Nonlinear effects on ultra-large scales

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Next Generation Large Scale Structure Surveys

- Future survey like HI intensity mapping will cover very large volumes.

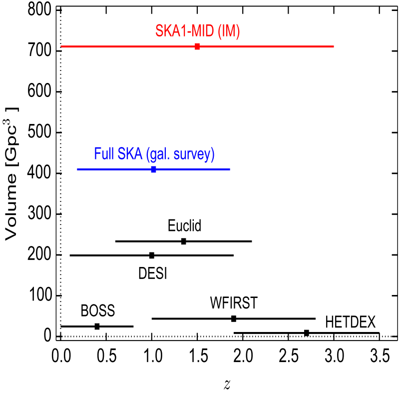


Figure 1: Santos et al (2015)

- Could possibly test
  - `Theories of gravity.`
  - Cosmological inflation
  - Supersymmetry
  - Bell inequalities
  - Add yours $\cdots$

Non-Gaussianity

Constraint on GR and non-Gaussianity

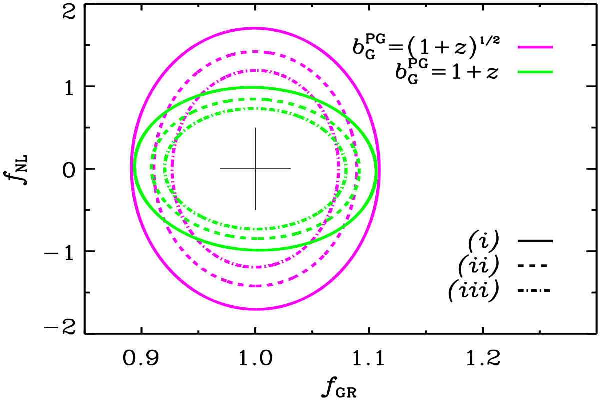
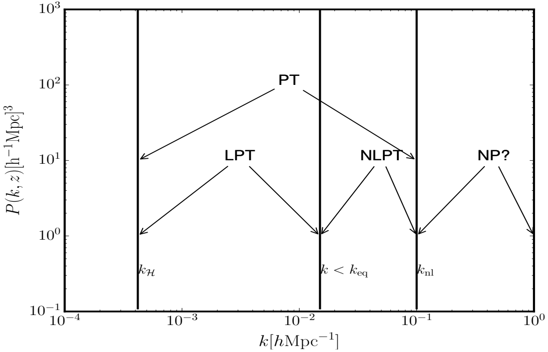


Figure 2: Fonseca et al (2015)

- Key assumptions:
  - On ultra-large scales, linear approximation is valid.
  - Gaussian bias is scale independent.

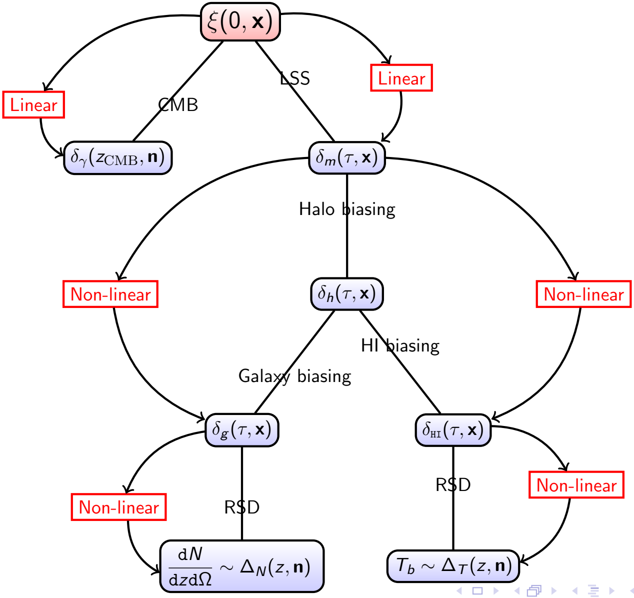
Methodology

- Range of validity of cosmological perturbation theory



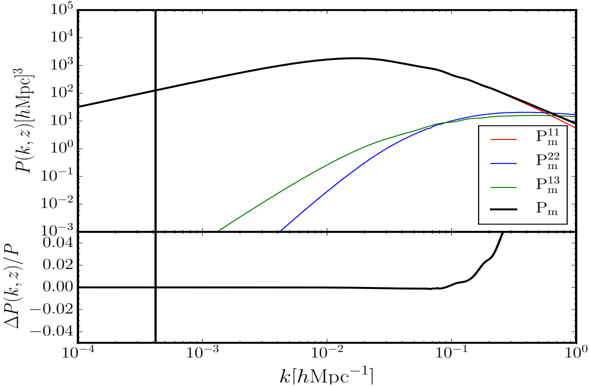
- This holds for:
  - Dark matter density field.
  - Tracers after renormalization.

Biased tracers on ultra-large scales



Dark matter density field

- Matter power spectrum: $P_m(k) = P_m^{11}(k) + \overbrace{P_m^{22}(k) + P_m^{13}(k)}^{\text{nonlinear}}$.



Tracer relationship with dark matter

- Assume that δ_{HI} is related to δ_m: $\delta_{\mathrm{HI}}(\tau, \mathbf{x}) = \mathcal{F}\left(\delta_m(\tau, \mathbf{x})\right)$.
- The form of the functional $\mathcal{F}$ is not exactly known?
- Employ series expansion:

$$\delta_{\mathrm{HI}}(\tau, \mathbf{x}) = \sum_{n=0}^{\infty} \frac{b_n^{(0)}}{n!} \delta_m^n(\tau, \mathbf{x})\,.$$

- Common assumption is that on large scales: $\delta_{\mathrm{HI}}(\tau, \mathbf{x}) = b_1 \delta_m^n(\tau, \mathbf{x})$.
- Important question: Does the series converge?.

Linear approximation is not enough?

- Taylor series expansion:

$$f(x) = f(x_0) + (x - x_0)f'(x_0) + \cdots \frac{(x - x_0)^n}{n!} f^{(n)}(x_o) + R_n \, .$$

- Linear approximation implies:

$$f(x) = f(x_0) + (x - x_0)f'(x_0) \, .$$

- The remainder is given by

$$R_n = f^{(n+1)}(x^*) \frac{(x - x_0)^{n+1}}{(n+1)!} = E_n \, .$$

- Error associated with linear approximation(theoretical systematic):

$$E_2(\eta, \mathbf{x}) = \frac{1}{2} b_2 (\delta_m(\eta, \mathbf{x}))^2 + \frac{1}{3!} b_3 (\delta_m(\eta, \mathbf{x}))^3 + \cdots$$

- Include the error in the likelihood function (Baldauf et al. 2016)

HI brightness temperature in real space

- In real space: $T \propto n_{\rm HI} = \bar{n}_{\rm HI}\left[1 + \delta_{\rm HI}\right]$ (i.e no RSD)
- Ensure that $\langle \delta_{\rm HI} \rangle = 0$ (McDonald 2006, Umeh et al. 2015)

$$\begin{aligned}\delta_{\rm HI} \;&=\; b_1\delta_m^{(1)} + \frac{1}{2}\left[b_1\delta_m^{(2)} + b_2\left((\delta_m^{(1)})^2 - \sigma_\Lambda^2\right)\right] \\ &\quad + \frac{1}{3!}\left[b_1\delta_m^{(3)} + 3b_2\delta_m^{(1)}\delta_m^{(2)} + b_3(\delta_m^{(1)})^3\right]\,.\end{aligned}$$

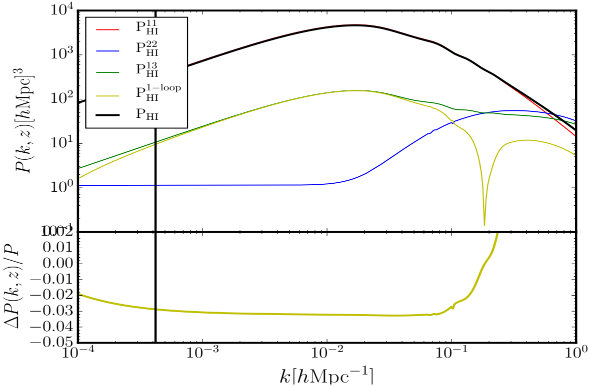
- Effective background number density

$$\bar{n}_{\rm HI}^{\rm FLRW} \to \bar{n}_{\rm HI} = \bar{n}_{\rm HI}^{\rm FLRW}\left(1 + b_0\right) = \bar{n}_{\rm HI}^{\rm FLRW}\left(1 + b_2\sigma_\Lambda^2/2\right)\,.$$

- where σ_Λ^2 is variance of matter density field smoothed at $\Lambda \sim k_{\rm nl}$.

HI power spectrum in real space

- HI power spectrum at $z = 1.5$: $P_{\rm HI}(k) = P^{11}_{\rm HI}(k) + \overbrace{P^{22}_{\rm HI}(k) + P^{13}_{\rm HI}(k)}^{\text{nonlinear correction}}$.



- Linear HI power spectrum not enough on ultra-large scales.

Bias renormalization:Gaussian initial conditions I

- On large scale, it is possible to obtain an effective linear theory:

$$P^L_{\text{HI}}(k) \approx \left[b_1 + \frac{1}{2}\left(b_3 + \frac{68}{21}b_2\right)\sigma^2_\Lambda\right]^2 P_m(k) + N_{\text{eff}}$$

- where N_{eff} is given by

$$N_{\text{eff}} = \frac{1}{2}b_2^2 \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} {P_m}^2(k_1)\,.$$

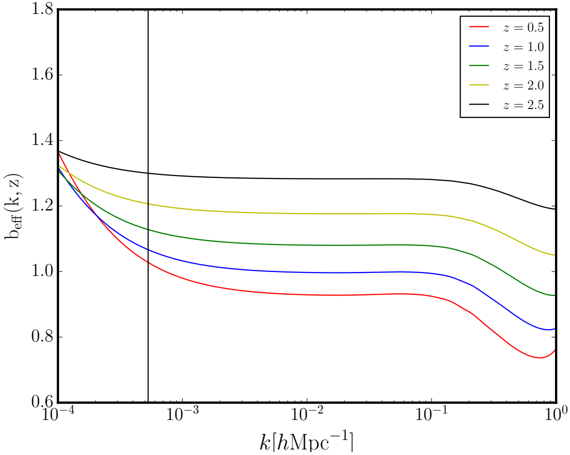
- Define an effective bias on large scales

$$b_{\text{eff}}(k,\Lambda) = \sqrt{\frac{P_{\text{HI}}(k)}{P_m(k)}} = \sqrt{b_1 + \frac{1}{2}\left(b_3 + \frac{68}{21}b_2\right)\sigma^2_\Lambda + \frac{N_{\text{eff}}}{P_m(k)}}\,.$$

- Effective bias acquires scale dependence due to nonlinear effects.

Bias renormalization:Gaussian initial conditions II

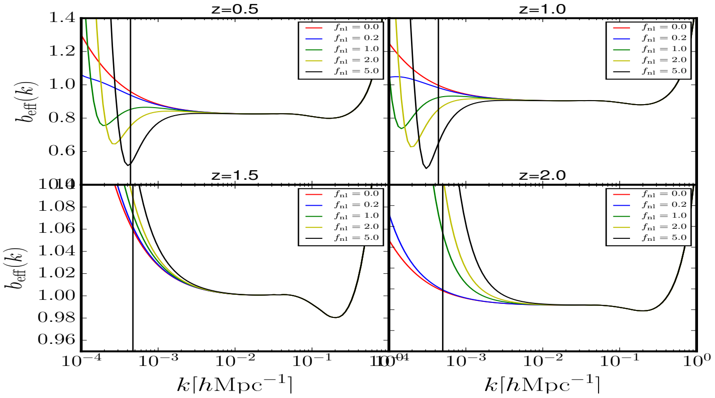
- Scale dependence of the effective bias on ultra-large scales.



Implications for primordial non-Gaussainity

- Non-Gaussainity in the bias (Dalal et al. 2007, Matarrese et al. 2008)

$$b_1 \mapsto b_1(k) = b_1 + \mathrm{f_{nl}} \left(\frac{\mathcal{H}}{k}\right)^2 \Delta b \,, \quad \Delta b \propto (b_1 - 1) \neq (b_{\rm eff} - 1)$$



Redshift space distortions

Let's include the effect of redshift space distortions.

General relativistic redshift space distortions I

- Real to redshift space map

$$s^i = x^i + \frac{n^i}{\mathcal{H}}\delta z\,,$$

- On large scales, perturbation theory is valid, so we expand

$$\delta z = \partial_{\parallel} v_s^{(1)} - \Phi_s^{(1)} - \int_0^{\chi_s} (\Phi^{(1)\prime} + \Psi^{(1)\prime})\mathrm{d}\chi + \frac{1}{2}\partial_{\parallel} v_s^{(2)} + \frac{1}{3!}\partial_{\parallel} v_s^{(3)}\,.$$

- The HI brightness temperature becomes

$$T^{\rm obs}(z,\mathbf{n}) \;=\; \bar{T}(z)\Big[1+\Delta_T(z,\mathbf{n})\Big]$$

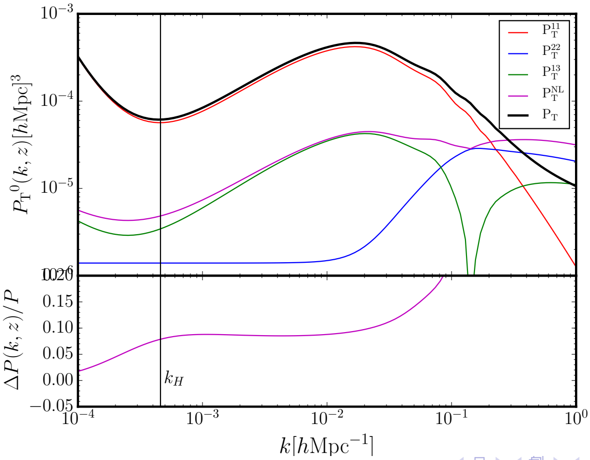
General relativistic redshift space distortions II

- Dominant terms on all scales:

$$
\begin{aligned}
\Delta_T(z, \mathbf{n}) \quad = \quad & \delta_{\rm HI} - \frac{1}{\mathcal{H}}\partial_\parallel^2 v \\
& + \overbrace{\left[\partial_\parallel v^{(1)}, \Psi^{(1)\prime}, \Phi^{(1)}\right]}^{\text{GRcorrection}} \\
& + \overbrace{\left[\delta^{(1)}_{\rm HI}\partial_\parallel^2 v^{(1)}, \left(\partial_\parallel^2 v^{(1)}\right)^2\right]}^{\text{nonlinear RSD}} \\
& + \overbrace{\left[\left(\frac{1}{\mathcal{H}}\partial_\parallel^2 v^{(1)}\right)^3, \delta^{(2)}_{\rm HI}\partial_\parallel^2 v^{(1)}\right]}^{\text{nonlinear RSD}} \\
& + \text{Lensing correction}
\end{aligned}
$$

HI power spectrum in redshift space

- Power spectrum: $P_T = P_T^{11}(k,\mu) + \overbrace{P_T^{22}(k,\mu) + P_T^{13}(k,\mu)}^{\text{nonlinear correction}}$



Effective HI power spectrum on large scales

- Large scale limit

$$P_T^{0\,L}(k) = \bar{T}^2 \Bigg[b_1 b_{\rm NL} + \frac{1}{3}\left(b_1 + b_{\rm NL}\right) f + \frac{1}{5} f^2 + \left(\frac{\mathcal{B}^2}{3} + \mathcal{A}\left(b_1 + b_{\rm NL}\right) + \frac{2}{3}\mathcal{A} f\right)\left(\frac{\mathcal{H}}{k}\right)^2 + \mathcal{A}^2\left(\frac{\mathcal{H}}{k}\right)^4 \Bigg] P_m(k) + N_{\rm eff} \,,$$

- where

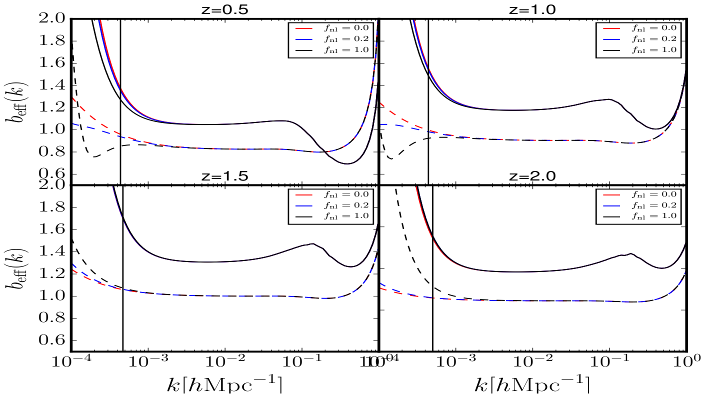
$$b_{\rm NL} = b_1 + \left(b_3 + \frac{68}{21} b_2\right)\sigma_\Lambda^2 + \mathcal{I}_R^0(k,\Lambda)\,.$$

- The effective bias in redshift space:

$$b_{\rm eff}^{\rm RS}(k,\Lambda) = \sqrt{\frac{P_T^0(k)}{P_m(k)}} = \sqrt{\frac{P_T^{0\,L}(k)}{P_m(k)}}\,.$$

Real/redshift space bias

- The assumption that $b_{\rm eff}^{\rm RS}(k,\Lambda) = b_{\rm eff}(k,\Lambda)$?



Angular Correlation function

What does the angular correlation function say?

Angular Correlation function I

- We can also compute the simpler part of the total angular power spectrum.

$$C_\ell(\nu_1, \nu_2) = C_\ell^{\mathrm{Lin}}(\nu_1, \nu_2) + C_\ell^{\mathrm{Nonlin}}(\nu_1, \nu_2),$$

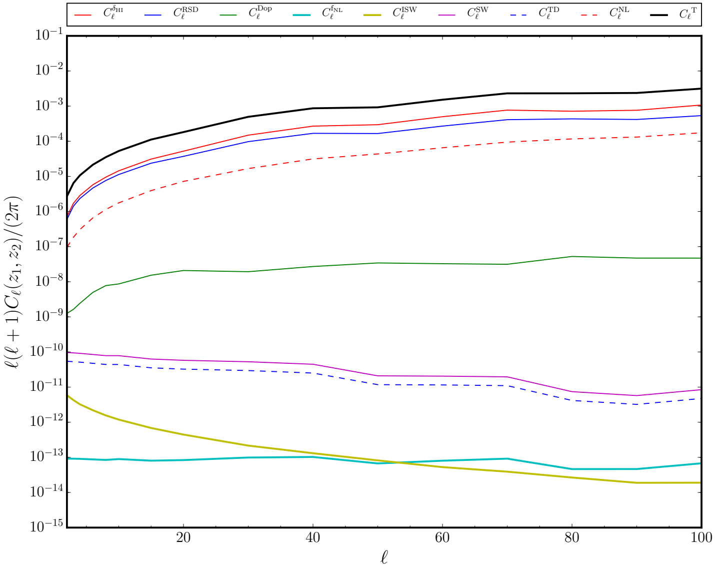
- where the linear order is given by

$$C_\ell^{\mathrm{L}}(\nu_1, \nu_2) = \frac{2}{\pi} \int k^2 \mathrm{d}k P_m(k) \mathcal{W}_\ell^{(1)}(\nu_1, k) \mathcal{W}_\ell^{(1)}(\nu_2, k) ,$$

- and the nonlinear part is given by

$$\begin{aligned} C_\ell^{\mathrm{NL}}(\nu_1, \nu_2) &= \frac{2}{\pi} \int k^2 \mathrm{d}k \Big[P_{22}(\nu_1, \nu_2, k) \\ &\quad + \frac{1}{2} \left(\mathcal{W}_\ell^{(1)}(\nu_1, k) P_{13}(\nu_2, k) + \mathcal{W}_\ell^{(1)}(\nu_2, k) P_{13}(\nu_1, k) \right) \Big] . \end{aligned}$$

Angular Correlation function II



Angular Correlation function III

Term	Effect	Scale dep.	z-dep	% contri.
Δ_{HI}	Clustering	$\sim \delta_m$	Local	≈ 30
Δ_{RSD}	Kaiser RSD	$\sim \delta_m$	Local	≈ 20
Δ_{Dop}	Doppler effect	$\sim \left(\frac{\mathcal{H}}{k}\right) \delta_m$	Local	≈ 0.023
Δ_{SW}	Sachs-Wolfe effect	$\sim \left(\frac{\mathcal{H}}{k}\right)^2 \delta_m$	Local	$< 10^{-3}$
Δ_{TD}	Time delay effect	$\sim \left(\frac{\mathcal{H}}{k}\right)^2 \delta_m$	Local	$< 10^{-3}$
Δ_{ISW}	ISW effect	$\sim \left(\frac{\mathcal{H}}{k}\right)^2 \delta_m$	Int.	$< 10^{-4}$
Δ_{NG}	Non-Gaussianity	$\sim \left(\frac{\mathcal{H}}{k}\right)^2 \delta_m$	Local	$< 10^{-3}$
Δ_{NL}	Nonlinear term	$\sim \left(\frac{\mathcal{H}}{k}\right)^{-2} (\delta_m)^{2*}$	Local	≈ 3.6

Conclusion

- Contamination: Bias is contaminated by GR and nonlinear effects.
- Scale dependence: Effective Gaussian bias is scale dependent on ultra-large scales.
- Scale dichotomy: For precision cosmology, there may not be a clear dichotomy between linear scales (where everything is fine and good) and nonlinear scales.

Thanks for listening.