

A light green world map is centered in the background, showing the continents. The text is overlaid on the map.

Overview of Indian-Ocean dynamics

Targeted Training Activity on:
Towards Improved Monsoon Simulations

International Centre for Theoretical Physics

Trieste, Italy

June 13–17, 2016



Dynamics of wind-driven circulations in the IO

**Targeted Training Activity on:
Towards Improved Monsoon Simulations**

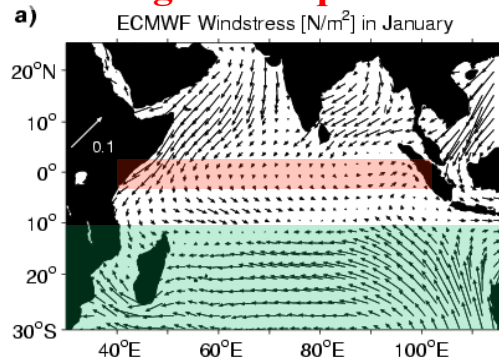
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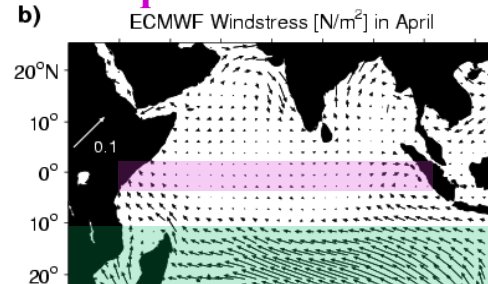
June 13–17, 2016

Climatological wind forcing

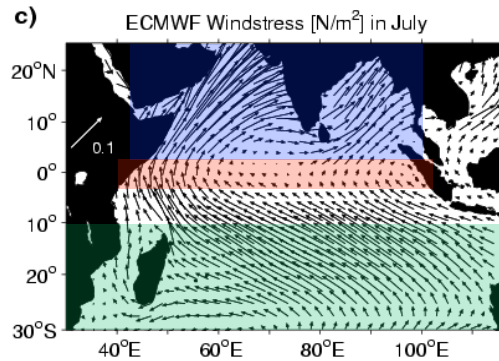
Reversing cross-equatorial winds



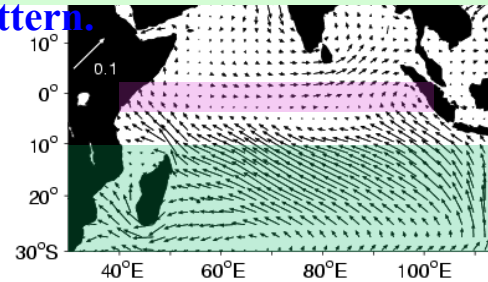
Equatorial westerlies



Upwelling-favorable annual-mean



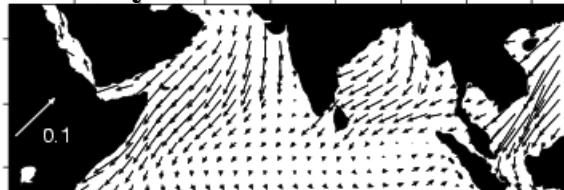
As a result, the **IO monsoon winds circulate clockwise (anticlockwise)** about the equator during the **summer (winter)**. The **annual-mean winds have the summer pattern.**



Relatively steady southeast tradewinds

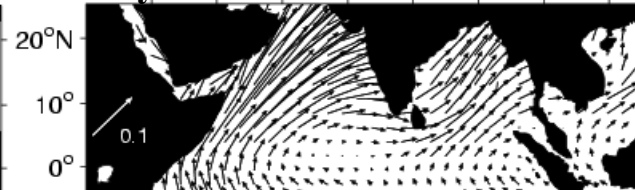
Wind-forced thermocline response

January

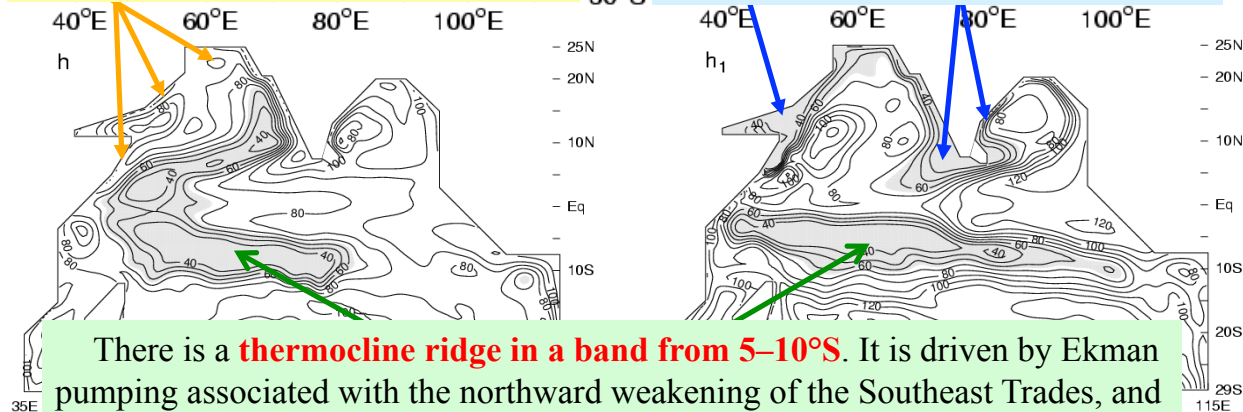


During the NEM, the **thermocline deepens** due to **alongshore winds off Somalia and Oman** and to **surface cooling in the northern AS**.

July



During the SWM, **upwelling favorable winds lift the thermocline** off Somalia and Oman, on the Indian coast, and around Sri Lanka.



There is a **thermocline ridge in a band from 5–10°S**. It is driven by Ekman pumping associated with the northward weakening of the Southeast Trades, and is **stronger in northern summer** when the Trades are stronger.

(1993)

Wave radiation

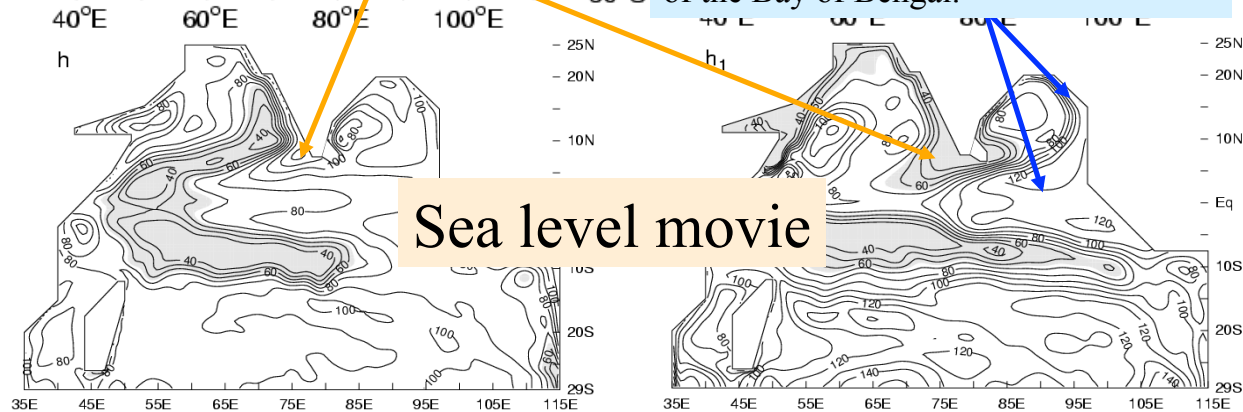
January

July

The **upwelling and downwelling responses** do not remain confined to their forcing regions. Rossby and Kelvin waves **radiate away from the forcing regions**, thereby impacting the ocean remotely.

Rossby waves radiate off the west coast of India during both seasons.

The **spring Wyrтки Jet reflects** from the IO eastern boundary as a **packet of Rossby waves**, and a **Kelvin (coastal) wave propagates** around the perimeter of the Bay of Bengal.



McCreary, Kundu, and Molinari (1993)

Goal

To provide an **introduction** to the dynamics of **wind-driven circulations in the North Indian Ocean (NIO)**, both for its **mean state and climatic variability**.

Approach

Our approach is to **split the complete IO response into smaller pieces** in the **interior, equatorial, and coastal oceans**. We then **discuss simple solutions** that illustrate the **dominant processes** at work in each region. These simpler solutions **provide the “language”** needed to discuss more complex problems.

Questions

What types of ocean models are useful for studying NIO phenomena?

model hierarchy: OGCMs, LCS model, layer models

What types of ocean waves impact NIO phenomena?

gravity & Rossby waves; Kelvin waves; (shelf waves)

How does the wind drive ocean circulations?

Ekman flow; Ekman pumping; excitation of Rossby and Kelvin waves; adjustment to Sverdrup balance

How do wind-driven dynamics differ in the interior, equatorial, and coastal oceans?

They are very similar, differing primarily in the types of waves that are generated.

References: ITCP and Dropbox

This lecture, [TTAlecture.pptx](#), and other files can be obtained from the ITCP website at:

<http://indico.ictp.it/event/7666/>

They can also be found at the Dropbox link:

<https://www.dropbox.com/home/Trieste2016>

at least for the next month.

Today's talk **focuses on physical concepts**, rather than mathematical solutions. There are **additional slides** at the end of [TTAlecture.pptx](#) that **derive some of the solutions** discussed today, as well as other **supportive material**.

References: NIOSS (2010)



The content & organization of this lecture were inspired by a set of lectures I gave at a **summer school held at NIO in 2010 (NIOSS)**. An overview of NIOSS can be found at the web site:

http://www.nio.org/index/option/com_newsdisplay/task/view/tid/4/sid/23/nid/255

References: NIOSS (2010)

All of the movies that I show were prepared during NIOSS, and they are stored in the **Tutorials folders** at the website

http://www.nio.org/index/option/com_eventdisplay/task/view/tid/4/sid/114/eid/143

They can also be obtained from the ICTP web site. I recommend that you **download all of these movies** onto your own computer.

A **description of each movie** is given in **ExperimentList.docx**, located in the **Tutorials folders** at the NIOSS website, as well as at the ICTP website.

References: INCOIS winter school (2015)



This lecture is a 1½-hour version of a set of lectures I gave during a **winter school held at INCOIS in 2016**. The complete set of lectures can be downloaded from the web site:

http://www.incois.gov.in/portal/ITCOcean/course_materials.jsp

A light green world map is visible in the background of the slide, showing the continents and oceans.

Organization

1) Hierarchy of ocean models

2) Midlatitude-ocean waves

3) Interior ocean

4) Equatorial ocean

5a) Equatorial waves

5b) Wind-forced solutions

[6) Coastal Ocean]

7) Summary



Hierarchy of ocean models:

OGCMs, LCS and layer models

Simpler ocean models

OGCMs are remarkably **good at simulating oceanic phenomena**.

At the same time, it is often **difficult to isolate basic processes** at work in OGCM solutions. Instead, **basic processes are illustrated better in simpler systems**. The simpler systems provide a **language for discussing phenomena and processes** in the more complicated ones. Moreover, **OGCM & simple solutions are often quite similar** to each other and to observations.

Here, I introduce equations for the **linear, continuously stratified (LCS) and 1½-layer models**, which are simpler equation sets that allow for **analytic solutions**. Most of our **understanding of ocean dynamics arises from analytic solutions** to these simpler equation sets.

OGCM equations

The following set of equations are the equations of motion that are solved in many OGCMs.

$$u_t + uu_x + vv_y + wu_z - fv + \frac{1}{\rho} p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + uv_x + vv_y + wv_z + fu + \frac{1}{\rho} p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

Many OGCMs adopt the **hydrostatic approximation**.
In most physical situations, it is an **EXCELLENT** one.

$$T_t + uT_x + vT_y + wT_z = (\kappa_T T_z)_z + \nu_h \nabla^2 T,$$

$$S_t + uS_x + vS_y + wS_z = (\kappa_S S_z)_z + \nu_h \nabla^2 S,$$

$$\nabla \cdot \mathbf{v} = 0,$$

$$\rho = \rho(S, T, p)$$

LCS model: mode equations

The LCS model **linearizes and simplifies the OGCM equations** until it is possible to express the **u , v , and p fields as the expansions**

$$u = \sum_{n=0}^{\infty} u_n \psi_n, \quad v = \sum_{n=0}^{\infty} v_n \psi_n, \quad \frac{p}{\rho} = \sum_{n=0}^{\infty} p_n \psi_n,$$

w/ The resulting equations for u_n , v_n , and p_n are

$$\begin{aligned} \left(\partial_t + \frac{A_\nu}{c_n^2} \right) u_n - f v_n + p_{nx} &= \frac{\tau^x}{\mathcal{H}_n} + \nu_h \nabla^2 u_n, \\ \left(\partial_t + \frac{A_\nu}{c_n^2} \right) v_n + f u_n + p_{ny} &= \frac{\tau^y}{\mathcal{H}_n} + \nu_h \nabla^2 v_n, \\ \left(\partial_t + \frac{A_\kappa}{c_n^2} \right) \frac{p_n}{c_n^2} + u_{nx} + v_{ny} &= \nu_h \nabla^2 \frac{p_n}{c_n^2} \end{aligned} \quad \mathcal{H}_n^{-1} = \frac{\int_{-D}^0 Z(z) dz}{\int_{-D}^0 \psi_n^2 dz} \quad (\text{A})$$

Th See additional slides and HIGNotes.pdf for a derivation of (A).
independent responses associated with each mode. They **interact only**
in the values of , the **Kelvin-wave speed** for each mode.

LCS model: baroclinic and barotropic modes

The functions **are the baroclinic and barotropic modes** of the ocean. They require that the **ocean bottom is flat** at $z = -D$ and that **depends only on z** . They are then solutions

to

$$\left(\partial_z \frac{1}{N_b^2} \partial_z \right) \psi_n = \left(\frac{1}{N_b^2} \psi_{nz} \right)_z = -\frac{1}{c_n^2} \psi_n \quad (1)$$

subject to the boundary conditions and normalization

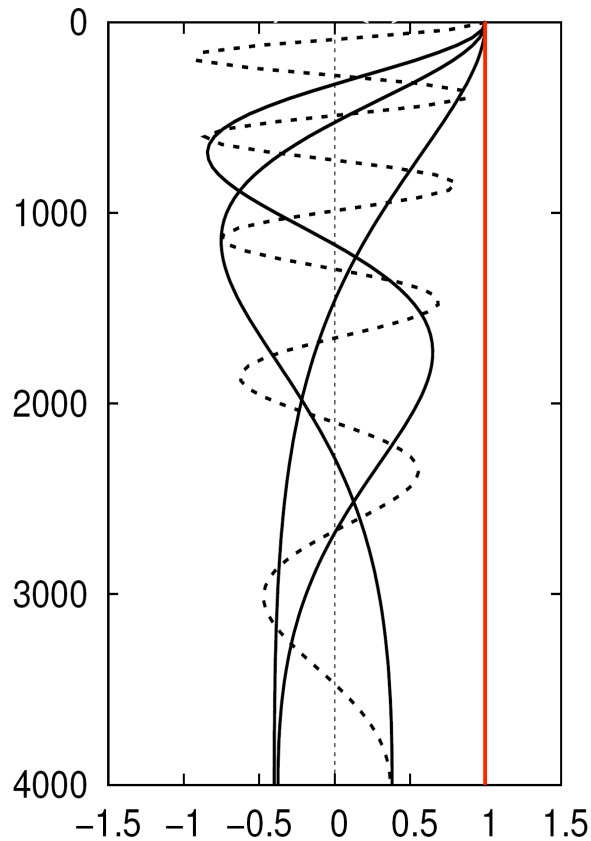
$$\psi_{nz}(-D) = \psi_{nz}(0) = 0, \quad \psi_n(0) = 1 \quad (2)$$

Integrating (1) over the water column gives

$$-\int_{-D}^0 \left(\frac{1}{N_b^2} \psi_{nz} \right)_z dz = -\frac{1}{N_b^2} \psi_{nz} \Big|_{-D}^0 = 0 = -\frac{1}{c_n^2} \int_{-D}^0 \psi_n dz \quad (3)$$

Constraint (3) can be satisfied in two ways. Either $c_n \rightarrow \infty$ **in which case $\psi_n(z) = 1$ (barotropic mode)** or c_n **is finite and its value is set so that the integral of ψ_n vanishes (baroclinic modes)**.

LCS model: baroclinic and barotropic modes



When $N \downarrow b \uparrow z$ **decreases with depth** like

$$N_b^2 = -g\rho_{bz}/\rho_0 = ge^{z/b}$$

and c_n **is finite**, solutions to (1) are similar, except their **wavelength increases and amplitude decreases with depth**.

The **values of c_n** are different from, but are **similar to, those for constant density**.

When c_n **is infinite**, the solution to (1) that satisfies boundary conditions (2) is

$$\psi_0(z) = 1$$

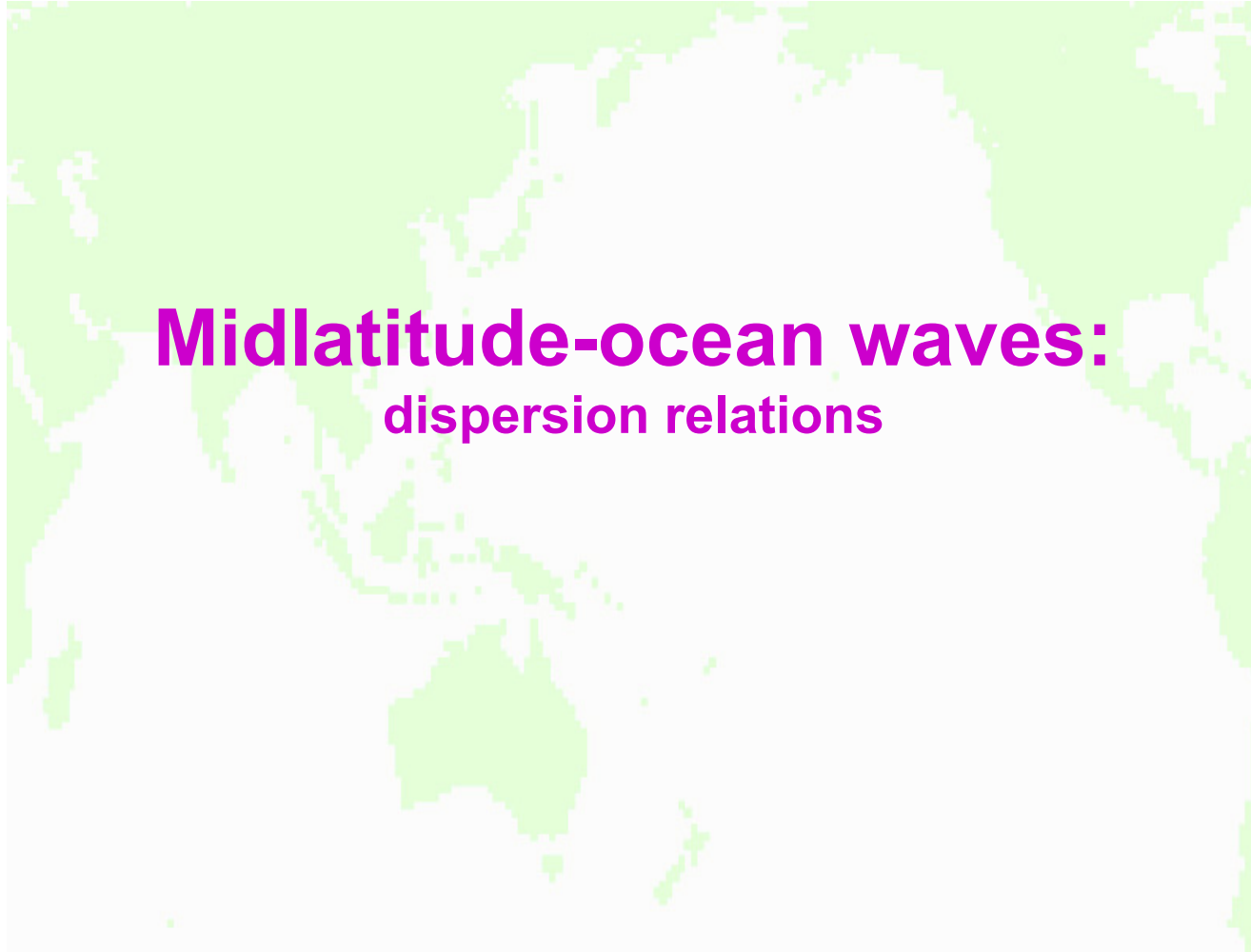
the **barotropic mode of the system**.

1½-layer model

If a particular phenomenon is **surface trapped**, it is often useful to study it with an **upper-layer model** that focuses on the surface flow. Such a model is the **1½-layer, reduced-gravity model**. In its **linear form**, its equations are

$$\begin{aligned}u_{nt} - fv_n + p_{nx} &= \frac{\tau^x}{\mathcal{H}_n} - \frac{A_\nu}{c_n^2}u_n + \nu_h \nabla^2 u_n, \\v_{nt} + fu_n + p_{ny} &= \frac{\tau^y}{\mathcal{H}_n} - \frac{A_\nu}{c_n^2}v_n + \nu_h \nabla^2 v_n, \\p_{nt} + c_n^2 (u_{nx} + v_{ny}) &= -\frac{A_\kappa}{c_n^2}p_n + \nu_h \nabla^2 p_n\end{aligned}$$

Most of the NIOSS movies are numerical **solutions to the LCS model for a single ($n = 1$) baroclinic mode** or, equivalently, **to a 1½-layer model**. A few movies are LCS solutions that are a **sum of a number of baroclinic modes**.



Midlatitude-ocean waves:
dispersion relations

$v \downarrow n$ equation

To focus on the free waves, we neglect **forcing**, **damping**, and **friction** terms in the equations for a mode of the LCS model (or 1½-layer model) to get

$$\begin{cases} \left(\partial_t + \right. & u_{nt} - f v_n + p_{nx} = 0, & \cancel{\nabla^2} u_n, \\ \left. \partial_t + \right) & v_{nt} + f u_n + p_{ny} = 0, & \cancel{\nabla^2} v_n, \\ & \frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0. \end{cases}$$

Waves associated with a **superposition of vertical modes**

$$u = \sum_{n=0}^N u_n \psi_n(z), \quad v = \sum_{n=0}^N v_n \psi_n(z), \quad p = \sum_{n=0}^N \bar{\rho} p_n \psi_n(z).$$

propagate both horizontally and vertically.

$v \downarrow n$ equation

Solving the unforced, inviscid equations for a **single equation in** v_n , and for convenience **dropping subscripts n** gives

$$v_{xxt} + v_{yyt} - \frac{1}{c^2} v_{ttt} - \frac{f^2}{c^2} v_t + \beta v_x = 0. \quad (1)$$

Solutions See additional slides for a derivation of (1). Use f is a **function of y** and the equation **includes y derivatives** ($v \downarrow yyt$ term). There are, however, useful analytic solutions to **approximate versions of (1)**.

Dispersion relation of free waves

The simplest approximation (**mid-latitude β -plane approximation**) simply **“pretends” that f and β are both constant**. Then, solutions to (1) have the form of **plane waves**,

$$\exp(ikx + i\ell y - i\sigma t).$$

Then, we can set $u = u_0 \exp(ikx + i\ell y - i\sigma t)$, $v = v_0 \exp(ikx + i\ell y - i\sigma t)$, and $\psi = \psi_0 \exp(ikx + i\ell y - i\sigma t)$ in (1), resulting in the dispersion relation,

$$u_{xxt} - \sigma \left(k^2 + \ell^2 - \frac{\sigma^2}{c^2} + \frac{f^2}{c^2} \right) = -k\beta. = 0.$$

The **dispersion relation provides a “biography”** for a model. It **describes everything about the waves** it supports.

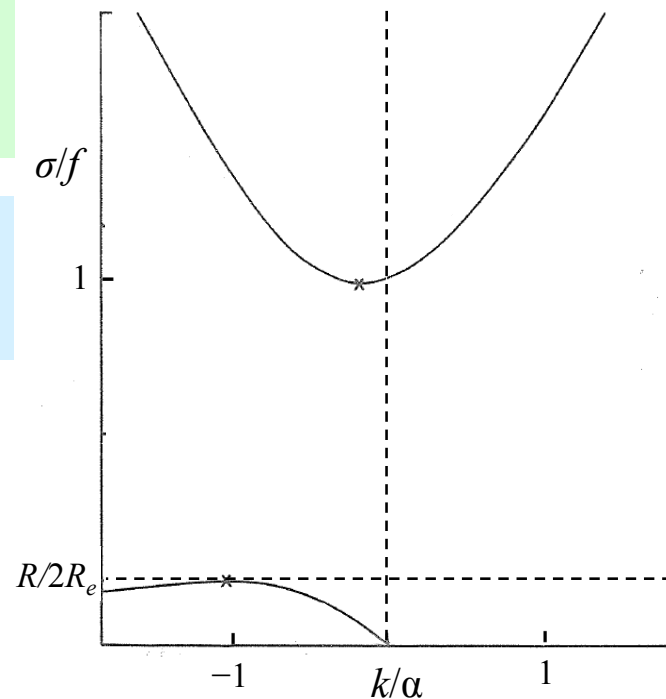
Gravity and Rossby waves

For convenience, the figure plots curves **when $\ell = 0$** .

When $\ell \neq 0$, the disp. rel. is a **circle for each σ** . So, the two curves become **circular bowls**.

The **top bowl (bottom bowl)** describes the **gravity waves (Rossby waves)** of the system.

$$\left(k + \frac{\beta}{2\sigma}\right)^2 + \ell^2 = \frac{\sigma^2 - f^2}{c^2} + \frac{\beta^2}{4\sigma^2}$$



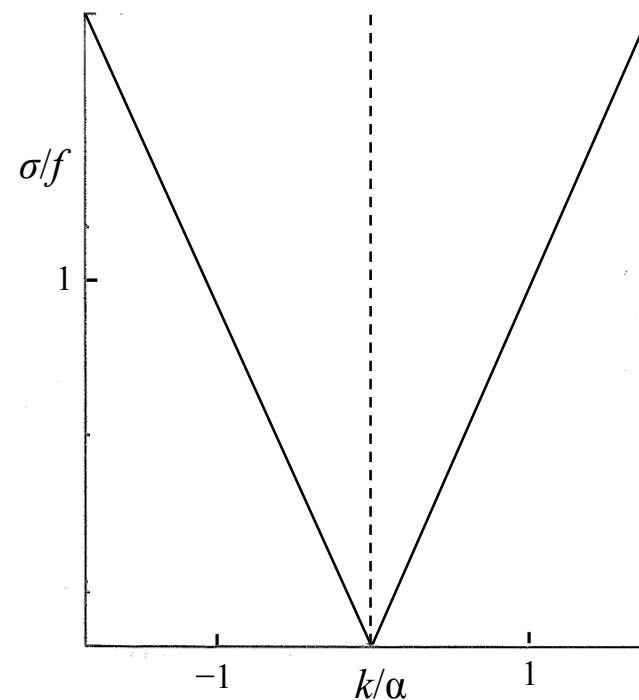
Kelvin waves

To derive the dispersion relation for GWs and RWs, we solved for a single equation in ν . So, we **missed a wave with $\nu = 0$** , the **coastal Kelvin wave**.

See the additional slides for a derivation of the Kelvin wave. **Kelvin waves along zonal boundaries**. KWs also exist along meridional boundaries.

The coastal KW propagates **along coasts at speed c** with the **coast to its right (in the NH)**, and **decays offshore with the scale $c/f = R$** , the Rossby radius of deformation.

$$\sigma = \pm ck$$



Phase and group speed

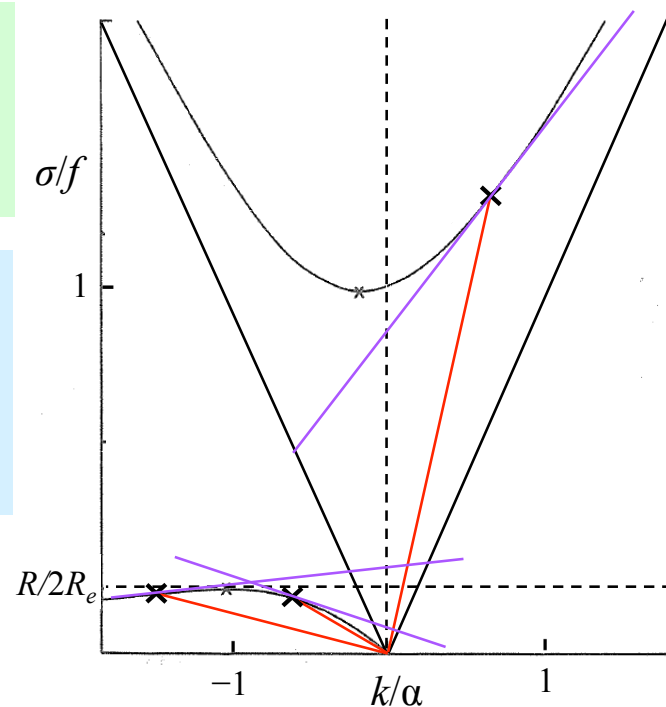
The figure shows the wave types that we have discussed.

$$\left(k + \frac{\beta}{2\sigma}\right)^2 + \ell^2 = \frac{\beta^2}{4\sigma^2} - \frac{f^2}{c^2}, \quad \sigma = \pm ck$$

The **phase speed** of a wave with wavenumber k and frequency σ is the **slope of the line that extends from $(0,0)$ to (σ,k)** .

The **group speed** of a wave with wavenumber k and frequency σ is the **slope of the line parallel to the dispersion curve at the point (σ,k)** .

Movies A1, A2, A3b





**Interior ocean:
Ekman pumping &
adjustment to Sverdrup balance**

Interior-ocean equations

Equations of motion for the u , v , and h fields of a single baroclinic mode are

$$\begin{cases} -fv + g'h = \tau^x / H, \\ fu + g'h_y = \tau^y / H, \\ h_t + H(u_x + v_y) = -\kappa(h - H) \end{cases}$$

This approximation is useful because it **filters out the gravity-wave response**. Thus, it only describes the **slowly varying parts** of the response, that is, its **Ekman-drift, Ekman pumping and Rossby wave (if $\beta \neq 0$) parts**.

ocean, and are only retained to represent western boundary currents.

Ekman pumping ($\beta=0$) with $\kappa = 0$ forced by $\tau \uparrow x$

Suppose the ocean is forced by a **zonal-wind stress** **switched on at** $t=0$, that **is constant** ($\tau^x = \tau_0$), and that there is **no damping** ($\kappa = 0$). Then, the solution is

$$h = H - w_{ek}t = H + \frac{\tau_y^x}{f}t$$

so that **h thickens (thins) continuously where $\tau_y^x > 0$ (< 0).**

With h known, the zonal and meridional velocities are

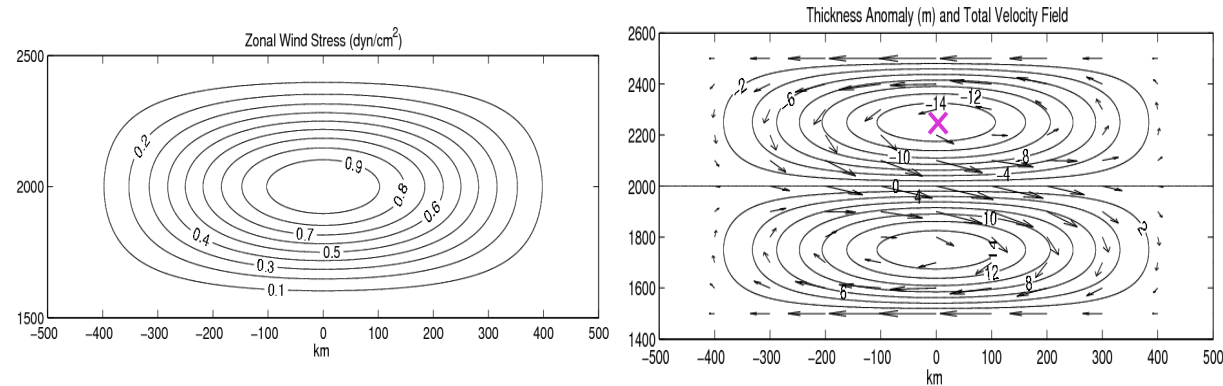
See additional slides for a derivation of this solution.

$$v = \frac{1}{f}g'h_x - \frac{\tau^w}{fH}, \quad u = -\frac{1}{f}g'h_y$$

a superposition of **Ekman and geostrophic currents**.

Note that, although the **geostrophic flow grows linearly in time**, the **Ekman flow switches on instantly at $t=0$ and thereafter remains constant**. It switches on instantly because the **interior-ocean equations filter out gravity (inertial) waves**.

Ekman pumping ($\beta=0$) with $\kappa = 0$ forced by $\tau \uparrow x$



For this wind, **north of 2000 km and h thin**, and the opposite change happens south of 2000 km. The **constant Ekman drift shifts water continually from the northern to the southern half** of the domain. **Counter-rotating geostrophic gyres spin-up** in response to h .

How long does it take for the **layer bottom to upwell to the surface**?

$$H = -\frac{\min(\tau_y^x)}{f} t = \frac{\pi}{2\Delta y} \frac{\tau_o}{f} t \quad \Rightarrow \quad t = \frac{2\Delta y f}{\pi \tau_o} H$$

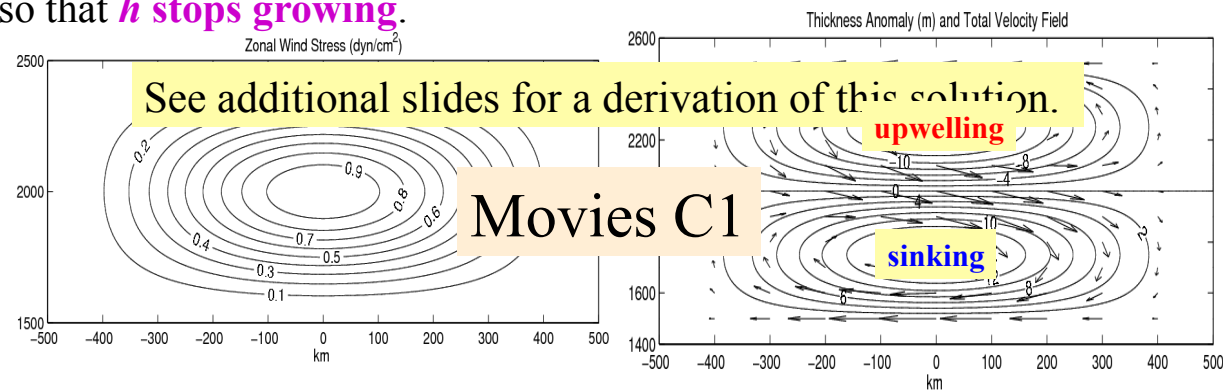
which for the above wind, $H = 100$ m, and $f = 10^{-4} \text{ s}^{-1}$ is **$t = 368$ days**.

Ekman pumping ($\beta=0$) with $\kappa \neq 0$ forced by $\tau \uparrow x$

Suppose the ocean is **forced by a zonal wind** (i.e., τ_x) and there is **damping** ($\kappa \neq 0$). Then, the solution is

$$h = H - \frac{1 - e^{-\kappa t}}{\kappa} w_{ek} = H + \frac{1 - e^{-\kappa t}}{\kappa} \frac{\tau_y^x}{f}$$

so that h stops growing.

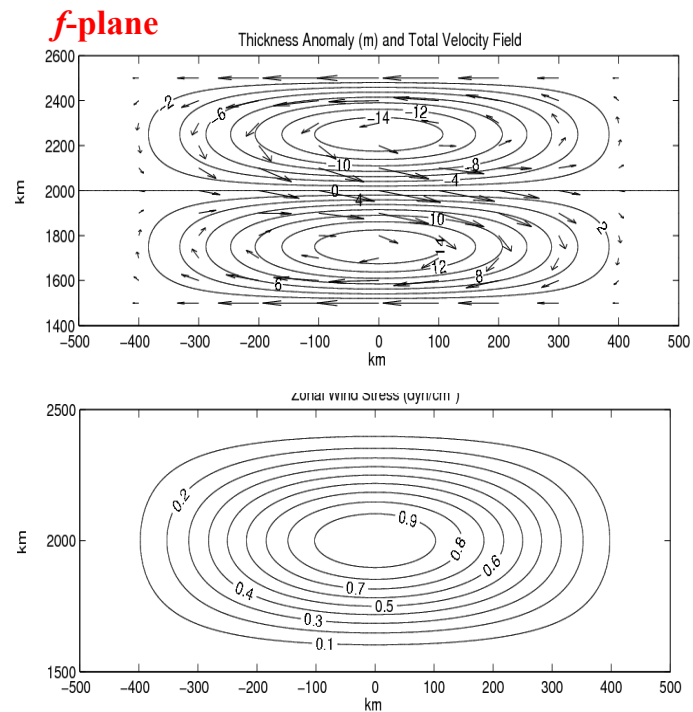


In steady state, **Ekman drift still flows from the northern to the southern half** of the domain. Water **entrains into the layer in the north to provide a source** for the Ekman drift and **detrains from the layer in the south to provide a sink**, forming an **overturning cell**.

Adjustment to Sverdrup balance ($\beta \neq 0$) forced by $\tau \uparrow x$

Consider the response to a switched-on patch when .

The **initial response** is the same as on the ***f*-plane**.
Ekman flow switches on instantly because gravity waves are filtered out of the system, and **wind curl drives Ekman pumping**.

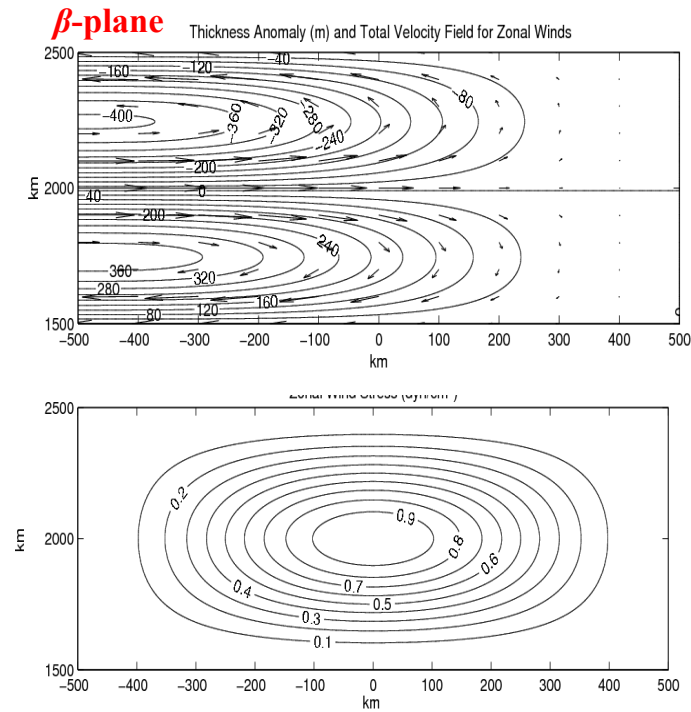


Adjustment to Sverdrup balance ($\beta \neq 0$) forced by $\tau \uparrow x$

Consider the response to a switched-on patch when .

Subsequently, **westward radiation of Rossby waves** extends the response west of the forcing region, and **adjusts the circulation to Sverdrup balance**.

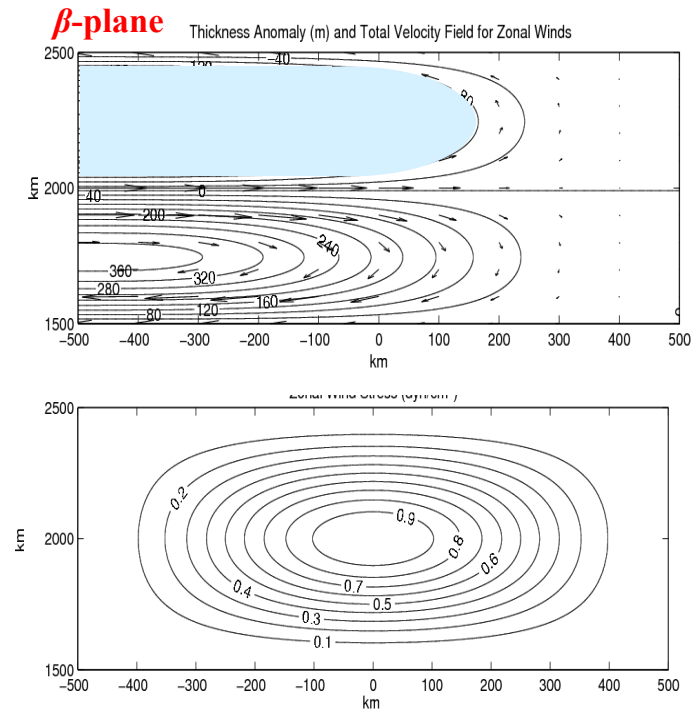
See additional slides for a derivation of this solution.



Adjustment to Sverdrup balance ($\beta \neq 0$) forced by $\tau \uparrow x$

Consider the response to a switched-on patch when .

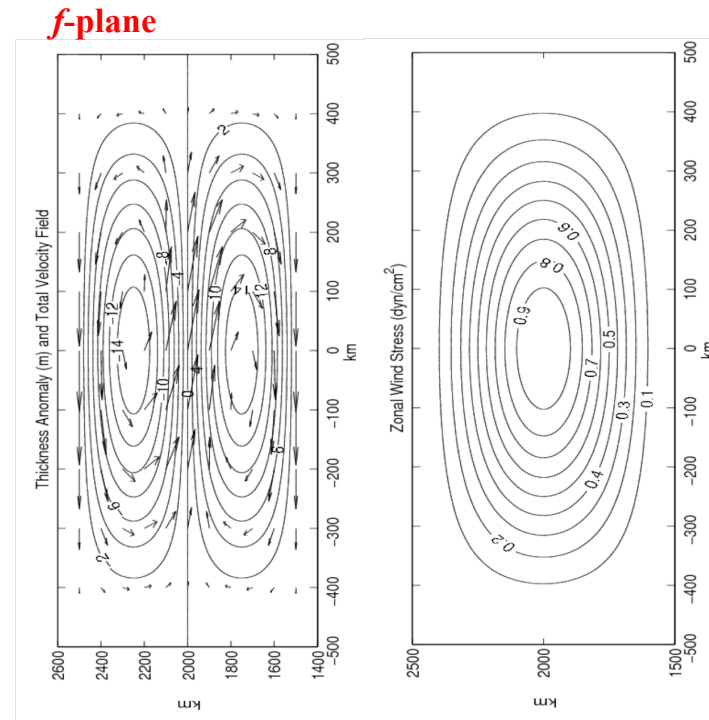
At any longitude, **Ekman pumping continues until the passage of Rossby waves.** Because they propagate slowly, the Ekman pumping can be **large enough for the bottom of the layer to rise to the surface (light blue area)**. In that case, the solution breaks down, and **there must be upwelling from the deep ocean.**



Adjustment to Sverdrup balance ($\beta \neq 0$) forced by $\tau \uparrow \gamma$

Consider the response to a switched-on **patch** when .

The **initial response** is the same as on the ***f*-plane**.
Ekman flow switches on instantly because gravity waves are filtered out of the system, and **wind curl drives Ekman pumping**.

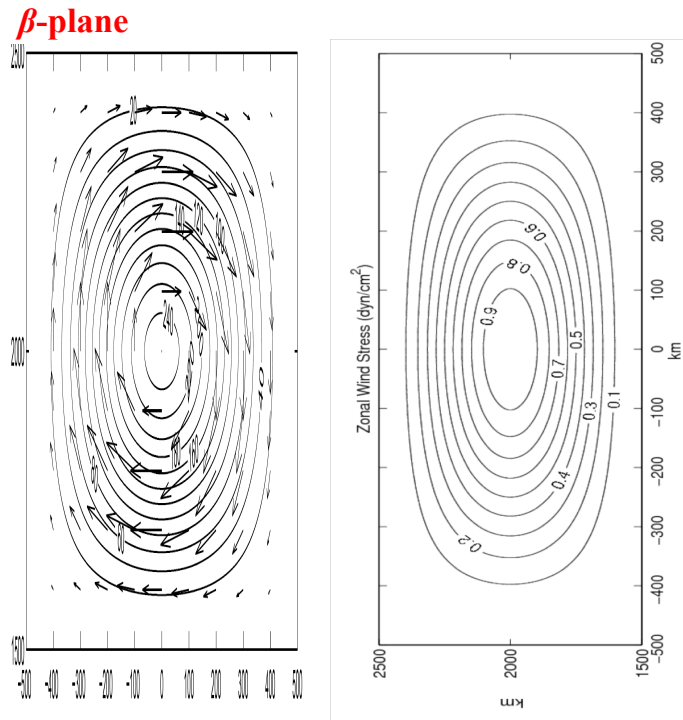


Adjustment to Sverdrup balance ($\beta \neq 0$) forced by $\tau \uparrow \gamma$

Consider the response to a switched-on **patch** when .

Subsequently, the **Rossby waves radiate westward** across the wind patch, but after their passage the adjusted **response remains confined to the forcing region**.

Movies C3 & C2a





**Equatorial ocean:
equatorially trapped waves &
wind-forced solutions**

Questions

What forcing mechanisms drive equatorial currents?

zonal and meridional wind stress

What are equatorial waves?

equatorial gravity, Rossby, and Kelvin waves;
mixed Rossby/gravity (Yanai) wave

How do they differ from midlatitude waves?

dynamically very similar; extra Yanai wave;
discreteness

What are the key differences between 2-d and 3-d theories of equatorial circulation?

Yoshida Jet; establishment of $p \downarrow x$ to balance $\tau \uparrow x$

How do equatorial waves reflect from basin boundaries?

Kelvin- and Rossby-wave reflections



**Equatorial ocean:
equatorially trapped waves**

Equatorial-ocean equations

Equations for the u_n , v_n , and p_n for a single baroclinic mode are

$$\begin{aligned} \left(\partial_t + \frac{A}{c_n^2} \right) u_n - f v_n + p_{nx} &= \tau^x / \mathcal{H}_n + \nu_h \nabla^2 u_n, \\ \left(\partial_t + \frac{A}{c_n^2} \right) v_n + f u_n + p_{ny} &= \tau^y / \mathcal{H}_n + \nu_h \nabla^2 v_n, \\ \left(\partial_t + \frac{A}{c_n^2} \right) \frac{p_n}{c_n^2} + u_{nx} + v_{ny} &= 0, \end{aligned} \quad (1)$$

Because f vanishes at the equator, **no terms can be dropped that allow for mathematically simple solutions** near the equator.

A useful assumption, though, is to set $f = \beta y$, known as the **equatorial β -plane approximation**. As a result, one can look for solutions as **expansions in Hermite functions**.

Equatorial gravity and Rossby waves

Equatorial waves are unforced () solutions to (1) of the form, , and **without damping** ($A = 0$). For convenience, we **drop the subscript n** . The **resulting v equation is**

$$\sigma k^2 v - \frac{\sigma^3}{c^2} v - \sigma \alpha_o^2 (\partial_{\eta\eta} - \eta^2) v + k\beta v = 0 \quad (2)$$

The mathematical c See additional interior-ocean version relation from (2) is that, because **f** slides for a derivation. **tor**, it is **not possible to set $\phi^\ell(\gamma) = \exp(i\ell\gamma)$** , like we did for the interior ocean. Rather, **$\phi^\ell(\gamma)$ is the set of solutions (eigenfunctions) that satisfy**

$$(\partial_{\eta\eta} - \eta^2) \phi_\ell = \lambda_\ell \phi_\ell = -(2\ell + 1) \phi_\ell, \quad (2)$$

and **vanish as $\eta \rightarrow \pm\infty$** , where **$\ell = 0, 1, 2, \dots$** . They are referred to as **Hermite functions**.

$^{-1}\text{s}^{-1}$, its value is **$R_1 = 331 \text{ km}$** .

Equatorial gravity and Rossby waves

The solutions to (2) can be represented as expansions in Hermite functions

$$v = \sum_{\ell=0}^{\infty} v_{\ell}(x) \phi_{\ell}(\eta) e^{ikx - i\sigma t}, \quad (3)$$

where v_{ℓ} is a wave amplitude. **Each term in expansion (3) is an individual equatorial wave.**

Inserting term v_{ℓ} in (3) into (2) and using (2) gives

$$\sigma k^2 v_{\ell} - \frac{\sigma^3}{c^2} v_{\ell} + \sigma \alpha_o^2 (2\ell + 1) v_{\ell} + k\beta v_{\ell} = 0,$$

which provides the **dispersion relation**

$$\sigma \left(k^2 + \alpha_{\ell}^2 - \frac{\sigma^2}{c^2} \right) + k\beta = 0, \quad \alpha_{\ell}^2 = \alpha_o^2 (2\ell + 1),$$

for equatorial, Rossby and gravity waves.

Equatorial gravity and Rossby waves

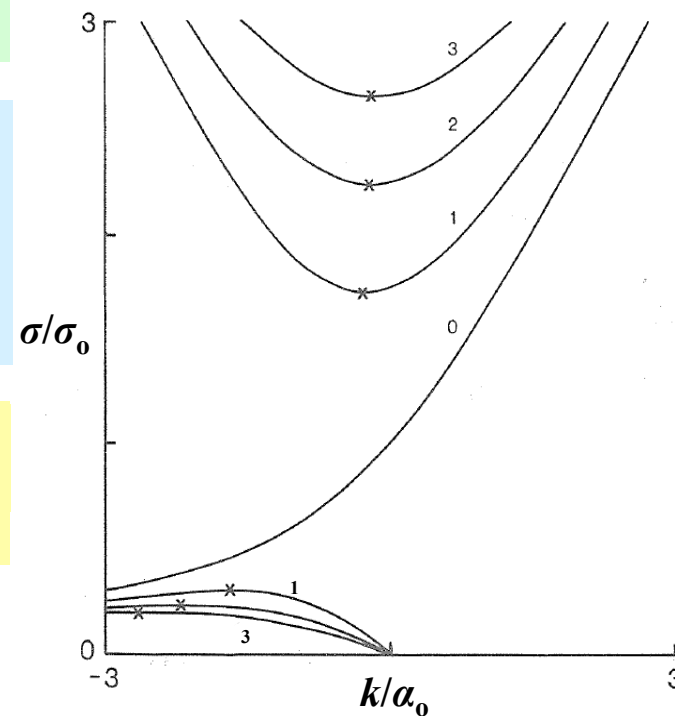
For each $\ell > 1$, there is a **gravity wave** (large σ) and a **Rossby wave** (small σ). The plot shows waves for $\ell = 1, 2$, and 3.

continuously for midlatitude

For $\ell = 0$, there is a new type of wave, the **mixed Rossby-gravity (Yanai) wave**, which behaves like a Rossby (gravity) wave for k positive (negative).

See additional slides for a deriv. of the Yanai-wave relation. dispersion curves.

$$\sigma \left(k^2 + \alpha^2 - \frac{\sigma^2}{c^2} \right) + k\beta = 0$$



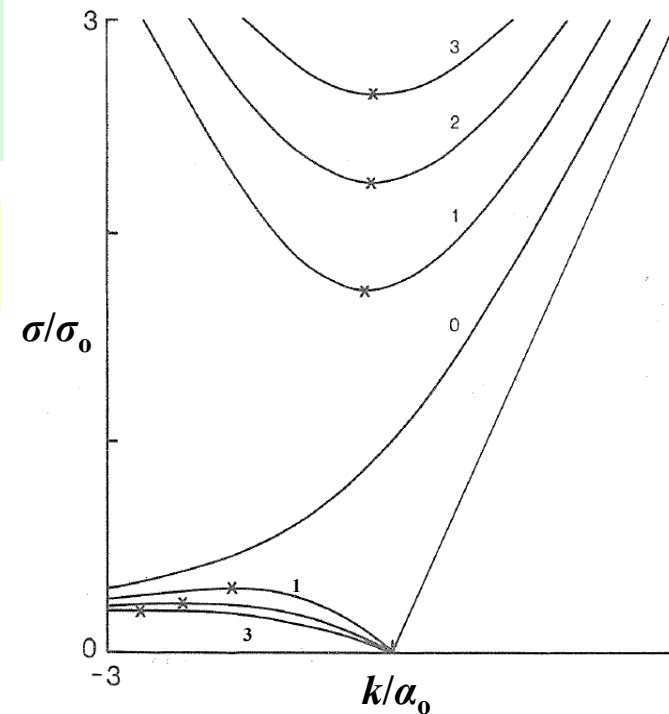
Theoretical equatorial waves

To derive the dispersion relation for GWs and RWs, we solved for a single equation in ν . So, we **missed a wave with $\nu=0$** , the **coastal Kelvin wave** with the **dispersion relation $\sigma=kc$** .

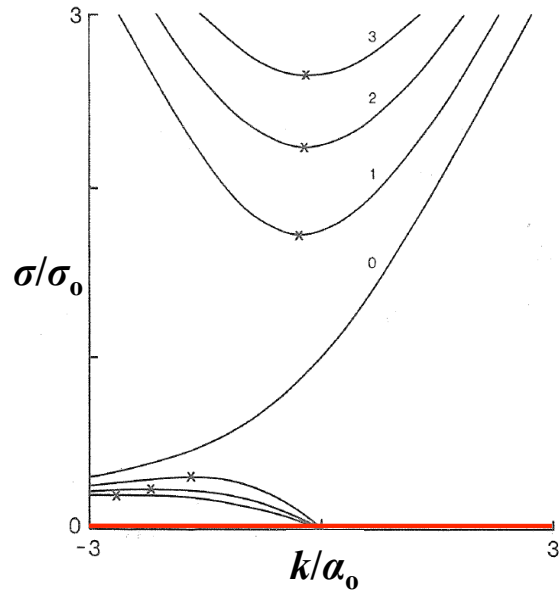
See additional slides for a deriv. of the equatorial KW.

and a **Rossby wave** (small σ). The plot indicates waves only for $\ell = 1, 2,$ and 3 . In addition, there is the **Yanai wave** for $\ell=0$, and the **equat. Kelvin wave** with $\nu=0$.

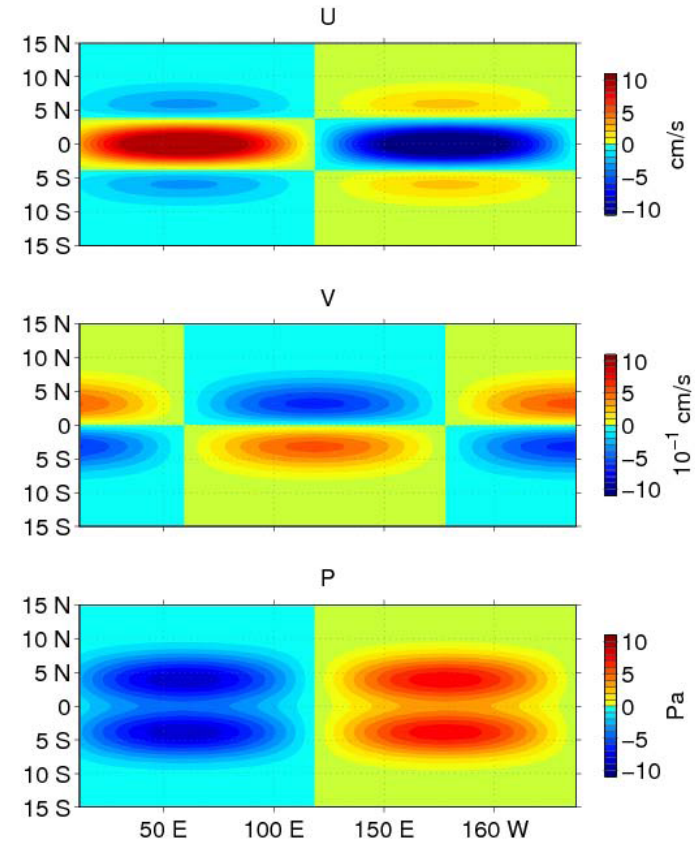
$$\sigma \left(k^2 + \alpha_\ell^2 - \frac{\sigma^2}{c^2} \right) + k\beta = 0, \quad \sigma = kc$$



Structure of equatorial Rossby waves

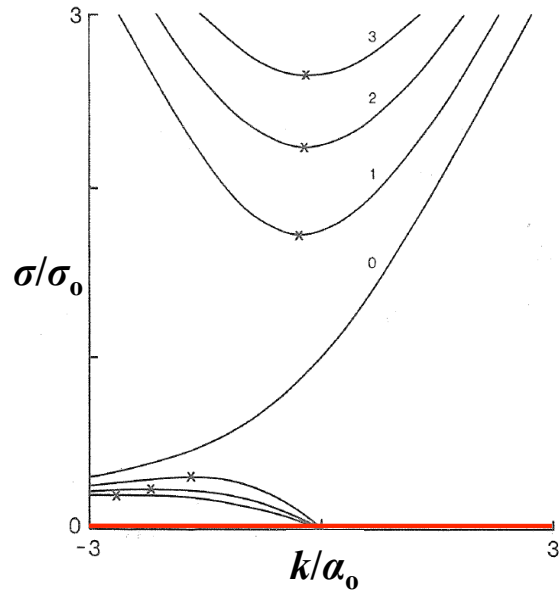


The u , v & p fields for an $\ell = 1$ Rossby wave when $c_n = 250$ cm/s and $P = 360$ days. For this P , $\sigma/\sigma_0 = .03$ and $\lambda = 240^\circ$.

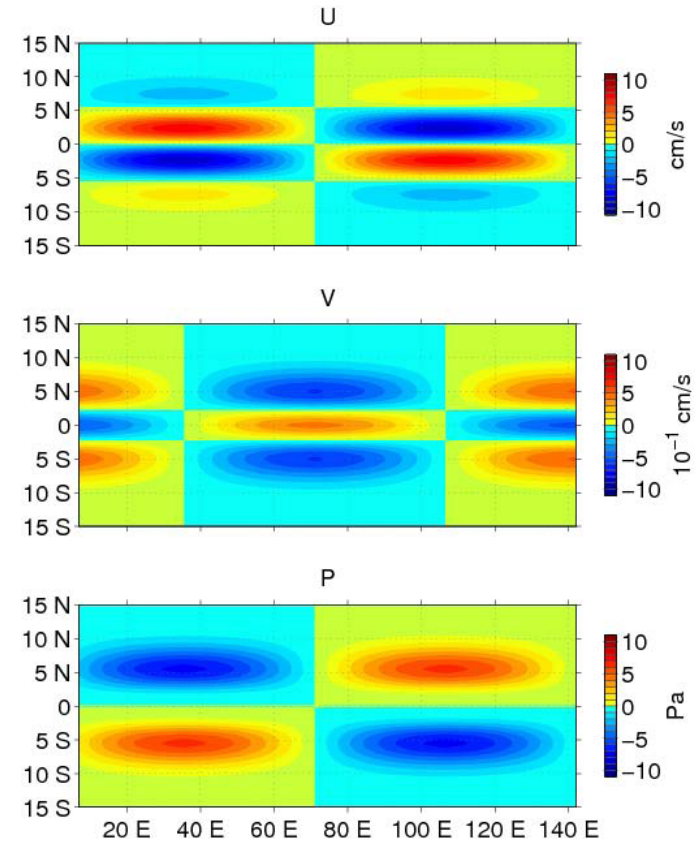


Courtesy of Francois Ascani

Structure of equatorial Rossby waves



The u , v & p fields for an $\ell = 2$ Rossby wave when $c_n = 250$ cm/s and $P = 360$ days. For this P , $\sigma/\sigma_0 = .03$ and $\lambda = 140^\circ$.



Courtesy of Francois Ascani

Observed equatorial waves

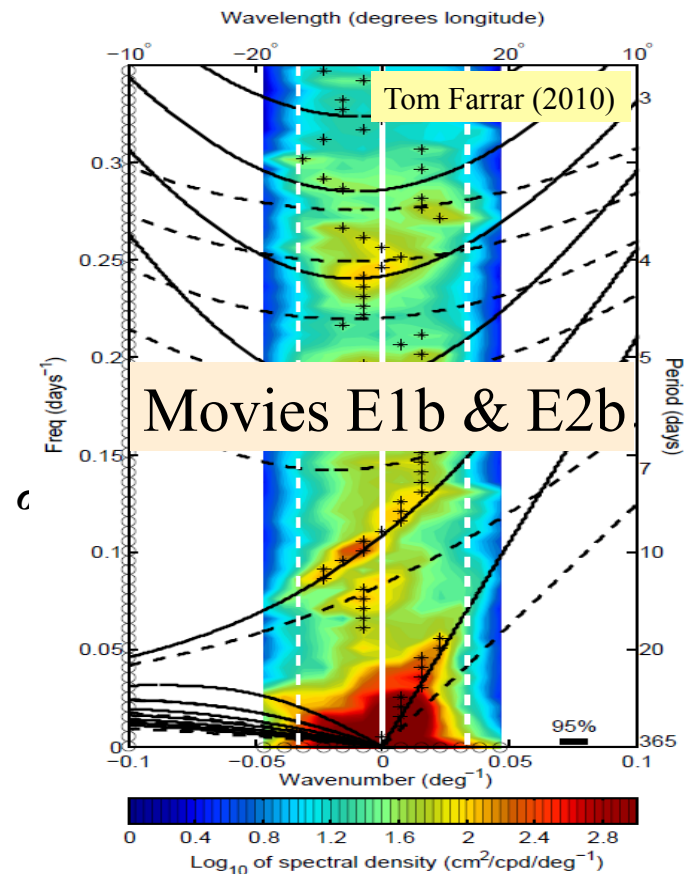
A lot of mathematics led to this set of dispersion curves. **Do any of these waves actually exist?!**

The first equatorially trapped waves to be discovered were **gravity waves** with periods of $O(10)$ days (Wunsch and Gill, 1976; *Deep-Sea Res.*).

The **equatorial Kelvin wave** was **discovered after it was predicted** (Knox and Halpern, 1982, *JMR*).

The **mixed Rossby-gravity (Yanai) wave** was first **observed in the atmosphere by Yanai**. In the ocean, it was (probably) **first detected in the Indian Ocean by Reverdin and Luyten (1986)** using altimeter data.

Who first detected an **equatorial Rossby wave**?



A world map with a light blue background and dark blue landmasses. The text "Equatorial ocean: wind-forced solutions" is overlaid in the center in a bold, black font. The text is centered horizontally and vertically, with "Equatorial ocean:" on the top line and "wind-forced solutions" on the bottom line.

**Equatorial ocean:
wind-forced solutions**

x-independent (2-d) Yoshida Jet

Kozo Yoshida wrote down the first solution for an **x-independent (2d) equatorial current driven by zonal winds**. The (more complete) theoretical solution developed somewhat later (Dennis Moore) has come to be called the “**Yoshida Jet**” (Jim O’Brien).

The basic dynamics of the Yoshida Jet can be understood from the zonal-momentum equation. Neglecting the βu term (the flow is assumed to be x -independent) and **mixing** terms in the zonal momentum equation gives

$$\left(\partial_t + \cancel{\frac{A}{c_n^2}} \right) \left[u_{nt} - \cancel{f} u_n = \tau^x / \mathcal{H}_n \right] + \cancel{\nu_h \nabla^2} u_n.$$

Offshore, Ekman balance ($f v \downarrow n = \tau^x / \mathcal{H} \downarrow n$) **holds**, whereas **at the equator** u_n **continues to accelerate** ($u \downarrow nt = \tau^x / \mathcal{H} \downarrow n$). The switch from one dynamical regime to the other occurs at $y \approx \alpha_{on}^{-1/2} = (\beta / c_n)^{-1/2}$.

Bounded (3-d) Yoshida Jet

In reality and models, equatorial zonal flows (Yoshida Jets) **don't continue to accelerate**. Why not?

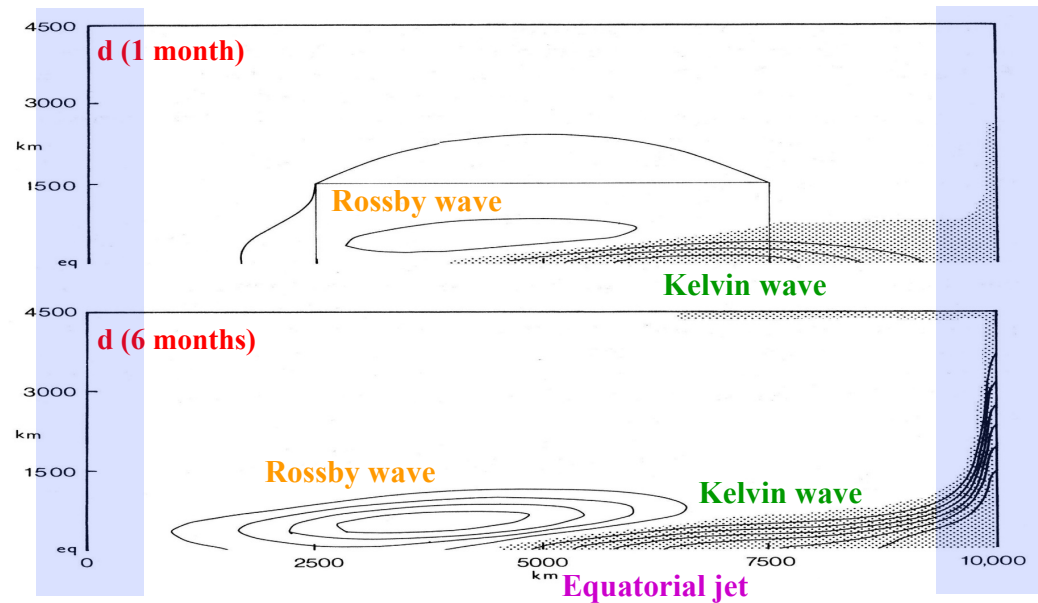
Because in the real world either the **wind forcing or the ocean basin is zonally bounded**, which **introduces x-dependence** into the solution. (An exception is the Southern Ocean, but we will not consider that case here.)

For convenience, we can still **drop the mixing terms** in the zonal momentum equation, and at the equator the **Coriolis term vanishes**. The boundaries, however, introduce x -dependence so we **cannot neglect the p_{nx} term**

$$\left(\partial_t + \cancel{\frac{A}{c_n^2}} \right) u_{nt} + \boxed{p_{nx} = \tau^x / \mathcal{H}_n} + \cancel{\nu_h \nabla^2} u_n.$$

In this case, the system **stops accelerating** by adjusting to a state where the **pressure gradient balances the wind**. It does so by **radiating equatorial Kelvin and Rossby waves**.

Bounded (3-d) Yoshida Jet

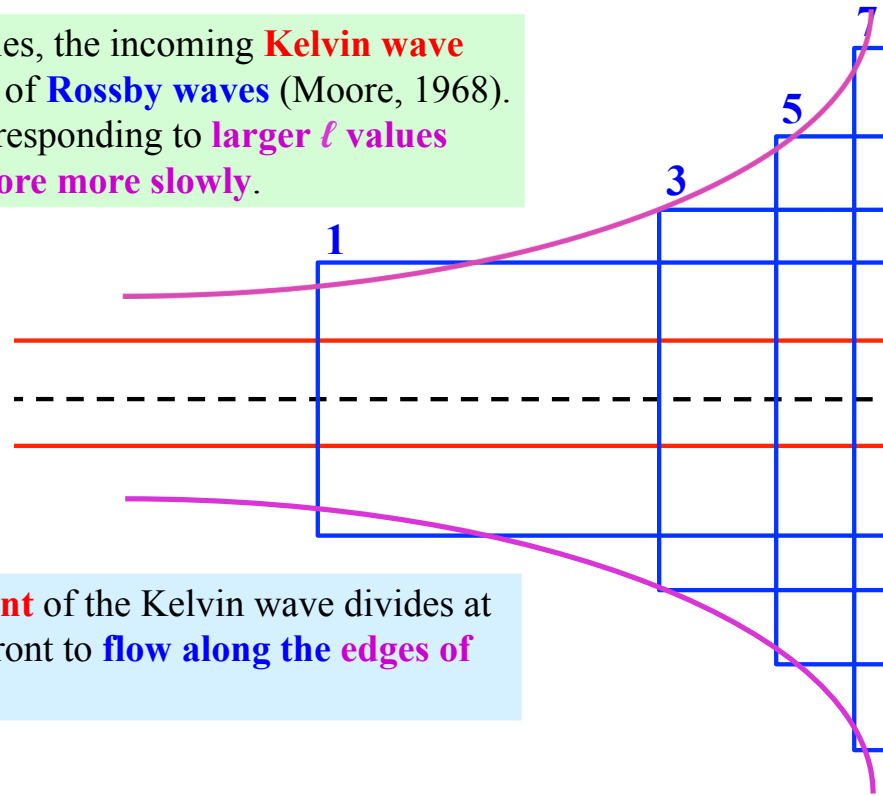


In response to forcing by a **patch of easterly wind**, an **accelerating Yoshida Jet initially develops** in the forcing region. Subsequently, **KWs and RWs radiate** from the forcing region. They **generate a steady, eastward, equatorial current** both east and west of the forcing region: the **bounded YJ**.

Eastern-boundary reflections

What happens when **basin boundaries are included**?

At low frequencies, the incoming **Kelvin wave** reflects as a packet of **Rossby waves** (Moore, 1968), with the waves corresponding to **larger ℓ values propagating offshore more slowly**.



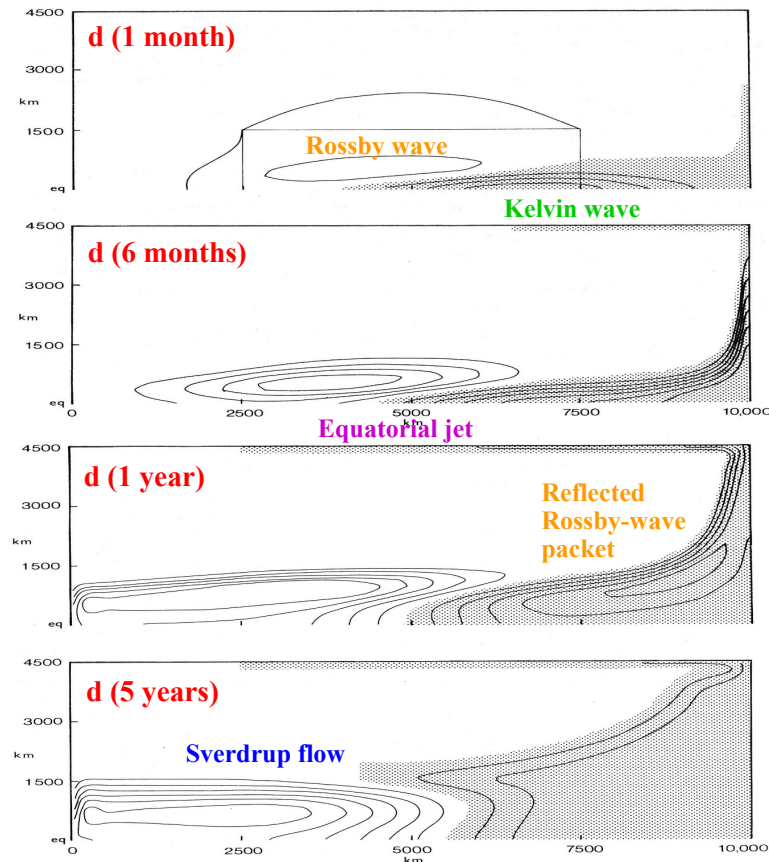
The **zonal current** of the Kelvin wave divides at the Rossby-wave front to **flow along the edges of the wave packet**.

Adjustment to steady state

In response to forcing by a **patch of wind in the interior ocean**, **KWs reflect from the eastern boundary** as a packet of RWs creating a characteristic **wedge-shaped pattern**. In addition, wind-generated **RWs reflect from the western boundary** to return to the interior ocean.

After multiple reflections, the solution eventually **adjusts to a steady state of Sverdrup balance**.

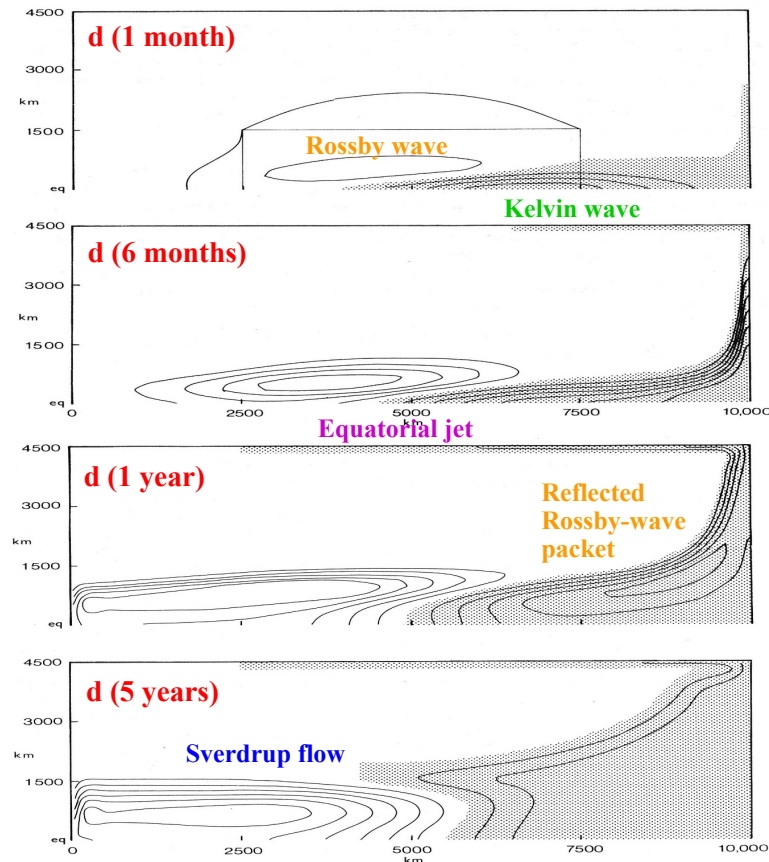
Movies F1b & F2



Multi-mode response to switched-on $\tau \uparrow x$

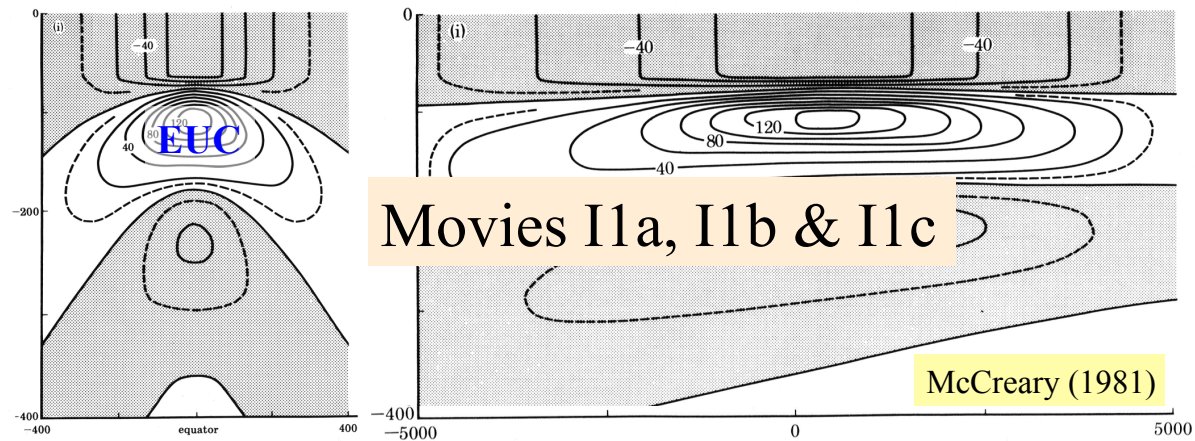
How does the LCS model adjust when **many baroclinic modes are included**?

With **damping**, the $n > 1$ responses are increasingly damped for larger n , since $\nu = A/c_n^2$. In that case, **waves that radiate from the forcing region are increasingly weakened** for larger n . **propagation speeds of eq. waves are slower** ($\propto c \downarrow n$ and $c_n < c_1$).



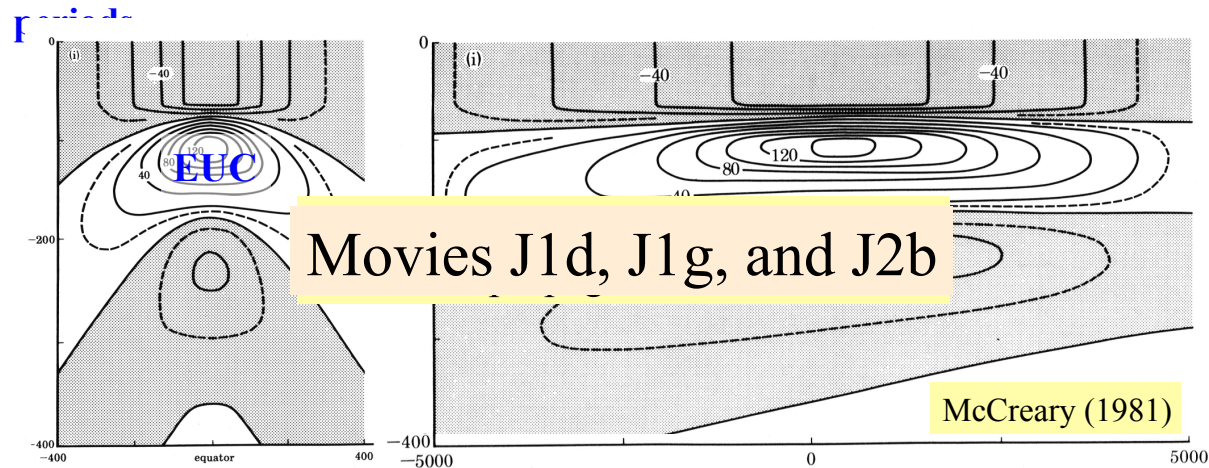
Multi-mode response to switched-on $\tau \uparrow x$

With damping (vertical mixing), the LCS model produces a realistic steady flow field near the equator with an EUC.



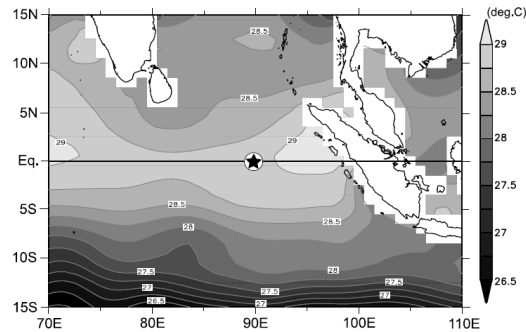
Multi-mode response to **periodic** $\tau \uparrow x$

In the IO, the **steady component** of equatorial $\tau \uparrow x$ is **weak**. Instead, $\tau \uparrow x$ tends to oscillate at **annual, semiannual, and other (e.g., intraseasonal)**



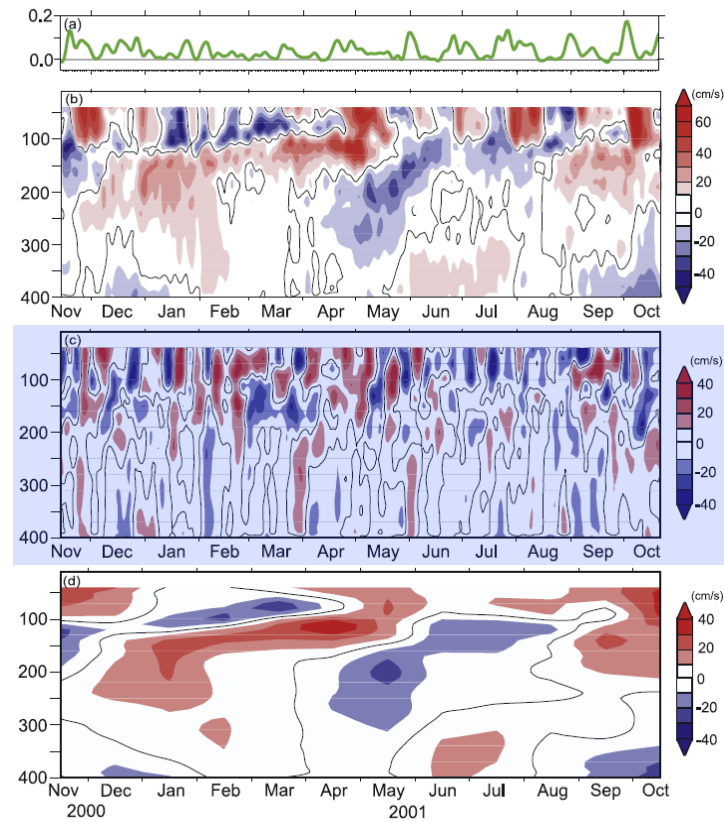
In response to periodic forcing, **equatorial waves** from a number of baroclinic modes superpose to **form beams that propagate vertically** as well as horizontally. **Kelvin (Rossby) beams** extend downward and **eastward (westward)** from the forcing region. Phase propagates **upwards (downwards)** across **downward-extending (upward-extending)** beams.

Upward phase propagation in the EEIO



The u field (b & d) shows a strong semiannual cycle.

Above 200 m, the phase of u propagates upwards, indicating that it is remotely forced (wave) signal!



Masumoto et al. (2005)

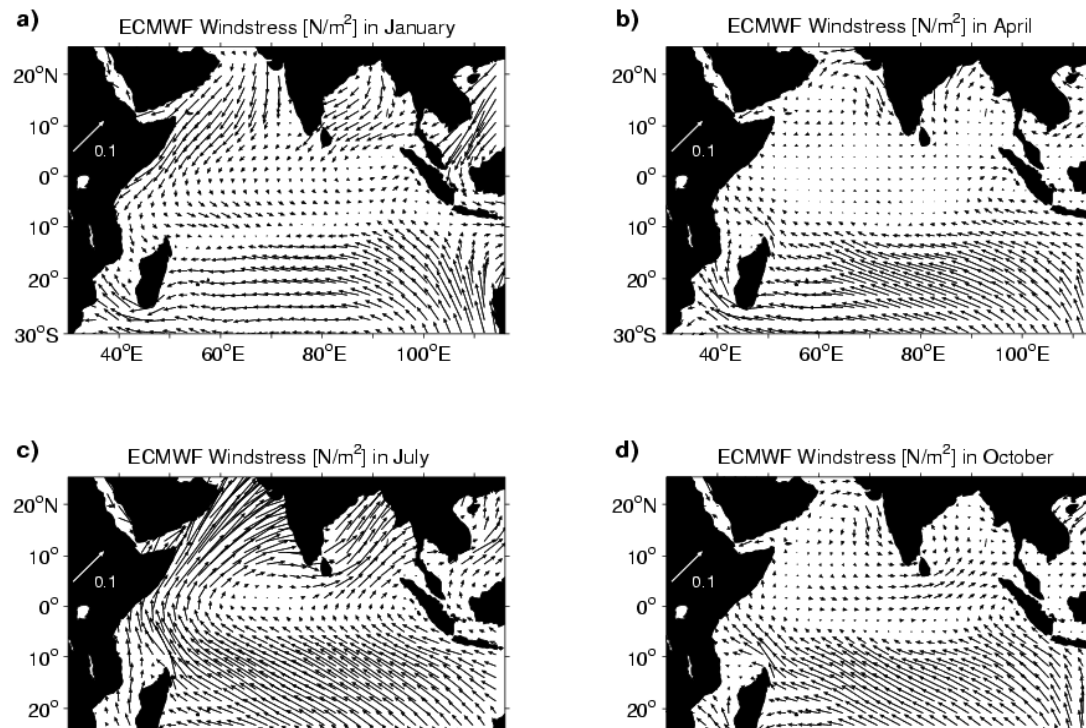


Coastal ocean:

2-d and 3-d solutions with
constant β ($\beta=0$) or variable β ($\beta \neq 0$)



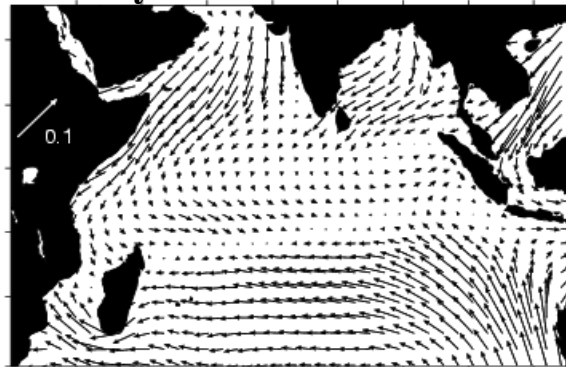
Indian Ocean phenomena



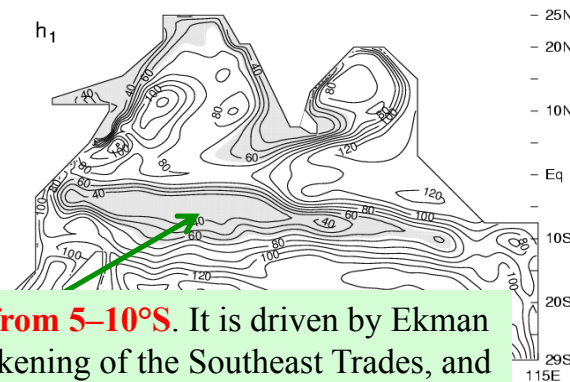
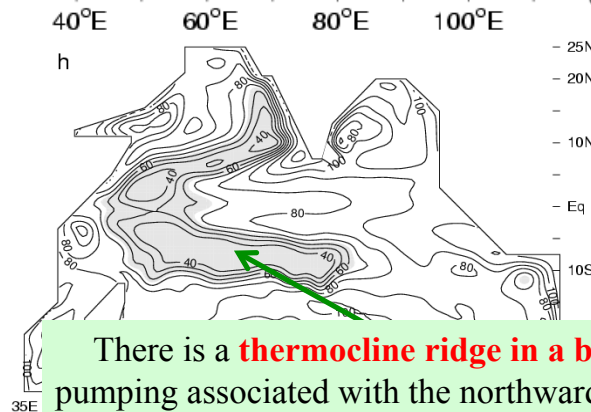
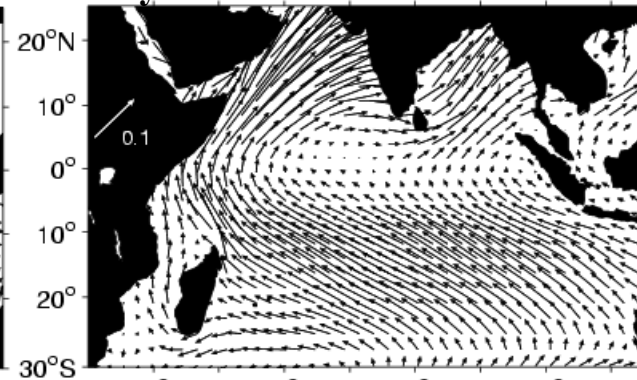
The **Indian Ocean winds** differ markedly from the those in the Pacific and Atlantic Oceans in that there are **no quasi-steady trades along the equator**.

Wind-forced thermocline response

January



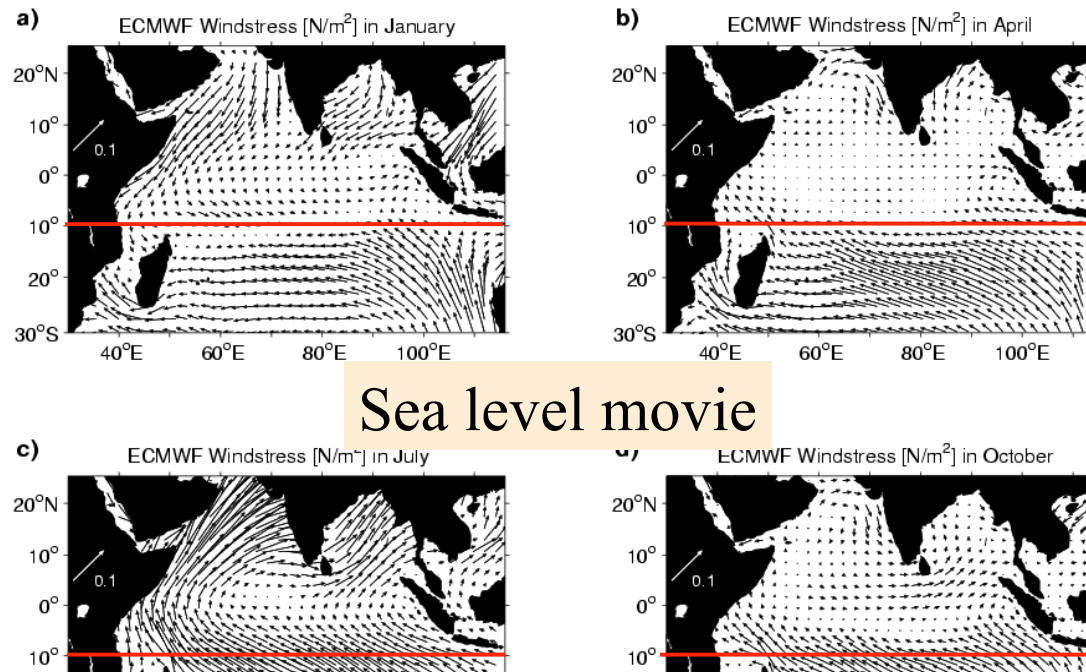
July



There is a **thermocline ridge in a band from 5–10°S**. It is driven by Ekman pumping associated with the northward weakening of the Southeast Trades, and is **stronger in northern summer** when the Trades are stronger.

(1993)

Indian Ocean phenomena



Sea level movie

The **winds in the North Indian Ocean** (north of $\sim 10^\circ\text{S}$), are **highly variable** because of the monsoon. As a result, there are no **steady currents**, and the **propagation of remotely-forced waves** around the basin **is apparent**.



Additional slides



Hierarchy of ocean models:

derivation of LCS model equations

See **HIGNotes.pdf** for a detailed discussion.

LCS model

u_t	$u_t - fv + \frac{1}{\rho} p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$	$\nu^2 u,$
v_t	$v_t + fu + \frac{1}{\rho} p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$	$\nu^2 v,$
$w_t + uw_x$	$p_z = -\rho g$	$\nu_h \nabla^2 w,$
$T_t + uT_x + vT_y + wT_z = (\kappa_T T_z)_z + \nu_h \nabla^2 T,$		$\nu_z)_z.$
$S_t + uS_x + vS_y + wS_z = (\kappa_S S_z)_z + \nu_h \nabla^2 S,$		the Vaisala
$\nabla \cdot \mathbf{v} = 0,$		a very
$\rho = \rho(S, T, p)$		ally

Impose

Dropping

frequency,

thin bound

unimportant

and can on

observations.

s **GOOD**.

fully with

Linearize the equation of state to

Then, set κ_T
single density
effects in the
double diffu
considered in

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}} p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$\rho_t + u\rho_x + v\rho_y + w\rho_z = (\kappa\rho_z)_z + \nu_h \nabla^2 \rho$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\nabla \cdot \mathbf{v} = 0,$$

$$\rho = \rho(S, T, p)$$

obtain a
density
 κ_S deletes
phenomena
OOD.

LCS model

$$\begin{aligned}
 u_t - fv + \frac{1}{\bar{\rho}} p_x &= (\nu u_z)_z + \nu_h \nabla^2 u, \\
 v_t + fu + \frac{1}{\bar{\rho}} p_y &= (\nu v_z)_z + \nu_h \nabla^2 v, \\
 p_z &= -\rho g \\
 \rho_t - w \frac{\bar{\rho}}{g} N_b^2 &= (\kappa \rho_z)_z + \nu_h \nabla^2 \rho \\
 \nabla \cdot \mathbf{v} &= 0
 \end{aligned}$$

The derivative ρ_{bz} is related to a **fundamental ocean frequency, the Vaisala frequency**, the square of which is

$$N_b^2(z) = -\frac{g}{\bar{\rho}} \rho_{bz}$$

Replace ρ_{bz} with N_b^2 .

LCS model

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = (\nu u_z)_z + \nu_h \nabla^2 u,$$

$$v_t + fu + \frac{1}{\bar{\rho}} p_y = (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$\rho_t - w \frac{\bar{\rho}}{g} N_b^2 = ((\kappa \rho)_{zz}) + \nu_h \nabla^2 \rho$$

$$u_x + v_y + w_z = 0$$

Modify the form of **vertical diffusion from $(\kappa \rho)_z$ to $(\kappa \rho)_{zz}$** . This assumption is essential to allow the **expansion of solutions into vertical (barotropic and baroclinic) modes**. Since the precise form of vertical diffusion is not known, it is **OKAY**.

LCS model

$$u_t - fv + \frac{1}{\bar{\rho}} p_x = \tau^x Z(z) + (\nu u_z)_z + \nu_h \nabla^2 u,$$
$$v_t + fu + \frac{1}{\bar{\rho}} p_y = \tau^y Z(z) + (\nu v_z)_z + \nu_h \nabla^2 v,$$

$$p_z = -\rho g$$

$$\rho_t - w \frac{\bar{\rho}}{g} N_b^2 = (\kappa \rho)_{zz} + \nu_h \nabla^2 \rho$$

$$u_x + v_y + w_z = 0$$

Wind stress enters the ocean in a surface mixed layer. To simulate this process in a simple way, we **introduce wind as a “body force” with the vertical profile $Z(z)$** . The body force differs from an actual mixed layer in that its **profile is uniform in space and constant in time**. This representation is **CONVENIENT** and **SENSIBLE**.

LCS model

$$\begin{aligned}
 u_t - fv + \frac{1}{\bar{\rho}} p_x &= \tau^x Z(z) + (\nu u_z)_z + \nu_h \nabla^2 u, \\
 v_t + fu + \frac{1}{\bar{\rho}} p_y &= \tau^y Z(z) + (\nu v_z)_z + \nu_h \nabla^2 v, \\
 - \left(\partial_z \frac{1}{N_b^2} \partial_z \right) \frac{p_t}{\bar{\rho}} + u_x + v_y &= - \left(\partial_z \frac{1}{N_b^2} \partial_z \right) \left[\left(\kappa \frac{p_z}{\bar{\rho}} \right)_z + \nu_h \nabla^2 \frac{p}{\bar{\rho}} \right] \\
 w &= - \frac{1}{\bar{\rho} N_b^2} \left[p_{zt} - (\kappa p_z)_{zz} - \nu_h \nabla^2 p_z \right] \\
 \rho &= - \frac{1}{g} p_z
 \end{aligned}$$

wind stress enters the ocean in a surface mixed layer. To simulate

Rewrite equations (1) – (3). First, **solve (1) for ρ** and **(2) for w in terms p_z** . Then, **insert both expressions into (3)**.
 mixed layer in that its **profile is uniform in space and constant in time**. This representation is **CONVENIENT** and **SENSIBLE**.

LCS model

$$\begin{aligned}
 u_t - fv + \frac{1}{\bar{\rho}} p_x &= \tau^x Z(z) + A_\nu \left(\partial_z \frac{1}{N_b^2} \partial_z \right) u + \nu_h \nabla^2 u, \\
 v_t + fu + \frac{1}{\bar{\rho}} p_y &= \tau^y Z(z) + A_\nu \left(\partial_z \frac{1}{N_b^2} \partial_z \right) v + \nu_h \nabla^2 v, \\
 - \left(\partial_z \frac{1}{N_b^2} \partial_z \right) \frac{p_t}{\bar{\rho}} + u_x + v_y &= -A_\kappa \left(\partial_z \frac{1}{N_b^2} \partial_z \right)^2 \frac{p}{\bar{\rho}} - \nu_h \left(\partial_z \frac{1}{N_b^2} \partial_z \right) \nabla^2 \frac{p}{\bar{\rho}} \\
 w &= -\frac{1}{\bar{\rho} N_b^2} [p_{zt} - (\kappa p_z)_{zz} - \nu_h \nabla^2 p_z] \\
 \rho &= -\frac{1}{g} p_z
 \end{aligned}$$

Finally, assume that

$$\nu = A_\nu / N_b^2, \quad \kappa = A_\kappa / N_b^2$$

In which case **all the z-operators have the same form**, a property **necessary to represent solutions as expansions in vertical modes**.

A world map with a light blue background and dark blue outlines of continents. The map is centered on the Atlantic Ocean.

Mid-latitude ocean waves:
derivation of ψ_{lm} equation

Derivation of v_n equation

$$u_{nt} - f v_n + p_{nx} = 0$$

$$v_{nt} + f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0$$

$$u_{ntx} - f v_{nx} + p_{nxx} = 0$$

$$(-1) \frac{p_{ntt}}{c_n^2} + u_{nxt} + v_{nyt} = 0$$

$$p_{nxx} - \frac{p_{ntt}}{c_n^2} = v_{nyt} + f v_{nx}$$

$$\left(\partial_{xx} - \frac{1}{c^2} \partial_{tt} \right) p_n = v_{nyt} + f v_{nx}$$

Derivation of v_n equation

$$u_{nt} - fv_n + p_{nx} = 0$$

$$v_{nt} + fu_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0$$

$$\left(\partial_{xx} - \frac{1}{c^2} \partial_{tt} \right) p_n = v_{nyt} + fv_{nx}$$

$$(-1/c_n^2) u_{ntt} - fv_{nt} + p_{nxt} = 0$$

$$\frac{p_{ntx}}{c_n^2} + u_{nxx} + v_{nyx} = 0$$

$$u_{nxx} - \frac{u_{ntt}}{c^2} = -\frac{f}{c^2} v_{nt} - v_{nyx}$$

$$\left(\partial_{xx} - \frac{1}{c^2} \partial_{tt} \right) u_n = -\frac{f}{c^2} v_{nt} - v_{nyx}$$

Derivation of v_n equation

$$u_{nt} - f v_n + p_{nx} = 0$$

$$v_{nt} + f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c_n^2} + u_{nx} + v_{ny} = 0$$

$$\left(\partial_{xx} - \frac{1}{c^2} \partial_{tt} \right) p_n = v_{nyt} + f v_{nx}$$

$$\left(\partial_{xx} - \frac{1}{c^2} \partial_{tt} \right) u_n = -\frac{f}{c^2} v_{nt} - v_{nyx}$$

$$\left(\partial_{xx} - \frac{1}{c^2} \partial_{tt} \right) v_{nt} + f \left(-\frac{f}{c^2} v_{nt} - v_{nyx} \right) + (v_{nyt} + f v_{nx})_y = 0$$

$$\left(\partial_{xx} - \frac{1}{c^2} \partial_{tt} \right) v_{nt} - \frac{f^2}{c^2} v_{nt} - f v_{nyx} + v_{nyyt} + f v_{nxy} + \beta v_x = 0$$

$$v_{xxt} + v_{yyt} - \frac{1}{c^2} v_{ttt} - \frac{f^2}{c^2} v_t + \beta v_x = 0$$

A world map with a light blue background and white landmasses. The map is centered on the Pacific Ocean, showing the Americas, Europe, Africa, and Australia.

Mid-latitude ocean waves:
derivation of coastal Kelvin wave

Derivation of KW solution

$$\begin{array}{l} u_t \\ v_t \\ \frac{p_t}{c} \end{array} \begin{array}{l} u_{nt} + p_{nx} = 0 \\ f u_n + p_{ny} = 0 \\ \frac{p_{nt}}{c^2} + u_{nx} = 0 \end{array} \begin{array}{l} 0 \\ 0 \\ 0 \end{array}$$

$$(-c^2) u_{ntx} + p_{nxx} = 0$$

$$p_{ntt} + c^2 u_{nxt} = 0$$

$$p_{ntt} - c^2 p_{nxx} = 0$$

Derivation of KW solution

$$u_{nt} + p_{nx} = 0$$

$$f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c^2} + u_{nx} = 0$$

$$p_{ntt} - c^2 p_{nxx} = 0$$

$$f u_{nt} + f p_{nx} = 0$$

$$\underline{(-1) f u_{nt} + p_{nyt} = 0}$$

$$f p_{nx} = p_{nyt}$$

Derivation of KW solution

$$u_{nt} + p_{nx} = 0$$

$$f u_n + p_{ny} = 0$$

$$\frac{p_{nt}}{c^2} + u_{nx} = 0$$

$$p_{ntt} - c^2 p_{nxx} = 0$$

$$f p_{nx} = p_{nyt}$$

Look for solutions proportional to **exp**
(ikx - iσt). Set **∂t = -iσ** and **∂x = ik**.

$$p_{ntt} - c^2 p_{nxx} = 0 \quad \Rightarrow \quad \sigma^2 = c^2 k^2 \quad \Rightarrow \quad \sigma = \pm ck$$

$$f p_{nx} = p_{nyt} \quad \Rightarrow \quad k f p_n = -\sigma p_{ny} \quad \Rightarrow \quad f p_n = \mp c p_{ny}$$

$$p_n = p_o \exp(\mp \alpha y) \exp[ik(x \mp ct)], \quad \alpha = \frac{f}{c}$$



**Interior ocean:
Ekman drift and inertial oscillations**

Ekman drift and inertial oscillations ($\beta = 0$)

The most fundamental forced motion in the ocean is Ekman drift. In an inviscid, single-mode (or 1½-layer) model, **Ekman drift** occurs at an angle of **90° to the right (left) of the wind** in the **northern (southern) hemisphere**.

To illustrate this response as simply as possible, we assume that the **ocean is unbounded**, **f is constant**, and the forcing is by a **spatially uniform τ^x** . Then, the equation (1) simplifies to

$v_{xyt} + v_{yxt} - \frac{1}{c^2}v_{ttt}$	$v_{tt} + f^2v = -fF$	$F_{yx} - \frac{1}{c^2}G_{tt} + G_{xx}$ (2)
------------------------------------------------------------------	-----------------------	-------------------------------------------------------------------

Why is it **“okay” to consider spatially uniform winds**? Because the typical **scale of wind the wind forcing** ($\sim 500\text{--}1000$ km) is **much greater than the Rossby radius of deformation** ($R \sim 25\text{--}50$ km).

Ekman drift and inertial oscillations ($\beta = 0$)

Suppose the **wind switches on at $t = 0$** . We split the solution into a **time-independent, particular solution**

$$\cancel{v_{p,tt}} + f^2 v_p = -fF \quad \Rightarrow \quad v_p = -\frac{F}{f}$$

and a **homogeneous solution that satisfies (2) with $F = 0$**

$$-v_{h,tt} + f^2 v_h = -\cancel{fF} \quad \Rightarrow \quad v_h = A \sin ft + B \cos ft$$

The **total solution** is then

$$v = -\frac{F}{f} + A \sin ft + B \cos ft$$

and **A and B are determined by applying initial conditions.**

Ekman drift and inertial oscillations ($\beta = 0$)

Assume that the ocean is at rest before the wind switches, so that appropriate **initial conditions are $u = v = 0$ at $t = 0$** .

We use the v momentum equation to **write the boundary condition for u in terms of v** . We have

$$v_t + fu = 0 \quad \Rightarrow \quad v_t = 0 \quad @ \quad t = 0$$

Applying the initial conditions gives

$$v(0) = -\frac{F}{f} + B = 0 \quad \Rightarrow \quad B = \frac{F}{f}$$

$$v_t(0) = fA = 0 \quad \Rightarrow \quad A = 0$$

so that

$$v = -\frac{F}{f} (1 - \cos ft)$$

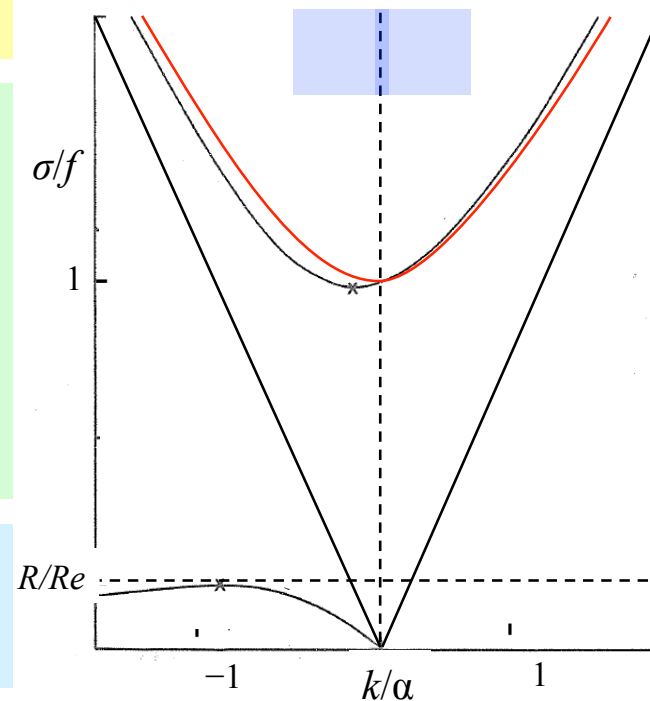
Ekman drift and inertial oscillations ($\beta = 0$)

The **steady-state solution is Ekman drift**, but **GWs at $\sigma = f$ are also generated** to satisfy the initial conditions.

Because $\beta = 0$ and there are **no coasts, only GWs are possible**. Because the **wind is spatially uniform**, only GWs with $k = \ell = 0$ can be excited. According to the disp. rel., the **waves with zero wavenumber are inertial waves with $\sigma = f$** .

If the **wind is not spatially uniform**, GWs with $k > 0$ and $\sigma > f$ can also be excited.

$$v = -\frac{F}{f} (1 - \cos ft)$$



Ekman drift and inertial oscillations ($\beta = 0$)

To summarize, the solutions for u and v when f is constant are

$$u = -\frac{v_t}{f} = \frac{F}{f} \sin ft, \quad v = -\frac{F}{f} + \frac{F}{f} \cos ft$$

a **steady, southward, Ekman drift** plus an **inertial oscillation** in which the **velocity vector rotates clockwise at a single frequency f** .

Ekman drift and inertial oscillations ($\beta \neq 0$)

To summarize, the solutions for u and v when f is constant are

$$u = -\frac{v_t}{f} = \frac{F}{f} \sin ft, \quad v = -\frac{F}{f} + \frac{F}{f} \cos ft$$

a steady, southward, Ekman drift plus an inertial oscillation in which the velocity vector rotates clockwise at a single frequency f .

Q: How does this simple response change when $\beta \neq 0$?

A: Frequency f and hence the clockwise **rotation of the velocity vector differ at each latitude**. Very quickly, **convergences (divergences) develop between different latitudes**, requiring water to downwell (upwell). This process **excites gravity waves with $\ell \neq 0$** , and is known as **β -dispersion**.

Movies B



Interior ocean:
derivation of Ekman pumping ($\beta = 0$)

Ekman pumping

Written **in terms of a 1½-layer model**, the interior-ocean equations are

$$\begin{aligned} -fv + g'h_x &= \frac{\tau^x}{H}, & fu + g'h_y &= \frac{\tau^y}{H}, \\ h_t + H(u_x + v_y) &= -\kappa(h - H) \end{aligned}$$

where

$$p_n \rightarrow g'(h - H), \quad c_n^2 \rightarrow g'H, \quad \mathcal{H}_n \rightarrow H, \quad \kappa = A/c_n^2$$

and the damping corresponds to **entrainment into or detrainment from the layer**.

Solving for a **single equation in h** gives

$$h_t - \beta \frac{g'H}{f^2} h_x + \left[h_t + \kappa(h - H) = -w_{ek} \left(\frac{-x}{f} \right)_y \right] \equiv -w_{ek}$$

where w_{ek} is the **Ekman-pumping velocity**, the **rate at which wind curl raises or lowers subsurface isopycnals**.

Ekman pumping

When there is no damping (), the solution is

$$h = H - w_{ek}t = H + \frac{\tau_y^x}{f}t$$

so that **h grows continuously in time**. With damping (), it is

$$h = H - \frac{1 - e^{-\kappa t}}{\kappa} w_{ek} = H + \frac{1 - e^{-\kappa t}}{\kappa} \frac{\tau_y^x}{f}$$

so that **h stops growing**.

With h known, the zonal and meridional velocities are

$$v = \frac{1}{f} g' h_x - \frac{\tau^x}{fH}, \quad u = -\frac{1}{f} g' h_y$$

a superposition of **Ekman and geostrophic currents**.



**Interior ocean:
adjustment to Sverdrup balance ($\beta \neq 0$)**

Adjustment when $\beta \neq 0$ and $\kappa = 0$

Suppose the model ocean allows f to vary ($\beta \neq 0$) and there is **no damping** ($\kappa = 0$). Then, h satisfies

$$\left. \begin{aligned} h_t - \beta \frac{g' H}{f^2} h_t - c_r h_x = -w_{ek}, \quad c_r = \beta \frac{g' H}{f^2} \end{aligned} \right] \equiv -w_{ek}$$

We obtain the solution by splitting it into **steady-state (particular, forced)** and **transient (homogenous, wave)** parts

$$h_p - H = \frac{1}{c_r} \int_{\infty}^x w_{ek} dx' \equiv \chi(x, y), \quad h_h = \Lambda(x + c_r t, y)$$

where $\Lambda(x, y)$ is an as yet unspecified function.

To satisfy the **initial condition that $h = H$ at $t = 0$** , we must choose $\Lambda(x, y) = -\chi(x, y)$, so that

$$h = h_p + h_h = H + \chi(x, y) - \chi(x + c_r t, y)$$

Initial adjustment

To determine the **response a short time after the wind switches on**, we **expand the Rossby-wave term in a Taylor series about $t = 0$** to get

$$\begin{aligned}\lim_{t \rightarrow 0} h &= H + \chi(x, y) - \lim_{t \rightarrow 0} \chi(x + c_r t, y) \\ &= H + \chi(x, y) - [\chi(x, y) + c_r \chi'(x, y) t + \dots] \\ &= H - c_r \chi'(x, y) t + \dots = H - w_{ek} t + \dots\end{aligned}$$

Thus, **at small times, the response is just Ekman pumping!**

The response does not change from Ekman pumping until the **Rossby waves have time enough to propagate significantly westward.**

Final adjustment

At longer times the solution for all the fields is

$$\begin{aligned}
 h &= H + \frac{1}{c_r} \int_{-\infty}^x \text{curl} \left(\frac{\tau}{f} \right) dx' - \chi(x + c_r t, y) \\
 v &= \frac{g'}{f} h_x - \frac{\tau^x}{fH} = \frac{\text{curl } \tau}{\beta H} - \frac{g'}{f} \chi_x(x + c_r t, y) \\
 u &= -\frac{g'}{f} h_y = -\frac{1}{H} \int_{-\infty}^x \left(\frac{\text{curl } \tau}{\beta} \right)_y dx' + \frac{g'}{f} \chi_y(x + c_r t, y)
 \end{aligned}$$

where

$$\text{curl } \tau = \tau_x^y - \tau_y^x, \quad \text{curl} \frac{\tau}{f} = \left(\frac{\tau^y}{f} \right)_x - \left(\frac{\tau^x}{f} \right)_y$$

A **packet of Rossby waves** propagates westward.

After their passage, the solution adjusts to a **steady-state Sverdrup balance**.

A world map with a light blue background. The continents are shown in a darker blue. The map is centered on the Pacific Ocean, with North America on the left and South America on the right. The text "Interior ocean: western boundary currents" is overlaid in the center of the Pacific Ocean.

**Interior ocean:
western boundary currents**

Western-boundary currents

When **long-wavelength Rossby waves** (LWRWs) propagate to a western-ocean boundary, **zonal flow** associated with them is **channeled into a western-boundary current** (WBC).

Without momentum mixing, the LWRWs reflect as a packet of **short-wavelength Rossby waves** (SWRWs) that **continuously thins**.

Movie MassSource(300days).fli

Western-boundary currents

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Without momentum mixing, the LWRWs reflect as a packet of **short-wavelength Rossby waves** (SWRWs) that **continuously thins**.

Movie MassSource(300days).fli

With momentum mixing, the **WBC thinning stops** (or never appears at all) and its offshore structure **adjusts to steady-state profile**.

Movies D

Western-boundary currents

To find the structure of the western-boundary current, **neglect time-dependent and vertical-mixing terms** and **forcing terms** in the equations of motion, and for convenience **drop subscripts n** .

$$\begin{aligned} \left(\frac{\partial}{\partial t}\right) -fv + p_x &= -\nu u + \nu_h \nabla^2 u, & u_n, \\ \left(\frac{\partial}{\partial t}\right) fu + p_y &= -\nu v + \nu_h \nabla^2 v, & v_n, \\ u_x + v_y &= 0 \end{aligned}$$

Solving for a single equation in v then gives

$$-\beta v_x = \nu \nabla^2 v + \nu_h \nabla^4 v$$

Western-boundary currents

$$-\beta v_x = \cancel{\nu \partial_{xx}} + \cancel{\nu_h \partial_{xxxx}} \quad (1)$$

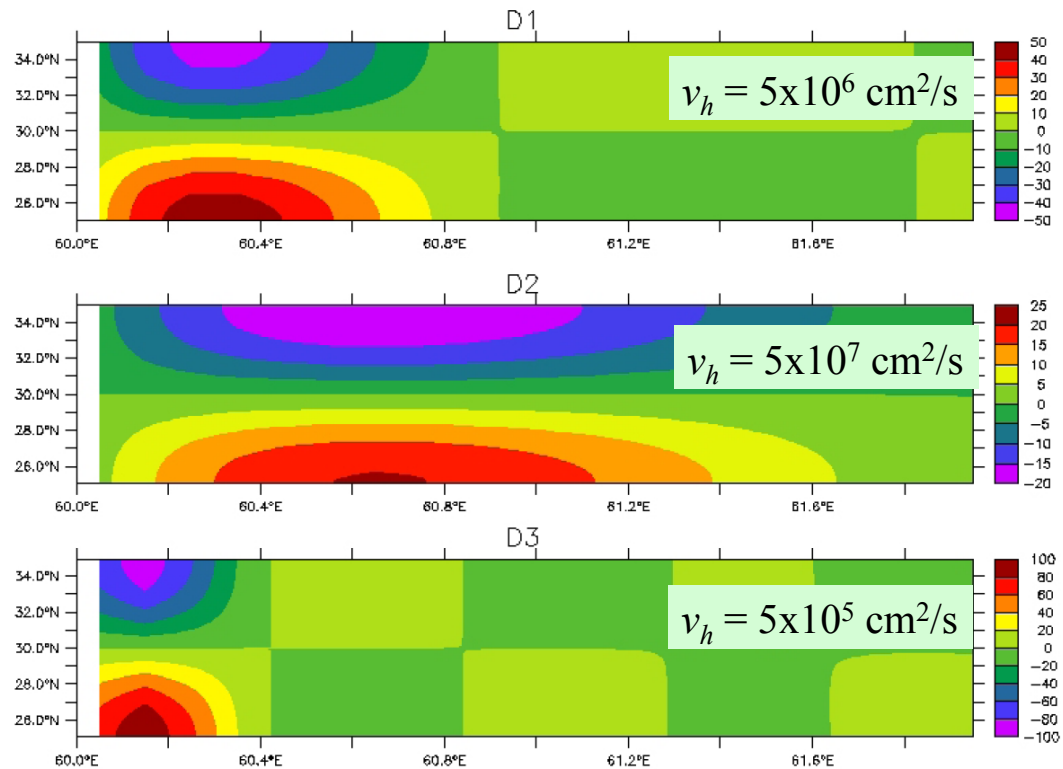
Adopting that **boundary-layer assumption that $L_y^2 \gg L_x^2$** , we **drop all y-derivative terms** from the right-hand side of (1).

With only Laplacian mixing ($\nu = 0$), then the solution to (1) is

$$v = v_0 \exp\left(-\frac{x}{r_m}\right) \sin\left(\frac{\sqrt{3}}{2} \frac{x}{r_m}\right), \quad r_m = \left(\frac{\nu_h}{\beta}\right)^{\frac{1}{3}}$$

a **Munk layer**. This layer oscillates, as well as decays, offshore.

Western-boundary currents



In Solutions D1 – D3, the **WBCs are Munk layers** that decay and oscillate offshore.



Equatorial ocean:
derivation of Yanai-wave dispersion relation

Mixed Rossby-gravity (Yanai) wave

The curious form of the Yanai-wave dispersion curve happens because it **factors into two parts when $\ell = 0$** . We have

$$\begin{aligned}\sigma \left(k^2 + \alpha_0^2 - \frac{\sigma^2}{c^2} \right) + k\beta &= 0 \\ \sigma \left(k^2 + \frac{\beta}{c} - \frac{\sigma^2}{c^2} \right) + k\beta &= 0 \\ \sigma \left(k^2 - \frac{\sigma^2}{c^2} \right) + \beta \left(k + \frac{\sigma}{c} \right) &= 0 \\ \left(k - \frac{\sigma}{c} + \frac{\beta}{\sigma} \right) \left(k + \frac{\sigma}{c} \right) &= 0\end{aligned}$$

Mixed Rossby-gravity (Yanai) wave

The curious form of the Yanai-wave dispersion curve happens because it **factors into two parts when $\ell = 0$** . We have

$$\left(k - \frac{\sigma}{c} + \frac{\beta}{\sigma}\right) \left(\cancel{k - \frac{\sigma}{c}}\right) = 0$$

The second factor describes a wave that **travels westward at the speed of a Kelvin wave**. It can be shown that this wave **blows up at $\pm\infty$, and so it must be discarded**.

The single dispersion relation for the Yanai wave is then

$$k - \frac{\sigma}{c} + \frac{\beta}{\sigma} = 0$$

For **small and large values of σ** , the relation simplifies to,

$$\lim_{\sigma \rightarrow 0} k = -\frac{\beta}{\sigma}, \quad \lim_{\sigma \rightarrow \infty} k = \frac{\sigma}{c}$$

the same **properties for Rossby and gravity waves**, respectively.

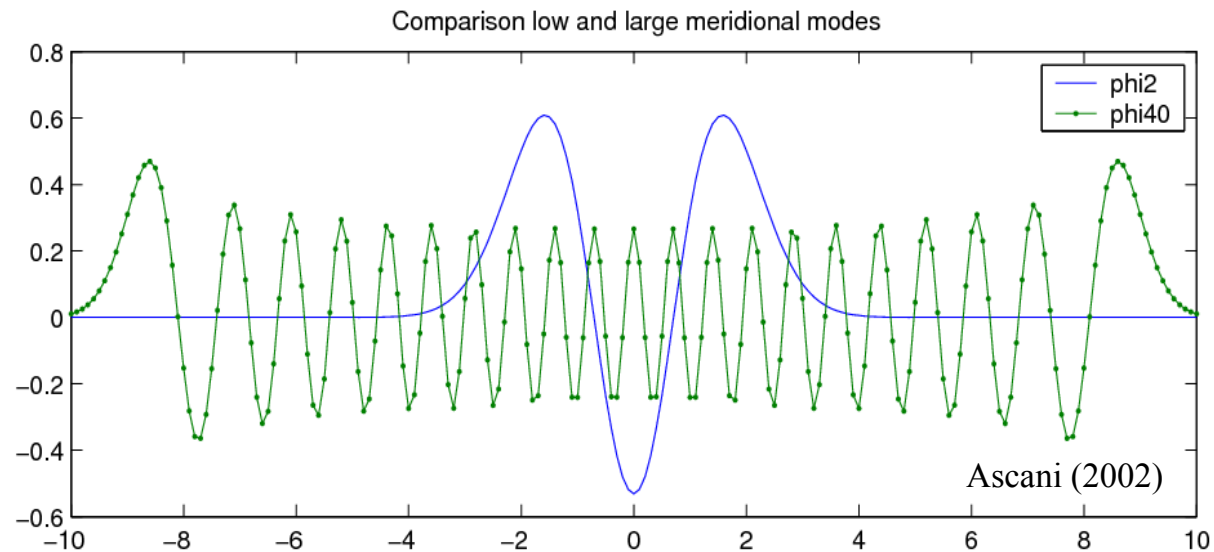


Equatorial ocean:
structures of Hermite functions and
equatorial gravity & Rossby waves

Hermite functions

The figure plots the **first six Hermite functions ϕ_ℓ ($\ell = 0-5$)**. The

For large ℓ , the Hermite functions resemble **cosine or sine curves near the equator**. They begin to **decay at latitudes higher than the “turning latitude.”** So, the **Hermite functions are equatorially trapped**.



Equatorial gravity and Rossby waves

The v_ℓ , u_ℓ , and p_ℓ fields for equatorially trapped **Rossby and gravity waves** are

$$v_\ell = \mathcal{V}_\ell \phi_\ell \exp(ik_j^\ell x - i\sigma t),$$

$$u_\ell = -ic\alpha_0 \mathcal{V}_\ell \left(\sqrt{\frac{\ell+1}{2}} \frac{\phi_{\ell+1}}{ck_j^\ell - \sigma} - \sqrt{\frac{\ell}{2}} \frac{\phi_{\ell-1}}{ck_j^\ell + \sigma} \phi_{\ell-1} \right) \exp(ik_j^\ell x - i\sigma t),$$

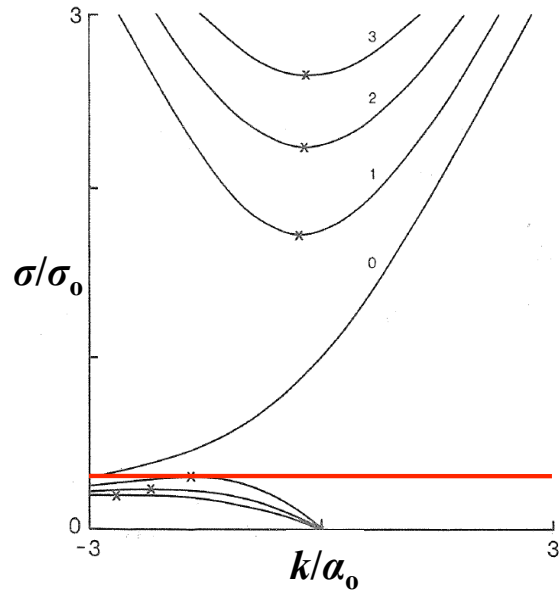
$$p_\ell = -ic^2\alpha_0 \mathcal{V}_\ell \left(\sqrt{\frac{\ell+1}{2}} \frac{\phi_{\ell+1}}{ck_j^\ell - \sigma} + \sqrt{\frac{\ell}{2}} \frac{\phi_{\ell-1}}{ck_j^\ell + \sigma} \phi_{\ell-1} \right) \exp(ik_j^\ell x - i\sigma t),$$

where \mathcal{V}_ℓ is a constant amplitude, ϕ_ℓ is a Hermite function,

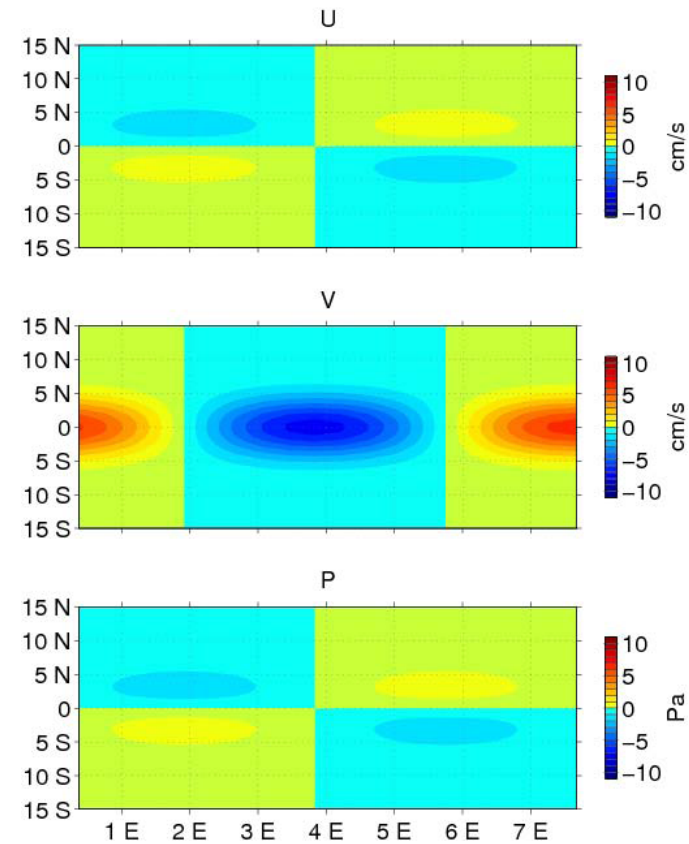
$$k_{1,2}^\ell = -\frac{\beta}{2\sigma} \left[1 \mp \sqrt{1 - 4\frac{\sigma^2}{\beta^2} \left(\alpha_\ell^2 - \frac{\sigma^2}{c^2} \right)} \right]$$

and $j = 1$ (**2**) corresponds to the $-$ (**+**) sign.

Structure of Yanai waves

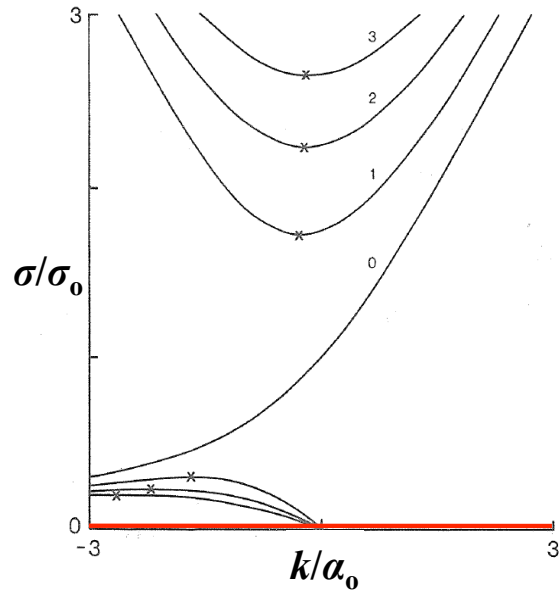


The u , v , and p fields for a **Yanai wave** when $c_n = 250$ cm/s and $P = 30$ days. For this P , $\sigma/\sigma_0 = .36$ and $\lambda = 7.3^\circ$.

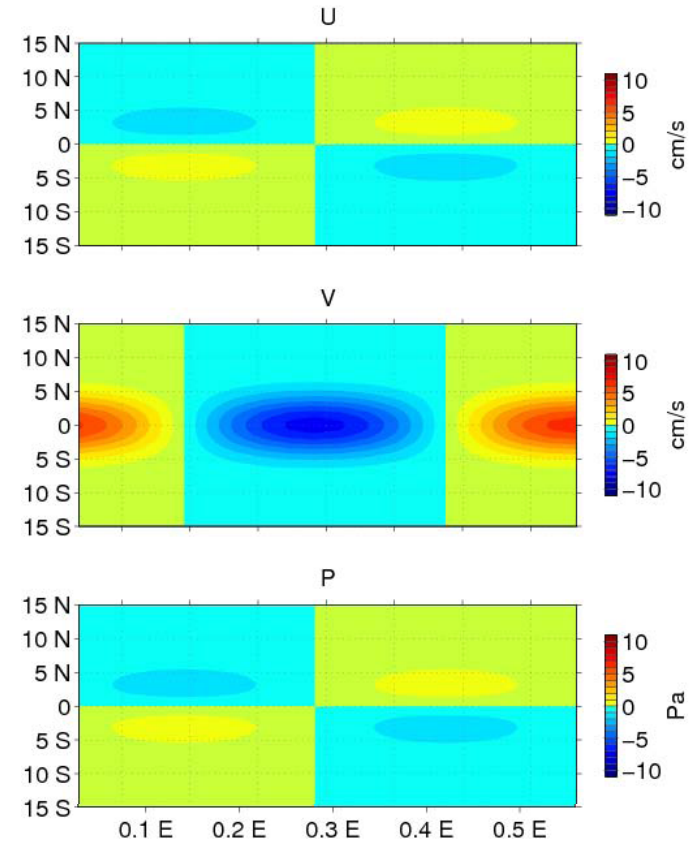


Courtesy of Francois Ascani

Structure of Yanai waves



The u , v , and p fields for a **Yanai wave** when $c_n = 250$ cm/s and $P = 360$ days. For this P , $\sigma/\sigma_0 = .03$ and $\lambda = 0.64^\circ$.



Courtesy of Francois Ascani



Equatorial ocean:
derivation of equatorial Kelvin wave

Equatorial Kelvin wave

The **equatorial Kelvin wave has $v = 0$** , and so was **missed in the preceding solutions**. To find it, **set $v = A = 0$ in (1)**, and look for a free-wave solution of the form

With these restrictions, equations (1) reduce to

$$-i\sigma u + ikp = 0, \quad fu + p_y = 0, \quad -i\sigma \frac{p}{c^2} + iku = 0.$$

The first and third equations imply

$$k = \pm \frac{\sigma}{c}, \quad u = \frac{k}{\sigma} p,$$

and the second then gives

$$p_y = -fu = -f \frac{k}{\sigma} p = \mp \frac{f}{c} p = \mp \alpha_o^2 y p \quad (4)$$

Equatorial Kelvin wave

The solution to (4) is

$$p = P'_o \exp\left(\mp \frac{1}{2} \alpha_o^2 y^2\right) \exp(ikx - i\sigma t).$$

The **solution that grows exponentially in y** , which corresponds to the root, $k = -\sigma/c$, **is physically unrealistic in an unbounded basin** and must be discarded. Therefore, the only possible wave is

$$p = P_o \phi_0(y) \exp\left[i \frac{\sigma}{c} (x - ct)\right], \quad \sigma = kc, \quad (5)$$

which describes the structure and dispersion relation for the **equatorial Kelvin wave**. In (5), I have used the property that

$$\phi_0(y) = \pi^{-1/4} \exp\left(-\frac{1}{2} \alpha_o^2 y^2\right),$$

and redefined the arbitrary constant amplitude to be $P_o = \pi^{1/4} P'_o$.

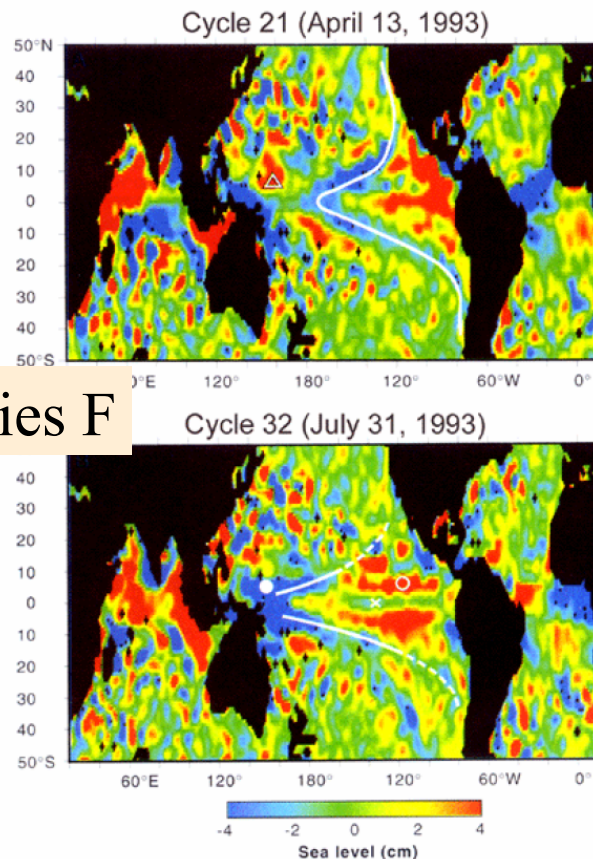
A world map with a light blue background. The equatorial region is highlighted in a darker blue, showing the Pacific, Indian, and Atlantic Oceans. The text "Equatorial ocean: eastern-boundary reflection" is overlaid in red on the Pacific Ocean.

**Equatorial ocean:
eastern-boundary reflection**

Eastern-boundary reflections

Remarkably, the characteristic **wedge shape and westward propagation is visible in satellite data**. The figure shows global maps of filtered sea level from TOPEX/Poseidon on April 13 and July 31, 1993. It shows **Rosby-wave packet generation** by the reflection of an **equatorial Kelvin wave forced by intraseasonal winds** in the western ocean. (After Chelton and Schlax, 1996.)

Movies F



Fedorov and Brown, 2007

A world map with a light blue background and dark blue landmasses. The text "Equatorial ocean: Vertical propagation" is overlaid in red, bold font, centered over the Pacific Ocean region.

Equatorial ocean:
Vertical propagation

Vertical propagation

Recall that the vertical structure of waves in the LCS model satisfy

$$\left(\frac{1}{N_b^2} \psi_{nz} \right)_z = -\frac{1}{c_n^2} \psi_n.$$

Rather than to look for solutions as expansions in vertical modes, $\psi_n(z)$, another way of studying solutions to the LCS model is to **look for approximate solutions of the form**,

$$\psi_n \propto \exp [im(z)z],$$

under the restriction that **the background stratification, $N_b(z)$ varies slowly with respect to the vertical wavelength of the wave, $m(z)$ (the WKB approximation)**. In that case,

$$\left(\frac{1}{N_b^2} \psi_{nz} \right)_z \approx -\frac{m^2}{N_b^2} \psi_n = -\frac{1}{c_n^2} \psi_n.$$

and c_n can be replaced by

$$c_n \rightarrow \frac{N_b}{|m|}$$

Vertical propagation (KW beams)

With this change, the dispersion relation for equatorial Kelvin waves is

$$\sigma = c_n k \quad \rightarrow \quad \sigma = N_b \frac{k}{|m|}.$$

Group theory states that a **packet of Kelvin waves** (that is, a superposition of several waves associated with different k and m values) **propagates at the “group” velocity**

$$c_{gx} = \sigma_k = \frac{N_b}{|m|}, \quad c_{gz} = \sigma_m = \mp N_b \frac{k}{m^2}, \quad m \geq 0$$

Thus, the **energy of the packet propagates to the east with the slope**

$$\frac{dz}{dx} = \frac{c_{gz}}{c_{gx}} = \frac{\sigma_m}{\sigma_k} = \mp \frac{\sigma}{N_b}, \quad m \geq 0$$

Since **coastal Kelvin waves have the same dispersion relation** as equatorial ones, they **propagate vertically in the same way**.

Vertical propagation (YW beams)

The dispersion relation for Yanai waves becomes

$$k - \frac{\sigma}{c} + \frac{\beta}{\sigma} = 0 \quad \rightarrow \quad k - \frac{|m|}{N_b} \sigma + \frac{\beta}{\sigma} = 0$$

Group theory states that a **packet of Yanai waves** (that is, a superposition of several waves associated with different k and m values) **propagates at the “group” velocity**

$$1 - \frac{|m|}{N_b} \sigma_k - \frac{\beta}{\sigma^2} \sigma_k = 0 \quad \Rightarrow \quad c_{gx} = \sigma_k = \left(\frac{|m|}{N_b} + \frac{\beta}{\sigma^2} \right)^{-1}$$

$$0 \mp \frac{\sigma}{N_b} - \frac{|m|}{N_b} \sigma_m - \frac{\beta}{\sigma^2} \sigma_m = 0 \quad \Rightarrow \quad c_{gz} = \sigma_m = \mp \frac{\sigma}{N_b} \left(\frac{m}{N_b} + \frac{\beta}{\sigma^2} \right)^{-1}$$

Thus, the **energy of the packet propagates to the east with the slope**

$$\frac{\partial z}{\partial x} = \frac{c_{gz}}{c_{gx}} = \mp \frac{\sigma}{N_b}, \quad m \geq 0$$

the **same slope as for Kelvin waves!**

Vertical propagation (long-wavelength RWs)

For the RW dispersion curves, as σ tends to zero so does k .

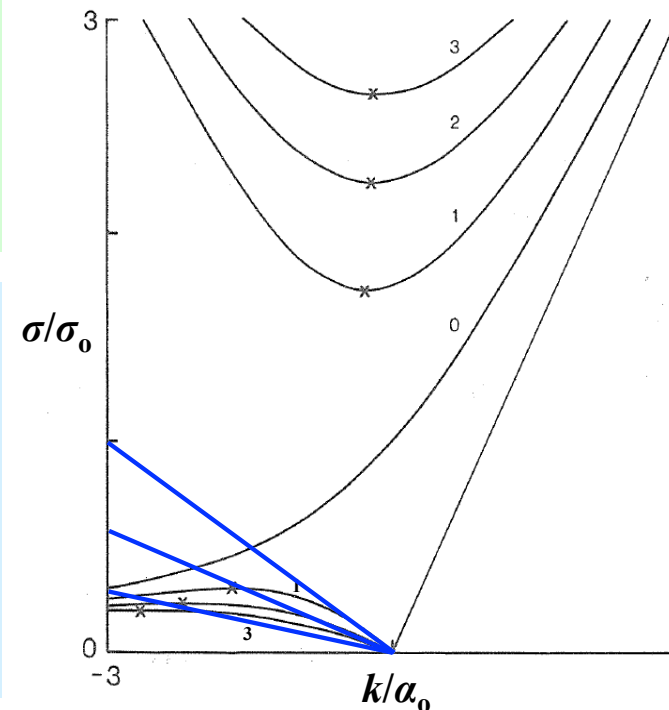
So, in the **low-frequency limit** the **RW disp. curves are non-dispersive**. This limit is known as the **long-wavelength approximation**.

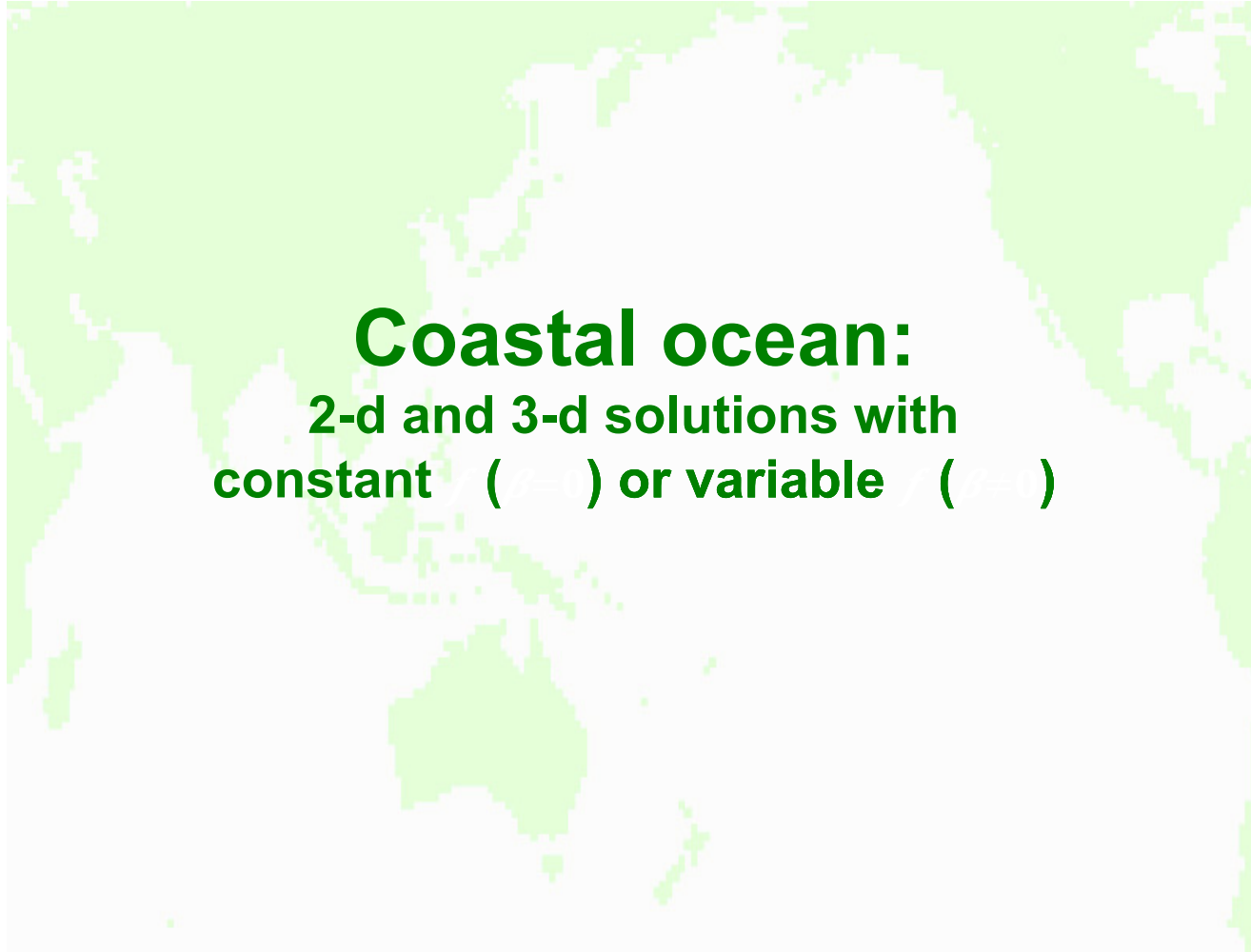
In this limit, **RWs propagate vertically with a slope**

$$\frac{dz}{dx} = \frac{\sigma_m}{\sigma_k} = \frac{\sigma}{N_b} (2\ell + 1)$$

with a **steeper slope, and in the opposite direction from, KW and YWs**.

$$\sigma \neq -k \frac{\beta}{\alpha_\ell^2} \neq -\frac{c}{2\ell + 1} k$$





Coastal ocean:
2-d and 3-d solutions with
constant β ($\beta=0$) or variable β ($\beta \neq 0$)

Questions

How does wind drive coastal currents?

across-shore Ekman flow driven by alongshore winds

What waves are generated at coasts?

Kelvin and Rossby waves; (shelf waves)

What are the key differences between 2-d and 3-d theories of coastal circulation?

wave radiation; establishment of $p \downarrow$ to balance $\tau \uparrow$

Why do eastern-boundary currents exist at all?

vertical mixing; (shelf trapping)

Coastal-ocean equations

A useful set of equations for the coastal ocean is

$$\begin{cases} -fv + g'h_x = 0, \\ v_t + fu + g'h_y = \tau^y/H - \lambda v, \\ h_t + H(u_x + v_y) = -\kappa(h - H) \end{cases}$$

As for the interior-ocean equations, this approximation is useful because it **filters out gravity waves**. Thus, it only describes the **slowly varying response**, that is, its **directly forced & Rossby-wave (if $\beta \neq 0$) parts**.

mixing terms. In this way, the **alongshore flow is in geostrophic balance**, a property consistent with observations.

Forcing by a band of alongshore wind $\tau \hat{y}$

All the coastal solutions discussed below are forced by a **band of alongshore winds** of the form,

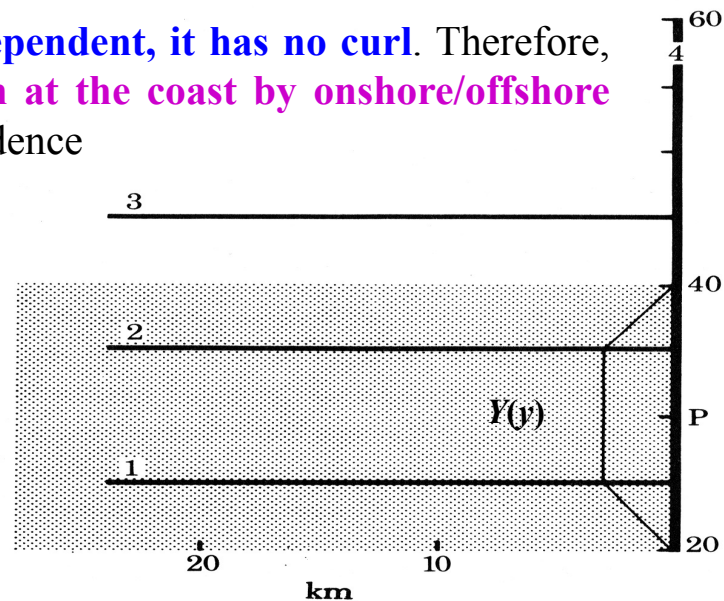
$$\tau^y(x, y, t) = \tau_o Y(y) T(t).$$

Since this **wind field is x -independent**, it has **no curl**. Therefore, the response is entirely **driven at the coast by onshore/offshore Ekman drift**. The time dependence is usually **switched-on**

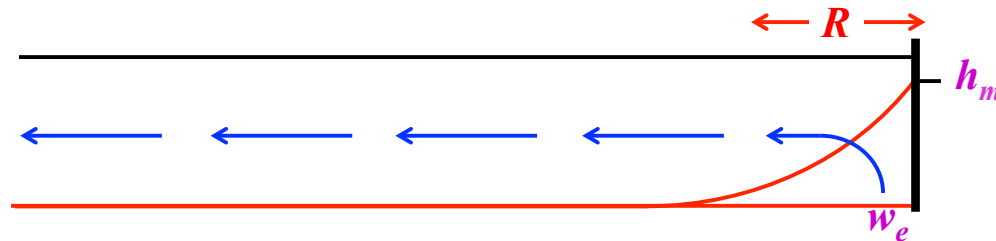
$$T(t) = \theta(t),$$

except for a few solutions when it is **periodic**

$$T(t) = e^{-i\sigma t}$$



2-d response to switched-on $\tau \hat{y}$



Consider the **2-dimensional** (x, h) coastal response of a **1½-layer model** when the wind is **independent of y** .

If the alongshore winds are directed **southward, they force offshore Ekman drift**. Since there can be no flow through the coast, the **thermocline must rise** to conserve mass. It rises until it **intersects the surface mixed layer**, and then **subsurface water entrains (upwells)** into surface layer.

The offshore decay scale of the circulation is the **Rossby radius of deformation, R** . There is a geostrophic **coastal current v in the direction of the wind**.

2-d response to switched-on τ^y

The solution to the 2-d coastal equations **with** τ^y is

$$h = H + \frac{\tau^y}{Rf} t e^{x/R}$$

For **southward winds** (), **h thins at the coast**, and the coastal response **weakens exponentially offshore with width scale R** .

How long d See additional slides for a derivation of (3). **e coast?** For the parameter choices

$$H = 100 \text{ m}, \quad f = 10^{-4} \text{ s}^{-1}, \quad R = 25 \text{ km}, \quad \tau^y = 1 \text{ dyn/cm}^2$$

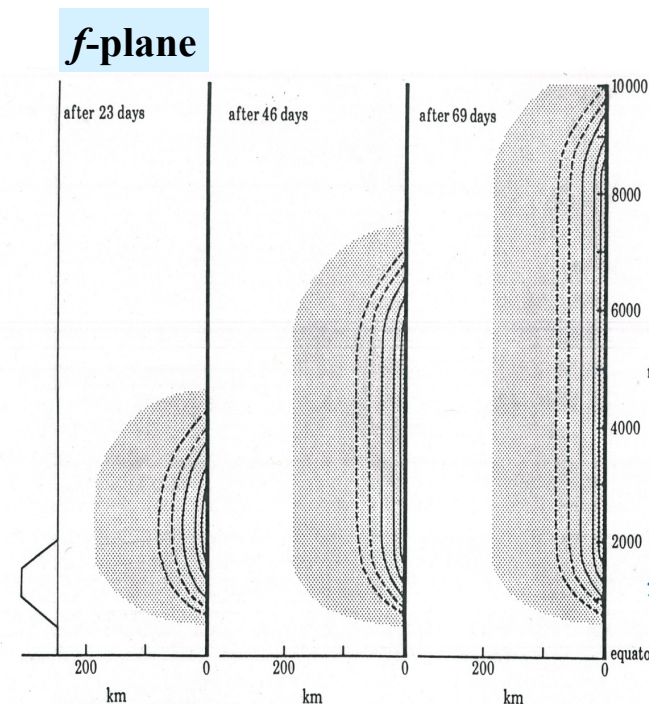
the time is **29 days**.

3-d response to switched-on $\tau \uparrow y$ ($\beta=0$)

Two-dimensional coastal upwelling is altered dramatically when 3-d processes are included. Specifically, the **propagation of Kelvin waves** along the coast **stops the rise of h** .

See additional slides for a derivation of this response. upwelling, **coastal Kelvin waves extend the response north** of the forcing region. The **pycnocline tilts** in the latitude band of the wind, creating a **pressure force that balances $\tau \uparrow y$** and stops the interface from rising further.

Movies H1a and H1b



3-d response to switched-on $\tau \uparrow y$ ($\beta \neq 0$)

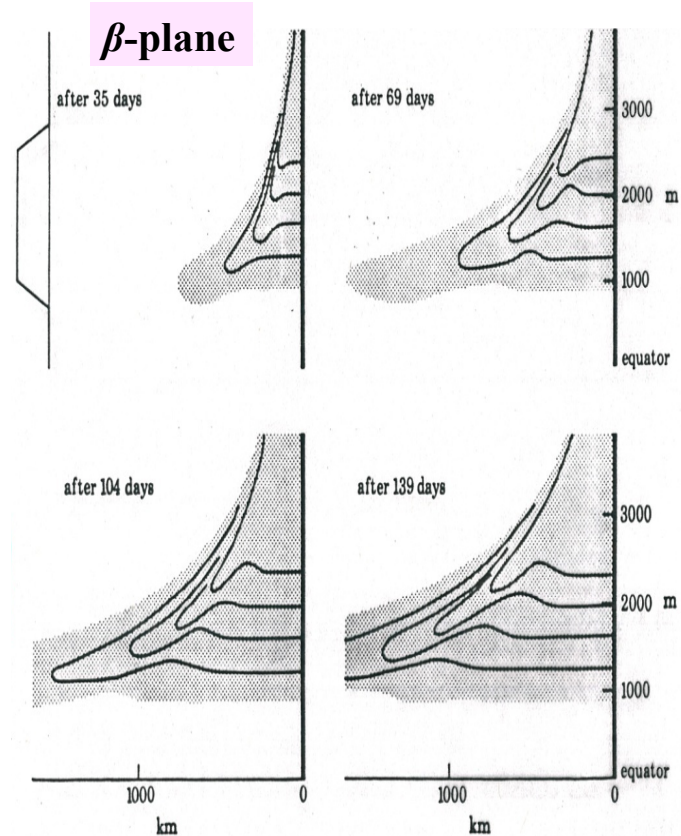
When $\beta \neq 0$, Rossby waves carry the coastal response offshore, leaving behind a state of rest in which $p \downarrow y$ balances $\tau \downarrow y$ everywhere.

The RW speed is

$$c_r = -\beta \frac{c^2}{f^2}$$

So, RWs propagate faster closer to the equator ($c \downarrow r \sim f \uparrow$).

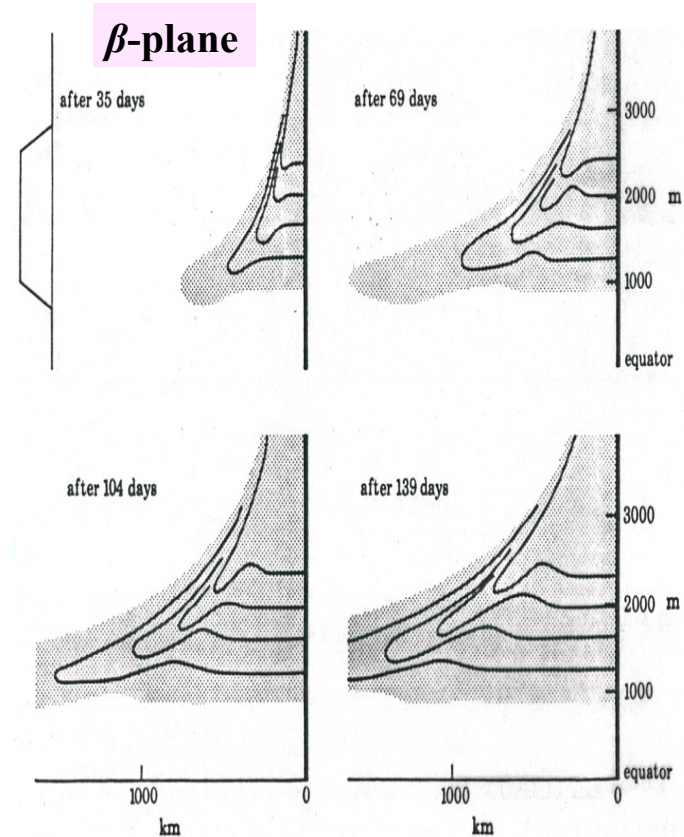
Movie H1c



Multi-mode response to switched-on $\tau \uparrow y$ ($\beta \neq 0$)

A **fundamental question** of coastal dynamics is: Since Rossby waves propagate offshore, **why do eastern-boundary currents exist at all?**

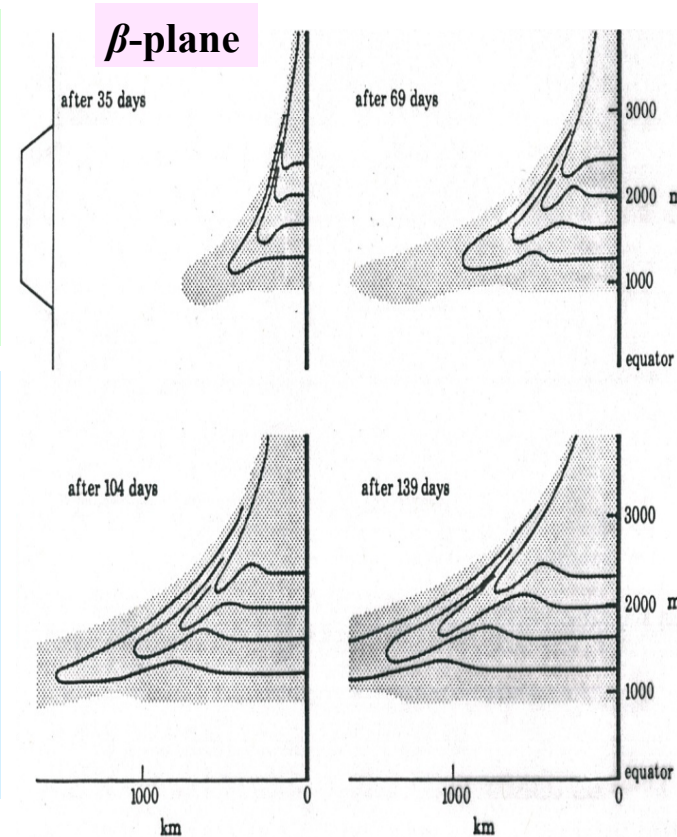
A possible answer is that **many baroclinic modes contribute** to the coastal response, and that the **RWs associated with them are damped** before they can propagate offshore.



Multi-mode response to switched-on $\tau \uparrow y$ ($\beta \neq 0$)

The plot shows the response of the $n = 1$ mode without damping. But, it also illustrates the $n > 1$ responses: the difference is that currents propagate offshore more slowly, since the RW propagation speed is $\propto c \downarrow n \uparrow -2$ and $c_n < c_1$.

With damping, the responses of the $n > 1$ modes are increasingly damped since $\nu = A/c_n^2$. In that case, the Kelvin and Rossby waves that radiate from the forcing region are weakened for larger n . For sufficiently large n , then, the response is confined to the forcing region.



Multi-mode response to switched-on $\tau \uparrow \gamma$ ($\beta \neq 0$)

McCreary (1981) obtained a **steady-state, coastal solution to the LCS model with damping.**

The model **allows Rossby waves to propagate offshore.** A **steady coastal circulation remains,** however, because they are **damped by vertical diffusion.**

There is **upwelling in the band of wind forcing.** There is a **surface current** in the direction of the wind, and a **subsurface CUC.**

Movies I2c and I3c

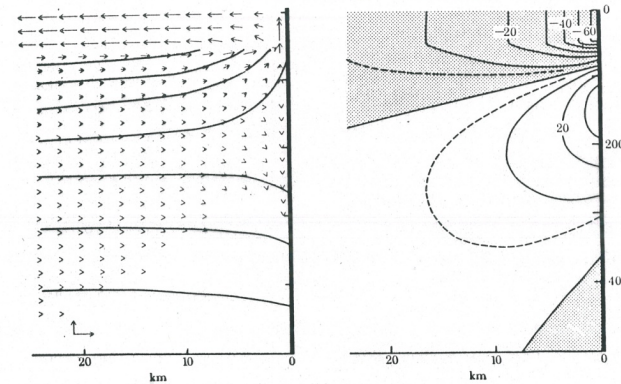
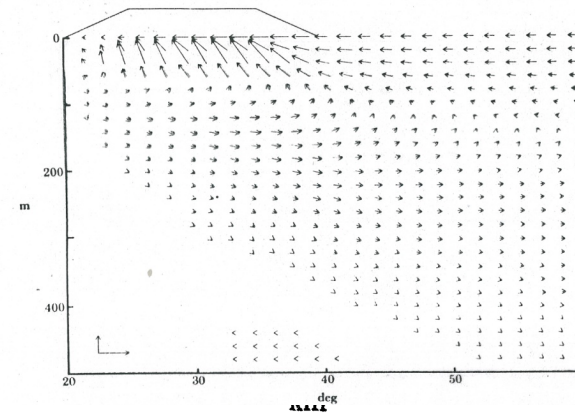
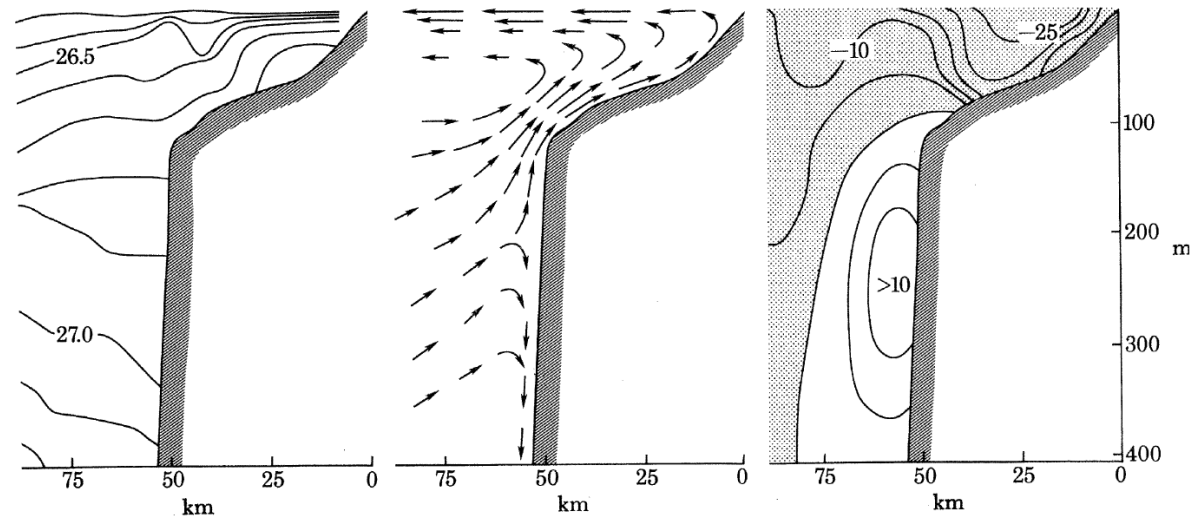


FIGURE 4b. As for figure 4a, but along section 2.



Observed eastern-coastal circulation



Mittelstadt et al. (1975), Huyer (1976)

The agreement with the McCreary (1981) model is striking. **Do eastern-boundary coastal in the real ocean exist due to diffusive damping?**

a **poleward undercurrent**, and **coastal upwelling**.

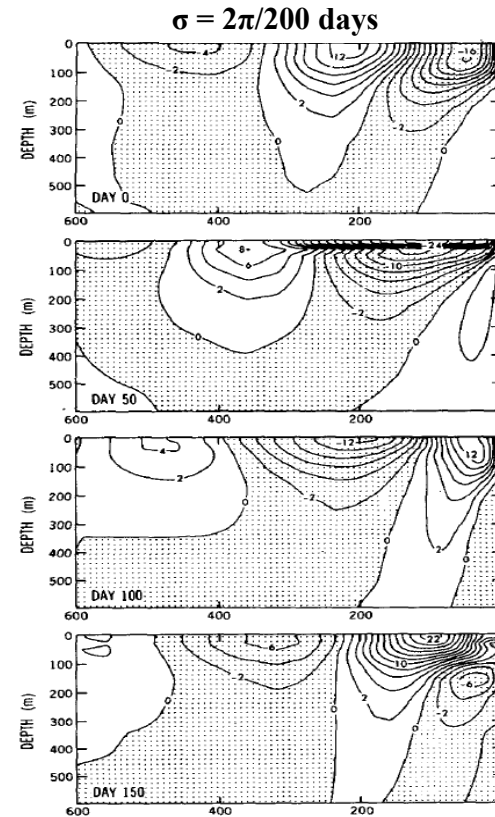
Multi-mode response to **periodic** $\tau \uparrow y$ ($\beta \neq 0$)

For a **switched-on** $\tau \uparrow y$, **Kelvin waves** (**Rossby waves**) radiate poleward (offshore), leaving behind a **steady-state coastal circulation**.

For a **periodic** $\tau \uparrow y$, coastal **KWs** and **RWs** are **continually generated**.

Kelvin-wave packets associated with a number of baroclinic modes form a **“beam”** that **carries energy downward**. There is **upward phase propagation** across the beam.

See **Movies K1e & K1d** for a description of Kelvin waves.



Philander and Yoon (1982)

A world map with a light blue background and dark blue landmasses, centered on the Pacific Ocean. The text is overlaid on the map.

Coastal ocean:
derivation of 2d coastal response

2-d response to switched-on τ^y

It is easy to solve the coastal equations for the initial rise of the thermocline. At that time, the **response is inviscid**, and the coastal equations **written in terms of a 2-d, 1½-layer model** are

$$\begin{array}{l}
 -fv + g'h_x = 0, \\
 v_{nt} + \frac{g'}{f}h_{xt} + fu = \frac{\tau^y}{H}, \\
 \frac{1}{c_n^2}h_t + Hu_x = 0
 \end{array}$$

~~$\frac{v}{c_n}$~~
 ~~$\frac{p_n}{c_n^4}$~~

Solving for a **single equation in h** gives

$$h_t - R^2 h_{xxt} = -\frac{\tau_x^y}{f} = 0, \tag{1}$$

where $R^2 = g'H/f^2$ is the square of Rossby radius of deformation. The **forcing term vanishes** because **τ^y is independent of x** .

2-d response to switched-on τ^y

The general solution to (1) is

$$h = H + A(t) e^{x/R} e^{-x/R}$$

The **coast is at $x = 0$** and the **ocean lies in the region $x < 0$** , so we have to **drop the B term** to ensure the solution is bounded as $x \rightarrow -\infty$.

To evaluate A , we **impose the boundary cond. that $u = 0$ at $x = 0$** . Using the v -momentum equation to **write u in terms of h** gives

$$f u = -v_t + \frac{\tau^y}{H} = -\frac{g'}{f} h_{xt} + \frac{\tau^y}{H} = 0 \quad @ \quad x = 0$$

and then

$$-\frac{g'}{f} \frac{A_t}{R} + \frac{\tau^y}{H} = 0 \quad \Rightarrow \quad A = \frac{\tau^y}{H} \frac{f}{g'} R t = \frac{\tau^y}{H} \frac{f^2}{g'} R \frac{t}{f} = \frac{\tau^y}{R f} t$$

2-d response to switched-on τ^y

The solution is then

$$h = H + \frac{\tau^y}{Rf} t e^{x/R}$$

For **southward winds** ($\tau^y < 0$), **h thins at the coast**, and the coastal response **weakens exponentially offshore with width scale R** .

There is a **meridional geostrophic current associated with h** ,

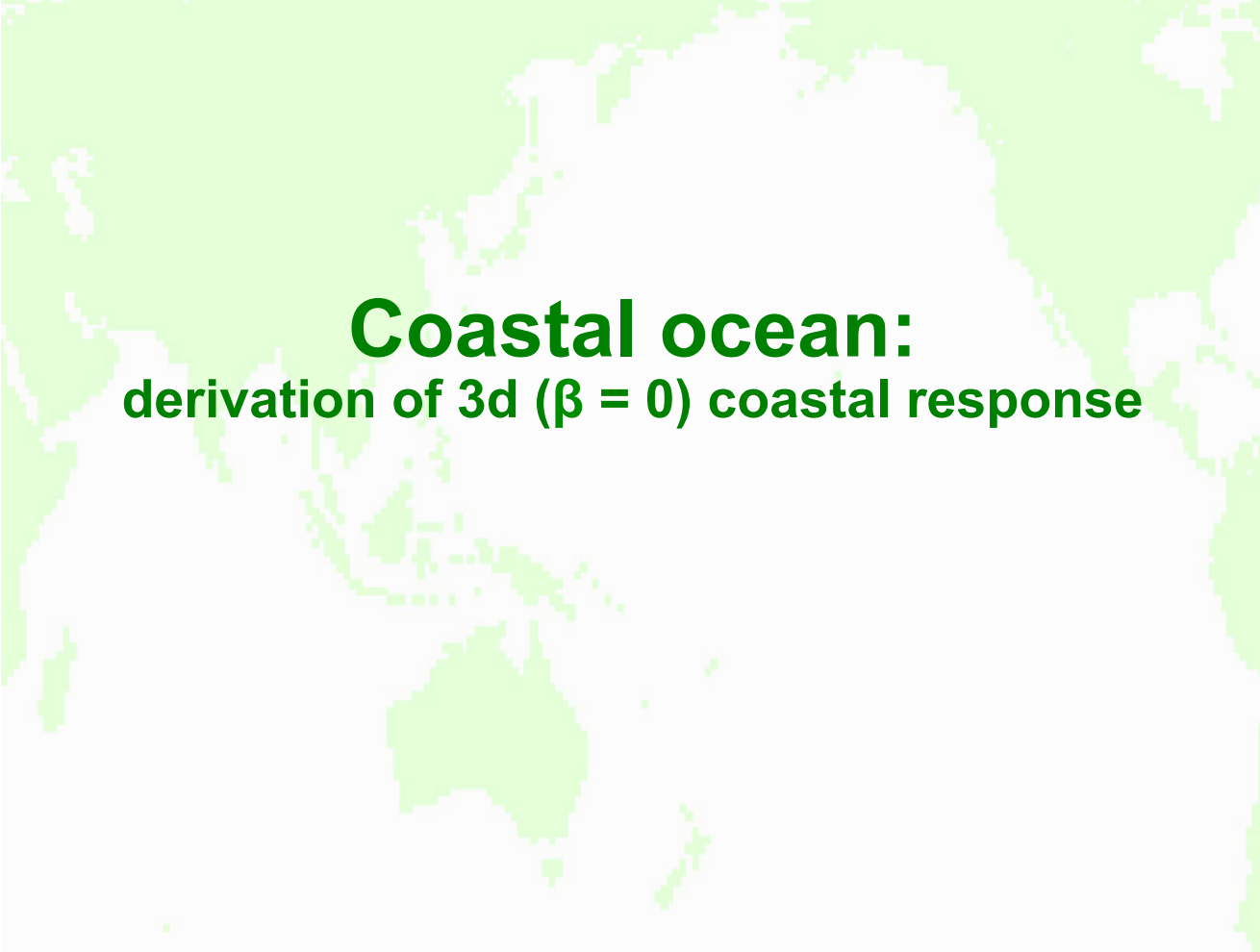
$$v = \frac{g'}{f} h_x = \frac{g' \tau^y}{R^2 f^2} t e^{x/R} = \frac{\tau^y}{H} t e^{x/R}$$

a **coastally trapped jet flowing in the direction of the wind**.

How long does it take for h to thin to the surface at the coast? For the parameter choices

$$H = 100 \text{ m}, \quad f = 10^{-4} \text{ s}^{-1}, \quad R = 25 \text{ km}, \quad \tau^y = 1 \text{ dyn/cm}^2$$

the time is **29 days**.

A world map with a light blue background and dark blue landmasses, centered on the Pacific Ocean. The map is slightly faded and serves as a background for the text.

Coastal ocean:
derivation of 3d ($\beta = 0$) coastal response

3-d response to switched-on τ^y

To see these properties, we solve the coastal equations **keeping the v_y and h_y terms**. Then, the **inviscid** coastal equations **written in terms of a 1½-layer model** are

$$\begin{array}{l}
 -fv + g'h_x = 0, \\
 v_t + fu + g'h_y = \frac{\tau^y}{H}, \\
 \frac{1}{c} h_t + H(u_x + v_y) = 0
 \end{array}$$

Solving for a **single equation in h** gives

$$h_t - R^2 h_{xxt} = -\frac{\tau_x^y}{f} = 0, \tag{1}$$

where $R^2 = g'H/f^2$ is the square of Rossby radius of deformation. The **forcing term vanishes** because **τ^y is independent of x** .

3-d response to switched-on τ^y

The general solution to (1) is

$$h = H \left(h = H + A(y, t) e^{x/R} \right) e^{-x/R} \quad (1)$$

The **coast is at $x = 0$** and the **ocean lies in the region $x < 0$** , so we have to **drop the B term** to ensure the solution is bounded as $x \rightarrow -\infty$.

To evaluate A , we **impose the boundary cond. that $u = 0$ at $x = 0$** . Using the v -momentum equation to **write u in terms of h** gives

$$f u = -\frac{g'}{f} h_{xt} - g' h_y + \frac{\tau^y}{H} = 0 \quad @ \quad x = 0$$

which, using (1), provides an equation for A ,

$$A_t + c A_y = \frac{\tau^y}{c}, \quad c = \sqrt{g' H}$$

3-d response to switched-on $\tau \hat{y}$

We obtain the solution for A by splitting it into **particular (steady-state)** and **homogeneous (Kelvin-wave) responses**,

$$A_p = \int_{-\infty}^y \frac{\tau y'}{g' H} dy' \equiv \chi(y), \quad A_h = \Lambda(y - ct)$$

where $\Lambda(x,y)$ is an as yet unspecified function.

To satisfy the **initial condition that $h = H$ at $t = 0$** , we must choose **$\Lambda(y) = -\chi(y)$** , so that

$$h = H + (A_p + A_h) e^{x/R} = H + \chi(y) e^{x/R} - \chi(y - ct) e^{x/R}$$

Initial adjustment

To determine the **response a short time after the wind switches on**, we **expand $\chi(y - ct)$ in a Taylor series about $t = 0$** to get

$$\begin{aligned}\lim_{t \rightarrow 0} h &= H + \chi(y) - \lim_{t \rightarrow 0} \chi(y - ct) \\ &= H + \chi(y) - [\chi(y) - c\chi'(y)t + \dots] \\ &= H + c\chi'(x, y)t + \dots = H + \frac{\tau^y}{c}t + \dots\end{aligned}$$

Thus, **at small times, the response is just the 2-d response!**

The response does not change from the 2-d response until the **Kelvin waves have propagated across the wind band.**

Final adjustment

At longer times the solution for all the fields is

$$\begin{aligned}
 h &= H - \left(\int_{-\infty}^y \frac{\tau^y}{g'H} dy' \right) e^{x/R} - \chi(y - ct) e^{x/R} \\
 v &= \frac{g'}{f} h_x = \frac{1}{R} \left(\int_{-\infty}^y \frac{\tau^y}{fH} dy' \right) e^{x/R} - \frac{c}{H} \chi(y - ct) e^{x/R} \\
 u &= -\frac{g'}{f^2} h_{xt} - \frac{g'}{f} h_y = \frac{\tau^y}{fH} (1 - e^{x/R})
 \end{aligned}$$

A **packet of Kelvin waves** propagates poleward. Note that, consistent with Kelvin waves, there is **no u field associated with the packet**.

After its passage, the solution adjusts to a **steady-state balance**.

Key properties of the steady solution are: 1) a **pressure gradient that balances the wind along the coast** ($x = 0$), that is, $p_y = g'h_y = \tau^y/H$; 2) a **coastal jet with a transport HRv** that supplies the Ekman transport from the coast; and 3) **Ekman drift that weakens to zero at the coast**.

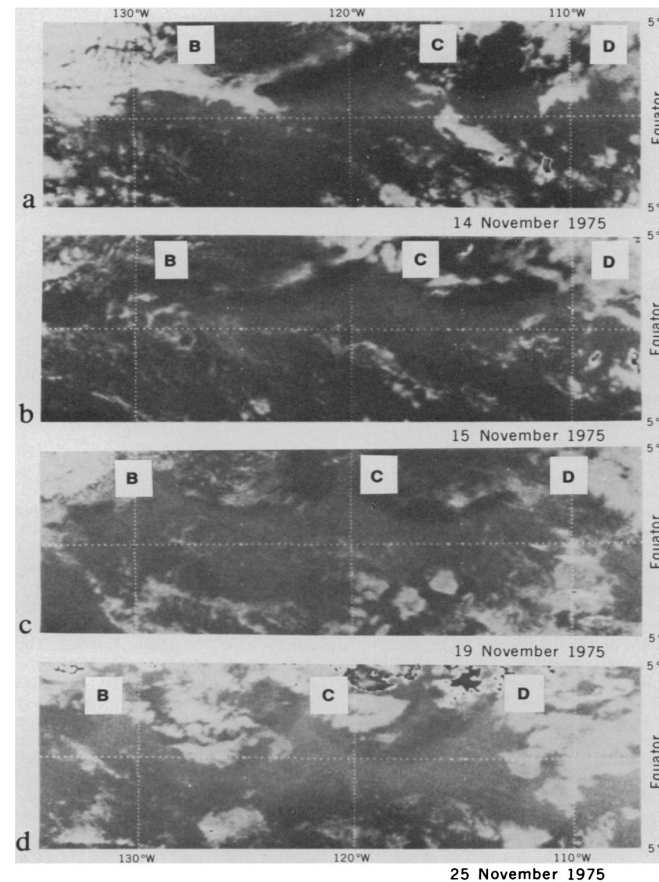


Intraseasonal variability

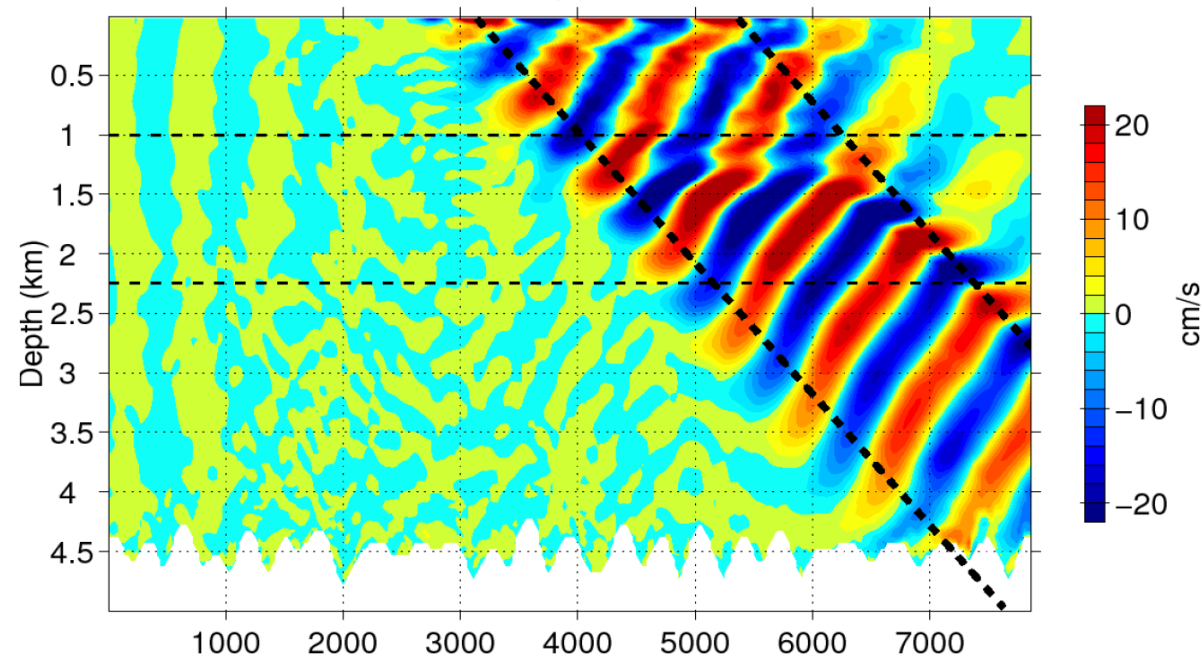
Tropical instability waves

Legeckis (1977, *Science*) first reported the presence of TIWs in the eastern, tropical Pacific. TIWs were soon shown to have a **large impact on the momentum and heat fluxes** in the region. Philander (1976, 1978, *JGR*) argued that TIWs were caused by **barotropic instability**. Yu *et al.* (1992, *Prog. Oceanogr.*) later suggested that an **instability of the temperature front** was involved. Luther and Johnson (1990) suggested that there was **more than one type of TIWs**.

Similar TIWs were soon observed in the Atlantic Ocean. Their dynamics are essentially the same as for the Pacific TIWs.

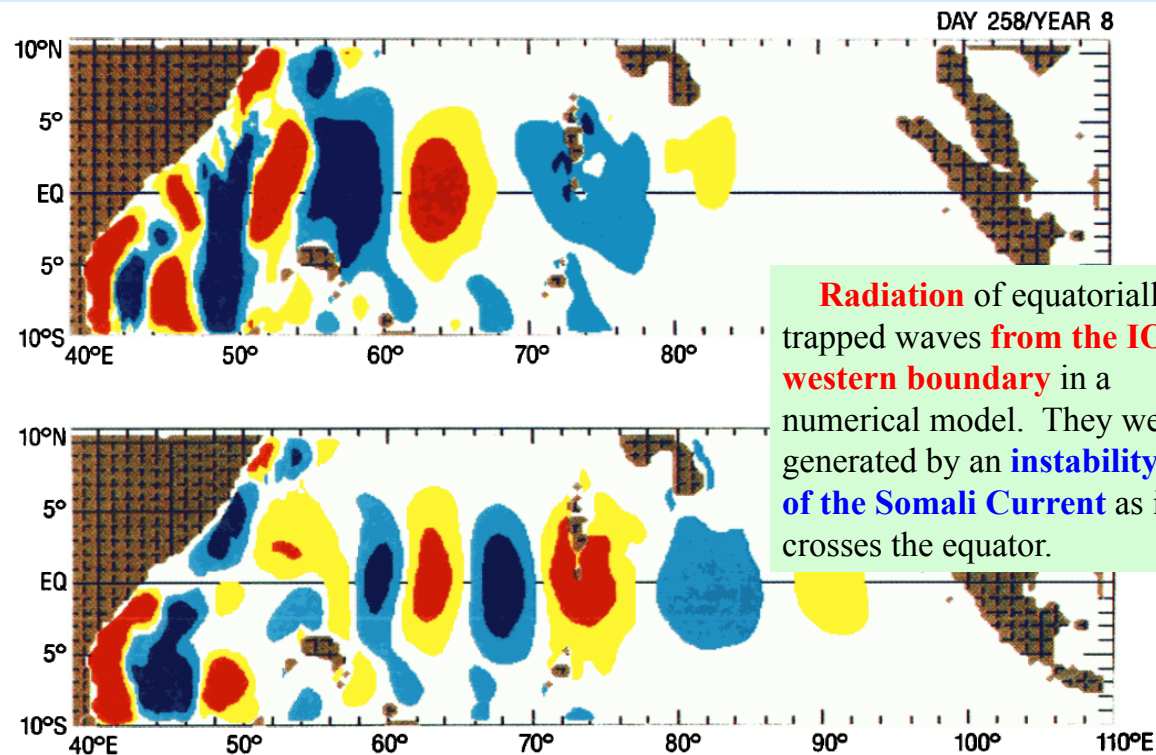


Tropical instability waves



Michael Cox (1980) reported a **Yanai-wave beam forced by surface TIWs in his OGCM solution**. Ascani & coworkers (2009) explored the idea that deep equatorial currents are caused by an **instability of the Yanai-wave beam generated by TIWs**. To simulate the effect of TIWs, they forced their OGCM by a **wind stress with the wavelength (~1000 km) and period (~30 days) of a typical TIW**, generating the Yanai-wave beam shown above.

Somali Current instability (27 days)

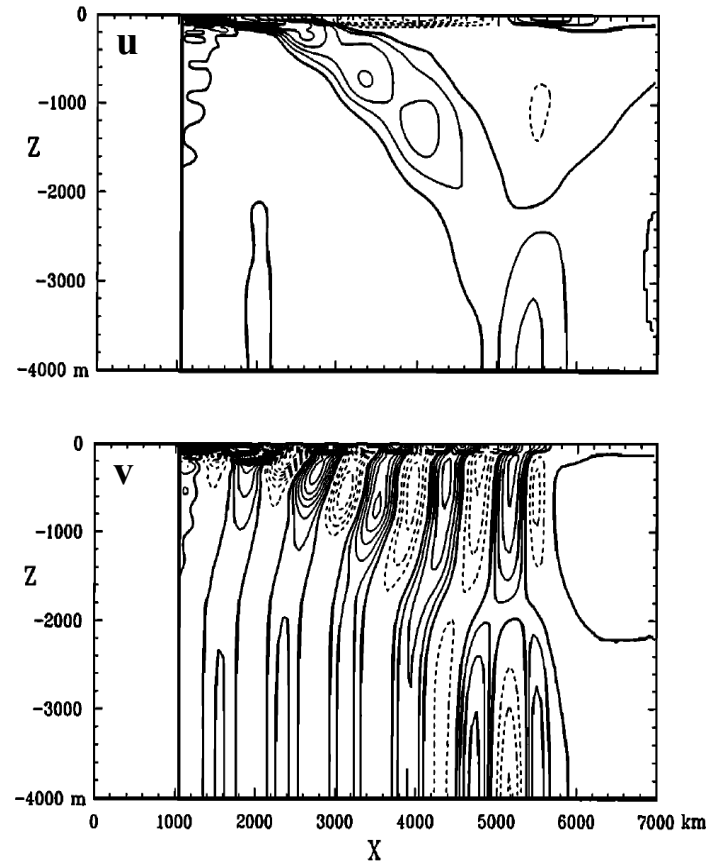


Their structure identifies the waves to be Yanai waves. These modeled waves were observed in altimetry by Tsai et al. (1992).

Somali Current instability (27 days)

When all the vertical modes are summed, **energy propagates downward** as well as eastward, **along paths parallel to the group velocity**. At the right of the plots, energy has **reflected off the bottom** to produce an **upward-propagating beam**.

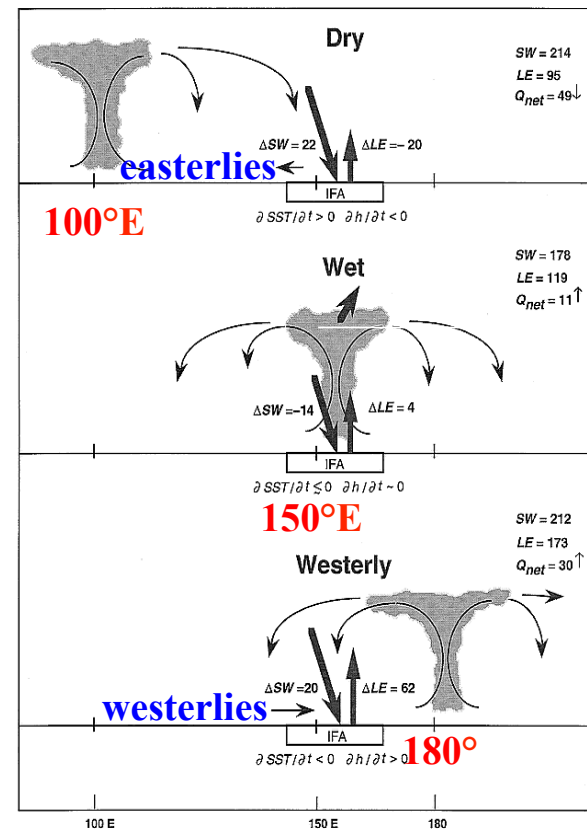
The presence of **intraseasonal variability at a depth of 750 m** in the Luyten and Roemmich (1982) observations thus appears to **result from the radiation of a beam of Yanai waves from the Somali coast**.



Madden Julian Oscillations (30–60 days)

In the central IO, oceanic ISV appears to be mostly wind-forced. A prominent forcing is by MJOs, eastward-propagating, convective disturbances, with periods of 30–60 days.

Their impacts on rainfall, oceanic surface fluxes, and SST are well documented.



Waliser, Murtugudde, Lucas (2003, 2004)

ISV in the EIO (~90 days)

Courtesy of Jerome Vialard

