Second order conformally invariant elliptic equations

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$\Gamma \subset \mathbb{R}^n$ open,convex,symmetric cone,vertex at origin

$$\Gamma_n \subset \Gamma \subset \Gamma_1$$

$$\Gamma_n := \{\lambda \in \mathbb{R}^n \mid \lambda_i > 0 \ \forall \ i\}, \quad \Gamma_1 := \{\lambda \in \mathbb{R}^n \mid \sum_{i=1}^{n} \lambda_i > 0\}$$

$f \in C^\infty(\Gamma) \cap C^0(\bar{\Gamma})$ symmetric, concave, homogeneous of degree 1

$$f_{\lambda_i} > 0 \text{ in } \Gamma \ \forall \ i, \quad f > 0 \text{ in } \Gamma, \quad f = 0 \text{ on } \partial\Gamma$$

(M^n, g), $n \geq 3$, Schouten tensor:

$$A_g = \frac{1}{n-2}\Big(Ric_g - \frac{1}{2(n-1)}\, R_g\, g\Big),$$

Proposition 1.
(N^n, g) complete, $\partial N \neq \emptyset$, $Ric_g \geq -(n-1)\alpha^2$, $\alpha \geq 0$ constant, mean curvature $H_{\partial N} > (n-1)c_0 > (n-1)\alpha$. Then, for all $x \in N$,

$$d_g(x, \partial N) \leq \begin{cases} \frac{1}{c_0} & \text{if } \alpha = 0, \\ \frac{1}{\alpha} \coth^{-1}\left(\frac{c_0}{\alpha}\right) & \text{if } \alpha > 0. \end{cases}$$

$(\coth^{-1}(y) = \frac{1}{2} y \ln(1 + \frac{2}{y-1})$

• Step 1.

$$u_i(x) \leq C d_g(x, x_i)^{-\frac{n-2}{2}} \text{ for all } x \in M \setminus \{x_i\}.$$

(in particular, x_∞ is the unique blow up point)

- Step 2.

$$|\nabla_g^k \ln u_i(x)| \leq C\, d_g(x, x_i)^{-k} \text{ for } x \neq x_i, k = 1, 2.$$

- Step 3. $u_i \to 0$ in $C^0_{\mathrm{loc}}(M \setminus \{x_\infty\})$.

- Step 4.

$$\lambda(A_{g_{v_\infty}}) \in \partial\Gamma \text{ in } M \setminus \{x_\infty\}.$$

- Step 5. There exists $0 \leq a < \infty$,

$$\lim_{x \to x_\infty} v_\infty(x)\ d_g(x, x_\infty)^{n-2} = a.$$

• Step 6. $a > 0$.

Proof divide into two cases. One case by constructing a barrier function, and the other case by constructing a barrier function and a generalization of Paul Levy's isoperemetric inequality due to Berard, Besson and Gallot.

- Step 7. Contradicting “(M, g) not standard sphere”

Thank you!