Quevedo-fest, ICTP, 2016

F-theory and Particle Physics

Mirjam Cvetič

Our scientific paths with Fernando overlapped at junior stages of our careers: co-authored et al.

M. Cvetič, A. Font, L. E. Ibañez, D. Lüst and F. Quevedo,
``Target space duality, supersymmetry breaking and the stability of classical string vacua,'' Nucl. Phys. B361 (1991) 194

M. Cvetič, F. Quevedo and S. J. Rey,
``Stringy domain walls and target space modular invariance,''
Phys. Rev. Lett. 67 (1991) 1836

Fernando went on to making numerous leading contributions in string theory compactification and its implications for particle physics & cosmology…
As a Director of the ICTP, he has been making tremendous impact on theoretical sciences in the developing world.

As a colleague and a friend he is generous, thoughtful and supportive, with not a shred of arrogance.

Wishing you a Happy 60th Birthday!

Outline:

I. F-theory Compactification: key ingredients

- i) non-Abelian gauge symmetries, matter, Yukawa couplings
- ii) Recentn Developments: Abelian & discrete symmetries in F-theory

II. Particle Physics Model Building

- i) building blocks via toric techniques
- ii) highlight concrete example of the first global example of three family supersymmetric Standard Model & Standard Model with R-parity

Emphasize geometric perspective & UPenn centric

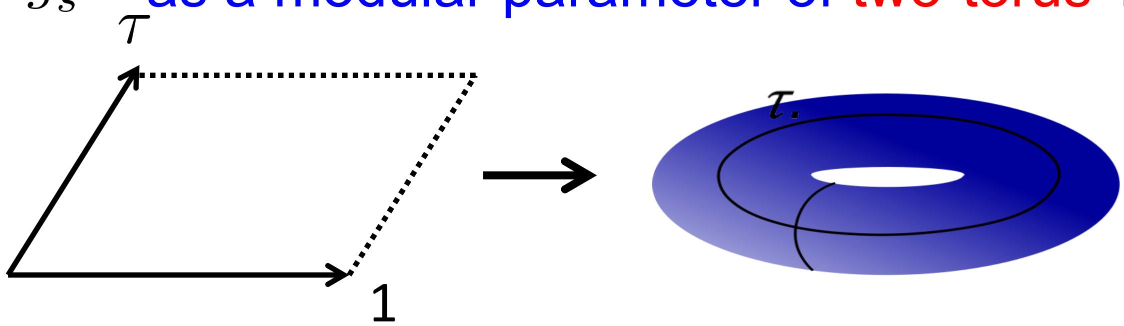
Type IIB perspective

F-THEORY BASIC INGREDIENTS

F-theory Compactification: Basic Ingredients

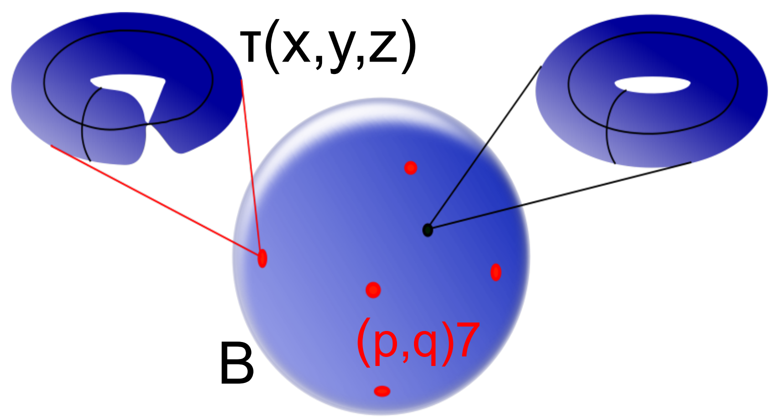
F-theory geometrizes the (Type IIB) string coupling (axio-dilaton)

$\tau \equiv C_0 + i g_s^{-1}$ as a modular parameter of two-torus $T^2(\tau)$ w/SL(2,Z)



[Vafa]

Compactifcation is a two-torus $T^2(\tau)$ (elliptic) fibration over a compact base space B:



Weierstrass form:

$$y^2 = x^3 + f x z^4 + g z^6$$

f, g- function fields on B

[z:x:y] homog. coords on $\mathbf{P}^2(1,2,3)$

F-theory Compactification: Basic Ingredients

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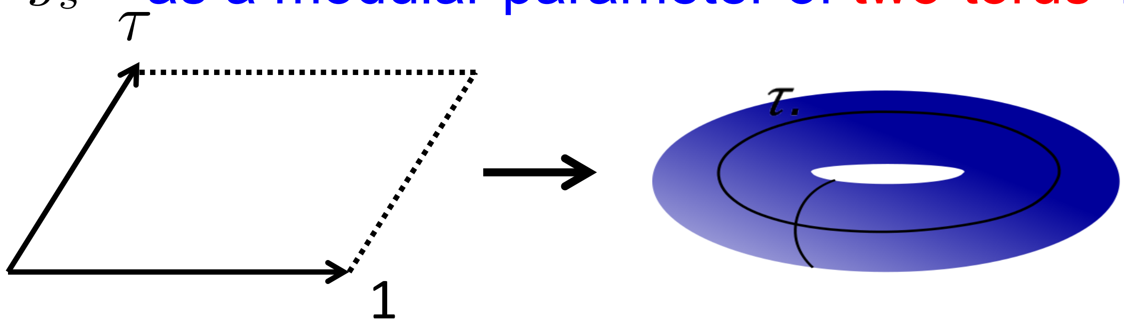

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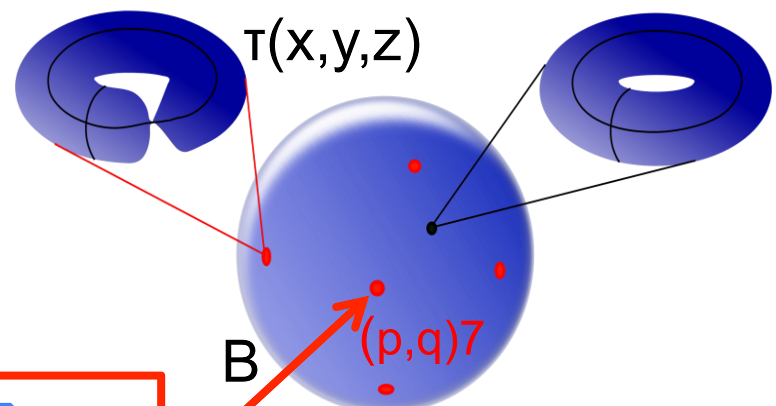


singular $T^2(\tau)$-fibr. → $g_s \rightarrow \infty$
location of (p,q) 7-branes

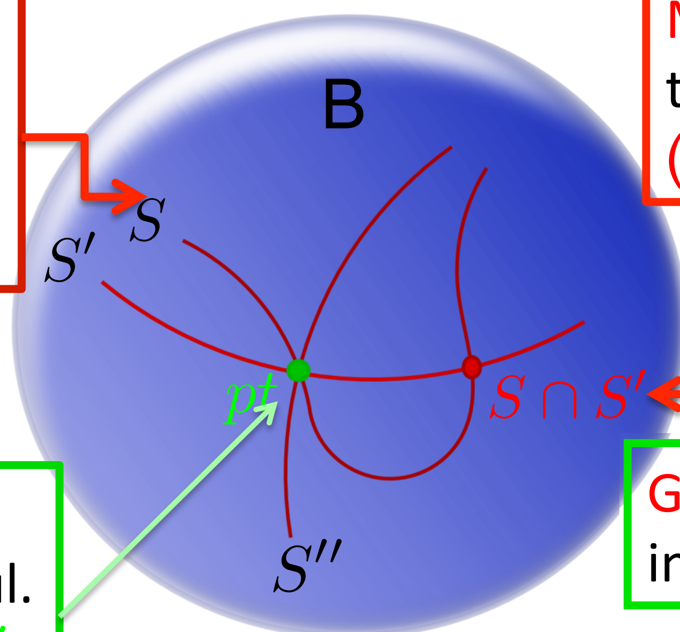
F-theory compactification: basic ingredients

- Total space of torus-fibration: singular elliptic Calabi-Yau manifold X

  D=4, N=1 vacua: fourfold X_4
- Singularities encode complicated set-up of intersecting D-branes:

Non-Abelian gauge symmetry in codimension-one singul. in B (divisors wrapped by branes)

Matter in codimension-two singul. in B (intersecting branes)



Yukawa couplings in codimension-three singul. in B $pt = S \cap S' \cap S''$

G_4-flux at intersection induces chiral 4D matter

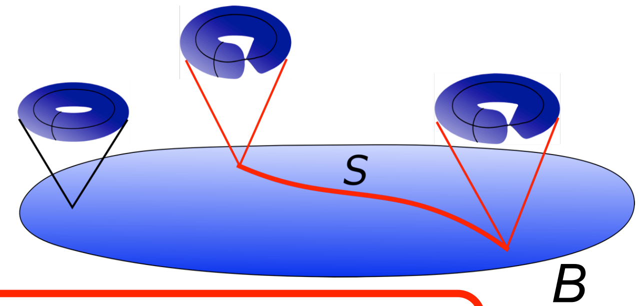
Highlights: Non-Abelian Gauge Symmetry

[Kodaira; Tate; Vafa; Morrison,Vafa;…]

1. Weierstrass form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$

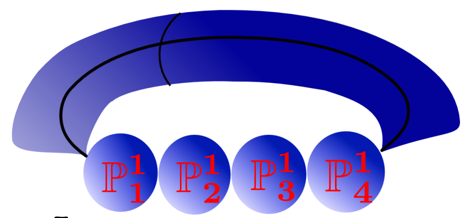
2. Severity of singularity along divisor S in B:



[$ord_S(f)$, $ord_S(g)$, $ord_S(\Delta)$] ←→ Singularity type of fibration of *X*

3. Resolution: singularity type ←→ structure of a tree of $\mathbb{P}^1$'s over *S*

I_n-singularity ←→ SU(n) Dynkin diagram



Deformation: [Grassi, Halverson, Shaneson]

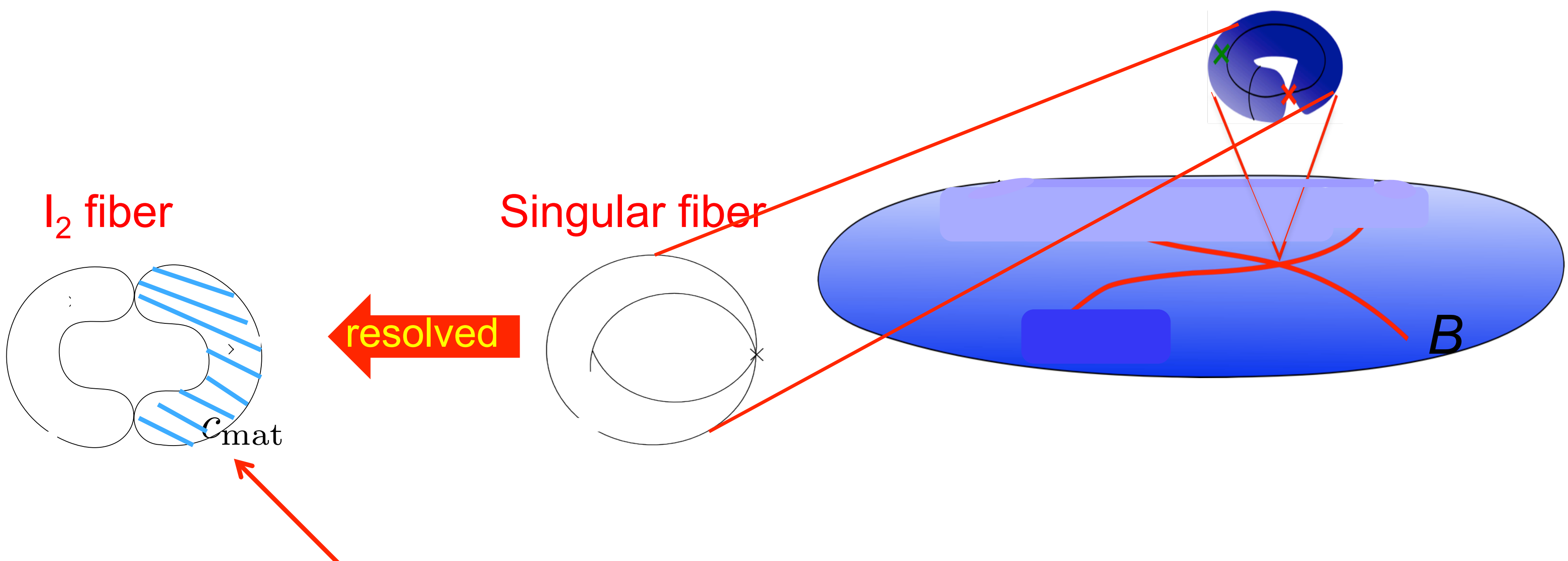
- Cartan generators for A^i gauge bosons: in M-theory via Kaluza-Klein (KK) reduction of C_3 potential along (1,1)-forms $\omega_i \leftrightarrow \mathbb{P}^1_i$ on X

$$C_3 \supset A^i \omega_i$$

- Non-Abelian generators: light M2-brane excitations on $\mathbb{P}^1$'s [Witten]

Highlights: Matter

Singularity at codimension-two in *B*:



w/isolated (M2-matter) curve wrapping $\mathbb{P}^1$ → charged matter
(determine via intersection theory)

II. U(1)-Symmetries in F-Theory

U(1)'s-Abelian Symmetry & Rational Sections

U(1) gauge bosons A^m should also arise via KK-reduction $C_3 \supset A^m \omega_m$

(1,1)-forms on X

[Morrison, Vafa]

$\boxed{\text{(1,1) - forms } \omega_m \Longleftrightarrow \text{ rational points}}$

Rational point Q on elliptic curve E with zero point P

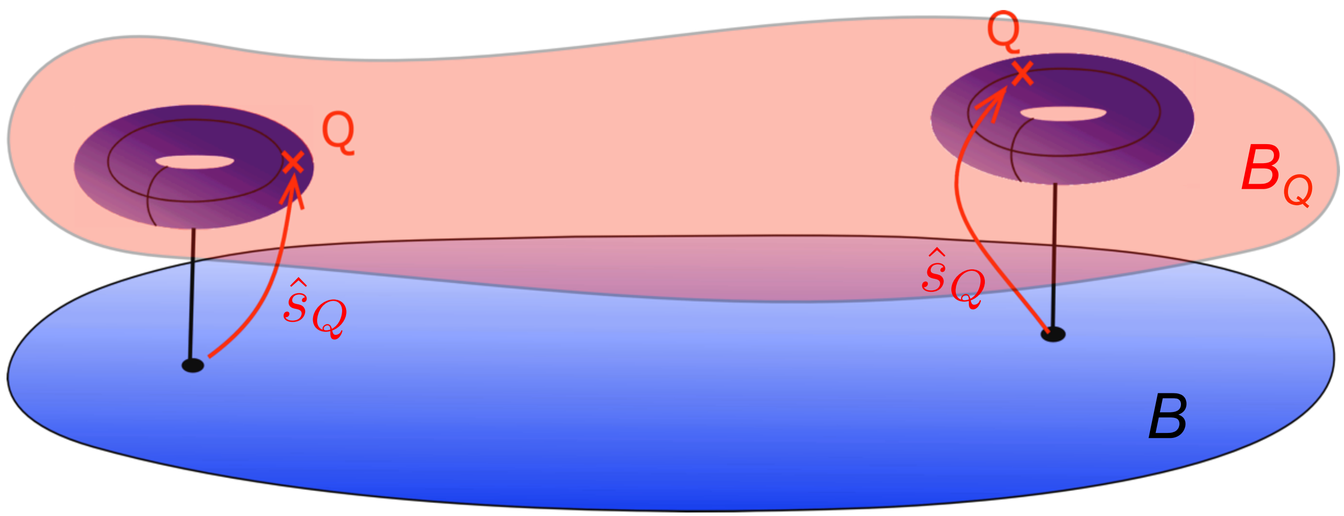
- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

- Rational points form group (addition) on E

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point **Q** induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⬌ rational section

Explicit Examples: (n+1)-rational sections – $U(1)^n$

via degree (n+1) - line bundle constr. *on elliptic curve E:*
CY one-fold on (blow-up) of $W\mathbb{P}^m$

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park'12]

n=2: with *P, Q, R* - specific example: generic CY in dP_2
[Borchmann,Mayerhofer,Palti,Weigand'13;
M.C.,Klevers,Piragua 1303.6970,1307.6425;
M.C.,Grassi,Klevers,Piragua 1306.0236]

generalization: nongeneric cubic in $\mathbb{P}^2[u:v:w]$
[M.C.,Klevers,Piragua,Taylor 1507.05954]

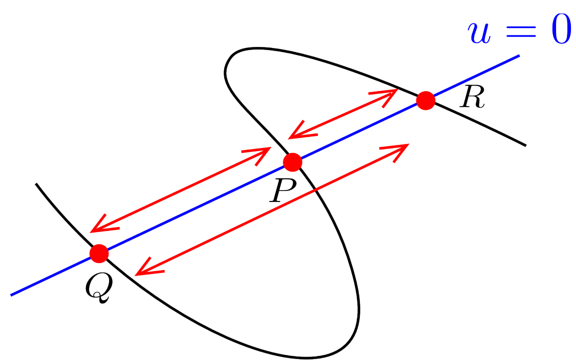
n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$ [M.C.,Klevers,Piragua,Song1310.0463]

n=4 determinantal variety in $\mathbb{P}^4$

higher *n,* not clear… ...

U(1)^2: Further Developments

General U(1)^2 construction: [M.C., Klevers, Piragua, Taylor 1507.05954]



non-generic cubic curve in $\mathbb{P}^2[u:v:w]$:

$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

$f_2(u,v,w)$ degree two polynomial in $\mathbb{P}^2[u:v:w]$

Study of non-Abelian enhancement (unHiggsing) by merging rational points P,Q,R [first symmetric rep. of SU(3)]

non-local horizontal divisors (Abelian) turn into local vertical ones (non-Abelian) →
both in geometry (w/ global resolutions) & field theory (Higgsing matter)

III. Discrete Symmetries in F-Theory

Discrete Symmetry:

Calabi-Yau geometries with genus-one fibrations

These geometries do not admit a section, but a multi-section

Earlier work: [Witten; deBoer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi;...]

Recent extensive efforts'14-'15: [Braun, Morrison; Morrison, Taylor; Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter; Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand; M.C.,Donagi,Klevers,Piragua,Poretschkin; Grimm, Pugh, Regalado]

Relation to models with U(1) symmetries via conifold transition

Geometries with n-section ⟺ Tate-Shafarevich Group Z_n

Extremely brief!

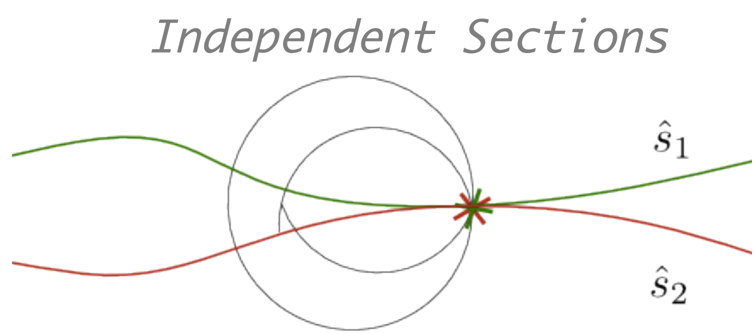
Z_2 [Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand'14]

Z_3 [M.C.,Donagi,Klevers,Piragua,Poretschkin 1502.06953]

Abelian & Discrete Gauge Symmetry in F-theory

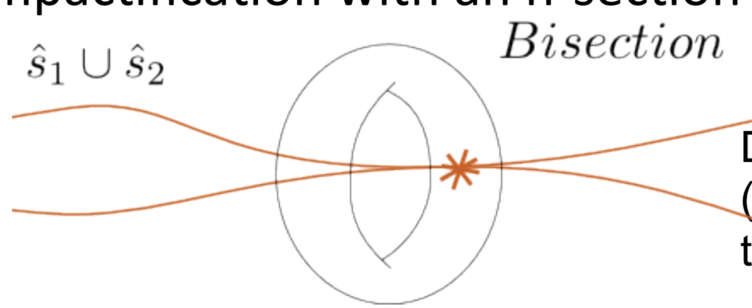
F-theory compactification with n sections (Abelian U(1)$^{(n-1)}$)

n=2 example



blow-down
(P^1 in the geometry with
multiple sections collapses)

F-theory compactification with an n-section (discrete Z$_n$ symmetry)



Deformation
(S^3 glues several sections
to a multi-section)

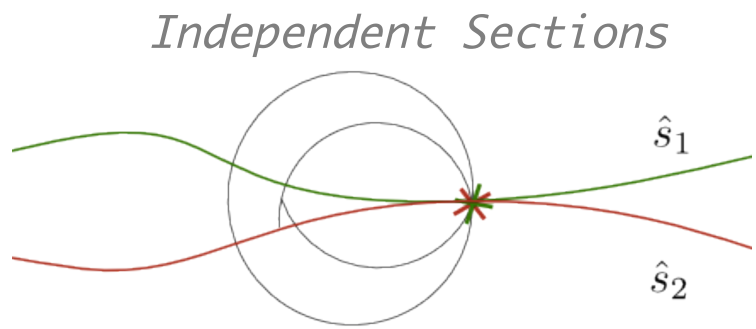
Conifold transition - Geometry

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example

Sing. codim locus

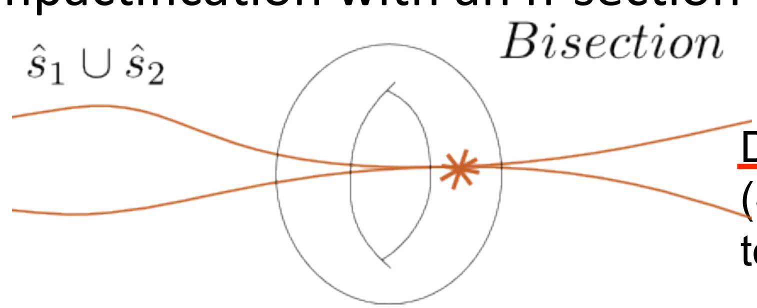


blow-down
(P^1 in the geometry with multiple sections collapses)

appearance of massless field

ϕ with charge 2

F-theory compactification with an n-section (discrete Z_n symmetry)



Deformation
(S^3 glues several sections to a multi-section)

massless field acquires VEV

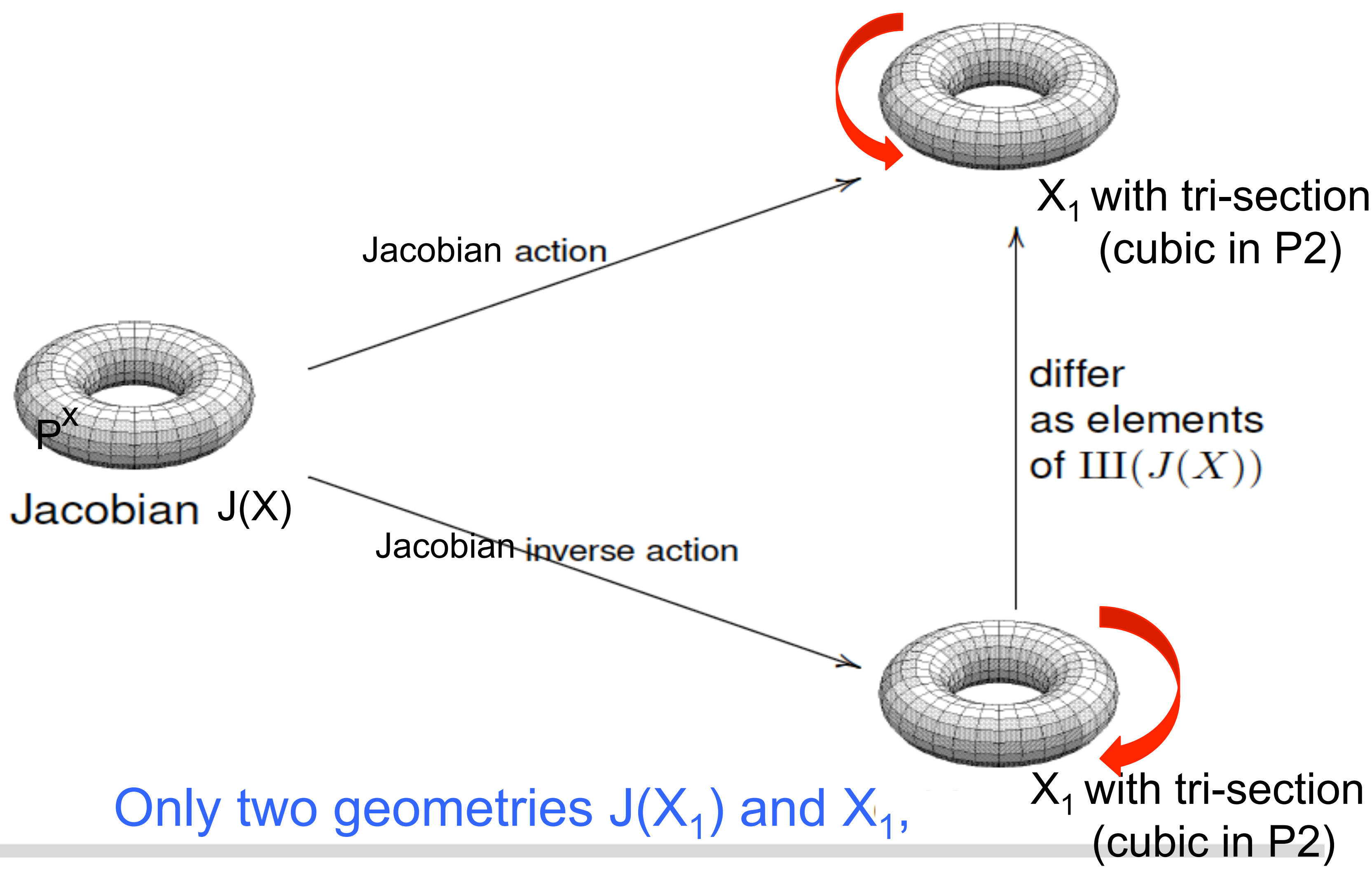
$\langle\phi\rangle \neq 0$

$U(1) \rightarrow Z_2$

Conifold transition - Effective theory

Tate-Shafarevich group and Z_3

[M.C.,Donagi,Klevers,Piragua,Poretschkin 1502.06953]



Only two geometries $J(X_1)$ and X_1,

but, three different elements of TS group!

IV. Particle Physics & F-theory

concrete examples

Initial focus: F-theory with SU(5) Grand Unification

[10 10 5 coupling,...] [Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

local [Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

global

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...

Recent progress on other Particle Physics Models:

Standard Model building blocks (via toric techniques)

[Lin,Weigand'14] SM x U(1) [1604.04292]

First Global 3-family Standard, Pati-Salam, Trinification Models
[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]
Standard Model with R-parity
[M.C., Klevers Oehlmann, Reuter 1605....]

→ highlights

Building Blocks:

i. Elliptic curve $\mathcal{C}$

Examples of constructions via toric techniques:

$\mathcal{C}_{F_i}$ as a Calabi-Yau hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$ [generalized projective spaces (blow-ups of $\mathbb{P}^2$)], associated with 16 reflexive polytops F_i:

[Klevers, Peña, Piragua, Oehlmann, Reuter'14]

$$\mathcal{C}_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

ii. Elliptically fibered Calabi-Yau space: X_{F_i}

Impose Calabi-Yau condition:
coordinates in $\mathbb{P}_{F_i}$ and coeffs. of $\mathcal{C}_{F_i}$ lifted to sections on (specific functions of) B

$$\begin{array}{ccc} \mathcal{C}_{F_i} \subset \mathbb{P}_{F_i} & \longrightarrow & X_{F_i} \\ & & \downarrow \\ & & B \end{array}$$

Fibration determined by two divisors $\mathcal{S}_7, \mathcal{S}_9$ on B

$$\begin{tikzcd} c_{mat} \arrow[r] & \boxed{\mathcal{C}_{\mathbf{R}}} \arrow[d] \\ & \Sigma_{\mathbf{R}} \end{tikzcd}$$

iii. Chiral index for D=4 matter:

$$\chi(\mathbf{R}) = \int_{\mathcal{C}_{\mathbf{R}}} G_4$$

a) construct G_4 flux by computing $H_V^{(2,2)}(\hat{X})$

b) determine matter surface $\mathcal{C}_{\mathbf{R}}$ (via resultant techniques)

[checked via M-theory duality (3D Chern-Simons terms)]

iv. Global consistency – D3 tadpole cancellation:

$$\frac{\chi(X)}{24} = n_{\mathrm{D3}} + \frac{1}{2}\int_X G_4 \wedge G_4$$

a) satisfied for integer and positive n_{D3} [and integer 3D CS terms]

→ quantization constraints on G_4 flux

b) check, all anomalies are cancelled

Model Building Strategy:

[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

i. Construction of X_{F_i} w/ Particle Physics

gauge symmetry (codim-1), matter reps. (codim-2) & Yukawas (codim-3)

$\boxed{F_{11} \text{ - Standard Model}}$ $\mathbf{SU(3) \times SU(2) \times U(1)}$ focus

Representation	$(\mathbf{3},\mathbf{2})_{1/6}$	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{1},\mathbf{1})_{-1}$

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

[hypersurface constraint in dP_4 ($\mathbb{P}^2$[u:v:w] with four blow-ups [e_1:e_2:e_3:e_4])

F_{13} - Pati-Salam Model

F_{16} - Trinification Model

Standard Model:

Base B = $\mathbb{P}^3$ Divisors in the base: $\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$

$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	$(27;16)$	–	–			
6	–	$(12;81)$	$(21;42)$	–	–		
5	–	–	$(12;57)$	$(30;8)$	–	$(3;46)$	
4	$(42;4)$	–	$(30;32)$	–	–	–	–
3	–	$(21;72)$	–	–	–	$(15;30)$	
2	$(45;16)$	$(24;79)$	$(21;66)$	$(24;44)$	$(3;64)$		
1	–	–	–	–			
0	–	–	$(12;112)$				
-1	$(36;91)$	$(33;74)$					
-2	–						

Pati-Salam Model

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
10	$(13;204)$						
9	–	$(11;140)$					
8	$(33;94)$	$(10;119)$	$(9;90)$				
7	–	$(9;100)$	$(6;77)$	$(14;48)$			
6	$(15;108)$	$(8;86)$	$(21;52)$	$(12;46)$	$(5;44)$		
5	$(6;106)$	$(35;44)$	–	$(30;16)$	–	$(3;44)$	
4	$(7;102)$	$(6;75)$	$(15;50)$	$(8;42)$	$(15;30)$	$(6;41)$	$(7;42)$
3	$(6;106)$	$(35;44)$	–	$(30;16)$	–	$(3;44)$	
2	$(15;108)$	$(8;86)$	$(21;52)$	$(12;46)$	$(5;44)$		
1	–	$(9;100)$	$(6;77)$	$(14;48)$			
0	$(33;94)$	$(10;119)$	$(9;90)$				
-1	–	$(11;140)$					
-2	$(13;204)$						

Trinification Model

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7	8	9	10
10	(5; 120)									
9	(3; 94)	(3; 94)								
8	(4; 72)	(8; 69)	(4; 72)							
7	(14; 48)	(7; 54)	(7; 54)	(14; 48)						
6	(5; 50)	(8; 44)	(3; 44)	(8; 44)	(5; 50)					
5	(5; 50)	(5; 42)	(10; 36)	(10; 36)	(5; 42)	(5; 50)				
4	(14; 48)	(8; 44)	(10; 36)	(16; 30)	(10; 36)	(8; 44)	(14; 48)			
3	(4; 72)	(7; 54)	(3; 44)	(10; 36)	(10; 36)	(3; 44)	(7; 54)	(4; 72)		
2	(3; 94)	(8; 69)	(7; 54)	(8; 44)	(5; 42)	(8; 44)	(7; 54)	(8; 69)	(3; 94)	
1	(5; 120)	(3; 94)	(4; 72)	(14; 48)	(5; 50)	(5; 50)	(14; 48)	(4; 72)	(3; 94)	(5; 120)

Yukawa Couplings (co-dimension 3 singularities)

Generically there for all gauge invariant couplings:

For the Standard Model example → μ-problem; R-parity violation

⬇

Standard Model with Z_2 symmetry

[M.C., Klevers,Oehlmann,Reuters,1605….]

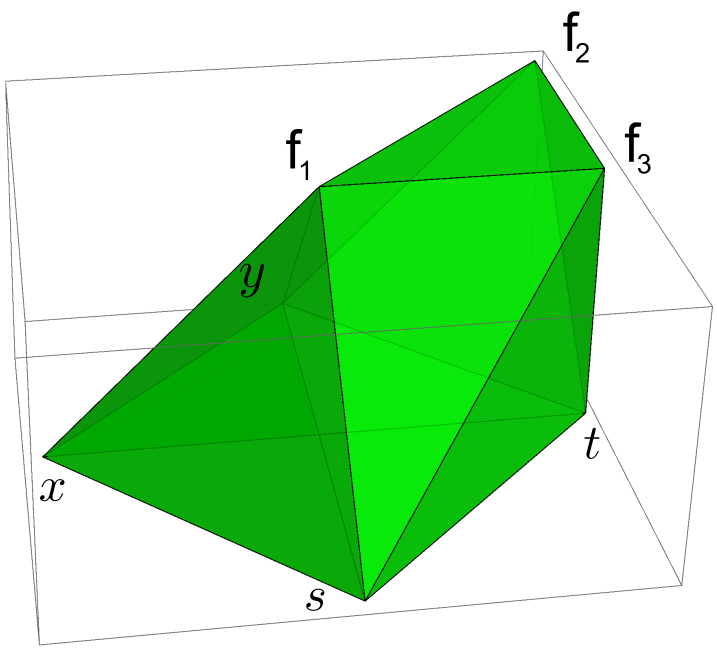
Standard Model with Z_2 symmetry

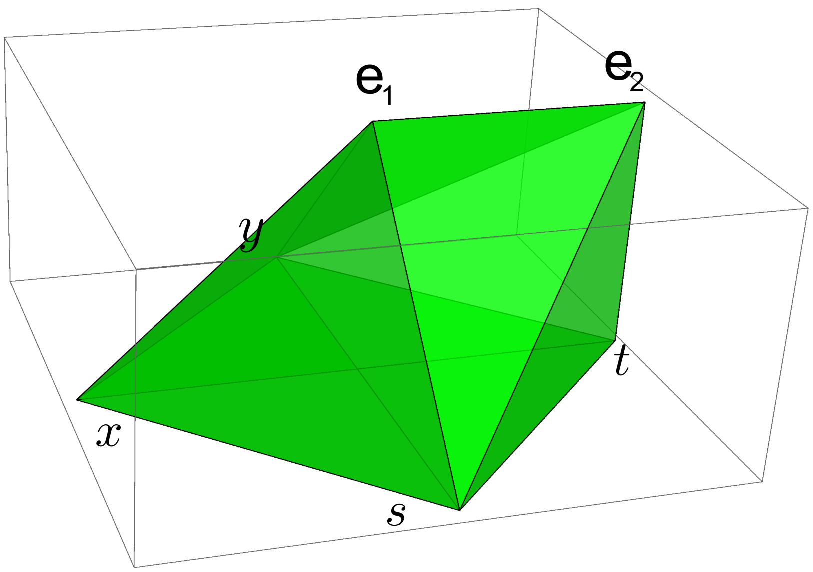
F_2: Genus one fibration

$$p_{F_2} = (b_1 y^2 + b_2 sy + b_3 s^2)x^2 + (b_5 y^2 + b_6 sy + b_7 s^2)xt + (b_8 y^2 + b_9 sy + b_{10} s^2)t^2$$

biquadric constraint in $\mathbb{P}^1 \times \mathbb{P}^1$

with U(1) x Z_2 generic symmetry & SU(2)xSU(3) by specializing b_i's





SU(3) & SU(2) tops over F_2.

Standard Model with Z_2 symmetry

Matter spectrum:

Fiber part	Matter Representation	MSSM
F_2	$(\mathbf{1},\mathbf{1})_{(0,-)}$	-
	$(\mathbf{1},\mathbf{1})_{(-1,-)}$	$\overline{E}_i$
	$(\mathbf{1},\mathbf{1})_{(-1,+)}$	-
SU(3) Top	$(\overline{\mathbf{3}},\mathbf{1})_{(-2/3,+)}$	-
	$(\overline{\mathbf{3}},\mathbf{1})_{(-2/3,-)}$	$\overline{u}_i$
	$(\overline{\mathbf{3}},\mathbf{1})_{(1/3,-)}$	$\overline{d}_i$
	$(\overline{\mathbf{3}},\mathbf{1})_{(1/3,+)}$	-
SU(2) Top	$(\mathbf{1},\mathbf{2})_{(-1/2,+)}$	H_u, H_d
	$(\mathbf{1},\mathbf{2})_{(1/2,-)}$	L_i
	$(\overline{\mathbf{3}},\mathbf{2})_{(1/6,-)}$	Q_i

Construct G_4 - chiral spectrum.
Working on tadpole constraints.

Z_2 allows for R-parity charges with no dimension 4 proton decay operators.

However, μ parameter allowed.

Detailed phenomenology may stilll be problemantic.

Summary and Outlook

- Highlights of F-theory Compactification
  Geometric perspective - discrete data:
  gauge symmetry; (chiral) matter reps and multiplicity; Yukawa couplings

- Conceptual developments:
  Abelian & Discrete Symmetries (related to respective MW & TS group)

- Construction of Particle Physics Models
  SU(5) GUT's & first examples of global
  three family Standard, Pati-Salam and Trinification models
  & Standard Model with R-parity (tip of the iceberg)

Issues: number of (Higgs) vector pairs (Chow group),
continuous data such as coupling magnitudes,…
moduli stabilization,…supersymmetry breaking,…

More work…

Thank you!

&

Happy Birthday, Fernando!