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# Partially Coherent Imaging & Phase Contrast Microscopy

Colin Sheppard

Nano-Physics Department

Italian Institute of Technology (IIT)

Genoa, Italy

[colinjrsheppard@gmail.com](mailto:colinjrsheppard@gmail.com)

# Perfect imaging

Object amplitude transmission

$$t(x, y) = a(x, y)e^{i\phi(x, y)}$$

$a(x, y)$  is modulus (amplitude), real

$\phi(x, y)$  is phase, real

Perfect image

$$I(x, y) = \left| a(x, y)e^{i\phi(x, y)} \right|^2 = a^2(x, y)$$

- No phase information in perfect image!

To see phase, need to have imperfect imaging system:

- Introduce phase (aberration)
- Introduce asymmetry

# 3 main methods of phase contrast

- Complex Pupil Function
  - Zernike phase contrast
  - Defocus
  - Transport of intensity equation (TIE)
- Phase gradient methods (asymmetry)
  - Schlieren
  - Hoffmann modulation contrast
  - Differential phase contrast (DPC)
  - Wavefront sensing (Shack-Hartmann)
  - Differential Interference Contrast (DIC)
- Interference methods
  - Interference microscopy
  - Digital holographic microscopy (DHM)

# Coherent vs. partially coherent

- 1 Coherent methods (Digital holographic microscopy)
  - Spatial frequencies only on Ewald sphere
  - Limited 3D imaging performance
  - But can get good 3D by holographic tomography
  - Limited spatial resolution
  - Speckle
  - Can reconstruct with Rytov approximation
  
- 2 Partially coherent methods
  - Improved image bandwidth
  - No speckle
  - More difficult to extract quantitative information

# Partially coherent image formation

Propagate mutual intensity through the system:

Image intensity

$$I'(u', v') = \left(\frac{1}{2\pi}\right)^2 \sum_m \sum_n \sum_p \sum_q C(m, n; p, q) a(m, n) a^*(p, q) e^{i[(m-p)u' + (n-q)v']}, \quad (17)$$

and (15) becomes

$$C(m, n; p, q) = 2\pi \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \gamma(x, y) f(x+m, y+n) f^*(x+p, y+q) dx dy. \quad (18)$$

↑
↑  
 source      pupil

$C(m, n; p, q)$  = transmission cross-coefficient (TCC)

$m, p$  are both spatial frequencies in  $x$  direction

$n, q$  are both spatial frequencies in  $y$  direction

- System and object separated.
- Although Hopkins propagated mutual intensity, he did not give mutual intensity of the image.

On the diffraction theory of optical images

BY H. H. HOPKINS

*Proc. R. Soc. Lond. A* **217**, 408 (1953)

# Imaging in a partially-coherent microscope

For non-periodic objects, replace sums by integrals:

$C$  = transmission cross-coefficient (TCC)

object spectrum

$$I(x_s, y_s) = \text{const.} \int \int \int \int_{-\infty}^{+\infty} C(m, n; p, q) T_0(m, n) T_0^*(p, q) \exp(2\pi j\{(m-p)x_s + (n-q)y_s\}) dm dn dp dq,$$

Conventional

$$C(m, n; p, q) = \int \int_{-\infty}^{+\infty} P_2(x, y) P_1(x + \tilde{m}, y + \tilde{n}) P_1^*(x + \tilde{p}, y + \tilde{q}) dx dy,$$

condenser    objective

Confocal microscope:

$$C(m, n; p, q) = \{P_1(\tilde{m}, \tilde{n}) \otimes P_2(\tilde{m}, \tilde{n})\} \{P_1^*(\tilde{p}, \tilde{q}) \otimes P_2^*(\tilde{p}, \tilde{q})\}$$

C. J. R. SHEPPARD and A. CHOUDHURY

**Image formation in the scanning microscope**

OPTICA ACTA, 1977, VOL. 24, NO. 10, 1051-1073

# Generalization of coherent imaging

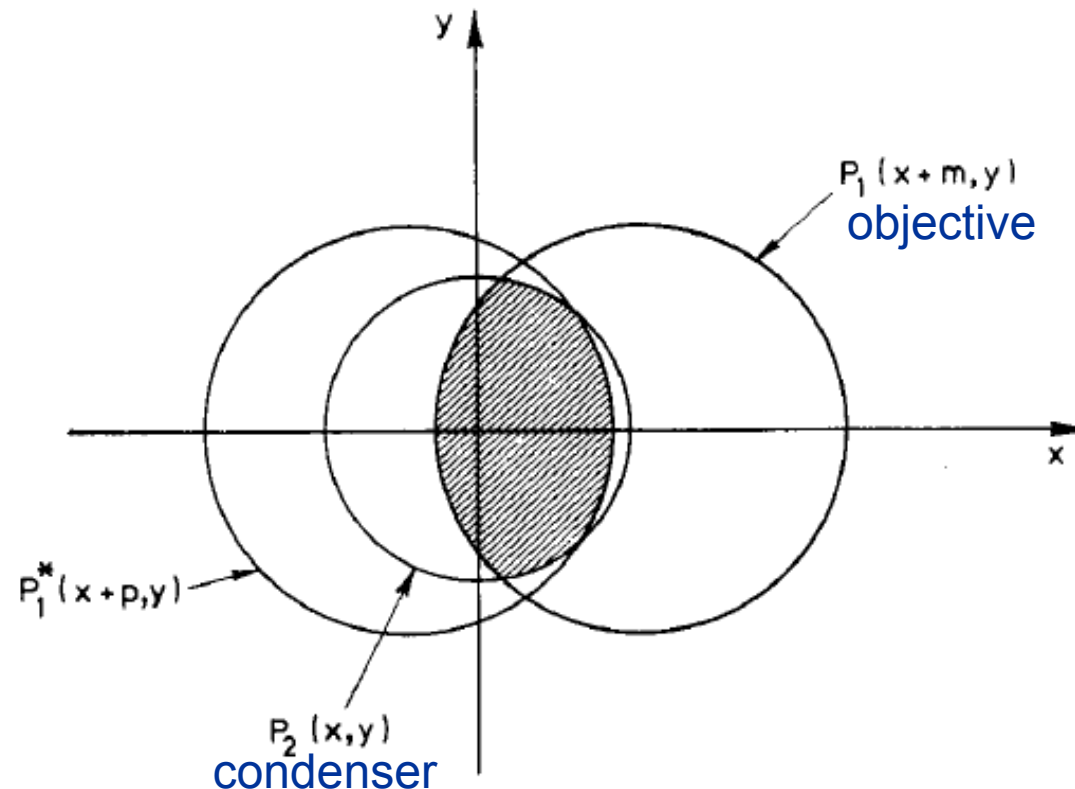


The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.

$$I(x) = \left| \int c(m)T(m) \exp[-2\pi imx] dm \right|^2$$
$$= \iint \underbrace{c(m_1)c(m_2)} T(m_1)T^*(m_2) \exp[-2\pi i(m_1 - m_2)x] dm_1 dm_2$$

For partially coherent,  $C(m_1, n_1; m_2, n_2)$  does not separate

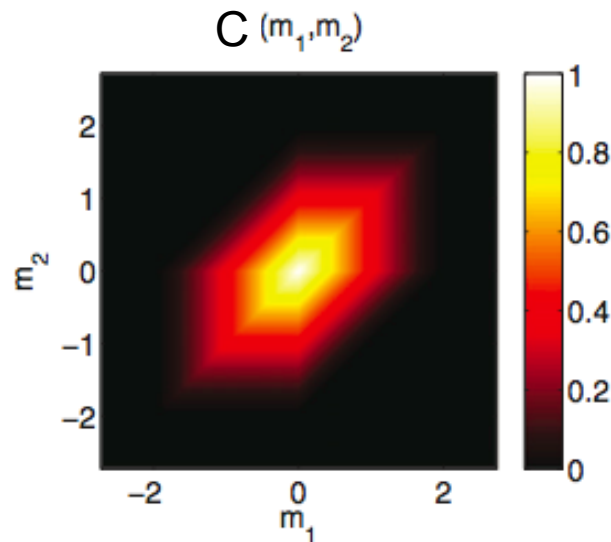
# $C(m, 0; p, 0)$ as area of overlap of three circles (conventional system)



C. J. R. SHEPPARD and A. CHOUDHURY  
**Image formation in the scanning microscope**  
OPTICA ACTA, 1977, VOL. 24, NO. 10, 1051-1073



# Transmission cross coefficient (TCC)



348 OPTICS LETTERS / Vol. 35, No. 3 / February 1, 2010

## Phase-space representation of partially coherent imaging systems using the Cohen class distribution

$$S = 1$$

$S$  is coherence ratio ( $NA_{\text{cond}}/NA_{\text{obj}}$ )

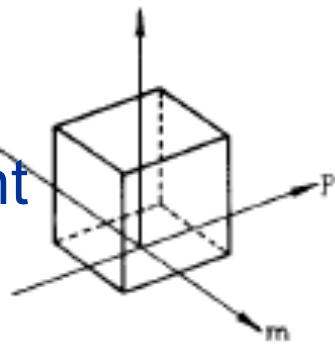
Shalin B. Mehta<sup>1,2,\*</sup> and Colin J. R. Sheppard<sup>1,2,3</sup>

Using the phase-space imager to analyze partially coherent imaging systems:  
bright-field, phase contrast, differential interference contrast, differential  
phase contrast, and spiral phase contrast

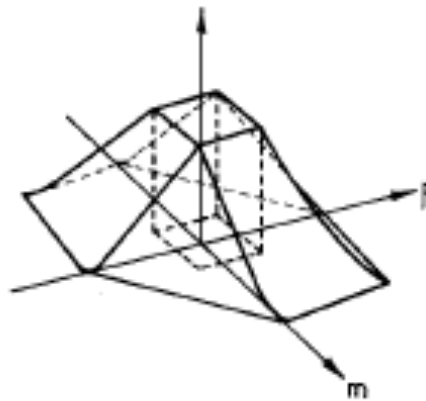
*J. Modern Optics* **57**, 718-739 (2010)

Shalin B. Mehta<sup>a,b\*</sup> and Colin J.R. Sheppard<sup>a,b,c</sup>

$S = 0$   
coherent



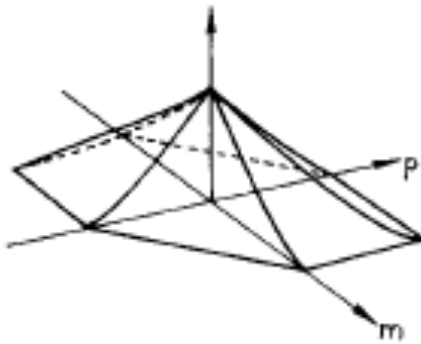
(a)  $a_2 = 0$



(b)  $a_2 < a_1$

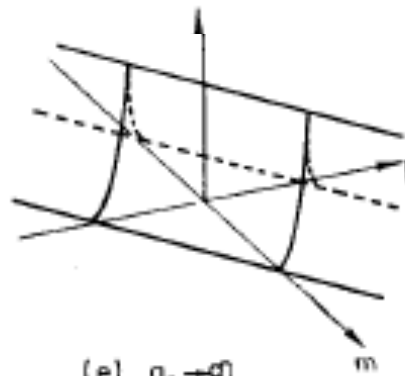
$C(m; p)$   
(conventional)

$S = 1$



(c)  $a_2 = a_1$

full, complete,  
or matched  
illumination



(e)  $a_2 \rightarrow \infty$

incoherent

$S \rightarrow \infty$

NB  $m, p$  are spatial frequencies  
both in the  $x$  direction

# Introduce central and difference coordinates



The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.

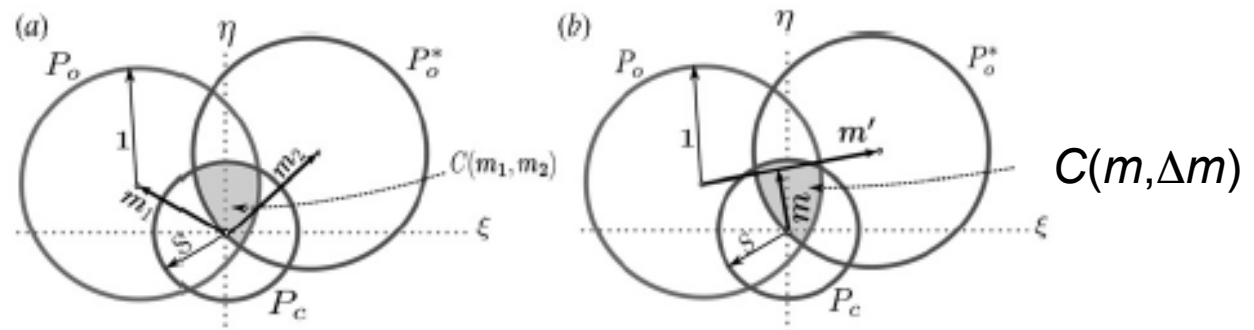
Introduce central and difference coordinates

$$x_1 = x + \Delta x / 2, x_2 = x - \Delta x / 2;$$

$$m_1 = m + \Delta m / 2, m_2 = m - \Delta m / 2$$

$$\Delta m x + m \Delta x = m_1 x_1 - m_2 x_2$$

$$dm_1 dm_2 = dm d\Delta m$$

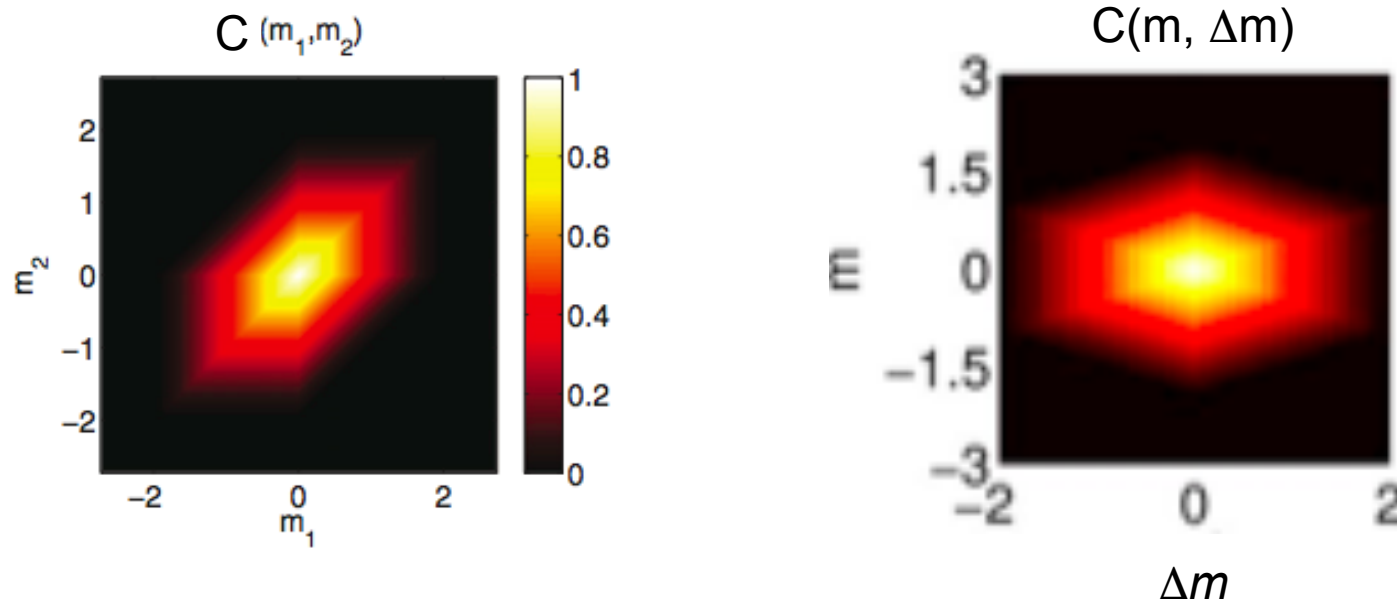


TCC

Transmission cross-coefficient

Area of overlap of source and 2 displaced pupils

# Transmission cross coefficient (TCC)



348 OPTICS LETTERS / Vol. 35, No. 3 / February 1, 2010

## Phase-space representation of partially coherent imaging systems using the Cohen class distribution

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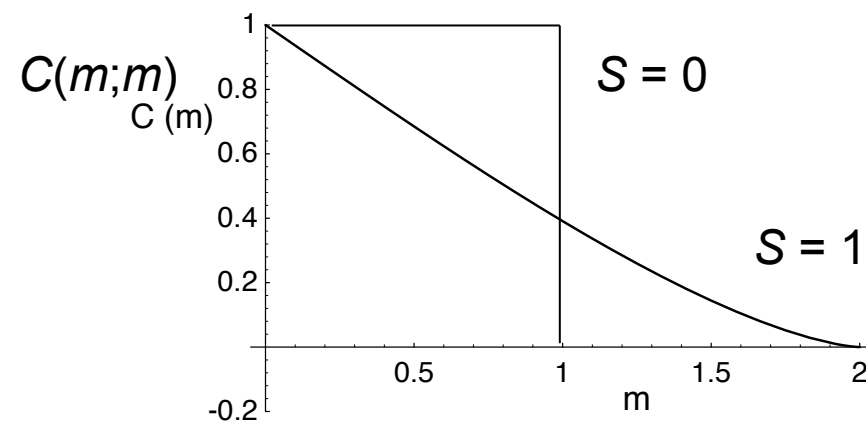
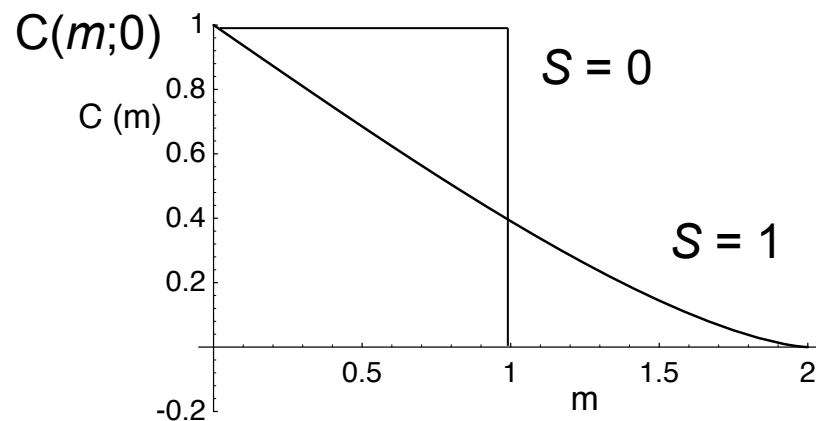
*J. Modern Optics* **57**, 718-739 (2010)

Shalin B. Mehta<sup>a,b,\*</sup> and Colin J.R. Sheppard<sup>a,b,c</sup>

# WOTF $C(m;0)$ and PGTF $C(m;m)$ for conventional microscope

Partially coherent imaging is complicated, but it becomes simpler for two special cases:

- Weak object (neglect interference of scattered light with scattered light)  
Can use if first Born approximation is satisfied.  
(But not necessarily the inverse)
- Slowly varying phase gradient  
Can use if Rytov approximation is valid



Weak object transfer function (**WOTF**)    Phase gradient transfer function (**PGTF**)

$S$  is coherence ratio ( $NA_{\text{cond}}/NA_{\text{obj}}$ )

# Weak object

$$t(x, y) = e^{b(x, y)} \quad b(x, y) \text{ complex}$$

- Weak object

$$t(x, y) \approx 1 + b(x, y)$$

Spectrum

$$T(m, n) = \delta(m)\delta(n) + B(m, n)$$

- $B$  is skew-Hermitian if  $b$  is imaginary

# Weak object transfer function (WOTF)

- Weak object ( $b$  is complex)

$$t(x) = 1 + b \cos 2\pi\nu x,$$

- For even  $C$ :

$$I(x_s) = 1 + 2 \operatorname{Re} \{ b C(\nu; 0) \} \cos 2\pi\nu x_s.$$

- Weak object transfer function (WOTF)
- Phase imaged by imaginary part of  $C(\nu; 0)$

**Fourier Imaging of Phase Information in Scanning and Conventional Optical Microscopes**

C. J. R. Sheppard; T. Wilson

*Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Volume 295, Issue 1415 (Feb. 7, 1980), 513-536.

# Weak phase object

- An Hermitian transfer function does not give contrast from a weak phase object
- Make pupil either
  - complex
  - asymmetric



# Defocus

- Earliest method of phase contrast
- Like Zernike, based on changing the phase of the signal
- Only works for a weak object
- Contrast opposite for different defocus directions
- Relative condenser aperture  $S$  cannot be too large
- For a coherent system,  
 $S = 0$ ,  $\arg[P(\rho)] = u\rho^2/2$ , so  $\arg[c(m)] = um^2/2$

# Defocused WOTF

$l$  is radial spatial frequency,  $l = (m^2+n^2)^{1/2}$

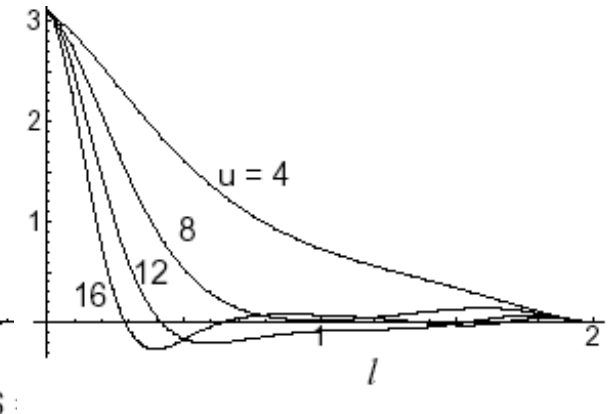
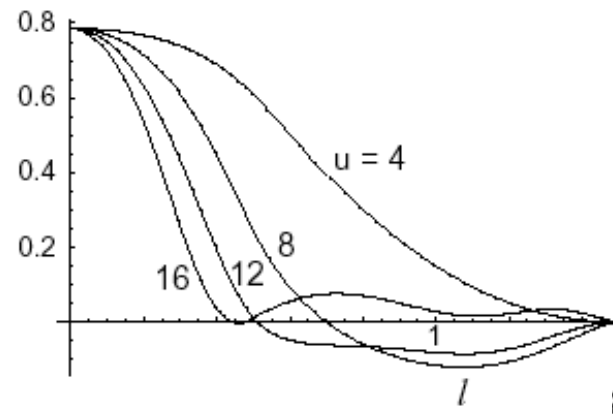
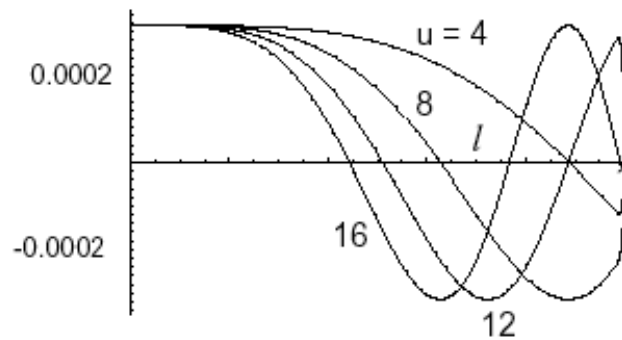
S=0.01

S=0.5

S=0.99

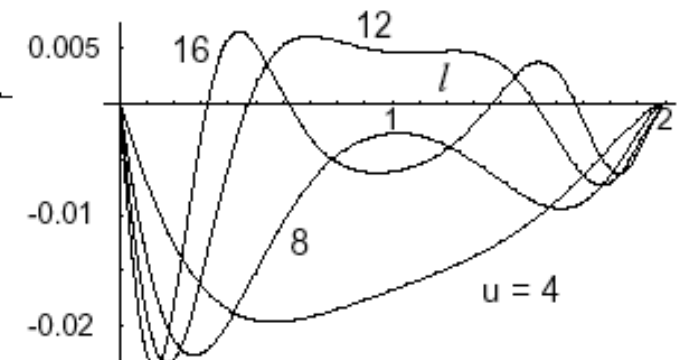
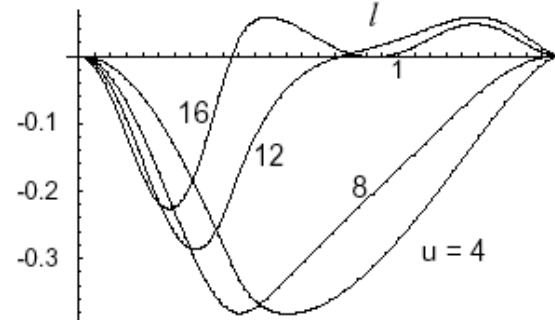
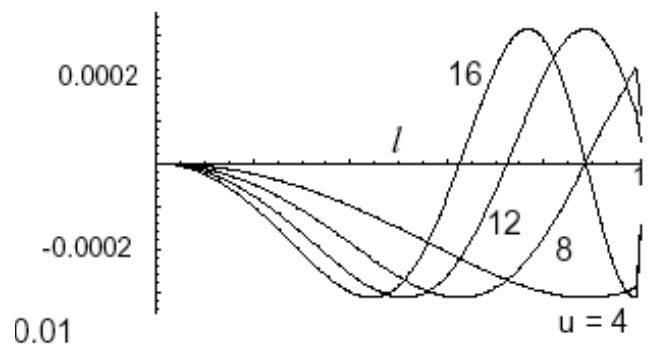
Real part

$C_{Wr}(l; u)$



Imaginary part

$C_{Wi}(l; u)$



Sheppard CJR

Defocused transfer function for a partially coherent microscope, and application to phase retrieval

*J. Opt. Soc. Am. A*, **21**, 828-831(2004)

# Small defocus: analytic expression

$$C_w = -\frac{1}{2} \pi u l^2 S^2, \quad 0 \leq l \leq 1 - S,$$

$$C_w = -\frac{u l^2}{2} \left[ S^2 \arccos\left(\frac{l^2 + S^2 - 1}{2lS}\right) - \arccos\left(\frac{l^2 - S^2 + 1}{2l}\right) \right]$$

$$-\frac{u}{6l} \left\{ \left[ S^2 - \left(\frac{l^2 + S^2 - 1}{2l}\right)^2 \right]^{1/2} \left( (1 - S^2)^2 - \frac{l^2}{2} (1 + l^2 + 7S^2) \right) \right. \quad 1 - S \leq l \leq 1 + S.$$

$$\left. - \left[ 1 - \left(\frac{l^2 - S^2 + 1}{2l}\right)^2 \right]^{1/2} \left( (1 - S^2)^2 - \frac{l^2}{2} (7 + l^2 + S^2) \right) \right\},$$

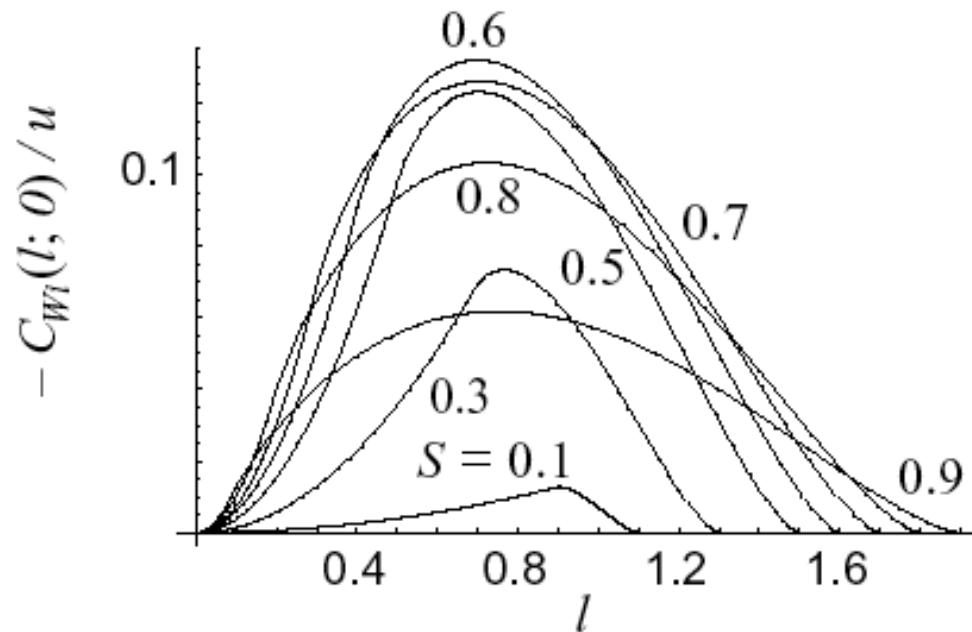
Sheppard CJR

Defocused transfer function for a partially coherent microscope, and application to phase retrieval

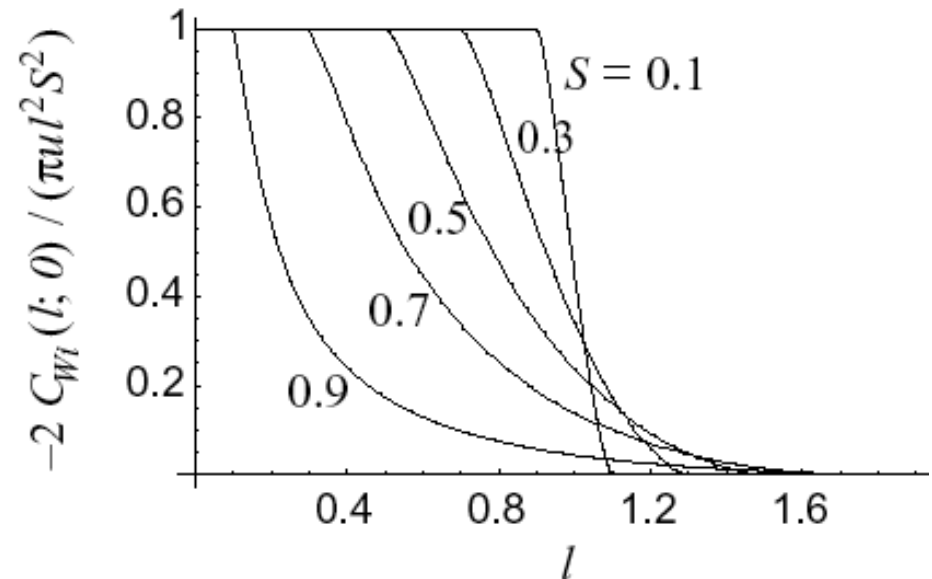
*J. Opt. Soc. Am. A*, **21**, 828-831(2004)

# WOTF for phase contrast image

$$I(\Delta u) - I(-\Delta u)$$



Inverse Laplacian:  
Phase restored up to  $l = 1 - S$

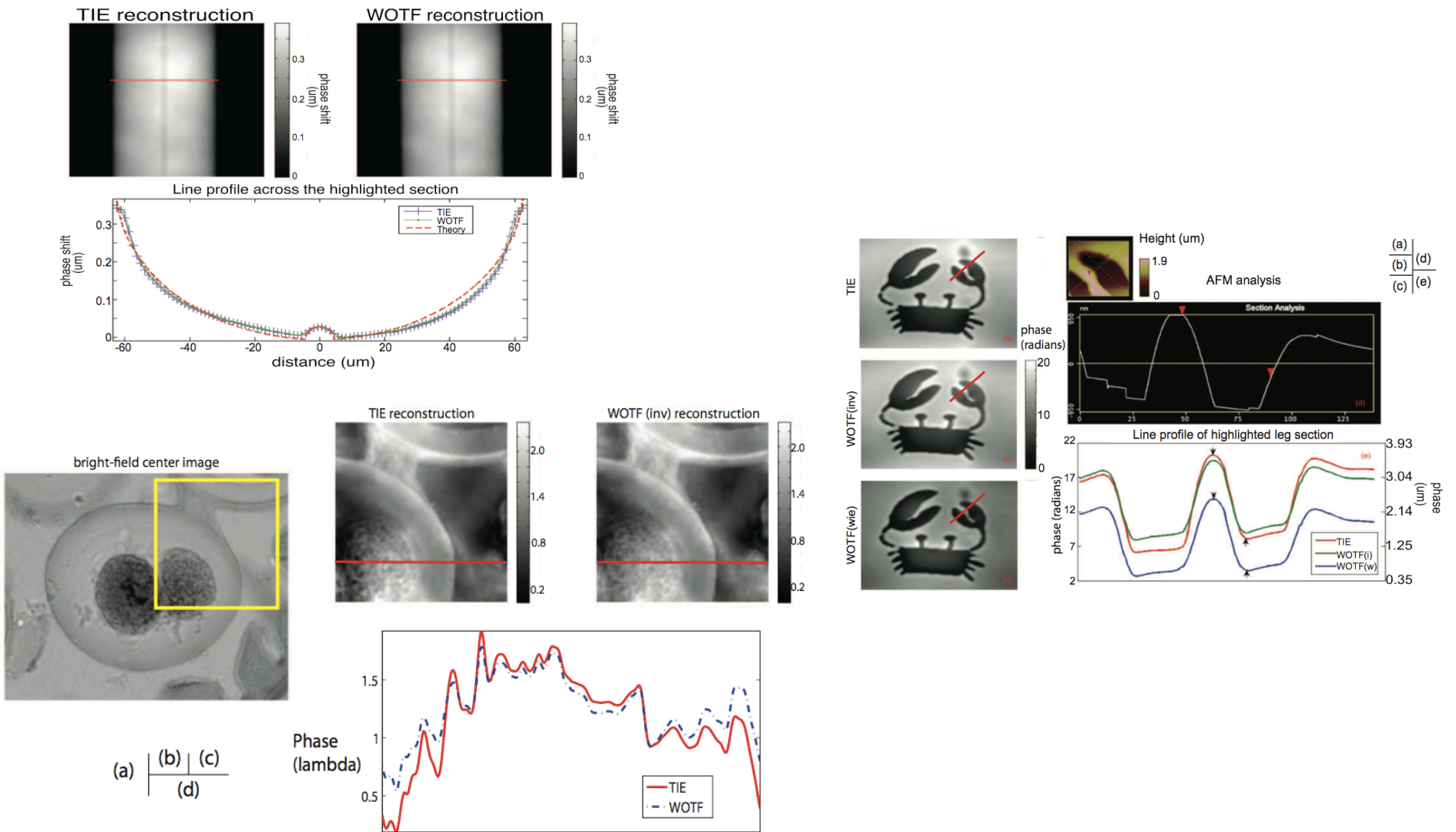


Parabolic for small  $l$

- Or use Wiener filter

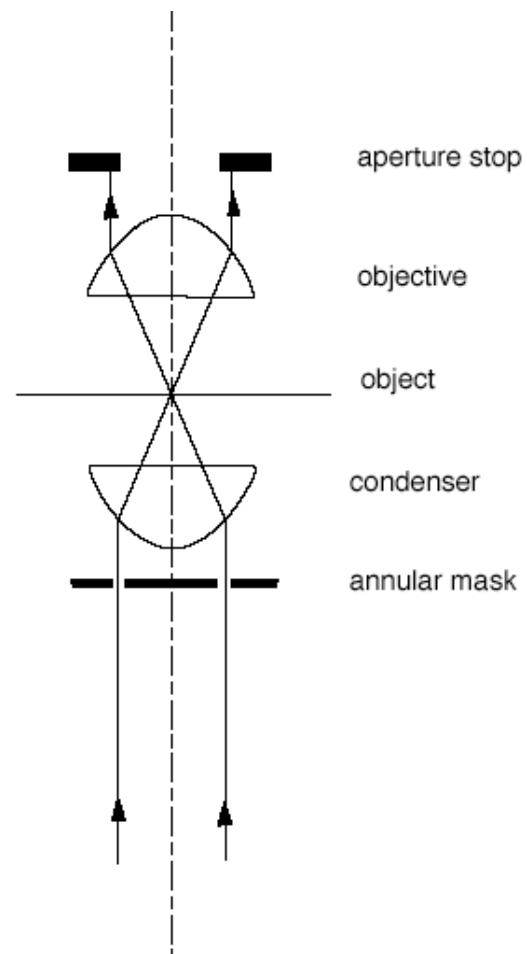
Sheppard CJR  
Defocused transfer function for a partially coherent  
microscope, and application to phase retrieval  
*J. Opt. Soc. Am. A*, **21**, 828-831(2004)

# Phase measurement using WOTF



Kou SS, Waller L, Barbastathis G, Marquet P, Depeursinge C, Sheppard CJR  
 Quantitative phase restoration by direct inversion using the optical transfer function,  
*Opt. Lett.* **36**, 2671-2673 (2011).

# Dark field microscope



- Direct light blocked
- Partially-coherent imaging

*Encyclopedia of Modern Optics*,  
RD Guenther, DG Steel, L Bayvel, eds,  
Elsevier, Oxford, **3**, pp. 103-110

## Phase Contrast Microscopy

**C J R Sheppard**, National University of Singapore,  
Singapore

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# Dark field

Weak phase object:

$$t(x, y) = 1 + id \cos(2\pi\nu x)$$

$d$  is a real constant

Object spectrum:

$$T(m, n) = \left[ \delta(m) + i\frac{d}{2}\delta(m - \nu) + i\frac{d}{2}\delta(m + \nu) \right] \delta(n)$$

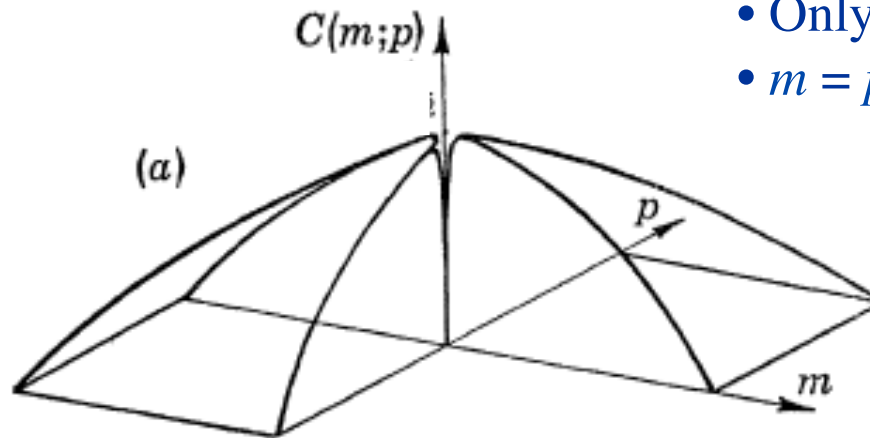
Dark field, no direct light:  $C(0, 0; 0, 0) = 0$ ,  $C(\nu, 0; 0, 0) = 0$

So only terms in  $C(\nu, \nu; 0, 0)$  and  $C(\nu, -\nu; 0, 0)$

$$I(x, y) = \frac{1}{2} d^2 \left[ C(\nu, \nu; 0, 0) + C(\nu, -\nu; 0, 0) \cos\left(\frac{4\pi\nu x}{M}\right) \right]$$

- Zero for annular dark field system
- Therefore no contrast for a single spatial frequency component

# Dark field



- Only difference frequencies imaged
- $m = p$  gives  $m - p = 0$

- Sum frequencies  $(m +ve, p -ve)$  not imaged

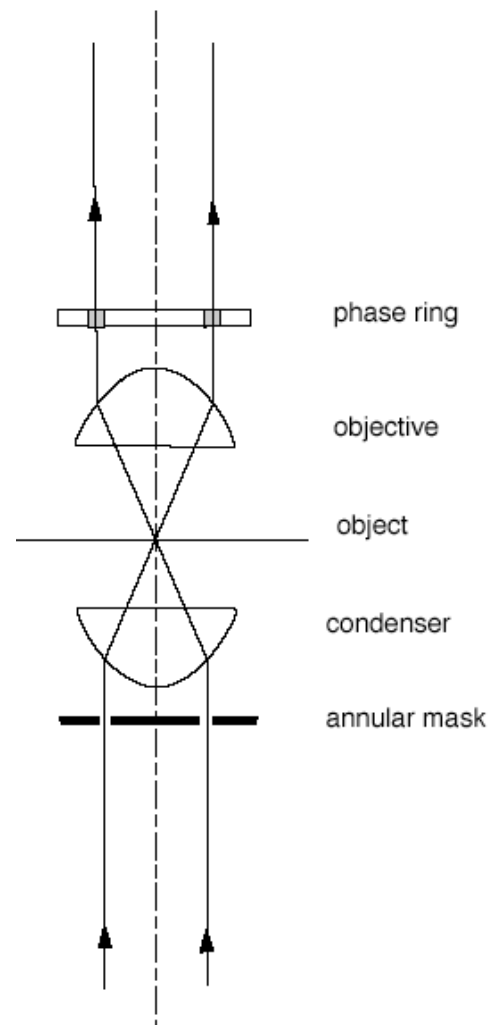
**Fourier Imaging of Phase Information in Scanning and Conventional Optical Microscopes**

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*Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Volume 295, Issue 1415 (Feb. 7, 1980), 513-536.



# Zernike phase contrast



F. Zernike, "Phase contrast, a new method for the microscopic observation of transparent object," *Physica* **9**, 686-693 (1942).

- Direct light changed in phase
- Partially-coherent imaging
- Direct light on annular cone increases resolution

*Encyclopedia of Modern Optics*,  
 RD Guenther, DG Steel, L Bayvel, eds,  
 Elsevier, Oxford, **3**, pp. 103-110

## Phase Contrast Microscopy

**C J R Sheppard**, National University of Singapore,  
 Singapore

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# Zernike phase contrast

Weak phase object

$$t(x, y) = 1 + id \cos(2\pi\nu x)$$

Object spectrum

$$T(m, n) = \left[ \delta(m) + i\frac{d}{2}\delta(m - \nu) + i\frac{d}{2}\delta(m + \nu) \right] \delta(n)$$

Phase imaging from imaginary part of  $C$ :

$$I(x, y) = 1 - dC_i(\nu, 0; 0, 0) \cos\left(\frac{2\pi\nu x}{M}\right)$$

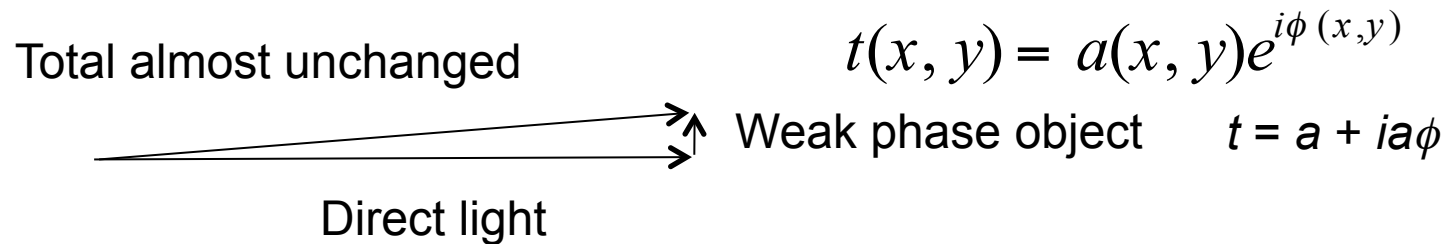
If  $c$  is the amplitude transmittance of the phase ring:

$$I(x, y) = 1 \pm \frac{2d}{c} C(\nu, 0; 0, 0) \cos\left(\frac{2\pi\nu x}{M}\right)$$

- Phase contrast is amplified

# Weak phase object, $\phi$ small

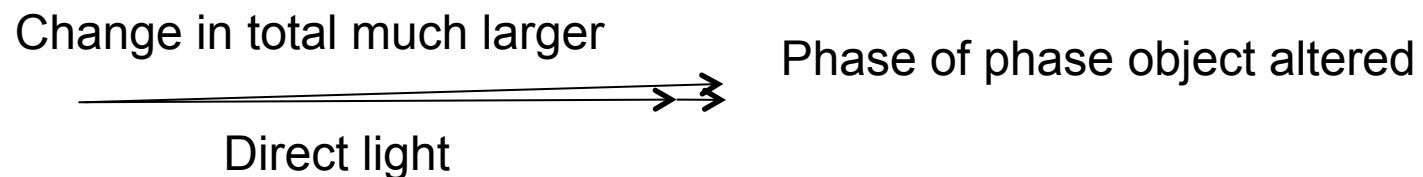
## Brightfield



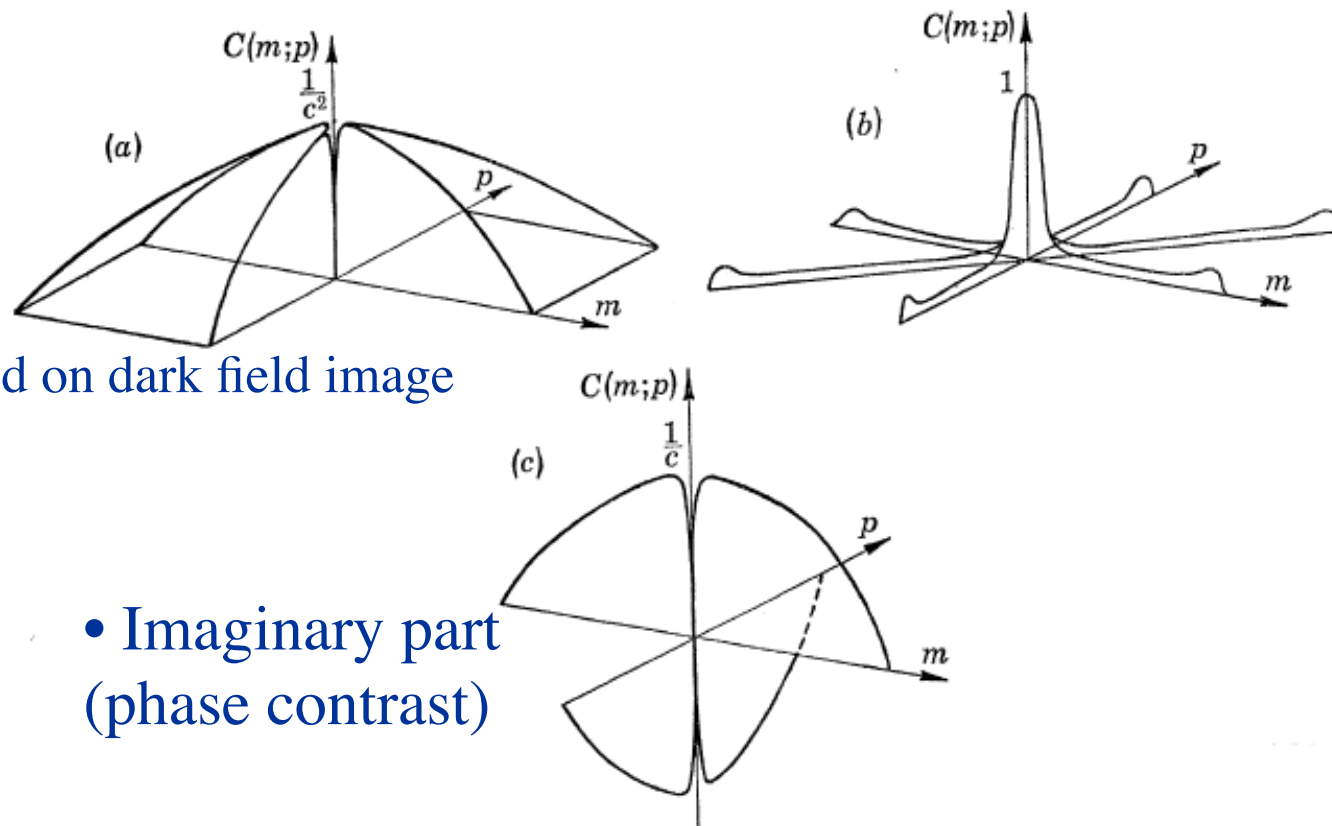
## Darkfield

↑ Weak phase object  
No direct light to reduce contrast

## Zernike



# Zernike phase contrast



- Superposed on dark field image

- Imaginary part  
(phase contrast)

FIGURE 11. Transfer function for conventional Zernike phase contrast.

**Fourier Imaging of Phase Information in Scanning and Conventional Optical Microscopes**

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# Zernike phase contrast

- Phase contrast amplified by transmittance of phase ring
- Can make +ve or –ve phase contrast from phase of phase ring
- Difficult to get quantitative information
- Haloes around phase changes

# Defocus

- Earliest method of phase contrast
- Like Zernike, based on changing the phase of the signal
- Only works for a weak object
- Contrast opposite for different defocus directions
- Relative condenser aperture  $S$  cannot be too large
- For a coherent system,  
 $S = 0$ ,  $\arg[P(\rho)] = u\rho^2/2$ , so  $\arg[c(m)] = um^2/2$

# Transport of intensity equation (TIE)

- Teague, *JOSA A* 1434, **73** (1983)
- Streibl, *Opt. Commun.* 6, **49** (1985)
- Barty, Nugent, Paganin, Roberts, *Opt. Lett.* 817, **23** (1998)

Amplitude in image space satisfies paraxial wave equation

$$k \frac{\partial I}{\partial z} = -\nabla_T \cdot (I \nabla_T \phi)$$

$$k \frac{\partial \ln I}{\partial z} = -\nabla_T^2 \phi - \nabla_T \ln I \cdot \nabla_T \phi$$

↑  
often small

- Similar to eikonal equation
- Wavefront curvature sensing

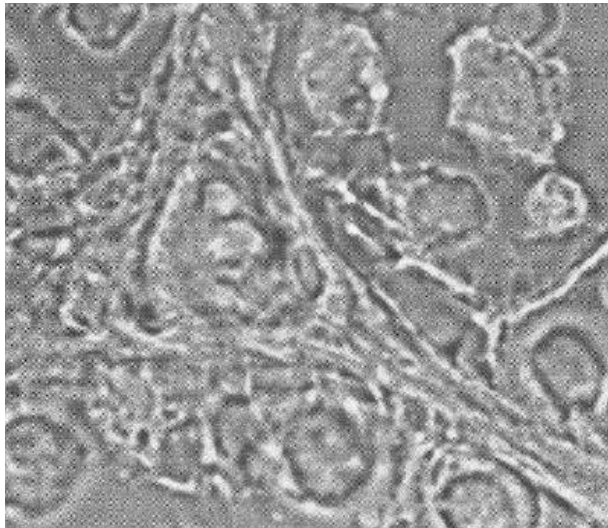
# Phase changes intensity



Photo:  
Miguel Porras



# Logarithmic derivative image

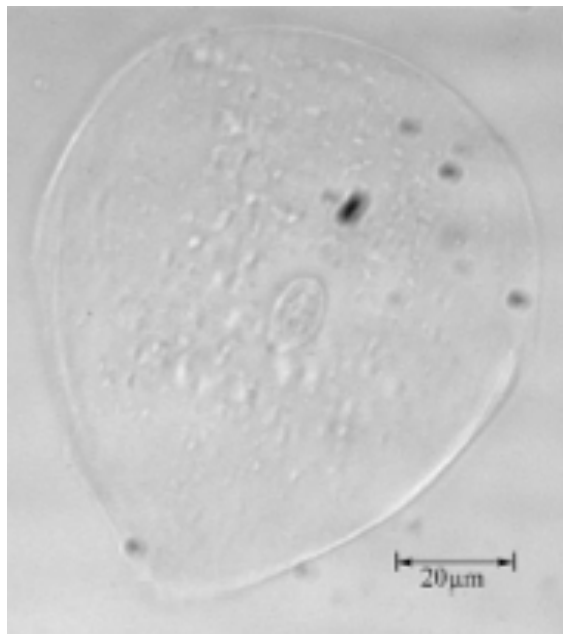


$$\frac{\partial I}{\partial z} = -\nabla_T \cdot (I \nabla_T \phi)$$

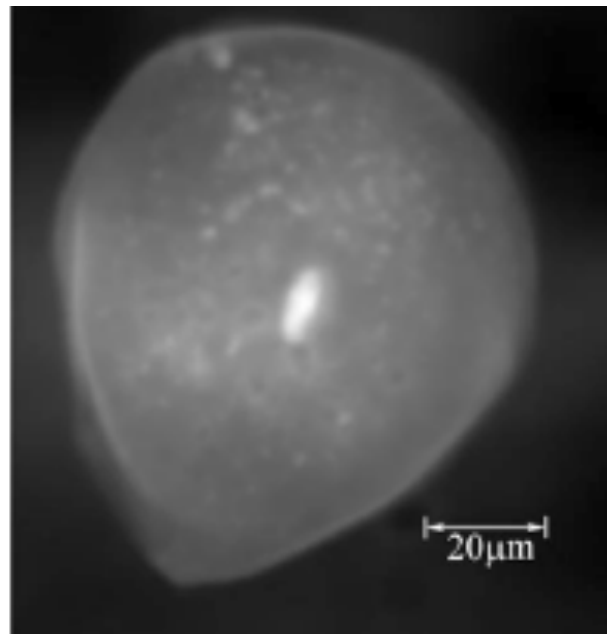
Logarithmic derivative:

$$\frac{\partial \ln I}{\partial z} = -\nabla_T^2 \phi - \nabla_T \ln I \cdot \nabla_T \phi$$

Testicle of rat, Streibl, *Opt. Commun.* **6**, 49 (1984)



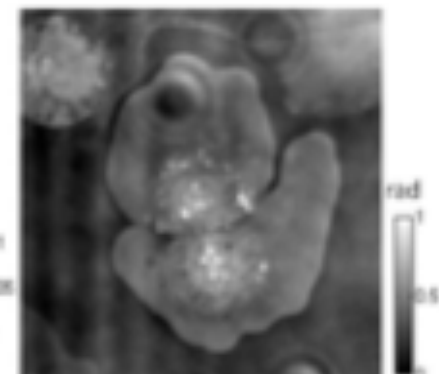
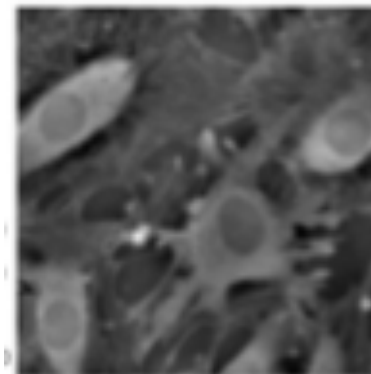
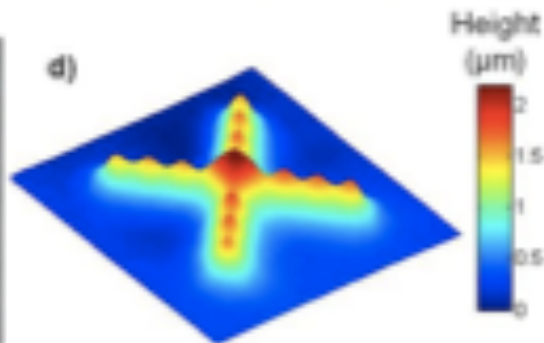
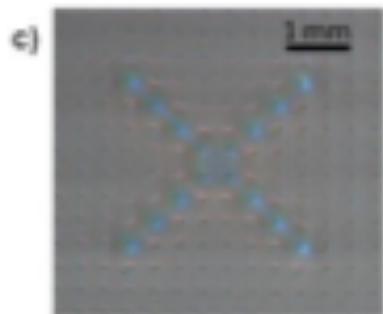
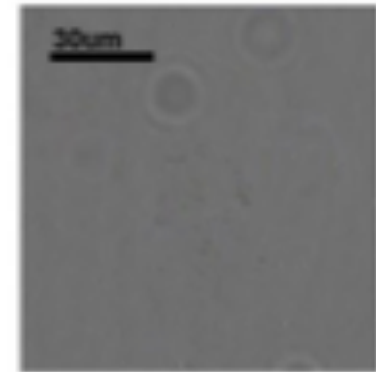
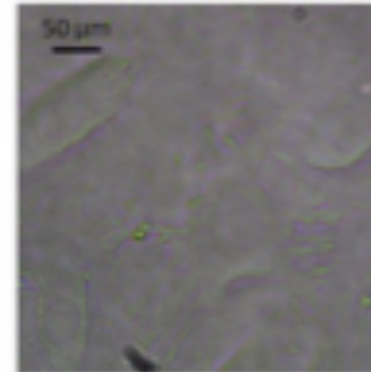
DIC



Barty, Nugent, Paganin,  
Roberts  
*Opt. Lett.* **23**, 817 (1998)

TIE phase image

# TIE from colour (single shot)



HMVEC cells

HeLa cells

## Phase from chromatic aberrations

Laura Waller,<sup>1,5,\*</sup> Shan Shan Kou,<sup>3,6</sup> Colin J. R. Sheppard,<sup>3</sup> and George Barbastathis<sup>2,4</sup>

· 2010 / Vol. 18, No. 22 / OPTICS EXPRESS 22817

# Quantitative phase imaging by TIE

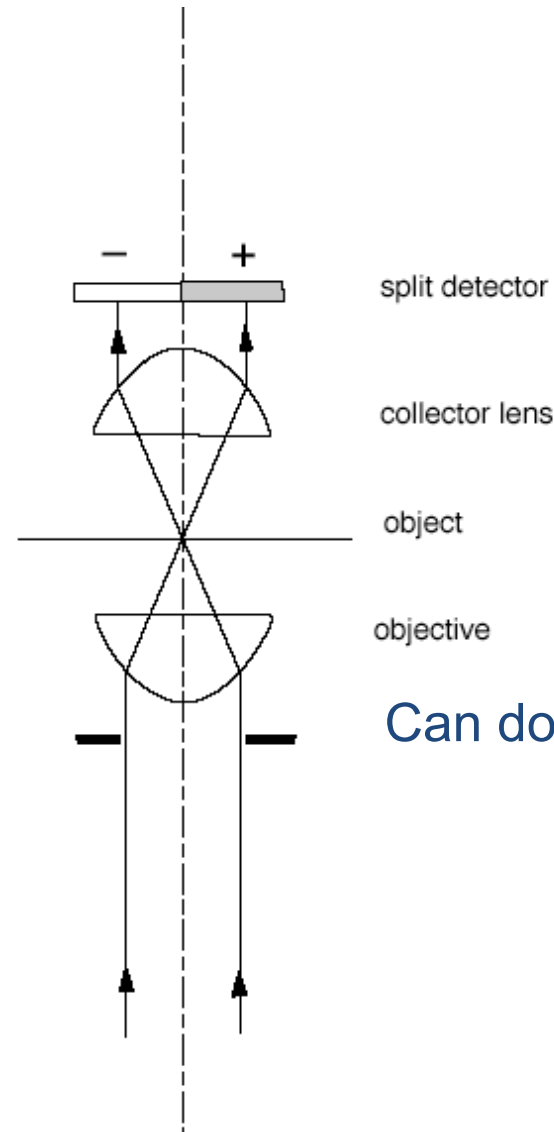
- IATIA system: measure  $\phi$  using TIE equation
- Can then simulate Zernike, DIC, etc. images

# Properties of TIE imaging

- Similar to defocus method for weak object, but not limited to weak phase
- Weak signal from low spatial frequencies
- $\Delta z$  small to approximate  $\partial/\partial z$ : weak signal
- Measures phase of image not object
- Not enough information to directly recover object phase for strong object
- Problem with 3D imaging:  
Measure  $\partial I / \partial z$  so no information on zero axial spatial frequency

Sheppard CJR (2002) Three-dimensional phase imaging with the intensity transport equation, *Appl. Opt.* 41, 5951-5955.

# Differential phase contrast (DPC)



Can do simply in confocal microscope

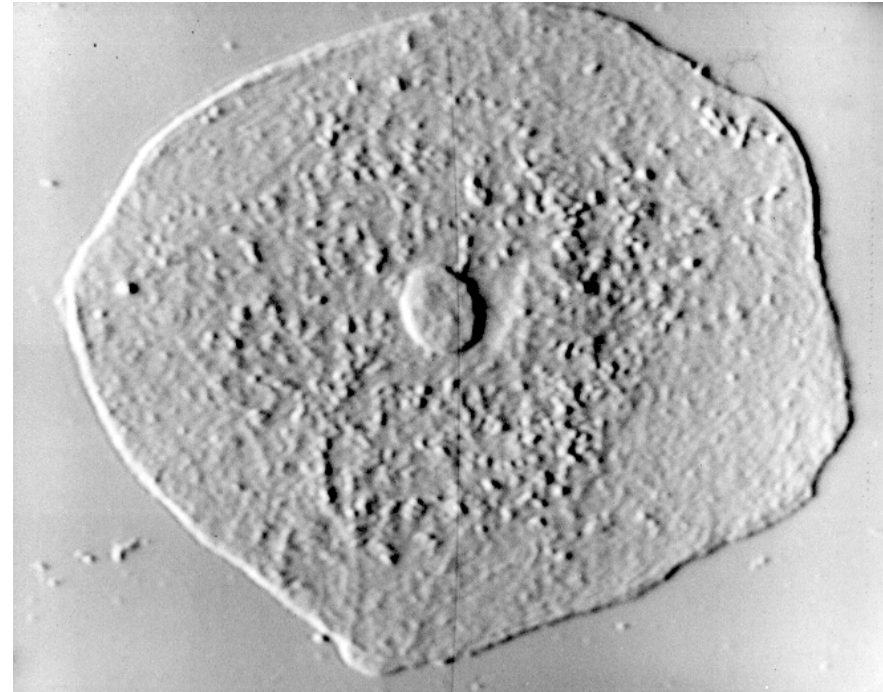
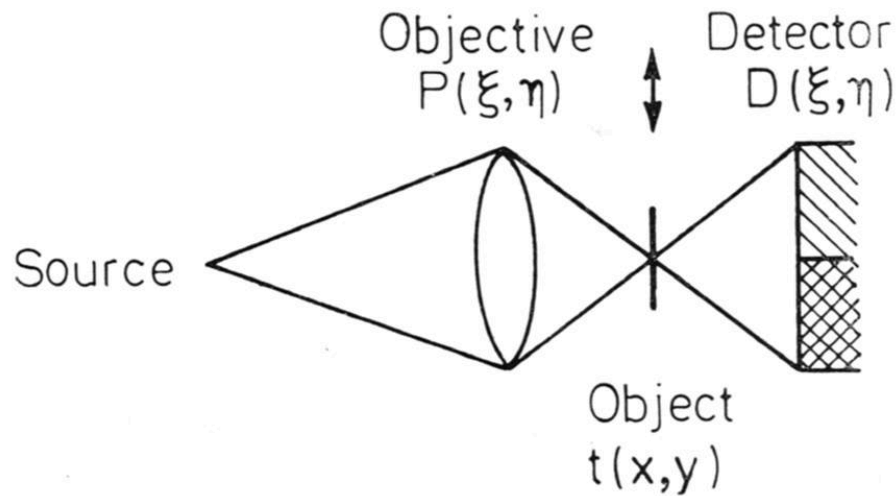
*Encyclopedia of Modern Optics*,  
RD Guenther, DG Steel, L Bayvel, eds,  
Elsevier, Oxford, **3**, pp. 103-110

## Phase Contrast Microscopy

**C J R Sheppard**, National University of Singapore,  
Singapore

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# Differential phase contrast (DPC)



DPC image of a cheek cell

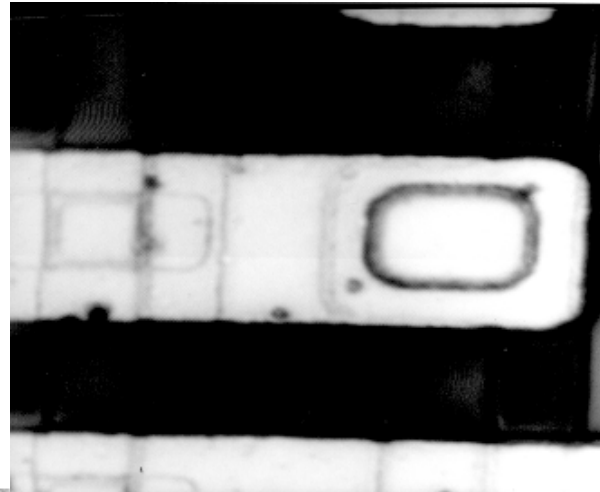
*Journal of Microscopy*, Vol. 133, Pt 1, January 1984, pp. 27–39.  
Received 1 November 1982; accepted 29 March 1983

D. K. HAMILTON *and* C. J. R. SHEPPARD,

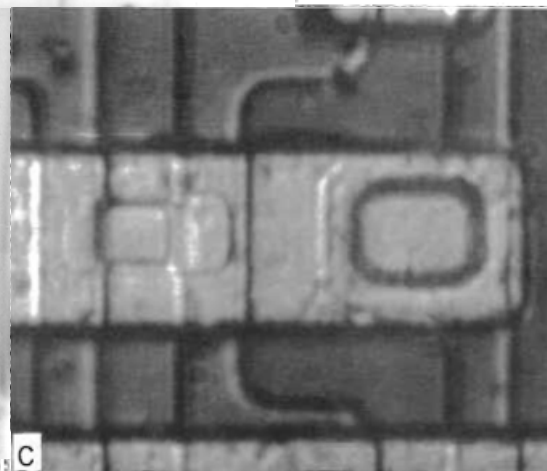
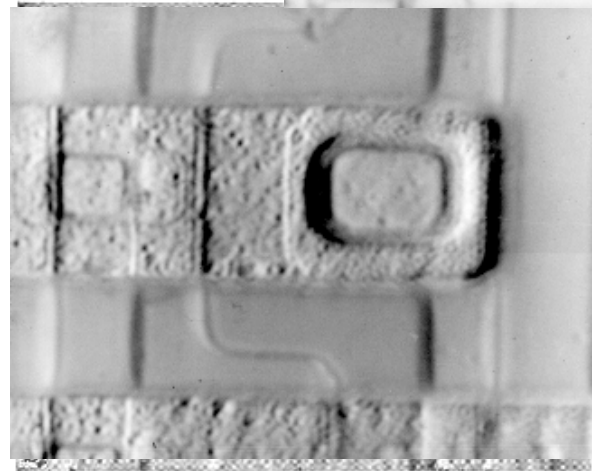
Differential phase contrast in scanning optical microscopy

# Can also do DPC in reflection

Brightfield image of an integrated circuit



DPC



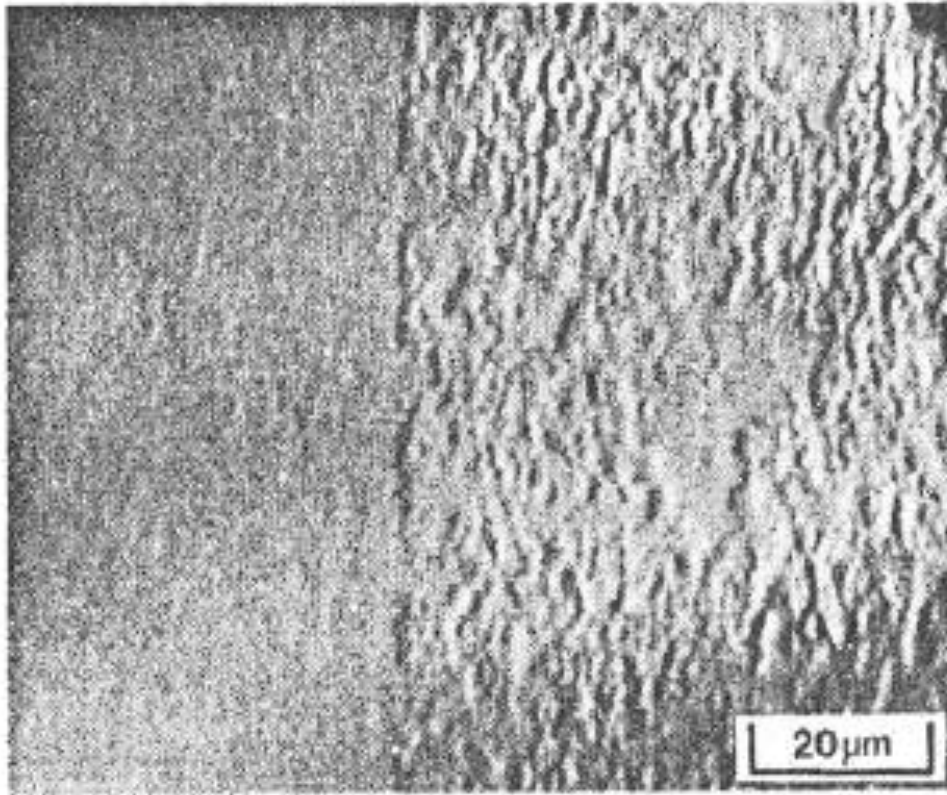
DIC

Fig. 2. An integrated circuit viewed in reflection: (a) amplitude image, (b) differential phase image, (c) a similar region viewed in a Zeiss microscope using Nomarski DIC.

Hamilton DK, Sheppard CJR (1984), *J. Microsc.* **133**, 27-39 (1984)



# DPC image of a single monolayer



**Fig. 2** A differential phase contrast image of a single monolayer.

- Very sensitive to weak phase changes

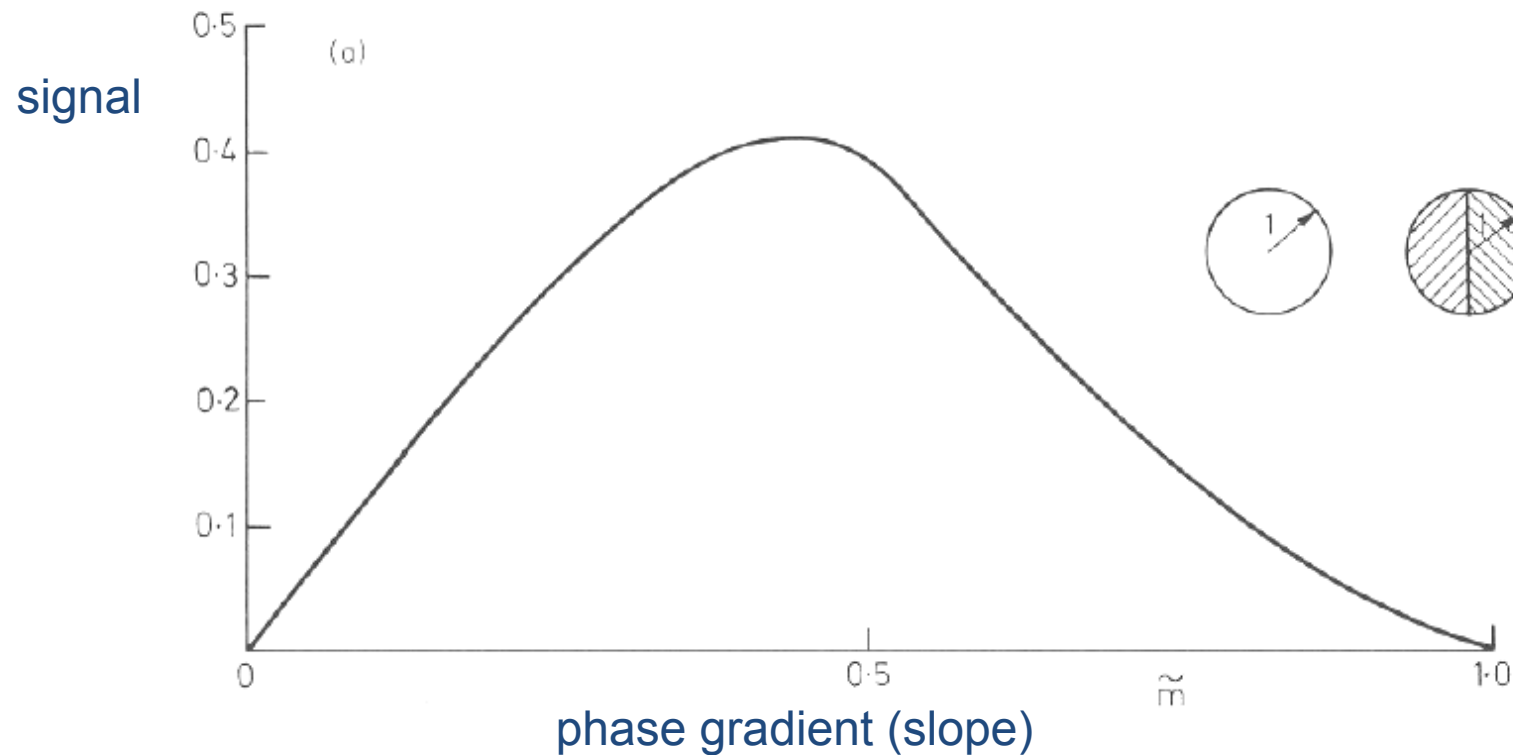
C.J.R. SHEPPARD  
D.K. HAMILTON  
H.J. MATTHEWS

Scanning optical microscopy  
of low-contrast samples

NATURE VOL. 334 18 AUGUST 1988 572



# Phase-gradient transfer function $C(m;m)$ (PGTF)

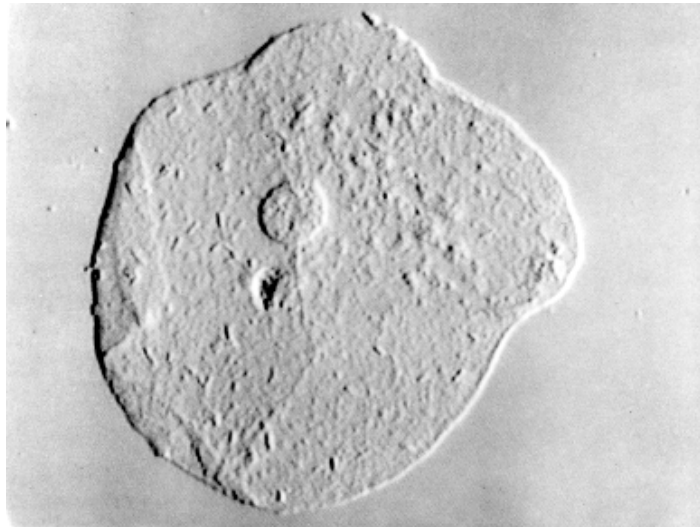


- Anti-symmetrical

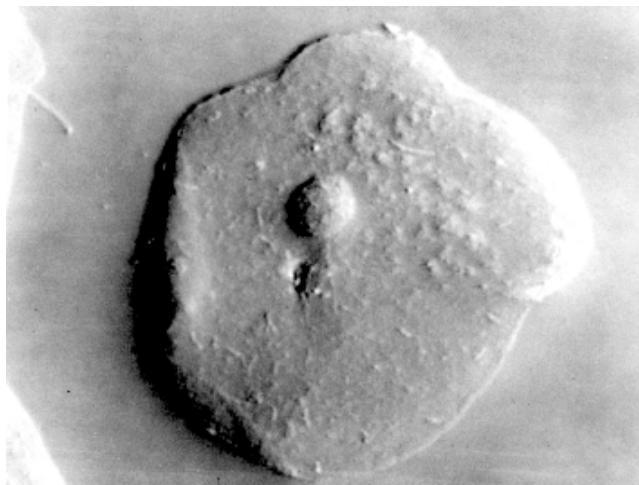
D. K. HAMILTON, C. J. R. SHEPPARD and T. WILSON,  
Improved imaging of phase gradients in scanning  
optical microscopy

*Journal of Microscopy*, Vol. 135, Pt 3, September 1984, pp. 275–286.

# DPC with an annular split detector

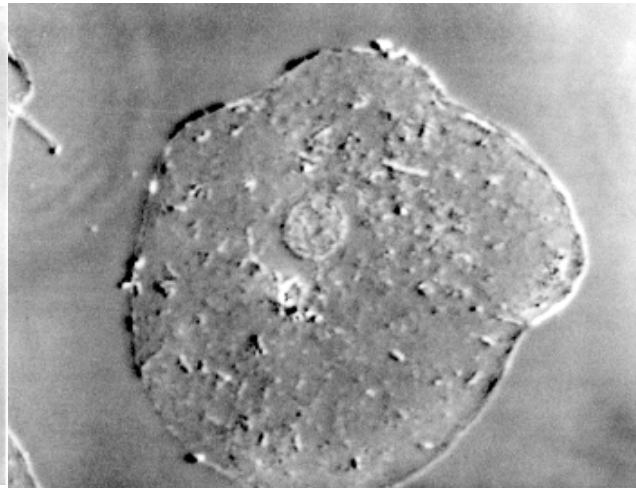


Hamilton DK, Sheppard CJR, Wilson T,  
*J. Microscopy* **153**, 275-286 (1984)



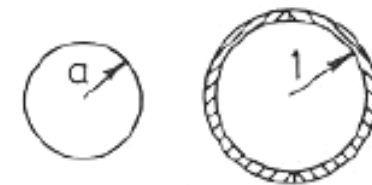
$$a = 1$$

- slow changes in slope



$$a = 0.7$$

- high spatial frequency response



- Can adjust contrast/  
resolution

# PGTF for DPC

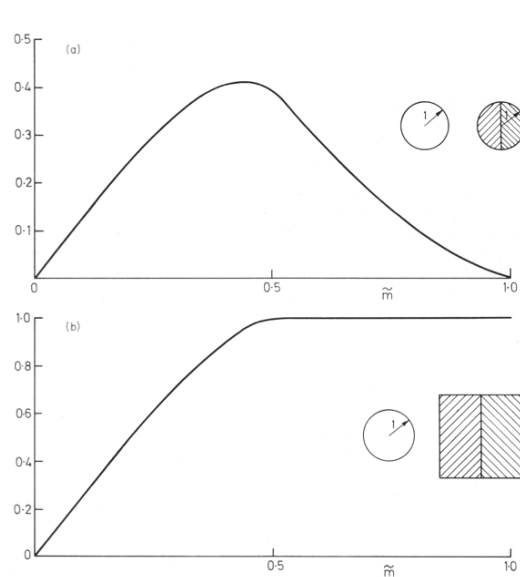


Fig. 2.  $C(m; 0)$  and  $C(m; m)$  for the transmission scanning optical microscope with an unobscured split detector. (a) Circular detector equal in size to the pupil; (b) square detector larger than the pupil.

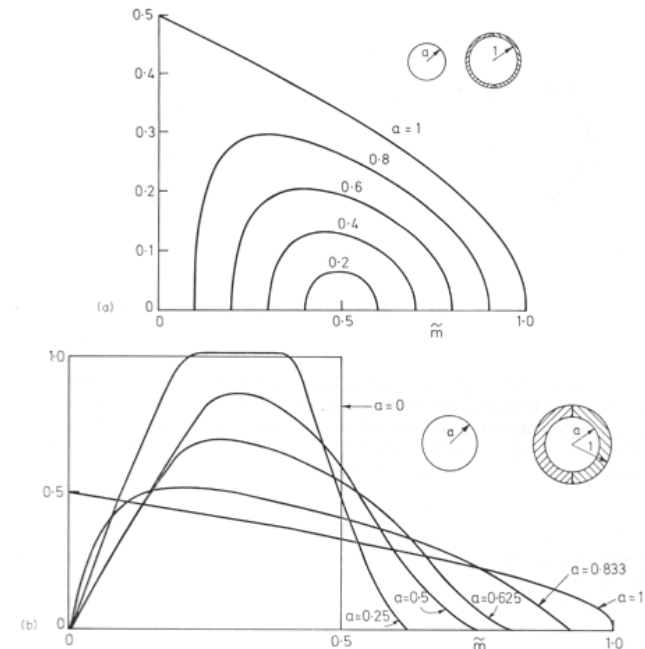


Fig. 4. (a)  $C(m; m)$  for the annular detector for various ratios  $a$  of pupil radius to annulus radius. When  $a = 1$   $C(m; 0) = C(m; m)$ ; otherwise  $C(m; 0)$  is zero. (b)  $C(m; m)$  for the circular split detector with a central obscured region equal in radius to the pupil, for various ratios of a pupil radius to detector radius. In this case  $C(m; 0)$  is always zero.

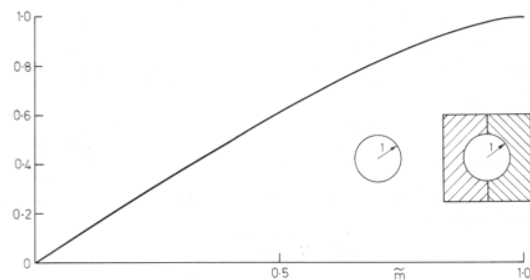


Fig. 5.  $C(m; m)$  for a large split detector with a central obscured region equal in radius to the pupil.  $C(m; 0) = 0$ .

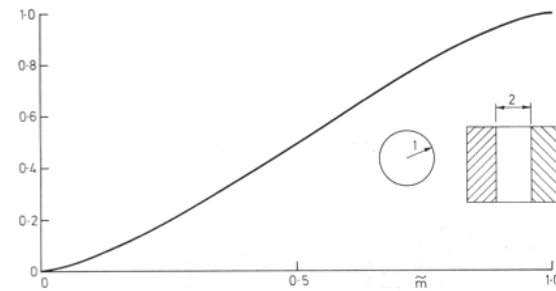
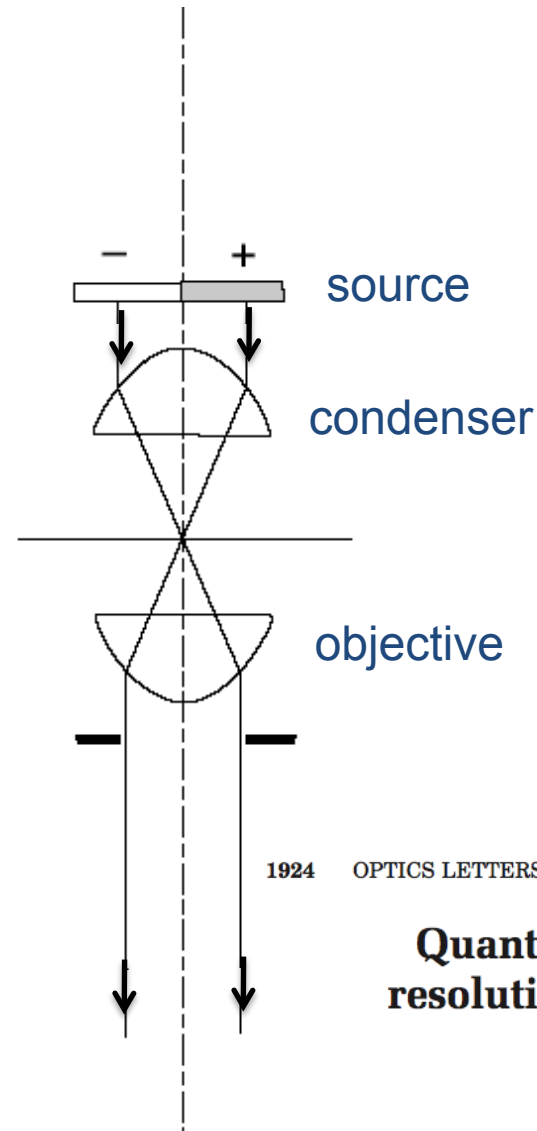


Fig. 7.  $C(m; m)$  for a split detector with a central obscured strip.

Often advantageous to have linear behaviour

Hamilton DK, Sheppard CJR, Wilson T,  
*Journal of Microscopy* **153**, 275-286 (1984)

# Asymmetric Illumination DPC (AI-DPC)



- Can also do in a conventional microscope

- Arrows reversed, source from each semicircle

- Need to take two images

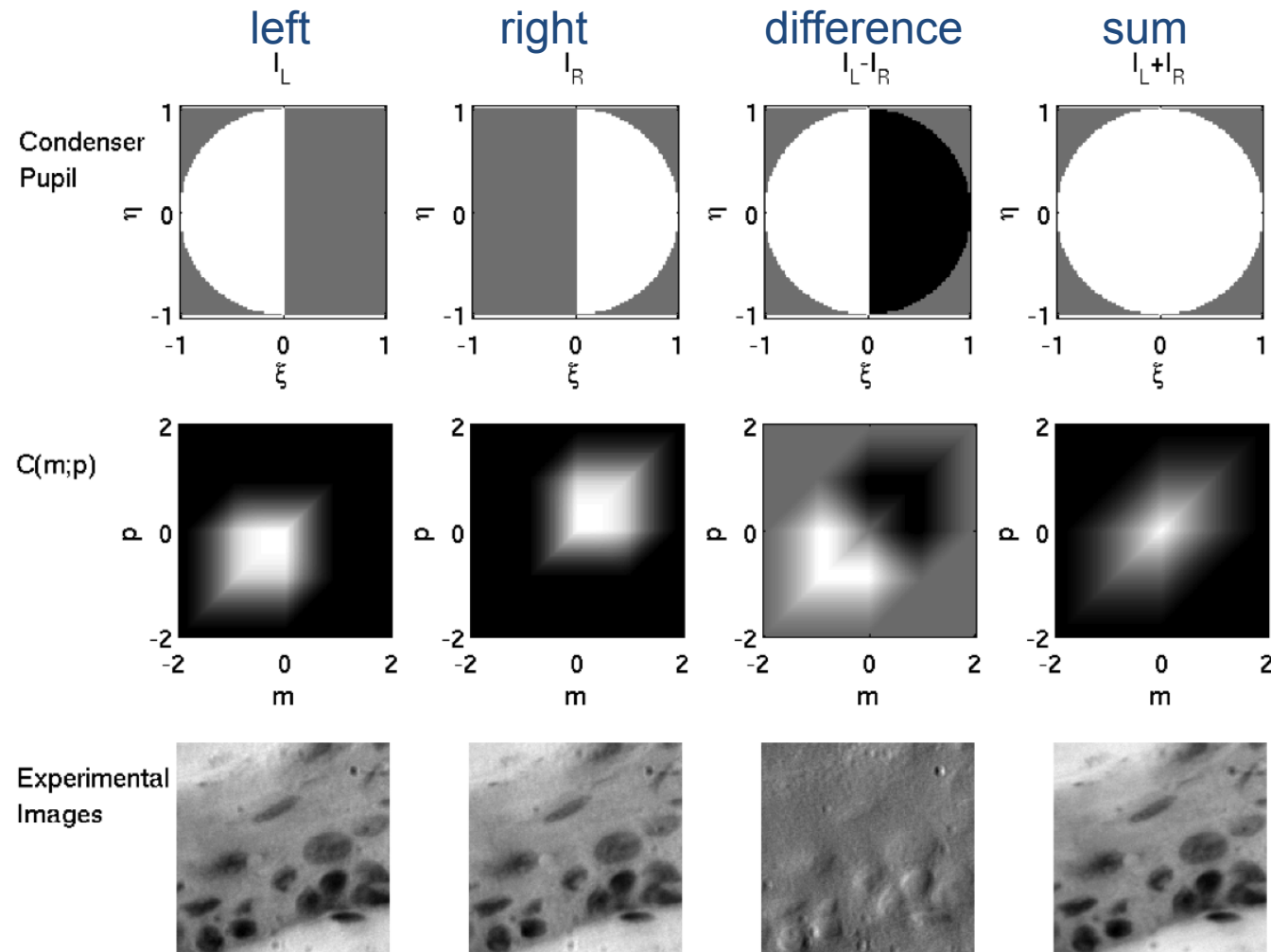
- Or 4 to get  $\partial\phi/\partial x$ ,  $\partial\phi/\partial y$

1924 OPTICS LETTERS / Vol. 34, No. 13 / July 1, 2009

**Quantitative phase-gradient imaging at high resolution with asymmetric illumination-based differential phase contrast**

Shalin B. Mehta<sup>1,2,4,\*</sup> and Colin J. R. Sheppard<sup>1,2,3</sup>

# Asymmetric illumination DPC (AI-DPC)



Condenser pupil structures (top row), partially coherent transfer function in direction of differentiation (middle row), and experimental images (bottom row) obtained with AIDPC.

The sample is skin H&E stained section courtesy Graham Wright, TLL and Declan Lunny, IMB.

# Phase measurement using DPC

- Integrate phase gradient to get phase (but still constant of integration)

$$\phi = \int \frac{\partial \phi}{\partial x} dx + const.$$

Measure  $\partial \phi / \partial x, \partial \phi / \partial y$

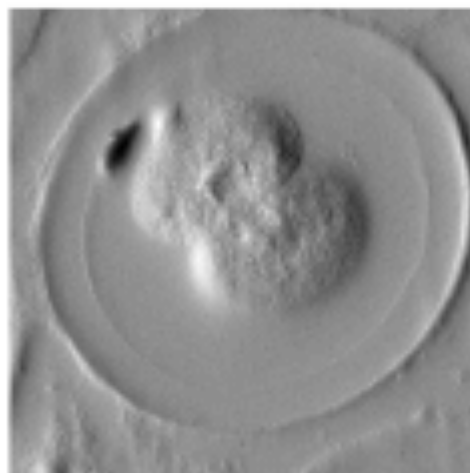
$$\phi(x, y) = F^{-1} \left[ \frac{F \left[ \frac{\partial \phi}{\partial x} + i \frac{\partial \phi}{\partial y} \right]}{2i(\sin 2\pi m \Delta + i \sin 2\pi n \Delta)} \right]$$

Arnison, Larkin, Sheppard, Smith, Cogswell, *J. Microsc.* **214**, 7-12 (2004)

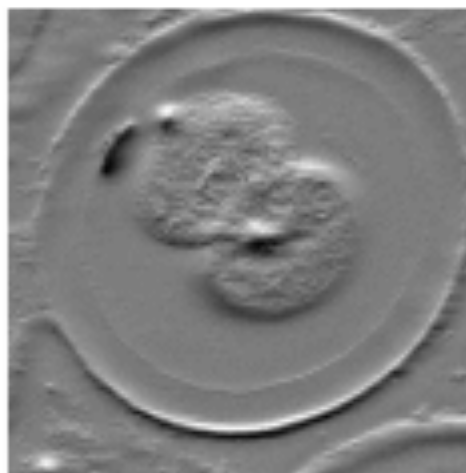
Similar to Frankot-Chellappa algorithm  
*IEEE Trans. Pattern Analysis* **10**, 439 (1988)  
 (Shape from shading)

# Phase reconstruction from AI-DPC

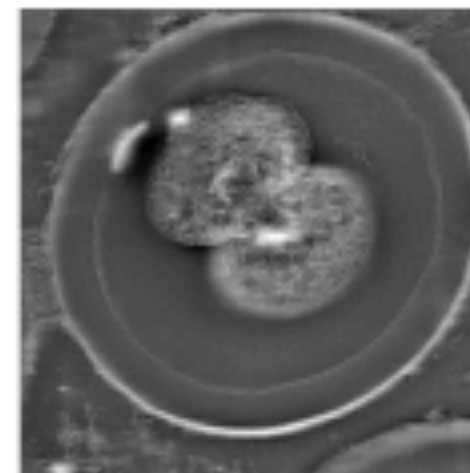
$\partial \phi / \partial x$



$\partial \phi / \partial y$



$\phi$



# History of DPC

First done in electron microscopy

N. H. Dekkers and H. de Lang, "Differential phase contrast in a STEM," *Optik* **41**, 452-456 (1974).

N. H. Dekkers and H. De Lang, "A detection method for producing phase and amplitude images simultaneously in a STEM," *Philips Tech. Review* **37**, 1 (1977)

**On differential phase contrast with an extended illumination source**

W. C. Stewart

813 *J. Opt. Soc. Am.*, Vol. 66, No. 8, August 1976

**United States Patent** [19]

[11] **4,255,014**

Ellis

[45] **Mar. 10, 1981**

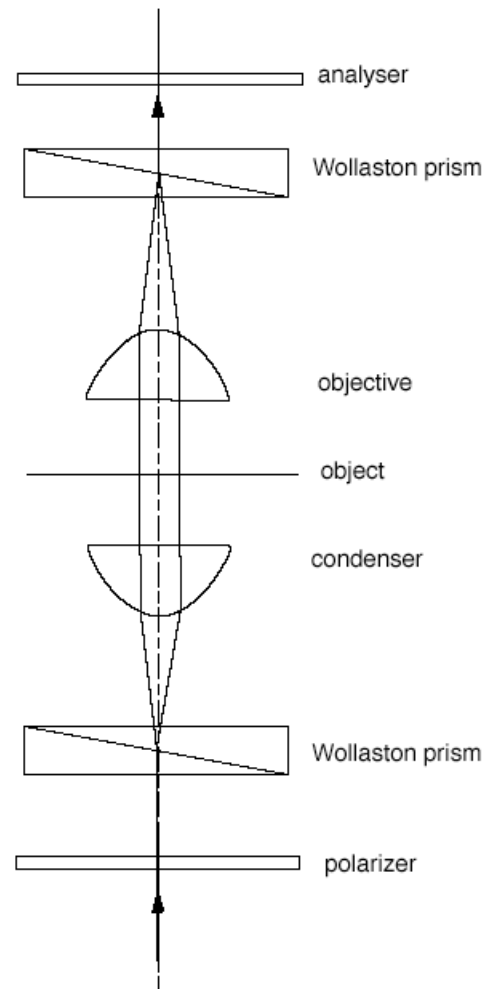
---

Hamilton DK, Sheppard CJR (1984), *J. Microsc.* **133**, 27-39 (1984)



# Nomarski Differential interference contrast (DIC)

- Phase difference (bias) between two images altered using compensator:
- Translate Wollaston prism
- Rotate polarizing elements



G. Nomarski,  
"Microinterferometrie  
differential a ondes  
polarisés," *J. Phys. Radium*  
**16**, 9-135 (1955)

*Encyclopedia of Modern Optics*,  
RD Guenther, DG Steel, L Bayvel, eds,  
Elsevier, Oxford, **3**, pp. 103-110

## Phase Contrast Microscopy

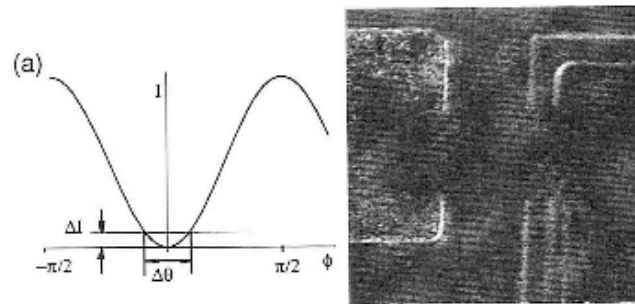
**C J R Sheppard**, National University of Singapore,  
Singapore

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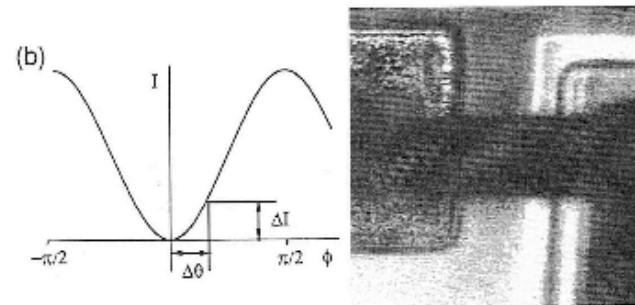
# Nomarski DIC

- Bias changed using shift of Wollaston prism or rotation of polarization  
(Sénarmont compensator)
- Uses polarization, so depends on birefringence of sample
- Can use in conventional or confocal mode

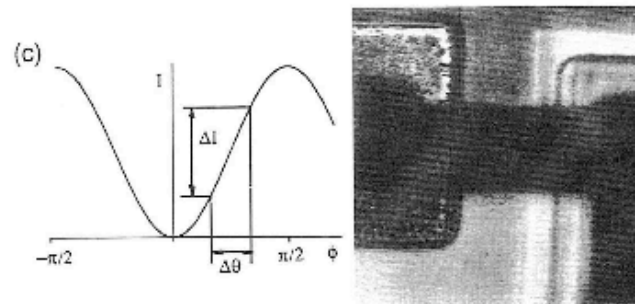
# Effect of bias



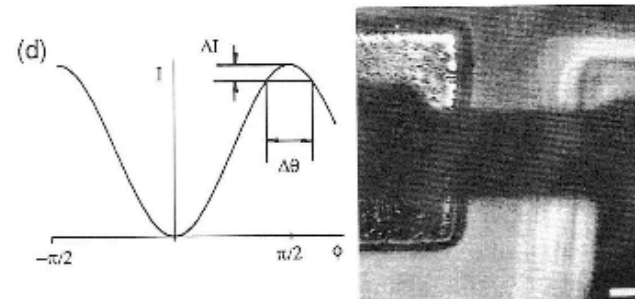
no bias (dark field)



small bias: used for visual observation



45° bias: used for CCD detection



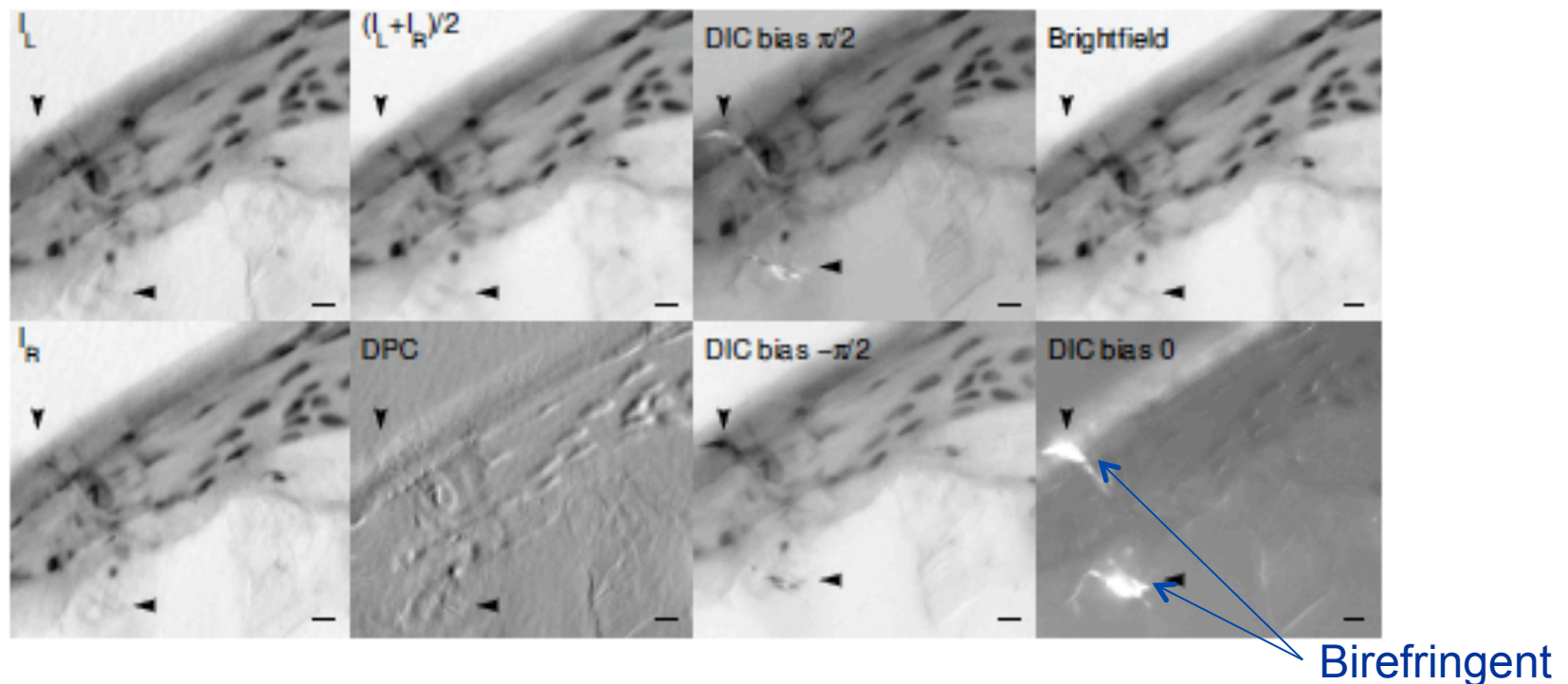
90° bias  
 (bright field)

*Journal of Microscopy*, Vol. 165, Pt 1, January 1992, pp. 81–101.  
 Received 7 December 1990; revised and accepted 6 March 1991

Confocal differential interference contrast (DIC) microscopy: including a theoretical analysis of conventional and confocal DIC imaging

by C. J. COGSWELL and C. J. R. SHEPPARD, Physical Optics Department,  
 School of Physics, University of Sydney, N.S.W. 2006, Australia

# Separation of absorption and thickness information



**Figure:** H&E preparation of  $7\mu m$  thick section of human skin. The average and DPC images provide clear separation of absorption and phase information. **Birefringent regions may be mistaken as absorbing features in DIC as visible at bias  $-\pi/2$ .** Such regions are indicated by arrow and appear bright under DIC with zero bias. Scale bar is  $5\mu m$ . **Sample courtesy:** Mr. Declan Lunny, Institute of Medical Biology & Dr. Graham Wright, Temasek Life Sciences Laboratory.

# Transfer function for DIC

$2\Delta$  is the shear

$2\phi_0$  is the bias compensation

$$C_{\text{eff}}(m,n; p,q) = 4C(m,n; p,q) \sin(2\pi m\Delta - \phi) \sin(2\pi p\Delta - \phi).$$

$$C_{\text{eff}}(m,n; p,q) = 2C(m,n; p,q) \{ \cos[2\pi(m-p)\Delta] - \cos 2\phi \cos[2\pi(m+p)\Delta] - \sin 2\phi \sin[2\pi(m+p)\Delta] \}.$$

Odd part: DPC term

Weak object transfer function  $p = 0$  (WOTF)

$$C_{\text{eff}}(m;0) = 2C(m;0) [ \cos(2\pi m\Delta)(1 - \cos 2\phi_0) - \sin(2\pi m\Delta) \sin 2\phi_0 ].$$

Strength depends on  $\phi_0$

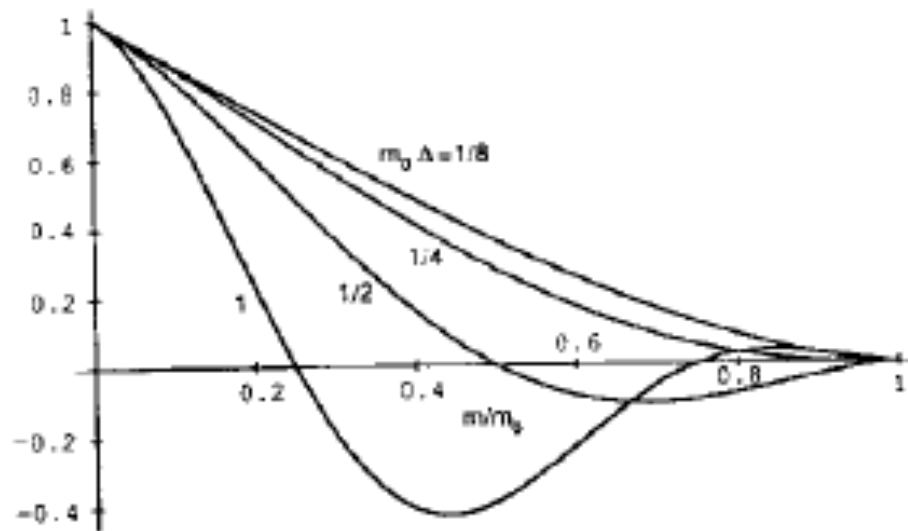
Phase-gradient transfer function  $p = m$  (PGTF)

$$C_{\text{eff}}(m;m) = C(m;m) \sin^2(2\pi m\Delta - \phi_0).$$

C. J. COGSWELL and C. J. R. SHEPPARD. Confocal differential interference contrast (DIC) microscopy: including a theoretical analysis of conventional and confocal DIC imaging

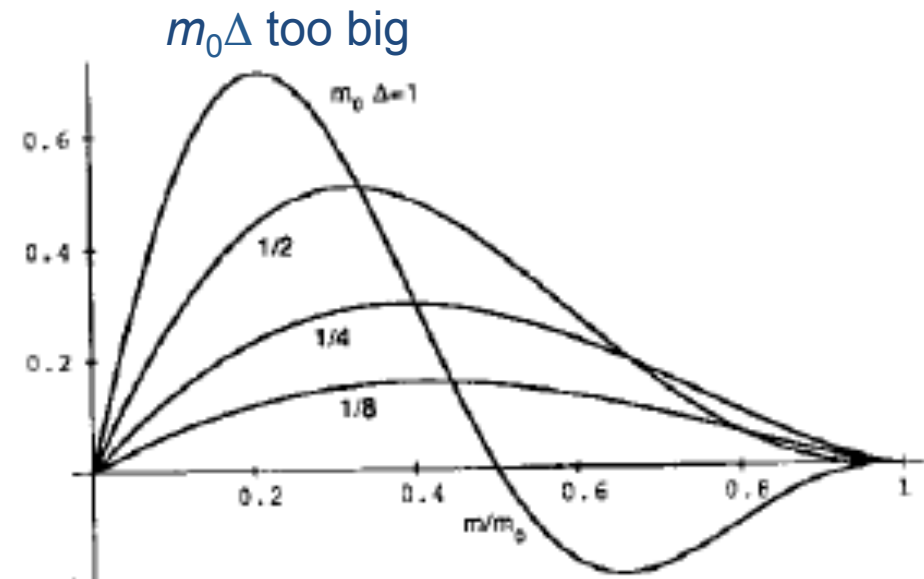
*Journal of Microscopy*, Vol. 165, Pt 1, January 1992, pp. 81–101.

# Weak object transfer function for DIC



Even part (amplitude contrast)

$m_0$  is spatial frequency cut-off



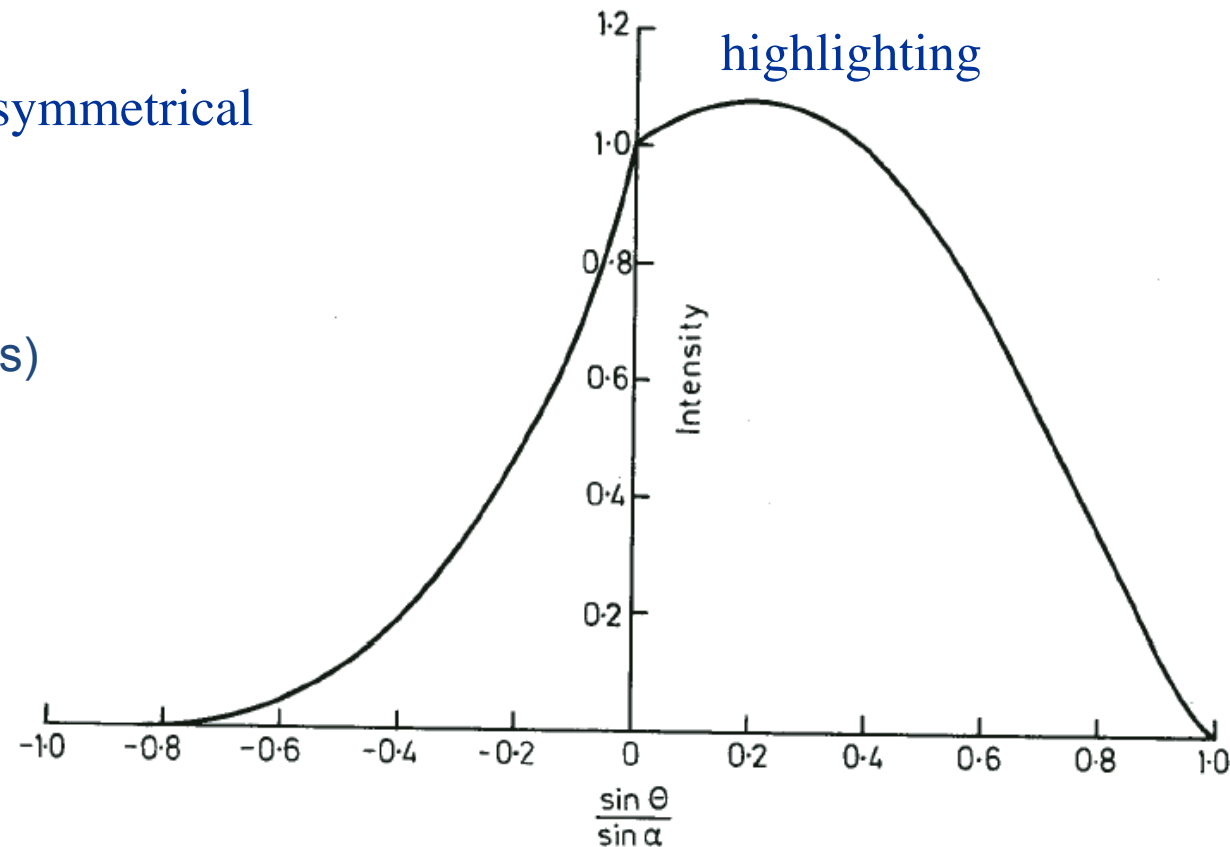
Odd part (DPC)

C. J. COGSWELL and C. J. R. SHEPPARD,  
Confocal differential interference contrast (DIC)  
microscopy: including a theoretical analysis of  
conventional and confocal DIC imaging

*Journal of Microscopy*, Vol. 165, Pt 1, January 1992, pp. 81-101.

# Phase-gradient transfer function for DIC

- Not anti-symmetrical
- (Small bias)



**Fig. 12.** The signal intensity for a surface at an angle  $\theta$ , Nomarski DIC with compensator set to just give no zeros in the pass band.

D. K. HAMILTON and C. J. R. SHEPPARD  
Differential phase contrast in scanning optical microscopy  
*Journal of Microscopy*, Vol. 133, Pt 1, January 1984, pp. 27–39



# PGTF for DIC

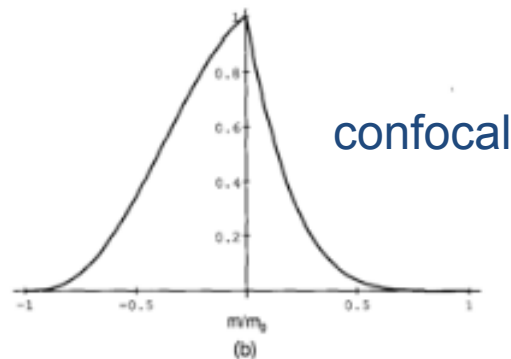
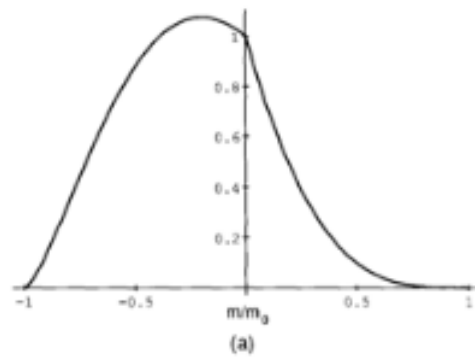


Fig. 5. The phase gradient transfer function for a DIC system with equal condenser and objective apertures, which gives the signal strength as a function of phase gradient  $\Psi' = 2\pi w$  for conditions  $\phi = 2\pi w_0 \Delta$  and  $\phi$  small: (a) conventional and (b) confocal.

small bias

- Non-linear behaviour

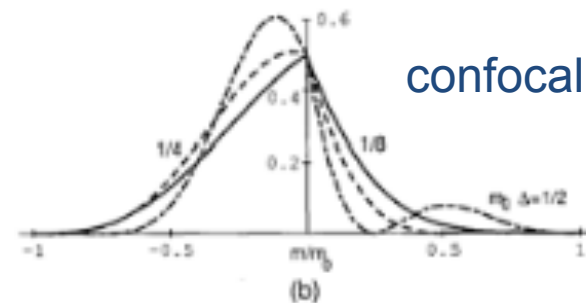
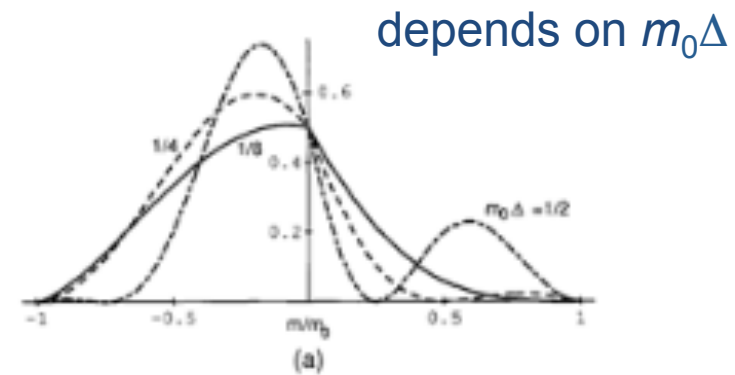


Fig. 6. The phase gradient transfer function for (a) conventional and (b) confocal DIC systems with  $\phi = \pi/4$  and values of  $m_0 \Delta = 1/2, 1/4$  and  $1/8$ .

bias =  $\pi/4$

C. J. COGSWELL and C. J. R. SHEPPARD,  
Confocal differential interference contrast (DIC)  
microscopy: including a theoretical analysis of  
conventional and confocal DIC imaging  
*Journal of Microscopy*, Vol. 165, Pt 1, January 1992, pp. 81–101.



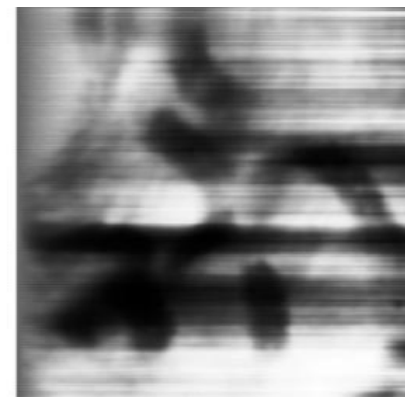
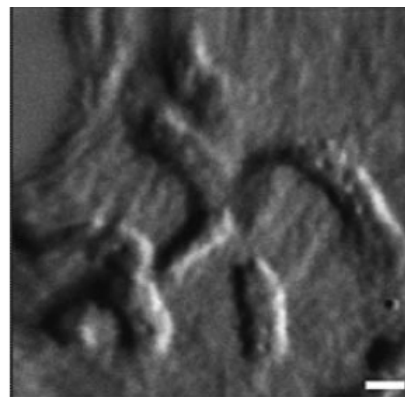
# Phase-stepping DIC

- Slowly-varying phase gradient  $\partial\phi / \partial x = 2\pi m(x,y)$
- Constant phase gradient deflects light through an angle (prism effect)

$$I = A + B \cos[2\pi m(x,y)\Delta - \phi_0] \quad \phi_0 \text{ is bias retardation}$$

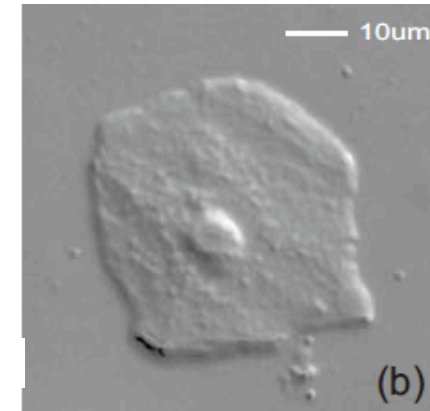
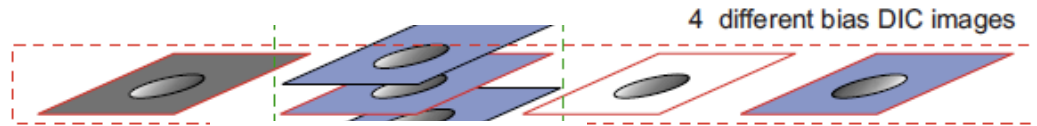
- Same form as normal interference pattern  
Measure  $I$  for different values of bias retardation  $\phi_0$
- Using phase-stepping algorithm, can recover phase gradient  $2\pi m(x,y)$   
Integrate phase gradient to get phase (but still constant of integration)

$$\phi = \int \frac{\partial\phi}{\partial x} dx + const.$$



Streaking  
artifact

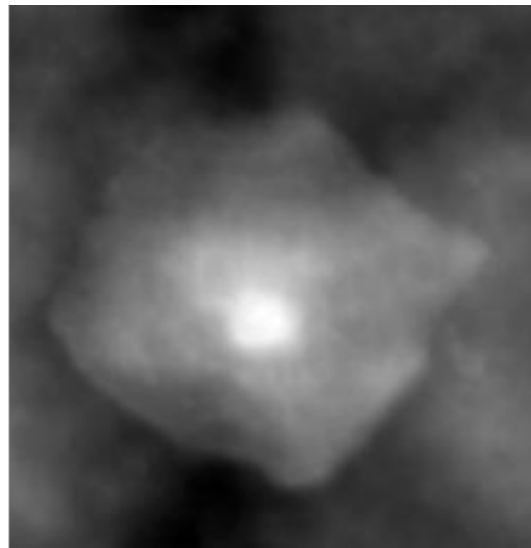
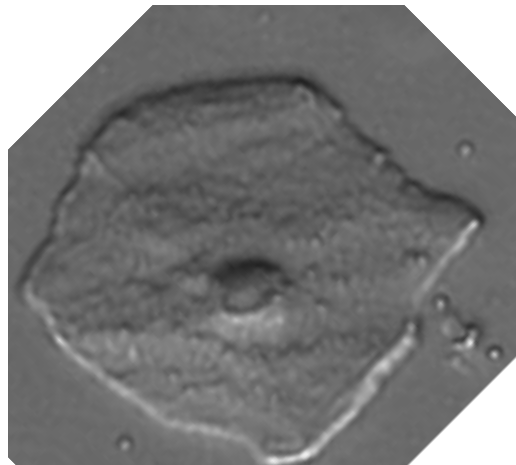
# Phase-stepping DIC



Phase gradient reconstructed from phase-shifting DIC

Shan Shan Kou,<sup>1,2,\*</sup> Laura Waller,<sup>3</sup> George Barbastathis,<sup>4,5</sup> and Colin J. R. Sheppard<sup>1,2,6</sup>

February 1, 2010 / Vol. 35, No. 3 / OPTICS LETTERS 447

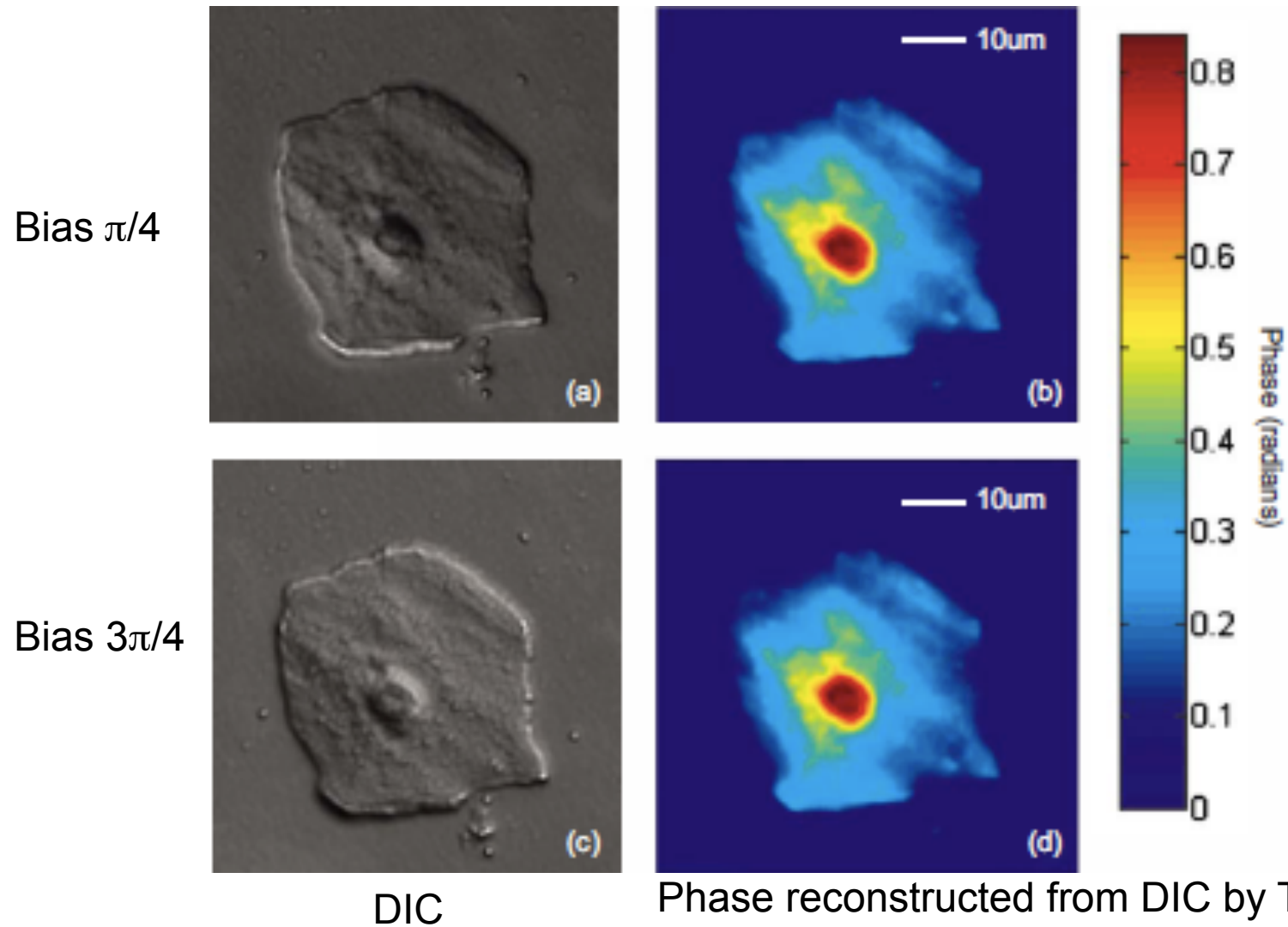


**Quantitative Phase Restoration in Differential Interference Contrast (DIC) Microscopy**

Shan Shan Kou<sup>ab</sup>, Colin J. R. Sheppard<sup>\*ab</sup>

Optical and Digital Image Processing, edited by Peter Schelkens, Touradj Ebrahimi, Gabriel Cristóbal, Frédéric Truchetet, Proc. of SPIE Vol. 7000, 700005, (2008) · 0277-786X/08/\$18 · doi: 10.1117/12.780912

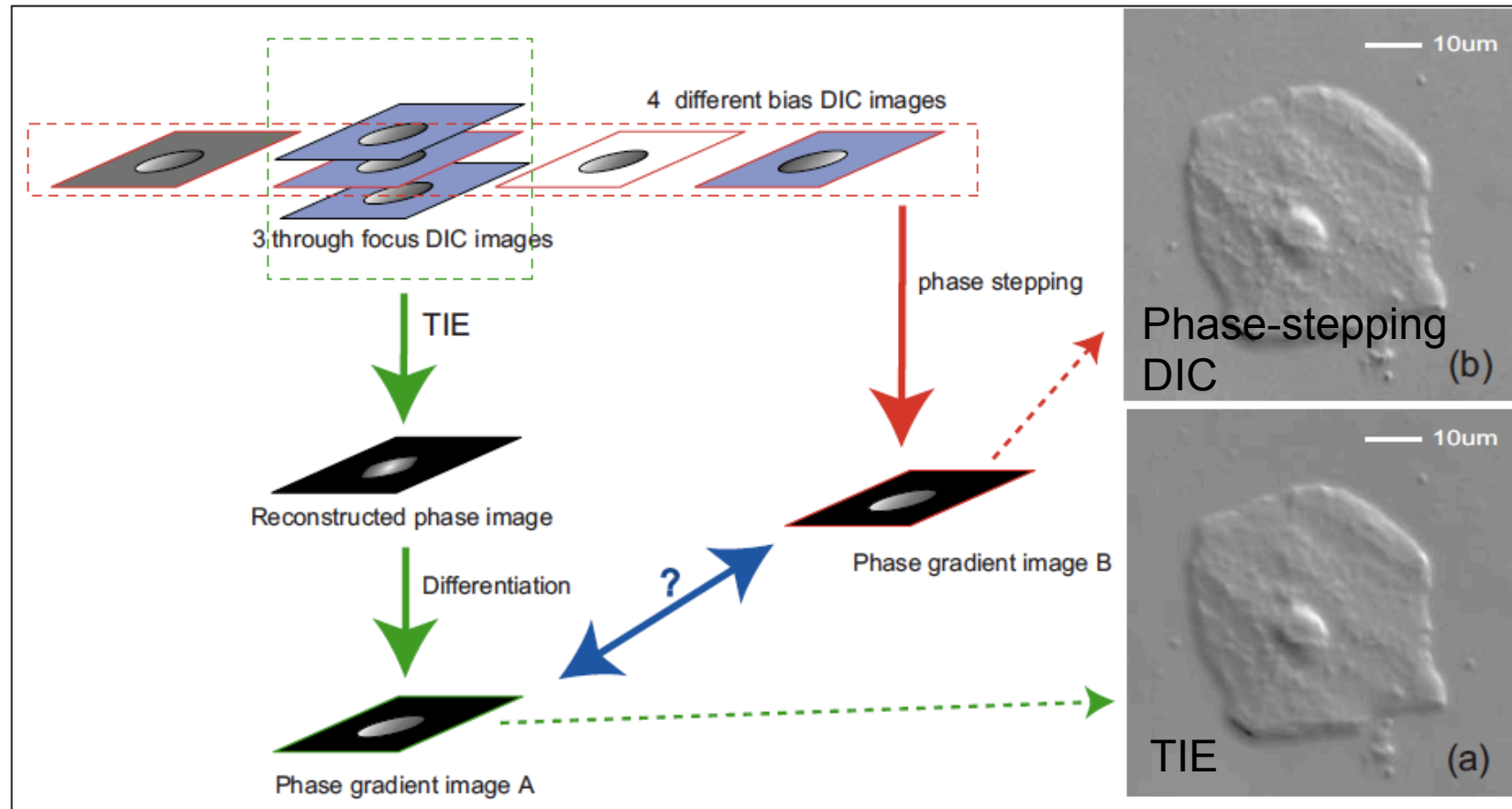
# Can use TIE on DIC (TI-DIC)



Shan Shan Kou,<sup>1,2,\*</sup> Laura Waller,<sup>3</sup> George Barbastathis,<sup>4,5</sup> and Colin J. R. Sheppard<sup>1,2,6</sup>

# PS-DIC and TI-DIC

Phase gradient reconstructed from phase-shifting DIC



Phase gradient reconstructed from DIC by TIE

Shan Shan Kou,<sup>1,2,\*</sup> Laura Waller,<sup>3</sup> George Barbastathis,<sup>4,5</sup> and Colin J. R. Sheppard<sup>1,2,6</sup>

# Quantitative phase from TIE-DIC

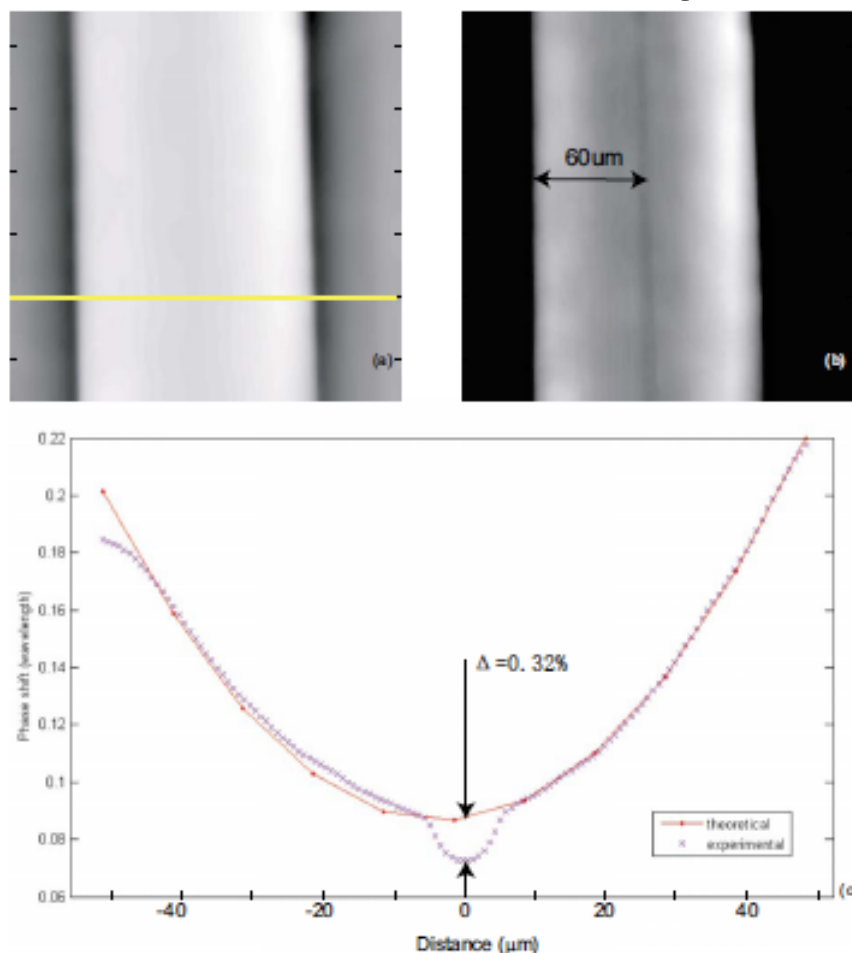


Fig. 3. (Color online) TI-DIC fiber reconstruction. (a) Reconstructed phase profile of the fiber. (b) Same profile after histogram adjustment. (c) Fitting of the fiber profile to theoretical data.

Shan Shan Kou,<sup>1,2,\*</sup> Laura Waller,<sup>3</sup> George Barbastathis,<sup>4,5</sup> and Colin J. R. Sheppard<sup>1,2,6</sup>

# Summary

- Zernike phase contrast
  - only for weak object
  - not quantitative
  - haloes
- Nomarski DIC
  - good 3D imaging
  - phase stepping for quantitative measurements
  - birefringence a problem (plastic slides)
- DPC
  - not good for 3D imaging
- Defocus
  - only for weak object
- TIE
  - not good for 3D imaging