D3-branes, Strings and F-Theory in Various Dimensions

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F-theory and D3-branes

F-theory geometrises the physics of 7-branes and D(-1)-instantons.

D3-branes probe this backreacted geometry.

Relevance of D3-branes in F-theory includes:

1) D3 on $\mathbb{R}^{1,3} \times \mathrm{pt}$	$\mathrm{pt} \subset \mathrm{CY}_4$	Spacetime-filling
2) D3 on $\mathrm{pt} \times D$	$D \subset B_3 \subset \mathrm{CY}_4$ divisor	Instanton
3) **D3 on $\mathbb{R}^{1,1} \times C$**	$C \subset B_{n-1} \subset \mathrm{CY}_n$ curve	**String**

We will focus on strings from wrapped D3-branes:

- $C \subset \mathrm{CY}_3$: self-dual string in 6d $\leftrightarrow$ relation to 6d SCFTs
- $C \subset \mathrm{CY}_4$: cosmic string in $\mathbb{R}^{1,3}$ - codimension-two object
- $C \subset \mathrm{CY}_5$: filling $\mathbb{R}^{1,1}$ and required by tadpoles

Aim:

Microscopic understanding of 2d QFT on string in various dimensions

D3-strings in F-theory

1) Extrinsic Motivation:

- 7-brane background for D3-strings as a means to geometrically engineer (new?) chiral 2d theories and SCFTs
- Methods to describe gauge theories with varying gauge coupling via topological duality twist [Martucci'14]
  $\Longrightarrow$ Go beyond topological twist of [Bershadsky,Johansen,Vafa,Sadov'95], [Benini,Bobev'13] ,. . .

2) Intrinsic Motivation:

D3-branes are exciting window into non-perturbative dynamics captured by F-theory

- Quantum Higgsing
- Mysterious 3-7 string sector

Outline

1) Topological (Duality) Twist on D3-brane on C

2) Massless Spectrum for Strings from D3-branes

3) Quantum Higgsing via F-theory

4) Anomalies and 3-7 Modes

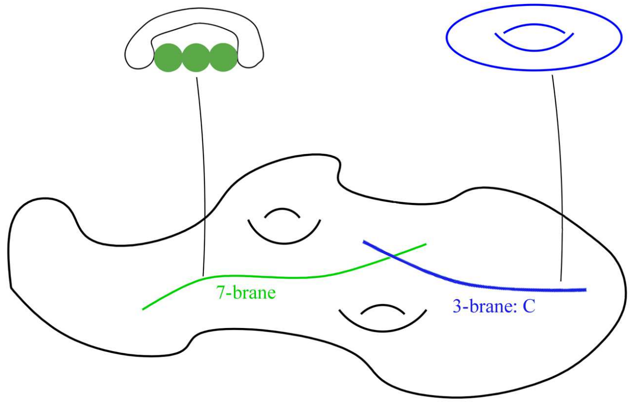
5) 2d (0,2) Gravity Sector

The general setup

F-theory on Y_n with base B_{n-1}

D3-brane on $\mathbb{R}^{1,1} \times C$

C a curve in base $C \subset B_{n-1}$



This talk: Single D3 with C not contained in discriminant locus Δ

- C is transverse to 7-branes on B_{n-1}
- C intersects 7-branes in isolated points on B_{n-1}

M-theory dual descriptions via T-duality see talk by S. Schäfer-Nameki

- transverse to D3-string on $\mathbb{R}^{1,1}$: M5-brane
- parallel to D3-string on $\mathbb{R}^{1,1}$: M2-brane

This talk: We will describe theory directly in language of F-Theory
via **topological duality twist** [Martucci'14]

Duality bundle

- 4d $N = 4$ SYM coupling on D3 $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$ $=$ F-theory axio-dilaton $C_0 + ie^{-\phi}$

- 7-brane background $\Longrightarrow$ τ-variation on $C \subset B_{n-1}$ monodromy around $C \cap$ (7-brane)

- Consistent due to $SL(2,\mathbb{Z})$ duality of $N = 4$ SYM:

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \qquad (F, F_D) \rightarrow M_{\mathrm{SL}(2,\mathbb{Z})}(F, F_D)$$

$$\text{SYM fields}: \Phi \rightarrow e^{iq\alpha}\,\Phi \quad \text{with} \quad e^{i\alpha} = \frac{c\tau + d}{|c\tau + d|}$$

q: $U(1)_D$ charge 'bonus symmetry' [Intriligator'98] [Kapustin,Witten'06]

- τ-variation on C described by non-trivial $SL(2,\mathbb{Z})$ bundle $\mathcal{L}_D$
  - connection $\mathcal{A} = \frac{\mathrm{d}\tau_1}{2\tau_2}$ $\qquad \tau = \tau_1 + i\tau_2$
  - as holomorphic bundle: $\mathcal{L}_D = K^{-1}_{B_{n-1}}|_C$ [Bianchi,Collinucci,Martucci'11] [Greene,Shapire,Vafa,Yau'89]

Duality Twist

- $G \supset SO(1,3)_L \times SU(4)_R \times U(1)_D$
- Supercharges: $Q_{\alpha I} \ : \ (\mathbf{2},\mathbf{1},\overline{\mathbf{4}})_{\mathbf{1}} \qquad \widetilde{Q}^I_{\dot\alpha} \ : \ (\mathbf{1},\mathbf{2},\mathbf{4})_{-\mathbf{1}}$

$\Longrightarrow$ Topological duality twist required due to τ variation [Martucci'14]

Ex: $C \subset B_2$ [Haghighat,Murthy,Vafa,Vandoren'15][Lawrie,S-Nameki,TW'16]

$$\begin{array}{rcl} G_{\text{total}} & \to & SO(4)_T \times SO(1,1)_L \times \mathbf{U(1)_R} \times \mathbf{U(1)_C} \times \mathbf{U(1)_D} \\ (\mathbf{2},\mathbf{1},\overline{\mathbf{4}})_1 & \to & (\mathbf{2},\mathbf{1})_{1;\mathbf{-1},\mathbf{1},\mathbf{1}} \oplus (\mathbf{2},\mathbf{1})_{-1;\mathbf{-1},\mathbf{-1},\mathbf{1}} \oplus (\mathbf{1},\mathbf{2})_{1;\mathbf{1},\mathbf{1},\mathbf{1}} \oplus (\mathbf{1},\mathbf{2})_{-1;\mathbf{1},\mathbf{-1},\mathbf{1}} \end{array}$$

$$T_C^{\text{twist}} = \tfrac{1}{2}(T_C + T_R), \qquad T_D^{\text{twist}} = \tfrac{1}{2}(T_D + T_R)$$

$$\begin{array}{rcl} G_{\text{total}} & \to & SO(4)_T \times SO(1,1)_L \times \mathbf{U(1)_C^{\text{twist}}} \times \mathbf{U(1)_D^{\text{twist}}} \\ (\mathbf{2},\mathbf{1},\overline{\mathbf{4}})_1 & \to & \underline{(\mathbf{2},\mathbf{1})_{1;\mathbf{0},\mathbf{0}}} \oplus (\mathbf{2},\mathbf{1})_{-1;\mathbf{-1},\mathbf{0}} \oplus (\mathbf{1},\mathbf{2})_{1;\mathbf{1},\mathbf{1}} \oplus \cancel{(\mathbf{1},\mathbf{2})_{-1;\mathbf{0},\mathbf{1}}} \\ (\mathbf{1},\mathbf{2},\mathbf{4})_{-1} & \to & \underline{(\mathbf{2},\mathbf{1})_{1;\mathbf{0},\mathbf{0}}} \oplus (\mathbf{2},\mathbf{1})_{-1;\mathbf{1},\mathbf{0}} \oplus (\mathbf{1},\mathbf{2})_{1;\mathbf{-1},\mathbf{-1}} \oplus \cancel{(\mathbf{1},\mathbf{2})_{-1;\mathbf{0},\mathbf{-1}}} \,. \end{array}$$

(4,4) broken to (0,4) by topological duality twist: **chiral theory**

F-theory Duality Twists

Applicable to all types of D3-brane strings in F-theory [Lawrie,S-Nameki,TW'16]

Spacetime dim d	8	6	4	2
CY$_n$	2	3	4	5
2d supersymmetry	$(0,8)$	$(0,4)$	$(0,2)$	$(0,2)$

$$
\begin{array}{llcl}
\text{CY}_2 & SU(4)_R & \to & SO(6)_T \\
\text{CY}_3 & SU(4)_R & \to & SO(4)_T \times \underline{U(1)_R} \\
\text{CY}_4 & SU(4)_R & \to & SO(2)_T \times SU(2)_R \times \underline{U(1)_R} \\
\text{CY}_5 & SU(4)_R & \to & SU(3)_R \times \underline{U(1)_R}
\end{array}
$$

- F-theory on K3 is an outlier: direct twist of $U(1)_C$ with $U(1)_D$

Twisted Bulk Spectrum

- $G_{\text{total}} = SO(1,3)_L \times SU(4)_R \times \mathbf{U(1)_D}$
- $A_\mu \ : \ (\mathbf{2},\mathbf{2},\mathbf{1})_{*} \quad \phi_i \ : \ (\mathbf{1},\mathbf{1},\mathbf{6})_{\mathbf{0}} \quad \Psi^I_\alpha \ : \ (\mathbf{2},\mathbf{1},\mathbf{4})_{\mathbf{1}} \quad \widetilde{\Psi}_{\dot{\alpha} I} \ : \ (\mathbf{1},\mathbf{2},\overline{\mathbf{4}})_{-\mathbf{1}}$

Strategy for ϕ_i, Ψ^I_α, $\widetilde{\Psi}_{\dot{\alpha} I}$ ($U(1)_D$ eigenstates!):

- Decompose $SU(4)_R \to SO(m)_T \times SU(k)_R \times U(1)_R$
- Determine representation under $SU(k)_R$ and $U(1)^{\text{twist}}_C$, $U(1)^{\text{twist}}_D$
- Deduce transformation of internal component as bundle valued form
- Determine e.o.m/BPS equations and obtain zero mode counting

Example: $C \subset \mathrm{CY}_4$ with $(0,2)$ SUSY

$$
\begin{array}{lll}
 & SU(2)_R \times U(1)^{\text{twist}}_C \times U(1)^{\text{twist}}_D & (q^{\text{twist}}_C, q^{\text{twist}}_D) = (-1,0): \quad \text{section of } K_C \\
\phi_i \ : & \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{2}_{\frac{1}{2},\frac{1}{2}} \oplus \mathbf{2}_{-\frac{1}{2},-\frac{1}{2}} & (q^{\text{twist}}_C, q^{\text{twist}}_D) = (0,-1): \quad \text{section of } \mathcal{L}_D
\end{array}
$$

$\Longrightarrow \mathbf{2}_{\frac{1}{2},\frac{1}{2}}$ section of N_{C/B_3}: $h^0(C, N_{C/B_3})$ zero modes

in agreement with $(q^{\text{twist}}_C, q^{\text{twist}}_D) = (-\frac{1}{2}, -\frac{1}{2})$ since $K_C = \mathcal{L}_D^{-1} \otimes \wedge^2 N_{C/B_3}$

Twisted Bulk Spectrum

4d $N = 4$ gauge field A_μ is not a $U(1)_D$ eigenstate

- Wilson line degree of freedom a (complex scalar):

  $\sqrt{\tau_2} a$ is $U(1)_D$ eigenstate: $\sqrt{\tau_2}\, \delta a = -2i\, \epsilon_- \underbrace{\tilde{\psi}_+}_{q_D^{\rm tw}=-1}$

- external gauge field v_+ and v_- no $U(1)_D$ eigenstates:

  $\sqrt{\tau_2}\, \delta v_- = 2i(\underbrace{\lambda_-}_{q_D^{\rm tw}=1} \tilde{\epsilon}_- + \underbrace{\tilde{\lambda}_-}_{q_D^{\rm tw}=-1} \epsilon_-)$ $\lambda, \tilde{\lambda}$: gauginos

  Counting proceeds via gauginos λ_-, $\tilde{\lambda}_-$

Strings in 6d from CY3

[Lawrie,Schäfer-Nameki,TW'16]

$(q_C^{\text{twist}}, q_D^{\text{twist}})$	Fermions		Bosons		$(0,4)$	Multiplicity
$(1,1)$ $(-1,-1)$	$(\mathbf{2},\mathbf{1})_1$ $(\mathbf{2},\mathbf{1})_1$	ψ_+ $\tilde{\psi}_+$	$(\mathbf{1},\mathbf{1})_0$, $(\mathbf{1},\mathbf{1})_0$ $(\mathbf{1},\mathbf{1})_0$, $(\mathbf{1},\mathbf{1})_0$	$\bar{a}, \bar{\sigma}$ $a,\ \sigma$	Hyper	$h^0(C, K_C \otimes \mathcal{L}_D)$ $= g - 1 + c_1(B_2) \cdot C$
$(0,0)$	$(\mathbf{1},\mathbf{2})_1$ $(\mathbf{1},\mathbf{2})_1$	μ_+ $\tilde{\mu}_+$	$(\mathbf{2},\mathbf{2})_0$	φ	Twisted Hyper	$h^0(C) = 1$
$(1,0)$ $(-1,0)$	$(\mathbf{1},\mathbf{2})_{-1}$ $(\mathbf{1},\mathbf{2})_{-1}$	$\tilde{\rho}_-$ ρ_-			Fermi	$h^1(C) = g$
$(0,1)$ $(0,-1)$	$(\mathbf{2},\mathbf{1})_{-1}$ $(\mathbf{2},\mathbf{1})_{-1}$	λ_- $\tilde{\lambda}_-$	$(\mathbf{1},\mathbf{1})_2$ $(\mathbf{1},\mathbf{1})_{-2}$	v_+ v_-	Vector	$h^1(C, K_C \otimes \mathcal{L}_D) = 0$

In agreement with previous analysis in [Haghighat,Murthy,Vafa,Vandoren'15]

Lots of recent work on 6d instanton strings:

including [del Zotto,Lockhart'16] and refs therein

2d (0,2) from D3 on CY5

[Lawrie,Schäfer-Nameki,TW'16]

Fermions	Bosons	(0,2) Multiplet	Zero-mode Cohomology
μ_+ $\tilde{\mu}_+$	φ $\bar{\varphi}$	Chiral Conjugate Chiral	$h^0(C, N_{C/B_4})$
$\tilde{\psi}_+$ ψ_+	a $\bar{a}$	Chiral Conjugate Chiral	$h^0(C, K_C \otimes \mathcal{L}_D) = g - 1 + c_1(B_4) \cdot C$
ρ_- $\tilde{\rho}_-$	— —	Fermi Conjugate Fermi	$h^1(C, N_{C/B_4}) = h^0(C, N_{C/B_4}) + g - 1 - c_1(B_4) \cdot C$
λ_- $\tilde{\lambda}_-$	v_+ v_-	Vector	$h^1(C, K_C \otimes \mathcal{L}_D) = 0$

U(1) Quantum Higgsing

massless vector multiplets: $h^0(C, \mathcal{L}_D^{-1})$ $\qquad$ $\mathcal{L}_D = K_B^{-1}|_C$

1. $C \cap \Delta = 0 \longleftrightarrow$ fibration over C is trivial
   $h^0(C, \mathcal{L}_D^{-1}) = h^0(C, \mathcal{O}) = 1 \to U(1)$ gauge group
2. $C \cap \Delta \neq 0 \leftrightarrow$ fibration over C non-trivial
   $h^0(C, \mathcal{L}_D^{-1}) = 0$ since $\mathcal{L}_D^{-1}$ is negative $\to U(1)$ broken

Type IIB: D3 on curve C_+ $\xleftrightarrow[\text{action } \sigma]{\text{orientifold}}$ D3' on curve C_-

- if $C_+ \neq C_-$: $U(1)$ gauge group - irresp. of 7-brane intersection!
- if $C_+ = C_-$: $U(1)$ broken

Suggests:

- In F-theory: Quantum higgsing of $U(1)$ due to strong coupling effects
- Claim: These are localised along the O7-plane and of same origin responsible for non-pert. splitting of O7-plane

U(1) Quantum Higgsing

Sen limit:

- $\Delta \simeq \epsilon^2 h^2 \underbrace{(\eta^2 - h\chi)}_{\text{D7-branes}} + \mathcal{O}(\epsilon^3)$ $\quad \epsilon \to 0:$ O7-plane at $h = 0$
- IIB double cover CY $X_{n-1} : \xi^2 = h \qquad \sigma : \xi \to -\xi$

Consider family of curves C_δ for D3-brane (e.g. n=3)

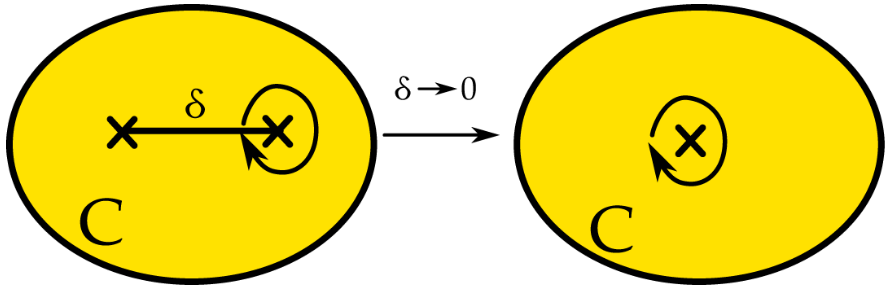
- on base B_2: $C_\delta : h = p_1^2 + \delta\, p_2 \subset B_2$
- on double cover X_2: $\tilde{C}_\delta : \xi^2 = p_1^2 + \delta\, p_2 \subset X_2$

Consider limit $\delta \to 0$ (in Sen limit $\epsilon \to 0$):

- On X_2: $\tilde{C}_0 = C_+ \cup C_- \qquad C_\pm : \xi = \pm p_1$
  at intersection $C_+ \cap C_-$ (on top of O-plane): 3-3' modes $\quad q_{U(1)} = 2$
  $=$ unHiggsing of U(1)
- On B_2: merely affects intersection points with O7-plane:
  $\{h = 0\} \cap C_\delta : \quad \{h = 0\} \cap \{p_1 = \pm\sqrt{\delta\, p_2}\}$

U(1) Quantum Higgsing

- Perturbative breaking of U(1) $=$ splitting of double-intersection with O7-plane
- Distance of intersection points $=$ order parameter for Higgsing (mass for 3-3' strings!)



Finally allow for $\epsilon \neq 0$ (full F-theory)

- Seiberg-Witten quantum splitting of O7-plane
  $\Longrightarrow$ non-pert. splitting of intersection with D3-brane - even for $\delta = 0$
- Distance of int. points: order parameter for non-pert. $U(1)$ Higgsing

Conclusion: [Lawrie,Schafer-Nameki,TW'16]

U(1) unbroken only if $\delta = 0$ (splitting) and in addition $\epsilon = 0$

Otherwise monodromy effects around intersection with O7-plane responsible for U(1) breaking

U(1) Quantum Higgsing

1) Fate of 3-3' strings:

- After quantum Higgsing one chiral multiplet gets absorbed by vector, one multiplet remains as modulus of D3 as part of bulk moduli
- Some of these bulk moduli can localize near O7-plane in perturbative limit [Harvey,Royston'07] [Cvetic,G-Extxebarria,Halverson'11]

2) Further application:

Same mechanism applied to D3-brane instantons explains why no distinction between $O(1)$ and $U(1)$ instanton in F-theory necessary

3-7 strings

Intersection points of C with 7-branes: Extra massless matter

Perturbative analysis:

1 complex chiral fermion per intersection point D3 $\cap$ D7 and no scalar (8 DN directions)

Challenge: Compute the spectrum for non-perturbative models

- D3 $\cap$ 7-br.: $[C] \cdot [\Delta] = 12\,[C] \cdot c_1(B_{n-1})$ intersection points
- This does not count the number of (independent) Fermi multiplets since not all 7-branes are of same (p,q)-type

3 ways to deduce correct counting: [Lawrie,Schafer-Nameki,TW'16]

1. by deforming to weak coupling - when possible
2. by anomaly inflow
3. by duality with M5-branes

   6d: cf. [Haghighat,Murthy,Vafa,Vandoren'15] see talk by Sakura Schafer-Nameki

3-7 strings - perturbatively

In perturbative limit

$$\Delta \simeq \epsilon^2 h^2 \underbrace{(\eta^2 - h\chi)}_{\mathrm{D7-branes}} + \mathcal{O}(\epsilon^3)$$

- $h = 0$: O7-plane
- $[\mathrm{D7-brane}] = 8c_1(B_{n-1})$

No independent 3-7 states at intersection with O7-plane

$$\#\ \text{of Fermis}: \quad 8\, c_1(B_{n-1}) \cdot [C]$$

Turns out: This is always the correct number of Fermi modes - uniquely and universally fixed by gauge and gravitational anomalies along the string

3-7 strings and anomalies

$$(n_{R,+} - n_{R,-})\, I_{4,R} + \mathcal{I}_4 = 0$$

- $I_{4,R}$: contribution to anomaly from 2d chiral fermions in repr. R
- $\mathcal{I}_4$: anomaly inflow from bulk CS terms

Anomaly inflow terms:

- For string in $\mathbb{R}^{1,d-1}$ $d \neq 6$: [Lawrie,S-Nameki,TW'16]
  $\mathcal{I}_4 = (p_1(T) + p_1(N))\left(-\frac{1}{4}\, c_1(B_n) \cdot C\right) - \sum_a \frac{1}{4}\, \mathrm{Tr} F_a^2 \left(D_a \cdot C\right)$
- For string in $\mathbb{R}^{1,5}$: 2 extra terms due to [Shimizu,Tachikawa'16]
  - self-duality $-\frac{1}{2}\,(C \cdot C)\, \chi_4(N) = -\frac{1}{2}\,(C \cdot C)\left(\frac{1}{2}\mathrm{tr} F_{T,2}^2 - \frac{1}{2}\mathrm{tr} F_{T,1}^2\right)$
  - $SU(2)_R$ symmetry $+\frac{1}{2}\mathrm{tr} F_I^2$

In all dimensions, normal and tangent bundle anomalies cancel iff

$\#$ of $3-7$ Fermis $=$ $8\, c_1(B_{n-1}) \cdot [C]$ [Lawrie,S-Nameki,TW'16]

3-7 strings and flavour

Universal flavour term: $\mathcal{I}_4 \supset -\sum_a \frac{1}{4}\,\mathrm{Tr}F_a^2(D_a \cdot C)$

a: non-abelian 7-brane stacks $\qquad$ $\mathrm{tr}_{\mathrm{fund}} F_a^2 = s_{G_a} \mathrm{Tr} F_a^2$

✓ First principle derivation possible for perturbative gauge groups

✓ Other cases: s_G completely fixed by 6d anomaly considerations

holds for all dim. [Grassi,Morrison'00] [Ohmori,Shimizu,Tachikawa,Yonekura'14]

G	$SU(k)$	$USp(k)$	$SO(k)$	G_2	F_4	E_6	E_7	E_8
s_G	$1/2$	$1/2$	1	1	3	3	6	30

$$\mathrm{Need}: (-\frac{1}{2}\mathrm{tr}_R F^2)(n_{R,+} - n_{R,-}) - \frac{1}{4}\,\mathrm{Tr}F_a^2(D_a \cdot C) \stackrel{!}{=} 0$$

✓ works for SU(k)/ USP(k) (complex) or SO(k)/G_2 (real) with $R = \mathrm{fund}$

X no solution for $G = E_{6,7,8}, F_4$

Flavour group must be broken in the UV due to monodromy effects!

Example 6d 'E-string': E_8 flavour group in IR $\rightarrow$ SO(16) in UV

2d (0,2) gravity

3-branes integral component of 2d $(0,2)$ from F-theory on 5-folds

[Schafer-Nameki,TW] [Apruzzi,Hassler,Heckman,Melnikov]'16

✓ curve class $[C]$ fixed by D3/M2-tadpole

✓ D3-sector crucial for cancellation of gauge/grav anomalies:

Sources for gravitational anomalies:

1. Charged 7-brane modes [Schafer-Nameki,TW][Apruzzi,Hassler,Heckman,Melnikov]'16
2. 2d (0,2) supergravity
3. 3-brane sector

E.g. for smooth Weierstrass model on Y_5: [Lawrie,Schafer-Nameki,TW'16

$$I_4(T) = -\frac{1}{24} p_1(T) \cdot \Big(-\tau(B_4) + \chi_1(Y_5) - 2\chi_1(B_4) + 24 + \mathfrak{a}_{D3} \Big) \equiv 0 \checkmark$$

Analysis of CS terms in dual 1d Super-Quantum Mechanics proves:

Gravitational Anomaly Cancellation $\Longleftrightarrow$ Cancellation of D3/M2 tadpole in F/M-Theory

2d (0,2) gravity

$(0,2)$ Multiplet	IIB Orientifold	F-theory	Origin in IIB/F-theory	SQM Multiplet
Chiral	$h^{1,1}_{+}(X_4) - 1$	$h^{1,1}(B_4) - 1$	J, C_4	$(1,2,1)$
	$h^{1,1}_{-}(X_4)$ $h^{1,0}_{-}(\hat{S})$	$h^{2,1}(Y_5) - h^{2,1}(B_4)$	B_2, C_2 Wilson lines	$(2,2,0)$
	1 $h^{3,1}_{-}(X_4)$ $h^{3,0}_{-}(\hat{S})$	$h^{4,1}(Y_5)$	C_0, φ cmplx. str. brane def.	$(2,2,0)$
	$h^{3,1}_{+}(X_4)$	$h^{3,1}(B_4)$	C_4	$(0,2,2)$
Fermi	$\tau_{+}(X_4)$	$\tau(B_4)$	C_4 (dualised)	$(0,2,2)$
	$h^{2,1}_{+}(X_4)$	$h^{2,1}(B_4)$	$-$	$(2,2,0)$
	$h^{2,1}_{-}(X_4)$ $h^{2,0}_{-}(\hat{S})$	$h^{3,1}(Y_5) - h^{3,1}(B_4)$	$-$ $-$	$(0,2,2)$
Gravity	1	1	$g_{\mu\nu}, \mathcal{V}$	$(1,2,1)$ $+$ 1d gravity

Conclusions

D3-branes on curve C in F-theory backgrounds define chiral string theories in various dimensions.

Spacetime dim d	8	6	4	2
CY$_n$	2	3	4	5
2d supersymmetry	$(0,8)$	$(0,4)$	$(0,2)$	$(0,2)$

Technical description via topological duality twist:
4d N=4 SYM with varying gauge coupling due to $SL(2,\mathbb{Z})$ duality

Next steps include:

- Generalisation to non-abelian D3-brane stacks - possibly via duality to M5-branes cf talk by Sakura Schäfer-Nameki
- Better understanding of mysterious 3-7 string sector possibly similar to [Grassi,Halverson,Ruehle,Shaneson'16]?