**Basic ocean processes, as illustrated in solutions to the LCS model**

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During June and July of 2010, a summer school, entitled “Dynamics of the North Indian Ocean,” took place at the National Institute of Oceanography (NIO), Goa, India. Fifty-eight participants, of whom 21 were from outside NIO, were admitted to the school. Dr. McCreary was the principal lecturer, with supporting presentations by Dr. S.R. Shetye, Prof. P.N. Vinayachandran, Dr. M. Ravichandran, and Dr. D. Shankar. The course began with an overview of prominent observational features in the NIO, but it focused on understanding the basic, dynamical processes that generate them. Theory was presented hierarchically, by first obtaining solutions to simple models in idealized situations and then using these solutions to interpret realistic solutions to state-of-the-art, ocean general circulation models (OGCMs).

Classes on theory consisted of two parts: morning lectures, in which analytic solutions illustrating basic processes were derived, and afternoon tutorials, where corresponding numerical solutions were obtained and movies made of them. All the solutions used the linear, continuously stratified (LCS) model, which represents solutions as expansions of barotropic and baroclinic modes. In each tutorial session, participants were given the model code, as well as ancillary codes used to generate movies. Time was set aside to run numerical experiments, and to prepare and discuss movies made from them. Feedback from the participants suggested that the visual aid provided by the movies was critical in helping them to understand the more abstract, mathematical, analytic solutions.

This document organizes and summarizes (polished versions of) the movies that were made during the tutorials. The movies illustrate basic processes that take place in the interior, equatorial, and coastal oceans, including Ekman drift, inertial waves, the adjustment to Sverdrup balance via Rossby-wave propagation, and vertical propagation. Each set of movies, designated by the letters A−K, covers a particular process. Most of the solutions are for a single baroclinic mode (Movies A–H), but a few were superpositions of a number of modes, thereby generating a fully three-dimensional, flow field (Movies I–K). We believe they provide a powerful pedagogical tool for teaching ocean dynamics.

It is best to view the movies with a viewer that can play them at variable (often much slower) speeds. The “VLC Media Player” and “mpeg\_play” are two such viewers, the former available for PCs and the latter for Linux and UNIX systems. Both viewers can be downloaded from the web at no cost.

Many people helped with the preparation and daily running of the course. Much of the effort went into assembling and maintaining the computational infrastructure needed for the tutorials. In this regard, we thank: G.S. Michael for coordinating the summer-school logistics; S.G. Aparna for developing user-friendly versions of the model code, and Sarvesh Chandra, Ashok Nulguda, NIO's Information Technology Group, and the Facilities Management staff for setting up the computing infrastructure. They were helped in this process by V. Mahalingam, N. Nasnodkar, A. Krishnakiran, A. Shirwaikar, and A. Phaldesai. The organisation of this tutorial infrastructure was coordinated by G.S. Michael. Finally, we thank the Council of Scientific and Industrial Research (CSIR), of which NIO is a constituent laboratory, for supporting Dr. McCreary’s visit under its Distinguished Foreign Scientist programme.

**A) Interior-ocean waves (June 23)**

This set of solutions illustrates properties of mid-latitude (off-equatorial) gravity, Rossby, and Kelvin waves. In each solution, an initial condition on *p*, and on *u* or *v* in two cases, is specified. The model is then integrated to determine how waves radiate from the initial state.

**Experiment A1**

**Domain:** 80ºE–100ºE, Eq.–20ºN

**Resolution:** 0.1º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 10ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial, *δ*-function-like, *p* field located in the center of the domain

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 15 minutes (each model time step)

**Description:**

The initial state of the ocean is a pressure anomaly,

*p*(*x*,*y*,0) = *p*0exp{[(*x* – 90º)2 + (*y* – 10º)2]/*R*2}, (A1)

where *R* = 4Δ*x*, Δ*x* = 0.1º and *p*0/*g* = 10,000/980 cm = 10.2 cm.In response, gravity waves radiate away from the center of the basin. Eventually, they reflect from basin boundaries, and propagate back into the interior of the basin.

According to the dispersion relation, gravity waves with the shorter (longer) wavelengths have faster (slower) group and phase speeds, with the fastest (slowest) speeds approaching *c*1 (0) as the wavelength goes to zero (infinity). As a result, at any point in the domain away from the forcing region, the wavelengths of the gravity waves *increase* in time, as faster-propagating, shorter-wavelength waves leave behind slower-propagating, longer-wavelength ones.

The radiation pattern in Exp A1 is consistent with these group-velocity properties. The leading wave front is composed of wavelengths of the order of the scale of the initial *p* field, *R*, and it advances at a speed very near c1. Subsequently, weaker oscillations with wavelengths that increase in time radiate from the basin center, as indicated by the expanding circles with different green shadings. These oscillations are weak because disturbances with long wavelengths are not a significant part of the initial, *δ*-function-like *p* field.

**Experiment A2a**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 10ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial, large-scale *p* field in the center of the domain

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

In this case, the initial *p* field has the form *p*(*x*,*y*,0) = *p*0*X*(*x*)*Y*(*y*), where

*X*(*x*) = 0.5{1 + cos[2*π*(*x* – 60º)/20º]}, 50º ≤ y ≤ 70º, (A2a)

*Y*(*y*) = 0.5{1 + cos[2*π*(*y* – 30º)/20º]}, 20º ≤ y ≤ 40º, (A2b)

and both *X*(*x*) and *Y*(*y*) are zero outside the designated ranges. This solution illustrates the adjustment of an initial, large-scale *p* field to geostrophic balance. When the movie is played very slowly, the radiation of weak gravity waves away from the initial disturbance is evident. Short-wavelength gravity waves are clearly followed by longer ones, consistent with group theory. After the radiation of the waves, there is an anticyclonic, geostrophic flow around the patch of high *p*. It decays very slowly due to horizontal viscosity, but that weak decay is not visible in the movie.

**Experiment A2b**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial, large-scale *p* field in the center of the domain

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 5 days

**Description:**

As for Experiment A2a, except on the *β*-plane. The response is initially very similar to that of Experiment A2a in that gravity waves radiate away from the initial region, leaving behind a geostrophically-balanced circulation. Subsequently, however, the geostrophic circulation propagates westward as a Rossby wave. The Rossby-wave propagation speed is faster closer to the equator, so the Rossby wave tilts as it propagates westward.

The solution also has more subtle and interesting properties. 1) Slow the movie to about 3% of its initial speed. Then, a striking packet of gravity waves is visible shortly after *t* = 0. It first propagates southward to the southern boundary, and then reflects there to propagate northward; subsequently, individual gravity waves reflect at their critical latitudes to propagate southward again, and so on. This process only occurs on the *β*-plane, and is considered further in Experiments B. 2) After some time, shorter-wavelength Rossby waves begin to appear on the eastern edge of the main packet, because their group speed is slower than that of the longer waves that make up the main Rossby-wave packet.

**Experiment A3a**

**Domain:** 80ºE–100ºE, 0ºN–20ºN

**Resolution:** 0.1º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 10ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial, *p* field for a Kelvin wave at the eastern boundary

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The initial *p* field has the form *p*(*x*,*y*,0) = *p*0*X*(*x*)*Y*(*y*), where

*X*(*x*) = exp[(*x*–100º)(*f*/*c*1)], (A3a)

*Y*(*y*) = 0.5{1+cos[2*π*(*y*–10º)/10º]}, 5º ≤ y ≤ 15º, (A3b)

and *Y*(*y*) = 0 outside the designated range. The offshore decay scale of *p*(*x*,*y*,0) is *R* = *c*1/*f*, the width of a coastal Kelvin wave.

Although *p*(*x*,*y*,0) has the structure of a coastal Kelvin wave, the condition that *v*(*x*,*y*,0) = 0 is not consistent with a Kelvin wave (see Experiment A3b). Therefore, the adjustment necessarily involves the radiation of gravity waves, as well as a coastal Kelvin wave. Gravity waves clearly radiate away from the forcing region, and, at a fixed distance from the forcing region, their wavelength increases in time. After the Kelvin and gravity waves have radiated from the initial region, a geostrophic circulation remains, circulating around a patch of high *p*.

**Experiment A3b**

**Domain:** 80ºE–100ºE, 0ºN–20ºN

**Resolution:** 0.1º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 10ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial, *p* and *v* fields for a Kelvin wave at the eastern boundary

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment A3a, except that the initial state also includes *v* = *p*/c1, the zonal velocity field that accompanies the Kelvin wave. With this addition, a pure Kelvin wave is produced, and no gravity waves and geostrophic circulation are generated.

**B) Ekman drift & inertial oscillations (June 24)**

These solutions illustrate the generation of Ekman drift and the excitation of inertial and gravity waves in response to a switched-on wind stress. To isolate these features, the wind stress lacks curl. As a result, there is no Ekman pumping in the interior ocean (except for the weak pumping in Experiment B2b), and hence no geostrophic flows are generated in the interior ocean.

**Experiment B1a**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 30ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, spatially uniform *τy*

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 hour 30 minutes (5X each model time step)

**Description:**

In response to the switched-on, uniform *τy*, an eastward Ekman drift plus inertial oscillations are generated. The velocity vectors circulate clockwise, as expected for inertial oscillations in the northern hemisphere. Initially, the length of the vectors decreases to zero each cycle, a result of destructive interference between the eastward Ekman drift and an equal westward flow associated with the negative (*u* < 0) phase of the inertial oscillations. Subsequently, damping slowly weakens the inertial oscillations and the cancellation is no longer complete. Furthermore, the oscillations are initially uniform throughout the basin, but as time passes they slowly change in different parts of the basin, a result of gravity waves that reflect from basin boundaries.

The wind has an alongshore component on the eastern and western boundaries of the basin. The coastal ocean responds there by radiating Kelvin waves, and after their passage an alongshore pressure gradient is established at the coast that balances the wind and stops the coastal jet from accelerating. The Kelvin waves wrap around the basin, complicating the response. Horizontal viscosity gradually broadens the coastal currents.

**Experiment B1b**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 30ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, spatially uniform *τx*

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 hour 30 minutes (5X each model time step)

**Description:**

As in Experiment B1a, except for *τx* forcing. The response is essentially the same as in Experiment B1a, except that the Ekman drift is directed southward.

**Experiment B2a**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, spatially uniform *τy*

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 hour 30 minutes (5X each model time step)

**Description:**

As in Experiment B1a, except on the *β*-plane. The response is *remarkably* different from the *f*-plane case. Because the inertial frequency changes with latitude, the velocity vectors spin slower closer to the equator. As a result, convergences and divergences develop across the interior ocean that alternately deepen and shallow the thermocline, allowing for *meridionally-propagating gravity waves*, a process referred to as *β*-dispersion.

The group velocity of all the gravity waves is initially directed equatorward, as indicated by the equatorward propagation of lines of zero contours (located between green bands). Eventually, though, the gravity waves reflect from the southern boundary to propagate northward to the latitude where they were generated, a process that is indicated by the presence of poleward-propagating zero contours in the movie. Towards the end of the movie, it is difficult to identify phase propagation in either direction because of the interference of both poleward- and equatorward-propagating waves.

Because of *β*-dispersion, the adjustment to steady Ekman flow happens much more rapidly than on the *f*-plane.

**Experiment B2b**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, spatially uniform *τx*

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 hour 30 minutes (5X each model time step)

**Description:**

As in Experiment B2a, except for *τx* forcing. The response is similar to that of Experiment B2a. A significant difference is that the Ekman pumping velocity, *wek* = –(*τx*/*f*)y = (*β*/*f*2)*τ*x > 0, is not zero, even though the wind curl, –(*τx*)y, is zero. Thus, *thermocline increases (sea level decreases)* slowly throughout the integration everywhere in the basin.

**Experiment B3**

**Domain:** 80ºE–100ºE, 10ºN–30ºN

**Resolution:** 0.1º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 20ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, spatially uniform *τx* confined east of 90ºE

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 1 hour 15 minutes (5x each model time step)

**Description:**

When the wind is cut off west of 90º so that the wind has an edge there, gravity waves with a short zonal wavelength radiate away from the edge, as well as reflect from basin boundaries. As in the previous movies in this set, the waves are indicated by the propagation of lines of zero contours (located between green bands). Waves that emanate from the edge reflect from the eastern and western boundaries of the basin, and return to the interior ocean. Because of the edge radiation, the solution adjusts to a steady Ekman balance faster than when the wind is uniform.

**Experiment B4**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, band of *τy*

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 1 hour (each model time step)

**Description:**

In this solution, the wind has the form *τy* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 1, *Y*(*y*) = *θ*[(*y* – 40º)(45º – *y*)] *T*(*t*) = *θ*(*t*), (B1)

*θ* is a step function, and *τ*o = 2 dyn/cm2, that is, *τy* is a band of wind confined between 40º and 45º. In response, a band of inertial oscillations develops that propagates equatorward because of *β*-dispersion. The band reflects from the southern boundary, propagates back to the latitude at which it was generated, and then returns equatorward once again toward the end of the movie. These reflections are evident from the direction of phase propagation of zero contours (between alternating green bands).

**C) Ekman pumping (June 25)**

These solutions illustrate how the ocean responds to changes in large-scale winds with curl. On the *f*-plane, Ekman pumping shallows or deepens the thermocline, and geostrophic currents circulate about the resulting regions of low and high pressure. On the *β*-plane, Rossby waves radiate from the forced region, and after their passage the ocean adjusts to a state of Sverdrup balance.

**Experiment C1a**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 30ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

The wind has the form *τx* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 0.5{1+cos[2*π*(*x*–60º)/20º]}, 50º ≤ x ≤ 70º (C1a)

*Y*(*y*) = 0.5{1+cos[2*π*(*y*–30º)/20º]}, 20º ≤ y ≤ 40º (C1b)

*T*(*t*) = *θ*(*t*) (C1c)

*θ* is a step function, *τo* = 2 dyn/cm2, and *X*(*x*) and *Y*(*y*) are both zero outside their designated ranges. Ekman drift drains water from the northern half of the wind patch and piles up water in the southern half, generating regions of low and high *p*, respectively. Geostrophic currents flow around the pressure regions and, in the absence of diffusion, they continually accelerate. When the movie is slowed (to 3%, say), initial gravity waves can be seen propagating away from the forcing region (green bands), with shorter wavelengths followed by longer ones.

**Experiment C1b**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 30ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τy* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

As in Experiment C1a, except forced by a *τy* wind patch. The response differs from Experiment C1a only in that the Ekman drift is directed eastward and the geostrophic gyres are oriented east-west.

**Experiment C2a**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τy* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 5 days

**Description:**

As in Experiment C1b, except that *β* ≠ 0. A Rossby-wave packet propagates westward, and after its passage the response is adjusted to Sverdrup balance. The Rossby packet reflects from the western boundary, radiates about the basin as a coastal Kevin wave, and propagates from the eastern boundary as a second Rossby wave packet. Eventually these processes spread the low-*p* values of the initial Rossby wave around the basin.

The inviscid, steady-state (Sverdrup) response to a *τ*y wind patch is a gyre confined entirely to the region of the forcing region. Note, however, that there is a weak extension of the circulation west of the wind patch (darker green band). Theory predicts that this extension results from damping or horizontal mixing. The next two experiments indicate that damping is the primary cause.

**Experiment C2b**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τy* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0

**Movie time step:** 5 days

**Description:**

As in Experiment C2a, except that there is no damping (*A* = 0). As expected, almost all of the green band west of the forcing region vanishes, and the response is very similar to the inviscid, analytic solution.

**Experiment C2c**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τy* wind patch

**Mixing:** *νh* = 106 cm2/s, *A* = 0

**Movie time step:** 5 days

**Description:**

As in Experiment C2a, except that *A* = 0 and *ν* = 106 cm2/s. The response is not much different from Experiment C2b, confirming that damping (*A* ≠ 0) is the primary cause of westward extension in Experiment C2a.

**Experiment C3**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y*0)*, y*0 *=* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τx* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 5 days

**Description:**

As in Experiment C1a, except that *β* ≠ 0. A Rossby-wave packet (Packet 1) propagates westward, and after its passage the response is adjusted to Sverdrup balance. In contrast to the response to *τy* forcing, the *p* and *u* fields extend from the forcing region all the way to the western boundary.

Packet 1 reflects from the western boundary, propagates about the basin as a coastal Kevin wave, and radiates from the eastern boundary as a second Rossby-wave packet (Packet 2). Because the southern part of Packet 1 with high *p* values arrives at the western boundary first, Packet 2 initially spreads high-*p* values around the basin. Eventually (after about ⅔ of the movie), the northern part of Packet 1 arrives at the western boundary, and subsequently Packet 2 begins to lower *p* in the interior ocean.

**Experiment C4**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***f*-plane:** *f* = 2*Ω*sin*θ*, *θ* = 30ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on oscillatory *τx* wind; *P* = 180 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

As in Experiment C1a, except that

*T*(*t*) = sin(*σt*)*θ*(*t*), (C2)

where *σ* = 2*π*/*P*, *P* = 180 days, so that the wind oscillates after it switches on. One expects that the solution should also oscillate from positive to negative values, but surprisingly it does not. Consider the response in the northern half of the wind patch. There, the response decreases to a minimum (blue shading), returns only to near-zero values, and initially does *not* increase to positive values. Subsequently, however, there is an indication that the response is *beginning* to swing to positive values, albeit weakly.

There is a simple explanation for this curious response. Consider the response of a 1½-model when *ut* and *vt* are neglected (*i.e.*, “interior-ocean” model) and the forcing is (C1) with *T*(*t*) given by (C2). On the *f*-plane, *h* satisfies

*ht* = (*τ*o/*ρ*o*f*)*Xx*(*x*)*Y*(*y*) sin(*σt*) *θ*(*t*), (C3)

which describes the Ekman pumping response to an oscillatory wind. It has the solution

*h* = (*τ*o/*ρ*o*f*)*Xx*(*x*)*Y*(*y*) [1 - cos(*σt*)]/σ *θ*(*t*). (C4)

Solution (C4) has two parts: a steady piece (the “1” term in brackets) and an oscillating part proportional to cos(σt). The steady piece is the “transient,” part of the spin-up of the response to switch on the winds. It slowly decays due to weak damping (a process neglected in Eq. C3), and as it does the positive values begin to appear in the northern half of the forcing region.

**Experiment C5**

**Domain:** 20ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.25º

***β*-plane:** *f* = *f*0 + *β*(*y – y*0)*, y*0 *=* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on oscillatory *τx* wind; *P* = 180 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

As in Experiment C4, except that *β* ≠ 0. In this case, the steady part of the solution of Experiment C4 does not remain in the forcing region; instead it propagates westward as a Rossby wave, leaving behind a local response that now oscillates from negative to positive values. (In Experiment C4, the only process that can eliminate the steady part is weak damping.)

Because the wind oscillates, it should continuously radiate Rossby waves to the west. Such Rossby waves, however, are not prominent in the response because part of the forcing region lies poleward of the critical latitude (for *θ* > *θcr* = 30°N) and because the wavelength of the Rossby waves are too short to be strongly excited by the large-scale wind (for *θ* < *θcr*). Towards the end of the movie, weak Rossby waves are indicated by green bands that radiate from the forcing region south of 30°N.

**D) Boundary layers (June 26)**

Boundary layers occur often in geophysical problems. In oceanography, two famous boundary layers are the Stommel and Munk layers, the latter forming the western-boundary currents in most OGCM solutions. Here, we look at the Munk boundary layers that develop in a solution forced by a *τx* wind patch, illustrating how their width changes as a function of the mixing coefficient, *νh*.

**Experiment D1**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β*(*y – y0*)*, y0 =* 30º, *β* = 2.28x10−13 cm−1s−1

**Beta=**2.28.x10-13 cm-1sec-1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τx* wind patch

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 5 days

**Description:**

The wind has the form *τx* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 0.5{1+cos[2*π*(*x*–80º)/20º]}, 70º ≤ x ≤ 90º (D1a)

*Y*(*y*) = 0.5{1+cos[2*π*(*y*–30º)/20º]}, 20º ≤ y ≤ 40º (D1b)

*T*(*t*) = *θ*(*t*) (D1c)

*θ* is a step function, *τo* = 1.5 dyn/cm2, and *X*(*x*) and *Y*(*y*) are both zero outside their designated ranges. There is a Munk layer in steady state, with a width of 0.35°, close to that predicted by theory (see discussion of Figure D).

**Experiment D2**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β*(*y – y*0)*, y*0 *=* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τx* wind patch

**Mixing:** *νh* = 5x107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 5 days

**Description:**

As in Experiment D1, but with *νh* = 5x107cm2/s. As expected, the Munk layer broadens by a factor of 10⅓ = 2.15 to 7°, consistent with theory (see discussion of Figure D).

**Experiment D3**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β*(*y – y*0)*, y*0 *=* 30º, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on, *τx* wind patch

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 5 days

**Description:**

As in Experiment D1, but with *νh* = 5x105 cm2/s. As expected, the Munk layer thins by a factor of 2.15 to 0.15°, consistent with theory (see discussion of Figure D).

**Figure D**

The movies for Experiments D1−D3 show the spin-up of solutions for 3 different values of the horizontal-viscosity coefficient, *νh*. To illustrate the structure of the western-boundary currents more clearly, Figure D plots maps of *v* very near the western boundary. Theoretically, the distance offshore *xm* where *v* attains its maximum value is

*xm* = 2*πrm*/(3√3) , (D1)

where *rm* = (*νh*/*β*)⅓. For *νh* = 5e5, 5e6, and 5e7 cm2/s, (D1) predicts *xm* = 0.14°, 30°, and 0.66°, respectively, in excellent agreement with the values of 0.15°, 0.35°, and 0.7° estimated from Figure D.

**E) Equatorial waves (June 28)**

These solutions illustrate the structures of equatorially trapped waves, particularly for Kelvin and Yanai waves, which are so prominent in altimeter (and other) observations from the Indian Ocean.

In all of the solutions, cyclic conditions are applied to the eastern and western boundaries of the basin in order to eliminate boundary reflections. As a result, waves with eastward group velocity exit the basin through the eastern boundary and at the same time enter it through the western boundary. Conversely, waves with westward group velocity exit the basin through the western boundary and enter it through the eastern boundary.

**Experiment E1a**

**Domain:** 20ºE–180ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial *p* for equatorial Kelvin wave

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

The initial *p* field has the form *p*(*x*,*y*,0) = *p*0*X*(*x*)*Y*(*y*), where

*X*(*x*) = 0.5{1 + cos[2*π*(*x* – 100º)/20º]}, 90º ≤ x ≤ 110º, (E1a)

*Y*(*y*) = exp[–½(*y*/*Ly*)2], (E1b)

*Ly* = (*c*1/*β*)½ = 351 km is the equatorial Rossby radius of deformation for the *n* = 1 mode, and *X*(*x*) = 0 outside the designated range. The *Y*(*y*) profile is proportional to lowest-order Hermite function, that is, the *y*-structure of a Kelvin wave.

Because only *p* is specified, however, other equatorially trapped waves are generated in addition to the Kelvin wave. They are the *ℓ* = 1 Rossby and gravity waves, which also involve the lowest-order Hermite function. The Kelvin (Rossby) waves have eastward (westward) group velocities, respectively. Because the zonal scale of the forcing is large with respect to the equatorial Rossby radius of deformation, *Ly*, the dominant gravity waves have small wavenumbers (*k* ≈ 0) with small group velocities, and hence they tend to remain in the region of the initial disturbance; however, gravity waves with other wavenumbers are also excited, albeit weakly, and are visible in the solution.

Slow the movie down (to 3%, say) and watch the initial adjustment until February 6, 1991. A prominent part of the response is the eastward propagation of a Kelvin-wave packet (red patch), which just reaches the eastern boundary at this time. A Rossby-wave packet (red patch) is also visible near 80°E. The other oscillations are gravity waves, with eastward (westward) phase and group velocities east (west) of the initial disturbance.

As time passes, the Kelvin-wave packet emerges from the western boundary (due to the cyclic conditions) and passes through the Rossby-wave packet. Similarly, the Rossby-wave packet emerges from the eastern boundary. Gravity waves with very long wavelengths (*k* ≈ 0) are also visible in the eastern ocean; because their group velocity is eastward and very slow, they remain there for a considerable time.

Eventually, it becomes difficult to identify individual waves, because they interfere so strongly with each other.

**Experiment E1b**

**Domain:** 20ºE–180ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial *p* and *u* fields for equatorial Kelvin wave

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

As in Experiment E1a, except that the initial state also includes *u* = *p*/c1, the zonal velocity field for the Kelvin wave. With this addition, a pure Kelvin wave is produced. Its zonal structure broadens slightly in time, likely a consequence of horizontal mixing (possibly numerical error).

**Experiment E2a**

**Domain:** 20ºE–180ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial *p* field for Yanai wave

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

The initial *p* field has the *X*(*x*) structure in (E1a) but *Y*(*y*) is changed to

*Y*(*y*) = (*y*/*Ly*)exp[–½(*y*/*Ly*)2], (E2)

the meridional structure of the Yanai wave. Because only *p* is specified, several equatorially trapped waves are generated in addition to Yanai waves; they are the *ℓ* = 2 Rossby and gravity waves.

Slow the movie down (to 3%, say) and watch the initial adjustment through February, 1991. The Yanai (Rossby) waves have eastward (westward) group velocities, respectively, and so appear east (west) of the initial disturbance. Because Yanai waves are dispersive, they are not as easy to identify as the Kelvin-wave packet in Experiment E1a (see the discussion of Experiment E2b). On February 28, the Rossby-wave packet (patch that is red north of the equator and blue south of it) is visible near 80°E. Gravity waves are also visible, with eastward (westward) phase and group velocities east (west) of the region of the initial disturbance. Because the zonal scale of the forcing is large with respect to *Ly*, gravity waves with small wavenumbers (k ≈ 0) and small, eastward, group velocities are preferentially excited, and they are visible just east of the region of the initial disturbance.

As time passes, the Yanai-wave packet emerges from the western boundary (due to the cyclic conditions) and passes through the Rossby-wave packet. Similarly, the Rossby-wave packet eventually emerges from the eastern boundary. Eventually, it becomes difficult to identify individual waves, because they interfere so strongly with each other.

**Experiment E2b**

**Domain:** 20ºE–180ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** initial *p* and *u* fields for Yanai wave

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** 2 days

**Description:**

As in Experiment E2a, except that the initial state also includes the zonal velocity field for the Yanai wave, *u* = *p*/c1. With this addition, only Yanai waves are produced.

Slow the movie down (to 3%, say) and watch the initial adjustment until April 1, 1991, by which time the leading edge of the Yanai-wave packet catches up with its trailing edge. The initial state contains Yanai waves with a variety of wavenumbers. These different waves can be seen in the response as the solution develops: Yanai waves with eastward phase velocity (*k* > 0, faster group velocity) emerge from the forcing region first, followed by waves with westward phase velocity (*k* < 0, slower group velocity); between these two waves groups, there is an elongated wave with zero phase velocity (*k* = 0). As a result, the phase velocity of waves in the packet change direction across the packet, from being westward (eastward) in the western (eastern) portion of the packet.

**Experiment E3a**

**Domain:** 20ºE–180ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** *τy* wind for 5 cycles; *X*(*x*) narrow; *P* = 15 days

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

In this sequence of solutions, a meridional wind field in the western ocean is used as a Yanai-wave generator. It has the form *τy* = *τoX*(*x*)*Y*(*y*)*T*(*t*), where *X*(*x*) is given in (E1), *Y*(*y*) by (E2), and

*T*(*t*) = sin(*σt*)*θ*(*t*)*θ*(5*P* – *t*), (E3)

where *σ* = 2*π*/*P*, *P* = 15 days. According to (E3), the forcing lasts only for 5 cycles. In this run, the wind oscillates with a period of 15 days. For this period, *σ′* = (2*π*/*P*)/(*βcn*)½  = 0.64. When σ′ < 1, the phase speed of Yanai waves is westward, a property that is apparent in the response.

Slow the movie down somewhat (to 33%) and watch the response until July 1, 1991, by which time the leading edge of the Yanai-wave packet catches up with the trailing edge. Because of the form of *T*(*t*), the dominant Yanai waves have a period of 15 days. Because the wind is switched on, however, the packet also contains Yanai waves at other periods. As a result, waves with shorter (longer) periods, longer (shorter) wavelengths, and faster (slower) group velocities are visible in front of (behind) the packet.

**Experiment E3b**

**Domain:** 20ºE–180ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** *τy* wind for 5 cycles; *X*(*x*) narrow; *P* = 9.34 days

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment E3a, except with *P* = 9.34 days. At this period, *σ’* = (2*π*/*P*)/(*βcn*)½  = 1. When *σ’* = 1, the zonal phase speed of Yanai waves is zero, that is, *k* = 0. The Yanai waves that radiate from the forcing region clearly have this property, simply flipping their sign across the equator with period *P*. Because the wind is switched on, however, the packet also contains Yanai waves at other periods. As a result, waves with shorter (longer) periods, faster (slower) group velocities, and eastward (westward) phase velocities are visible in front of (behind) the packet.

**Experiment E3c**

**Domain:** 20ºE–180ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** *τy* wind for 5 cycles; *X*(*x*) narrow; *P* = 6 days

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment E3a, except with *P* = 6 days. At this period, *σ′* = (2*π*/*P*)/(*βcn*)½  = 1.61 > 1. When *σ′* > 1, the phase speed of Yanai waves is eastward, a property that is apparent in the response. Because the wind is switched on, however, the packet also contains Yanai waves at other periods. As a result, waves with shorter (longer) periods and faster (slower) group velocities are visible in front of (behind) the packet. The shorter-period waves all have eastward phase velocity. The longer-period waves can have eastward, westward, or zero (k ≈ 0) phase velocities.

**F) Yoshida jet (June 29)**

The Yoshida Jet is perhaps the most fundamental, forced equatorial response, one that distinguishes equatorial from midlatitude dynamics. The Yoshida Jet exists because *f* vanishes at the equator, so that the zonal momentum equation reduces to *ut* +*px* = *τx*/*H*. For *x*-independent forcing (idealized), *px* = 0 and the Yoshida Jet continuously accelerates (*ut* = *τx*/*H*). When the forcing is zonally bounded (realistic), the radiation of equatorial Kelvin and Rossby waves establishes the balance *px* = *τx*/*H*, and the jet stops accelerating to form a “bounded” Yoshida Jet. No Yoshida Jet is generated by *τy* winds.

The steady-state currents for all the solutions found here are Sverdrup flows. It is noteworthy that they are *exactly the same* as they are off the equator, a consequence of the Sverdrup circulation not depending on *f*, but only *β*.

**Experiment F1a**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013

**Movie time step:** 2 days

**Description:**

The wind has the form *τx* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = cos[2*π*(*x*–60º)/30º], 45º ≤ x ≤ 75º (F1a)

*Y*(*y*) = (1 + *y2*/*Ly2*)exp(-*y2*/*Ly*2), (F1b)

*T*(*t*) = *θ*(*t*), (F1c)

where *Ly* = 10º, *τ*o = 1.5 dyn/cm2, and *X*(*x*) is zero outside the designated range The form of *Y*(*y*) ensures that the wind has zero curl at the equator.

In order to see the development of the bounded Yoshida Jet, slow down your video player as much as possible (3%, say). As soon as the wind switches on, the equatorial jet accelerates. Within a month or so, wind-generated Kelvin and Rossby waves have propagated across the wind patch, a pressure gradient develops to balance the wind, and the jet stops accelerating, locally forming a “bounded” Yoshida Jet. Subsequently, the radiation of Kelvin and Rossby waves extends the jet farther from the forcing region, more rapidly in the eastern ocean since the Kelvin wave speed is 3x faster than the fastest Rossby (ℓ = 1) wave.

At later times, ℓ = 1 Rossby waves, which reflect from the eastern boundary, propagate across the basin and reflect there as Kelvin waves that then return to the eastern ocean. The time it takes for an ℓ = 1 Rossby wave to cross the basin and a Kelvin wave to return is 4*L*/*c*1 = 154 days, the natural ringing time for the equatorial ocean. The curved bands of radiating away from the eastern boundary are a set of reflected Rossby waves generated by this process (also see the discussion of Exp G1).

Eventually, reflected waves from both the eastern and western boundaries eliminate the equatorial jet. As a result, the steady-state response is the same Sverdrup circulation that develops at midlatitudes. Note that the Sverdrup gyres are located off the equator, a consequence of *Y*(*y*) having zero curvature there.

**Experiment F1b**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013

**Movie time step:** 2 days

**Description:**

As in Experiment F1a, except that

*Y*(*y*) = 0.5{1+cos[2*πy*/30º]}. –15º ≤ *y* ≤ 15º, (F2)

and vanishes outside the designated range. The response is similar to that of Experiment F1a, except that, because there is wind curvature near the equator, the steady-state Sverdrup gyres are joined together.

**Experiment F1c**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*5 = 59.7 cm/s

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013

**Movie time step:** 2 days

**Description:**

As in Experiment F1a, except for the *n* = 5 baroclinic mode. The terms proportional to *A*/*cn*2 damp equatorial waves. The terms are small for the first baroclinic mode, but not for intermediate modes like *n* = 5. The impact of damping in the response of the *n* = 5 baroclinic mode is striking. For example, the eastern-boundary Rossby waves are strongly damped before they can propagate into the interior ocean to eliminate the equatorial jet. As a result, the equatorial jet remains a strong feature of the flow in steady state. Surprisingly, then, damping actually *strengthens* the equatorial current.

**Experiment F2**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on *τy* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013

**Movie time step:** daily

**Description:**

As in Experiment F1a, except for a meridional wind of the form *τy* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where *X*(*x*), *Y*(*y*), and *T*(*t*) are given in (F1). In response to the switched-on *τy* wind, Yanai and Rossby waves radiate away from the patch, and the response quickly adjusts to Sverdrup balance. There are no strong flows because *τy* winds do not drive a Yoshida Jet.

The Yanai-wave packet has a remarkable structure. Fourier analysis allows the packet to be viewed as a superposition of sinusoidal waves with wavelengths *k* that are both positive and negative. Because the group velocity of Yanai waves with positive *k* (eastward phase velocity) is greater than those with negative *k* (westward phase velocity), the leading (trailing) edge of the packet has eastward (westward) phase velocity. Between these two parts, the Yanai wave becomes quite elongated because *k* ≈ 0.

**G) Boundary reflections (June 30)**

Equatorial waves reflect from the eastern and western boundaries of the basin. This set of experiments focuses on reflections from the eastern boundary, which can impact the ocean well off the equator. The solutions consider the reflections of a Kelvin-wave pulse (Experiment G1) and of Kelvin waves oscillating at a frequency *σ* (Experiments G2 and G3). At an eastern boundary, the boundary reflections consist of a set of equatorially-trapped waves (designated by the index *ℓ*), all linked by Moore’s chain rule.

For the oscillating solutions, there is a critical value of *ℓ*, given by

*ℓcr* = (*a* – *a*-1)2/4 (G1)

where *a* = 2½*σ′* and *σ′* = (*βcn*)½ is the equatorial inertial period of the *n*th baroclinic mode. Boundary waves with ℓ values smaller (larger) than ℓ*cr* have real (complex) zonal wavenumbers. As a result, only the lower-order modes of the set (ℓ < ℓ*cr*) propagate offshore, and they sum to form a packet of reflected Rossby waves; the higher-order waves (ℓ > ℓ*cr*) all decay offshore, and they sum to generate a *β*-plane, coastal Kelvin wave that propagates poleward along the boundary.

It is often useful to restate the concept of ℓ*cr* in terms of a critical latitude

*ycr* = *cn*/(2*σ*). (G2)

The critical latitude defines the location where reflected waves change from being Rossby-like (*y* < *ycr*) to Kelvin-like (*y* > *ycr*).

**Experiment G1**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The wind has the form *τx* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 0.5{1+cos[2*π*(*x*–37.5º)/15º]}, 30º ≤ x ≤ 45º, (G3a)

*Y*(*y*) = 0.5{1+cos[2*π*(*y*–0º)/30º]}, –15º ≤ y ≤ 15º, (G3b)

*τ*o = 1.5 dyn/cm2, and *X*(*x*) and *Y*(*y*) are zero outside their designated ranges. The wind is located in the western ocean to allow Kelvin waves and Rossby waves reflected at the eastern boundary to be more visible. To weaken inertial oscillations, the time dependence is taken to be the smooth function,

*T*(*t*) = 0.5[1 – cos(2*πt*/*δt*)]*θ*(*δt* − *t*). (G3c)

According to (G3c), the wind smoothly rises to a maximum at 15 days, weakens to zero at 30 days, and is switched off thereafter.

To see the initial response, slow down the viewer considerably (to 6%, say). A Kelvin-wave *pulse* (red) radiates from the forcing region, and begins to reflect from the eastern boundary by February. In addition, wind-forced Rossby waves have reached the western boundary of the basin where they reflect as a Kelvin wave (darker green and blue), and during February it crosses the basin. At this time, there are also small-scale oscillations behind (west of) the Kelvin-wave pulse, which are weak inertial oscillations that are still excited despite the smoothness of (G3c).

The first Rossby wave to emerge from the eastern boundary is an ℓ = 1 Rossby wave. By March, it is clearly visible as two westward-propagating (red) patches. At that time, the western-boundary Kelvin wave has crossed the basin, and also begins to reflect from the eastern boundary as an ℓ = 1 Kelvin wave. By mid-May, both of these Rossby waves have propagated to the middle of the basin. Closer to the eastern boundary, the structure of response is more complex, having more reversals in latitude and spreading farther from the equator. These properties indicate the presence of higher-order Rossby waves (ℓ > 1), but individual waves are difficult to identify because they are separate from each other.

Two other interesting features are apparent when the movie is played at full speed (100%). First, there are alternating bands of Rossby waves radiating away from the eastern boundary. They are clearly generated from the equator by multiple reflections of equatorial Kelvin waves and ℓ = 1 Rossby waves: The period of the bands is *P* = 4*L*/*c*1, where *L* is the basin width, that is, the time it takes a Kelvin wave to cross the basin and an ℓ = 1 Rossby wave to return. Second, short-wavelength Rossby waves with westward group velocity reflect from the western boundary. They have eastward group velocity, and as time passes they extend farther into the interior ocean. They exist in the solution because horizontal mixing is weak, so that they are not strongly damped.

**Experiment G2a**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** oscillatory *τx* wind; *P* = 15 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment G1, except that

*T*(*t*) = sin(*σt*)*θ*(*t*), (G2)

where *σ* = 2*π*/*P*. Accordingly, the wind switches on at *t* = 0 and thereafter oscillates with a period of *P* = 15 days.

At this period, *ℓcr* = .016, so that *no* boundary-reflected waves with real wavenumbers (*i.e.*, Rossby waves) are possible at this frequency. As a result, the eastern-boundary reflected waves all superpose to form a *β*-plane coastal Kelvin wave. Note, however, that there are weak *transient* Rossby waves, generated because the wind is not a pure sinusoidal oscillation in time, but is switched on. They are visible during the first half of the movie, radiating from the eastern boundary as a sequence of positive and negative pulses (see the discussion of Experiment F1a).

**Experiment G2b**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** oscillatory *τx* wind; *P* = 30 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment G2a, except with *P* = 30 days. At this frequency, *ℓcr* = 0.83, so again no boundary-reflected waves with real wavenumbers (*i.e.*, Rossby waves) are possible, and hence the eastern-boundary reflected waves all decay offshore (*i.e.*, are evanescent), and superpose to form a *β*-plane coastal Kelvin wave. At this longer period, however, the decay of the lowest-order (*ℓ* = 1) evanescent wave is weak. Consequently, it is visible in the movie as a westward-decaying packet at two locations: west of the wind-forced region, and near the eastern boundary. There are also transient Rossby waves like those in Experiment G2a, radiating off the eastern boundary; they are almost decayed away by the time the movie is half over.

**Experiment G2c**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** oscillatory *τx* wind; *P* = 60 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment G2a, except with *P* = 60 days. In this case, ℓ*cr* = 4.6, so the ℓ = 1 and ℓ = 3 boundary-reflected waves have real wavenumbers (*i.e.*, they are Rossby wave). (Boundary waves with even values of *ℓ* are not allowed because they are not symmetric about the equator.) All the other eastern-boundary reflected waves have complex wavenumbers (ℓ = 5, 7, 9, ∙∙∙), and they superpose to form a *β*-plane coastal Kelvin wave. The reflected Rossby waves interfere so strongly with the incoming Kelvin wave that it is difficult (impossible) to see the Kelvin wave in the equatorial response.

Since there are several reflected Rossby waves at this period, the concept of a critical latitude now makes sense. Its value is *ycr* = 1090 km = 9.8°, in good agreement with the latitudinal extent of the response in the movie. (The response appears to extend a bit beyond *ycr* in the longitude band from 30° to 40°, but that is due to the directly-forced response, not to the wave packet radiating from the eastern boundary.)

**Experiment G2d**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** oscillatory *τx* wind; *P* = 120 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment G2a, except with *P* = 120 days. In this case, *ℓcr* = 20, so that eastern-boundary reflected waves with *ℓ* ≤ 19 have real wavenumbers (*i.e.*, are Rossby waves). As a result, there is a critical latitude at *ycr* = 2180 km = 20º. Equatorward of *ycr*, the eastern-boundary reflections radiate back into the interior ocean as Rossby waves, whereas poleward of *ycr* they decay to the west and, hence, superpose to form a *β*-plane coastal Kelvin wave.

Surprisingly, the response does not show a clear critical latitude at 20°, below which the ocean is filled with Rossby waves. Instead, the Rossby-wave packet appears to bend equatorward as it propagates away from the eastern boundary. Indeed, group theory suggests the packet does bend equatorward, to an approximate focal point on the equator a distance,

*xf* = (*π*/4)(*c*1/*σ*) = *c*1*P*/8, (G4)

from the eastern boundary. For *P* = 120 days, *xf* = 3420 km = 31°, and this property is consistent with the response in the movie. (During the summer school, the concept of a focal point was mentioned but we did not cover this topic in any detail. See Problem 18 of the Equatorial Notes for a detailed discussion. Also see Experiment H2d.)

**Experiment G3a**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** oscillatory *τy* wind; *P* = 15 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment G2a, except for a meridional wind of the form *τy* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where the structure functions are given in (G1a), (G1b) and (G2). With *P* = 15 days, *ℓcr* = 0.016 and no boundary reflected waves with real wavenumbers (*i.e.*, Rossby waves) are possible. As a result, the eastern-boundary reflected waves superpose to form a *β*-plane coastal Kelvin wave.

Because *σ′* = 0.62 < 1, the Yanai waves generated by the wind have *eastward* group velocity. Initially, however, a transient packet of Yanai waves radiates eastward, the leading edge of which has *eastward* phase velocity (see the discussion of Experiment F2).

**Experiment G3b**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** oscillatory *τy* wind; *P* = 30 days

**Mixing:** *νh* = 107 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment G3a, except that *P* = 30 days. At this period, *ℓcr* = 0.83 so that no boundary reflected waves with real wavenumbers (*i.e.*, Rossby waves) are possible. As a result, the eastern-boundary reflected waves superpose to form a *β*-plane coastal Kelvin wave.

There is a prominent packet of transient Yanai waves, much more visible than in Experiment G3a. Its leading edge has eastward phase velocity, its trailing edge has westward group velocity, and its center is a region with a large wavelength where *k* ≈ 0 (see the discussion of Experiment F2).

Because *σ* = 0.31 < 1, the Yanai waves in the equilibrium response have *eastward* group velocity. Their amplitude is much weaker than in Experiment G3a because their wavelength, *λ* = 2*π*/*k* = 2*π*/(*σ*/*c*1 – *β*/*σ*) = 6.7°, is much less than the zonal width of the forcing region (15°).

There is a curious pulsing of the Yanai waves in the equilibrium response. Such a pulsing can only occur due to interference between the Yanai waves and waves with a different wavelength. The most likely possible waves are the ℓ = 2 evanescent waves, which have the weakest damping rates.

**H) Coastal dynamics (July 1)**

These solutions illustrate how the coastal ocean responds to a midlatitude (off-equatorial) band of *τy* winds when vertical mixing (nu = A/c12) is weak. On the *f*-plane, the balance, *py* = *τy*/*H*, is established along the coast after the passage of coastal Kelvin waves, and there is a coastal current in the direction of the wind. On the *β*-plane, the eastern-boundary flow field propagates offshore as a Rossby wave, and in its wake *py* = *τy*/*H* everywhere. When the wind oscillates, there is a critical latitude, *ycr*, that divides the coastal response into Rossby-like (*y* < *ycr*) and Kelvin-like (*y* > *ycr*) regimes.

For all experiments except Experiment H1a, there is a damper in the *p* equation that relaxes *p* to zero within 2º of the southern boundary. It has the form, –*γp*, where *γ* increases linearly toward the boundary from zero at 12ºN to a maximum of (1 day)-1 at 10ºN. The damper ensures that Kelvin waves along the western boundary do not propagate along the southern boundary to interfere with the eastern-boundary response. On the other hand, the southern damper does not stop the eastern-boundary coastal response from impacting *western* boundary. Therefore, we focus on the eastern-boundary response in this set of solutions.

**Experiment H1a**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***f*-plane:** *f* = *f(y*0), *y*0 = 30ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** no southern damper

**Movie time step:** 5 days

**Description:**

The wind has the form *τy* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 1, *Y*(*y*) = *θ*[(*y* – 20º)(40º – *y*)] *T*(*t*) = *θ*(*t*), (H1)

where *θ* is a step function, *Y*(*y*) is a “top hat” function, and *τ*o = 2 dyn/cm2. According to (H1), the forcing is a band of meridional wind confined between 20ºN and 40ºN. The initial response to the *τy* wind band is westward Ekman drift across the basin plus inertial oscillations (see Experiments B). Shortly thereafter, coastal Kelvin waves radiate poleward (equatorward) along the eastern (western) coasts. After their passage, a meridional pressure gradient is established along the coasts that balances *τy* (*i.e.*, *py* = *τy*/*H*), and equatorward (poleward) coastal jets provide the water for the offshore (onshore) Ekman drift. As time passes, the coastal currents begin to develop weak reverse flows on their offshore flanks (indicated by bands of green shading that continually broaden), a result of horizontal viscosity.

Note that the western-boundary response is quickly carried to the eastern boundary by coastal Kelvin waves along the southern boundary (and vice versa along the northern boundary). This process complicates the eastern-boundary response, and so for the other solutions in this section we introduce the southern-boundary damper to avoid this problem.

**Experiment H1b**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***f*-plane:** *f* = *f(y*0), *y*0 = 30ºN

**Characteristic speed:** *c*1 = 264 cm/s

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** *p* relaxed to zero within a day near the southern boundary

**Movie time step:** 5 days

**Description:**

As in Experiment H1a, except with the damper along the southern boundary. In this case, western-boundary Kelvin waves are damped before they can interfere with the eastern-boundary response.

**Experiment H1c**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

**Characteristic speed:** *c*1 = 264 cm/s

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 1

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** *p* relaxed to 0 in a day near the southern boundary

**Movie time step:** daily

**Description:**

As in Experiment H1b, except with *β* ≠ 0. (This solution is essentially the same as Experiment B4, and it is useful to compare the two responses.) The initial response is much the same as in ExpH1b. Subsequently, however, the coastal response propagates offshore as a packet of Rossby waves. Eventually, the solution adjusts to a steady-state response in which *py* balances *τy* *everywhere* in the basin. This simple response is the Sverdrup flow for the wind-band forcing.

Interestingly, the transient Rossby-wave packet develops an oscillation (waviness) as it propagates offshore. The oscillation exists because initially the packet has a sharp edge. Consequently, it is not composed only of long-wavelength Rossby waves, but also includes shorter ones. The longer waves propagate with a speed of close to *cr* = *β*(*c*1/*f*)2, but the shorter waves have a slower group velocity and so lag behind the front of the packet to form the oscillations.

**Experiment H2a**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

**Characteristic speed:** *c*1 = 264 cm/s

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 1

**Forcing:** switched-on, oscillatory *τy* wind band, *P* = 15

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** *p* relaxed to 0 in a day near the southern boundary

**Movie time step:** daily

**Description:**

As in Experiment H1b, except with

*T*(*t*) = sin(*σt*)*θ*(*t*), (H2)

where *σ* = 2*π*/*P*, *P* = 15 days. At this period, the critical latitude is *ycr* = 272 km = 2.5° (see Eq. G2), so that no periodic Rossby waves exist in the basin, and the coastal response consists entirely of Kelvin waves that continually radiate poleward along the eastern boundary.

**Experiment H2b**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

**Characteristic speed:** *c*1 = 264 cm/s

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 1

**Forcing:** switched-on, oscillatory *τy* wind band, *P* = 30

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** *p* relaxed to 0 in a day near the southern boundary

**Movie time step:** 5 days

**Description:**

As in Experiment H2a, except with *P* = 30 days. At this period, the critical latitude is *ycr* = 545 km = 4.9° (see Eq. G2), so that still no periodic Rossby waves exist in the basin, and the coastal response consists entirely of Kelvin waves that continually radiate poleward along the eastern boundary. Transient Rossby waves, however, are visible propagating off the eastern boundary during the first half of the movie.

**Experiment H2c**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

**Characteristic speed:** *c*1 = 264 cm/s

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 1

**Forcing:** switched-on, oscillatory *τy* wind band, *P* = 60

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** *p* relaxed to 0 in a day near the southern boundary

**Movie time step:** 5 days

**Description:**

As in Experiment H2a, except with *P* = 60 days. At this period, the critical latitude is *ycr* = 1090 km = 9.8° (see Eq. G2), so that still no periodic Rossby waves exist in the basin, and the coastal response consists entirely of Kelvin waves that continually radiate poleward along the eastern boundary. Transient Rossby waves, however, are visible propagating away from the eastern boundary during the first half of the movie.

**Experiment H2d**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

**Characteristic speed:** *c*1 = 264 cm/s

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 1

**Forcing:** switched-on, oscillatory *τy* wind band, *P* = 180

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** *p* relaxed to 0 in a day near the southern boundary

**Movie time step:** 5 days

**Description:**

As in Experiment H2a, except with *P* = 180 days. In this case, the critical latitude is *ycr* = 3270 km = 29°, so that Rossby waves are present in the solution for *y* ≤ *ycr*. Poleward of *ycr*, the boundary waves decay to the west and superpose to form a *β*-plane, coastal Kelvin wave. Transient Rossby waves, however, are visible propagating off the eastern boundary during the first quarter of the movie, even north of *ycr*.

Surprisingly, in the equilibrium (non-transient) response, there is not a sharp distinction between the responses north and south of *ycr*, likely because of mixing in the numerical model. Even more surprising, the group velocity of the Rossby-wave packet south of *ycr* is not directed only westward, but also has a significant southward component. The dynamics of this feature are the same as those that generate the focal point in Experiment G2d. Indeed, if the basin of Experiment H2d were extended to the equator, the Rossby-wave packet should focus near the longitude given by (G4), *xf* = 5130 km = 46°. Note that the packet reflects off the southern boundary (despite the presence of the damper).

There are also short-wavelength Rossby waves (with eastward group velocity) radiating off the western boundary. Their theoretical wavelength is *λ* = 2*π*(*σ*/*β*) = 110 km = 1°, somewhat shorter than the oscillations in the solution likely because of horizontal mixing.

**Experiment H2e**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

**Characteristic speed:** *c*1 = 264 cm/s

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 1

**Forcing:** switched-on, oscillatory *τy* wind band, *P* = 360

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Damper:** *p* relaxed to 0 in a day near the southern boundary

**Movie time step:** daily

**Description:**

As in Experiment H2a, except with *P* = 360 days. In this case, the critical latitude is *ycr* = 6540 km = 58°, so that Rossby waves exist at all latitudes in the basin.

As in Experiment H2d, the group velocity of the packet of eastern-boundary Rossby waves has a southward component, as demonstrated by the absence of Rossby waves in the northwest corner of the basin in the equilibrium response. As expected from (G4), the southward bending is not as much as in Experiment H2d, since *xf* is located twice as far from the eastern boundary.

There are also short-wavelength Rossby waves (with eastward group velocity) radiating off the western boundary. Their theoretical wavelength is *λ* = 2*π*(*σ*/*β*) = 55 km = .5°, considerably shorter than the oscillations in the solution likely because of horizontal mixing.

**I) EUC and CUC (July 2)**

Undercurrents are prominent aspec*t*s of both equatorial and coastal circulations. In the North Indian Ocean, for example, there are coastal undercurrents along both coasts of India, as well as along Somalia and Oman. Here, we obtain solutions that illustrate the three-dimensional spin-up of the Equatorial Undercurrent (EUC) and an eastern-boundary Coastal Undercurrent (CUC) to switched-on winds, by summing the responses of *N* = 25 baroclinic modes, a number that ensures the solution is well converged. To prevent the western-coastal solution from propagating to the eastern boundary, the CUC solution includes the damper described in the introduction to Section H.

**Experiment I1a**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The wind has the form *τx* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = cos[2*π*(*x*–60º)/40º], 40º ≤ *x* ≤ 80º, (I1a)

*Y*(*y*) = (1 + *y2*/*Ly2*)exp(-*y2*/*Ly*2), (I1b)

*T*(*t*) = *θ*(*t*), (I1c)

*Ly* = 10º, *τ*o = −2 dyn/cm2, and *X*(*x*) = 0 outside the designated range. The movie shows the adjustment of the LCS model to steady state for *N* = 25 modes, plotting a surface map of *p* (expressed in centimeters as sea level) and (*u*,*v*) vectors. See Experiments F1a and F1c for movies of the surface maps for the *n* = 1 and *n* = 5 modes separately.

If the movie is slowed (to 3%, say), the initial development of the Yoshida Jet is visible. Subsequently, Kelvin and Rossby waves radiate from the wind patch, and reflect from basin boundaries. Equatorial inertial oscillations are also excited, which last throughout the movie; towards the end of the movie, they appear as a pulsing of the equatorial currents.

In steady state, the low-order baroclinic modes (*n* ≈ 1−3), for which damping is weak, adjust to Sverdrup balance, as in Experiment F1a. Essentially, all the sea-level response is generated by the low-order modes. Note that in steady state there is a zonal pressure gradient (sea-level tilt) along the equator that balances *τx*. The intermediate modes (*n* ≈ 4−8) adjust to a state with a strong bounded Yoshida Jet as in Experiment F1c, so they are the modes that generate the strong equatorial current. For the higher-order modes (*n* ≥ 8), the Yoshida Jet is weak: These modes add up to generate the upwelling circulation discussed in the next two movies.

**Experiment I1b**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The same solution as in Experiment I1a, except showing an equatorial (*x*,*z*) section of *u* (shading) and (*u*,*w*) vectors. Slow the movie down and step through the first few frames. In the first frames, there are prominent inertial oscillations that extend throughout the water column, there is surface upwelling driven by divergent Ekman drift in the mixed layer, and the Yoshida Jet accelerates at the ocean surface. As Kelvin and Rossby waves propagate across the wind patch, a near-surface pressure gradient is established that stops the Yoshida Jet from accelerating and drives the EUC. As the waves propagate farther from the forcing region, so does the EUC.

The reflection of Rossby waves from the eastern boundary is apparent after the arrival of the Kelvin waves. As the Rossby waves propagate away from the eastern boundary, phase tends to propagate upward (an indication that energy is radiating downwards). Eventually, the reflected waves generate a westward current beneath the EUC, the model’s Equatorial Intermediate Current (EIC). Even at the end of the movie, inertial oscillations remain, visible as a pulsing of the currents.

**Experiment I1c**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τx* wind patch

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The same solution as in Experiment I1a, except showing a meridional (*y*,*z*) section of *u* (shading) and (*v*,*w*) vectors at *x* = 60º. The spin-up of the solution is as described in Experiments I1a and I1b. The meridional structure of the equatorial currents can be seen. At the surface, there is a meridional overturning circulation, the “Tropical Cell” (TC). It consists of equatorward, geostrophic flow into the core of the EUC, equatorial upwelling, and poleward Ekman drift at the ocean surface. The cell is closed by downwelling in the tropics (not shown). (The strong TC is a limitation of this solution since the source of the water in the EUC arises from downwelling in the subtropical ocean to for the “Subtropical Cell.”)

**Experiment I2a**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The wind has the form *τy* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 1, *Y*(*y*) = *θ*[(*y* – 20º)(40º – *y*)] *T*(*t*) = *θ*(*t*), (I1)

*θ* is a step function, *Y*(*y*) is a “top hat” function, and *τ*o = –2 dyn/cm2, a band of wind confined between 20ºN and 40ºN. The movie shows the adjustment of the LCS model to steady state using *N* = 25 modes, plotting a surface map of *p* (expressed in centimeters as sea level) and (*u*,*v*) vectors. See Experiment H1c for a movie of the surface map for only the *n* = 1 mode.

If the movie is slowed (to 3%, say), the radiation of inertial oscillations from the forcing band via *β*-dispersion is apparent (see Experiments H1c and B4). Subsequently, Kelvin radiate poleward along the eastern coast, and Rossby waves radiate offshore. The structure of the *n* = 1 Rossby-wave packet is the same as that in Experiment H1c, including the trailing oscillations of shorter-wavelength Rossby waves. The *n* = 2 packet clearly separates from the eastern boundary about April, 1992, and the *n* = 3 packet separates about July, 1993. At the end of the movie, coastal currents still remain because the offshore propagation speed of higher-order baroclinic Rossby waves is so slow. Since there is no damping, however, Rossby waves for all the modes will eventually propagate offshore (at a time much longer than the movie length), and no coastal currents will remain.

**Experiment I2b**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The same solution as in Experiment I2a, except showing an alongshore (*y*,*z*) section of *v* (shading) and (*v*,*w*) vectors. Slow the movie down (to 3%, say) and view the solution through February, 1991. During January, inertial oscillations are prominent in the response, and their meridional scale becomes increasingly small due to *β*-dispersion; during February the inertial oscillations weaken, and they are essentially gone by March 1.

At the same time, an alongshore current in the direction of the wind (southward) develops. As Kelvin waves propagate poleward along the coast, an alongshore pressure gradient develops to balance *τy*, the surface current stops accelerating, and a CUC appears at depth. As it strengthens, the CUC rises in the water column, a consequence of coastal adjustments associated with slower-propagating, higher-order-baroclinic Kelvin waves. After an initial period of growth, the coastal circulation weakens throughout the rest of the movie, a consequence of the continual offshore propagation of Rossby waves.

**Experiment I2c**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

The same solution as in Experiment I2a, except showing an across-shore (*x*,*z*) section of *v* (shading) and (*u*,*w*) vectors. The spin-up of the solution is as described in Experiments I1a and I1b. During the adjustment to equilibrium, upward phase propagation of the coastal currents is apparent, an indication of the passage Kelvin modes associated with higher-order baroclinic modes. In addition, the offshore radiation of low-order Rossby waves can be seen, the *n* = 1 Rossby (no zero crossing in the upper 1000 m), then the *n* = 2 wave (one zero crossing), and finally the *n* = 3 wave (two zero crossings).

**Experiment I3a**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00065 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment I2a, except with damping that weakens higher-order-mode Rossby waves before they can very far propagate offshore. In this movie, for example, only the *n* = 1 Rossby-wave packet is visible, whereas in Experiment I2a packets for the first 3 modes can be seen.

In steady state, the low-order baroclinic modes (*n* ≈ 1−2), for which damping is weak, adjust to Sverdrup balance, as in Experiments F1a and H1c: The coastal currents associated with these modes propagate offshore, and in their wake a meridional pressure gradient (sea-level tilt) is established that balances *τy*. Virtually all the sea-level response is generated by the low-order modes. For the intermediate modes (*n* ≈ 3−8), damping is strong enough for the Rossby waves to decay *before* they propagate very far offshore; as a result, the coastal current associated with these modes remain coastally trapped, and they are the modes that generate the alongshore currents. For the higher-order modes (*n* ≥ 8), the alongshore currents are weak: They add up to generate the upwelling circulation discussed in the next two movies.

**Experiment I3b**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00065 cm2/s3

**Movie time step:** daily

**Description:**

The same solution as in Experiment I3a, except showing an alongshore (*y*,*z*) section of *v* (shading) and (*v*,*w*) vectors. The spin-up of the coastal currents is initially the same as in Experiment I2b. In contrast to Experiment I2b, however, the coastal currents do not later weaken; instead, they adjust to a steady state because higher-order-mode Rossby waves are damped before they can propagate offshore. In steady state, the surface jet and CUC extend well poleward of the forcing band (40°N), due to the Kelvin-wave propagation.

**Experiment I3c**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on *τy* wind band

**Mixing:** *νh* = 106 cm2/s, *A* = 0.00065 cm2/s3

**Movie time step:** daily

**Description:**

The same solution as in Experiment I3a, except showing an across-shore (*x*,*z*) section of *v* (shading) and (*u*,*w*) vectors. The spin-up of the solution is similar to that described in Experiments I1c, with upward phase propagation indicating the passage Kelvin waves associated with higher-order baroclinic modes. In contrast to Experiment I2c, however, only the offshore radiation of the *n* = 1 Rossby wave (no zero crossing in the upper 1000 m) and the *n* = 2 wave (one zero crossing) are visible, owing to the damping of the higher-order baroclinic modes. Furthermore, the coastal currents do not continue to shallow but rather adjust to a steady-state profile, again due to Rossby-wave damping. In steady state, there is a subsurface, onshore, geostrophic flow generated by the Rossby waves. At the coast, part of it upwells to feed the offshore Ekman drift, another part bends northward to supply water for the CUC, and the rest supplies water for weak coastal downwelling below about 300 m.

**J) Equatorial beams (July 21)**

Waves generated by periodic forcing propagate vertically as well as horizontally. Such vertically-propagating signals have been observed at several locations in the equatorial ocean. One property of vertically-propagating waves is that the propagation directions of phase and energy are *opposite*: phase propagates upward (downward) when energy propagates downward (upward). To obtain these “beam-like” solutions, we sum the responses to *N* = 25 baroclinic modes. The solutions essentially are a repeat of the EUC solution of Section I, except forced by oscillating winds.

**Experiment J1a**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τx* wind; *P* = 15 days

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

The wind has the form *τx* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 0.5{1+cos[2*π*(*x*–37.5º)/15º]}, 30º ≤ *x* ≤ 45º, (J1a)

*Y*(*y*) = 0.5{1+cos[2*π*(*y*–30º)/30º]}, –15º ≤ *y* ≤ 15º, (J1b)

*T*(*t*) = sin(*σt*), (J1c)

where *σ* = 2*π*/*P*, *P* = 15 days, *τ*o = 1.5 dyn/cm2, and *X*(*x*) and *Y*(*y*) vanish outsider their designated ranges. The movie shows the response of the LCS model with *N* = 25 modes, plotting an equatorial (*x*,*z*) section of *u* (shading) and (*u*,*w*) vectors. It is perhaps best to view this movie at a slow speed (3%) to see the spin-up of the Kelvin-wave beam and at an intermediate speeds (12%, say) to view the equilibrium response.

During the spin-up, Kelvin waves associated with several low-order baroclinic modes radiate eastward from the forcing region, and they superpose to form a beam. The beam is fully established by the end of February, 1991, by which time all the Kelvin waves have reached the eastern boundary. In equilibrium, the Kelvin beam radiates downward from the forcing region, reflects off the ocean bottom to return to the ocean surface, then makes another top-to-bottom transit, and finally almost another bottom-to-top transit. The angle of energy propagation is given by *θ* = ±*σ*/*Nb*, where *Nb*(*z*) is the background Vaisala frequency. Note that |*θ*| increases with depth because *Nb* decreases. Consistent with theory, phase propagates upward (downward) for beams in which energy propagates downward (upward). In addition, there is a phase shift of 2*π* across each beam (*i.e.*, one positive and one negative region).

Also present in the movie are gravity waves associated with high-order baroclinic modes. They tend to be standing oscillation (no vertical obvious propagation), with eastward (westward) phase velocity east (west) of the wind. Because their group velocities are small, they tend to remain near the forcing region.

**Experiment J1b**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τx* wind; *P* = 30 days

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment J1a, except with *P* = 30 days. In this case, a Kelvin beam radiates downward from the forcing region at half the angle of the one in Experiment J1a, reflects off the ocean bottom, and returns to the ocean surface. At the eastern boundary, the beam reflects as a packet of boundary waves. All of the reflected waves are boundary trapped (*i.e.*, *ℓcr* defined in Eq. G1 is less than 1 for all values of *n*). The *n* = 1, *ℓ* = 1 Rossby wave, however, is weakly damped (see Experiment G2b), and is visible in the solution. It strengthens for 30º ≤ *x* ≤ 45º, a result of direct forcing by the wind.

Because it propagates at a shallower angle, the beam is narrower than in Experiment J1a. To generate a narrower beam requires the superposition of Kelvin waves for considerably more baroclinic modes. As a result, the beam spins up more slowly, and is not well developed until June, 1991.

**Experiment J1c**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τx* wind; *P* = 60 days

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment J1a, except with *P* = 60 days. In this case, the Kelvin beam radiates downward from the forcing region at one quarter the angle of the one in Experiment J1a, and just reaches the bottom at the eastern boundary. At this period, several Rossby waves exist (see Experiment G2c), and they are visible in the solution.

**Experiment J1d**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τx* wind; *P* = 180 days

**Mixing:** *νh* = 5x106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment J1a, except with *P* = 180 days and

*X*(*x*) = cos[2*π*(*x*–60º)/40º] –40º ≤ x ≤ 80º, (J2)

where *X*(*x*) = 0 outside its designated range. It is useful to view the movie slowly (3%) to see the initial response and at intermediate speeds (25%, say) to see the equilibrium state. At this period, both Kelvin and Rossby beams are apparent.

Initially, a small-vertical-scale oscillation develops underneath the forcing region, clearly visible during March, 1991. It is a resonant, equatorial inertial oscillation with near-zero group velocity; as a result, it remains underneath the forcing region, and, because it is such a high-order baroclinic mode, it decays in time. At the same time, Rossby and Kelvin waves are also visible propagating from the forcing region, and the waves associated with a number of vertical modes quickly superpose to form beams. Because *P* = 180 days, the angle at which the Kelvin wave descends into the ocean is small, and it intersects the eastern boundary at a depth of only about 200 m. The Rossby beam that is visible west of the forcing region is a superposition of *ℓ* = 1 Rossby waves for a number of vertical modes; its structure is similar to the Kelvin beam, except that it descends into the ocean westward and at a steeper angle, *θ* = −(*σ*/*Nb*)/3.

Subsequently, Rossby waves reflect from the eastern boundary and begin to superpose to form beams. The most obvious beam leaves the eastern boundary near 200 m, reaches the bottom near 50°, and reflects from the bottom as an upward-propagating beam that intersects the western boundary near a depth of about 1200 m. An analogous beam of *ℓ* = 3 Rossby waves is less clear; it extends from the eastern boundary at 200 m, descends into the deep ocean at the steeper angle, *θ* = −(*σ*/*Nb*)/5, and intersects the bottom near 85°.

**Experiment J2a**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 15 days

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

An equatorial section of *v* and (*v*,*w*) vectors, showing the response of the LCS model with *N* = 25 modes when *P* = 15 days. Compare the response to that of Experiment J1a.

As for the Kelvin beam, the angle of energy propagation for a Yanai beam is *θ* = ±*σ*/*Nb*, where *Nb*(*z*) is the background Vaisala frequency, and phase propagates upward (downward) for beams in which energy propagates downward (upward). As a result, the ray paths of Yanai beams are *identical* to those for Kelvin beams.

Although less clear than the Kelvin-wave beam in Experiment J1a, a Yanai beam radiates downward from the forcing region, reflects off the ocean bottom to return to the ocean surface, and then makes another top-to-bottom transit and partial bottom-to-top transit. (To pick out the beam, view the movie at full speed and look for regions of clear upward and downward phase propagation. They all lie in the beam path.)

The Yanai waves that contribute to the beam are those that are strongly excited by the wind. Since the wind is large-scale with respect to the equatorial Rossby radius, essentially only Yanai waves with large zonal scale are excited. For *P* = 15 days, the waves that have large zonal scale are the low-order baroclinic modes. Consistent with this property, the dominant mode in the beam appears to be *n* = 3, as indicated by the number of zero crossings in the beam from the top to the bottom of the ocean.

Gravity waves associated with high-order baroclinic modes are also visible in the movie. They appear to be standing oscillations (*i.e.*, no vertical obvious propagation), with eastward (westward) phase velocity east (west) of the wind.

**Experiment J2b**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Fo*rcing:*** switched-on, oscillatory *τy* wind; *P* = 30 days

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment J2a, except when *P* = 30 days. Compare the response to that of Experiment J1b.

A well-formed Yanai beam radiates downward from the forcing region, reflects off the ocean bottom, and returns to the ocean surface in the top-right corner of the movie. For *P* = 30 days, the waves that have a large zonal scale and hence are strongly excited by the wind, are associated with intermediate (*n* ≈ 12) baroclinic modes. This property is apparent in that the beam has 12 zero crossings from the top to the bottom of the ocean.

There are also weak gravity waves associated with high-order baroclinic modes. They are much weaker than in Experiment J1b, because their mode number is high so that they are strongly damped. There is no indication of waves that reflect off the eastern boundary because all of them are boundary trapped (*i.e.*, *ℓcr* defined in Eq. G1 is less than 2 for all values of *n*).

**Experiment J2c**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 60 days

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment J2a, except when *P* = 60 days. Compare the response to the Kelvin-beam of Experiment J1c.

The spin-up process is interesting and best viewed at an intermediate speed (25%, say). Initially, a Yanai beam quickly forms, and at later times its propagation angle, *θ*, continues to decrease; at the same time, the beam develops a smaller vertical scale and weakens in amplitude, eventually disappearing.

For *P* = 60 days, the waves that have a large zonal scale and hence are strongly excited by the wind, are associated with very high baroclinic modes. For these very high-order modes, the equatorial Rossby radius of deformation, (*cn*/*β*)½, is so small that it cannot be resolved by the numerical grid (Δ*y* = 25 km). So, the vanishing of the Yanai beam is an artifact of the numerical model. (On the other hand, the high-order Yanai beam cannot exist in a model with realistic damping; see Experiment J2d.)

At *P* = 60 days, *ℓcr* is large enough for Rossby waves to exist for the *n* =1 and *n* = 2 baroclinic modes. Wind-generated, longer-wavelength, Rossby waves (with westward group velocity) propagate to the western boundary, where they reflect as shorter-wavelength, Rossby waves with eastward group velocity. As time progresses, the shorter-wavelength Rossby propagate into the interior ocean where they dominate the response.

Weak gravity waves associated with high-order baroclinic modes are also visible in the solution in the longitude range of the forcing.

**Experiment J2d**

**Domain:** 20ºE–100ºE, 30ºS–30ºN

**Resolution:** 0.25º

**Equatorial *β*-plane:** *f* = *βy*, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 60 days

**Mixing:** *νh* = 5x105 cm2/s, *A* = 0.00013 cm2/s3

**Movie time step:** daily

**Description:**

As in Experiment J2c, except with damping. In this case, the Yanai beam decays by explicit damping (*A* ≠ 0) as well as by numerical damping. In addition, the reflected shorter-wavelength Rossby waves are efficiently damped.

**K) Coastal beams (July 21)**

Waves generated by periodic forcing propagate vertically as well as horizontally. Such vertically propagating signals have been observed at several locations in the equatorial ocean. One property of such waves is the presence of upward phase propagation, an indication that energy is propagating downward. To obtain these “beam-like” solutions, we sum the responses to *N* = 25 baroclinic modes. The solutions are essentially repeats of the CUC solution of Section I, except that they are forced by oscillating winds.

**Experiment K1a**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 15 days

**Mixing:** *νh* = 106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

The wind has the form *τy* = *τ*o*X*(*x*)*Y*(*y*)*T*(*t*), where

*X*(*x*) = 1, *T*(*t*) = sin(σ*t*)θ(t), (K1a)

*Y*(*y*) = *θ*[(*y* – 20º)(40º – *y*)] cos[2*π*(y–60º)/40º], (K1b)

where *θ* is a step function, *σ* = 2*π*/*P*, *P* = 15 days, and *τ*o = –2 dyn/cm2, a band of wind confined between 20ºN and 40ºN. The movie shows the response of the LCS model with *N* = 25 modes, plotting an alongshore (*y*,*z*) section of *p* and (*v*,*w*) vectors.

During the spin-up, Kelvin waves associated with several low-order baroclinic modes radiate northward along the coast. They superpose to form a beam that is fully established by the end of February, 1991, by which time all the Kelvin waves have reached the northern boundary. In equilibrium, the Kelvin beam radiates downward from the forcing region, and reaches the ocean bottom near the northern edge of the basin (50°N). The angle of energy propagation is given by *θ* = ±*σ*/*Nb*, where *Nb*(*z*) is the background Vaisala frequency. Note that |*θ*| increases with depth because *Nb* decreases. Consistent with theory, phase propagates upward (downward) for beams in which energy propagates downward (upward). In addition, there is a phase shift of 2*π* across each beam (*i.e.*, one positive and one negative region).

**Experiment K1b**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 30 days

**Mixing:** *νh* = 106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment K1a, except with *P* = 30 days. In this case, the Kelvin beam radiates downward from the forcing region at half the angle of the one in Experiment K1a, and intersects the northern boundary near 1000−1500 m.

**Experiment K1c**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 60 days

**Mixing:** *νh* = 106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment K1a, except with *P* = 60 days. The Kelvin beam radiates downward from the forcing region at a quarter of the angle of the one in Experiment K1a, and intersects the northern boundary near 400−500 m.

**Experiment K1d**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 180 days

**Mixing:** *νh* = 106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment K1a, except with *P* = 180 days. The Kelvin beam intersects the northern boundary near the ocean surface (about 200 m). There is clear vertical phase propagation across the beam, which is visible even to the ocean bottom.

**Experiment K1e**

**Domain:** 60ºE–100ºE, 10ºN–50ºN

**Resolution:** 0.1º

***β*-plane:** *f* = *f*0 + *β(y – y*0*), y*0 = 30ºN, *β* = 2.28x10−13 cm−1s−1

**Number of modes:** *N* = 25

**Forcing:** switched-on, oscillatory *τy* wind; *P* = 180 days

**Mixing:** *νh* = 106 cm2/s, *A* = 0

**Movie time step:** daily

**Description:**

As in Experiment K2a, except showing an across-shore (*x*,*z*) section at 40ºN. The transient and equilibrium responses have very different properties, namely, the former propagates offshore and the latter remains trapped at the coast.

To see the equilibrium response most clearly, view the second half of the movie by which time most of the transient response has propagated offshore. At *P* = 180 days, the critical latitude associated with mode-*n* baroclinic waves is *ycr* = *cn*/(2*σ*) ≤ *c*1/(2*σ*) = 3270 km = 29°. Since the latitude of this section is at 40°N, it follows that no Rossby waves are present in the equilibrium solution (see the discussion of Eq. G2), and the coastal response is damped offshore. At the coast, phase propagates upward, indicating the presence of a Kelvin-wave beam.

During the transient response, Rossby waves propagate offshore at the speeds, *crn* = *βcn*2/*f*2. (The transient response does not remain coastally trapped. Only the oscillating, equilibrium response does.) Since *cn* decreases with *n*, first-baroclinic-mode Rossby waves propagate offshore first followed by higher-order ones. For example, on January 1, 1992, the response west of about 98°E has one zero crossing in the vertical, indicating the presence of an *n* = 1 baroclinic mode. Subsequently, the offshore response is much weaker (visible only as two shades of green) and develops more zero crossings. On January 1, 1995, it has two zero crossings (an *n* = 2 mode), and near July, 1998, it has three (an *n* = 3 mode). Toward the end of the movie, there are indications of an offshore signal with 4 zero crossings.