Optical beam configuration for manipulation of micro and nano particles

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Light as an instrument for small particle manipulation

History: from Kepler's hypothesis to Ashkin's proposal and beyond

- Kepler's *De Cometis* (1619): the light radiation pressure pushes objects along the beam propagation direction yielding the deflection of the comet tails pointing away from the sun
- Lebedev (1901), Nichols and Hull (1901): The first laboratory demonstrations of the radiation pressure force
- Ashkin (1970): The radiation pressure can be used for optical manipulation of microparticles
- Actual applications: Micro/nano particle control (confinement, transportation, sorting), cell sugery, molecular motors, atom cooling, etc.

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Optical tweezers for biomedical applications

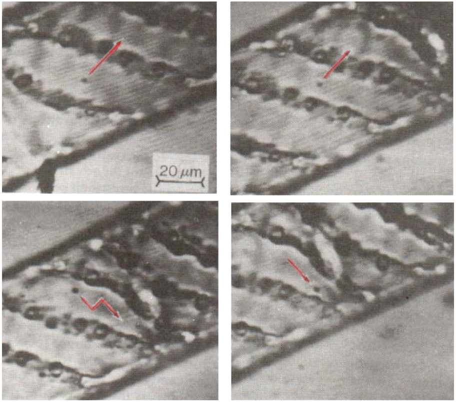
Non-contact forces in pN-nN region

- Single molecule studies:
  - motor proteins
  - RNA and DNA mechanics
  - DNA–protein interaction

  [Lecture C. Bustamante: Single Molecule Manipulation in Biochemistry]
- Single cell confinement in a static or fluid flow environment
  - for measurement of volume changes
  - mechanical characterization
  - cell surgery
- Single or multiple cell transportation, sorting, assembling and organizing
  [H. Zhang and K.-K. Liu, J R Soc Interface 5, 671 (2008)]

Optical manipulation into cell

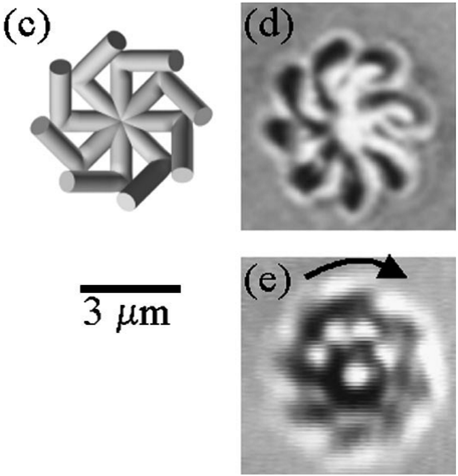
Manipulation of particles within cytoplasm of a cell of spirogyra



A. Ashkin et al, Nature 330, 769 (1987)

Micromachines driven by light

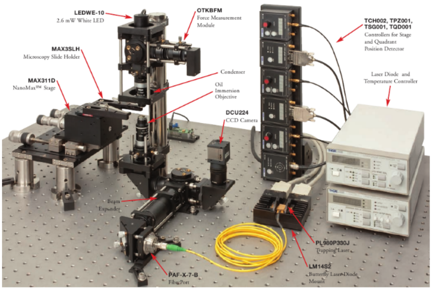
The light deflected by the trapped particle exerts the torque to drive the rotation



P. Galajada & P. Ormos, Appl. Phys. Lett. 78, 249 (2001)

Comercial optical twezeers (OTs)

OTs allow precise, contact-free cell manipulation as well as to trap, move, and sort microscopic particles



Zeiss company THORLABS company

Trapping forces

Depend on the particle size, form, refractive index and optical beam structure

Optical forces exerted by beam with complex field amplitude (scalar picture) $E(\mathbf{r}) = A(\mathbf{r})\exp(i\varphi(\mathbf{r}))$:

- scattering forces proportional to optical current $\mathbf{F}_{\nabla\varphi} \propto I\nabla\varphi$, where $I = |E(\mathbf{r})|^2$
- gradient forces proportional to intensity gradient $\mathbf{F}_{\nabla I} \propto \nabla I$

The light interaction with particles is divided into three regimes:

- Mie regime ($d \gg \lambda$): The model of momentum conservation is applicable (acceptable limit $d \geq 10\lambda$).
- Rayleigh regime ($d \ll \lambda$): The dipole model is applicable (acceptable limit $d \leq \lambda$, for calculation of transverse force).
- Intermediate size (Lorentz-Mie) regime [J. Lock, Appl. Opt. 43, 2532 (2004)]

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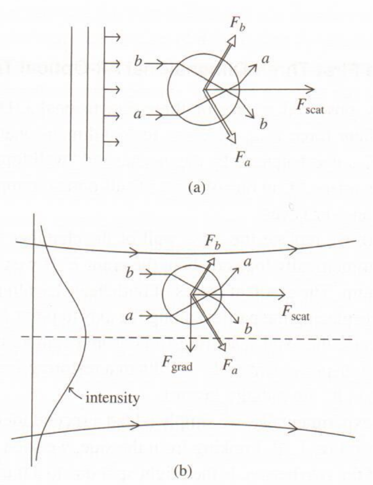
Optical trapping by focused beam

Ray optics model for Mie dielectric particles ($n_p > n_m$)



A. Ashkin, Optical Trapping and Manipulation of Neutral Particles Using Lasers (2006)

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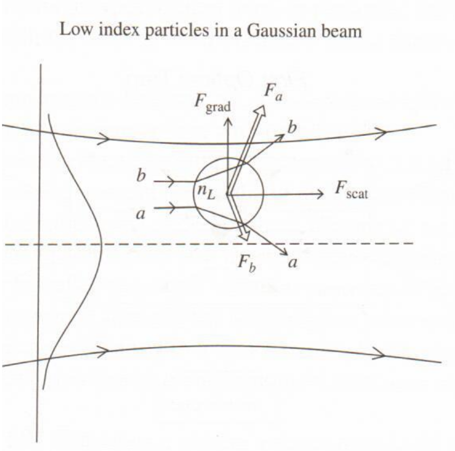

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Forces on submicrometer Rayleigh particles

A. Ashkin et all Opt. Lett. 11, 288 (1986), Y. Roichman et al, Phys. Rev. Lett. 100, 013602 (2008)

- Gradient forces ("lenslike properties of the scatterer")

$$\mathbf{F}_{\nabla I} \propto \frac{n_m}{2}\left(\frac{n_p^2-n_m^2}{n_p^2+2n_m^2}\right) r^3 \nabla I$$

its sign depends on the sign $n_p - n_m$

- Scattering forces (the momentum transfer from the external radiation field to the particle by scattering and absorption)

$$\mathbf{F}_{\nabla\varphi} \propto \frac{128\pi^5 n_m}{3\lambda^4 c}\left(\frac{n_p^2-n_m^2}{n_p^2+2n_m^2}\right)^2 r^6 I \nabla\varphi,$$

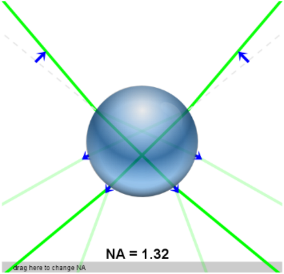
where r is a particle radius, n_p and n_m are refractive indices of a particle and surrounding medium, λ is a wavelength.

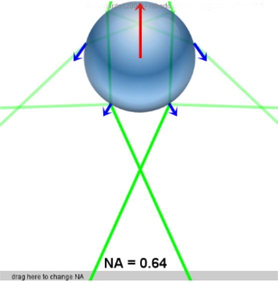
Conditions for optical trapping

Ray optics model for dielectric particles $d \gg \lambda$ ($n_p > n_m$)

Stable and efficient trapping needs high 3D intensity gradients (axial and transverse).

Strongly focused beams $\Longrightarrow$ objective lens $NA > 1$





http://glass.phys.uniroma1.it/dileonardo/content/apps/trapforces.php

Outline

- **How to move particles?**
  - Focal spot movement or Beam shaping?
- **How to draw a 2D light curve of arbitrary form ?**
  - Requirements for particle confinement and transport
  - All-optical freestyle trap design
- **Propelling micro and nano particle along 2D trayectories**
  - Polymorphyc beam concept
- **From 2D to 3D curves**
  - Tractor beams
- **Trap creation**
  - Computer generated holograms
- **Concluding remarks**

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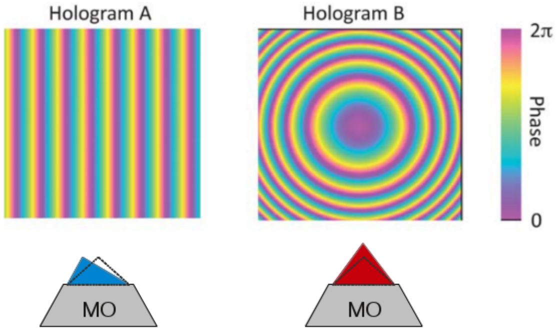
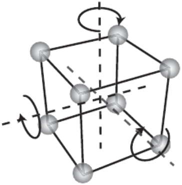
- Lateral or axial focal spot movement by introducing temporally changing linear $\phi_l = 2\pi(x/l_x + y/l_y)$ or quadratic $\phi_z = \pi(x^2+y^2)/\lambda l_z$ phase, respectively

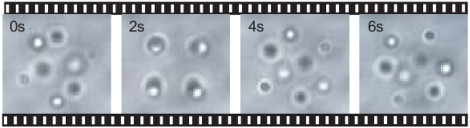


Multiple particles manipulation

Individual particle control

- Movement of several particles requires the corresponding manipulation of several focal spots





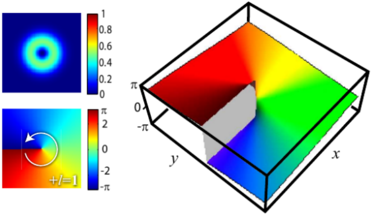
[J. Leach et al, Opt. Express, 12, (2004)]

Particle movement by a Gaussian vortex beam

Transverse intensity gradient forces trap particles while phase gradient forces move them along a ring

- Optical vortex beam, helical phase $\exp(il\theta)$, exerts torque over the trapped particle
- **Large $|l| \Longrightarrow$ High phase gradient $\Longrightarrow$ high rotation speed**
- It is not all-optical trap: particles are confined against the coversilp glass

Example:
Intensity and phase distribution of a Laguerre-Gaussian beam with $l = 1$.

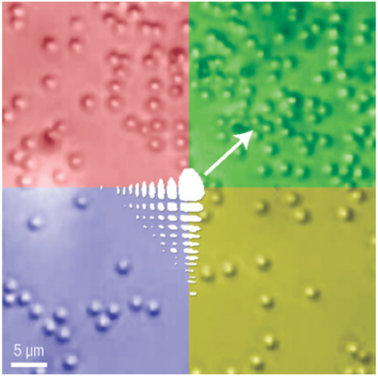


[M. Padgett & R. Bowman, Nature Photonics Focus Review 2011]

Particle movement along non-circular trajectory

It is not beam orbital angular momentum, but phase gradient is responsible for particle propelling

- An Airy beam incident from the third quadrant move microparticles along parabolic trayectory into the first quadrant [J. Baumgartl et al, Nature Phot. 2, 675 (2008)]



Beam shaping: problems to solve

Instead of using the beam of known form - design a beam on demand!

Requirements for particle confinement and transport:

- Light intensity distribution in form of an arbitrary curve
- Stability of confinement = high intensity gradients
- Manipulation of single or multiple particles
- Manipulation of objects with different optical properties of a large size-range (nano and micro particles)
- Control of particle speed (= phase distribution) along the trajectory
- All-optical 3D trap (far away from chamber walls)
- Motion planning according with current situation (detecting the target positions and avoiding of obstacles)

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Drawing of arbitrary 2D curves

Method for structurally stable (spiral) beam construction [E. Abramochkin and V. Volostnikov, Physics-Uspekhi, 2004]

Complex field amplitude of beam $G(\mathbf{r})$, $[\mathbf{r} = (x, y)]$, in plane $z = \text{constant}$, in the form plane curve $\mathbf{c}(t) = (R(t)\cos t, R(t)\sin t)$:

$$G(\mathbf{r}) = \frac{1}{L}\int_0^T g(\mathbf{r}, t) \exp\left[\frac{\mathrm{i}}{w_0^2} R(t)\,(x\cos t + y\sin t)\right] \mathrm{d}t,$$

where

$$g(\mathbf{r}, t) = \left|\mathbf{c}'(t)\right| \exp\left(-\frac{[\mathbf{r} - \mathbf{c}(t)]^2}{2w_0^2}\right) \exp\left(\pm\frac{\mathrm{i}}{w_0^2}\int_0^t R^2(\tau)\mathrm{d}\tau\right)$$

- $|\mathbf{c}'(t)|$ provides uniform intensity distribution along the curve
- $\exp\left(-\frac{[\mathbf{r}-\mathbf{c}(t)]^2}{2w_0^2}\right)$ (Gaussian brush) provides smooth curve profile
- $\exp\left(\pm\frac{\mathrm{i}}{w_0^2}\int_0^t R^2(\tau)\mathrm{d}\tau\right)$ is responsible for a particular phase distribution along the curve that guarantees stability

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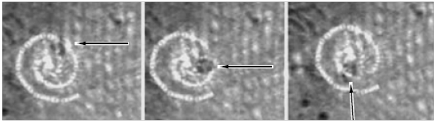
Spiral beams
[E. Abramochkin and V. Volostnikov, Physics-Uspekhi, 2004]

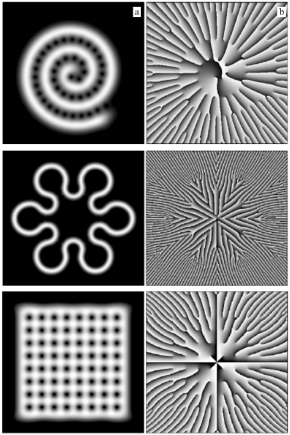
Spiral beam stability: Intensity distribution does not change its form a part of scaling and rotation during propagation

Laguerre-Gaussian beams belongs to this family

Archimedes spiral beam trap absorbing cetylpyridine bromide particles of $2.2\mu\mathrm{m}$ diameter size



[E. G. Abramochkin et al, Laser Phys. 16, 842 (2006)]



From spiral beam to polymorphic beam
Gaussian brush is changed to spot brush

The method of curved beam construction is useful, but it has to be adapted for optical trapping applications:

- Beam with high transverse and axial intensity gradients is needed

Solution: Change the Gaussian brush to the spot one $\exp\left(-\frac{[\mathbf{r}-\mathbf{c}(t)]^2}{2w_0^2}\right) \Rightarrow \mathbf{1}$

- Phase gradient distribution along the curve is fixed and depends of the curve size that impeeds independent control of particle velocity

Solution: Change to $\exp\left(\pm\frac{\mathrm{i}}{w_0^2}\int_0^t R^2(\tau)\mathrm{d}\tau\right) \Rightarrow \exp\left[\mathrm{i}\frac{2\pi l}{S(T)}S(t)\right]$, where l defines the phase accumulation along the entire curve.
After these changes beam spiral character is lost, but it is not required for trapping applications

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Solution: Change to $\exp\left(\pm\frac{\mathrm{i}}{w_0^2}\int_0^t R^2(\tau)\mathrm{d}\tau\right) \Rightarrow \exp\left[\mathrm{i}\frac{2\pi l}{S(T)}S(t)\right]$, where l defines the phase accumulation along the entire curve. After these changes beam spiral character is lost, but it is not required for trapping applications

From spiral beam to polymorphic beam

Gaussian brush is changed to spot brush

The method of curved beam construction is useful, but it has to be adapted for optical trapping applications:

- Beam with high transverse and axial intensity gradients is needed

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After these changes beam spiral character is lost, but it is not required for trapping applications

Polymorphic beam (PB)

PB has a form of highly focused curve in trapping plane and a vortex lattice structure in Fourier conjugated plane

- PB complex field amplitude in Fourier domain (codified by hologram):

$$E(x,y) = \int_0^T g(t) \exp\left[-\mathrm{i}\frac{k}{\mathrm{f}} R(t)\,(x\cos t + y\sin t)\right] \mathrm{d}t.$$

  - $g(t) = |g(t)| \exp(\mathrm{i}\Psi)$ is a weight function of plane waves
  - f is an objective focal distance, k is a wavenumber, T is the maximum value of the azimuthal angle t ($w_0^2 \Rightarrow \mathrm{f}/k$)

- Complex field amplitude of light curve (PB focused by objective):

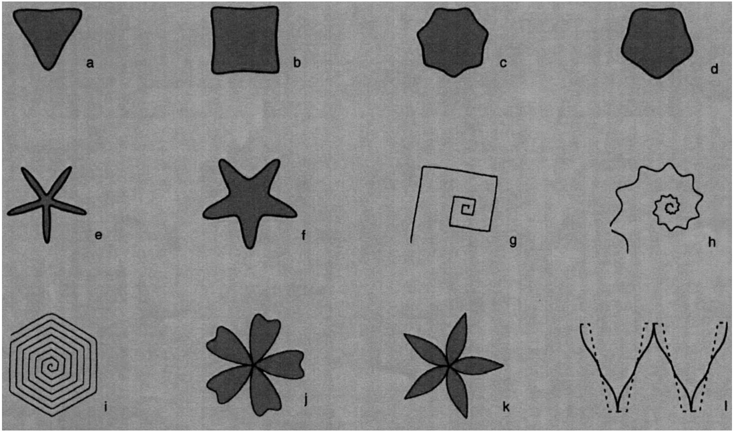
$$\widetilde{E}(u,v) = \frac{1}{\mathrm{i}\lambda \mathrm{f}} \int E(x,y) \exp\left[-\mathrm{i}\frac{k}{\mathrm{f}}(xu + yv)\right] \mathrm{d}x\mathrm{d}y$$

$$= \mathrm{i}^{-1}\lambda \mathrm{f} \int_0^T g(t)\,\delta(u + R(t)\cos t)\,\delta(v + R(t)\sin t)\,\mathrm{d}t$$

Superformula for a variety of 2D curves

[J. Gielis, Am. J. Bot. 90, 333 (2003)]

A Superformula describes shapes of plants, micro-organisms (e.g.: cells, bacteria and diatoms), small animals (e.g.: starfish), crystals, etc.



Superformula and Nature inspired circuits for optical current

Superformula can be used for PBs generation

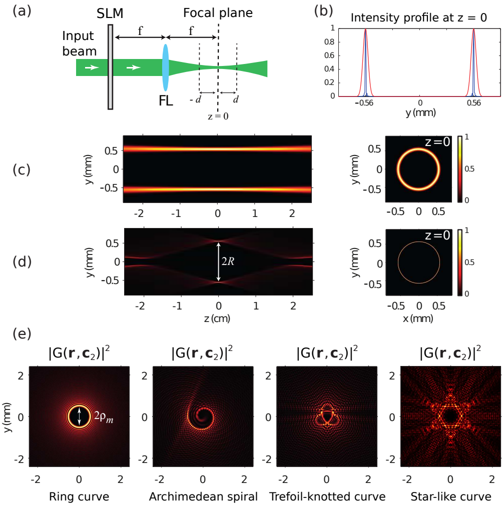
The Superformula gives the radius of the curve $R(t)$

$$R(t) = \rho\,(t)\left[\left|\frac{1}{a}\cos\left(\frac{m}{4}t\right)\right|^{n_2} + \left|\frac{1}{b}\sin\left(\frac{m}{4}t\right)\right|^{n_3}\right]^{-1/n_1}$$

as a function of the polar angle t, where the real numbers in $\mathbf{q} = (a, b, n_1, n_2, n_3, m)$ are the design parameters of the curve and $\rho(t)$ is a non-periodic function of t required for the construction of asymmetric and spiral-like curves (e.g.: $\rho(t) \propto e^{\alpha t}$ or $\rho(t) \propto t^{\alpha}$).
For $\rho(t) = \rho_0$ and $\mathbf{q} = (1,1,1,1,1,0)$, where $t \in [0, 2\pi]$, a circle with radius $R(t) = \rho_0$
[J. Gielis, Am. J. Bot. 90, 333 (2003)]

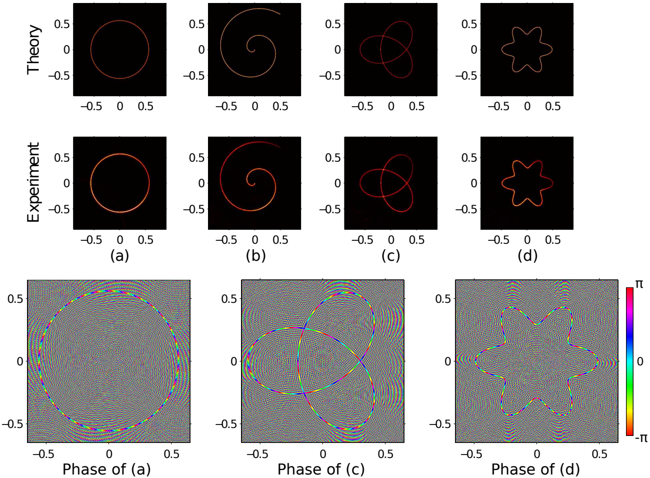
3D traps shaped in arbitrary 2D curves

Polymorphic beams versus spiral beams [J. A. Rodrigo et al, Opt. Express 2013]



3D traps shaped in 2D curves

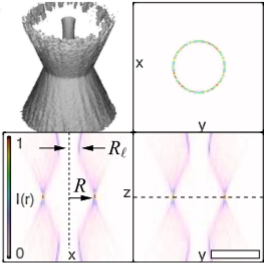
Phase gradient along 2D curve [J. A. Rodrigo et al, Opt. Express 2013]:

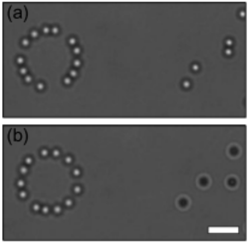


True 3D optical vortex trap

First demonstration [Y. Roichman et al, SPIE 2007 and PRL 2008]

Helical Bessel beams $(l = 30)$ are true 3D vortex ring trap. 2μm silica particles suspended in water are trapped and moved far away from the chamber walls





True 3D traps have to be strong enough to compensate axial light radiation pressure

Single-beam optical trap based on a light curve

Independent design of the light curve's shape and phase gradient prescribed along it

- $R(t)$ is the 2D curve equation in polar coordinates.
- Non-difractive beams (ex. Bessel beams) are also PBs $(R(t) = R)$ [J. Durnin, J. Opt. Soc. Am. A 4, 651 (1987)]
- **2D curve optical trap is created by focusing a polymorphic beam:**

$$E(x,y) = \int_0^T g(t) \exp\left[-\mathrm{i}kR(t)\left(x\cos t + y\sin t\right)/\mathrm{f}\right] \mathrm{d}t,$$

- $g(t) = |g(t)| \exp[i\Psi(t)] = |g(t)| \exp\left[\mathrm{i}\frac{2\pi l}{S(T)} S(t)\right]$ defines the field amplitude and phase distribution along the curved beam $\widetilde{E}(\mathbf{r})$
- Uniform intensity distribution along the curve: $|g(t)| = \sqrt{R'(t)^2 + R(t)^2}$

[J. A. Rodrigo & T. Alieva, Sci. Rep. 6, 35341 (2016)]

Propelling forces of curved optical trap

Independent choice of form and size of light curve and phase gradient

- Propelling forces $\propto \mathbf{j} = |\widetilde{E}|^2 \nabla \Psi$ (optical current) are controlled by the phase of the function $g(t) = |g(t)| \exp[i\Psi(t)]$:

$$\Psi(t) = \frac{2\pi l}{S(T)} S(t),$$

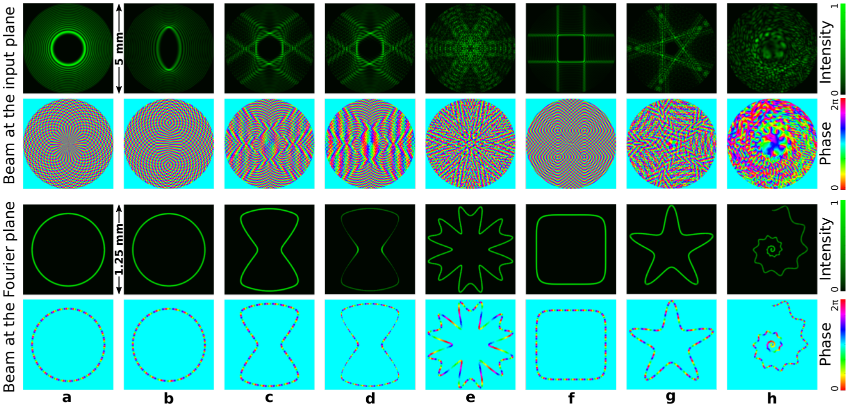
  l defines the total phase accumulation along the curve

- The phase gradient (tangential to the curve) can be
  - Uniform if $S_{\mathrm{u}}(t) = \int_0^t \sqrt{R'(\tau)^2 + R(\tau)^2} \mathrm{d}\tau$
  - Non-uniform as for example $S_{\mathrm{n/u}}(t) = \int_0^t R^2(\tau) \mathrm{d}\tau$

  [J. A. Rodrigo & T. Alieva, Sci. Rep. 6, 35341 (2016)]

Variety of forms and functionality

Independent control of shape and phase gradient

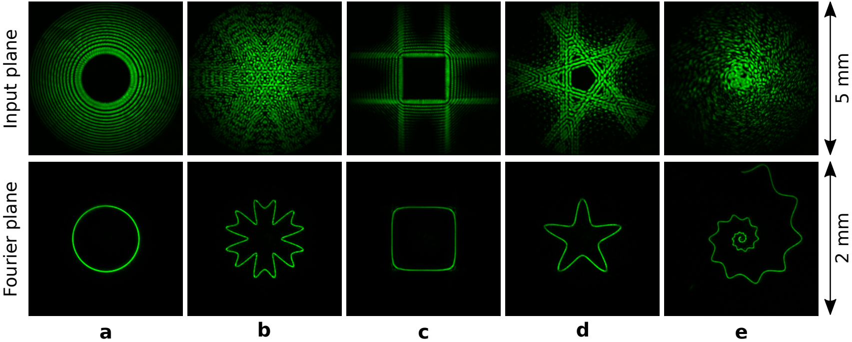


[J. A. Rodrigo & T. Alieva, Sci. Rep. 6, 35341 (2016)]

Experimental generation of the polymorphic beam

The polymorphic beam is holographically encoded onto a programmable SLM

Intensity distributions of several polymorphic beams and the corresponding light curves created by focusing them:

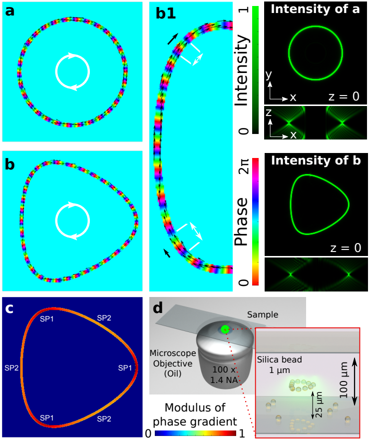


[J. A. Rodrigo & T. Alieva, Sci. Rep. 6, 35341 (2016)]

True 3D traps for arbitrary 2D curves

Laser trap was created $25\,\mu\mathrm{m}$ deep in the sample

Phase gradient distribution $S(t) = \int_0^t R^2(\tau)\mathrm{d}\tau$

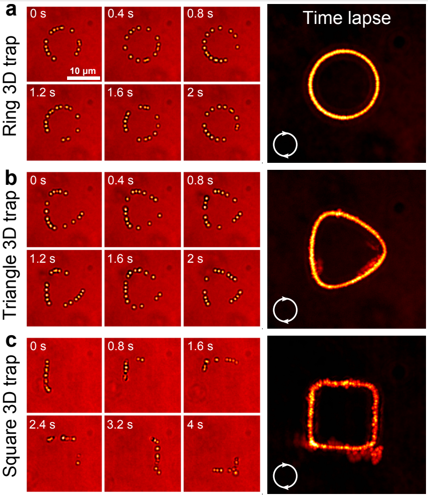


[J. A. Rodrigo & T. Alieva, Optica 2, 812 (2015)]

All optical propelling microparticles along 2D curves

$1\,\mu$m silica spherical particles in deionized water

Laser trap shaped in (a) ring with $R = 5\,\mu$m, $l = 30$ (b) triangle $l = 34$ and (c) square $l = 34$ induce particle rotation with rates of 0.5 Hz, 0.25 Hz and 0.14 Hz, respectively.



[J. A. Rodrigo & T. Alieva, Optica 2, 812 (2015)]

Radiation induced forces exerted on nanoparticles

Intensity and phase gradient forces [A. Urban et al, Nanoscale (2014)]

- A dipolar NP with size $d \lesssim \lambda$ experiences radiation-induced forces transverse to the beam propagation direction:

$$\mathbf{F} = \mathbf{F}_{\nabla I} + \mathbf{F}_{\nabla \varphi} \propto \varepsilon \left[\frac{1}{4} \alpha^{\prime}(\lambda) \nabla I + \frac{1}{2} \alpha^{\prime\prime}(\lambda) I \nabla \varphi \right],$$

  - λ is a laser wavelength
  - I and φ are a beam intensity and phase distributions
  - ε is the permittivity of the surrounding medium
  - $\alpha(\lambda) = \alpha^{\prime}(\lambda) + i\alpha^{\prime\prime}(\lambda)$ is the particle polarizability

- $\alpha^{\prime\prime}(\lambda) > 0$, then the direction of the phase gradient forces $\mathbf{F}_{\nabla \varphi}$ always coincides with the direction $\nabla \varphi$

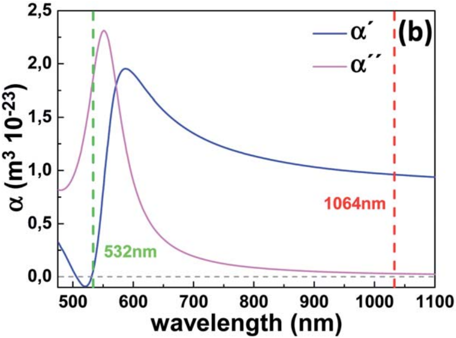
- $\alpha^{\prime}(\lambda)$ can take negative values if λ is near and on the blue side of the localized surface plasmon resonance (LSPR)

Radiation induced forces exerted on NPs

Intensity and phase gradient forces [A. Urban et al, Nanoscale (2014)]

- Example: Polarizability of gold NP sphere $d = 80\,\mathrm{nm}$

$$\mathbf{F} = \mathbf{F}_{\nabla I} + \mathbf{F}_{\nabla \varphi} \propto \varepsilon \left[\frac{1}{4} \alpha'(\lambda) \nabla I + \frac{1}{2} \alpha''(\lambda) I \nabla \varphi \right]$$



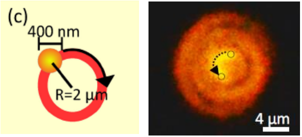
- Depending on λ, the force $\mathbf{F}_{\nabla I}$ can be too weak for optical confinement of the NP

Optical transport of metallic NPs

State of the art: Experiments with Gaussian vortex and line traps have been reported

2D trapping due to intensity gradient force $\mathbf{F}_{\nabla I}$ while $\mathbf{F}_{\nabla \varphi}$ propels NPs

Classical Gaussian vortex beam



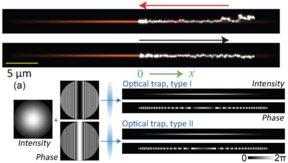
(Too big to be considered as a true NP)

Spherical gold particle 400nm rotated by focused Gaussian vortex (l=8, λ=830nm)

This is created by using a spiral phase plate

[A. Lehmuskero et al, Opt. Express 22, 4349 (2014)]

Classical line trap



Spherical silver NPs of 150 nm assembled by a line trap (λ=830nm)

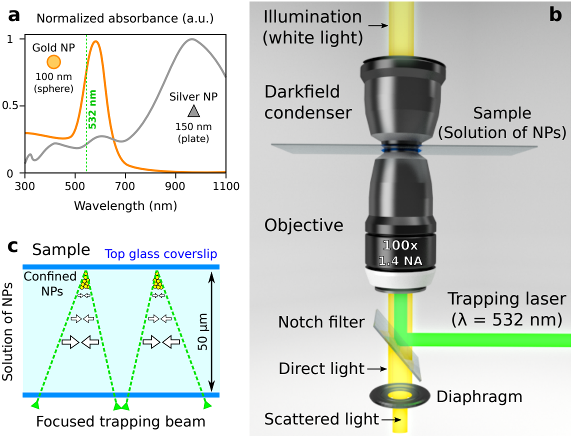
This is created by using a cylindrical lens

[Z. Yan et al, PRL 114, 1-5 (2015)]

Our proposal: Application of phase gradient forces for both confinement and transport of NPs along **arbitrary and reconfigurable** trajectories

Light-driven transport of plasmonic NPs
2D trapping against the coverslip

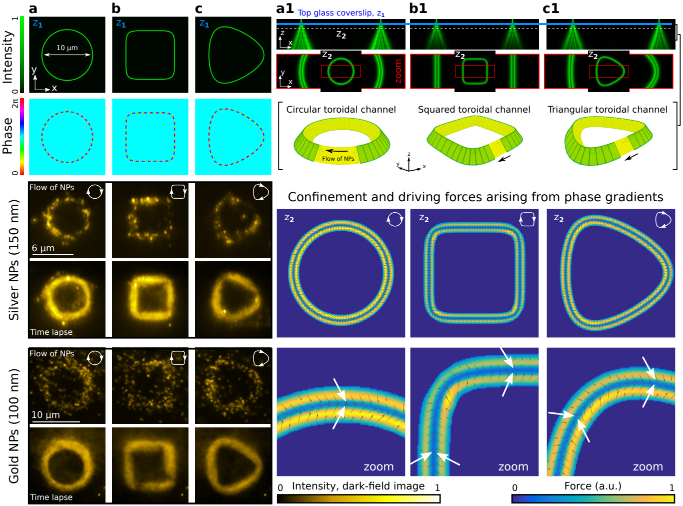
- Polymorphic beam is focused by the objective lens on the top sample coverslip (inverted dark-filed microscope)



[J. A. Rodrigo & T. Alieva, Sci. Rep. 6, 33729 (2016)]

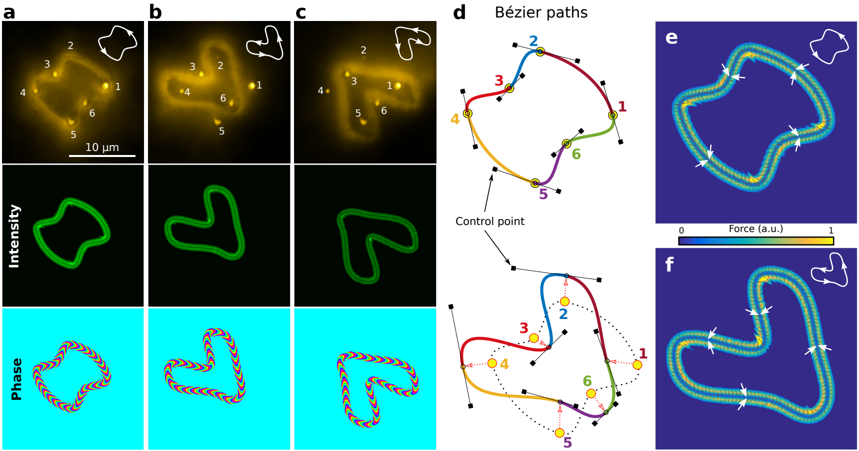
Scattering force NPs confinement and transport

NPs confinement by phase gradient forces in channels created by PBs near focusing plane: $\mathbf{F}_{\nabla\varphi} \propto \mathbf{u}_{\perp}\partial_{\perp}\varphi + \mathbf{u}_{\shortparallel}\partial_{\shortparallel}\varphi$. [J. A. Rodrigo & T. Alieva, Sci. Rep. **6**, 33729, 2016]



Programmable transport routing of NPs

2D trajectories programmed on demand to around or impact on objects in the host environment. Experimental results:



[J. A. Rodrigo & T. Alieva, Sci. Rep. 6, 33729 (2016)]

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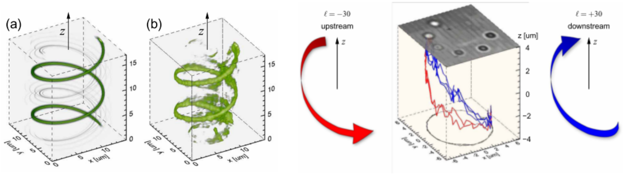
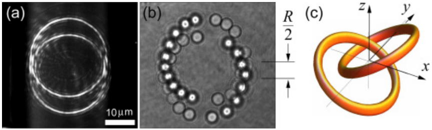
Superposition of Helical Bessel beams $(m = 30)$



S. Lee et al., Opt. Express 2010



E. Shanblatt & D. Grier, Opt. Express 2011

How to construct arbitrary 3D trap?

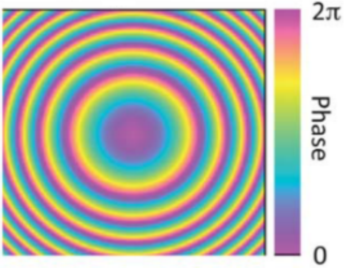
Polymorphic beam in 3D

3D parametric curve $(x_0(t), y_0(t), z_0(t))$ [J. A. Rodrigo et al, Opt. Express 2013; J. A. Rodrigo & T. Alieva, Optica 2, 812 (2015)]

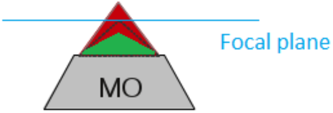
Add the quadratic phase function to $\Psi(t)$

$$\varphi(\mathbf{r}, t) =$$

$$\exp\left(i\pi \frac{[x - x_0(t)]^2 + [y - y_0(t)]^2}{\lambda f^2} z_0(t)\right),$$



yields a *defocusing* distance $z_0(t)$ defined along the curve projection.
e.g. $z_0(t) > 0$ **and** $z_0(t) < 0 \Longrightarrow$

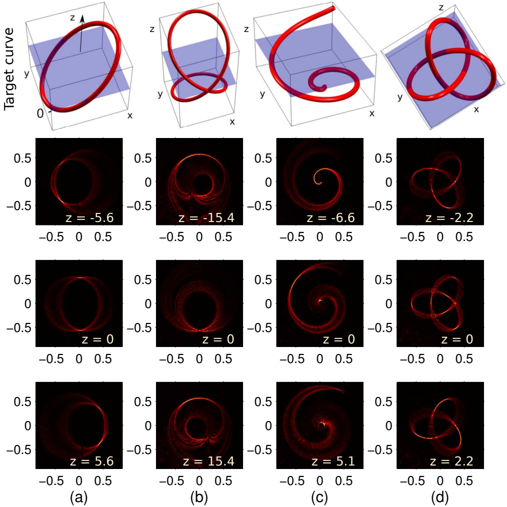


The 3D curved beam comprises focused spots coherently combined in phase and amplitude

3D traps shaped in arbitrary curves

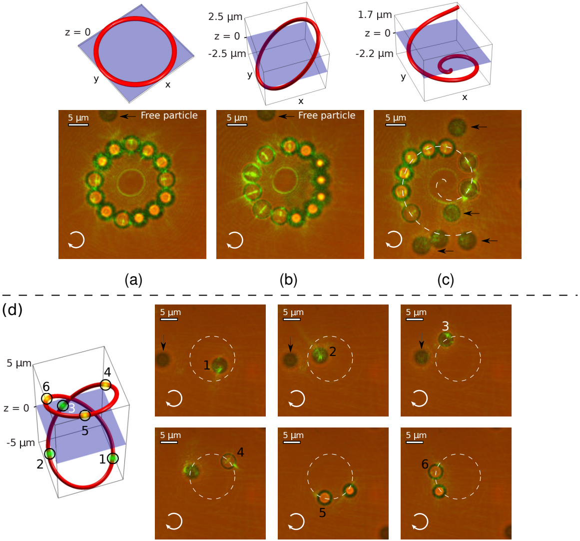
High intensity gradients along prescribed trajectories

Experimental results [J. A. Rodrigo et al, Opt. Express 2013]



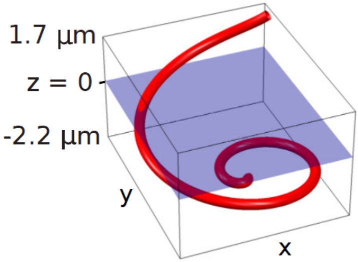
3D curved trapping beams in action

4.7μm diameter polystyrene sphere (ring-vortex of radius R =10μm, $l = 30$, contained in the focal plane and inclined at 14^0) [J. A. Rodrigo et al, Opt. Express 2013]



3D curved trapping beams in action

Switchable tractor beams: $4.7\mu\mathrm{m}$ diameter polystyrene sphere [J. A. Rodrigo et al, Opt. Express 2013]



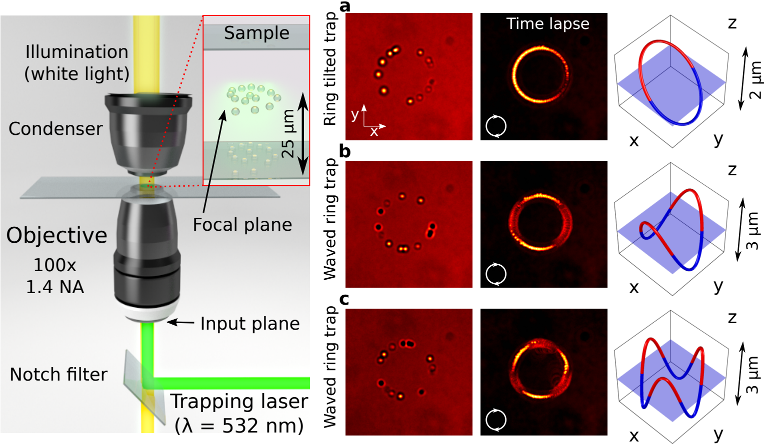
Tilted ring: $\sigma = +1$ or -1 $\Rightarrow$ clockwise/anti-clockwise motion

Archimedean spiral: $\sigma = +1$ or -1 $\Rightarrow$ for upstream retrograde (tractor beam) or downstream motion

3D curved beams driving 1μm silica particles

Switchable tractor beams

Freestyle laser traps and dielectric microparticles



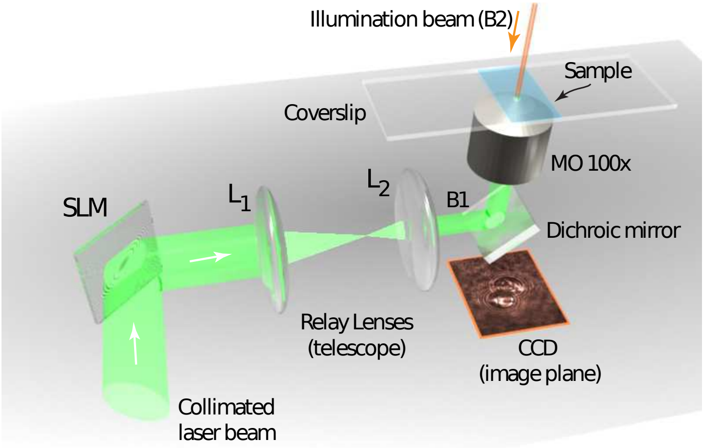
[J. A. Rodrigo & T. Alieva, Optica 2, 812 (2015)]

3D curved beams driving $1\mu\mathrm{m}$ silica particles

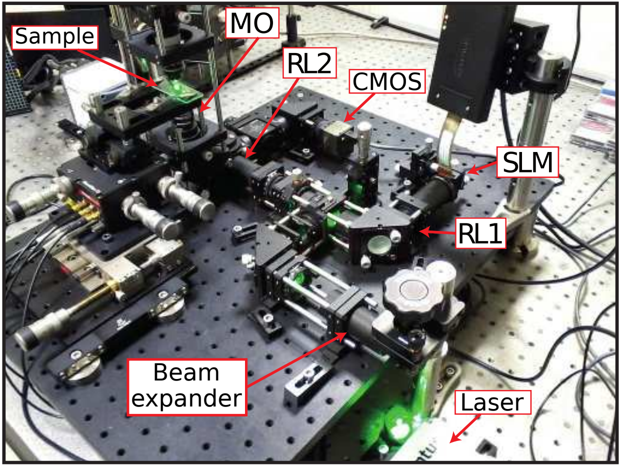
Switchable tractor beams [J. A. Rodrigo et al, Optica 2, 812 (2015)]

How to create 2D and 3D curved trap?

Comptuter Generated Hologram (CGH) implemented by Spatial Light Modulator (SLM)



How to create 2D and 3D curved trap?
Optical setup



[J. A. Rodrigo et al, Opt. Express 2013]

How to encode polymorphic beam?

Computer generated holograms (CGHs)

- The goal of CGH is to create a desired light distribution in the observation plane.
- The operation of CGH is based on the diffraction of light.
- The CGHs are implemented by diffractive optical element (DOE), spatial light modulators (SLMs) which can modulate field amplitude, phase or both of them
- Applications: optical lithography and fabrication, lenses, zone plates, diffraction gratings, array illuminators, phase spatial filters).
- First steps: Detour phase hologram, Kinoform, Phase contour holograms, etc

[B. R. Brown & A. W. Lohmann, Appl. Opt. 5, 967 (1966). A. W. Lohmann & D. P. Paris, Appl. Opt. 7, 1739 (1967). W. H. Lee, Appl. Opt. 13, 1677 (1974)]

Phase-only hologram encoding: general approach

V. Arrizón, et al., J. Opt. Soc. Am. A 24, 3500 (2007); J. P. Kirk & A. L. Jones, J. Opt. Soc. Am. 61, 1023 (1971); J. Davis et al, Appl. Opt . 38, 5005 (1999)

- Complex function to encode $f(x,y) = a(x,y)\exp[i\phi(x,y)]$, amplitude $a(x,y) \in [0,1]$ and phase $\phi(x,y) \in [-\pi,\pi]$
- Corresponding phase-only function $h(x,y) = \exp[i\psi(a,\phi)]$
- The representation of $h(x,y)$ by a Fourier series in the domain of ϕ

$$h(x,y) = \sum_{n=-\infty}^{\infty} h_n(x,y) = \sum_{n=-\infty}^{\infty} c_n(a)\exp[in\phi]$$

where

$$c_n(a) = (2\pi)^{-1}\int_{-\pi}^{\pi} \exp[i\psi(a,\phi)]\exp[-in\phi]\,d\phi$$

The signal $f(x,y)$ is recovered from the first-order term $h_1(x,y)$ if $c_n(a) = Aa$, A is a positive constant, $\max A = 1$.

CGHs: different methods of codifications

V. Arrizón, et al., J. Opt. Soc. Am. A 24, 3500 (2007)

- Necessary and sufficient conditions:

$$\int_{-\pi}^{\pi} \sin\left[\psi(a,\phi) - \phi\right] d\phi = 0$$
$$\int_{-\pi}^{\pi} \cos\left[\psi(a,\phi) - \phi\right] d\phi = 2\pi A a$$

- Type I: $\psi(a,\phi) = s(a)\phi \Rightarrow c_n(a) = \mathrm{sinc}[n - s(a)]$.
  $s(a)$ is obtained by numerically inverted the equation
  $\mathrm{sinc}[1 - s(a)] = a$ $(A = 1)$

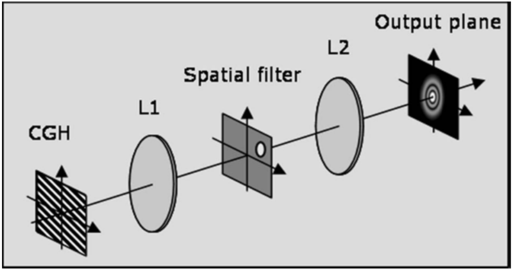
- Type II: $\psi(a,\phi) = \phi + s(a)\sin\phi \Rightarrow c_n(a) = J_{n-1}[s(a)]$.
  $s(a)$ is obtained by numerically inverted the equation
  $J_0[s(a)] = a$ $(A = 1)$

- Type III: $\psi(a,\phi) = s(a)\sin\phi \Rightarrow c_n(a) = J_n[s(a)]$.
  $s(a)$ is obtained by numerically inverted the equation
  $J_1[s(a)] = Aa$ $(\max A \cong 0,5819)$, J_n is a Bessel function of order n.

CGHs: phase carrier

V. Arrizón, et al., J. Opt. Soc. Am. A 24, 3500 (2007)

- The modified hologram transmittance $h(x,y) = \sum_{n=-\infty}^{\infty} h_n(x,y)\exp\left[i2\pi n(u_0 x + v_0 y)\right]$ to separate and filter the $h_1(x,y)$ in Fourier domain
- Fourier spectrum $H(u,v) = \sum_{n=-\infty}^{\infty} H_n(u - nu_0, v - nv_0)$



- Polymorphic beam provide
  - diversity of forms with inherent biomimicry
  - high intensity gradients in focal region
  - independent phase gradient control
  - arbitrary design of the optical current along the circuit

- Obstacle avoidance technique based on re-configurable Bézier light curves paves the way for optical micro-robotics.

- Transport of micro and nano particles applications: cell scale phototherapy, drug delivery, micro-rheology, multiparticle dynamic study, etc.

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THANK YOU!

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Image by J A Rodrigo