Warped effective theories and holography

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based on: 1610.02403

1411.2623

1603.04470 with Alberto Zaffaroni

Plan

- Part I: Effective theory of warped flux compactifications

1610.02403

1411.2623

- Part II: Holographic effective field theories

1603.04470

with Alberto Zaffaroni

Part I:

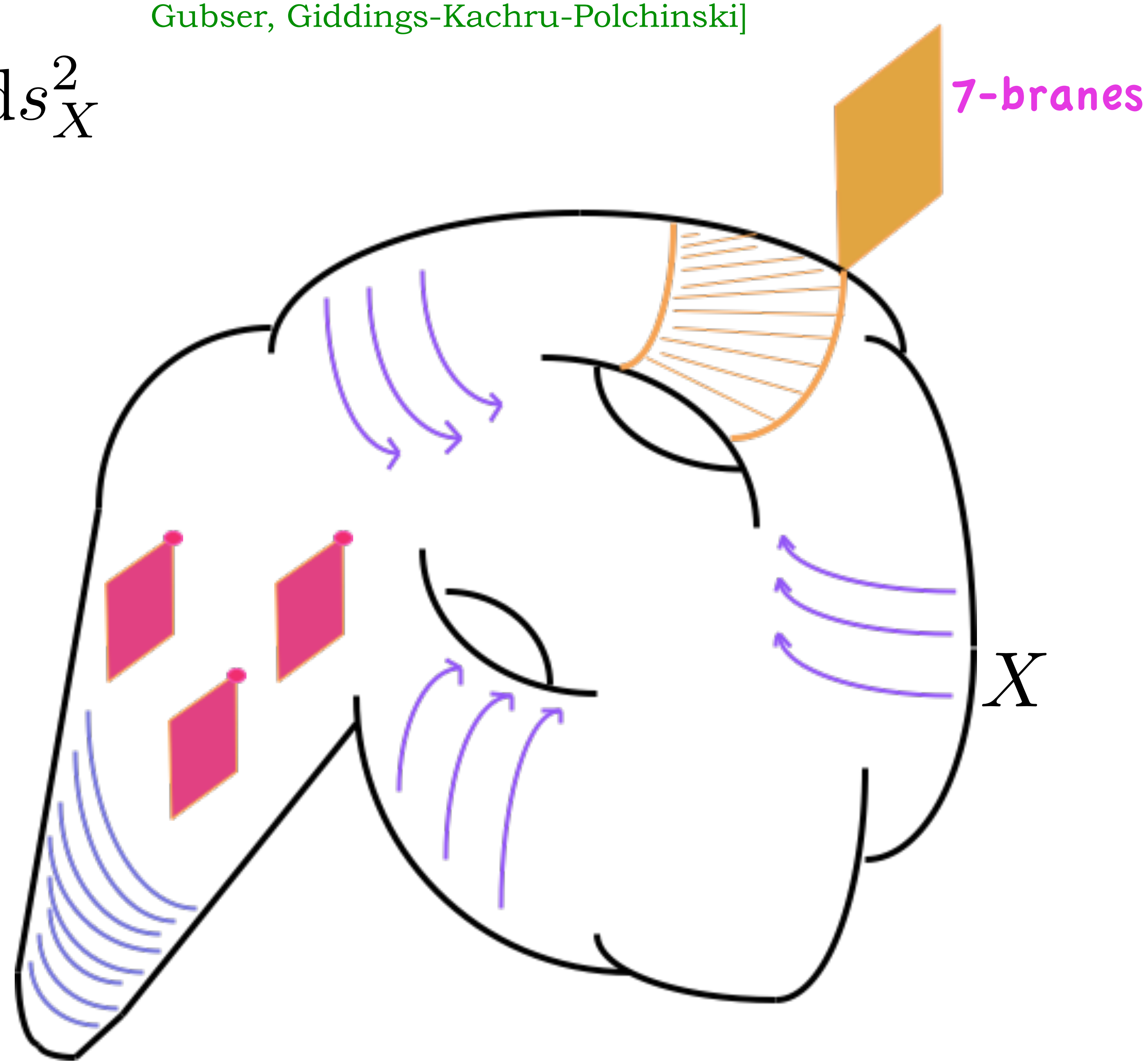
Effective theory of warped flux compactifications

The question

- F/M-theory compactifications are generically warped

[Becker-Becker,Grana-Polchinski, Gubser, Giddings-Kachru-Polchinski]

$$ds^2_{10} = e^{2A(y)} ds^2_4 + e^{-2A(y)} ds^2_X$$



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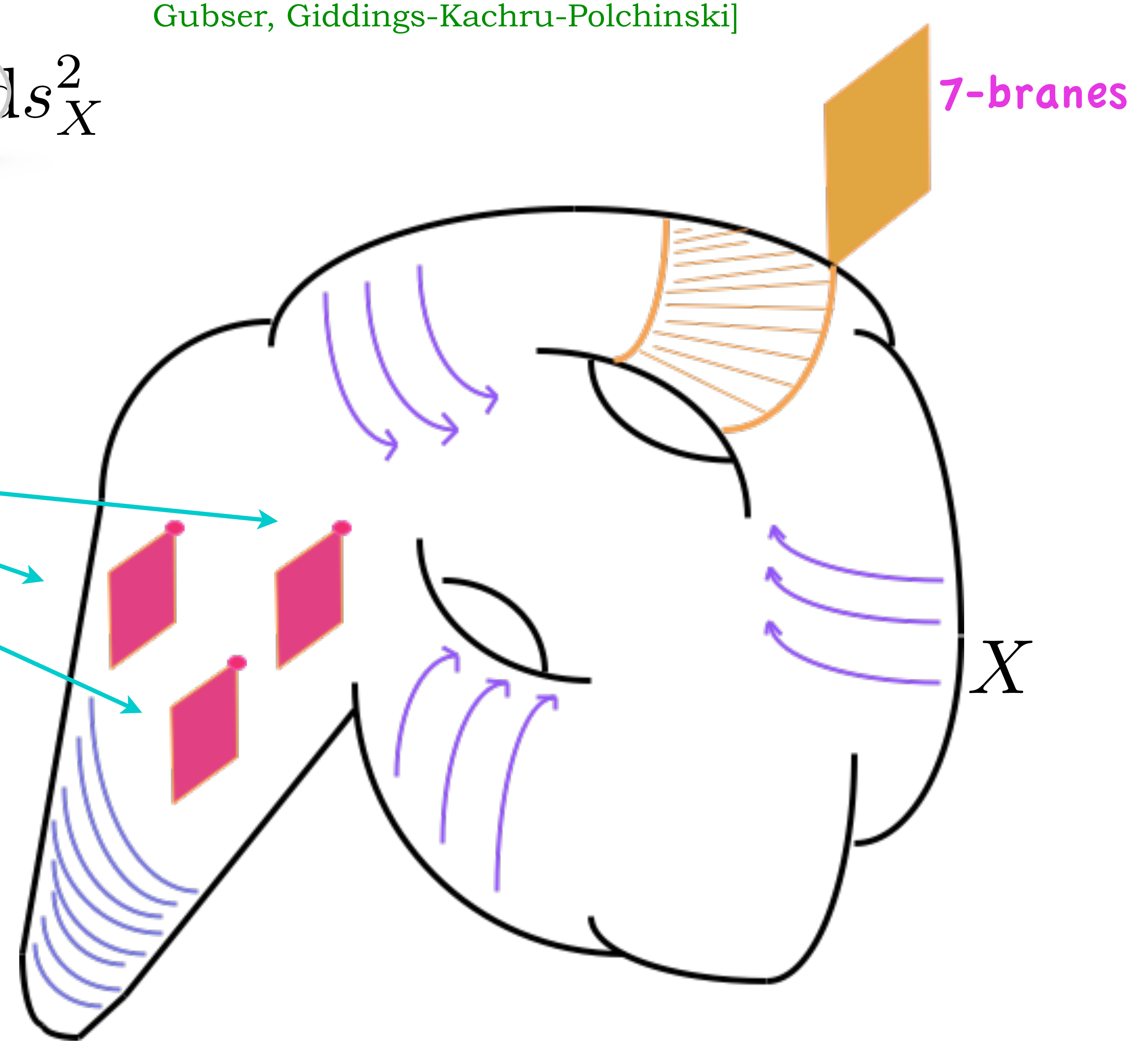
warping generated by:

- mobile D3-branes



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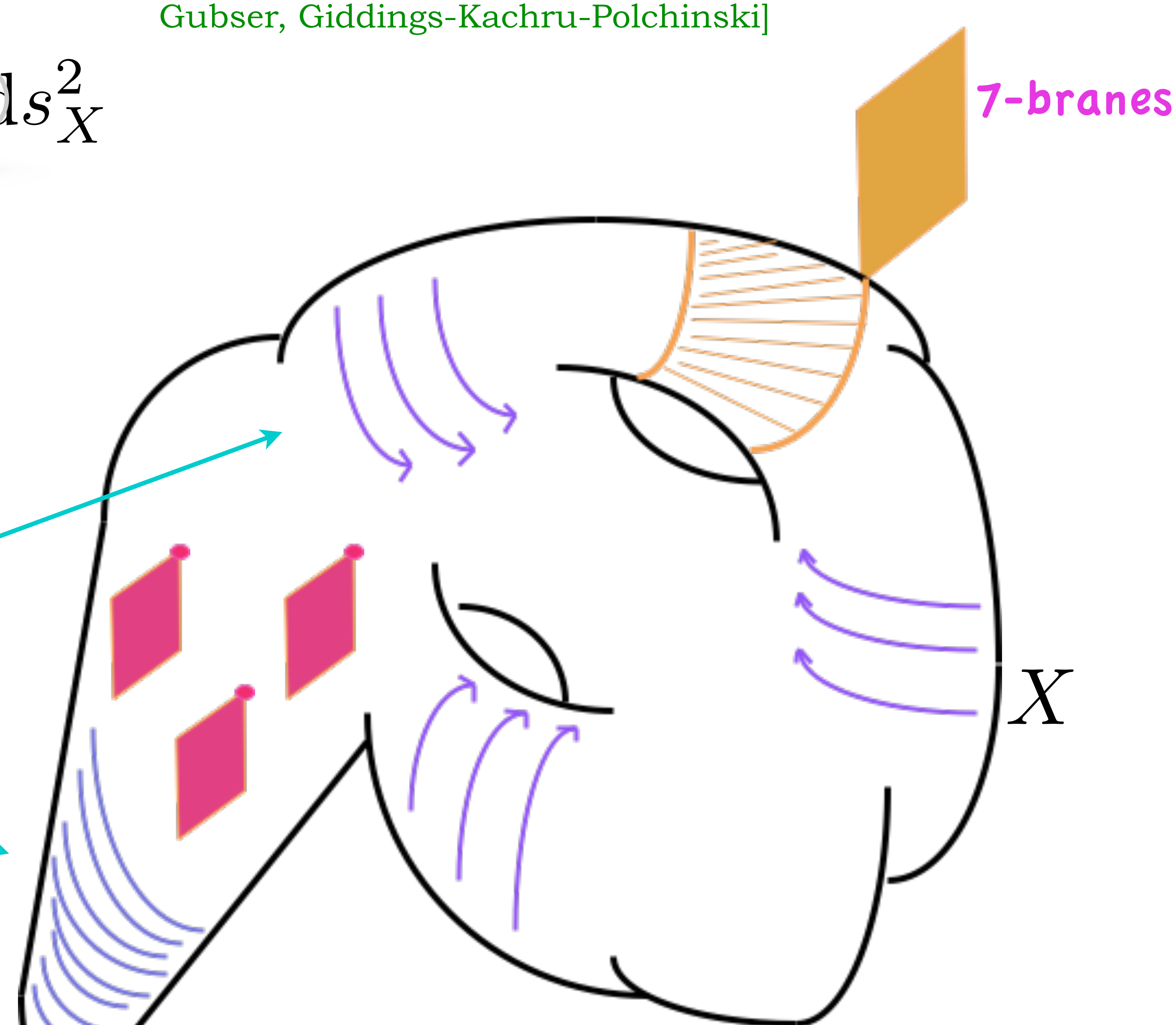
warping generated by:

* mobile D3-branes

* ISD fluxes

moduli stabilisation, SUSY breaking, strongly warped throats, ...



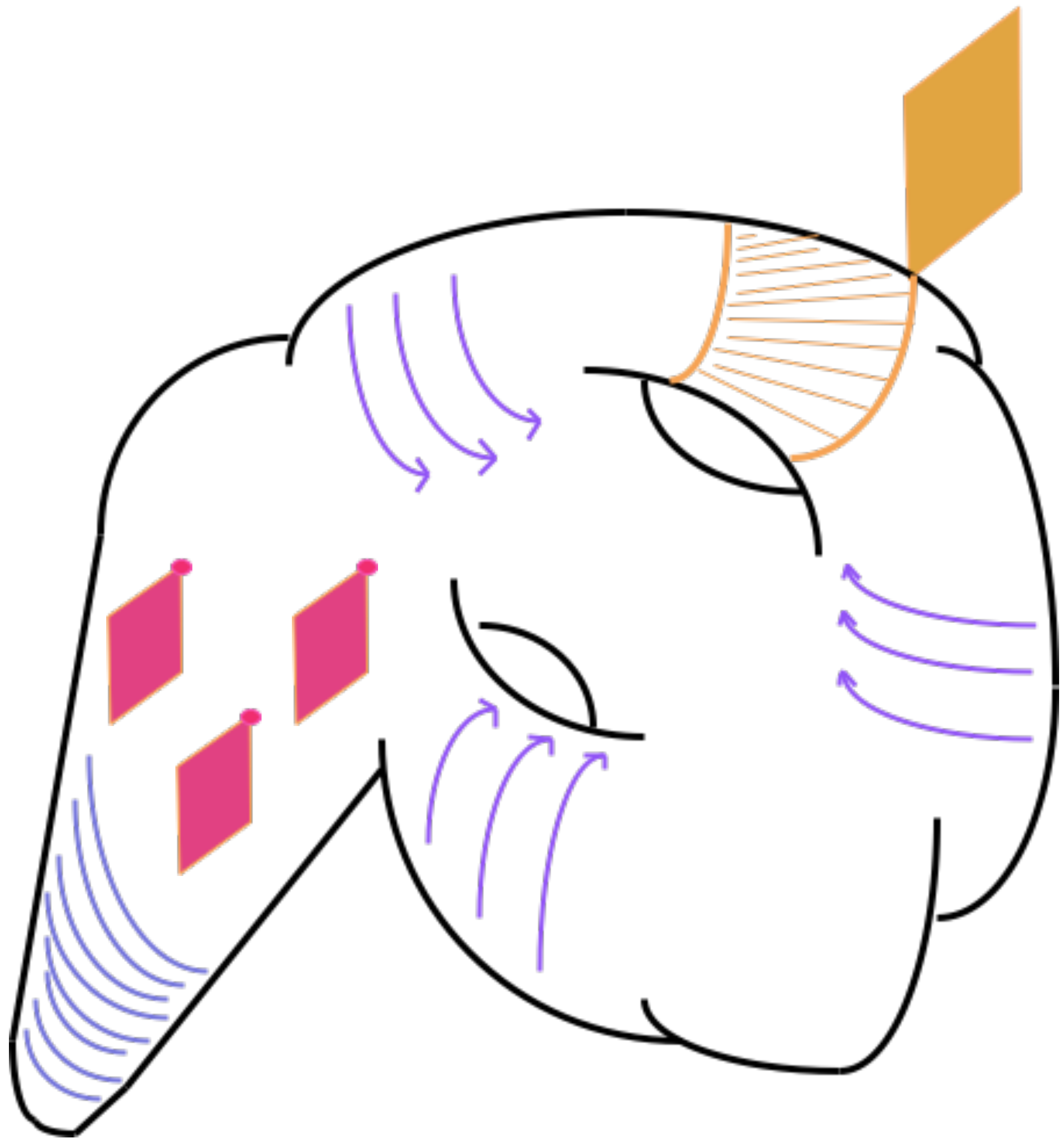
$$e^{A_{\rm min}} \ll e^{A_{\rm bulk}}$$

[Klebanov-Strassler]

The question

- (Perturbative) moduli include
  - D3-brane moduli
  - Kähler moduli
  - axions

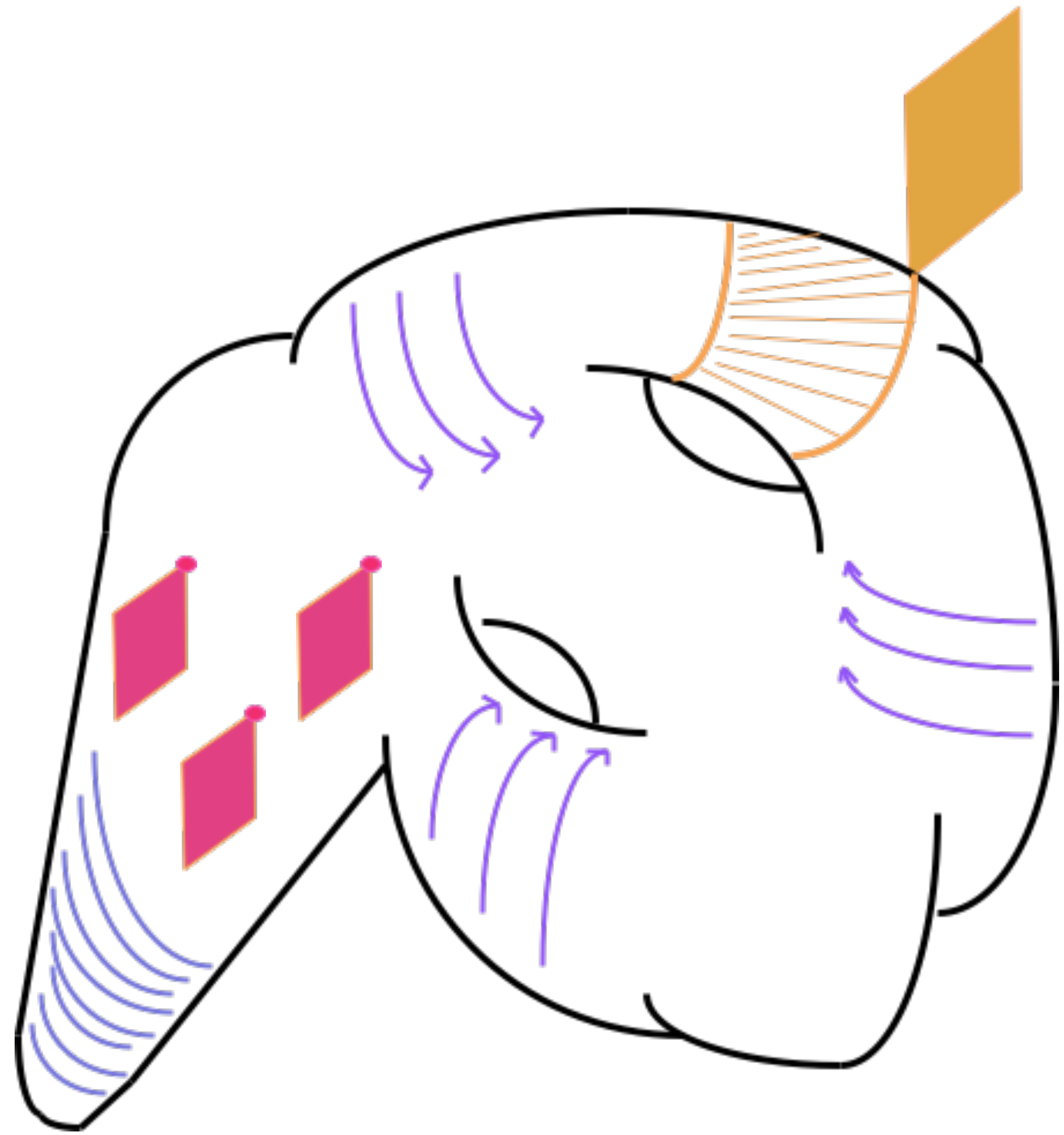
(Complex structure, axion-dilaton and 7-brane moduli assumed stabilised by fluxes)



The question

- (Perturbative) moduli include
  * D3-brane moduli
  * Kähler moduli
  * axions

- Fully coupled effective theory?
  * D3-branes beyond probe approximation?
  * impact of fluxes?



Universal modulus and Kähler potential

- The universal Kähler modulus:

$$\Delta_6 e^{-4A} = *_6 Q_{\rm D3} \quad \Longrightarrow \quad e^{-4A(y)} = a + e^{-4A_0(y)}$$

[Giddings-Maharana]

D3-branes, ISD fluxes, ...

UNIVERSAL MODULUS

$$\int_X e^{-4A_0} \mathrm{dvol}_X = 0$$

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[Giddings-Maharana]

D3-branes, ISD fluxes, ...

UNIVERSAL MODULUS

$$\int_X e^{-4A_0} {\rm dvol}_X = 0$$

- Natural superconformal structure fixes [LM `14]

$$K = -3 \log a$$

simple but implicit!

Chiral coordinates

- The moduli include
  - * D3 positions: z_I^i good chiral coordinates
  - * universal Kähler modulus a
  - * non-universal Kähler moduli $J = v^a \omega_a$ with $\frac{1}{3!}\int J \wedge J \wedge J = 1$
    basis of harmonic 2-forms
  - * C_4 -moduli
  - (* B_2, C_2 -moduli ignored in this talk)

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  - * non-universal Kähler moduli $J = v^a \omega_a$ with $\frac{1}{3!}\int J \wedge J \wedge J = 1$
    - ω_a: basis of harmonic 2-forms
  - * C_4-moduli

↓

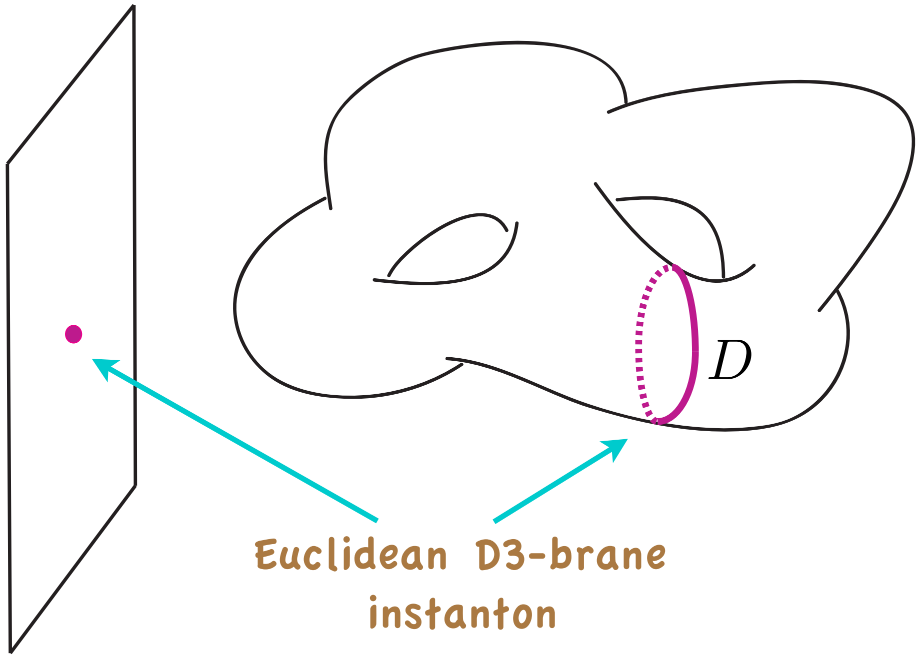
chiral coordinates: $\rho_a \qquad a = 1, \ldots, h_{1,1}$ Explicit form?

Chiral coordinates

- Probe SUSY D3 instantons

F-terms $\sim \quad e^{-S_{\mathrm{E3}}}$

must be holomorphic in ρ_a , z_I^i



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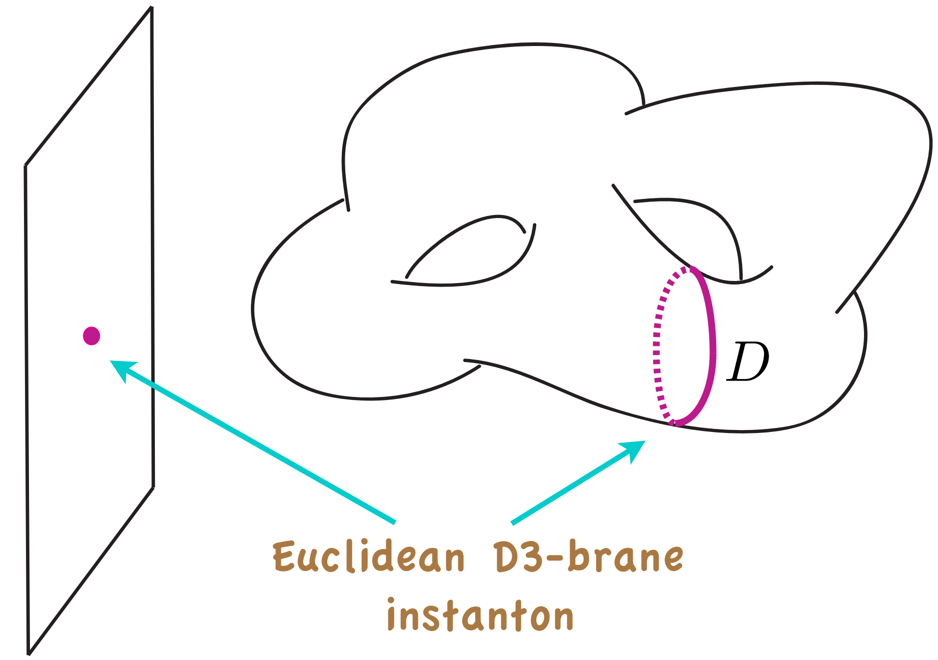

- $$S_{\rm E3} = \frac{1}{2}\int_D e^{-4A} J\wedge J - \frac{1}{2}\int_D \mathrm{Im}\,\tau\,\mathcal{F}\wedge\mathcal{F} + \ldots$$ $\left(\mathcal{F} = F_{\rm E3} - B_2\right)$

Chiral coordinates

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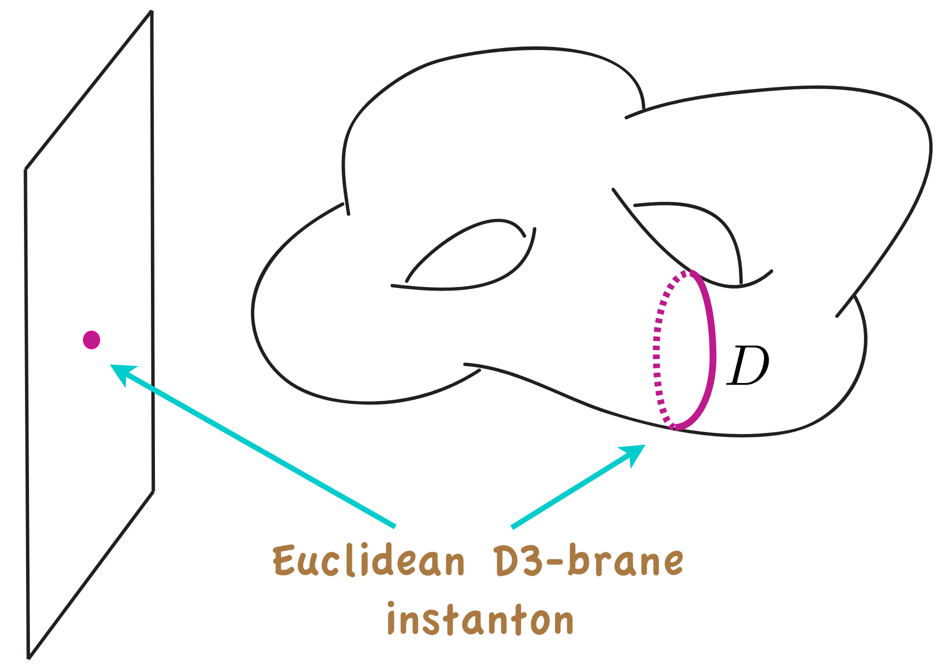

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Kähler moduli, mobile D3-branes, fluxes

cf. [Giddings-Maharana]

Chiral coordinates

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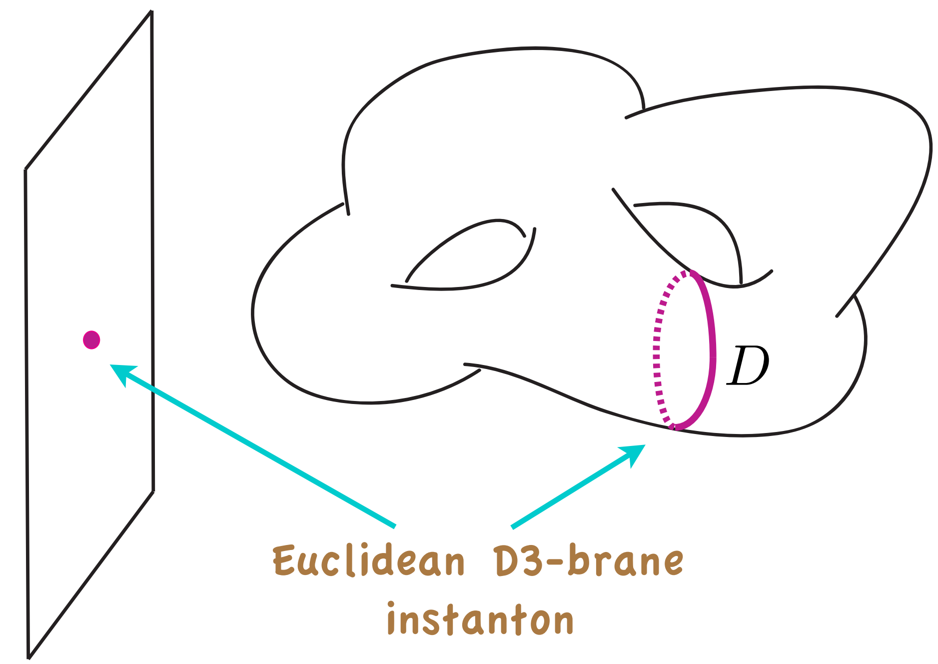

- $$S_{\rm E3} = \frac{1}{2}\int_D e^{-4A} J \wedge J - \frac{1}{2}\int_D {\rm Im}\,\tau \mathcal{F} \wedge \mathcal{F} + \dots \qquad (\, \mathcal{F} = F_{\rm E3} - B_2 \,)$$

Kähler moduli, mobile D3-branes, fluxes

cf. [Giddings-Maharana]

hidden dependence on non-universal Kähler moduli [LM `16]

$$J \wedge G_3 = 0 \quad \Longrightarrow \quad \delta G_3 = \delta v^a \, \partial\bar{\partial}\Lambda^{1,0}_a$$

with $\Delta_6 \Lambda^{1,0}_a = -2 *_6 (\omega_a \wedge G_3) \qquad (\, \tau = {\rm const.} \,)$

Chiral coordinates

- Perturbative C_4-axionic symmetry: $\mathrm{Im}\,\rho_a \to \mathrm{Im}\,\rho_a + \mathrm{const.}$

- We can focus on the real part:

$$\mathrm{Re}\,\rho_a = \frac{1}{2} a\,\mathcal{I}_{abc} v^b v^c + \frac{1}{2}\sum_I \kappa_a(z_I, \bar{z}_I; v) + h_a(v)$$
$$- \frac{1}{2\mathrm{Im}\,\tau}\int_{D_a}\left[\mathrm{Re}\left(b^{1,0}\wedge \bar{G}_3\right) - \frac{1}{2}\bar{\partial} b^{1,0}\wedge \partial \bar{b}^{0,1}\right]$$

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1

1. unwarped contribution [Grimm & Louis `04]

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1. unwarped contribution [Grimm & Louis `04]

2. $\mathrm{i}\partial\bar{\partial}\kappa_a = \omega_a = [D_a]_{\mathrm{harm}}$

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1. unwarped contribution [Grimm & Louis `04]

2. $\mathrm{i}\partial\bar{\partial}\kappa_a = \omega_a = [D_a]_{\mathrm{harm}}$

3. $h_a(v) \equiv \int_X \log\left(e^{-2\pi\kappa_a}|\zeta_a|^2\right)\left(\frac{\mathrm{i}G_3\wedge \bar{G}_3}{2\mathrm{Im}\,\tau} - Q_{\mathrm{D3}}^{\mathrm{nd}}\right)$

4. $G_3 = G_3^{(0)} + \partial\bar{\partial} b^{1,0}(v)$

Effective theory

$$K = -3\log a$$

implicit function of z_I^i and $\mathrm{Re}\,\rho_a$

$\Downarrow$

$$\mathcal{L}_{\rm bos} = \frac{1}{2}R_4 * 1 - \mathcal{G}^{ab}\nabla\rho_a \wedge *\nabla\bar{\rho}_b - \frac{1}{2\mathrm{v}_0 a}\sum_{I=1}^{N_{\rm D3}} g_{i\bar{\jmath}}(z_I, \bar{z}_I; v)\mathrm{d}z_I^i \wedge *\mathrm{d}\bar{z}_I^{\bar{\jmath}}$$

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D3-branes kinetic terms

matching probe approximation

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inverse of

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modifications due to warping and fluxes

$$\left(\Delta_6 \Lambda_a^{1,0} = -2 *_6 (\omega_a \wedge G_3)\right)$$

cf. [Coenden, Frey, David Marsh, Underwood `16]
[Frey, Roberts `14]

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D3-branes kinetic terms

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$$\left(\Delta_6\Lambda_a^{1,0} = -2 *_6 (\omega_a \wedge G_3)\right)$$

Furthermore:

no-scale $K^{A\bar{B}}K_A K_{\bar{B}} = 3$

cf. [Coenden, Frey, David Marsh, Underwood `16]
[Frey, Roberts `14]

Comments

- Dynamical higher derivative contributions ignored

  cf. [Grimm, Pugh, Weissenbacher `14,`15]

- (Co)homological structure of unwarped theory seems lost

- Explicit form requires explicit knowledge of CY metric

  ⇓

  Investigate implications in non-compact models, as in

  * local pheno models [Aldazabal, Ibanez, Quevedo, Uranga`00] ... [Donagi, Wijnholt - Beasley, Heckman, Vafa `08]
  * holography — strong warping! [LM, Zaffaroni `16]

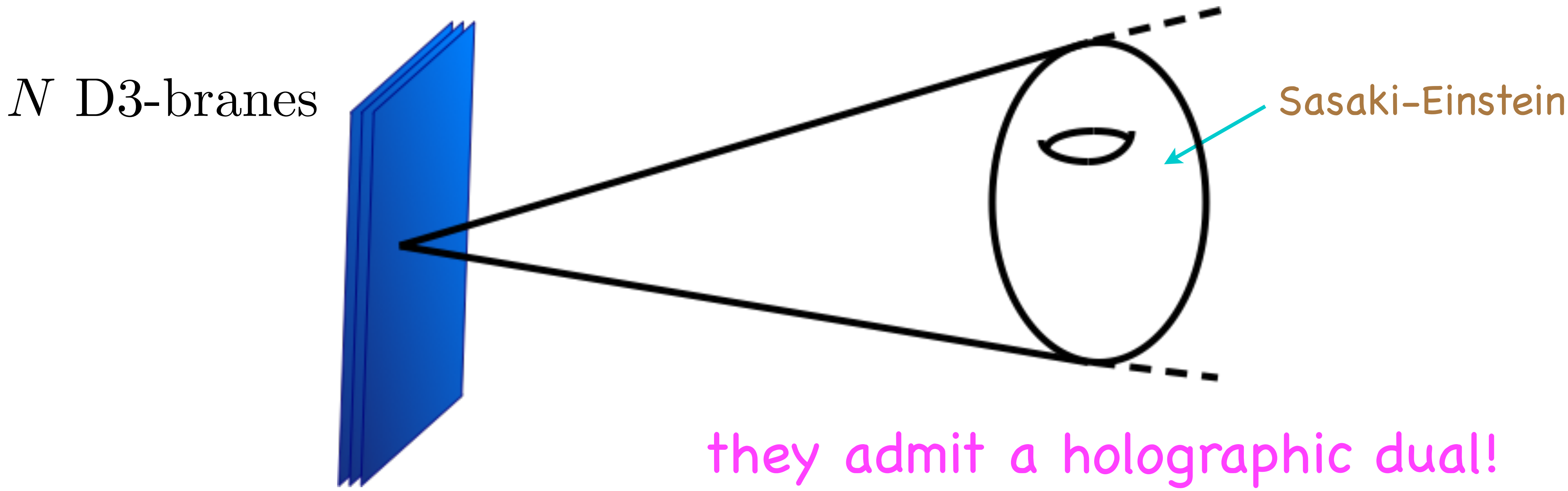
Part II:

Holographic Effective Field Theories

(HEFT)

IIB local holographic models

- Focus on 4d $\mathcal{N}=1$ SCFT's which are strongly coupled IR fixed points of



they admit a holographic dual!

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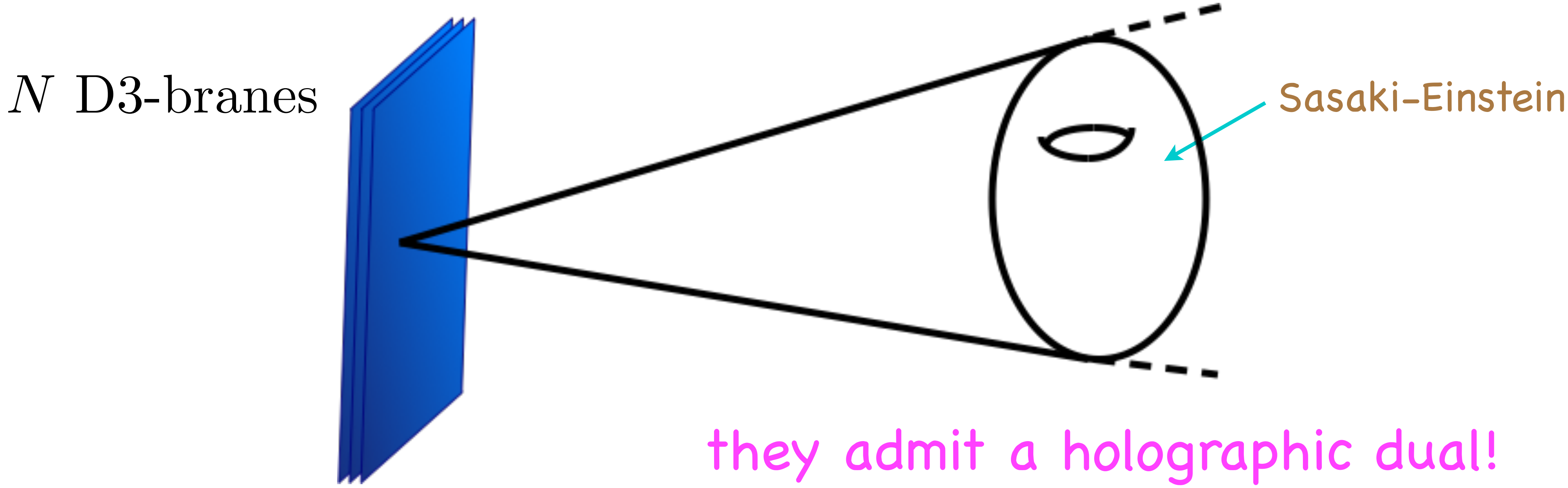
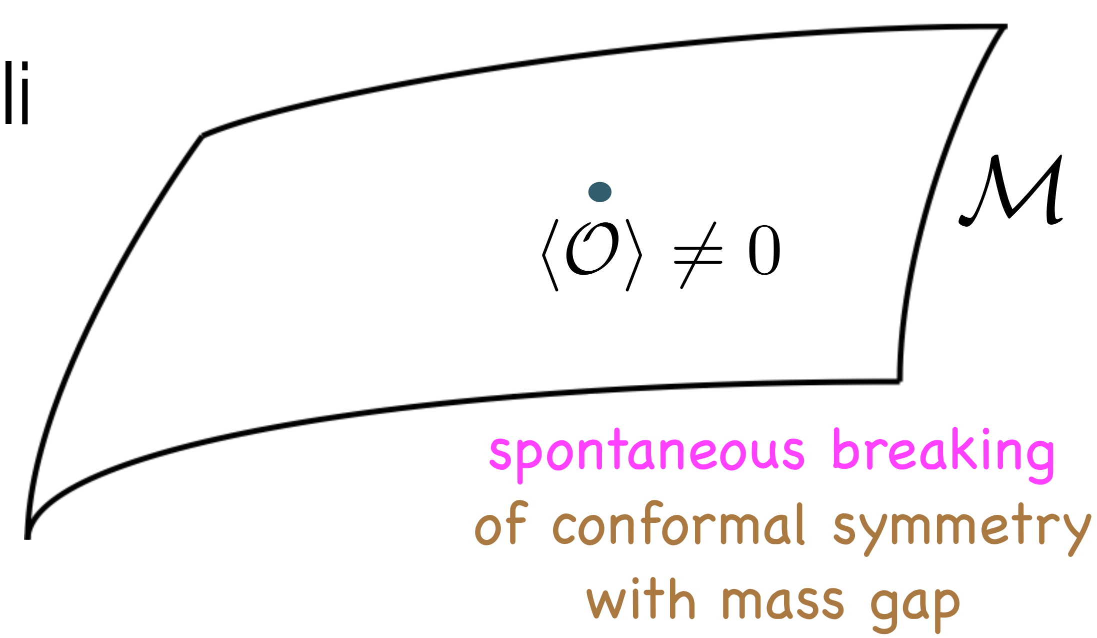


- These SCFTs have non-trivial moduli space of susy vacua



- Low-energy effective theory inaccessible in QFT

IIB local holographic models

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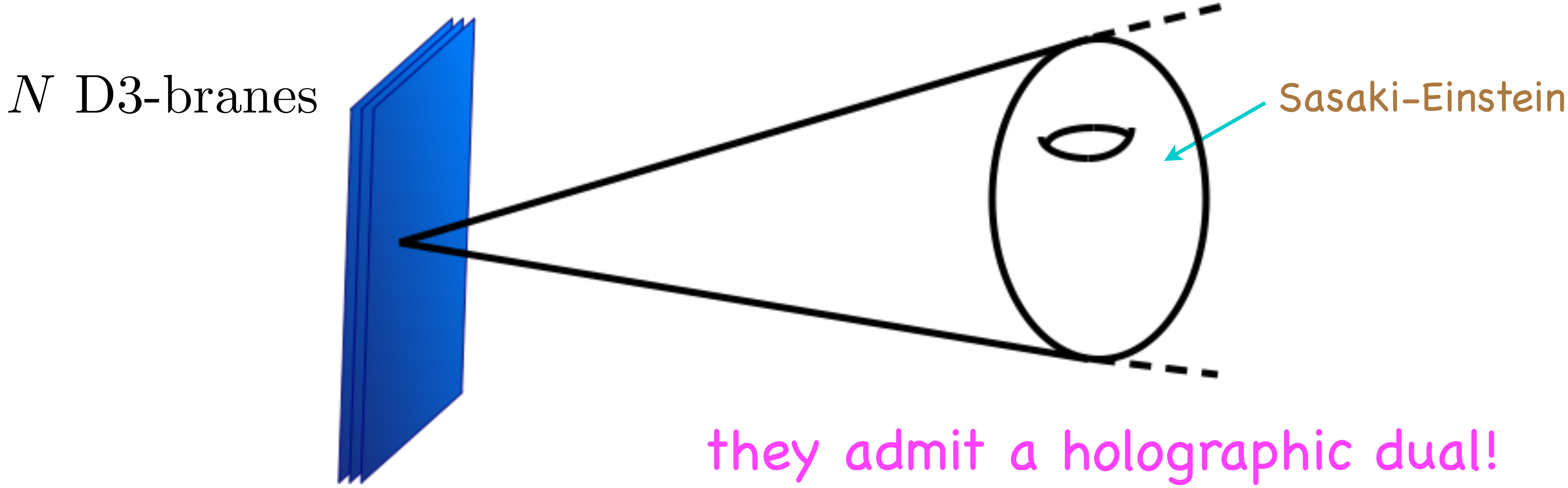

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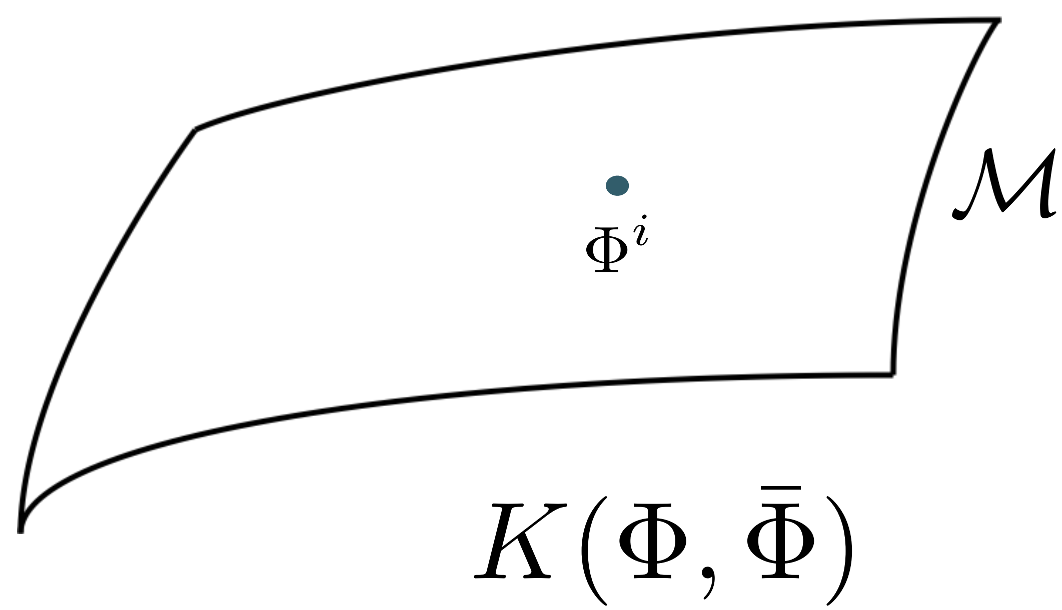
- Use holography to determine EFT:

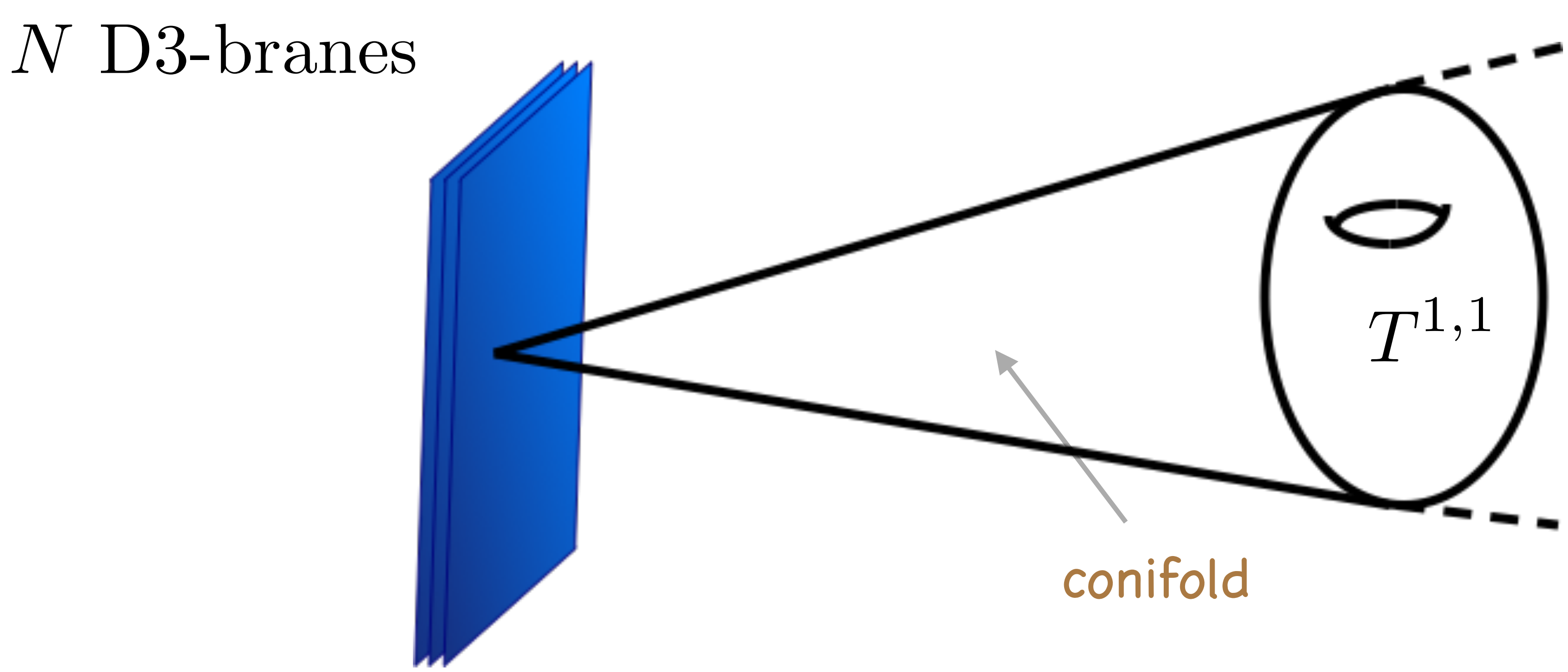
holographic effective field theory (HEFT)



Prototype: the Klebanov-Witten model

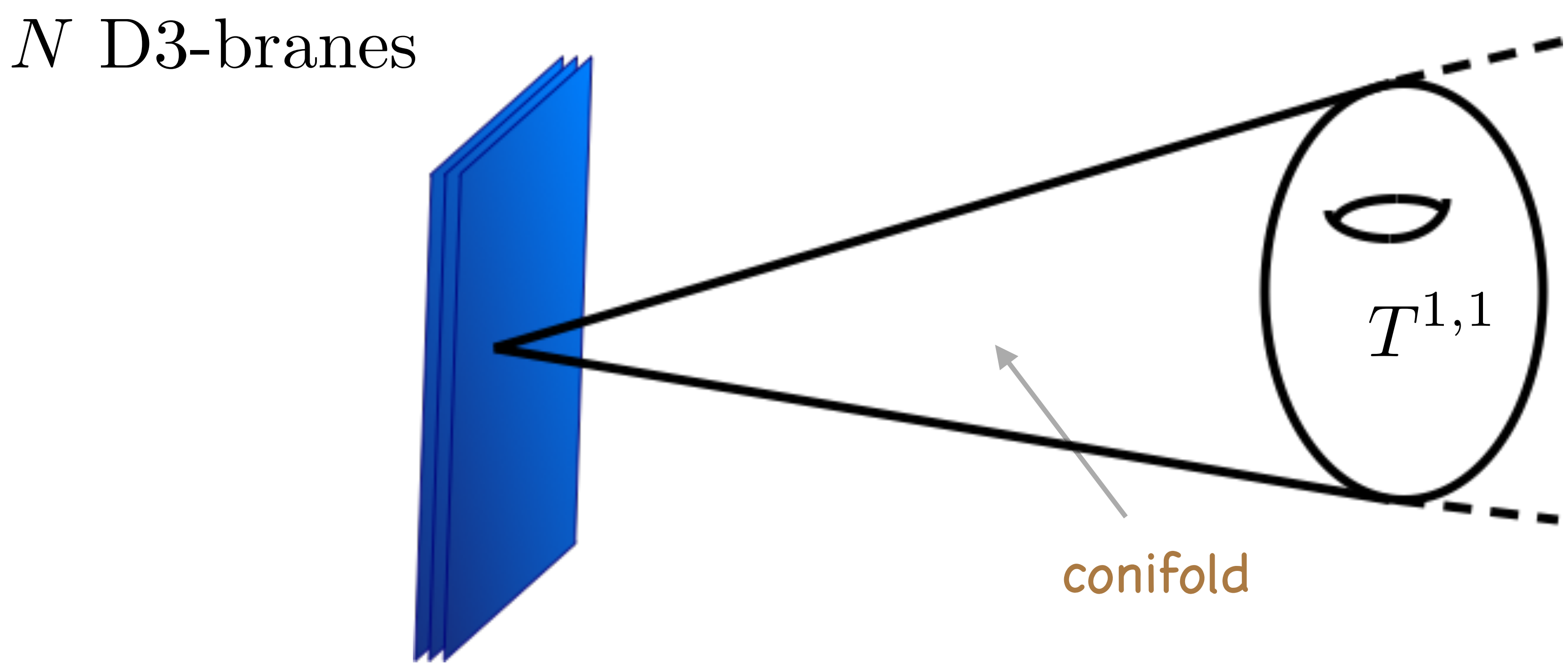
Klebanov-Witten model

[Klebanov & Witten , `98-`99]

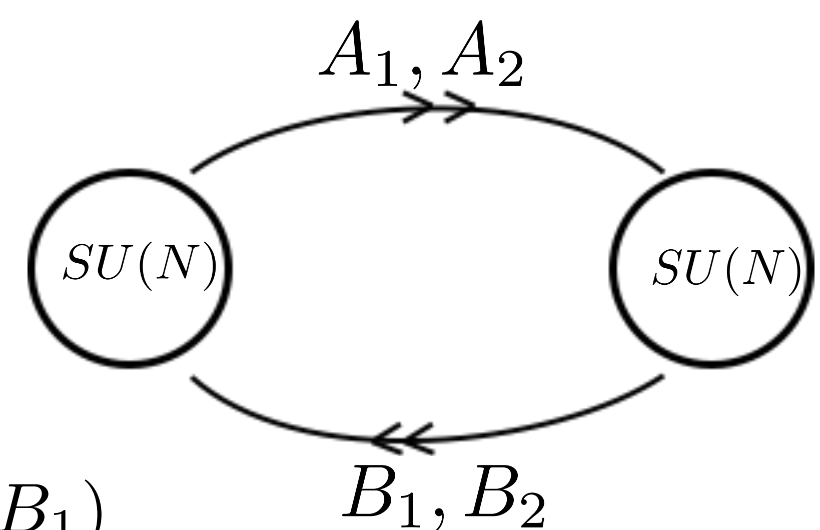


Klebanov-Witten model

[Klebanov & Witten , `98-`99]

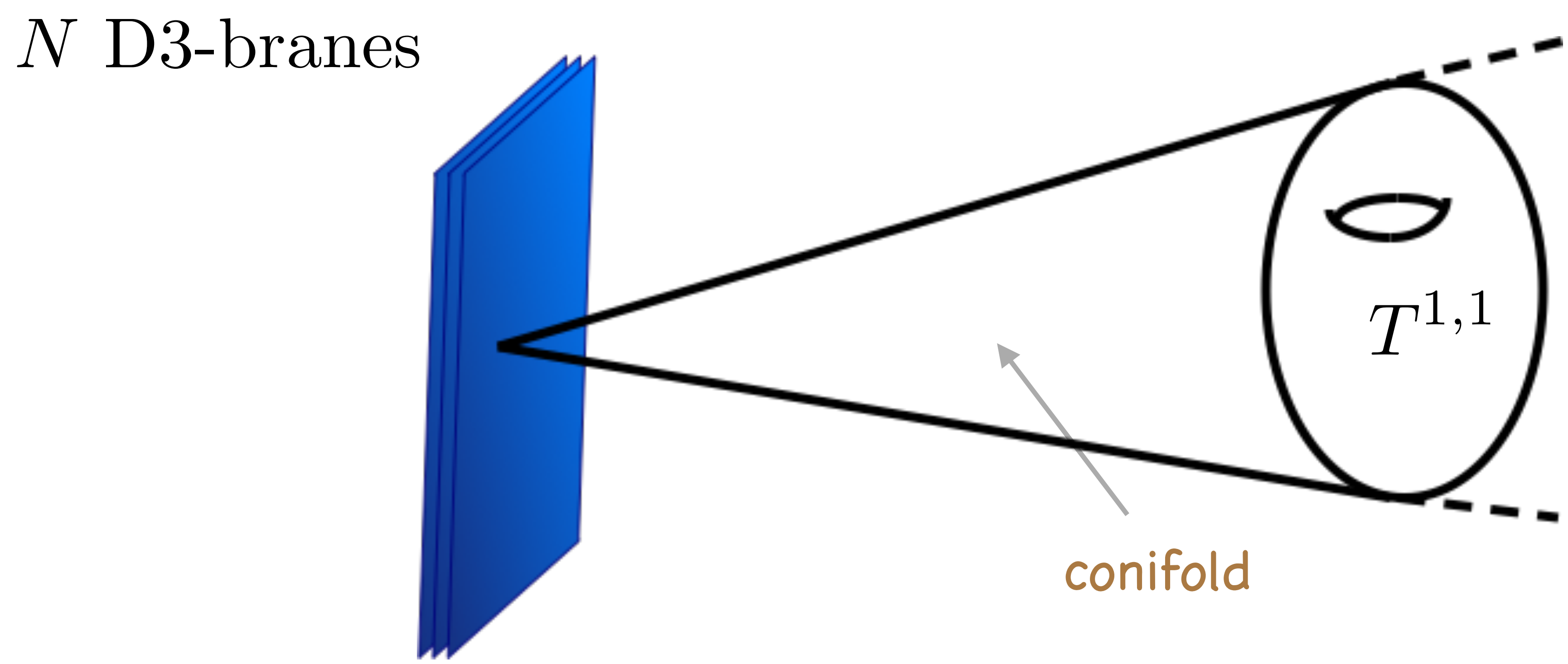


- UV quiver gauge theory:
  - gauge group: $SU(N) \times SU(N)$
  - chiral matter: $A_1, A_2 \in (\mathbf{N}, \bar{\mathbf{N}})$, $B_1, B_2 \in (\bar{\mathbf{N}}, \mathbf{N})$
  - superpotential: $W = h \; \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)$



Klebanov-Witten model

[Klebanov & Witten , `98-`99]



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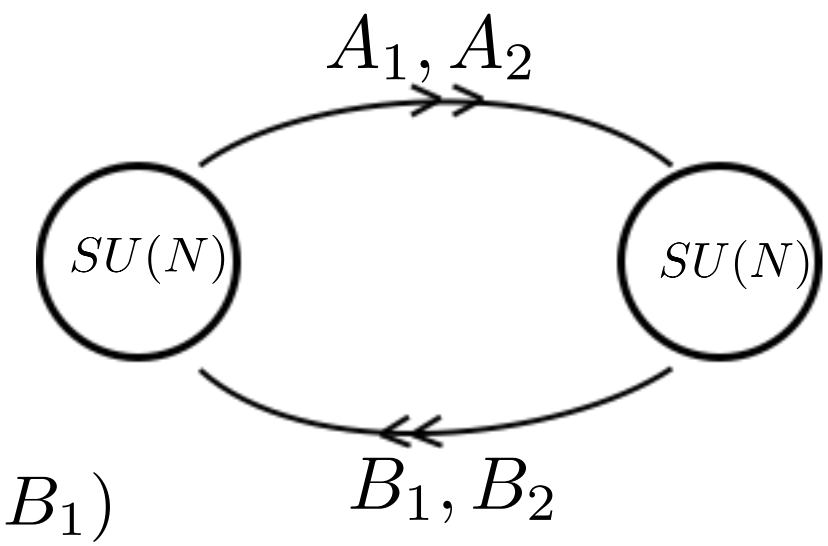
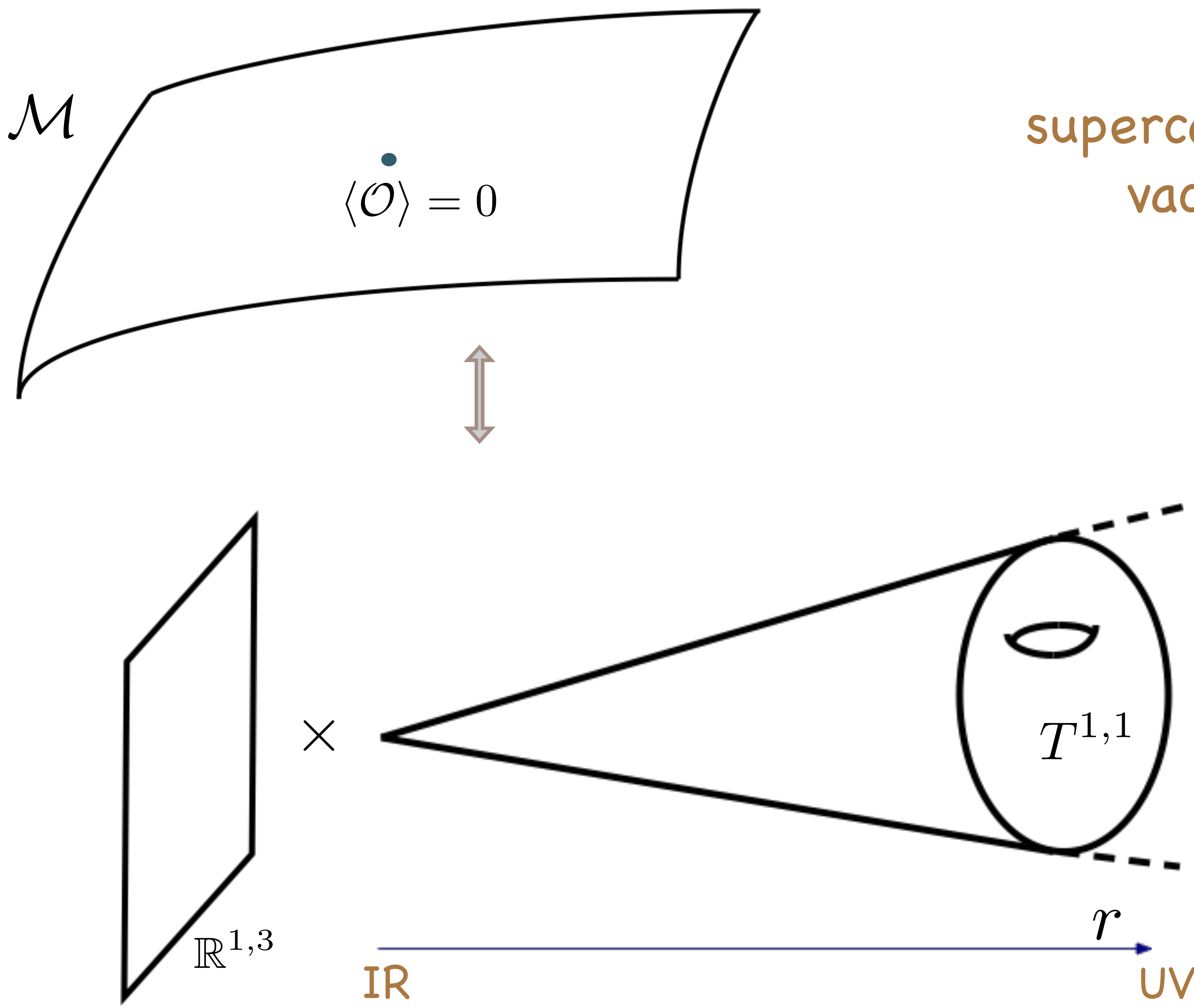


UV	RG-flow	IR
Quiver gauge theory	→	$\mathcal{N} = 1$ SCFT

Klebanov-Witten model

[Klebanov & Witten , `98-`99]

SCFT and gravity dual have non-trivial moduli space



superconformal vacuum

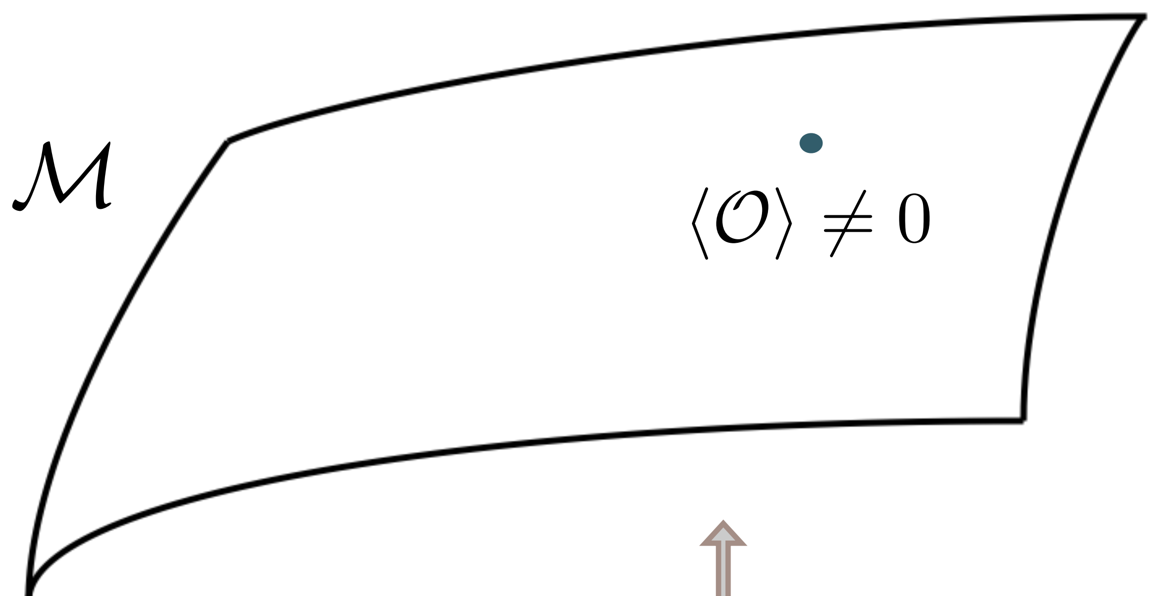
$\text{AdS}_5 \times T^{1,1}$

$$e^{-4A} = \frac{N}{r^4}$$

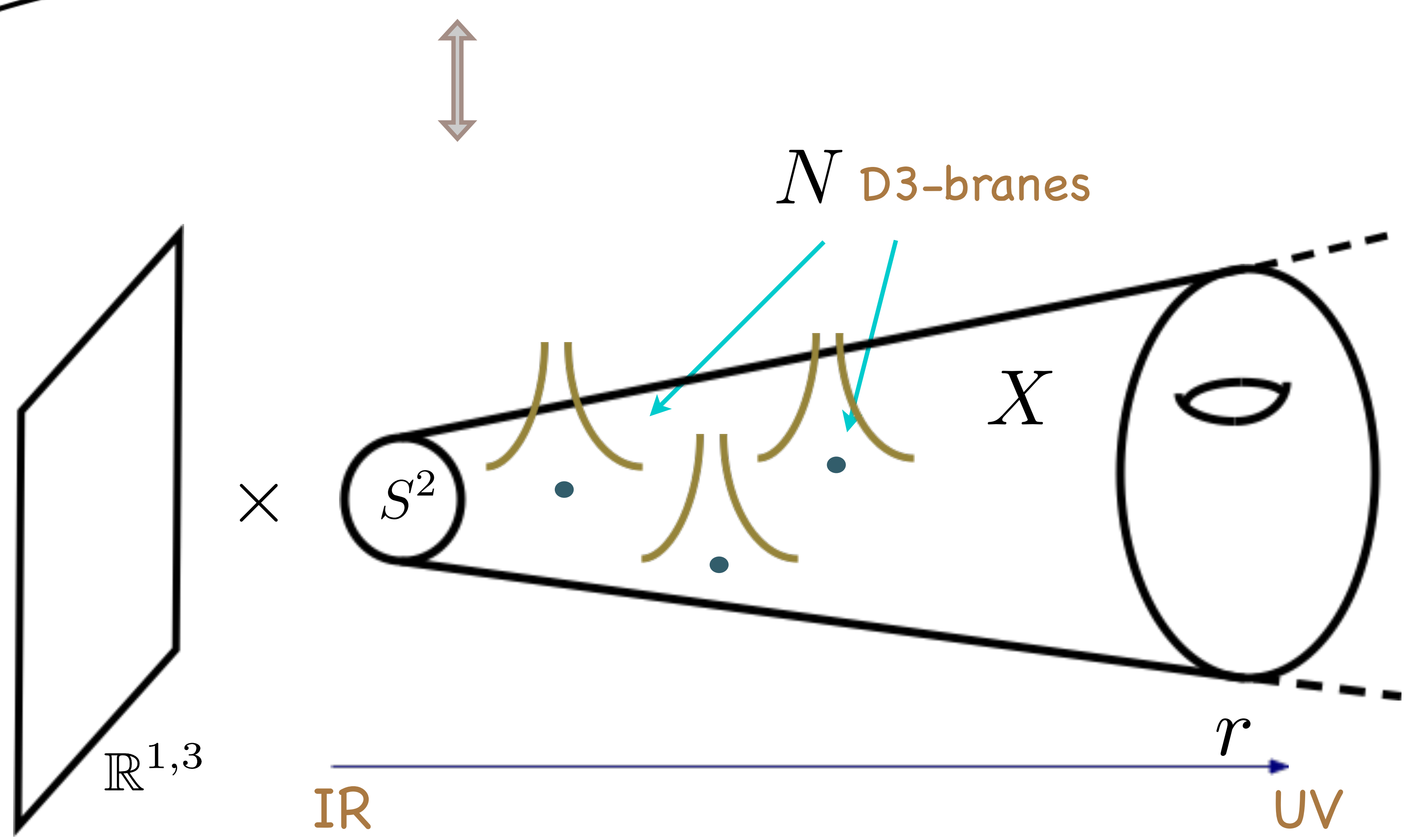
Klebanov-Witten model

[Klebanov & Witten , `98-`99]

SCFT and gravity dual have non-trivial moduli space



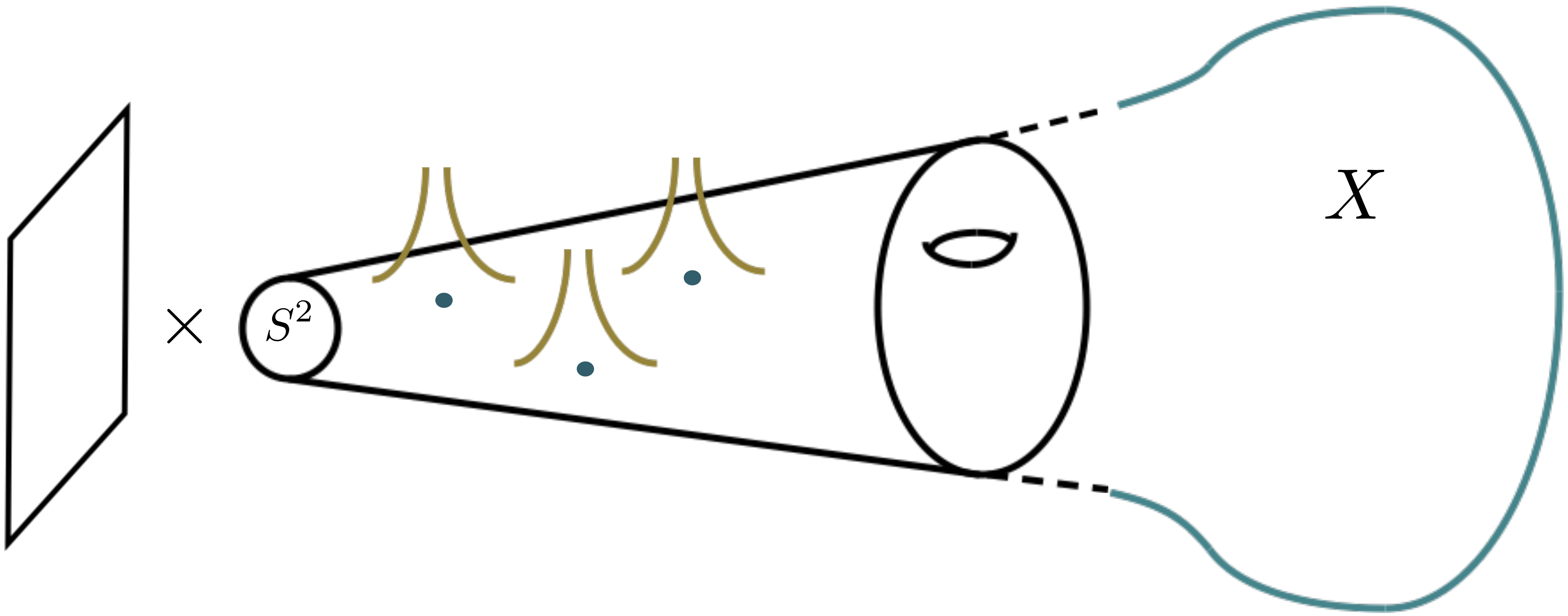
spontaneous breaking of conformal symmetry



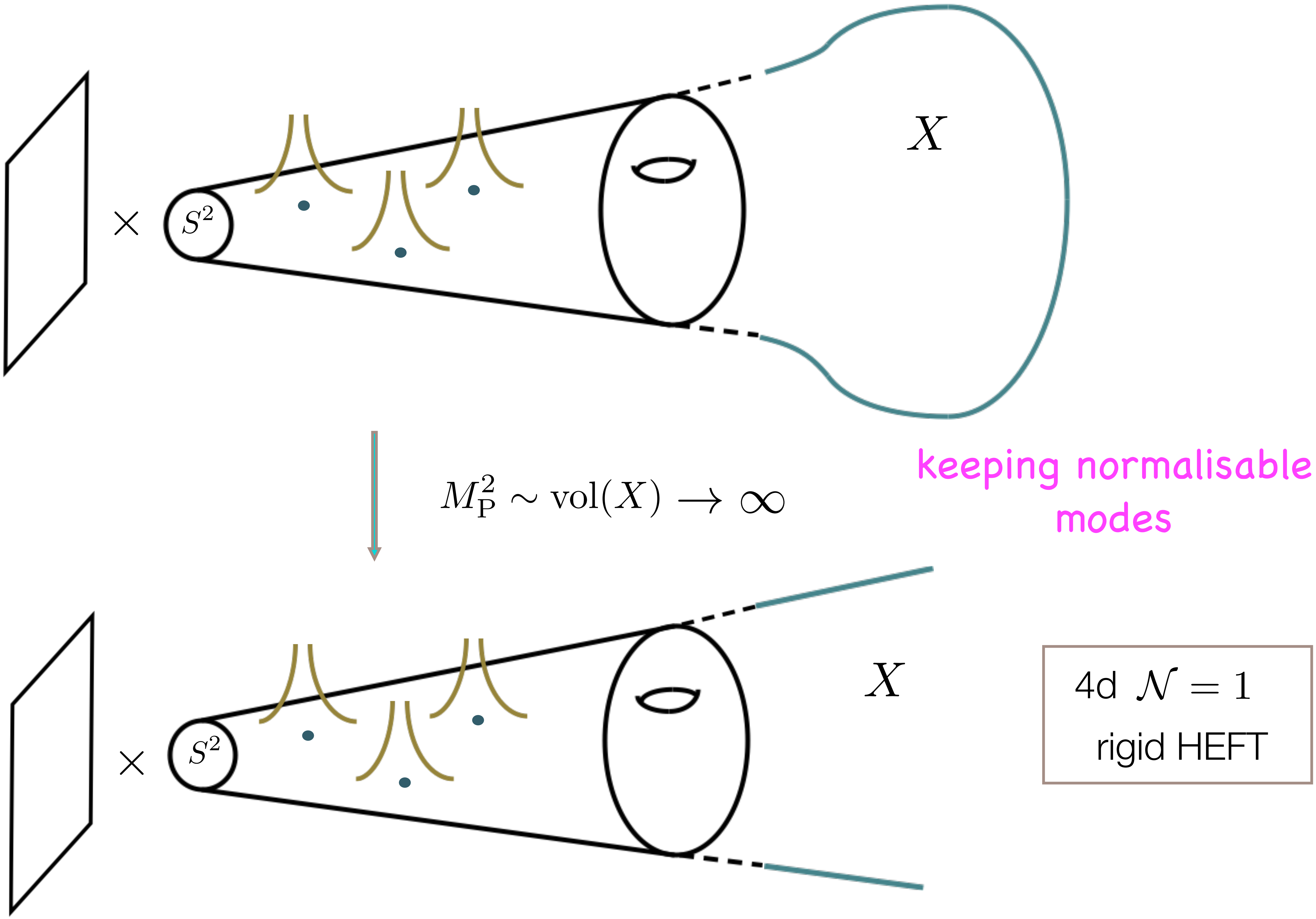
$$e^{-4A} \sim \frac{N}{r^4}$$

for $r \to \infty$

Deriving the HEFT from rigid limit

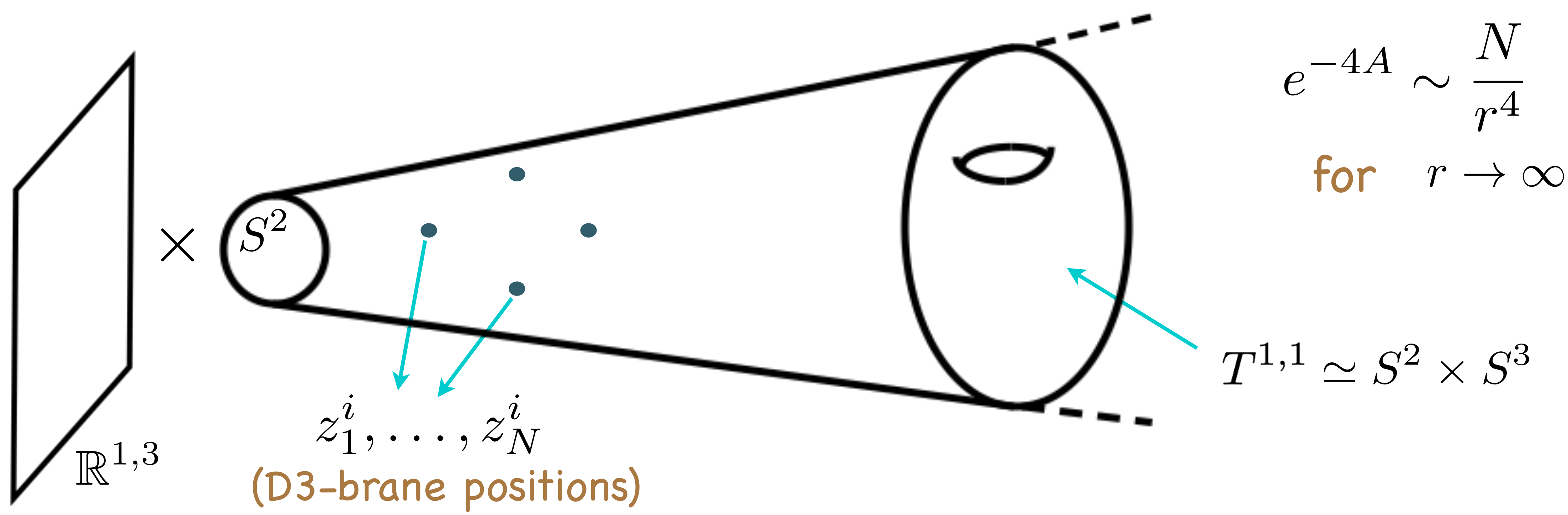


Deriving the HEFT from rigid limit



4d $\mathcal{N} = 1$
rigid HEFT

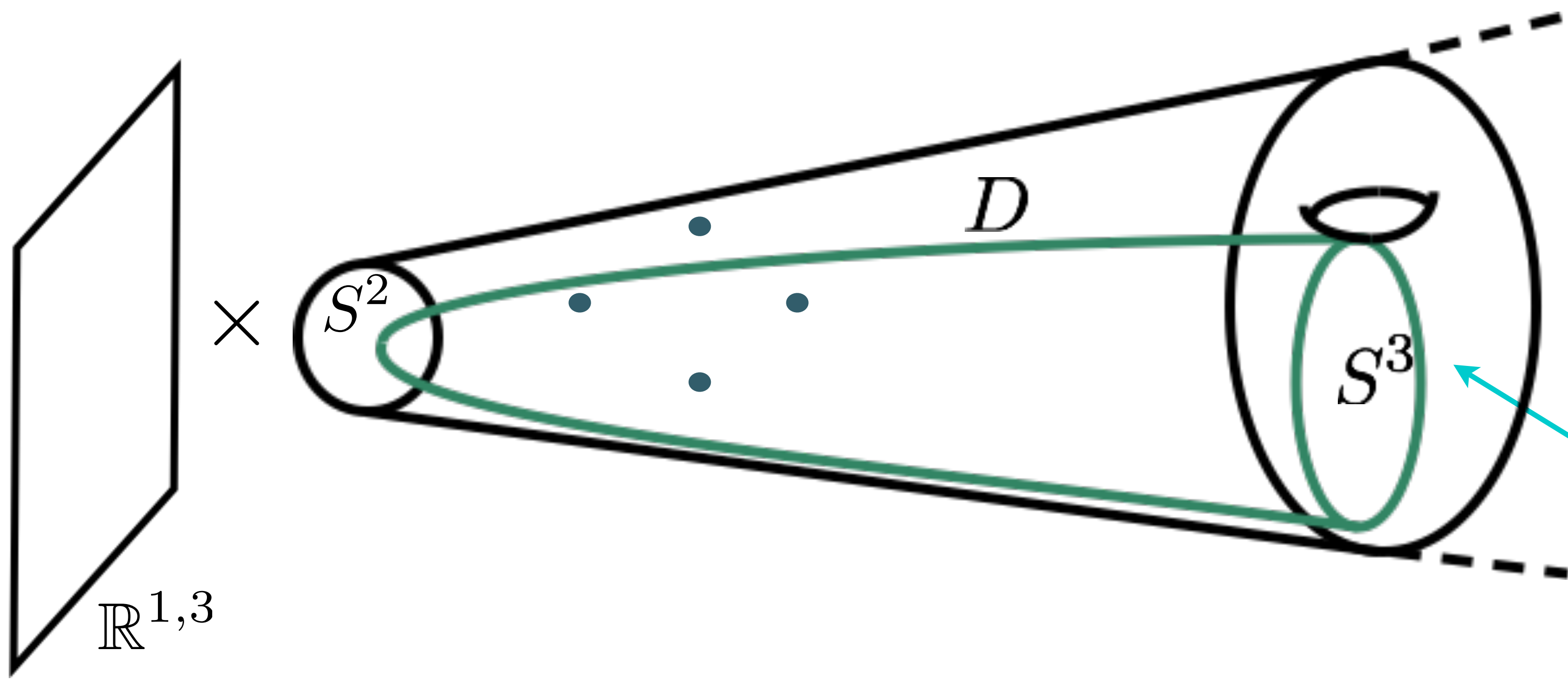
HEFT chiral fields



- D3-brane positions → z^i_I (SCFT mesonic branch)
- $v = \mathrm{vol}(S^2)$ + $c = \int_D C_4$ → $\rho = \mathrm{Re}\,\rho + \mathrm{i}c$ (SCFT baryonic branch)

  (Kähler modulus)

HEFT chiral fields



$e^{-4A} \sim \frac{N}{r^4}$ for $r \to \infty$

$T^{1,1} \simeq S^2 \times S^3$

- D3-brane positions → z_I^i (SCFT mesonic branch)
- $v = \text{vol}(S^2)$ (Kähler modulus) + $c = \int_D C_4$ → $\rho = \text{Re}\,\rho + \text{i}c$ (SCFT baryonic branch)

$[D]_{\text{harm}} = \omega$ with $\int \omega \wedge *_6 \omega = \infty$ BUT

$\int e^{-4A} \omega \wedge *_6 \omega < \infty$ ⟶ ρ is dynamical modulus

Some more explicit formulas

$$\mathcal{L}^{\text{bos}}_{\text{HEFT}} = -\frac{\pi}{\mathcal{G}(\rho,\bar\rho,z,\bar z)} \nabla_\mu\rho\nabla^\mu\bar\rho - 2\pi \sum_I g_{i\bar\jmath}(z_I,\bar z_I;v)\partial_\mu z_I^i \partial^\mu \bar z_I^{\bar\jmath}$$

$$\int_X e^{-4A}\omega\wedge *\omega \qquad \partial_\mu\rho - \frac{\partial\kappa}{\partial z_I^i}\partial_\mu z_I^i \qquad \text{CY metric on } X$$

- $\mathrm{Re}\,\rho = \frac{1}{2}\sum_{I\in \mathrm{D3's}} \kappa(z_I,\bar z_I;v)$ $\qquad K = 2\pi \sum_{I\in\mathrm{D3's}} k_0(z_I,\bar z_I;v)$
- $z^i = (\lambda, U, Y)$ local complex patch, $s^2 = (1+|\lambda|^2)(|U|^2+|Y|^2)$ radial coordinate
- $k_0(z,\bar z;v) = \frac{3}{4}G(s^2;v) + \frac{3}{8\pi}v$
- $\kappa(z,\bar z;v) = -\frac{1}{4}\int_0^{s^2}\frac{\mathrm{d}x}{x}\frac{G(x;v)}{\pi G(x;v)+v} + \frac{1}{2\pi}\log(1+|\lambda|^2) - \frac{3}{8\pi}\log v$
- $\mathcal{G}(\rho,\bar\rho,z,\bar z) = \frac{3}{4}\sum_I \frac{1}{v+\pi G(s_I^2;v)}$

where $G(s^2;v) = -\frac{1}{2\pi}v + \frac{v^2}{4\pi^2}\mathcal{N}^{-\frac{1}{3}}(s^2;v) + \mathcal{N}^{\frac{1}{3}}(\chi^2;v)$

with $\mathcal{N}(s^2;v) = \frac{1}{2}\left(s^4 - \frac{v^3}{4\pi^3} + s^2\sqrt{s^4 - \frac{v^3}{2\pi^3}}\right)$

Complete HEFT

Superconformal symmetry

- Each HEFT field has its definite scaling dimension:

$$\longrightarrow \qquad \Delta_{K(\rho,\bar{\rho},z,\bar{z})} = 2$$

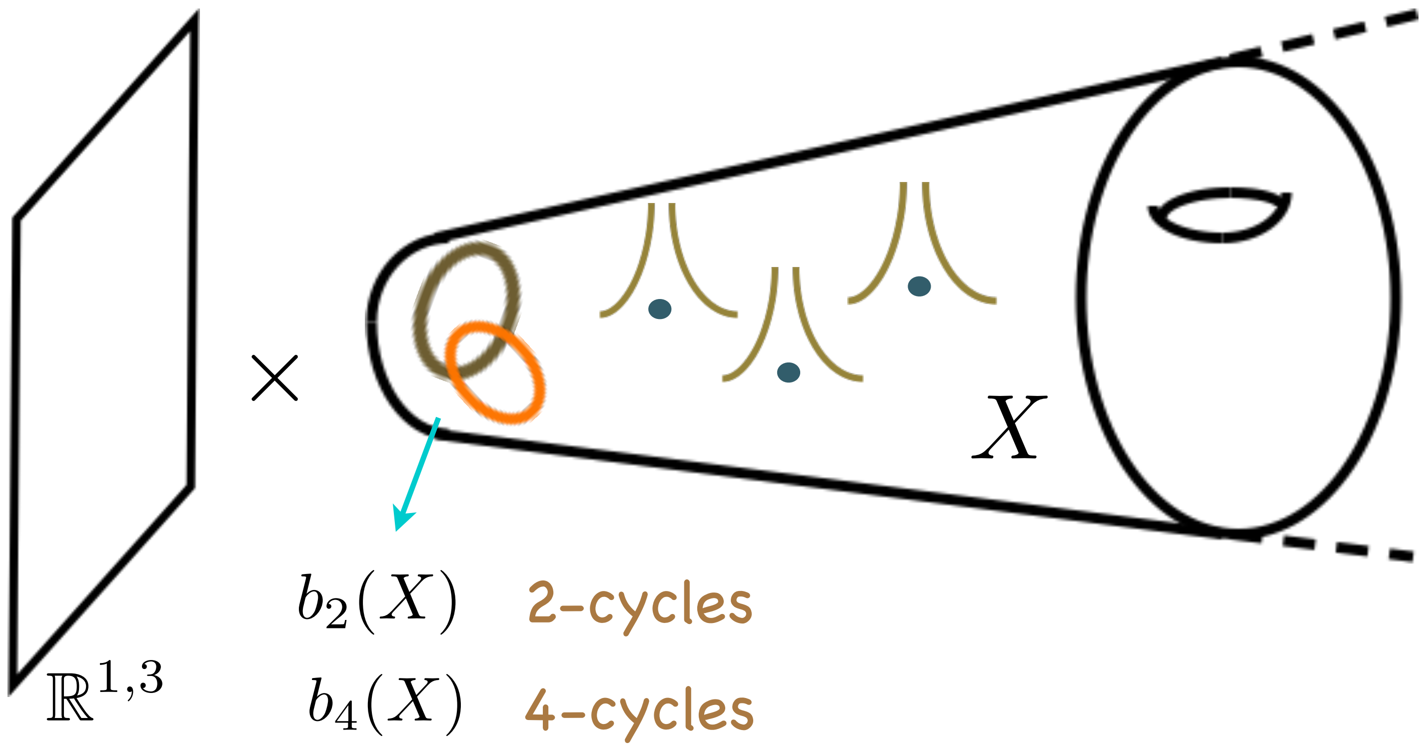
$$\Downarrow$$

$$\mathcal{L}_{\rm HEFT} = \int {\rm d}^4\theta \, K(\rho,\bar{\rho},z,\bar{z}) \qquad \mathcal{N}=1$$ superconformal

Non-linearly realised superconformal symmetry!

Final remarks

- The procedure is valid for more general IIB models



[Martelli, Sparks `08]

* Quiver gauge group: $SU(N)^{1+b_2(X)+b_4(X)}$

- Explicit form of the HEFT uses the explicit CY metric

cases-by-case study apparently needed

- Extension to $\mathrm{AdS}_4/\mathrm{CFT}_3$ models rather natural!

[Cremonesi, LM, Garcia-Etxebarria, in prep.]

Final remarks

- Marginal deformations to non-CY geometries?

- Non-conformal settings (e.g. baryonic branch of Klebanov-Strassler) ?

- Results valid for $N \gg 1$, $\lambda_{\text{'t Hooft}} \sim g_{\rm s} N \gg 1$ subleading effects?

Thanks