

# **THE FINAL INDO-EUROPEAN CONFERENCE ON MODULI**

**DEC 11-15, 2017**

**INDIAN NATIONAL SCIENCE ACADEMY, NEW DELHI**



# THE FINAL INDO EUROPEAN CONFERENCE OF THE PROJECT MODULI

December 11-15, 2017

Venue:

Indian National Science Academy (INSA), New Delhi, India

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Abstracts of Invited Speakers

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The project MODULI is an International Research Staff Exchange Scheme (IRSES) of the Marie Curie Programme of the European Union. MODULI involves four institutions in Europe:

CSIC (Madrid, Spain), CNRS (Paris, France), University of Oxford (Oxford, UK) and University of Aarhus (Denmark);

and four institutions in India:

The Indian Institute of Science (Bengaluru), Chennai Mathematical Institute (Chennai), Institute of Mathematical Sciences (Chennai) and Tata Institute for Fundamental Research (Mumbai).

This booklet consists of the title and abstracts for all the talks.

We thank Dr. A N Thakur, DED-I (P), Mr. S P Mishra, DED-I (Sc), Dr. Sudhanshu Aggarwal, Production Editor of Indian Journal of Pure and Applied Mathematics and Mr. Manish Thakran, Assistant-I, for their help. The financial support from the project MODULI, ICTP, (Trieste, Italy), GEAR (NSF, USA) and the Indian National Science Academy (New Delhi), are gratefully acknowledged.

**Gadadhar Misra**

**Member, Organizing Committee**

**The Final Conference of the**

**Indo European Project MODULI**

## **List of Participants**

- **Steve Bradlow** (UIUC, Urbana-Champaign), [sbradlow@gmail.com](mailto:sbradlow@gmail.com)
- **Oscar Garcia-Prada** (ICMAT, Madrid), [oscar.garcia-prada@icmat.es](mailto:oscar.garcia-prada@icmat.es)
- **Tomas Gomez** (ICMAT, Madrid), [tomas.gomez@icmat.es](mailto:tomas.gomez@icmat.es)
- **Artur de Araujo** (ICMAT, Madrid), [ad.dearaujo@gmail.com](mailto:ad.dearaujo@gmail.com)
- **Luis Angel Calvo** (ICMAT, Madrid), [luisangelcalvopascual@gmail.com](mailto:luisangelcalvopascual@gmail.com)
- **Michel Waldschmidt** (Université Pierre et Marie Curie (Paris 6) )  
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- **Suratno Basu** (IMSc, Chennai), [suratno.math@gmail.com](mailto:suratno.math@gmail.com)
- **Indranil Biswas** (TIFR, Mumbai) [indranil@math.tifr.res.in](mailto:indranil@math.tifr.res.in)
- **C. S. Rajan** (TIFR, Mumbai) [rajanconj@gmail.com](mailto:rajanconj@gmail.com)
- **V. Srinivas** (TIFR, Mumbai), [srinivas@math.tifr.res.in](mailto:srinivas@math.tifr.res.in)
- **Krishnendu Gongopadhyay** (IISER, Mohali) [krishnendug@gmail.com](mailto:krishnendug@gmail.com)
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- **Gadadhar Misra** (IISc, Bangalore), [gm@math.iisc.ernet.in](mailto:gm@math.iisc.ernet.in)
- **S. Ramanan** (CMI, Chennai), [sramanan@cmi.ac.in](mailto:sramanan@cmi.ac.in)
- **T. N. Venkataramana** (TIFR, Mumbai), [venky@math.tifr.res.in](mailto:venky@math.tifr.res.in)
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- **Shibananda Biswas** (IISER, Kolkata), [shibananda@gmail.com](mailto:shibananda@gmail.com)

## **PROGRAMME:**

### **Monday, 11 December**

12.30 -- 12.45: Opening Remarks

M. S. Narasimhan (IISc, Bangalore)

14:00 -- 15:00: T. N. Venkataramana (TIFR, Mumbai)

15:30 -- 16:30: Artur de Araujo (ICMAT, Madrid)

### **Tuesday, 12 December**

10:00 -- 11:00: Tomas Gomez (ICMAT, Madrid)

11:30 -- 12:30: Harish Seshadri (IISc, Bangalore)

15:00 -- 16:00: Indranil Biswas (TIFR, Mumbai)

### **Wednesday, 13 December**

10:00 -- 11:00: Michel Waldschmidt (Université Pierre et Marie Curie  
-- Paris 6)

11:30 -- 12:30: Vamsi Pingali (IISc, Bangalore)

15:00 -- 16:00: C. S. Rajan (TIFR, Mumbai)

### **Thursday, 14 December**

10:00 -- 11:00: Oscar Garcia-Prada (ICMAT, Madrid),

11:30 -- 12:30: S. Ramanan (CMI, Chennai)

15:00 -- 16:00: Steve Bradlow (UIUC, Urbana-Champaign)

16:30 -- 17:30: Krishnendu Gongopadhyay (IISER, Mohali)

### **Friday, 15 December**

10:00 -- 11:00: E. Ghate (TIFR, Mumbai)

11:30 -- 12:30: V. Srinivas (TIFR, Mumbai)

## **TITLES AND ABSTRACTS**

### **Quillen metrics and perturbed equations**

**Vamsi Pingali**

IISc, Bangalore, [vamsipritham@gmail.com](mailto:vamsipritham@gmail.com)

We describe three natural perturbations of already existing equations in complex differential geometry and mathematical physics. We interpret them as the zeroes of a moment map on infinite-dimensional symplectic manifolds whose symplectic form is the curvature of a natural line bundle - the Quillen bundle. The three equations are - the generalised Monge-Ampere equation, the almost Hermite-Einstein equation, and the Calabi-Yang Mills equation.

### **A universal Torelli theorem for elliptic surfaces**

**C. S. Rajan**

TIFR, Mumbai, [rajanconj@gmail.com](mailto:rajanconj@gmail.com)

Given two semistable elliptic surfaces over a curve  $C$  defined over a field of characteristic zero or finitely generated over its prime field, we show that any compatible family of effective isometries of the Néron-Severi lattices of the base changed elliptic surfaces for all finite separable maps  $B \rightarrow C$  arises from an isomorphism of the elliptic surfaces. Without the effectivity hypothesis, we show that the two elliptic surfaces are isomorphic.

We also determine the group of universal automorphisms of a semistable elliptic surface. In particular, this includes showing that the Picard-Lefschetz transformations corresponding to an irreducible component of a singular fibre, can be extended as universal isometries. In the process, we get a family of homomorphisms of the affine Weyl group associated to  $\tilde{A}_{n-1}$  to that of  $\tilde{A}_{dn-1}$ , indexed by natural numbers  $d$ , which are closed under composition.

## **A relative version of Gieseker's problem on stratified vector bundles**

**V. Srinivas**

TIFR, Mumbai, [srinivas@math.tifr.res.in](mailto:srinivas@math.tifr.res.in)

This talk is a report on joint work with Hélène Esnault. We show that if a morphism of smooth projective varieties in char.  $p$  induces the trivial map on étale fundamental groups, then the pullback of any stratified vector bundle is trivial, as a stratified bundle.

## **Two invariants related to two conjectures due to Nagata**

**Michael Waldschmidt**

Université Pierre et Marie Curie (Paris 6)

Seshadri's constant is related to a conjecture due to Nagata. Another conjecture, also due to Nagata and solved by Bombieri in 1970, is related with algebraic values of meromorphic functions. The main part of Bombieri's proof leads to a Schwarz Lemma in several variables, the proof of which gives rise to another invariant associated with symbolic powers of the ideal of functions vanishing on a finite set of points. This invariant is an asymptotic measure of the least degree of a polynomial in several variables with given order of vanishing on a finite set of points. Recent works on the resurgence of ideals of points and the containment problem compare powers and symbolic powers of ideals.

## **Geometry of Teichmüller domains**

**Harish Seshadri**

IISc, Bangalore, [harish@iisc.ac.in](mailto:harish@iisc.ac.in)

It is known, by the work of L. Bers, that the Teichmüller space of a compact Riemann surface of genus at least 2 can be biholomorphically embedded as a domain in complex Euclidean space. In this talk, I will discuss recent developments concerning the geometry of such domains, focusing on convexity of these domains.

## **Grothendieck Closure of integral linear groups**

**T. N. Venkatramana**

TIFR, Mumbai, [venky@math.tifr.res.in](mailto:venky@math.tifr.res.in)

In a joint work with Alex Lubotzky, we answer a question of Grothendieck of characterizing integral linear groups which are equal to their own (Grothendieck) closure; As an application, we show that the free group on two generators has infinite index in its closure, settling an old problem.

## **Reductions of Galois Representations and the Monomial Lattice**

**Eknath Ghate**

TIFR, Mumbai, [eghate@math.tifr.res.in](mailto:eghate@math.tifr.res.in)

We briefly describe our ongoing work (with various coauthors) on the reductions of Galois representations. This will motivate us to ask what seems to be an open problem in classical modular representation theory: to describe the lattice generated by the monomials in the symmetric power representations of the general linear group over a finite field. We end with some preliminary results that shed some light on this lattice.

## **Exotic components of surface group representation varieties, and their Higgs bundle avatars**

**Steve Bradlow**

UIUC, Urbana-Champaign, [sbradlow@gmail.com](mailto:sbradlow@gmail.com)

Moduli spaces of Higgs bundles on a Riemann surface correspond to representation varieties for the surface fundamental group. For representations into complex semisimple Lie groups, the components of these spaces are labeled by obvious topological invariants. This is no longer true if one restricts to real forms of the complex groups. Factors other than the obvious invariants lead to the existence of extra 'exotic' components which can have special significance. All formerly known instances of such exotic components were attributable to one of two distinct mechanisms and collectively comprise so-called "higher Teichmüller" components. Recent Higgs bundle results for the groups  $SO(p, q)$  shed new light on these phenomena and reveal new examples outside the scope of the two known mechanisms. This talk will describe the new  $SO(p, q)$  results.



## **Kaehler-Yang-Mills equations**

**Oscar Garcia-Prada**

ICMAT, Madrid, [oscar.garcia-prada@icmat.es](mailto:oscar.garcia-prada@icmat.es)

In this talk we first introduce the Kaehler-Yang-Mills equations on a holomorphic bundle over a compact complex manifold. They emerge from a natural extension of the theories for constant scalar curvature Kaehler metrics and Hermitian-Yang-Mills connections. We construct solutions to these equations by applying dimensional reduction methods to the product of the complex projective line with a compact Riemann surface. The resulting equations, that we call gravitating vortex equations, describe abelian vortices on the Riemann surface coupled to a metric (based on joint work with L. Alvarez-Consul, M. Garcia-Fernandez and V. Pingali).

## **Stability of the Poincare bundle for the stack of principal $G$ -bundles**

**Tomas Gomez**

ICMAT, Madrid, [tomas.gomez@icmat.es](mailto:tomas.gomez@icmat.es)

Let  $X$  be a smooth projective curve of genus at least two. Let  $G$  be an almost simple affine algebraic group. The universal principal  $G$ -bundle is called the Poincaré bundle. It is a principal  $G$ -bundle on the product of the curve and the moduli stack of principal  $G$ -bundles. We prove that it is stable with respect to any polarization (joint work with I. Biswas and N. Hoffmann).

## **Representations of Generalized Quivers**

**Artur de Araujo**

ICMAT, Madrid, [ad.dearaujo@gmail.com](mailto:ad.dearaujo@gmail.com)

We will motivate and define generalized quivers, and the corresponding bundle version. We will formulate the general moduli problem for dimensional representations, and present an inductive formula for their Poincare polynomial. We will then briefly discuss the stability problem for the bundle case.

## **Rank 2 Higgs bundles**

**S. Ramanan**

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