i\sigma t\right),$$

$$p_\ell = -ic^2\alpha_0 \mathcal{V}_\ell \left(\sqrt{\frac{\ell+1}{2}} \frac{\phi_{\ell+1}}{ck_j^\ell - \sigma} + \sqrt{\frac{\ell}{2}} \frac{\phi_{\ell-1}}{ck_j^\ell + \sigma} \phi_{\ell-1}\right) \exp\left(ik_j^\ell x - i\sigma t\right),$$

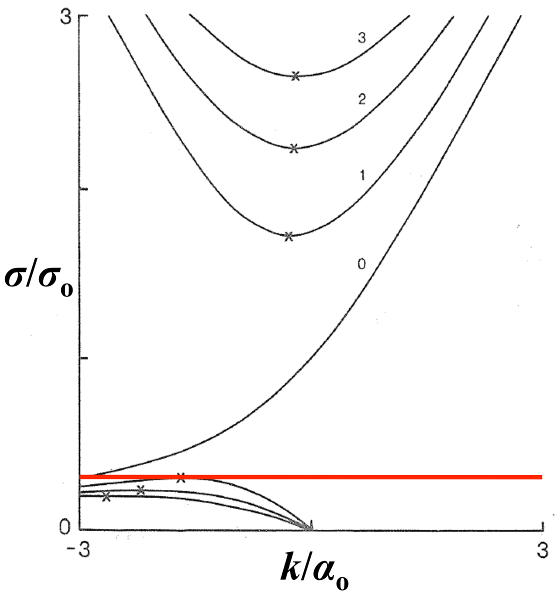
where ***V_ℓ* is a constant amplitude**, **is a Hermite function**,

,

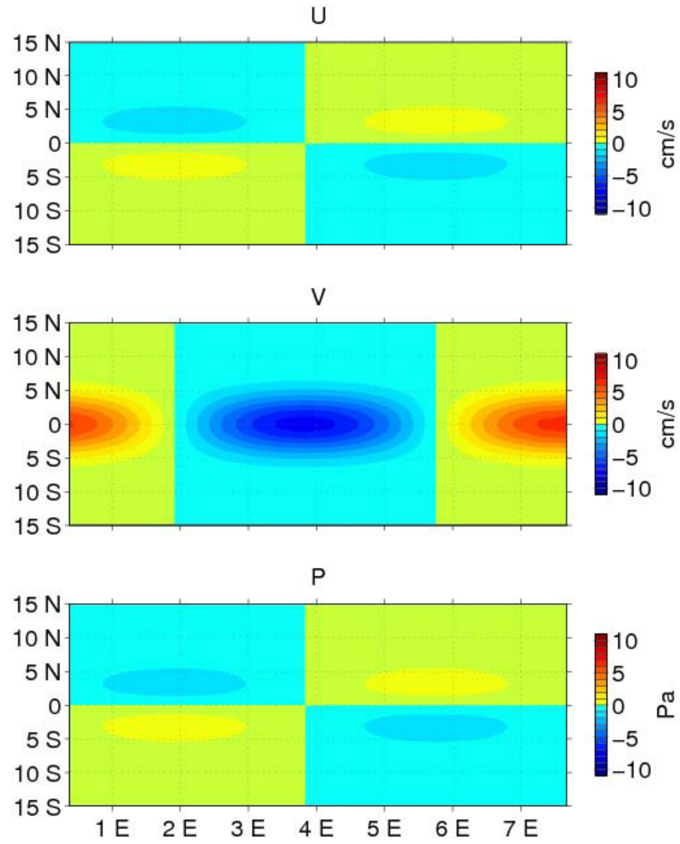
$$k_{1,2}^\ell = -\frac{\beta}{2\sigma}\left[1 \mp \sqrt{1 - 4\frac{\sigma^2}{\beta^2}\left(\alpha_\ell^2 - \frac{\sigma^2}{c^2}\right)}\right]$$

and ***j* = 1 (2)** corresponds to the **− (+) sign**.

Structure of Yanai waves

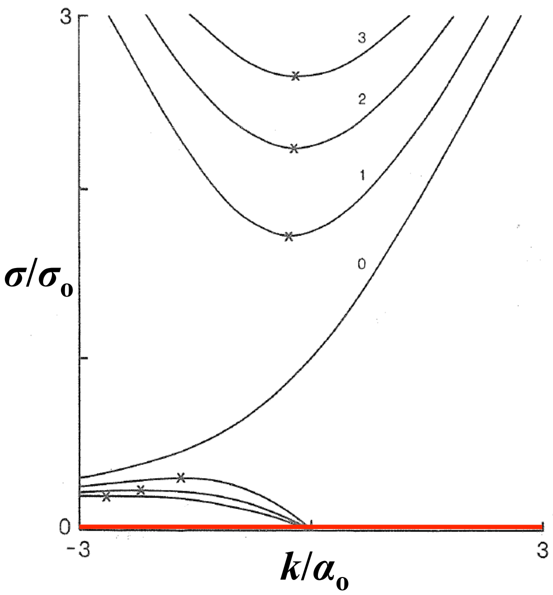


The *u*, *v*, and *p* fields for a **Yanai wave** when c_n **= 250 cm/s** and **P = 30 days**. For this P, $\sigma/\sigma_0 = .36$ and $\lambda = 7.3^o$.

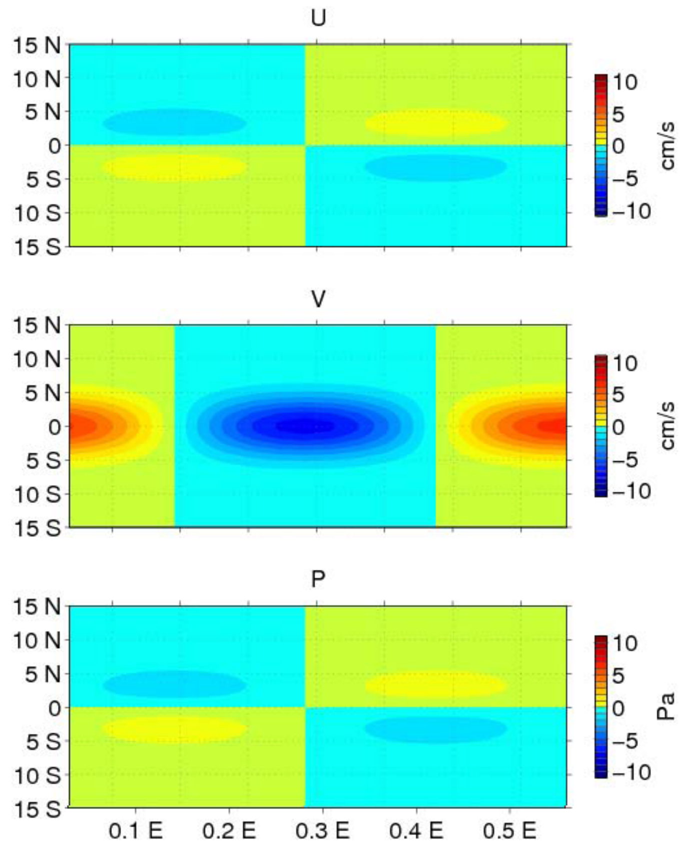


Courtesy of Francois Ascani

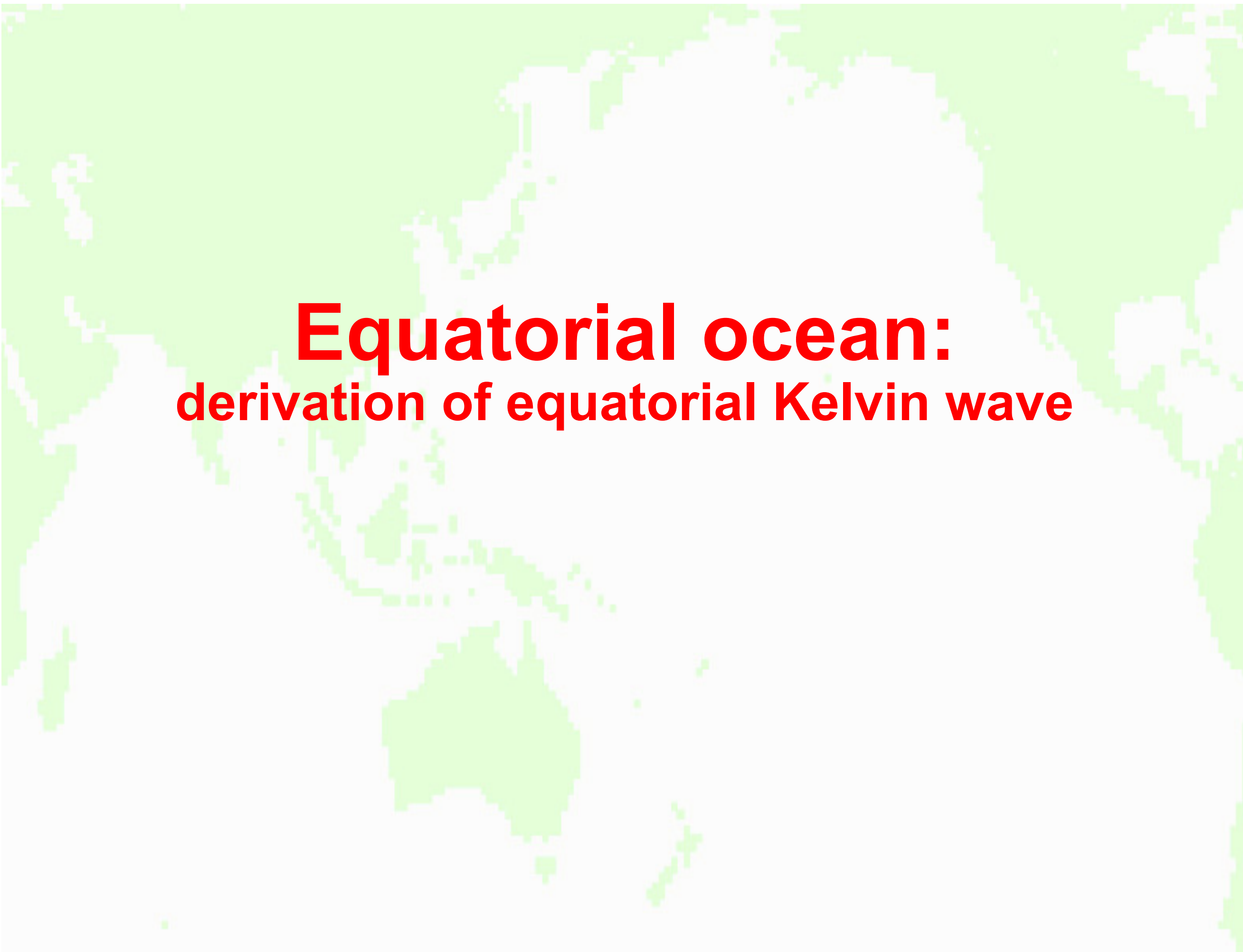
Structure of Yanai waves



The u, v, and p fields for a **Yanai wave** when $\boldsymbol{c_n}$ **= 250 cm/s** and **P = 360 days**. For this P, $\boldsymbol{\sigma/\sigma_o}$ **= .03** and $\boldsymbol{\lambda}$ **= 0.64º**.



Courtesy of Francois Ascani



Equatorial ocean:

derivation of equatorial Kelvin wave

Equatorial Kelvin wave

The **equatorial Kelvin wave has $v = 0$**, and so was **missed in the preceding solutions**. To find it, **set $v = A = 0$ in (1)**, and look for a free-wave solution of the form .

With these restrictions, equations (1) reduce to

$$-i\sigma u + ikp = 0, \qquad fu + p_y = 0, \qquad -i\sigma \frac{p}{c^2} + iku = 0.$$

The first and third equations imply

$$k = \pm\frac{\sigma}{c}, \qquad u = \frac{k}{\sigma}p,$$

and the second then gives

$$p_y = -fu = -f\frac{k}{\sigma}p = \mp\frac{f}{c}p = \mp\alpha_o^2 yp \tag{4}$$

Equatorial Kelvin wave

The solution to (4) is

$$p = P'_o \exp\left(\mp\frac{1}{2}\alpha_o^2 y^2\right)\exp\left(ikx - i\sigma t\right).$$

The **solution that grows exponentially in *y***, which corresponds to the root, $k = -\sigma/c$, **is physically unrealistic in an unbounded basin** and must be discarded. Therefore, the only possible wave is

$$p = P_o \phi_0(y) \exp\left[i\frac{\sigma}{c}(x - ct)\right], \qquad \sigma = kc, \tag{5}$$

which describes the structure and dispersion relation for the **equatorial Kelvin wave**. In (5), I have used the property that

$$\phi_0(y) = \pi^{-\frac{1}{4}} \exp\left(-\frac{1}{2}\alpha_o^2 y^2\right),$$

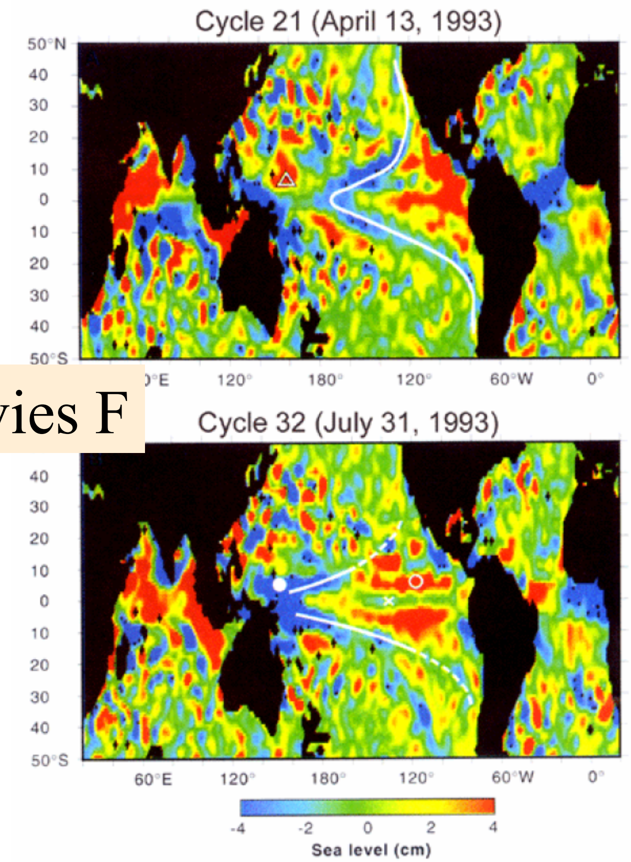
and redefined the arbitrary constant amplitude to be $P_o = \pi^{1/4} P'_o$.

Equatorial ocean:
eastern-boundary reflection

Eastern-boundary reflections

Remarkably, the characteristic **wedge shape and westward propagation is visible in satellite data**. The figure shows global maps of filtered sea level from TOPEX/Poseidon on April 13 and July 31, 1993. It sho **Rossby-wave packet gener by the reflection of an equatorial Kelvin wave forced by intraseasonal winds** in the western ocean. (After Chelton and Schlax, 1996.)

Movies F



Fedorov and Brown, 2007

Equatorial ocean:

Vertical propagation

Vertical propagation

Recall that the vertical structure of waves in the LCS model satisfy

$$\left(\frac{1}{N_b^2}\psi_{nz}\right)_z = -\frac{1}{c_n^2}\psi_n.$$

Rather than to look for solutions as expansions in vertical modes, $\psi_n(z)$, another way of studying solutions to the LCS model is to **look for approximate solutions of the form**,

$$\psi_n \propto \exp\left[im(z)z\right],$$

under the restriction that **the background stratification, $N_b(z)$ varies slowly with respect to the vertical wavelength of the wave, $m(z)$** (**the WKB approximation**). In that case,

$$\left(\frac{1}{N_b^2}\psi_{nz}\right)_z \approx -\frac{m^2}{N_b^2}\psi_n = -\frac{1}{c_n^2}\psi_n.$$

and **c_n can be replaced by**

$$c_n \rightarrow \frac{N_b}{|m|}$$

Vertical propagation (KW beams)

With this change, the dispersion relation for equatorial Kelvin waves is

$$\sigma = c_n k \quad \rightarrow \quad \sigma = N_b \frac{k}{|m|}.$$

Group theory states that a **packet of Kelvin waves** (that is, a superposition of several waves associated with different *k* and *m* values) **propagates at the "group" velocity**

$$c_{gx} = \sigma_k = \frac{N_b}{|m|}, \qquad c_{gz} = \sigma_m = \mp N_b \frac{k}{m^2}, \qquad m \gtrless 0$$

Thus, the **energy of the packet propagates to the east with the slope**

$$\frac{dz}{dx} = \frac{c_{gz}}{c_{gx}} = \frac{\sigma_m}{\sigma_k} = \mp \frac{\sigma}{N_b}, \qquad m \gtrless 0$$

Since **coastal Kelvin waves have the same dispersion relation** as equatorial ones, they **propagate vertically in the same way**.

Vertical propagation (YW beams)

The dispersion relation for Yanai waves becomes

$$k - \frac{\sigma}{c} + \frac{\beta}{\sigma} = 0 \quad \rightarrow \quad k - \frac{|m|}{N_b}\sigma + \frac{\beta}{\sigma} = 0$$

Group theory states that a **packet of Yanai waves** (that is, a superposition of several waves associated with different *k* and *m* values) **propagates at the "group" velocity**

$$1 - \frac{|m|}{N_b}\sigma_k - \frac{\beta}{\sigma^2}\sigma_k = 0 \quad \Rightarrow \quad c_{gx} = \sigma_k = \left(\frac{|m|}{N_b} + \frac{\beta}{\sigma^2}\right)^{-1}$$

$$0 \mp \frac{\sigma}{N_b} - \frac{|m|}{N_b}\sigma_m - \frac{\beta}{\sigma^2}\sigma_m = 0 \quad \Rightarrow \quad c_{gz} = \sigma_m = \mp\frac{\sigma}{N_b}\left(\frac{m}{N_b} + \frac{\beta}{\sigma^2}\right)^{-1}$$

Thus, the **energy of the packet propagates to the east with the slope**

$$\frac{\partial z}{\partial x} = \frac{c_{gz}}{c_{gx}} = \mp\frac{\sigma}{N_b}, \qquad m \gtrless 0$$

the **same slope as for Kelvin waves!**

Vertical propagation (long-wavelength RWs)

For the RW dispersion curves, as ***σ* tends to zero so does *k***.

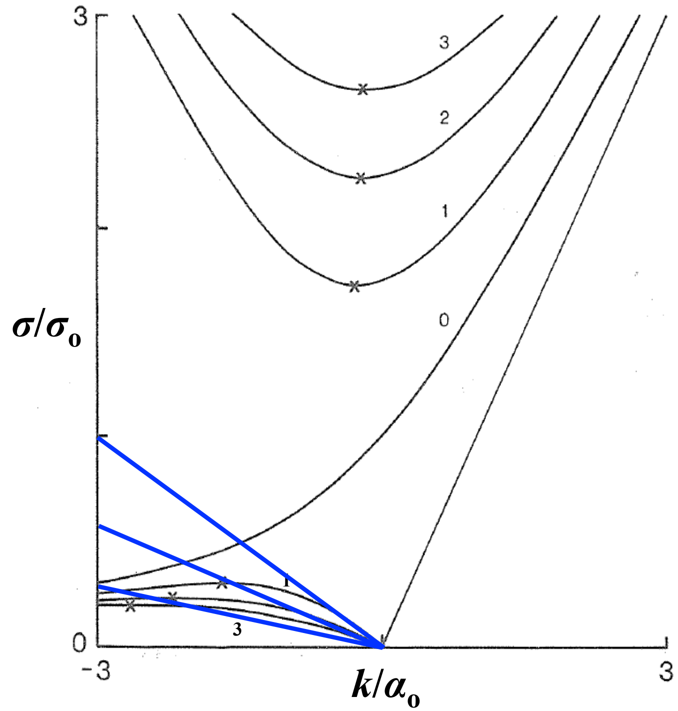
So, in the **low-frequency limit** the **RW disp. curves are non-dispersive**. This limit is known as the **long-wavelength approximation**.

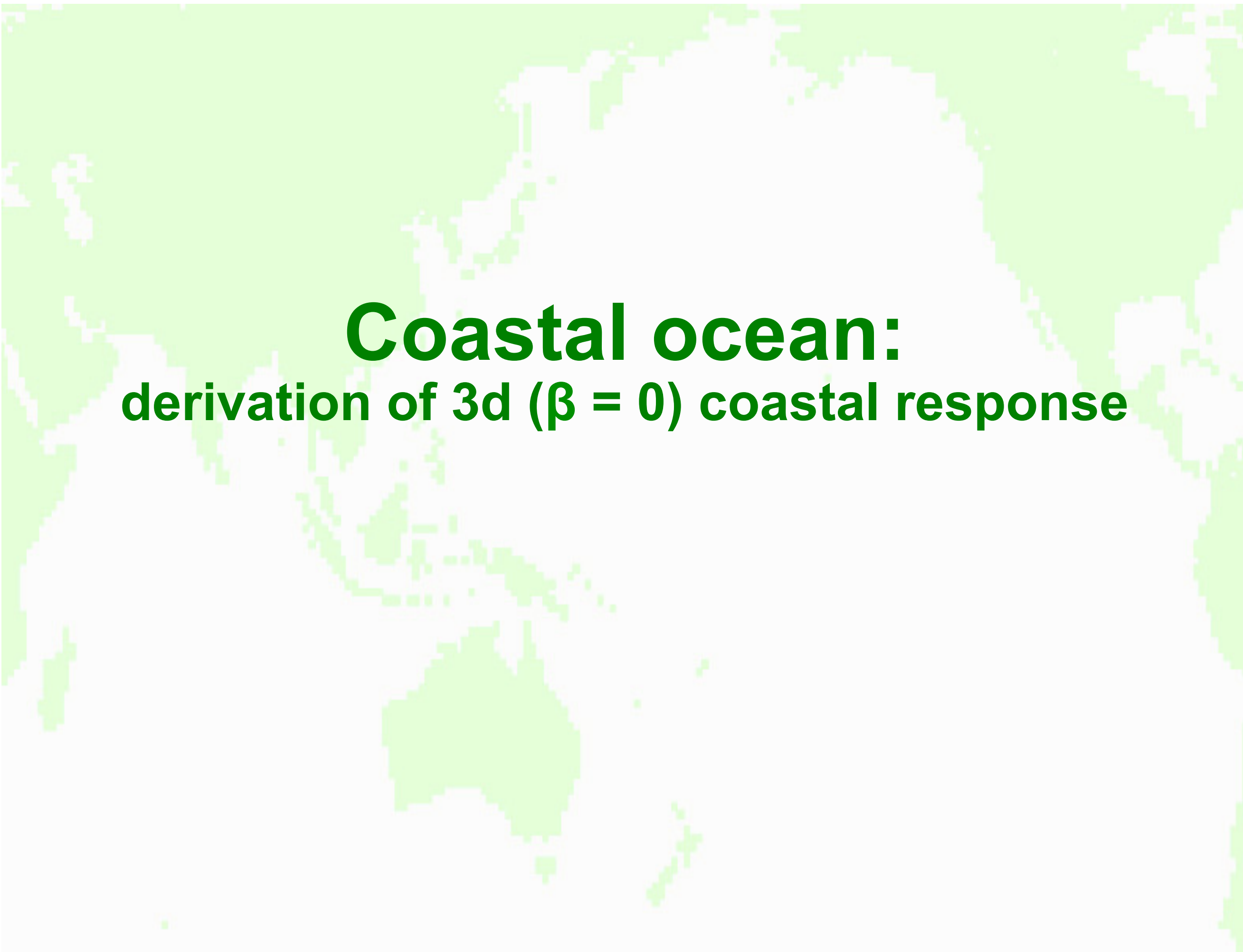
In this limit, **RWs propagate vertically with a slope**

$$\frac{dz}{dx} = \frac{\sigma_m}{\sigma_k} = \frac{\sigma}{N_b}(2\ell + 1)$$

with a **steeper slope, and in the opposite direction from, KW and YWs**.

$$\sigma = -k\frac{\beta}{\alpha_\ell^2} = -\frac{c}{2\ell+1}k$$





Coastal ocean:

derivation of 3d ($\beta = 0$) coastal response

3-d response to switched-on τ^y

To see these properties, we solve the coastal equations **keeping the v_y and h_y terms**. Then, the **inviscid** coastal equations **written in terms of a 1½-layer model** are

$$-fv + g'h_x = 0,$$

$$v_t + fu + g'h_y = \frac{\tau^y}{H},$$

$$h_t + H\left(u_x + v_y\right) = 0$$

Solving for a **single equation in *h*** gives

$$h_t - R^2 h_{xxt} = -\frac{\tau^y_x}{f} = 0, \qquad (1)$$

where $R^2 = g'H/f^2$ is the square of Rossby radius of deformation. The **forcing term vanishes** because **τ^y is independent of x**.

3-d response to switched-on τ^y

The general solution to (1) is

$$h = H \; \boxed{h = H + A\left(y,t\right) e^{x/R}} \;)\, e^{-x/R} \qquad (1)$$

The **coast is at $x = 0$** and the **ocean lies in the region $x < 0$**, so we have to **drop the B term** to ensure the solution is bounded as $x \to -\infty$.

To evaluate A, we **impose the boundary cond. that $u = 0$ at $x = 0$**. Using the v-momentum equation to **write u in terms of h** gives

$$fu = -\frac{g'}{f} h_{xt} - g' h_y + \frac{\tau^y}{H} = 0 \quad @ \quad x = 0$$

which, using (1), provides an equation for A,

$$\boxed{A_t + cA_y = \frac{\tau^y}{c}, \qquad c = \sqrt{g'H}}$$

3-d response to switched-on τ^y

We obtain the solution for A by splitting it into **particular (steady-state)** and **homogeneous (Kelvin-wave) responses**,

$$A_p = \int_{-\infty}^{y} \frac{\tau^y}{g'H}\, dy' \equiv \chi(y)\,, \qquad A_h = \Lambda(y - ct)$$

where $\Lambda(x,y)$ is an as yet unspecified function.

To satisfy the **initial condition that $h = H$ at $t = 0$**, we must choose **$\Lambda(y) = -\chi(y)$**, so that

$$h = H + (A_p + A_h)\, e^{x/R} = H + \chi(y)\, e^{x/R} - \chi(y - ct)\, e^{x/R}$$

Initial adjustment

To determine the **response a short time after the wind switches on**, we **expand $\chi(y - ct)$ in a Taylor series about $t = 0$** to get

$$\lim_{t\to 0} h = H + \chi(y) - \lim_{t\to 0} \chi(y - ct)$$

$$= H + \chi(y) - [\chi(y) - c\chi'(y)\,\boldsymbol{t} + \cdots]$$

$$= H + c\chi'(x, y)\,t + \cdots = H + \frac{\tau^y}{c} t + \cdots$$

Thus, **at small times, the response is just the 2-d response!**

The response does not change from the 2-d response until the **Kelvin waves have propagated across the wind band**.

Final adjustment

At longer times the solution for all the fields is

$$h = H + \left(\int_{-\infty}^{y} \frac{\tau^y}{g' H} \, dy' \right) e^{x/R} - \chi\,(y - ct)\, e^{x/R}$$

$$v = \frac{g'}{f} h_x = \frac{1}{R} \left(\int_{-\infty}^{y} \frac{\tau^y}{f H} \, dy' \right) e^{x/R} - \frac{c}{H} \chi\,(y - ct)\, e^{x/R}$$

$$u = -\frac{g'}{f^2} h_{xt} - \frac{g'}{f} h_y = \frac{\tau^y}{f H} \left(1 - e^{x/R}\right)$$

A **packet of Kelvin waves** propagates poleward. Note that, consistent with Kelvin waves, there is **no *u* field associated with the packet**.

After its passage, the solution adjusts to a **steady-state balance**.

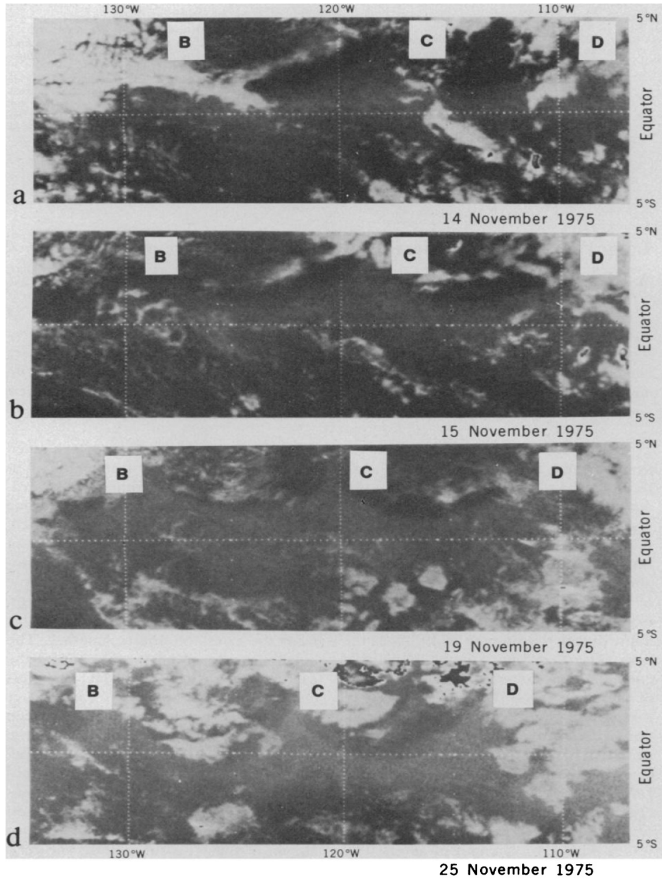
Key properties of the steady solution are: 1) a **pressure gradient that balances the wind along the coast** ($x = 0$), that is, $p_y = g' h_y = \tau^y / H$; 2) a **coastal jet with a transport *HRv*** that supplies the Ekman transport from the coast; and 3) **Ekman drift that weakens to zero at the coast**.

Intraseasonal variability

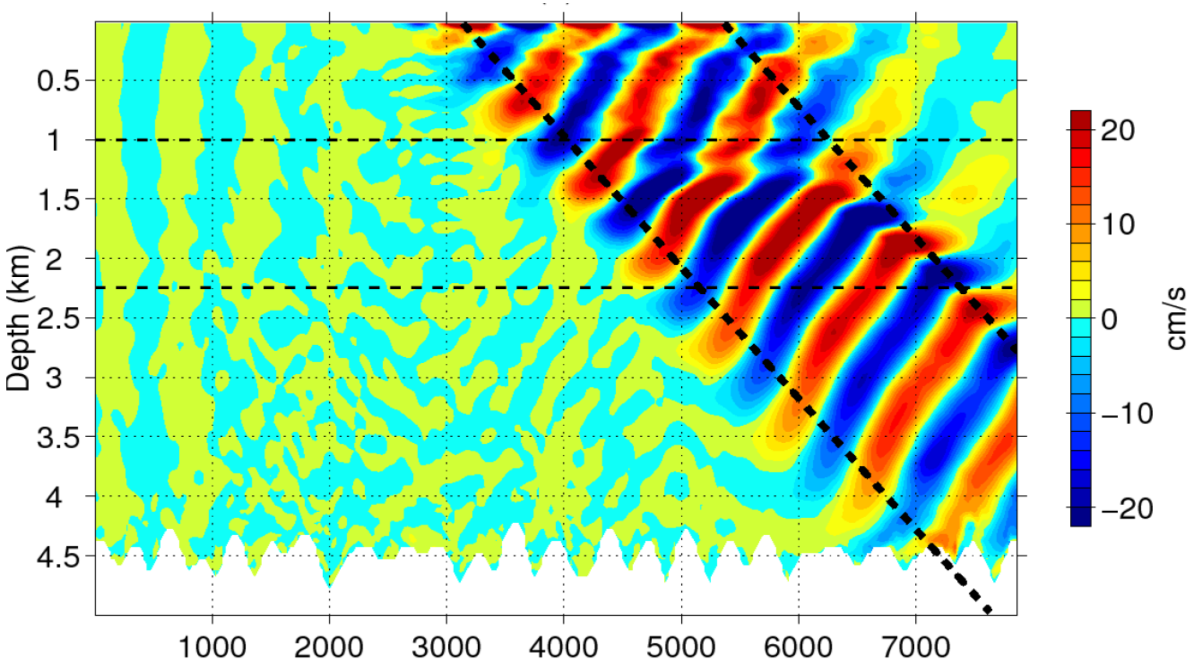
Tropical instability waves

Legeckis (1977, *Science*) first reported the presence of TIWs in the eastern, tropical Pacific. TIWs were soon shown to have a **large impact on the momentum and heat fluxes** in the region. Philander (1976, 1978, *JGR*) argued that TIWs were caused by **barotropic instability**. Yu *et al.* (1992, *Prog.* Oceanogr.) later suggested that an **instability of the temperature front** was involved. Luther and Johnson (1990) suggested that there was **more than one type of TIWs**.

Similar TIWs were soon observed in the Atlantic Ocean. Their dynamics are essentially the same as for the Pacific TIWs.

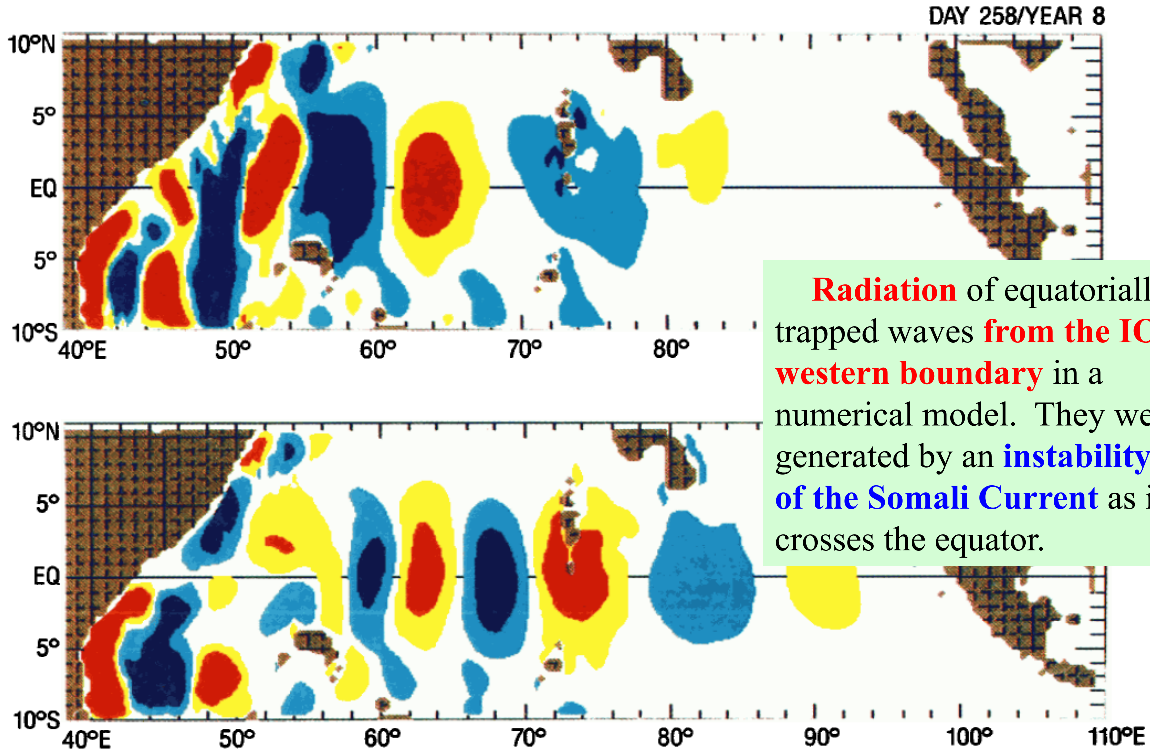


Tropical instability waves



Michael Cox (1980) reported a **Yanai-wave beam forced by surface TIWs in his OGCM solution**. Ascani & coworkers (2009) explored the idea that deep equatorial currents are caused by an **instability of the Yanai-wave beam generated by TIWs**. To simulate the effect of TIWs, they forced their OGCM by a **wind stress with the wavelength (~1000 km) and period (~30 days) of a typical TIW**, generating the Yanai-wave beam shown above.

Somali Current instability (27 days)



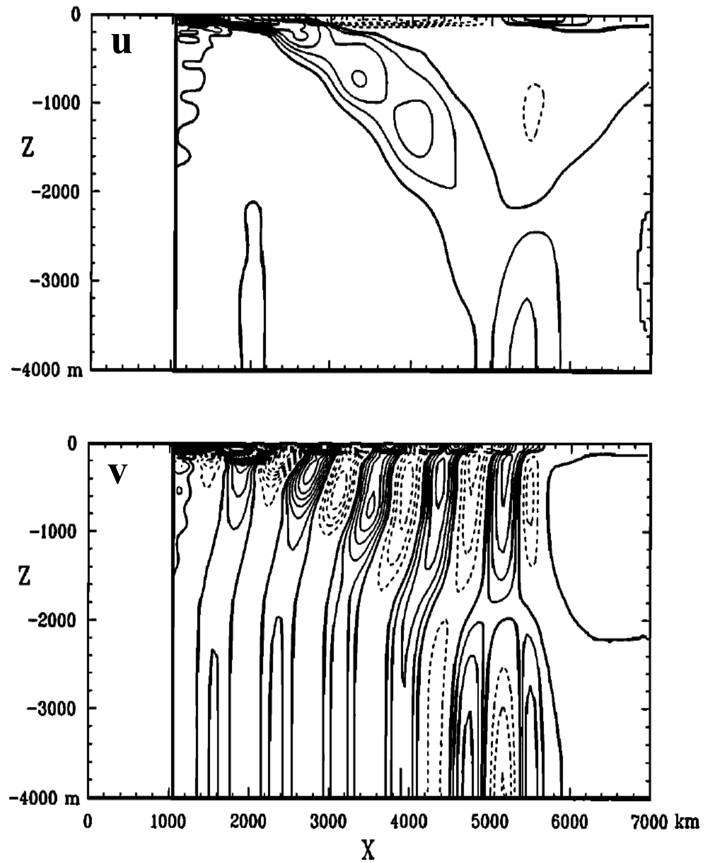
Radiation of equatorially trapped waves **from the IO western boundary** in a numerical model. They were generated by an **instability of the Somali Current** as it crosses the equator.

Their **structure identifies the waves to be Yanai waves**. These modeled waves were **observed in altimetry by Tsai et al. (1992)**.

Somali Current instability (27 days)

When all the vertical modes are summed, **energy propagates downward** as well as eastward, **along paths parallel to the group velocity**. At the right of the plots, energy has **reflected off the bottom** to produce an **upward-propagating beam**.

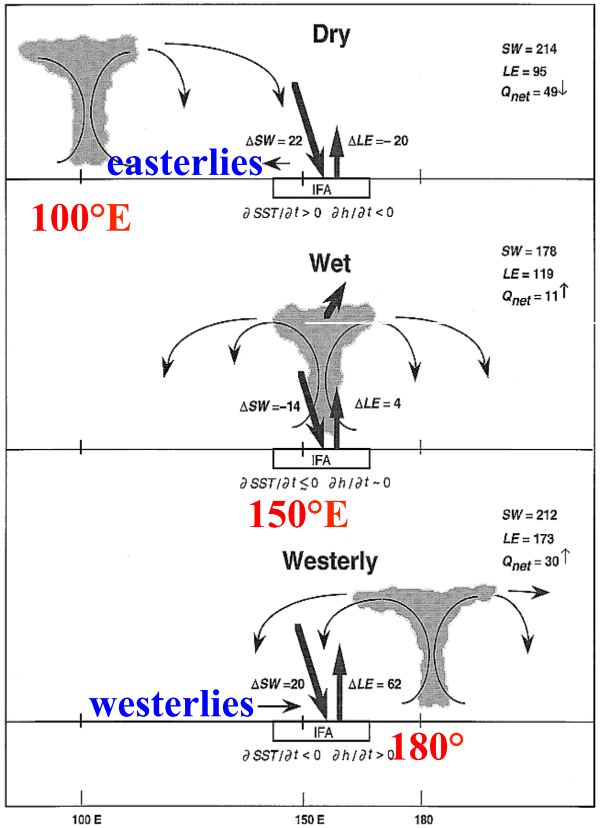
The presence of **intraseasonal variability at a depth of 750 m** in the Luyten and Roemmich (1982) observations thus appears to **result from the radiation of a beam of Yanai waves from the Somali coast**.



Madden Julian Oscillations (30−60 days)

In the central IO, oceanic ISV appears to be mostly wind-forced. A prominent forcing is by **MJOs, eastward-propagating, convective disturbances**, with **periods of 30–60 days**.

Their **impacts on rainfall, oceanic surface fluxes, and SST** are well documented.



Waliser, Murtugudde, Lucas (2003, 2004)

ISV in the EIO (~90 days)

Courtesy of Jerome Vialard