The Abdus Salam
International Centre
for Theoretical Physics

Winter College on Optics: Advanced Optical Techniques for Bio-imaging

POLARIZATION MICROSCOPY: BIOMEDICAL IMAGING AND DIAGNOSTICS

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Date & time:

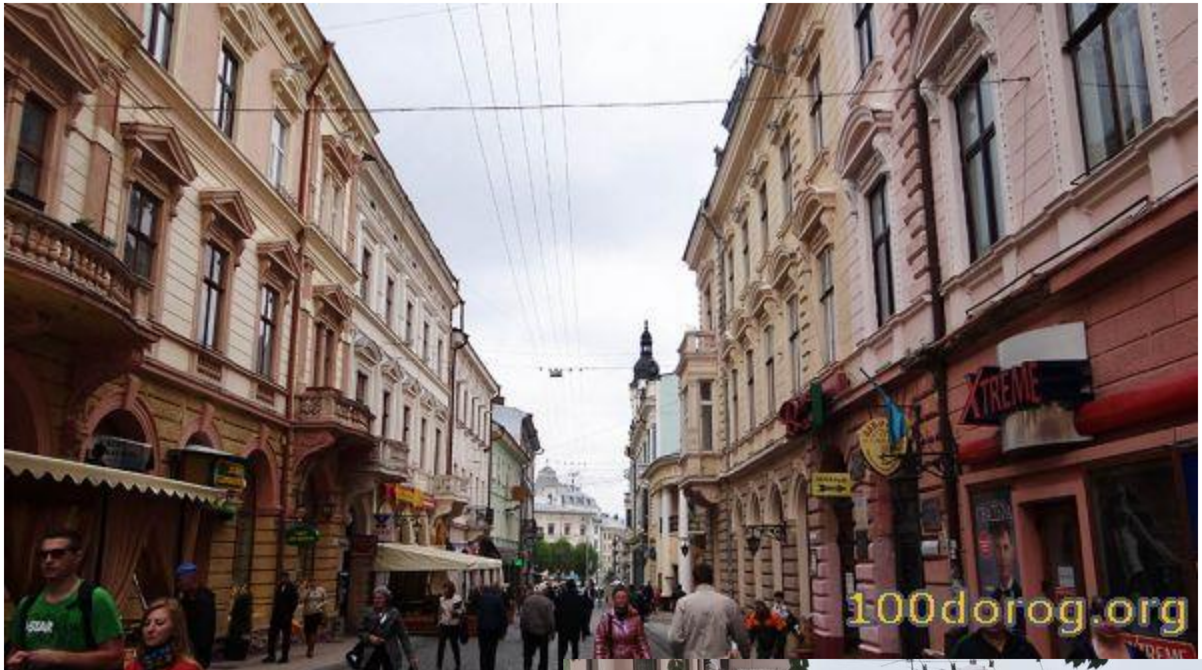
(Lecture) February 17, 2017 (Friday), 15.30
(Experiment) February 23 , 2017 (Thursday), 14.00
Room: Leonardo Building - Budinich Lecture Hall

List of Lectures

- **Introduction.**
- **Lecture 1. Basic concepts. Polarization. Stokes vector. Mueller matrix. Basics of laser polarimetry.**
- **Lecture 2.** Basics of model description of structure and optical anisotropy of biological tissues.
- **Lecture 3.** Methods and resources of analysis and processing of biological tissues polarization-inhomogeneous images.
- **Lecture 4.** Principles and methods of polarization and Mueller-matrix mapping.


SKYANDMETHOD.COM
Drone aerial photo and video imaging


УНІВЕРСИТЕТ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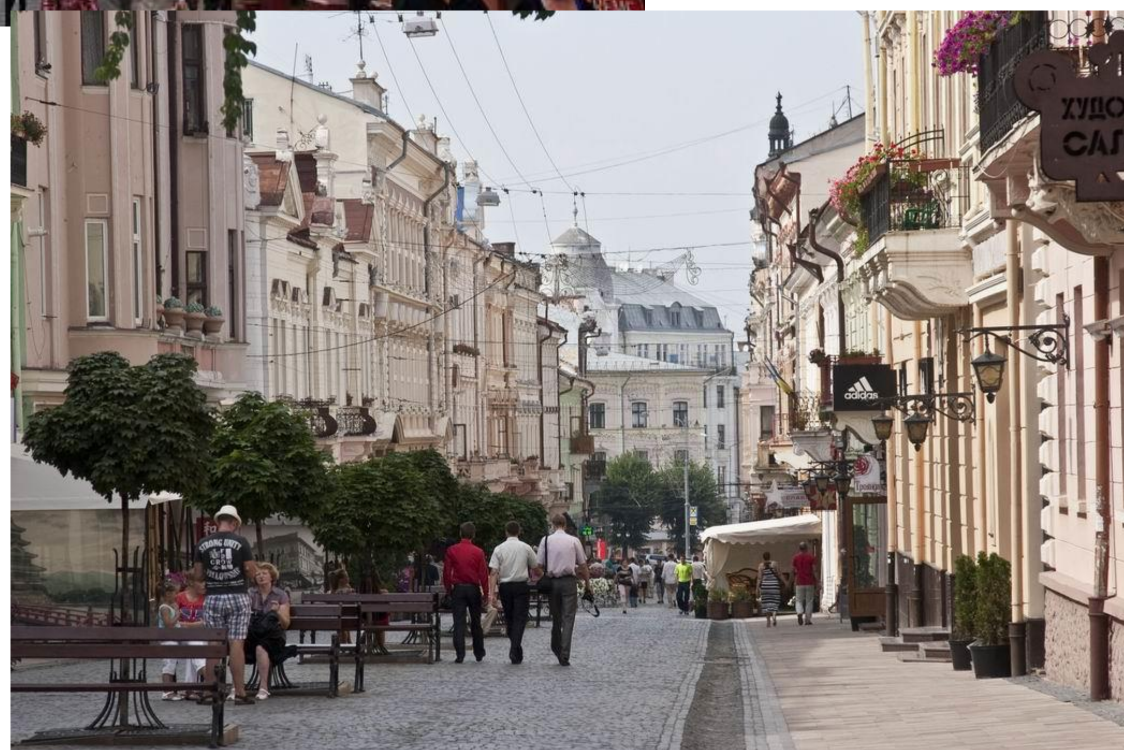
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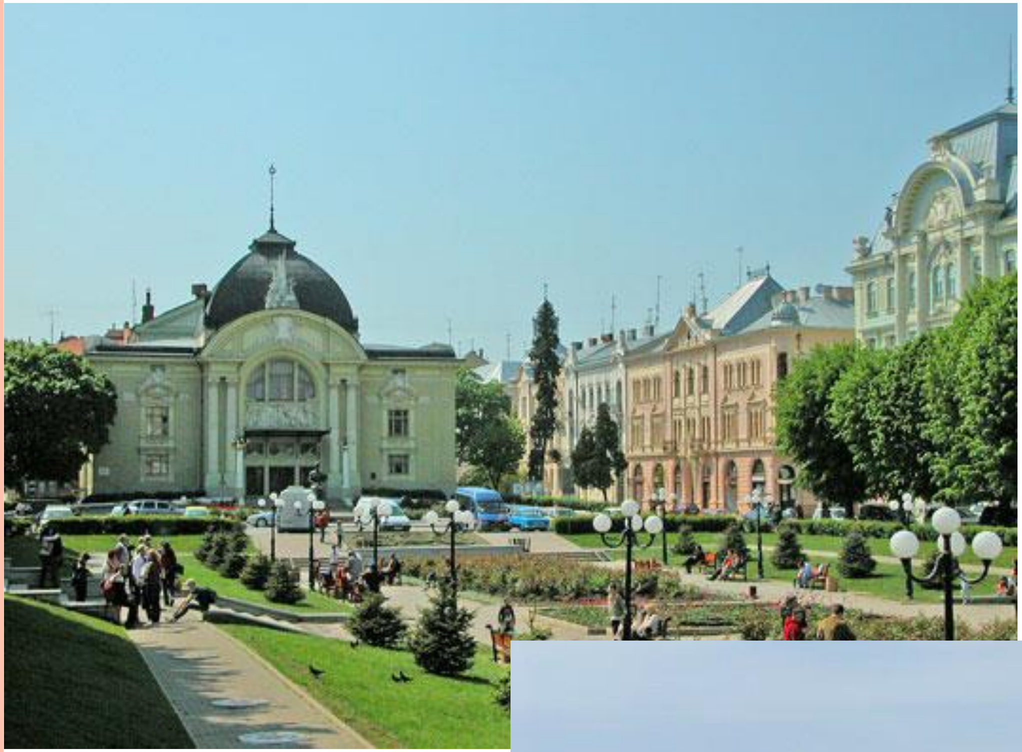
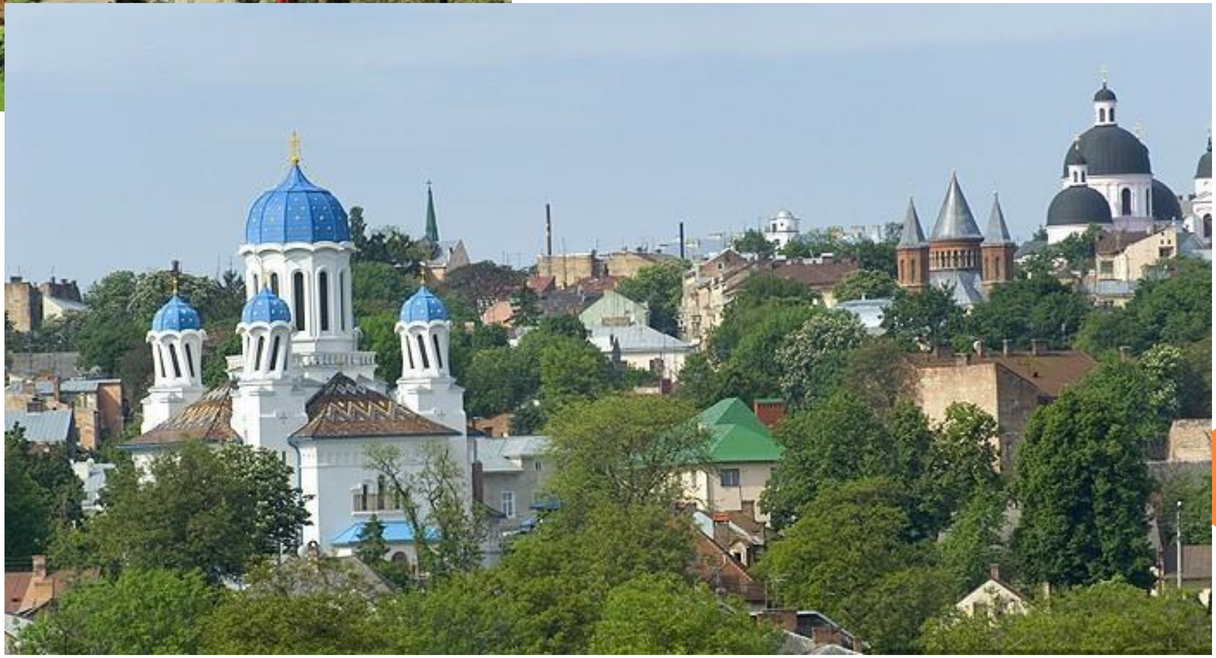



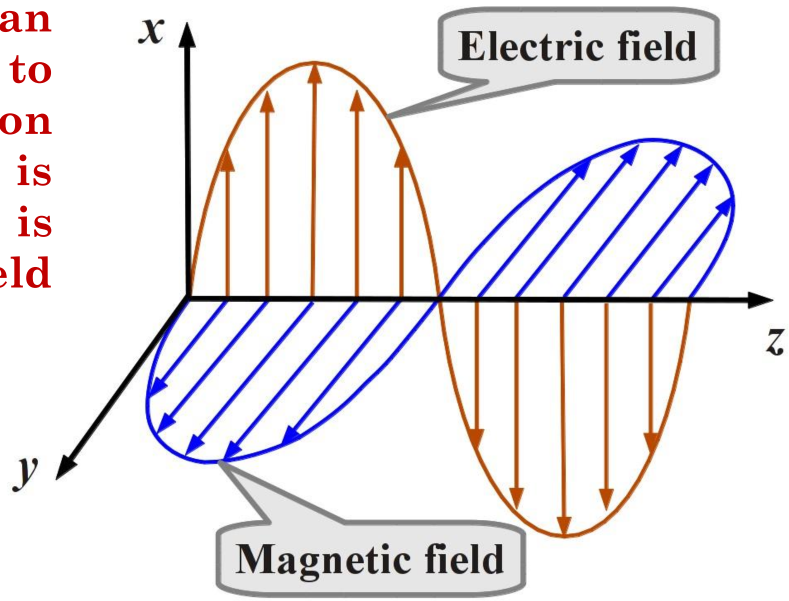
INTRODUCTION

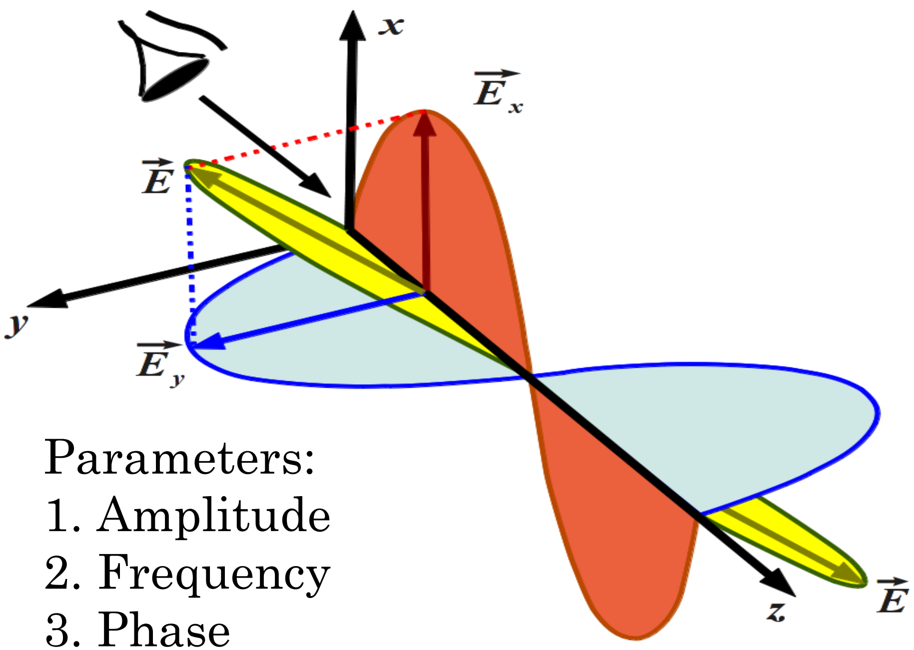
- Optical methods of diagnostics of biological objects and visualization of their structures occupy a leading position thanks to their high information content, multi-functional capabilities (photometric, spectral, and polarization correlation).
- It should be stated that new scientific direction - optics of biological tissues and fluids was finally formed and rapidly developing. The main areas of basic research are the results of theoretical and experimental studies of photon transport in biological tissues and fluids.
- A separate direction in optics of biological tissues formed polarimetric investigations. Analysis of polarization characteristics of the scattered radiation allows to obtain qualitatively new results on morphological and physiological state of biological tissues.
- A new step in the development of methods of optical diagnostics of biological tissues was successful unification of polarimetric and fluorescent techniques.

LECTURE 1. BASIC CONCEPTS. POLARIZATION. STOKES VECTOR. MUELLER MATRIX. BASICS OF LASER POLARIMETRY.

Light as transverse electromagnetic wave

The electric and magnetic fields of an electromagnetic wave are perpendicular to each other and transverse to the direction of propagation. An electromagnetic wave is propagating along z-axis. Its electric field is aligned to the x-axis and magnetic field along the y-axis.





$$\vec{E}_x(z,t) = E_{0x}\cos(kz - \omega t)\,\vec{x}$$
$$\vec{E}_y(z,t) = E_{0y}\cos(kz - \omega t + \delta)\,\vec{y}$$

Parameters:
1. Amplitude
2. Frequency
3. Phase
4. **Polarization**

$\omega = 2\pi\nu$ – *angular frequency*

$k = 2\pi/\lambda$ – *wave number*

δ – *phase (initial)*

Polarization of electromagnetic wave

Polarization is a important property of electromagnetic waves. In communications, completely polarized waves are used. In radio astronomy un-polarized components exist. The techniques to analyze polarization known as ***polarimetry*.**

The complete polarization types of electromagnetic waves are:

(*i*) *Linear Polarization.*
(*ii*) *Circular Polarization.*
(*iii*) *Elliptical Polarization.*

Electromagnetic waves from of radio astronomical sources may posses:

(*i*) *Random polarization (also known as un-polarized waves).*
(*ii*) *Partial polarization (completely polarized + un-polarized)*

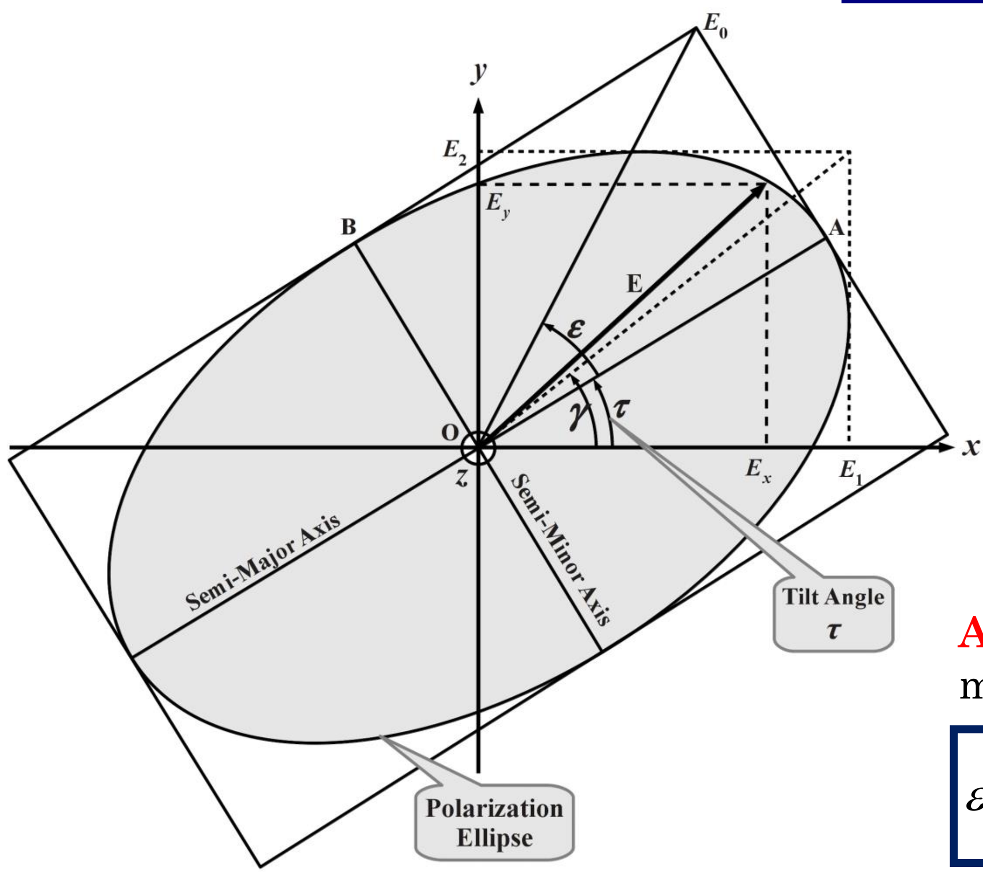
Polarization of electromagnetic wave. Graphical representation. Polarization ellipse

$$\vec{E}_x(z,t) = E_{0x}\cos(kz - \omega t)\,\vec{x}$$
$$\vec{E}_y(z,t) = E_{0y}\cos(kz - \omega t + \delta)\,\vec{y}$$

Some algebra

$$\boxed{\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta = \sin^2\delta}$$

equation of an ellipse



An ellipse can be characterized by:

1. size of minor axis
2. size of major axis
3. orientation (tilt angle, azimuth)
4. Axial ratio (ellipticity)
5. sense (CW, CCW)

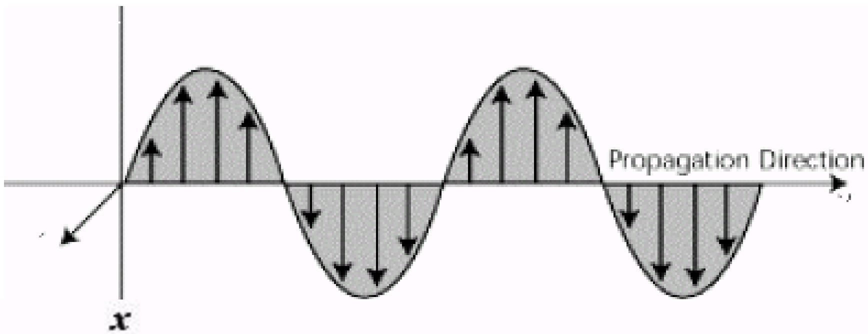
Axial ratio - is a ratio of length of minor to the length of major axis.

$$\boxed{\varepsilon = \pm\arctan\left(\frac{OB}{OA}\right)}$$ **- ellipticity (angle)**

Polarization of electromagnetic wave. Types of polarization. Linear polarization

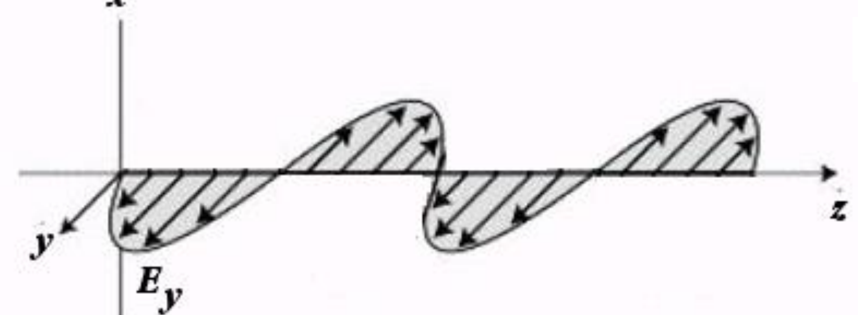
Any form of complete polarization resulting from a coherent source can be analyzed using *polarization ellipse* !!!

$$\vec{E}_x(z,t) = E_{0x}\cos(kz - \omega t)\,\vec{x}$$
$$\vec{E}_y(z,t) = E_{0y}\cos(kz - \omega t + \delta)\,\vec{y}$$



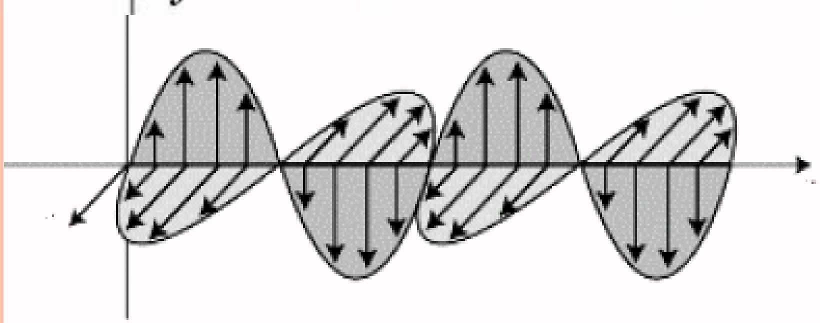
$\tau = 0^0$

If there is no amplitude in y (E_{0y} = 0), there is only one component, in x (vertical).

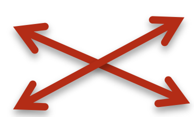


$\tau = 90^0$

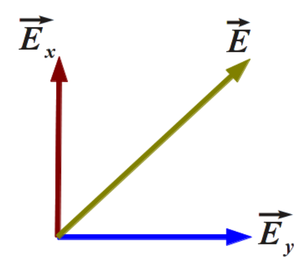
If there is no amplitude in x (E_{0x} = 0), there is only one component, in y (horizontal).



$\tau = \pm 45^0$



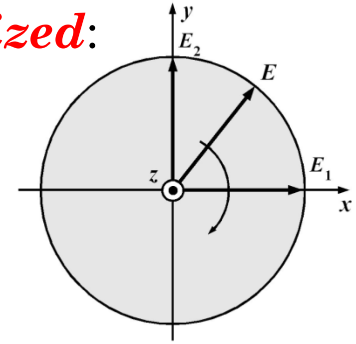
Phase difference ($\delta = 0; \pi$) and $E_{0x} = E_{0y}$, then $\mathbf{E}_x = \mathbf{E}_y$



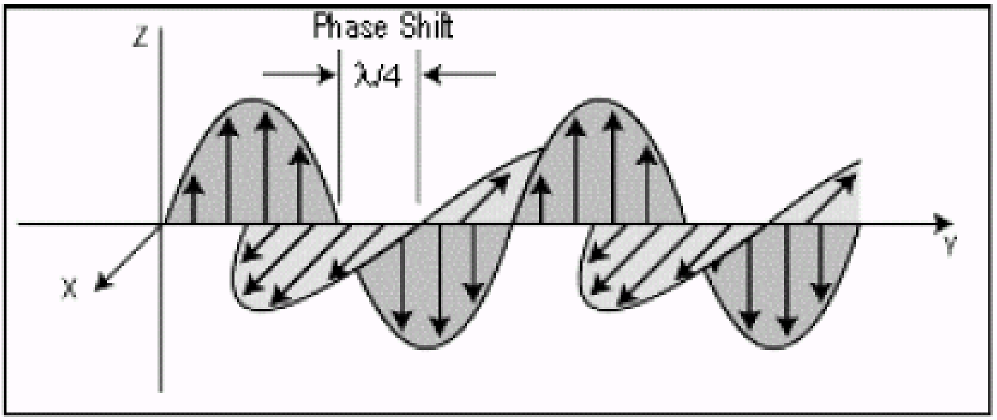
Polarization of electromagnetic wave. Types of polarization. Circular polarization

$$\vec{E}_x(z,t) = E_{0x}\cos(kz - \omega t)\,\vec{x}$$
$$\vec{E}_y(z,t) = E_{0y}\cos(kz - \omega t + \delta)\,\vec{y}$$

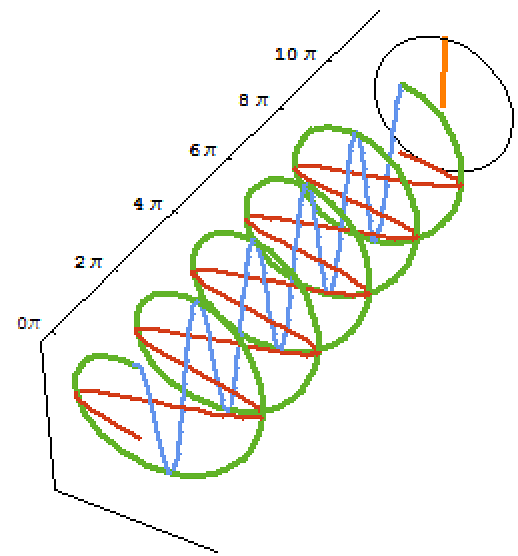
If the phase difference is $\delta = \pm 90^0$ and $E_{0x} = E_{0y}$ then: $\mathbf{E}_x / \mathbf{E}_{0x} = \cos\Theta$, $\mathbf{E}_y / \mathbf{E}_{0y} = \sin\Theta$ and we get the equation of a **circle** with CW or CCW rotation and wave is said to be ***circularly polarized***:



$$\boxed{\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 = \cos^2\Theta + \sin^2\Theta = 1}$$

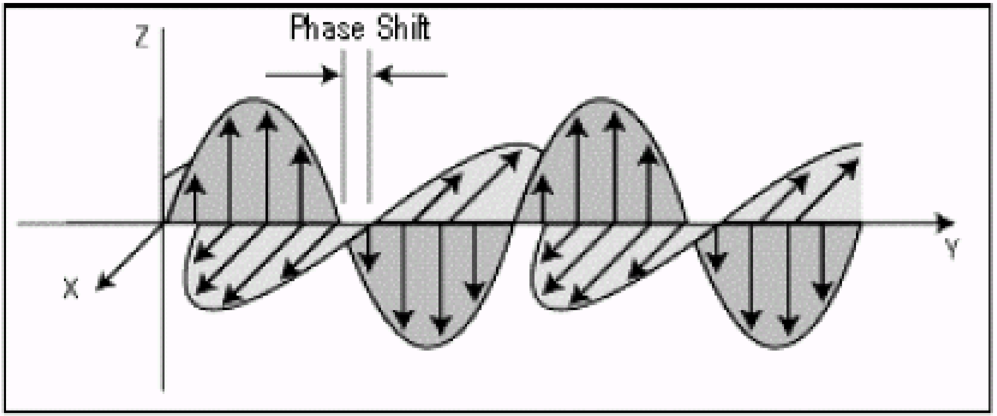


C. Circularly Polarized Light

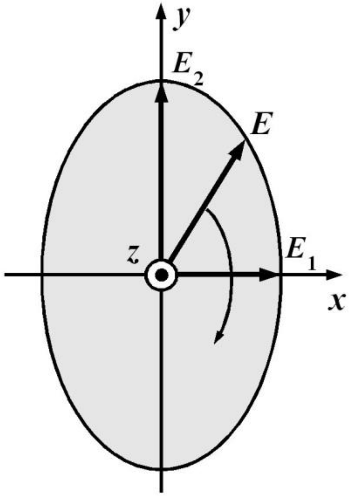


Polarization of electromagnetic wave. Types of polarization. Elliptical polarization.

If the magnitudes of **E**x and **E**y are not equal, and there exists a phase difference between the two, the tip of the electric field vector describes an ellipse and wave is said to be ***elliptically polarized***.



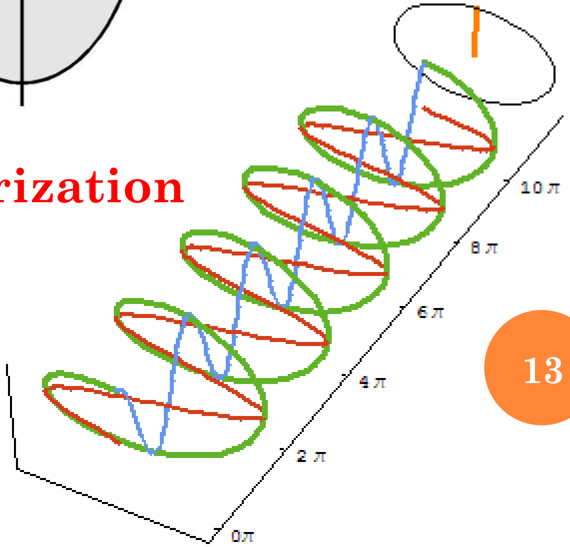
D. Elliptically Polarized Light



Linear + circular polarization = elliptical polarization

Any wave may be written as a superposition of the two polarizations

ANIMATIONS



Stokes parameters

1852: Sir George Gabriel Stokes took a **very different approach** and discovered that polarization can be described in terms of **observables** using an **experimental definition.**

The polarization ellipse is only valid at a given instant of time (function of time)!!!

$$\left(\frac{E_x}{E_{0x}}\right)^2+\left(\frac{E_y}{E_{0y}}\right)^2-2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta=\sin^2\delta$$

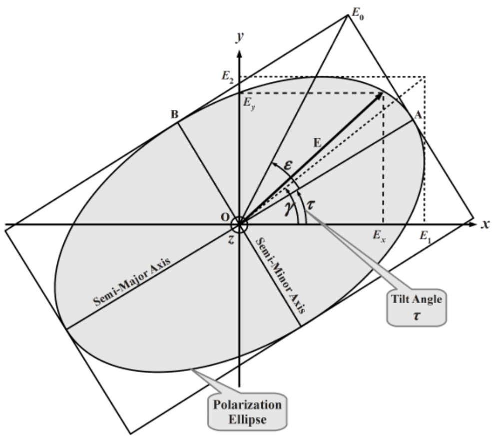
To get the Stokes parameters, do a time average (integral over time) and a little bit of algebra...

$$\left(E_{0x}^2+E_{0y}^2\right)^2-\left(E_{0x}^2-E_{0y}^2\right)^2-\left(2E_{0x}E_{0y}\cos\delta\right)^2=\left(2E_{0x}E_{0y}\sin\delta\right)^2$$

The 4 Stokes parameters are:

$$\boxed{\begin{aligned} S_0 &= I = E_{0x}^2+E_{0y}^2 \\ S_1 &= Q = E_{0x}^2-E_{0y}^2 \\ S_2 &= U = 2E_{0x}E_{0y}\cos\delta \\ S_3 &= V = 2E_{0x}E_{0y}\sin\delta \end{aligned}}$$

Stokes parameters described in geometrical terms. Stokes vector



$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ \cos 2\varepsilon \cos 2\tau \\ \cos 2\varepsilon \sin 2\tau \\ \sin 2\varepsilon \end{pmatrix}$$

The Stokes parameters can be arranged in a Stokes vector:

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos\delta \\ 2E_{0x}E_{0y}\sin\delta \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ I(0°) - I(90°) \\ I(45°) - I(135°) \\ I(RCP) - I(LCP) \end{pmatrix}$$

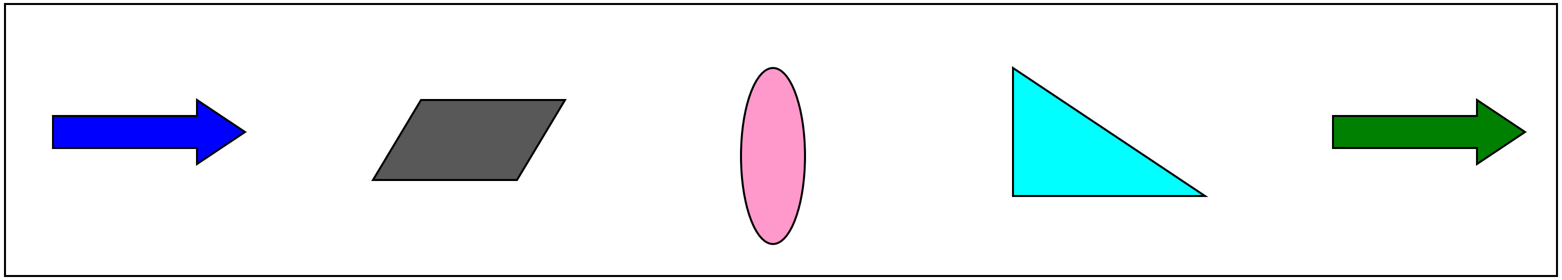
- Linear polarization $Q \neq 0, U \neq 0, V = 0$
- Circular polarization $Q = 0, U = 0, V \neq 0$
- Fully polarized light $I^2 = Q^2 + U^2 + V^2$

Mueller matrices

If light is represented by Stokes vectors, optical components are then described with Mueller matrices:

[output light] = [Muller matrix] [input light]

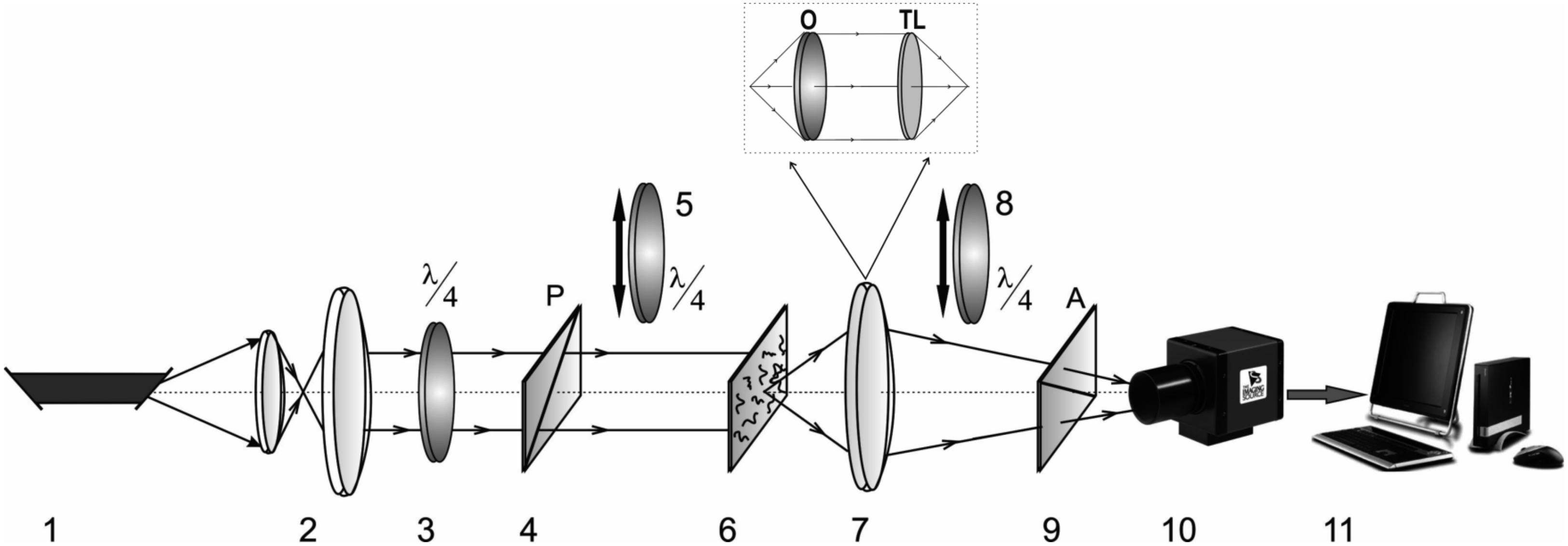
$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$



Element 1 M_1 Element 2 M_2 Element 3 M_3

$$\boxed{S' = M_3 M_2 M_1 S}$$

Basics of laser polarimetry



Need to be measured

$$I_{\min}\begin{pmatrix} r_1,\ldots r_m \\ \ldots\ldots\ldots \\ r_n,\ldots r_m \end{pmatrix}; I_{\max}\begin{pmatrix} r_1,\ldots r_m \\ \ldots\ldots\ldots \\ r_n,\ldots r_m \end{pmatrix}$$

$$\Theta\begin{pmatrix} r_1,\ldots r_m \\ \ldots\ldots\ldots \\ r_n,\ldots r_m \end{pmatrix}\left(I\begin{pmatrix} r_1,\ldots r_m \\ \ldots\ldots\ldots \\ r_n,\ldots r_m \end{pmatrix} \equiv \min \right)$$

Polarization maps (calculation)

$$\alpha\begin{pmatrix} r_1,\ldots r_m \\ \ldots\ldots\ldots \\ r_n,\ldots r_m \end{pmatrix} = \Theta\left(I\left(r_i\right) \equiv \min\right) - \frac{\pi}{2};$$

$$\beta\begin{pmatrix} r_1,\ldots r_m \\ \ldots\ldots\ldots \\ r_n,\ldots r_m \end{pmatrix} = arctg\,\frac{I\left(r_i\right)_{\min}}{I\left(r_i\right)_{\max}}.$$

List of Lectures

- **Introduction**.
- **Lecture 1.** Basic concepts. Polarization. Stokes vector. Mueller matrix. Basics of laser polarimetry.
- **Lecture 2. Basics of model description of structure and optical anisotropy of biological tissues.**
- **Lecture 3.** Methods and resources of analysis and processing of biological tissues polarization-inhomogeneous images.
- **Lecture 4.** Principles and methods of polarization and Mueller-matrix mapping.

Optical anisotropy

Optical anisotropy **- difference in the optical properties of a medium as a function of the direction of propagation of optical radiation (light) in the medium and of the state of polarization of the radiation.**

Amplitude anisotropy (dichroism)

- crystals may similarly show absorption which depends upon polarization.

1. **Linear dichroism**
2. **Circular dichroism**

Phase anisotropy (birefringence)

- asymmetry in crystal structure causes two different refractive indices;
- opposite polarizations follow different paths through crystal.

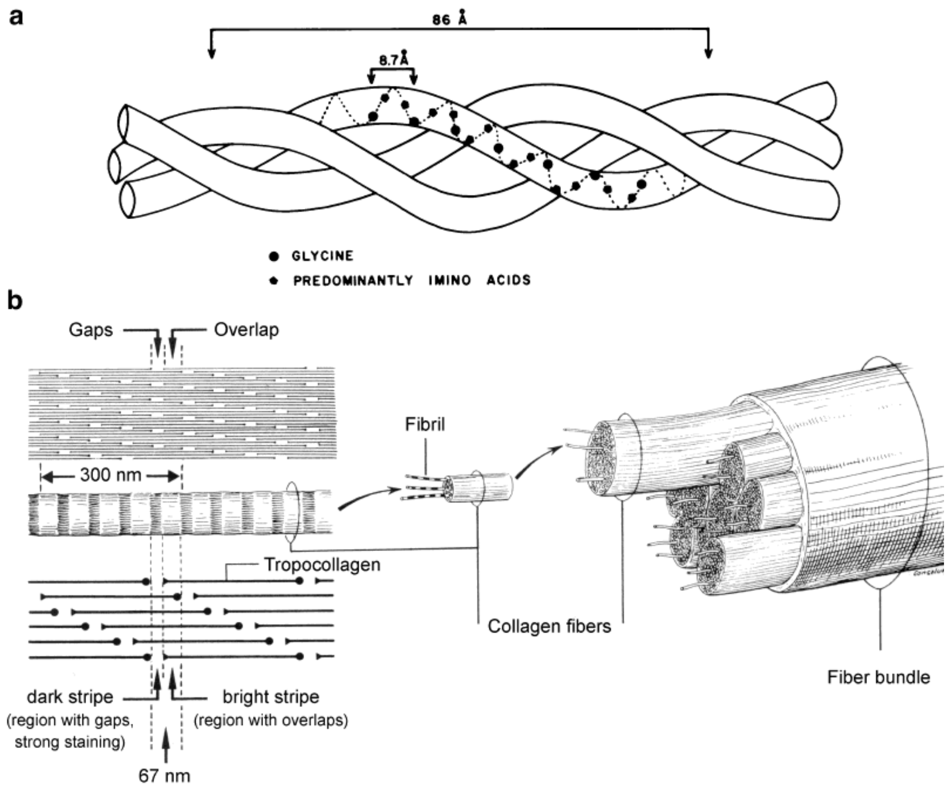
1. **Linear birefringence**
2. **Circular birefringence (optical activity)**

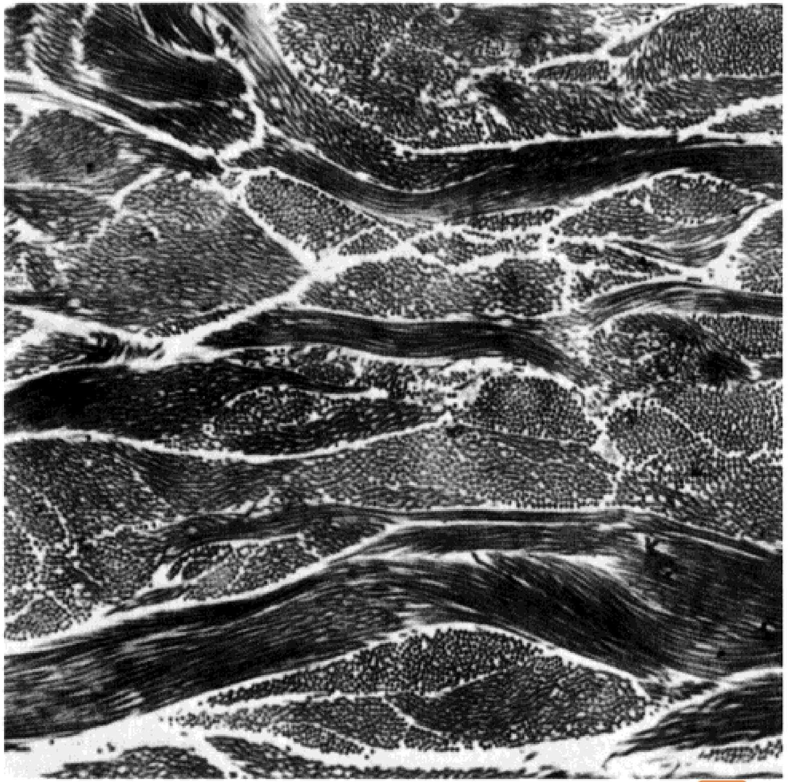
ANIMATIONS

LECTURE 2. BASICS OF MODEL DESCRIPTION OF STRUCTURE AND OPTICAL ANISOTROPY OF BIOLOGICAL TISSUES.

Mueller-matrix approach for description of biological layers with amplitude and phase anisotropies

Soft tissue structure





Biological tissues reveal self-similar (fractal) structure as a result of growth processes

Transmission electron micrograph of human skin (dermis), showing collagen fibers sectioned both longitudinally and transversely. Magnification 4900x.

Mueller-matrix approach for description of biological layers with amplitude and phase anisotropies

Optically anisotropic biological layer			
Phase anisotropy		Amplitude anisotropy	
Optical activity	Linear birefringence	Circular dichroism	Linear dichroism
Parameters			
Polarization plane rotation angle - θ	Phase shift between the orthogonal components of amplitude - δ	Circular dichroism index - Δg	Linear dichroism index - $\Delta\tau$
Partial Mueller matrix - $\{\Omega\}$	Partial Mueller matrix - $\{D\}$	Partial Mueller matrix - $\{\Phi\}$	Partial Mueller matrix - $\{\Psi\}$
Mueller matrix of generalized anisotropy $\{M\}=\{\Omega\}\times\{D\}\times\{\Phi\}\times\{\Psi\}$			
Azimuthally independent Mueller-matrix elements and invariants $M_{ik}(\Theta)=const\ ;\ \Delta M_{ik}(\Theta)=const$			
Algorithms of Mueller-matrix reconstruction of parameters of optical anisotropy			
$\theta=w\left(M^*_{ik},\Delta M^*_{ik}\right)$	$\delta=u\left(M^*_{ik},\Delta M^*_{ik}\right)$	$\Delta g=h\left(M^*_{ik},\Delta M^*_{ik}\right)$	$\Delta\tau=v\left(M^*_{ik},\Delta M^*_{ik}\right)$

Algorithm of Mueller-matrix modeling of biological layer anisotropy

Mueller-matrix operators of mechanisms of phase and amplitude anisotropy

Circular birefringence

$$\{\Omega\} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \omega_{22} & \omega_{23} & 0 \\ 0 & \omega_{32} & \omega_{33} & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix} \qquad \omega_{ik} = \begin{cases} \omega_{22} = \omega_{33} = \cos 2\theta, \\ \omega_{23} = -\omega_{32} = \sin 2\theta. \end{cases}$$

Circular dichroism

$$\{\Phi\} = \begin{Vmatrix} 1 & 0 & 0 & \phi_{14} \\ 0 & \phi_{22} & 0 & 0 \\ 0 & 0 & \phi_{33} & 0 \\ \phi_{41} & 0 & 0 & 1 \end{Vmatrix} \qquad \phi_{ik} = \begin{cases} \phi_{22} = \phi_{33} = \dfrac{1-\Delta g^2}{1+\Delta g^2}, \\ \phi_{14} = \phi_{41} = \pm \dfrac{2\Delta g}{1+\Delta g^2}. \end{cases}$$

Linear birefringence

$$\{D\} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & d_{22} & d_{23} & d_{24} \\ 0 & d_{32} & d_{33} & d_{34} \\ 0 & d_{42} & d_{43} & d_{44} \end{Vmatrix} \qquad d_{ik} = \begin{cases} d_{22} = \cos^2 2\rho + \sin^2 2\rho \cos\delta, \\ d_{23} = d_{32} = \cos 2\rho \sin 2\rho (1-\cos\delta), \\ d_{33} = \sin^2 2\rho + \cos^2 2\rho \cos\delta, \\ d_{42} = -d_{24} = \sin 2\rho \sin\delta, \\ d_{34} = -d_{43} = \cos 2\rho \sin\delta, \\ d_{44} = \cos\delta. \end{cases}$$

Linear dichroism

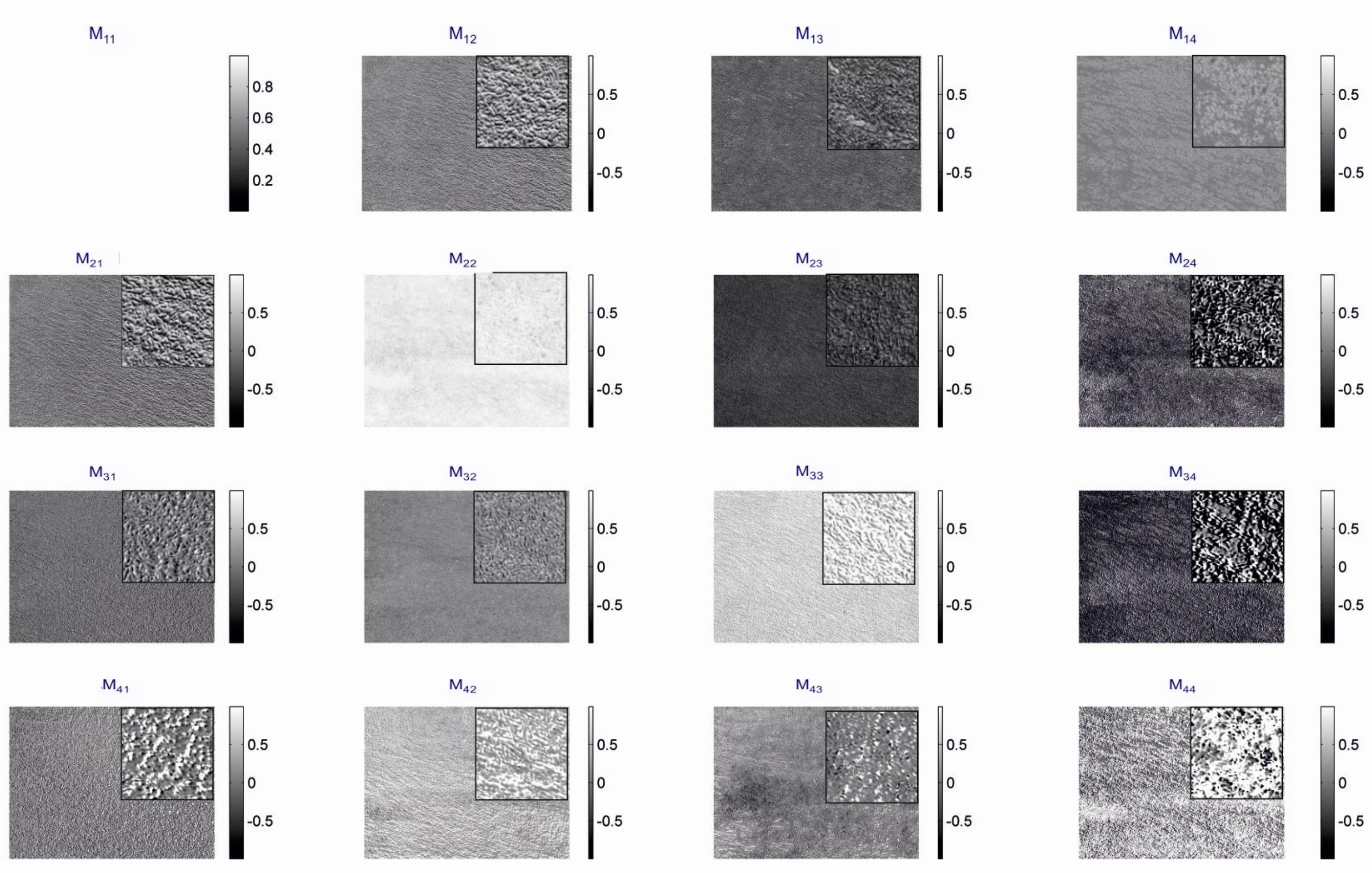
$$\{\Psi\} = \begin{Vmatrix} 1 & \psi_{12} & \psi_{13} & 0 \\ \psi_{21} & \psi_{22} & \psi_{23} & 0 \\ \psi_{31} & \psi_{32} & \psi_{33} & 0 \\ 0 & 0 & 0 & \psi_{44} \end{Vmatrix} \qquad \varphi_{ik} = \frac{1}{1+\Delta\tau} \times \begin{cases} \psi_{12} = \psi_{21} = (1-\Delta\tau)\cos 2\gamma; \\ \psi_{13} = \psi_{31} = (1-\Delta\tau)\sin 2\gamma; \\ \psi_{22} = (1+\Delta\tau)\cos^2 2\gamma + 2\sqrt{\Delta\tau}\sin^2 2\gamma; \\ \psi_{23} = \psi_{32} = \left(1-\sqrt{\Delta\tau}\right)^2 \cos 2\gamma \sin 2\gamma; \\ \psi_{33} = (1+\Delta\tau)\sin^2 2\gamma + 2\sqrt{\Delta\tau}\cos^2 2\gamma; \\ \psi_{44} = 2\sqrt{\Delta\tau}. \end{cases}$$

Generalized Mueller matrix of biological tissue optical anisotropy

$$\boxed{\{M\} = \{\Omega\} \times \{D\} \times \{\Phi\} \times \{\Psi\}}$$

$$\boxed{\{M\} = M_{11}^{-1} \times \begin{Vmatrix} 1 & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{Vmatrix}}$$

Mueller-matrix approach for description of biological layers with amplitude and phase anisotropies



$$M_{ik} \neq 0$$

Mueller-matrix images of skeletal muscle

Information content of Mueller-matrix elements

$\boxed{M_{i=1;k=1;2;3;4}}$ - Mechanisms of optically anisotropic absorption

$\boxed{M_{i=2;3;k=1;2;3;4}}$ - Phase modulation (δ, θ) of laser radiation on the background of optically anisotropic absorption ($\Delta g, \Delta \tau$)

$\boxed{M_{i=4;k=1;2;3;4}}$ - Complex information about superposition of mechanisms of linear birefringence and dichroism

Mueller-matrix invariants

$$\begin{cases} M_{11}(\Theta) = const; M_{14}(\Theta) = const, \\ M_{41}(\Theta) = const; M_{44}(\Theta) = const, \end{cases}$$

$$\frac{M_{23} - M_{32}}{M_{22} + M_{33}} = \Delta M = const$$

$\Rightarrow$

$$\boxed{M_{14} \propto \Delta \tau}$$

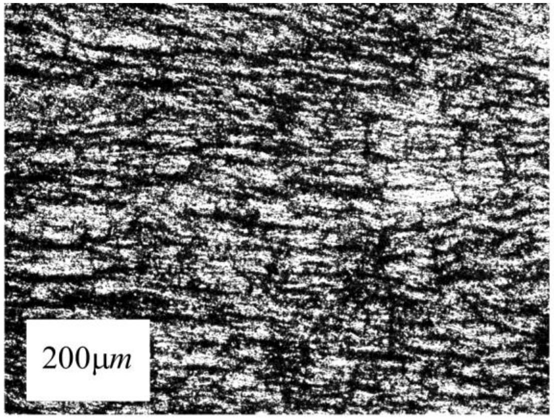
$$\boxed{M_{41} \propto \Delta g}$$

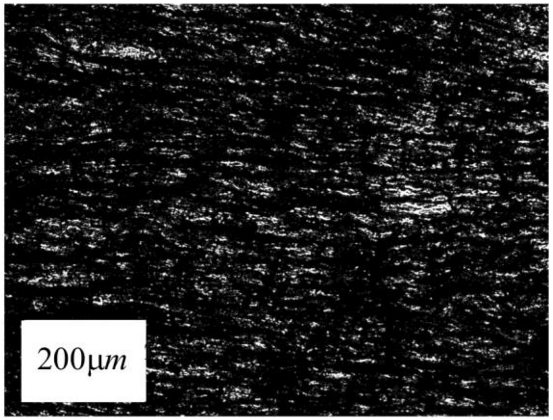
$$\boxed{M_{44} = \cos \delta}$$

$$\boxed{\Delta M = tg 2\theta.}$$

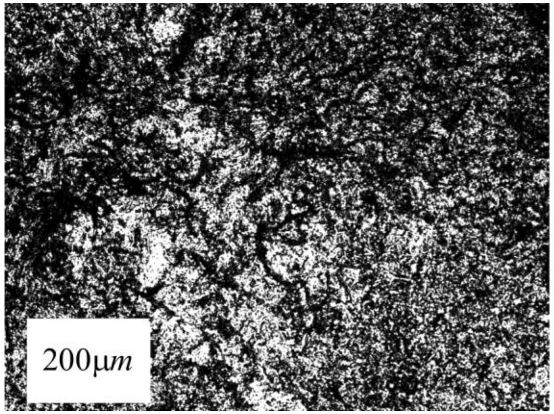
Samples structure (histological sections)

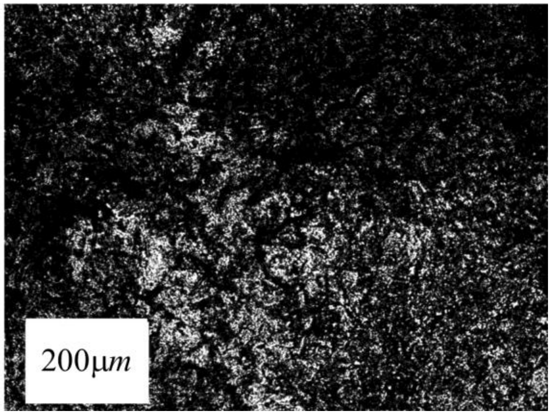
1. Tissue with ordered and disordered structure





Myocardium tissue in coaxial and crossed polarizer-analyzer

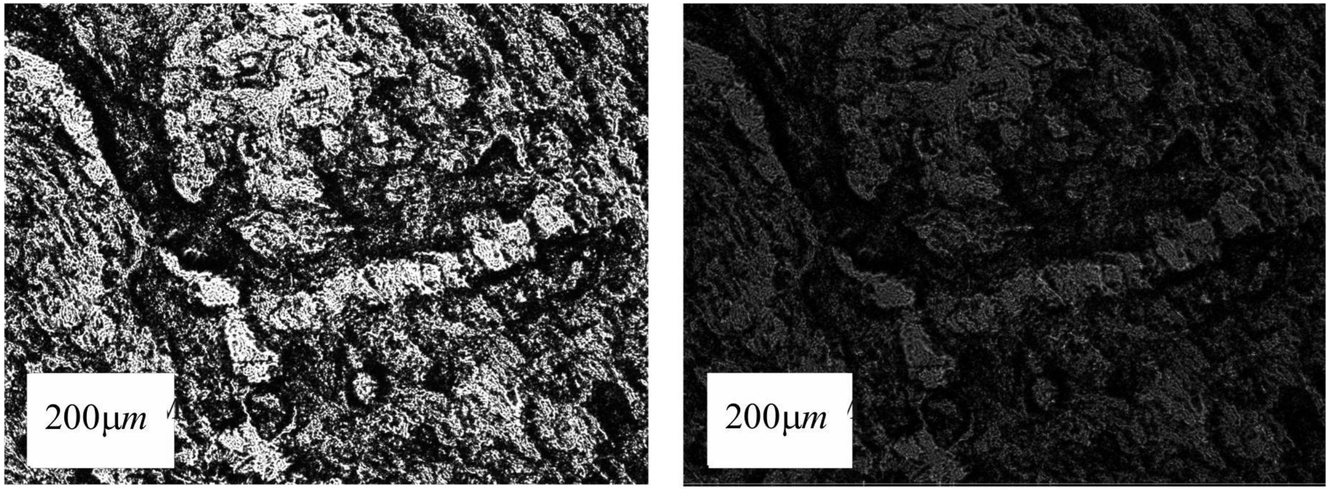




Brain tissue in coaxial and crossed polarizer-analyzer

Samples structure (histological sections)

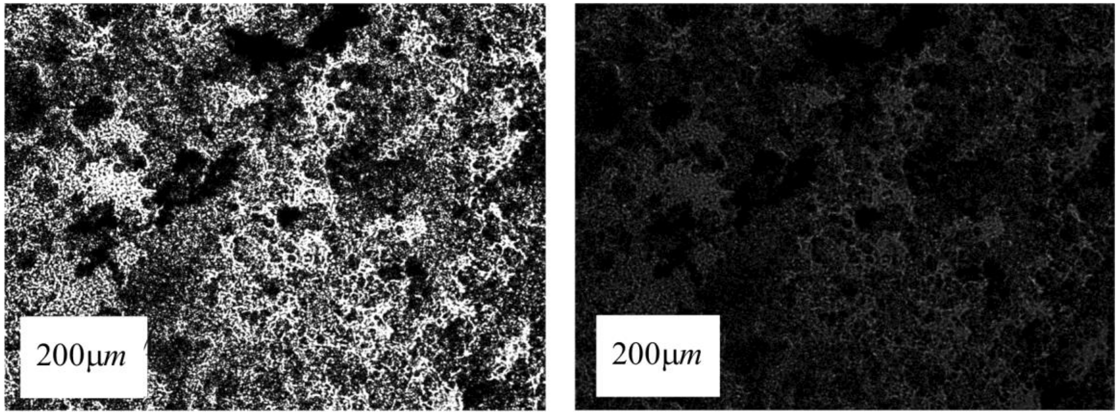
2. Parenchymatous tissue with cluster (disordered) structure



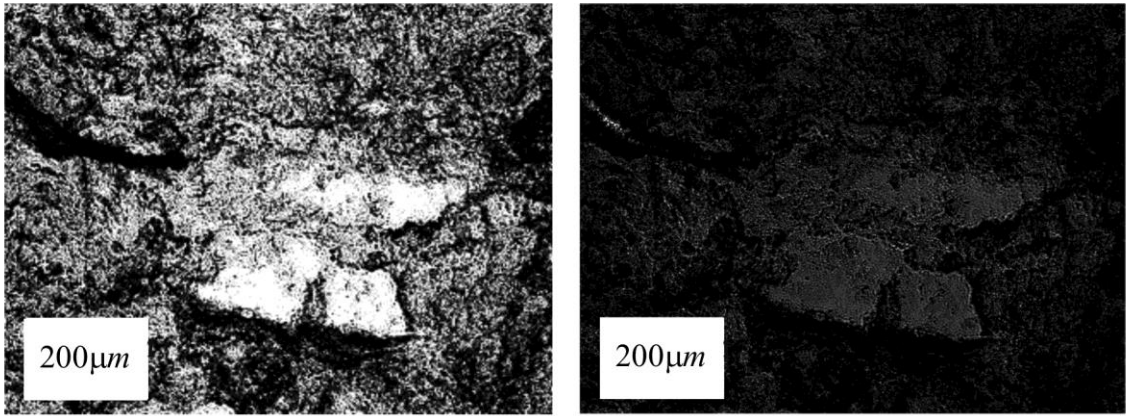
Benign tumor (adenoma) of prostate gland tissue in coaxial and crossed polarizer-analyzer

Samples structure (histological sections)

3. Tissues with benign and malignant formations

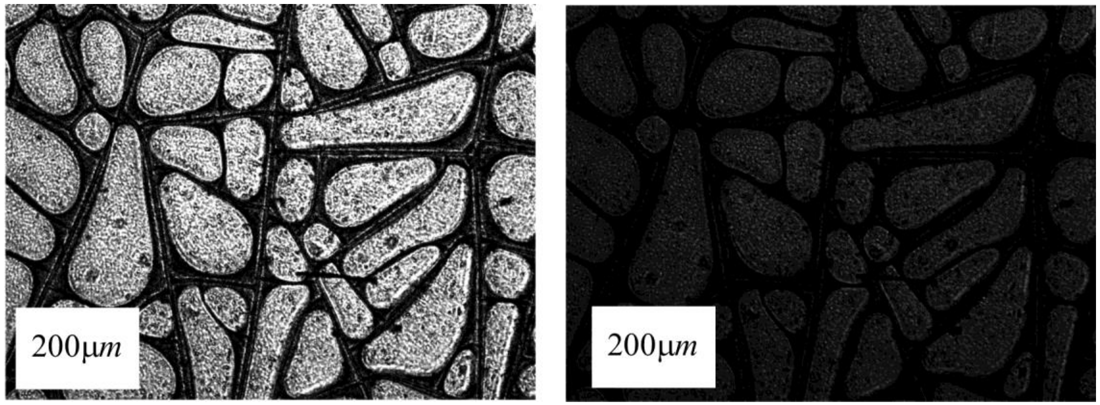


Precancer (dysplasia) of *cervix uteri* tissue

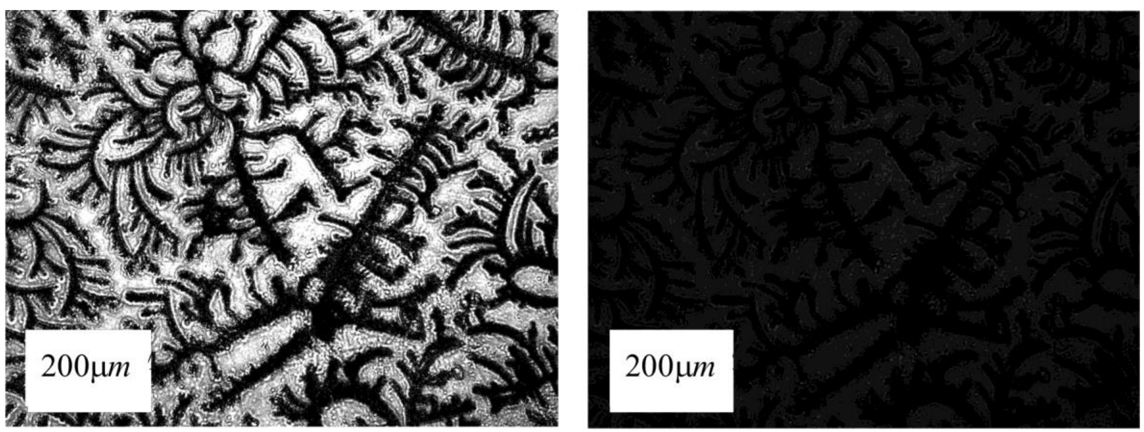


Malignant formation (adenocarcinopma) of *cervix uteri* tissue

Samples structure (dried smears of biological fluids)



Donor blood plasma in coaxial and crossed polarizer-analyzer



Synovial fluid of joint with rheumatoid arthritis in coaxial and crossed polarizer-analyzer

List of Lectures

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Objective criteria

All the data and parameters (*q*) presented in previous lectures need to be quantitatively analyzed!!! It was proposed to use:

1. Statistic analysis

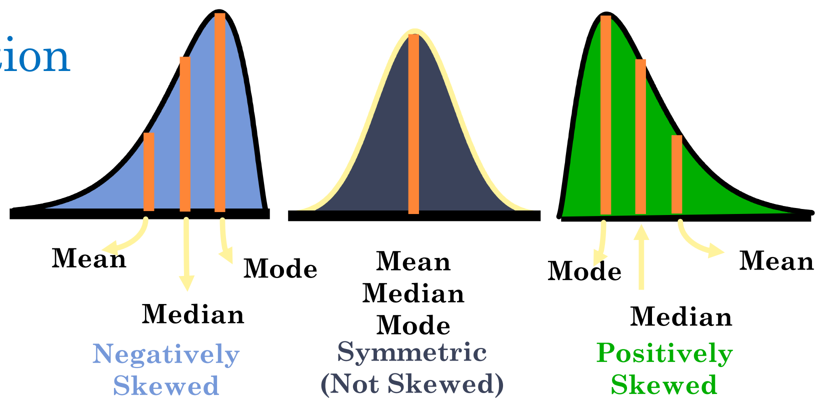
A moment is a quantitative measure of the shape of a set of points.

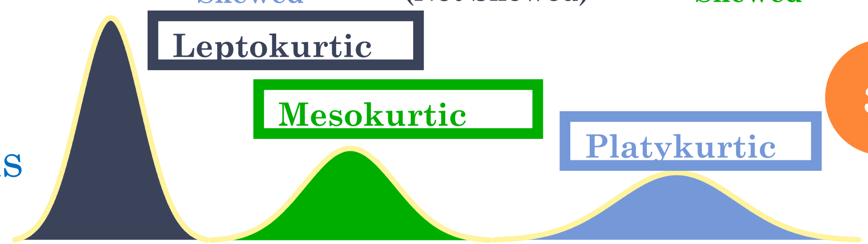
$$Z_1 = \frac{1}{N}\sum_{i=1}^{N}(q)_i$$ - Mean value

$$Z_2 = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(q - Z_1)_i^{\;2}}$$ - Standard deviation

$$Z_3 = \frac{1}{M_2^{\;3}}\frac{1}{N}\sum_{i=1}^{N}(q - Z_1)_i^{\;3}$$ - Skewness

$$Z_4 = \frac{1}{M_2^{\;4}}\frac{1}{N}\sum_{i=1}^{N}(q - Z_1)_i^{\;4}$$ - Kurtosis





Objective criteria

All the data and parameters (*q*) presented in previous lectures need to be quantitatively analyzed!!! It was proposed to use:

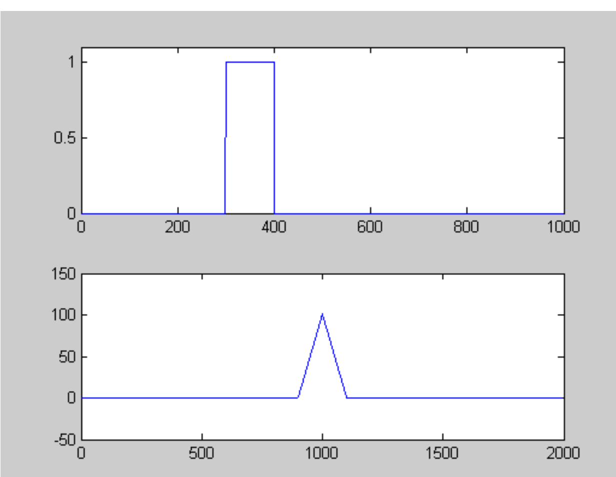
2. Correlation analysis

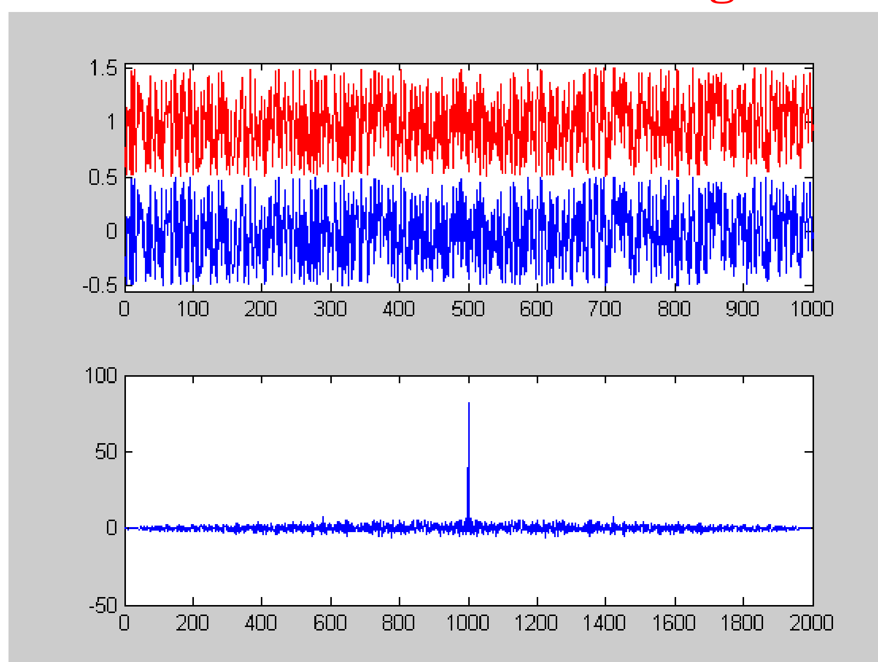
Correlation plays a central role in the study of time series. In general, correlation gives a ***quantitative estimate of the degree of similarity between two functions.*** The correlation of functions g and f both with N samples is defined as:

$$r_k = \frac{1}{N} \sum_{i=0}^{N-k-1} g_i f_{k+i}$$

$$k = 0,1,2,\ldots,N-1$$

Auto-correlation – correlation of a signal with itself.



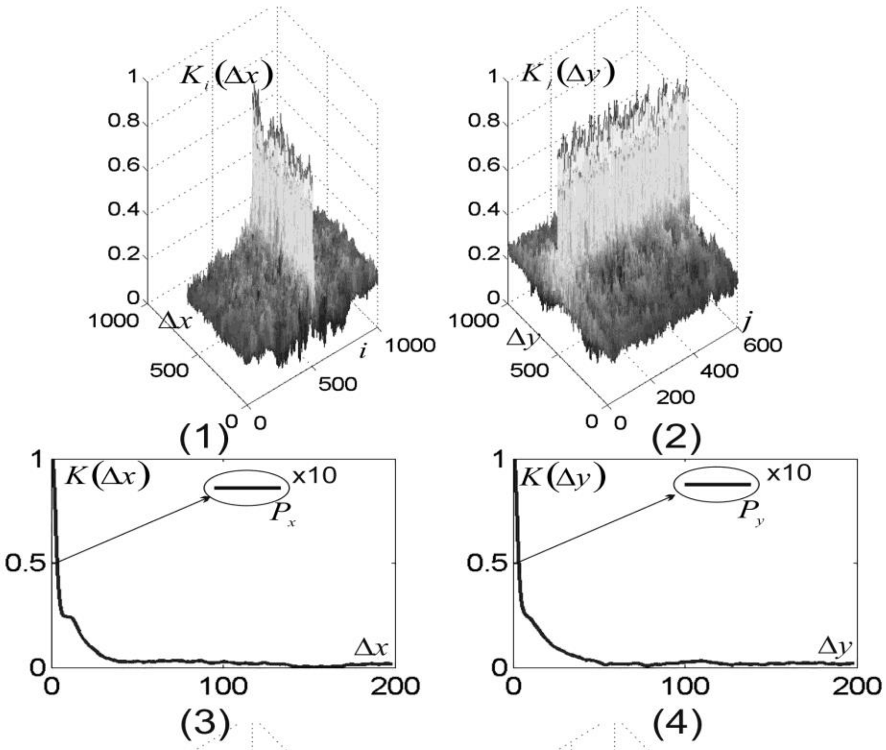


Objective criteria

All the data and parameters (*q*) presented in previous lectures need to be quantitatively analyzed!!! It was proposed to use:

2. Correlation analysis

Any azimuthally asymmetric distribution can be evaluated by correlation analysis in two perpendicular directions. Based on this, we used the following methodology of autocorrelation processing of the distribution of values *q*:



$$
\begin{cases}
K(W,X,y) \rightarrow \begin{pmatrix} K_{y=1}(\Delta x); \Delta x = 1,\ldots,m \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ K_{y=n}(\Delta x); \Delta x = 1,\ldots,m \end{pmatrix} \rightarrow \overline{K}(\Delta x) = \frac{1}{n}\sum_{i=1}^{n} K_i(\Delta x); \\
K(W,x,Y) \rightarrow \begin{pmatrix} K_{x=1}(\Delta y); \Delta y = 1,\ldots,n \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ K_{y=m}(\Delta y); \Delta y = 1,\ldots,n \end{pmatrix} \rightarrow \overline{K}(\Delta y) = \frac{1}{m}\sum_{i=1}^{n} K_i(\Delta y).
\end{cases}
$$

Half-width of autocorrelation dependency is an important diagnostical characteristic!!!

$$\boxed{\xi = \frac{P_{max}}{P_{min}}}$$ **- Asymmetry coefficient**

Objective criteria

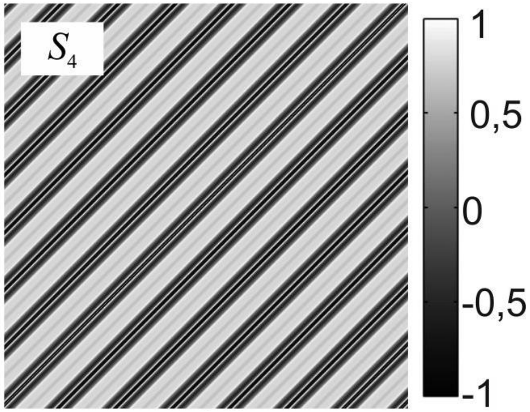
All the data and parameters (q) presented in previous lectures need to be quantitatively analyzed!!! It was proposed to use:

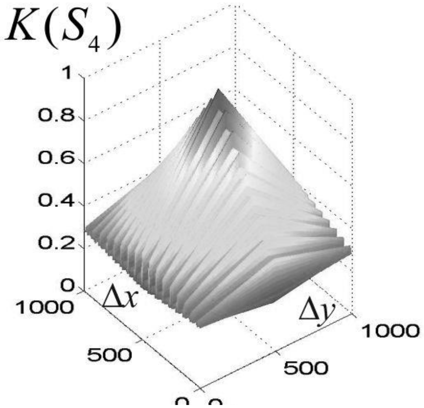
3. Fractal (self-similarity) analysis

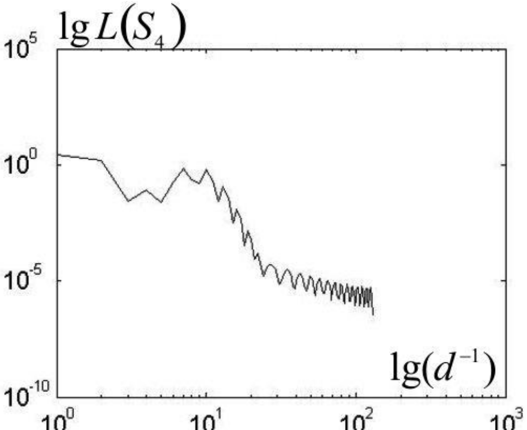
Fractal anlysis is based on the calculation of logarithmic dependencies of power spectra of values q: $\lg L(q) - \lg(d^{-1})$. Further mentioned dependecies are approximated by least squares method in a curves $\Phi(\eta)$.

Due to the form of curves $\Phi(\eta)$ one can classify:

1. Distributions *q* **are fractal** when there is one stable inclination angle $\eta = const$ within 2-3 decades of sizes changes.
2. Distributions *q* **are multifractal** when there is several stable inclination angles exist.
3. Distributions *q* **are random** when there is no stable inclination angles.







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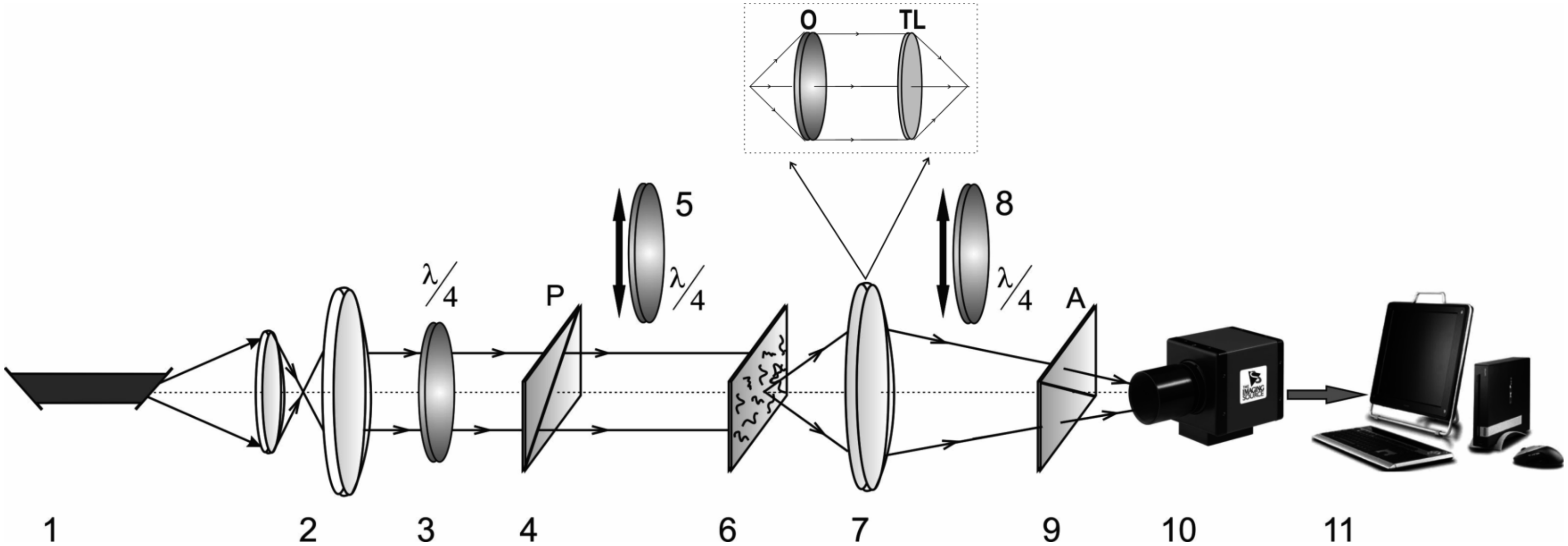
Experimental setup and measuring technique



Illumination

Analysis

Linear polarization with azimuths 0^0, 45^0, 90^0

Right circular polarization

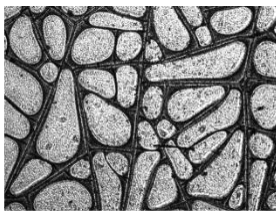
azimuths 0^0, 45^0, 90^0, 135^0, right and left circular

Experimental setup and measuring technique

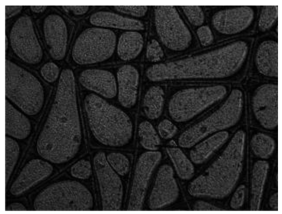
Let the probing beam will be linearly polarized with azimuth $\alpha_0 = 0^0$.

First step: set transmission plane of analyzer 9 at an angle of $\Theta = 0^0$ and measure $I_0(m \times n)$
set transmission plane of analyzer 9 at an angle of $\Theta = 90^0$ and measure $I_{90}(m \times n)$

$I_0(m \times n)$



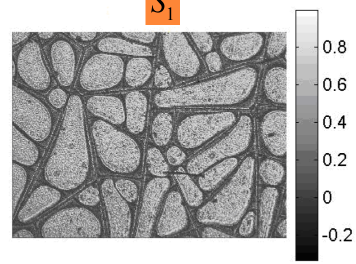
$I_{90}(m \times n)$



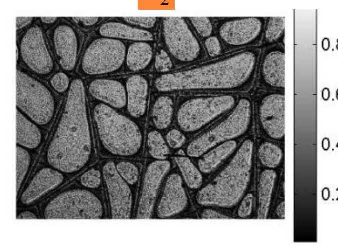
$$S_1 = I_0 + I_{90}$$

$$S_2 = (I_0 - I_{90}) / S_1$$

S_1

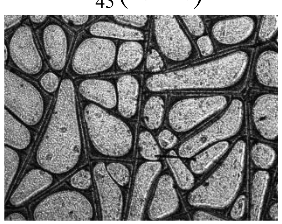


S_2

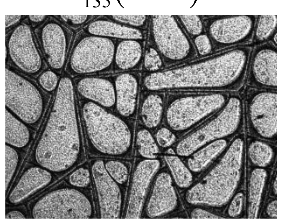


Second step: set transmission plane of analyzer 9 at an angle of $\Theta = 45^0$ and measure $I_{45}(m \times n)$
set transmission plane of analyzer 9 at an angle of $\Theta = 135^0$ and measure $I_{135}(m \times n)$

$I_{45}(m \times n)$



$I_{135}(m \times n)$



$$S_3 = (I_{45} - I_{135}) / S_1$$

S_3

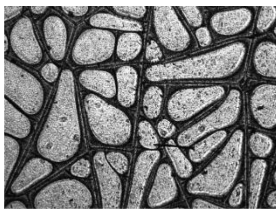
Experimental setup and measuring technique

Third step: insert quarter-wave plate 8

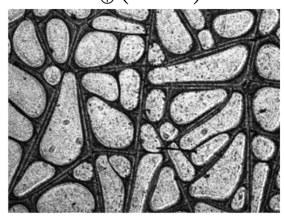
set transmission plane of analyzer 9 at an angle of $\Theta = 45^0$ and measure $I_{\otimes}(m \times n)$

set transmission plane of analyzer 9 at an angle of $\Theta = 135^0$ and measure $I_{\oplus}(m \times n)$

$I_{\otimes}(m \times n)$

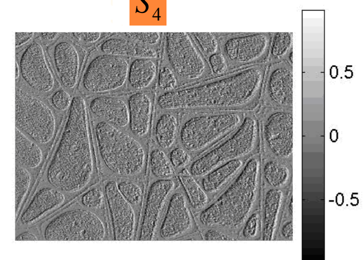


$I_{\oplus}(m \times n)$



$$\boxed{S_4 = I_{\otimes} - I_{\oplus}}$$

S_4



Similarly one can calculate other Stokes vectors: $\boxed{S_{i=1;2;3;4}^{45,90,\otimes}(m \times n)}$

Algorithm for Mueller matrix calculation

Stokes vector parameters

$$S_{i=1}^{0;45;90;\otimes} = I_0^{0;45;90;\otimes} + I_{90}^{0;45;90;\otimes};$$
$$S_{i=2}^{0;45;90;\otimes} = I_0^{0;45;90;\otimes} - I_{90}^{0;45;90;\otimes};$$
$$S_{i=3}^{0;45;90;\otimes} = I_{45}^{0;45;90;\otimes} - I_{135}^{0;45;90;\otimes};$$
$$S_{i=4}^{0;45;90;\otimes} = I_{\otimes}^{0;45;90;\otimes} + I_{\oplus}^{0;45;90;\otimes}.$$

Mueller-matrix elements

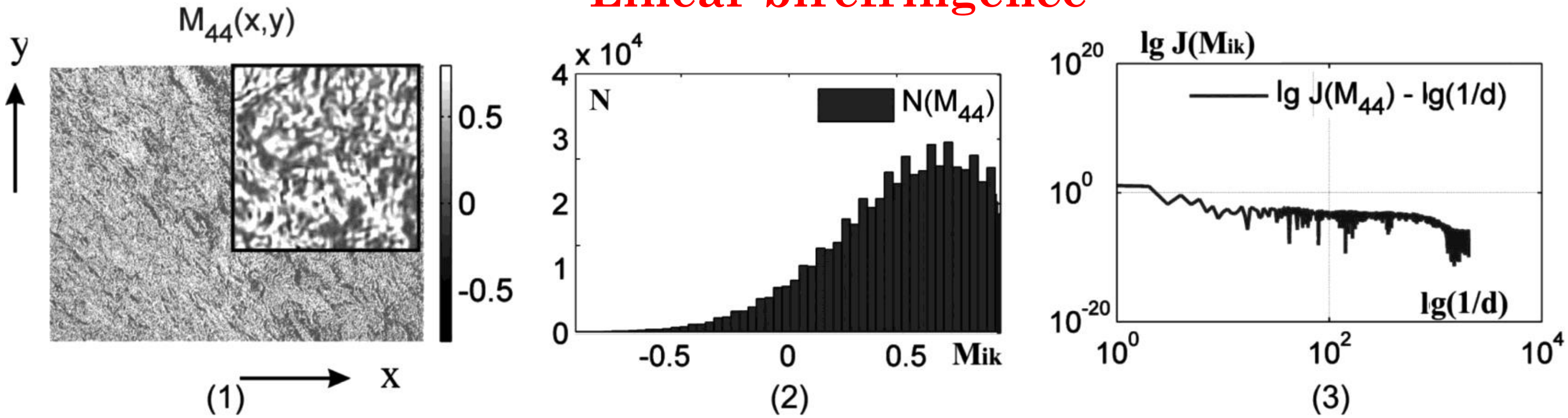
$$M_{11} = 0.5(S_1^0 + S_1^{90});$$
$$M_{12} = 0.5(S_1^0 - S_1^{90});$$
$$M_{13} = S_1^{45} - f_{11};$$
$$M_{14} = S_1^{\otimes} - f_{11};$$
$$M_{21} = 0.5(S_2^0 + S_2^{90});$$
$$M_{22} = 0.5(S_2^0 - S_2^{90});$$
$$M_{23} = S_2^{45} - f_{21};$$
$$M_{24} = S_2^{\otimes} - f_{21};$$
$$M_{31} = 0.5(S_3^0 + S_3^{90});$$
$$M_{32} = 0.5(S_3^0 - S_3^{90});$$
$$M_{33} = S_3^{45} - f_{31};$$
$$M_{34} = S_3^{\otimes} - f_{31};$$
$$M_{41} = 0.5(S_4^0 + S_4^{90});$$
$$M_{42} = 0.5(S_4^0 - S_4^{90});$$
$$M_{43} = S_4^{45} - f_{41};$$
$$M_{44} = S_4^{\otimes} - f_{41}.$$

Polarization parameters

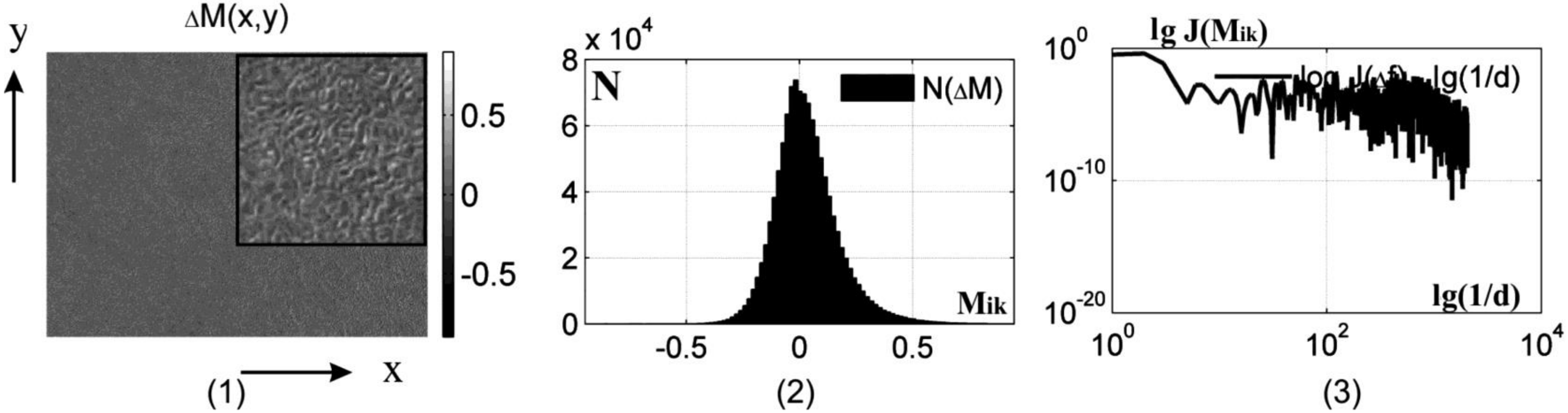
$$\alpha^* = 0.5 \arctan \frac{S_{i=3}}{S_{i=2}};$$
$$\beta^* = 0.5 \arcsin \frac{S_{i=4}}{S_{i=1}}.$$

Mueller-matrix invariants and optical anisotropy parameters

Linear birefringence

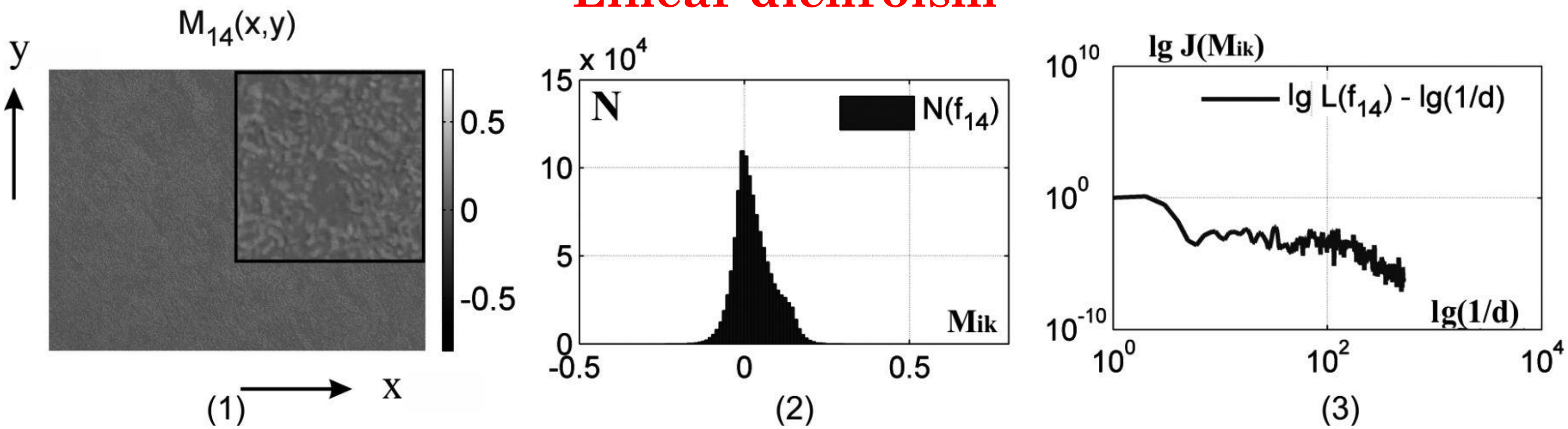


Circular birefringence

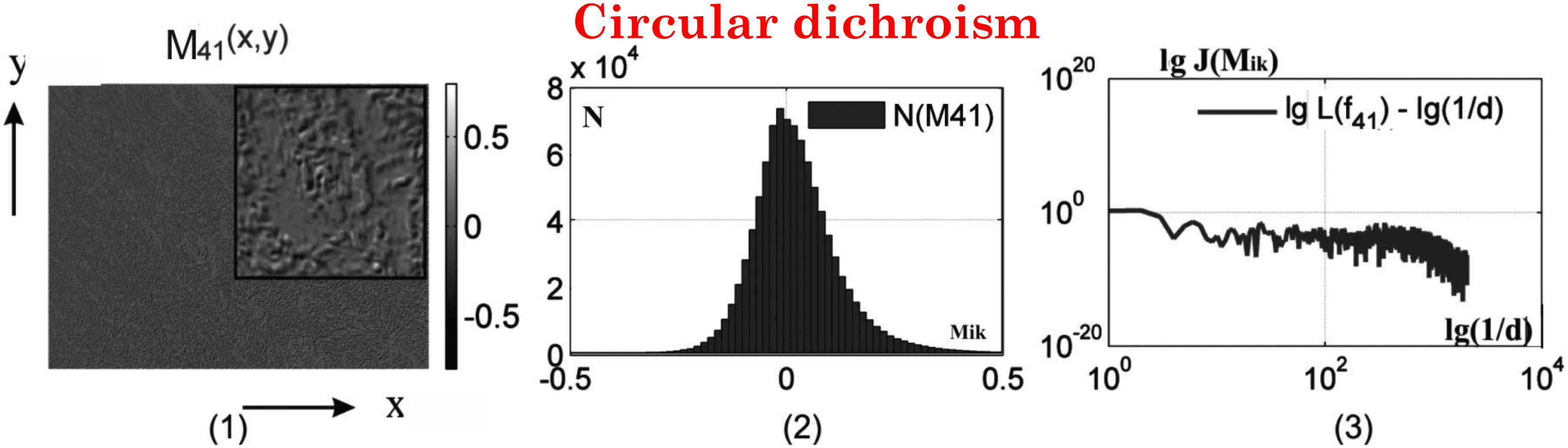


Mueller-matrix invariants and optical anisotropy parameters

Linear dichroism



Circular dichroism



Mueller-matrix invariants and optical anisotropy parameters

Parameters	M_{44}	ΔM	M_{14}	M_{41}
Z_1	0.46	0.12	0.73	0.16
Z_2	0.29	0.13	0.19	0.11
Z_3	0.48	0.23	0.57	1.14
Z_4	0.47	0.61	0.41	0.93
D	0.23	0.29	0.26	0.22