Higher Derivative Supergravity and Moduli Stabilization

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Quevedo Fest, May 2016

Motivation

- ➪ higher derivative interactions exist in string theory
  and are expected generically in any effective QFT
- ➪ in supersymmetric theories: correction to potential possible
- ➪ in general: subleading correction to two-derivative action
- ➪ **unless:** flat directions exist

  $\Rightarrow$ higher derivative corrections can be dominant contribution for

  moduli stabilization and inflation

- ➪ for early work see: **[Cecotti, Ferrara, Girardello]**
  more recently: **[Khoury, Koehn, Lehners, Ovrut, ...]**

Example in $d=4, N=1$ global supersymmetry

$$\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}_{(1)} \ , \qquad \Phi^i = A^i + \theta\chi^i + \theta^2 F^i$$

$$\mathcal{L}_{(0)} = \int d^4\theta\, K(\Phi, \Phi^\dagger) + \int d^2\theta\, W(\Phi) + \text{h.c.}$$

$$\mathcal{L}_{(1)} = \frac{1}{16} \int d^4\theta\, \mathbf{T_{ij\bar{k}\bar{l}}(\Phi, \Phi^\dagger)}\, D^\alpha\Phi^i D_\alpha\Phi^j \bar{D}_{\dot\alpha}\Phi^{\dagger\bar{k}} \bar{D}^{\dot\alpha}\Phi^{\dagger\bar{l}}$$

$\mathcal{L}_{(1)}$: special higher derivative operator in that

- induces corrections to potential
- does not introduce kinetic terms for F^i

$\Rightarrow$ recent systematic analysis by **[Ciupke]**

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$$\mathcal{L}_{\text{bos}} = -G_{i\bar{j}}\,\partial_\mu A^i\partial^\mu \bar{A}^{\bar{j}} + G_{i\bar{j}}\, F^i\bar{F}^{\bar{j}} + F^i W_i + \bar{F}^{\bar{i}}\,\bar{W}_{\bar{i}} +$$
$$\mathbf{T_{ij\bar{k}\bar{l}}(A, \bar{A})}\big[(\partial_\mu A^i\partial^\mu A^j)(\partial_\nu \bar{A}^{\bar{k}}\partial^\nu \bar{A}^{\bar{l}}) - 2F^i\bar{F}^{\bar{k}}(\partial_\mu A^j\partial^\mu \bar{A}^{\bar{l}}) + F^iF^j\bar{F}^{\bar{k}}\bar{F}^{\bar{l}}\big]$$

Note:

- $G_{i\bar{j}} = \partial_i\partial_{\bar{j}} K$ standard Kähler metric
- $\mathcal{L}_{\text{bos}}$ quartic in F $\Rightarrow$ e.o.m. cubic

$$G_{i\bar{k}}F^i + \bar{W}_{\bar{k}} + 2F^i(F^j\bar{F}^{\bar{l}} - \partial_\mu A^j\partial^\mu \bar{A}^{\bar{l}})\mathbf{T_{ij\bar{k}\bar{l}}} = 0$$

Cubic equation of motion for auxiliary F

$$G_{i\bar{k}}F^i + \bar{W}_{\bar{k}} + 2F^i(F^j\bar{F}^{\bar{l}} - \partial_\mu A^j\partial^\mu\bar{A}^{\bar{l}})\mathbf{T_{ij\bar{k}\bar{l}}} = 0 \ .$$

Solutions:

- (always) one solution analytic in $\mathbf{T}$
- all others: non-analytic in $\mathbf{T}$ — diverge as $\mathbf{T} \to 0$

  $\Rightarrow$ discard as artefact of approximation/truncation of infinite series of higher-derivative terms

  confirmed in exact solution of one-loop WZ-model

  [Buchbinder,Kuzenko,Yarevskaya,Tyler]

Aside:

kinetic terms for F can be discarded by same reasoning [Ciupke]

Analytic solution

$$F^i = -G^{i\bar{l}}\,\bar{W}_{\bar{l}} + 2\,\mathbf{T}^{\bar{\mathbf{k}}\bar{\mathbf{l}}\mathbf{ij}}\,\bar{W}_{\bar{k}}\,\bar{W}_{\bar{l}}\,W_j - 2\mathbf{T}^{\bar{\mathbf{k}}}{}_{\mathbf{j}}{}^{\mathbf{i}}{}_{\bar{\mathbf{l}}}\,(\partial_\mu A^j \partial^\mu \bar{A}^{\bar{l}})\,\bar{W}_{\bar{k}}\ .$$

Inserted into $\mathcal{L}_{\text{bos}}$ (at order $\mathcal{O}(\mathbf{T})$)

$$\mathcal{L}_{\text{bos}} = -\left(G_{i\bar{k}} + 2\mathbf{T}^{\bar{\mathbf{l}}}{}_{\mathbf{i}}{}^{\mathbf{j}}{}_{\bar{\mathbf{k}}}\,W_j\,\bar{W}_{\bar{l}}\right)\partial_\mu A^i \partial^\mu \bar{A}^{\bar{k}} + \mathbf{T}_{\mathbf{ij}\bar{\mathbf{k}}\bar{\mathbf{l}}}\,(\partial_\mu A^i \partial^\mu A^j)(\partial_\mu \bar{A}^{\bar{k}} \partial^\mu \bar{A}^{\bar{l}}) - V(A, \bar{A})$$

$$V = G^{i\bar{j}} W_i \bar{W}_{\bar{j}} - \mathbf{T}^{\mathbf{ij}\bar{\mathbf{k}}\bar{\mathbf{l}}} W_i W_j \bar{W}_{\bar{k}} \bar{W}_{\bar{l}}\ .$$

Note:

- no ghosts
- non-Kähler correction to metric & correction to V
- supersymmetric minima: $F^i = 0 = W_i \Rightarrow \langle A^i \rangle$ unchanged
- non-supersymmetric minima: $F^i \neq 0 \Rightarrow \langle A^i \rangle$ can shift

Generalization to supergravity

$$\mathcal{L}_{\text{bos}} = -\tfrac{1}{2}\mathcal{R} - \big(G_{i\bar{k}} + 2\mathrm{e}^{K}\mathbf{T^{\bar{l}}{}_{i}{}^{j}{}_{\bar{k}}}\, D_j W\, \bar{D}_{\bar{l}}\bar{W}\big)\partial_\mu A^i \partial^\mu \bar{A}^{\bar{k}}$$
$$+\, \mathbf{T_{ij\bar{k}\bar{l}}}(\partial_\mu A^i \partial^\mu A^j)(\partial_\nu \bar{A}^{\bar{k}} \partial^\nu \bar{A}^{\bar{l}}) - V(A, \bar{A})$$

where

$$V = \mathrm{e}^{K}\big(G^{i\bar{j}} D_i W \bar{D}_{\bar{j}}\bar{W} - 3|W|^2\big) \; - \mathrm{e}^{2K}\mathbf{T^{\bar{i}\bar{j}kl}}\bar{D}_{\bar{i}}\bar{W}\bar{D}_{\bar{j}}\bar{W} D_k W D_l W$$

no-scale example:

$$K = -p\ln(A + \bar{A})\ , \qquad W = \text{const.}\ , \qquad T < 0$$

minimum of V at

$$\langle A + \bar{A}\rangle = \big(-\tfrac{27}{2}p\mathbf{T}|W_0|^2\big)^{1/3}$$

Computation of $\mathbf{T}$ in CY orientifold compactifications of type IIB

strategy:

compactify $d = 10, R^4$-term and read off $\mathbf{T}$ from $\mathbf{T}(\partial A)^4$ coupling

leading order result:

$$\mathbf{T_{ij\bar{k}\bar{l}}} = \lambda(\alpha')^3 g_s^{-3/2} (\mathbf{\Pi_m \, t^m}) \, K_{(0),i} K_{(0),j} K_{(0),\bar{k}} K_{(0),\bar{l}} \ ,$$

where

$$\mathbf{\Pi_m t^m} \ = \int_{CY} c_2 \wedge J(t)$$

$\lambda =$ (uncomputed) combinatorial number, $c_2 =$ 2nd Chern class,

$\mathbf{t^m} =$ Kähler moduli of CY, $\mathbf{\Pi_m} =$ topological numbers ,

$J(t) =$ Kähler form of CY

Minimization of V

➪ **correction to V implied:**

$$V = a(\chi)\,\mathcal{V}^{-3}|W_0|^2 + b(\lambda)\,\mathcal{V}^{-4}|W_0|^4(\mathbf{\Pi_m\, t^m})\ ,$$

$W_0 =$ flux superpotential, $\quad \mathcal{V} =$ volume of CY,
a, b numerical factors

➪ **minimization:**

for $\lambda < 0,\ \chi > 0$:

- V has non-supersymmetric AdS minimum fixing all $\langle \mathbf{t^m} \rangle$ without non-perturbative W!
- $m_{\frac{3}{2}} \sim |W_0|^{-2},\ \mathcal{V} \sim |W_0|^3$
  $\Rightarrow m_{\frac{3}{2}}$ small and $\mathcal{V}$ large for large $|W_0|$
- "orthogonal" to LVS as $\chi > 0$

Conclusion/Outloolk

- ➪ application to inflation

  [Broy,Ciupke,Pedro,Westphal; Cicoli,Muia,Pedro,...]

- ➪ application to moduli stabilization in LVS

  [Cicoli,Ciupke,de Alwis,Muia, to appear]

- ➪ similar analysis for DBI-action

  [Bielleman,Ibanez,Pedro,Valenzuela,Wieck]

!!Happy Birthday Fernando!!

and thank you for your friendship and your genuine interest in my work throughout the past 30 years