# The Standard Model and Flavor Physics*

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## Abstract

This is a written version of a series of lectures aimed at graduate students in particle physics. Starting from symmetry principles, we construct the Standard Model, and derive the elementary particles and fundamental interactions of the model. We describe the phenomenological predictions and some of their experimental tests. Then, we focus on flavor physics. We explain the reasons for the interest in flavor physics. We describe flavor physics and the related CP violation within the Standard Model, and explain how the B-factories proved that the CKM (KM) mechanism dominates the flavor changing (CP violating) processes that have been observed in meson decays. We explain the implications of flavor physics for new physics, with emphasis on the "new physics flavor puzzle", and present the idea of minimal flavor violation as a possible solution. We explain the "standard model flavor puzzle," and present the Froggatt-Nielsen mechanism as a possible solution. We show that measurements of the Higgs boson decays may provide new opportunities for making progress on the various flavor puzzles. We briefly discuss two sets of measurements and their possible theoretical implications: BR($h \to \tau\mu$) and $R(D^{(*)})$.

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## I. THE STANDARD MODEL

A model of elementary particles and their interactions is defined by the following ingredients: (i) The symmetries of the Lagrangian and the pattern of spontaneous symmetry breaking (SSB); (ii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows:

- The symmetry is a local

$$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{1}$$

which is spontaneously broken into

$$G_{\rm SM} \to SU(3)_C \times U(1)_{\rm EM} \quad (Q_{\rm EM} = T_3 + Y). \tag{2}$$

- There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3}, \quad L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1} \quad (i = 1,2,3). \tag{3}$$

There is a single scalar field,

$$\phi(1,2)_{+1/2}. \tag{4}$$

## II. THE SM LAGRANGIAN

The most general renormalizable Lagrangian with scalar and fermion fields can be decomposed into

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi. \tag{5}$$

Here $\mathcal{L}_{\rm kin}$ describes free propagation in spacetime, as well as gauge interactions, $\mathcal{L}_\psi$ gives fermion mass terms, $\mathcal{L}_{\rm Yuk}$ describes the Yukawa interactions, and $\mathcal{L}_\phi$ gives the scalar potential. We now find the specific form of the Lagrangian made of the fermion fields $Q_{Li}$, $U_{Ri}$, $D_{Ri}$, $L_{Li}$ and $E_{Ri}$ (3), and the scalar field (4), subject to the gauge symmetry (1) and leading to the SSB of Eq. (2).

### A. $\mathcal{L}_{\rm kin}$

The local symmetry requires that we introduce the following gauge boson degrees of freedom:

$$G_a^\mu(8,1)_0, \quad W_a^\mu(1,3)_0, \quad B^\mu(1,1)_0. \tag{6}$$

The corresponding field strengths are given by

$$\begin{aligned} G_a^{\mu\nu} &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu, \\ W_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu, \\ B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu, \end{aligned} \tag{7}$$

where $f_{abc}$ ($\epsilon_{abc}$) are the structure constants of $SU(3)$ ($SU(2)$). The covariant derivative is

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' B^\mu Y, \tag{8}$$

where the $L_a$'s are $SU(3)_C$ generators (the $3\times 3$ Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the $T_b$'s are $SU(2)_L$ generators (the $2\times 2$ Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the $Y$'s are the $U(1)_Y$ charges. Explicitly, the covariant derivatives acting on the various scalar and fermion fields are given by

$$\begin{aligned} D^\mu \phi &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{2} g' B^\mu\right)\phi, \\ D^\mu Q_{Li} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{6} g' B^\mu\right) Q_{Li}, \\ D^\mu U_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{2i}{3} g' B^\mu\right) U_{Ri}, \\ D^\mu D_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a - \frac{i}{3} g' B^\mu\right) D_{Ri}, \\ D^\mu L_{Li} &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b - \frac{i}{2} g' B^\mu\right) L_{Li}, \\ D^\mu E_{Ri} &= \left(\partial^\mu - i g' B^\mu\right) E_{Ri}. \end{aligned} \tag{9}$$

$\mathcal{L}_{\rm kin}$ is given by

$$\begin{aligned}\mathcal{L}_{\rm kin}^{\rm SM} &= -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_b^{\mu\nu}W_{b\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ &\quad -i\overline{Q_{Li}}\not{D}Q_{Li} - i\overline{U_{Ri}}\not{D}U_{Ri} - i\overline{D_{Ri}}\not{D}D_{Ri} - i\overline{L_{Li}}\not{D}L_{Li} - i\overline{E_{Ri}}\not{D}E_{Ri} \\ &\quad -(D^\mu\phi)^\dagger(D_\mu\phi).\end{aligned} \tag{10}$$

This part of the interaction Lagrangian is flavor-universal. In addition, it conserves CP.

### B. $\mathcal{L}_\psi$

There are no mass terms for the fermions in the SM. We cannot write Dirac mass terms for the fermions because they are assigned to chiral representations of the gauge symmetry. We cannot write Majorana mass terms for the fermions because they all have $Y \neq 0$. Thus,

$$\mathcal{L}_\psi^{\rm SM} = 0. \tag{11}$$

### C. $\mathcal{L}_{\rm Yuk}$

The Yukawa part of the Lagrangian is given by

$$\mathcal{L}_{\rm Y}^{\rm SM} = Y_{ij}^d\overline{Q_{Li}}\phi D_{Rj} + Y_{ij}^u\overline{Q_{Li}}\tilde{\phi}U_{Rj} + Y_{ij}^e\overline{L_{Li}}\phi E_{Rj} + {\rm h.c.}, \tag{12}$$

where $\tilde{\phi} = i\tau_2\phi^\dagger$, and the $Y^f$ are general $3\times 3$ matrices of dimensionless couplings. This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

Without loss of generality, we can use a bi-unitary transformation,

$$Y^e \to \hat{Y}_e = U_{eL}Y^eU_{eR}^\dagger, \tag{13}$$

to change the basis to one where $Y^e$ is diagonal and real:

$$\hat{Y}^e = {\rm diag}(y_e, y_\mu, y_\tau). \tag{14}$$

In the basis defined in Eq. (14), we denote the components of the lepton $SU(2)$-doublets, and the three lepton $SU(2)$-singlets, as follows:

$$\begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}, \qquad \begin{pmatrix}\nu_{\mu L}\\ \mu_L\end{pmatrix}, \qquad \begin{pmatrix}\nu_{\tau L}\\ \tau_L\end{pmatrix}; \qquad e_R, \qquad \mu_R, \qquad \tau_R, \tag{15}$$

where $e, \mu, \tau$ are ordered by the size of $y_{e,\mu,\tau}$ (from smallest to largest).

Similarly, without loss of generality, we can use a bi-unitary transformation,

$$Y^u \to \hat{Y}_u = V_{uL} Y^u V_{uR}^\dagger, \tag{16}$$

to change the basis to one where $\hat{Y}^u$ is diagonal and real:

$$\hat{Y}^u = \mathrm{diag}(y_u, y_c, y_t). \tag{17}$$

In the basis defined in Eq. (17), we denote the components of the quark $SU(2)$-doublets, and the quark up $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_L \\ d_{uL} \end{pmatrix}, \quad \begin{pmatrix} c_L \\ d_{cL} \end{pmatrix}, \quad \begin{pmatrix} t_L \\ d_{tL} \end{pmatrix}; \qquad u_R, \qquad c_R, \qquad t_R, \tag{18}$$

where $u, c, t$ are ordered by the size of $y_{u,c,t}$ (from smallest to largest).

We can use yet another bi-unitary transformation,

$$Y^d \to \hat{Y}_d = V_{dL} Y^d V_{dR}^\dagger, \tag{19}$$

to change the basis to one where $\hat{Y}^d$ is diagonal and real:

$$\hat{Y}^d = \mathrm{diag}(y_d, y_s, y_b). \tag{20}$$

In the basis defined in Eq. (20), we denote the components of the quark $SU(2)$-doublets, and the quark down $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_{dL} \\ d_L \end{pmatrix}, \quad \begin{pmatrix} u_{sL} \\ s_L \end{pmatrix}, \quad \begin{pmatrix} u_{bL} \\ b_L \end{pmatrix}; \qquad d_R, \qquad s_R, \qquad b_R, \tag{21}$$

where $d, s, b$ are ordered by the size of $y_{d,s,b}$ (from smallest to largest).

Note that if $V_{uL} \neq V_{dL}$, as is the general case, then the interaction basis defined by (17) is different from the interaction basis defined by (20). In the former, $Y^d$ can be written as a unitary matrix times a diagonal one,

$$Y^u = \hat{Y}^u, \quad Y^d = V \hat{Y}^d. \tag{22}$$

In the latter, $Y^u$ can be written as a unitary matrix times a diagonal one,

$$Y^d = \hat{Y}^d, \quad Y^u = V^\dagger \hat{Y}^u. \tag{23}$$

In either case, the matrix $V$ is given by

$$V = V_{uL} V_{dL}^{\dagger}, \tag{24}$$

where $V_{uL}$ and $V_{dL}$ are defined in Eqs. (16) and (19), respectively. Note that $V_{uL}$, $V_{uR}$, $V_{dL}$ and $V_{dR}$ depend on the basis from which we start the diagonalization. The combination $V = V_{uL} V_{dL}^{\dagger}$, however, does not. This is a hint that $V$ is physical. Indeed, below we see that it plays a crucial role in the charged current interactions.

### D. $\mathcal{L}_\phi$

The scalar potential is given by

$$\mathcal{L}_\phi^{\rm SM} = -\mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2. \tag{25}$$

This part of the Lagrangian is also CP conserving.

Choosing $\mu^2 < 0$ and $\lambda > 0$ leads to the required spontaneous symmetry breaking. Defining

$$v^2 = -\frac{\mu^2}{\lambda}, \tag{26}$$

we can rewrite Eq. (25) as follows (up to a constant term):

$$\mathcal{L}_\phi = -\lambda \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2. \tag{27}$$

The scalar potential (27) implies that the scalar field acquires a VEV, $|\langle\phi\rangle| = v/\sqrt{2}$. We have to make a choice of the direction of $\langle\phi\rangle$, and we choose it in the real direction of the down component,

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}. \tag{28}$$

This VEV breaks the $SU(2) \times U(1)$ symmetry down to a $U(1)$ subgroup. This statement corresponds to the fact that there is one (and only one) linear combination of generators that annihilates the vacuum state. With our specific choice, Eq. (28), it is $T_3 + Y$. The unbroken subgroup is identified with $U(1)_{\rm EM}$, and hence its generator, $Q$, is identified as

$$Q = T_3 + Y. \tag{29}$$

### E. Summary

The renormalizable part of the Standard Model Lagrangian is given by

$$
\begin{aligned}
\L_{\rm SM} = &- \frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_b^{\mu\nu}W_{b\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - (D^\mu\phi)^\dagger(D_\mu\phi) \\
&- i\overline{Q_{Li}}\not{D}Q_{Li} - i\overline{U_{Ri}}\not{D}U_{Ri} - i\overline{D_{Ri}}\not{D}D_{Ri} - i\overline{L_{Li}}\not{D}L_{Li} - i\overline{E_{Ri}}\not{D}E_{Ri} \\
&+ \left(Y_{ij}^u\overline{Q_{Li}}U_{Rj}\,\tilde{\phi} + Y_{ij}^d\overline{Q_{Li}}D_{Rj}\,\phi + Y_{ij}^e\overline{L_{Li}}E_{Rj}\,\phi + \text{h.c.}\right) \\
&- \lambda\left(\phi^\dagger\phi - v^2/2\right)^2 ,
\end{aligned}
\tag{30}
$$

where $i, j = 1, 2, 3$.

## III. THE SM SPECTRUM

### A. Scalars: back to $\mathcal{L}_\phi$

Let us denote the four real components of the scalar doublet as three phases, $\theta_a(x)$ ($a = 1, 2, 3$), and one magnitude, $h(x)$. We choose the three phases to be the three "would be" Goldstone bosons. In the SM, the broken generators are $T_1$, $T_2$, and $T_3 - Y$, and thus we write

$$\phi(x) = \exp\left[(i/2)\left(\sigma_a\theta_a(x) - I\theta_3(x)\right)\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}. \tag{31}$$

The local $SU(2)_L \times U(1)_Y$ symmetry of the Lagrangian allows one to rotate away the explicit dependence on the three $\theta_a(x)$. They represent the three would-be Goldstone bosons that are eaten by the three gauge bosons that acquire masses as a result of the SSB. In this gauge $\phi(x)$ has one degree of freedom (DoF):

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}. \tag{32}$$

The scalar $h$ is the Higgs boson. Its mass can be obtained by plugging (32) into (27), and is given by

$$m_h^2 = 2\lambda v^2. \tag{33}$$

Experiment gives [1]

$$m_h = 125.09 \pm 0.24\ \text{GeV}. \tag{34}$$

### B. Vector bosons: back to $\mathcal{L}_{\rm kin}(\phi)$

Since the symmetry that is related to three out of the four generators is spontaneously broken, three of the four vector bosons acquire masses, while one remains massless. To see how this happens, we examine $(D_\mu\langle\phi\rangle)^\dagger(D^\mu\langle\phi\rangle)$. Using Eq. (9) for $D^\mu\phi$, we obtain:

$$D^\mu\langle\phi\rangle=\frac{i}{\sqrt{8}}\left(gW_a^\mu\sigma_a+g'B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}=\frac{i}{\sqrt{8}}\begin{pmatrix}gW_3^\mu+g'B^\mu & g(W_1^\mu-iW_2^\mu)\\ g(W_1^\mu+iW_2^\mu) & -gW_3^\mu+g'B^\mu\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}. \tag{35}$$

The mass terms for the vector bosons are thus given by

$$\mathcal{L}_{M_V}=\frac{1}{8}(0\ v)\begin{pmatrix}gW_{3\mu}+g'B_\mu & g(W_1-iW_2)_\mu\\ g(W_1+iW_2)_\mu & -gW_{3\mu}+g'B_\mu\end{pmatrix}\begin{pmatrix}gW_3^\mu+g'B^\mu & g(W_1-iW_2)^\mu\\ g(W_1+iW_2)^\mu & -gW_3^\mu+g'B^\mu\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}. \tag{36}$$

We define an angle $\theta_W$ via

$$\tan\theta_W\equiv\frac{g'}{g}. \tag{37}$$

We define four gauge boson states:

$$W_\mu^\pm=\frac{1}{\sqrt{2}}(W_1\mp iW_2)_\mu,\quad Z_\mu^0=\cos\theta_W W_{3\mu}-\sin\theta_W B_\mu,\quad A_\mu^0=\sin\theta_W W_{3\mu}+\cos\theta_W B_\mu. \tag{38}$$

The $W_\mu^\pm$ are charged under electromagnetism (hence the superscripts $\pm$), while $A_\mu^0$ and $Z_\mu^0$ are neutral. In terms of the vector boson fields of Eq. (38), Eq. (36) reads

$$\mathcal{L}_{M_V}=\frac{1}{4}g^2v^2W^{+\mu}W_\mu^-+\frac{1}{8}(g^2+g'^2)v^2Z^{0\mu}Z_\mu^0. \tag{39}$$

We learn that the four states of Eq. (38) are the mass eigenstates, with masses-squared

$$m_W^2=\frac{1}{4}g^2v^2,\qquad m_Z^2=\frac{1}{4}(g^2+g'^2)v^2,\qquad m_A^2=0. \tag{40}$$

(Recall that for a complex field $\phi$ with mass $m$ the mass term is $m^2|\phi|^2$ while for a real field it is $m^2\phi^2/2$.) Three points are worth emphasizing:

1. As anticipated, three vector boson acquire masses.
2. $m_A^2=0$ is not a prediction, but rather a consistency check on our calculation.
3. The angle $\theta_W$ represents a rotation angle of the two neutral vector bosons from the interaction basis, where fields have well-defined transformation properties under the full gauge symmetry, $(W_3,B)$, into the mass basis for the vector bosons, $(Z,A)$.

SSB leads to relation between observables that would have been independent in the absence of a symmetry. One such important relation involves the vector-boson masses and their couplings:

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2 + g'^2}. \tag{41}$$

This relation is testable. The left hand side can be derived from the measured spectrum, and the right hand side from interaction rates. It is conventional to express this relation in terms of $\theta_W$, defined in Eq. (37):

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1. \tag{42}$$

The $\rho = 1$ relation is a consequence of the SSB by $SU(2)$-doublets. It thus tests this specific ingredient of the SM.

The experimental values of the weak gauge boson masses are given by [1]

$$m_W = 80.385 \pm 0.015 \text{ GeV}; \qquad m_Z = 91.1876 \pm 0.0021 \text{ GeV}. \tag{43}$$

We can then use the $\rho = 1$ relation to determine $\sin^2\theta_W$:

$$\frac{m_W}{m_Z} = 0.8815 \pm 0.0002 \quad \Longrightarrow \quad \sin^2\theta_W = 1 - (m_W/m_Z)^2 = 0.2229 \pm 0.0004. \tag{44}$$

Measurements determinate $\sin^2\theta_W$ by various interaction rates. The $\rho = 1$ relation is indeed realized in Nature (within experimental errors, and up to calculable quantum corrections).

### C. Fermions: back to $\mathcal{L}_{\rm Yuk}$

Since the SM allows no bare mass terms for the fermions, their masses can only arise from the Yukawa part of the Lagrangian, which is given in Eq. (12). Indeed, with $\langle\phi^0\rangle = v/\sqrt{2}$, Eq. (12) has a piece that corresponds to charged lepton masses:

$$m_e = \frac{y_e v}{\sqrt{2}}, \qquad m_\mu = \frac{y_\mu v}{\sqrt{2}}, \qquad m_\tau = \frac{y_\tau v}{\sqrt{2}}, \tag{45}$$

a piece that corresponds to up-type quark masses,

$$m_u = \frac{y_u v}{\sqrt{2}}, \qquad m_c = \frac{y_c v}{\sqrt{2}}, \qquad m_t = \frac{y_t v}{\sqrt{2}}, \tag{46}$$

and a piece that corresponds to down-type quark masses,

$$m_d = \frac{y_d v}{\sqrt{2}}, \qquad m_s = \frac{y_s v}{\sqrt{2}}, \qquad m_b = \frac{y_b v}{\sqrt{2}}. \tag{47}$$

We conclude that all charged fermions acquire Dirac masses as a result of the spontaneous symmetry breaking. The key to this feature is that, while the charged fermions are in chiral representations of the full gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, they are in vector-like representations of the $SU(3)_C \times U(1)_{\rm EM}$ group:

- The LH and RH charged lepton fields, $e$, $\mu$ and $\tau$, are in the $(1)_{-1}$ representation.
- The LH and RH up-type quark fields, $u$, $c$ and $t$, are in the $(3)_{+2/3}$ representation.
- The LH and RH down-type quark fields, $d$, $s$ and $b$, are in the $(3)_{-1/3}$ representation.

On the other hand, the neutrinos remain massless:

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0. \tag{48}$$

This is the case in spite of the fact that the neutrinos transform as $(1)_0$ under the unbroken gauge group, allowing in principle for Majorana masses. As we discuss below, their masslessness is related to an accidental symmetry of the SM.

The experimental values of the charged fermion masses are [1]

$$\begin{aligned} &m_e = 0.510998946(3)\ \text{MeV}, \qquad m_\mu = 105.6583745(24)\ \text{MeV}, \qquad m_\tau = 1776.86(12)\ \text{MeV}, \\ &m_u = 2.2^{+0.6}_{-0.4}\ \text{MeV}, \qquad m_c = 1.27 \pm 0.03\ \text{GeV}, \qquad m_t = 173.2 \pm 0.09\ \text{GeV}, \\ &m_d = 4.7^{+0.5}_{-0.4}\ \text{MeV}, \qquad m_s = 96^{+8}_{-4}\ \text{MeV}, \qquad m_b = 4.18^{+0.04}_{-0.03}\ \text{GeV}, \end{aligned} \tag{49}$$

where the $u$-, $d$- and $s$-quark masses are given at a scale $\mu = 2$ GeV, the $c$- and $b$-quark masses are the running masses in the $\overline{\rm MS}$ scheme, and the $t$-quark mass is derived from direct measurement.

### D. Summary

The mass eigenstates of the SM, their $SU(3)_C \times U(1)_{\rm EM}$ quantum numbers, and their masses in units of the VEV $v$, are presented in Table I. All masses are proportional to the VEV of the scalar field, $v$. For the three massive gauge bosons, and for the fermions, this is expected: In the absence of spontaneous symmetry breaking, the former would be protected from acquiring masses by the gauge symmetry and the latter by their chiral nature. For the Higgs boson, the situation is different, as a mass-squared term does not violate any

TABLE I: The SM particles

| particle | spin | color | $Q$ | mass $[v]$ |
|---|---|---|---|---|
| $W^\pm$ | 1 | (1) | $\pm 1$ | $\frac{1}{2}g$ |
| $Z^0$ | 1 | (1) | 0 | $\frac{1}{2}\sqrt{g^2+g'^2}$ |
| $A^0$ | 1 | (1) | 0 | 0 |
| $g$ | 1 | (8) | 0 | 0 |
| $h$ | 0 | (1) | 0 | $\sqrt{2\lambda}$ |
| $e,\mu,\tau$ | 1/2 | (1) | $-1$ | $y_{e,\mu,\tau}/\sqrt{2}$ |
| $\nu_e,\nu_\mu,\nu_\tau$ | 1/2 | (1) | 0 | 0 |
| $u,c,t$ | 1/2 | (3) | $+2/3$ | $y_{u,c,t}/\sqrt{2}$ |
| $d,s,b$ | 1/2 | (3) | $-1/3$ | $y_{d,s,b}/\sqrt{2}$ |

symmetry: $m_h \propto v$ is just a manifestation of the fact that the SM has a single dimensionful parameter, which can be taken to be $v$, and therefore all masses must be proportional to this parameter.

## IV. THE SM INTERACTIONS

In this Section, we discuss the interactions of the fermion and scalar mass eigenstates of the SM.

### A. EM and strong interactions

By construction, a local $SU(3)_C \times U(1)_{\rm EM}$ symmetry survives the SSB. The SM has thus the photon and gluon massless gauge fields. All charged fermions interact with the photon:

$$\mathcal{L}_{{\rm QED},\psi} = -\frac{2e}{3}\overline{u_i}\not{A}u_i + \frac{e}{3}\overline{d_i}\not{A}d_i + e\overline{\ell_i}\not{A}\ell_i, \tag{50}$$

where $u_{1,2,3} = u,c,t$, $d_{1,2,3} = d,s,b$ and $\ell_{1,2,3} = e,\mu,\tau$. We emphasize the following points:

1. The photon couplings are *vector-like* and *parity conserving.*

2. *Diagonality:* The photon couples to $e^+e^-$, $\mu^+\mu^-$ and $\tau^+\tau^-$, but not to $e^\pm\mu^\mp$, $e^\pm\tau^\mp$ or $\mu^\pm\tau^\mp$ pairs, and similarly in the up and down sectors.

3. *Universality:* The couplings of the photon to different generations are universal.

All colored fermions (namely, quarks) interact with the gluon:

$$\mathcal{L}_{\rm QCD,\psi} = -\frac{g_s}{2}\overline{q}\lambda_a \not{G}_a q, \tag{51}$$

where $q = u, c, t, d, s, b$. We emphasize the following points:

1. The gluon couplings are *vector-like* and *parity conserving*.

2. *Diagonality:* The gluon couples to $\bar{t}t$, $\bar{c}c$, *etc.*, but not to $\bar{t}c$ or any other flavor changing pair.

3. *Universality:* The couplings of the gluon to different quark generations are universal.

The universality of the photon and gluon couplings is a result of the $SU(3)_C \times U(1)_{\rm EM}$ gauge invariance, and thus holds in any model, and not just within the SM.

### B. $Z$-mediated weak interactions

All SM fermions couple to the $Z$-boson:

$$\begin{aligned}\mathcal{L}_{Z,\psi} \;=\; & \frac{e}{s_W c_W}\left[-\left(\frac{1}{2}-s_W^2\right)\,\overline{e_{Li}}\not{Z}e_{Li} + s_W^2\,\overline{e_{Ri}}\not{Z}e_{Ri} + \frac{1}{2}\,\overline{\nu_{L\alpha}}\not{Z}\nu_{L\alpha}\right. \\ & \left. +\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\,\overline{u_{Li}}\not{Z}u_{Li} - \frac{2}{3}s_W^2\,\overline{u_{Ri}}\not{Z}u_{Ri} - \left(\frac{1}{2}-\frac{1}{3}s_W^2\right)\,\overline{d_{Li}}\not{Z}d_{Li} + \frac{1}{3}s_W^2\,\overline{d_{Ri}}\not{Z}d_{Ri}\right].\end{aligned} \tag{52}$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. We emphasize the following points:

1. The $Z$-boson couplings are *chiral* and *parity violating*.

2. *Diagonality:* The $Z$-boson couples diagonally and, as a result of this, there are no $Z$-mediated flavor changing neutral current (FCNC) processes.

3. *Universality:* The couplings of the $Z$-boson to different fermion generations are universal.

The universality is a result of a special feature of the SM: all fermions of given chirality and given charge come from the same $SU(2)_L \times U(1)_Y$ representation.

As an example to experimental tests of diagonality and universality, we can take the leptonic sector. The branching ratios of the $Z$-boson into charged lepton pairs [1],

$$\begin{aligned} \mathrm{BR}(Z \to e^+e^-) &= (3.363 \pm 0.004)\%\,, \\ \mathrm{BR}(Z \to \mu^+\mu^-) &= (3.366 \pm 0.007)\%\,, \\ \mathrm{BR}(Z \to \tau^+\tau^-) &= (3.370 \pm 0.008)\%\,. \end{aligned} \tag{53}$$

beautifully confirms universality:

$$\begin{aligned} \Gamma(\mu^+\mu^-)/\Gamma(e^+e^-) &= 1.001 \pm 0.003, \\ \Gamma(\tau^+\tau^-)/\Gamma(e^+e^-) &= 1.002 \pm 0.003. \end{aligned}$$

Diagonality is also tested by the following experimental searches:

$$\begin{aligned} \mathrm{BR}(Z \to e^+\mu^-) &< 7.5 \times 10^{-7}, \\ \mathrm{BR}(Z \to e^+\tau^-) &< 9.8 \times 10^{-6}, \\ \mathrm{BR}(Z \to \mu^+\tau^-) &< 1.2 \times 10^{-5}. \end{aligned} \tag{54}$$

Omitting common factors, particularly, a factor of $e^2/(4s_W^2c_W^2)$, and phase-space factors, we obtain the following predictions for the $Z$ decays into a one-generation fermion-pair of each type:

$$\begin{aligned} \Gamma(Z \to \nu\bar{\nu}) &\propto 1, \\ \Gamma(Z \to \ell\bar{\ell}) &\propto 1 - 4s_W^2 + 8s_W^4, \\ \Gamma(Z \to u\bar{u}) &\propto 3\left(1 - \frac{8}{3}s_W^2 + \frac{32}{9}s_W^4\right), \\ \Gamma(Z \to d\bar{d}) &\propto 3\left(1 - \frac{4}{3}s_W^2 + \frac{8}{9}s_W^4\right). \end{aligned} \tag{55}$$

Putting $s_W^2 = 0.225$, we obtain

$$\Gamma_\nu : \Gamma_\ell : \Gamma_u : \Gamma_d = 1 : 0.51 : 1.74 : 2.24. \tag{56}$$

Experiments measure the following average branching ratio into a single generation of each fermion species:

$$\mathrm{BR}(Z \to \nu\bar{\nu}) = (6.67 \pm 0.02)\%,$$

$$\begin{aligned} \mathrm{BR}(Z \to \ell\bar{\ell}) &= (3.37 \pm 0.01)\%, \\ \mathrm{BR}(Z \to u\bar{u}) &= (11.6 \pm 0.6)\%, \\ \mathrm{BR}(Z \to d\bar{d}) &= (15.6 \pm 0.4)\%, \end{aligned} \tag{57}$$

which, using central values, give

$$\Gamma_\nu : \Gamma_\ell : \Gamma_u : \Gamma_d = 1 : 0.505 : 1.74 : 2.34, \tag{58}$$

in very nice agreement with the predictions.

### C. $W$-mediated weak interactions

We now study the couplings of the charged vector bosons, $W^\pm$, to fermion pairs. For the lepton mass eigenstates, things are simple, because there exists an interaction basis that is also a mass basis. Thus, the $W$ interactions must be universal also in the mass basis:

$$\mathcal{L}_{W,\ell} = -\frac{g}{\sqrt{2}} \left( \overline{\nu_{eL}} \not{W}^+ e_L^- + \overline{\nu_{\mu L}} \not{W}^+ \mu_L^- + \overline{\nu_{\tau L}} \not{W}^+ \tau_L^- + \text{h.c.} \right). \tag{59}$$

Eq. (59) reveals some important features of the model:

1. Only left-handed leptons take part in charged-current interactions. Consequently, parity is violated.

2. *Diagonality*: the charged current interactions couple each charged lepton to a single neutrino, and each neutrino to a single charged lepton. Note that a global $SU(2)$ symmetry would allow off-diagonal couplings; It is the local symmetry that leads to diagonality.

3. *Universality*: the couplings of the $W$-boson to $\tau\bar{\nu}_\tau$, to $\mu\bar{\nu}_\mu$ and to $e\bar{\nu}_e$ are equal. Again, a global symmetry would have allowed an independent coupling to each lepton pair.

All of these predictions have been experimentally tested. As an example of how well universality works, consider the decay rates of the $W$-bosons to the three lepton pairs [1]:

$$\begin{aligned} \mathrm{BR}(W^+ \to e^+\nu_e) &= (10.71 \pm 0.16) \times 10^{-2}, \\ \mathrm{BR}(W^+ \to \mu^+\nu_\mu) &= (10.63 \pm 0.15) \times 10^{-2}, \\ \mathrm{BR}(W^+ \to \tau^+\nu_\tau) &= (11.38 \pm 0.21) \times 10^{-2}. \end{aligned} \tag{60}$$

You must be impressed by the nice agreement!

As concerns quarks, things are more complicated, since there is no interaction basis that is also a mass basis. In the interaction basis where the down quarks are mass eigenstates (21), the $W$ interactions have the following form:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}} \left( \overline{u_{dL}} \not{W}^+ d_L + \overline{u_{sL}} \not{W}^+ s_L + \overline{u_{bL}} \not{W}^+ b_L + \text{h.c.} \right). \tag{61}$$

The Yukawa matrices in this basis have the form (23), and in particular, for the up sector, we have

$$\mathcal{L}^u_{\text{Yuk}} = (\overline{u_{dL}}\ \overline{u_{sL}}\ \overline{u_{bL}}) V^\dagger \hat{Y}^u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}, \tag{62}$$

which tells us straightforwardly how to transform to the mass basis:

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = V \begin{pmatrix} u_{dL} \\ u_{sL} \\ u_{bL} \end{pmatrix}. \tag{63}$$

Using Eq. (63), we obtain the form of the $W$ interactions (61) in the mass basis:

$$-\frac{g}{\sqrt{2}} \left( \overline{u_L}\ \overline{c_L}\ \overline{t_L} \right)\ V \not{W}^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}. \tag{64}$$

You can easily convince yourself that we would have obtained the same form starting from any arbitrary interaction basis. We remind you that $V = V_{uL} V^\dagger_{dL}$ is basis independent.

Eq. (64) reveals some important features of the model:

1. Only left-handed quarks take part in charged-current interactions. Consequently, parity is violated by these interactions.

2. The $W$ couplings to the quark mass eigenstates are neither universal nor diagonal. The universality of gauge interactions is hidden in the unitarity of the matrix $V$.

The (hidden) universality within the quark sector is tested by the prediction

$$\Gamma(W \to uX) = \Gamma(W \to cX) = \frac{1}{2}\Gamma(W \to \text{hadrons}). \tag{65}$$

Experimentally,

$$\Gamma(W \to cX)/\Gamma(W \to \text{hadrons}) = 0.49 \pm 0.04. \tag{66}$$

The matrix $V$ is called the CKM matrix [2, 3]. The form of the CKM matrix is not unique. First, there is freedom in defining $V$ in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of $V$ are therefore written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{67}$$

Omitting common factors (particularly, a factor of $g^2/4$) and phase-space factors, we obtain the following predictions for the $W$ decays:

$$\begin{aligned} \Gamma(W^+ \to \ell^+ \nu_\ell) &\propto 1, \\ \Gamma(W^+ \to u_i \overline{d_j}) &\propto 3|V_{ij}|^2 \qquad (i = 1, 2;\ j = 1, 2, 3). \end{aligned} \tag{68}$$

The top quark is not included because it is heavier than the $W$ boson. Taking this fact into account, and the CKM unitarity relations

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1, \tag{69}$$

we obtain

$$\Gamma(W \to \text{hadrons}) \approx 2\Gamma(W \to \text{leptons}). \tag{70}$$

Experimentally,

$$\text{BR}(W \to \text{leptons}) = (32.40 \pm 0.27)\%, \qquad \text{BR}(W \to \text{hadrons}) = (67.41 \pm 0.27)\%, \tag{71}$$

which leads to

$$\Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) = 2.09 \pm 0.01, \tag{72}$$

in good agreement with the SM prediction.

### D. Interactions of the Higgs boson

The Higgs boson has self-interactions, weak interactions, and Yukawa interactions:

$$\begin{aligned} \mathcal{L}_h &= \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4 \\ &+ m_W^2 W_\mu^- W^{\mu+}\left(\frac{2h}{v} + \frac{h^2}{v^2}\right) + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \left(\frac{2h}{v} + \frac{h^2}{v^2}\right) \end{aligned} \tag{73}$$

$$
\begin{aligned}
- \; & \frac{h}{v}\left(m_e\,\overline{e_L}\,e_R + m_\mu\,\overline{\mu_L}\,\mu_R + m_\tau\,\overline{\tau_L}\,\tau_R\right.\\
&\left. + m_u\,\overline{u_L}\,u_R + m_c\,\overline{c_L}\,c_R + m_t\,\overline{t_L}\,t_R + m_d\,\overline{d_L}\,d_R + m_s\,\overline{s_L}\,s_R + m_b\,\overline{b_L}\,b_R + \text{h.c.}\right).
\end{aligned}
$$

Note that the Higgs boson couples diagonally to the quark mass eigenstates. The reason for this is that the Yukawa couplings determine both the masses and the Higgs couplings to the fermions. Thus, in the mass basis the Yukawa interactions are also diagonal. A formal derivation, starting from an arbitrary interaction basis, goes as follows:

$$
\begin{aligned}
h\overline{D_L}Y^d D_R &= h\overline{D_L}(V_{dL}^\dagger V_{dL})Y^d(V_{dR}^\dagger V_{dR})D_R \\
&= h(\overline{D_L}V_{dL}^\dagger)(V_{dL}Y^d V_{dR}^\dagger)(V_{dR}D_R) \\
&= h(\overline{d_L}\ \overline{s_L}\ \overline{b_L})\hat{Y}^d(d_R\ s_R\ b_R)^T. \qquad (74)
\end{aligned}
$$

We conclude that the Higgs couplings to the fermion mass eigenstates have the following features:

1. *Diagonality.*
2. *Non-universality.*
3. *Proportionality* to the fermion masses: the heavier the fermion, the stronger the coupling. The factor of proportionality is $m_\psi/v$.

Thus, the Higgs boson decay is dominated by the heaviest particle which can be pair-produced in the decay. For $m_h \sim 125$ GeV, this is the bottom quark. Indeed, the SM predicts the following branching ratios for the leading decay modes:

$$
\text{BR}_{\bar{b}b} : \text{BR}_{WW^*} : \text{BR}_{gg} : \text{BR}_{\tau^+\tau^-} : \text{BR}_{ZZ^*} : \text{BR}_{c\bar{c}} = 0.58 : 0.21 : 0.09 : 0.06 : 0.03 : 0.03. \qquad (75)
$$

The following comments are in order with regard to Eq. (75):

1. From the six branching ratios, three $(b, \tau, c)$ stand for two-body tree-level decays. Thus, at tree level, the respective branching ratios obey $\text{BR}_{\bar{b}b} : \text{BR}_{\tau^+\tau^-} : \text{BR}_{c\bar{c}} = 3m_b^2 : m_\tau^2 : 3m_c^2$. QCD radiative corrections somewhat suppress the two modes with the quark final states $(b, c)$ compared to one with the lepton final state $(\tau)$.

2. The $WW^*$ and $ZZ^*$ modes stand for the three-body tree-level decays, where one of the vector bosons is on-shell and the other off-shell.

TABLE II: The SM fermion interactions

| interaction | fermions | force carrier | coupling | flavor |
|---|---|---|---|---|
| Electromagnetic | $u, d, \ell$ | $A^0$ | $eQ$ | universal |
| Strong | $u, d$ | $g$ | $g_s$ | universal |
| NC weak | all | $Z^0$ | $\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$ | universal |
| CC weak | $\bar{u}d/\bar{\ell}\nu$ | $W^\pm$ | $gV/g$ | non-universal/universal |
| Yukawa | $u, d, \ell$ | $h$ | $y_q$ | diagonal |

3. The Higgs boson does not have a tree-level coupling to gluons since it carries no color (and the gluons have no mass). The decay into final gluons proceeds via loop diagrams. The dominant contribution comes from the top-quark loop.

4. Similarly, the Higgs decays into final two photons via loop diagrams with small ($\text{BR}_{\gamma\gamma} \sim 0.002$), but observable, rate. The dominant contributions come from the $W$ and the top-quark loops which interfere destructively.

Experimentally, the decays into final $ZZ^*$, $WW^*$, $\gamma\gamma$ and $\tau^+\tau^-$ have been established [47]. Normalized to the SM rate, we have

$$\begin{aligned} \mu_{ZZ^*} &= 1.17 \pm 0.23, \\ \mu_{WW^*} &= 0.99 \pm 0.15, \\ \mu_{\gamma\gamma} &= 1.14 \pm 0.14, \\ \mu_{\tau\tau} &= 1.09 \pm 0.23. \end{aligned} \tag{76}$$

### E. Summary

Within the SM, the fermions have five types of interactions. These interactions are summarized in Table II.

## V. THE ACCIDENTAL SYMMETRIES OF THE SM

In the absence of the Yukawa matrices, $\mathcal{L}_{\rm Yuk} = 0$, the SM has a large $U(3)^5$ global symmetry:

$$G^{\rm SM}_{\rm global}(Y^{u,d,e}=0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5, \tag{77}$$

where

$$\begin{aligned} SU(3)^3_q &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\ SU(3)^2_\ell &= SU(3)_L \times SU(3)_E, \\ U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\rm PQ} \times U(1)_E. \end{aligned} \tag{78}$$

Out of the five $U(1)$ charges, three can be identified with baryon number ($B$), lepton number ($L$) and hypercharge ($Y$), which are respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and $D_R, E_R$ fields have opposite charges, and with a global rotation of $E_R$ only.

The point that is important for our purposes is that $\mathcal{L}_{\rm kin}$ respects the non-Abelian flavor symmetry $SU(3)^3_q \times SU(3)^2_\ell$, under which

$$Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \tag{79}$$

where the $V_i$ are unitary matrices. The Yukawa interactions (12) break the global symmetry,

$$G^{\rm SM}_{\rm global}(Y^{u,d,e}\neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \tag{80}$$

Under $U(1)_B$, all quarks (antiquarks) carry charge $+1/3$ ($-1/3$), while all other fields are neutral. It explains why proton decay has not been observed. Possible proton decay modes, such as $p \to \pi^0 e^+$ or $p \to K^+\nu$, are not forbidden by the $SU(3)_C \times U(1)_{\rm EM}$ symmetry. However, they violate $U(1)_B$, and therefore do not occur within the SM. The lesson here is quite general: The lightest particle that carries a conserved charge is stable. The accidental $U(1)_B$ symmetry also explains why neutron-antineutron oscillations have not been observed.

Note that $U(1)_B$ as well as each of the lepton numbers are anomalous. The combination of $B-L$, however, is anomaly free. Due to the anomaly, baryon and lepton number violating processes occur non-perturbatively. However, the non-perturbative effects obey $\Delta B = \Delta L = 3n$, with $n =$integer, and thus do not lead to proton decay. Moreover, they are very small, and can be neglected in almost all cases we study, and thus we do not discuss them any further.

The accidental symmetries of the renormalizable part of the SM Lagrangian also explain the vanishing of neutrino masses. A Majorana mass term violates the accidental $B - L$ symmetry by two units. Thus, the symmetry prevents mass terms not only at tree level but also to all orders in perturbation theory. Moreover, since the $B - L$ symmetry is non-anomalous, Majorana mass terms do not arise even at the non-perturbative level. We conclude that the renormalizable SM gives the *exact* prediction:

$$m_\nu = 0. \tag{81}$$

We see that the transformations of Eq. (79) are not a symmetry of $\mathcal{L}_{\rm SM}$. Instead, they correspond to a change of the interaction basis. These observations also provide a definition of flavor physics: it refers to interactions that break the $SU(3)^5$ symmetry (79). Thus, the term "**flavor violation**" is often used to describe processes or parameters that break the symmetry.

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)^3_q$ symmetry (but are neutral under $U(1)_B$),

$$Y^u \sim (3, \bar{3}, 1)_{SU(3)^3_q}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)^3_q}, \tag{82}$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)^2_\ell$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$Y^e \sim (3, \bar{3})_{SU(3)^2_\ell}. \tag{83}$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression factors (see Section XI), and the idea of minimal flavor violation (see Section XI C).

### A. Counting parameters

How many independent parameters are there in $\mathcal{L}^q_{\rm Yuk}$? The two Yukawa matrices, $Y^u$ and $Y^d$, are $3 \times 3$ and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of $G_{\rm global}$ breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three $3 \times 3$ unitary matrices minus the phase related to $U(1)_B$). For example, we can use the unitary transformations $Q_L \to V_Q Q_L$, $U_R \to V_U U_R$ and $D_R \to V_D D_R$, to lead to the following interaction basis:

$$Y^d = \lambda_d, \quad Y^u = V^\dagger \lambda_u, \tag{84}$$

where $\lambda_{d,u}$ are diagonal,

$$\lambda_d = \mathrm{diag}(y_d, y_s, y_b), \quad \lambda_u = \mathrm{diag}(y_u, y_c, y_t), \tag{85}$$

while $V$ is a unitary matrix that depends on three real angles and one complex phase. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we identify the nine real parameters as six quark masses and three mixing angles, while the single phase is $\delta_{\rm KM}$.

How many independent parameters are there in $\mathcal{L}^\ell_{\rm Yuk}$? The Yukawa matrix $Y^e$ is $3 \times 3$ and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two $3\times3$ unitary matrices minus the phases related to $U(1)^3$). For example, we can use the unitary transformations $L_L \to V_L L_L$ and $E_R \to V_E E_R$, to lead to the following interaction basis:

$$Y^e = \lambda_e = \mathrm{diag}(y_e, y_\mu, y_\tau). \tag{86}$$

We conclude that there are 3 real lepton flavor parameters. In the mass basis, we identify these parameters as the three charged lepton masses. We must, however, modify the model when we take into account the evidence for neutrino masses.

## VI. BEYOND THE SM

The SM is not a full theory of Nature. It is only a low energy effective theory, valid below some scale $\Lambda \gg m_Z$. Then, the SM Lagrangian should be extended to include all non-renormalizable terms, suppressed by powers of $\Lambda$:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda} O_{d=5} + \frac{1}{\Lambda^2} O_{d=6} + \cdots, \tag{87}$$

where $O_{d=n}$ represents operators that are products of SM fields, transforming as singlets under the SM gauge group, of overall dimension $n$ in the fields. For physics at an energy scale

$E$ well below $\Lambda$, the effects of operators of dimension $n > 4$ are suppressed by $(E/\Lambda)^{n-4}$. Thus, in general, the higher the dimension of an operator, the smaller its effect at low energies.

In previous sections, we studied the SM mainly at tree level and with only renormalizable terms. We can classify the effects of including loop corrections and nonrenormalizable terms into three broad categories:

1. *Forbidden processes:* Various processes are forbidden by the accidental symmetries of the Standard Model. Nonrenormalizable terms (but not loop corrections!) can break these accidental symmetries and allow the forbidden processes to occur. Examples include neutrino masses and proton decay.

2. *Rare processes:* Various processes are not allowed at tree level. These effects can often be related to accidental symmetries that hold within a particular sector of, but not in the entire, SM. Here both loop corrections and nonrenormalizable terms can contribute. Examples include flavor changing neutral current (FCNC) processes.

3. *Tree level processes:* Often tree level processes in a particular sector depend on a small subset of the SM parameters. This situation leads to relations among different processes within this sector. These relations are violated by both loop effects and nonrenormalizable terms. Here, precision measurements and precision theory calculations are needed to observe these small effects. Examples include electroweak precision measurements (EWPM).

As concerns the last two types of effects, where loop corrections and nonrenormalizable terms may both contribute, their use in phenomenology can be divided to two eras. Before all the SM particles have been directly discovered and all the SM parameters measured, one could assume the validity of the renormalizable SM and indirectly measure the properties of the yet unobserved SM particles. Indeed, the charm quark, the top quark and the Higgs boson masses were predicted in this way. Once all the SM particles have been observed and the parameters measured directly, the loop corrections can be quantitatively determined, and effects of nonrenormalizable terms can be unambiguously probed. Thus, at present, all three classes of processes serve to search for new physics. In what comes we focus on flavor physics as a way to probe physics beyond the SM.

## VII. INTRODUCTION TO FLAVOR PHYSICS

### A. What is flavor?

The term "**flavors**" is used, in the jargon of particle physics, to describe several copies of the same gauge representation, namely several fields that are assigned the same quantum charges. Within the Standard Model, when thinking of its unbroken $SU(3)_\mathrm{C} \times U(1)_\mathrm{EM}$ gauge group, there are four different types of particles, each coming in three flavors:

- Up-type quarks in the $(3)_{+2/3}$ representation: $u, c, t$;
- Down-type quarks in the $(3)_{-1/3}$ representation: $d, s, b$;
- Charged leptons in the $(1)_{-1}$ representation: $e, \mu, \tau$;
- Neutrinos in the $(1)_0$ representation: $\nu_1, \nu_2, \nu_3$.

The term "**flavor physics**" refers to interactions that distinguish between flavors. By definition, gauge interactions, namely interactions that are related to unbroken symmetries and mediated therefore by massless gauge bosons, do not distinguish among the flavors and do not constitute part of flavor physics. Within the Standard Model, flavor-physics refers to the weak and Yukawa interactions.

The term "**flavor parameters**" refers to parameters that carry flavor indices. Within the Standard Model, these are the nine masses of the charged fermions and the four "mixing parameters" (three angles and one phase) that describe the interactions of the charged weak-force carriers ($W^\pm$) with quark-antiquark pairs. If one augments the Standard Model with Majorana mass terms for the neutrinos, one should add to the list three neutrino masses and six mixing parameters (three angles and three phases) for the $W^\pm$ interactions with lepton-antilepton pairs.

The term "**flavor universal**" refers to interactions with couplings (or to parameters) that are proportional to the unit matrix in flavor space. Thus, the strong and electromagnetic interactions are flavor-universal.[1] An alternative term for "flavor-universal" is "**flavor-blind**".

[1] In the interaction basis, the weak interactions are also flavor-universal, and one can identify the source of all flavor physics in the Yukawa interactions among the gauge-interaction eigenstates.

The term "**flavor diagonal**" refers to interactions with couplings (or to parameters) that are diagonal, but not necessarily universal, in the flavor space. Within the Standard Model, the Yukawa interactions of the Higgs particle are flavor diagonal in the mass basis.

The term "**flavor changing**" refers to processes where the initial and final flavor-numbers (that is, the number of particles of a certain flavor minus the number of anti-particles of the same flavor) are different. In "flavor changing charged current" processes, both up-type and down-type flavors, and/or both charged lepton and neutrino flavors are involved. Examples are (i) muon decay via $\mu \to e\bar{\nu}_e\nu_\mu$, (ii) $K^- \to \mu^-\bar{\nu}_\mu$ (which corresponds, at the quark level, to $s\bar{u} \to \mu^-\bar{\nu}_\mu$), and (iii) $B \to \psi K$ ($b \to c\bar{c}s$). Within the Standard Model, these processes are mediated by the $W$-bosons and occur at tree level. In "**flavor changing neutral current**" (FCNC) processes, either up-type or down-type flavors but not both, and/or either charged lepton or neutrino flavors but not both, are involved. Example are (i) muon decay via $\mu \to e\gamma$, (ii) $K_L \to \mu^+\mu^-$ (which corresponds, at the quark level, to $s\bar{d} \to \mu^+\mu^-$), and (iii) $B \to \phi K$ ($b \to s\bar{s}s$). Within the Standard Model, these processes do not occur at tree level, and are often highly suppressed.

Another useful term is "**flavor violation**". It refers to interactions or parameters that violate the $[SU(3)]^5$ symmetry of Eq. (78).

### B. Why is flavor physics interesting?

Flavor physics is interesting, on one hand, as a tool for discovery and, on the other hand, because of intrinsic puzzling features:

- Flavor physics can discover new physics or probe it before it is directly observed in experiments. The strongest sensitivity comes from flavor changing neutral current processes. Here are some examples from the past:
  - The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;
  - The size of $\Delta m_K$ led to a successful prediction of the charm mass;
  - The size of $\Delta m_B$ led to a successful prediction of the top mass;
  - The measurement of $\varepsilon_K$ led to predicting the third generation.
  - The measurement of neutrino flavor transitions led to the discovery of neutrino masses.

TABLE III: Measurements related to neutral meson mixing

| Sector | CP-conserving | CP-violating |
|---|---|---|
| $sd$ | $\Delta m_K/m_K = 7.0 \times 10^{-15}$ | $\epsilon_K = 2.3 \times 10^{-3}$ |
| $cu$ | $\Delta m_D/m_D = 8.7 \times 10^{-15}$ | $A_\Gamma/y_{\rm CP} \lesssim 0.2$ |
| $bd$ | $\Delta m_B/m_B = 6.3 \times 10^{-14}$ | $S_{\psi K} = +0.67 \pm 0.02$ |
| $bs$ | $\Delta m_{B_s}/m_{B_s} = 2.1 \times 10^{-12}$ | $S_{\psi\phi} = +0.01 \pm 0.04$ |

- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi-Maskawa phase $\delta_{\rm KM}$ [2]. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.

- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter imply that there exists new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle.*

- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle.* The puzzle became even deeper after neutrino masses and mixings were measured because, so far, neither smallness nor hierarchy in these parameters have been established.

## VIII. FLAVOR CHANGING NEUTRAL CURRENT (FCNC) PROCESSES

A very useful class of FCNC is that of neutral meson mixing. Nature provides us with four pairs of neutral mesons: $K^0 - \overline{K}^0$, $B^0 - \overline{B}^0$, $B_s^0 - \overline{B}_s^0$, and $D^0 - \overline{D}^0$. Mixing in this context refers to a transition such as $K^0 \to \overline{K}^0$ ($\bar{s}d \to \bar{d}s$).[2] The experimental results for CP

[2] These transitions involve four-quark operators. When calculating the matrix elements of these operators between meson-antimeson states, approximate symmetries of QCD are of no help. Instead, one uses lattice

conserving and CP violating observables related to neutral meson mixing (mass splittings and CP asymmetries in tree level decays, respectively) are given in Table III.

Our aim in this section is to explain the suppression factors that affect FCNC within the SM.

(a) **Loop suppression.** The $W$-boson cannot mediate FCNC processes at tree level, since it couples to up-down pairs, or to neutrino-charged lepton pairs. Obviously, only neutral bosons can mediate FCNC at tree level. The SM has four neutral bosons: the gluon, the photon, the $Z$-boson and the Higgs-boson. As concerns the massless gauge bosons, the gluon and the photon, their couplings are flavor-universal and, in particular, flavor-diagonal. This is guaranteed by gauge invariance. The universality of the kinetic terms in the canonical basis requires universality of the gauge couplings related to the unbroken symmetries. Hence neither the gluon nor the photon can mediate flavor changing processes at tree level. The situation concerning the $Z$-boson and the Higgs-boson is more complicated. In fact, the diagonality of their tree-level couplings is a consequence of special features of the SM, and can be violated with new physics.

The $Z$-boson, similarly to the $W$-boson, does not correspond to an unbroken gauge symmetry (as manifest in the fact that it is massive). Hence, there is no fundamental symmetry principle that forbids flavor changing couplings. Yet, as mentioned in Section IV B, in the SM this does not happen. The key point is the following. For each sector of mass eigenstates, characterized by spin, $SU(3)_C$ representation and $U(1)_{\rm EM}$ charge, there are two possibilities:

1. All mass eigenstates in this sector originate from interaction eigenstates in the same $SU(2)_L \times U(1)_Y$ representation.

2. The mass eigenstates in this sector mix interaction eigenstates of different $SU(2)_L \times U(1)_Y$ representations (but, of course, with the same $T_3 + Y$).

Let us examine the $Z$ couplings in the interaction basis in the subspace of all states that mix within a given sector of mass eigenstates:

1. In the first class, the $Z$ couplings in this subspace are universal, namely they are proportional to the unit matrix (times $T_3 - Q\sin^2\theta_W$ of the relevant interaction eigen-

calculations to relate, for example, the $B^0 \to \overline{B}^0$ transition to the corresponding quark process, $\bar{b}d \to \bar{d}b$.

states). The rotation to the mass basis maintains the universality: $V_{fM}\times\mathbf{1}\times V_{fM}^\dagger=\mathbf{1}$ ($f=u,d,e$; $M=L,R$).

2. In the second class, the $Z$ couplings are only "block-universal". In each sub-block $i$ of $m_i$ interaction eigenstates that have the same $(T_3)_i$, they are proportional to the $m_i\times m_i$ unit matrix, but the overall factor of $(T_3)_i-Q\sin^2\theta_W$ is different between the sub-blocks. In this case, the rotation to the mass basis, $V_{fM}\times\mathrm{diag}\{[(T_3)_1-Qs_W^2]\mathbf{1}_{m_1},[(T_3)_2-Qs_W^2]\mathbf{1}_{m_2},\ldots\}\times V_{fM}^\dagger$, does not maintain the universality, nor even the diagonality.

The special feature of the SM fermions is that they belong to the first class: All fermion mass eigenstates of a given chirality and $SU(3)_C\times U(1)_{\rm EM}$ representation come from the same $SU(3)_C\times SU(2)_L\times U(1)_Y$ representation.[3] For example, all the left-handed up quark mass eigenstates, which are in the $(3)_{+2/3}$ representation, come from interaction eigenstates in the $(3,2)_{+1/6}$ representation. This is the reason that the SM predicts universal $Z$ couplings to fermions. If, for example, Nature had left-handed quarks in the $(3,1)_{+2/3}$ representation, then the $Z$ couplings in the left-handed up sector would be non-universal and the $Z$ could mediate FCNC.

The Yukawa couplings of the Higgs boson are not universal. In fact, in the interaction basis, they are given by completely general $3\times3$ matrices. Yet, as explained in Section IV D, in the fermion mass basis they are diagonal. The reason is that the fermion mass matrix is proportional to the corresponding Yukawa matrix. Consequently, the mass matrix and the Yukawa matrix are simultaneously diagonalized. The special features of the SM in this regard are the following:

1. All the SM fermions are chiral, and therefore there are no bare mass terms.

2. The scalar sector has a single Higgs doublet.

In contrast, either of the following possible extensions would lead to flavor changing Higgs couplings:

1. There are quarks or leptons in vector-like representations, and thus there are bare mass terms.

[3] This is not true for the SM bosons. The vector boson mass eigenstates in the $(1)_0$ representation come from interaction eigenstates in the $(1,3)_0$ and $(1,1)_0$ representations ($W_3$ and $B$, respectively).

2. There is more than one $SU(2)_L$-doublet scalar.

We conclude that within the SM, all FCNC processes are loop suppressed. However, in extensions of the SM, FCNC can appear at the tree level, mediated by the $Z$ boson or by the Higgs boson or by new massive bosons.

(b) **CKM suppression.** Obviously, all flavor changing processes are proportional to off-diagonal entries in the CKM matrix. A quick look at the absolute values of the off-diagonal entries of the CKM matrix (A12) reveals that they are small. A rough estimate of the CKM suppression can be acquired by counting powers of $\lambda$ in the Wolfenstein parametrization (A4): $|V_{us}|$ and $|V_{cd}|$ are suppressed by $\lambda$, $|V_{cb}|$ and $|V_{ts}|$ by $\lambda^2$, $|V_{ub}|$ and $|V_{td}|$ by $\lambda^3$.

For example, the amplitude for $b \to s\gamma$ decay comes from penguin diagrams, dominated by the intermediate top quark, and suppressed by $|V_{tb}V_{ts}| \sim \lambda^2$. As another example, the $B^0 - \overline{B}^0$ mixing amplitude comes from box diagrams, dominated by intermediate top quarks, and suppressed by $|V_{tb}V_{td}|^2 \sim \lambda^6$.

(c) **GIM suppression.** If all quarks in a given sector were degenerate, then there would be no flavor changing $W$-couplings. A consequence of this fact is that FCNC in the down (up) sector are proportional to mass-squared differences between the quarks of the up (down) sector. For FCNC processes that involve only quarks of the first two generations, this leads to a strong suppression factor related to the light quark masses, and known as Glashow-Iliopoulos-Maiani (GIM) suppression.

Let us take as an example $\Delta m_K$, the mass splitting between the two neutral $K$-mesons. We have $\Delta m_K = 2|M_{K\bar{K}}|$, where $M_{K\bar{K}}$ corresponds to the $\overline{K}^0 \to K^0$ transition and comes from box diagrams. The top contribution is CKM-suppressed compared to the contributions from intermediate up and charm, so we consider only the latter:

$$M_{K\bar{K}} \simeq \sum_{i,j=u,c} \frac{G_F^2 m_W^2}{16\pi^2} \langle K^0|(\bar{d}_L\gamma^\mu s_L)^2|\overline{K}^0\rangle (V_{is}V_{id}^* V_{js}V_{jd}^*) \times F(x_i,x_j), \tag{88}$$

where $x_i = m_i^2/m_W^2$. If we had $m_u = m_c$, the amplitude would be proportional to $(V_{us}V_{ud}^* + V_{cs}V_{cd}^*)^2$, which vanishes in the two generation limit. We conclude that $\Delta m_K \propto (m_c^2 - m_u^2)/m_W^2$, which is the GIM suppression factor.

For the $B^0 - \overline{B}^0$ and $B_s - \overline{B}_s$ mixing amplitudes, the top-mediated contribution is not CKM suppressed compared to the lighter generations. The mass ratio $m_t^2/m_W^2$ enhances, rather than suppresses, the top contribution. Consequently, the $M_{B\bar{B}}$ amplitude is dominated by the top contribution:

$$M_{B\bar{B}} \simeq \frac{G_F^2 m_W^2}{16\pi^2} \langle B^0 | (\bar{d}_L \gamma^\mu b_L)^2 | \overline{B}^0 \rangle (V_{tb} V_{td}^*)^2 \times F\left(x_t, x_t\right). \tag{89}$$

### A. SM1.5: FCNC at tree level

Consider a model with the SM gauge group and pattern of SSB, but with only three quark flavors: $u$, $d$, $s$. Such a situation cannot fit into a model with all left-handed quarks in doublets of $SU(2)_L$. How can we incorporate the interactions of the strange quark in this picture? The solution that we now describe is wrong. Yet, it is of historical significance and, moreover, helps us to understand some of the unique properties of the SM described above. In particular, it leads to FCNC at tree level. We define the three flavor Standard Model (SM1.5) as follows (we ignore the lepton sector):

- The symmetry is a local

  $$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{90}$$

  It is spontaneously broken into

  $$SU(3)_C \times U(1)_{\rm EM} \quad (Q_{\rm EM} = T_3 + Y). \tag{91}$$

- The colored fermion representations are the following:

  $$Q_L(3,2)_{+1/6}, \quad D_L(3,1)_{-1/3}, \quad U_R(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3} \quad (i=1,2). \tag{92}$$

  There is a single scalar field,

  $$\phi(1,2)_{+1/2}. \tag{93}$$

We point out two important ingredients that are different from the SM:

1. There are quarks in a vector-like representation ($D_L + D_R$);
2. Not all $(3)_{-1/3}$ quarks come from the same type of $SU(2)_L \times U(1)_Y$ representations.

We first note that $D_L$ does not couple to the $W$-bosons:

$$\mathcal{L}_W = \frac{g}{2} \overline{Q_L} \not{W}_b \tau_b Q_L. \tag{94}$$

The Yukawa interactions are given by

$$\mathcal{L}_{\rm Yuk} = -y_u \overline{Q_L}\tilde{\phi}U_R - Y_i^d \overline{Q_L}\phi D_{Ri} + {\rm h.c.}. \tag{95}$$

Unlike the SM, we now have bare mass terms for fermions:

$$\mathcal{L}_q = -m_{di}\overline{D_L}D_{Ri} + {\rm h.c.}. \tag{96}$$

Given that there is a single up generation, the interaction basis is also the up mass basis. Explicitly, we identify the up-component of $Q_L$ with $u_L$ (and denote the down component of the doublet as $d_{uL}$), and $U_R$ with $u_R$. With the SSB, we have the following mass terms:

$$-\mathcal{L}_{\rm mass} = (\overline{d_{uL}}\ \overline{D_L}) \begin{pmatrix} Y_{d1}\frac{v}{\sqrt{2}} & Y_{d2}\frac{v}{\sqrt{2}} \\ m_{d1} & m_{d2} \end{pmatrix} \begin{pmatrix} D_{R1} \\ D_{R2} \end{pmatrix} + y_u \frac{v}{\sqrt{2}} \overline{u_L} u_R + {\rm h.c.}. \tag{97}$$

We now rotate to the down mass basis:

$$V_{dL} \begin{pmatrix} Y_{d1}\frac{v}{\sqrt{2}} & Y_{d2}\frac{v}{\sqrt{2}} \\ m_{d1} & m_{d2} \end{pmatrix} V_{dR}^\dagger = \begin{pmatrix} m_d & \\ & m_s \end{pmatrix}. \tag{98}$$

The resulting mixing matrix for the charged current interactions is a $1 \times 2$ matrix:

$$-\mathcal{L}_{W,q} = \frac{g}{\sqrt{2}} \overline{u_L} \not{W}^+ (\cos\theta_C\ \ \sin\theta_C) \begin{pmatrix} d_L \\ s_L \end{pmatrix} + {\rm h.c.}, \tag{99}$$

where $\theta_C$ is the rotation angle of $V_{dL}$. The neutral current interactions in the left-handed down sector are neither universal nor diagonal:

$$\begin{aligned} \mathcal{L}_{Z,q} = & \frac{g}{c_W}\left[\left(\frac{1}{2} - \frac{2}{3}s_W^2\right) \overline{u_L}\not{Z}u_L - \frac{2}{3}s_W^2\ \overline{u_R}\not{Z}u_R + \frac{1}{3}s_W^2(\overline{d_L}\not{Z}d_L + \overline{s_L}\not{Z}s_L + \overline{d_R}\not{Z}d_R + \overline{s_R}\not{Z}s_R)\right] \\ & - \frac{g}{2c_W}(\overline{d_L}\ \overline{s_L})\not{Z}\begin{pmatrix} \cos^2\theta_C & \cos\theta_C\sin\theta_C \\ \cos\theta_C\sin\theta_C & \sin^2\theta_C \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}. \end{aligned} \tag{100}$$

The Higgs interactions in the down sector are neither proportional to the mass matrix nor diagonal:

$$\mathcal{L}_{\rm Yuk}^q = y_u h \overline{u_L} u_R + h(\overline{d_L}\ \overline{s_L}) \left[V_{dL} \begin{pmatrix} Y_{d1} & Y_{d2} \\ 0 & 0 \end{pmatrix} V_{dR}^\dagger\right] \begin{pmatrix} d_R \\ s_R \end{pmatrix} + {\rm h.c.}. \tag{101}$$

Thus, in this model, both the $Z$-boson and the $h$-boson mediate FCNC at tree level. For example, $K_L \to \mu^+\mu^-$ and $K^0 - \overline{K}^0$ mixing get $Z$- and $h$-mediated tree-level contributions.

## B. 2HDM: FCNC at tree level

Consider a model with two Higgs doublets. The symmetry structure, the pattern of spontaneous symmetry breaking, and the fermion content are the same as in the SM. However, the scalar content is extended:

- The scalar representations are

$$\phi_i(1,2)_{+1/2}, \quad i = 1, 2. \tag{102}$$

We are particularly interested in the modification of the Yukawa terms:

$$\mathcal{L}_{\rm Yuk} = (Y_k^u)_{ij}\overline{Q_{Li}}U_{Rj}\,\phi_k + (Y_k^d)_{ij}\overline{Q_{Li}}D_{Rj}\,\phi_k + (Y_k^e)_{ij}\overline{L_{Li}}E_{Rj}\,\phi_k + \text{h.c.}. \tag{103}$$

Without loss of generality, we can work in a basis (commonly called "the Higgs basis") $(\phi_A, \phi_M)$, where one of the Higgs doublets carries the VEV, $\langle\phi_M\rangle = v/\sqrt{2}$, while the other has zero VEV, $\langle\phi_A\rangle = 0$. In this basis, $Y_M^f$ is known and related to the fermions masses in the same way as the Yukawa matrices of the SM:

$$Y_M^f = \sqrt{2}M_f/v. \tag{104}$$

The entries Yukawa matrices $Y_A^f$ are, however, free parameters and, in general, unrelated to the fermion masses. The rotation angle from the Higgs basis to the basis of neutral CP-even Higgs states, $(\phi_h, \phi_H)$, is denoted by $(\alpha - \beta)$. The Yukawa matrix of the light Higgs field $h$ is given by

$$Y_h^f = c_{\alpha-\beta}Y_A^f - s_{\alpha-\beta}Y_M^f. \tag{105}$$

Given the arbitrary structure of $Y_A^f$, the Higgs boson can have couplings that are neither proportional to the mass matrix nor diagonal.

It is interesting to note, however, that not all multi Higgs doublet models lead to flavor changing Higgs couplings. If all the fermions of a given sector couple to one and the same doublet, then the Higgs couplings in that sector would still be diagonal. For example, in a model with two Higgs doublets, $\phi_1$ and $\phi_2$, and Yukawa terms of the form

$$\mathcal{L}_{\rm Yuk} = Y_{ij}^u\overline{Q_{Li}}U_{Rj}\,\phi_2 + Y_{ij}^d\overline{Q_{Li}}D_{Rj}\,\phi_1 + Y_{ij}^e\overline{L_{Li}}E_{Rj}\,\phi_1 + \text{h.c.}, \tag{106}$$

the Higgs couplings are flavor diagonal:

$$Y_h^u = (c_\alpha/s_\beta)Y_M^u, \quad Y_h^d = -(s_\alpha/c_\beta)Y_M^d, \quad Y_h^e = -(s_\alpha/c_\beta)Y_M^e, \tag{107}$$

where $\beta$ $[\alpha]$ is the rotation angle from the $(\phi_1, \phi_2)$ basis to the $(\phi_A, \phi_M)$ $[(\phi_h, \phi_H)]$ basis. In the physics jargon, we say that such models have *natural flavor conservation* (NFC).

## IX. CP VIOLATION

There are two main reasons for the interest in CP violation:

- CP asymmetries provide some of the theoretically cleanest probes of flavor physics. The reason for that is that CP is a good symmetry of the strong interactions. Consequently, for some hadronic decays, QCD-related uncertainties cancel out in the CP asymmetries.

- There is a cosmological puzzle related to CP violation. The baryon asymmetry of the Universe is a CP violating observable, and it is many orders of magnitude larger than the SM prediction. Hence, there must exist new sources of CP violation beyond the single phase of the CKM matrix.

In this section we explain why CP violation is related to complex parameters of the Lagrangian. Based on this fact, we prove that CP violation in a two generation SM is impossible, while CP violation in a three generation SM requires a long list of conditions on its flavor parameters in order to occur.

### A. CP violation and complex couplings

The CP transformation combines charge conjugation C with parity P. Under C, particles and antiparticles are interchanged by conjugating all internal quantum numbers, *e.g.*, $Q \to -Q$. Under P, the handedness of space is reversed, $\vec{x} \to -\vec{x}$. Thus, for example, a left-handed electron $e_L^-$ is transformed under CP into a right-handed positron $e_R^+$.

At the Lagrangian level, CP is a good symmetry if there is a basis where all couplings are real. Let us provide a simple explanation of this statement. Consider a theory with a single scalar, $\phi$, and two sets of $N$ fermions, $\psi_L^i$ and $\psi_R^i$ $(i = 1, 2, ..., N)$. The Yukawa interactions are given by

$$-\mathcal{L} = Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + Y_{ij}^*\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \qquad (108)$$

where we write the two hermitian conjugate terms explicitly. The CP transformation of the fields is defined as follows:

$$\phi \to \phi^\dagger, \qquad \psi_{Li} \to \overline{\psi_{Li}}, \qquad \psi_{Ri} \to \overline{\psi_{Ri}}. \tag{109}$$

Therefore, a CP transformation exchanges the operators

$$\overline{\psi_{Li}}\phi\psi_{Rj} \stackrel{CP}{\longleftrightarrow} \overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{110}$$

but leaves their coefficients, $Y_{ij}$ and $Y^*_{ij}$, unchanged. This means that CP is a symmetry of $\mathcal{L}$ if $Y_{ij} = Y^*_{ij}$.

In practice, things are more subtle, since one can define the CP transformation in a more general way than Eq. (109):

$$\phi \to e^{i\theta}\phi^\dagger, \qquad \psi^i_L \to e^{i\theta_{Li}}\overline{\psi^i_L}, \qquad \psi^i_R \to e^{i\theta_{Ri}}\overline{\psi^i_R}, \tag{111}$$

with $\theta, \theta_{Li}, \theta_{Ri}$ convention-dependent phases. Then, there can be complex couplings, yet CP would be a good symmetry. The correct statement is that CP is violated if, using all freedom to redefine the phases of the fields, one cannot find any basis where all couplings are real.

Let us examine the situation in the mass basis of the SM. The couplings of the gluons, the photon and the $Z$-boson are all real, as are the two parameters of the scalar potential. As concerns the fermion mass terms (or, equivalently, the Yukawa couplings) and the weak gauge interactions, the relevant CP transformation laws are

$$\bar{\psi}_i\psi_j \to \bar{\psi}_j\psi_i, \quad \bar{\psi}_i\gamma^\mu W^+_\mu(1-\gamma_5)\psi_j \to \bar{\psi}_j\gamma^\mu W^-_\mu(1-\gamma_5)\psi_i. \tag{112}$$

Thus the mass terms and CC weak interaction terms are CP invariant if all the masses and couplings are real. We can always choose the masses to be real. Then, let us focus on the couplings of $W^\pm$ to quarks:

$$-\frac{g}{\sqrt{2}}\left(V_{ij}\bar{u}_i\gamma^\mu W^+_\mu(1-\gamma_5)d_j + V^*_{ij}\bar{d}_j\gamma^\mu W^-_\mu(1-\gamma_5)u_i\right). \tag{113}$$

The CP operation exchanges the two terms, except that $V_{ij}$ and $V^*_{ij}$ are not interchanged. Thus CP would be a good symmetry of the SM only if there were a mass basis and choice of phase convention where all masses and entries of the CKM matrix are real.

### B. SM2: CP conserving

Consider a two generation Standard Model, SM2. This model is similar to the one defined in Section I, which in this section will be referred to as SM3, except that there are two, rather than three fermion generations. Many features of SM2 are similar to SM3, but there is one important difference: CP is a good symmetry of SM2, but not of SM3. To see how this difference comes about, let us examine the accidental symmetries of SM2. We follow here the line of analysis of SM3 in Section V A.

If we set the Yukawa couplings to zero, $\mathcal{L}_{\rm Yuk}^{\rm SM2} = 0$, SM2 gains an accidental global symmetry:

$$G_{\rm SM2}^{\rm global}(Y^{u,d,e} = 0) = U(2)_Q \times U(2)_U \times U(2)_D \times U(2)_L \times U(2)_E, \tag{114}$$

where the two generations of each gauge representation are a doublet of the corresponding $U(2)$. The Yukawa couplings break this symmetry into the subgroup

$$G_{\rm SM2}^{\rm global} = U(1)_B \times U(1)_e \times U(1)_\mu. \tag{115}$$

A-priori, the Yukawa terms depend on three $2 \times 2$ complex matrices, namely $12_R + 12_I$ parameters. The global symmetry breaking, $[U(2)]^5 \to [U(1)]^3$, implies that we can remove $5 \times (1_R + 3_I) - 3_I = 5_R + 12_I$ parameters. Thus the number of physical flavor parameters is 7 real parameters and no imaginary parameter. The real parameters can be identified as two charged lepton masses, four quark masses, and the single real mixing angle, $\sin\theta_c = |V_{us}|$.

The important conclusion for our purposes is that all imaginary couplings can be removed from SM2, and CP is an accidental symmetry of the model.

### C. SM3: Not necessarily CP violating

A-priori, CP is not necessarily violated in SM3. If two quarks of the same charge had equal masses, one mixing angle and the phase could be removed from $V$. This can be written as a condition on the quark mass differences. CP violation requires

$$(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \neq 0. \tag{116}$$

Likewise, if the value of any of the three mixing angles were 0 or $\pi/2$, then the phase can be removed. Finally, CP would not be violated if the value of the single phase were 0 or $\pi$.

These last eight conditions are elegantly incorporated into one, parametrization-independent condition. To find this condition, note that the unitarity of the CKM matrix, $VV^\dagger = 1$, requires that for any choice of $i, j, k, l = 1, 2, 3$,

$$\mathcal{I}m[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln}. \tag{117}$$

Then the conditions on the mixing parameters are summarized by

$$J \neq 0. \tag{118}$$

The quantity $J$ is of much interest in the study of CP violation from the CKM matrix. The maximum value that $J$ could assume in principle is $1/(6\sqrt{3}) \approx 0.1$, but it is found to be $\sim 4 \times 10^{-5}$.

The fourteen conditions incorporated in Eqs. (116) and (118) can all be written as a single requirement on the quark mass matrices in the interaction basis:

$$X_{CP} \equiv \mathcal{I}m\left\{\det\left[M_d M_d^\dagger, M_u M_u^\dagger\right]\right\} \neq 0 \;\Leftrightarrow\; \text{CP violation}. \tag{119}$$

This is a convention independent condition.

## X. TESTING CKM

Measurements of rates, mixing, and CP asymmetries in $B$ decays in the two B factories, BaBar and Belle, and in the two Tevatron detectors, CDF and D0, (and, more recently, at the LHCb experiment) signified a new era in our understanding of flavor physics and CP violation. The progress has been both qualitative and quantitative. Various basic questions concerning CP and flavor violation have received, for the first time, answers based on experimental information. These questions include, for example,

- Is the Kobayashi-Maskawa mechanism at work (namely, is $\delta_{\rm KM} \neq 0$)?
- Does the KM phase dominate the observed CP violation?
- Does the CKM mechanism dominate FCNC?

As a first step, one may assume the SM and test the overall consistency of the various measurements. However, the richness of data from the B factories allow us to go a step further and answer these questions model independently, namely allowing new physics to contribute to the relevant processes. We here explain the way in which this analysis proceeds.

### A. $S_{\psi K_S}$

The CP asymmetry in $B \to \psi K_S$ decays plays a major role in testing the KM mechanism. Before we explain the test itself, we should understand why the theoretical interpretation of the asymmetry is exceptionally clean, and what are the theoretical parameters on which it depends, within and beyond the Standard Model.

The CP asymmetry in neutral $B$ meson decays into final CP eigenstates $f_{CP}$ is defined as follows:

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]} . \tag{120}$$

A detailed evaluation of this asymmetry is given in Appendix B. It leads to the following form:

$$\begin{aligned} \mathcal{A}_{f_{CP}}(t) &= S_{f_{CP}} \sin(\Delta m_B t) - C_{f_{CP}} \cos(\Delta m_B t), \\ S_{f_{CP}} &\equiv \frac{2\,\mathcal{I}m(\lambda_{f_{CP}})}{1+|\lambda_{f_{CP}}|^2}, \quad C_{f_{CP}} \equiv \frac{1-|\lambda_{f_{CP}}|^2}{1+|\lambda_{f_{CP}}|^2} , \end{aligned} \tag{121}$$

where

$$\lambda_{f_{CP}} = e^{-i\phi_B}(\overline{A}_{f_{CP}}/A_{f_{CP}}) . \tag{122}$$

Here $\phi_B$ refers to the phase of $M_{B\bar{B}}$ [see Eq. (B23)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) . \tag{123}$$

The decay amplitudes $A_f$ and $\overline{A}_f$ are defined in Eq. (B1).

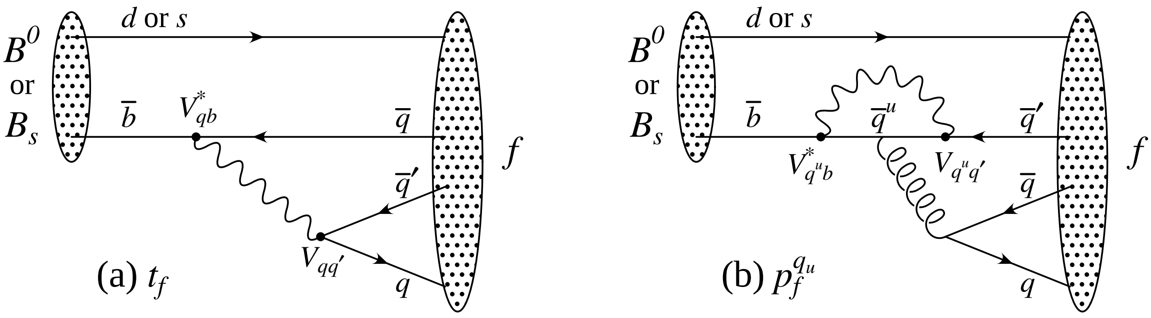
The $B^0 \to J/\psi K^0$ decay [4, 5] proceeds via the quark transition $\bar{b} \to \bar{c}c\bar{s}$. There are contributions from both tree ($t$) and penguin ($p^{q_u}$, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 1) which carry different weak phases:

$$A_f = (V_{cb}^* V_{cs})\, t_f + \sum_{q_u=u,c,t} \left(V_{q_u b}^* V_{q_u s}\right) p_f^{q_u} . \tag{124}$$

(The distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, ref. [6].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations:

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u, \tag{125}$$

FIG. 1: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.




where $T_{\psi K} = t_{\psi K} + p^c_{\psi K} - p^t_{\psi K}$ and $P^u_{\psi K} = p^u_{\psi K} - p^t_{\psi K}$. A subtlety arises in this decay that is related to the fact that $B^0 \to J/\psi K^0$ and $\overline{B}^0 \to J/\psi \overline{K}^0$. A common final state, *e.g.* $J/\psi K_S$, can be reached via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral $K$ mixing, $e^{-i\phi_K} = (V^*_{cd}V_{cs})/(V_{cd}V^*_{cs})$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb}V^*_{cs})\, T_{\psi K} + (V_{ub}V^*_{us})\, P^u_{\psi K}}{(V^*_{cb}V_{cs})\, T_{\psi K} + (V^*_{ub}V_{us})\, P^u_{\psi K}} \times \frac{V^*_{cd}V_{cs}}{V_{cd}V^*_{cs}}. \tag{126}$$

The crucial point is that, for $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c}c\bar{s}$ processes, we can neglect the $P^u$ contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$|P^u_{\psi K}/T_{\psi K}| \times |V_{ub}/V_{cb}| \times |V_{us}/V_{cs}| \sim (\text{loop factor}) \times 0.1 \times 0.23 \lesssim 0.005. \tag{127}$$

Thus, to an accuracy better than one percent,

$$\lambda_{\psi K_S} = \left(\frac{V^*_{tb}V_{td}}{V_{tb}V^*_{td}}\right)\left(\frac{V_{cb}V^*_{cd}}{V^*_{cb}V_{cd}}\right) = -e^{-2i\beta}, \tag{128}$$

where $\beta$ is defined in Eq. (A9), and consequently

$$S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0\ . \tag{129}$$

(Below the percent level, several effects modify this equation [7–10].)

**Exercise 3:** *Show that, if the $B \to \pi\pi$ decays were dominated by tree diagrams, then $S_{\pi\pi} = \sin 2\alpha$.*

**Exercise 4:** *Estimate the accuracy of the predictions $S_{\phi K_S} = \sin 2\beta$ and $C_{\phi K_S} = 0$.*

When we consider extensions of the SM, we still do not expect any significant new contribution to the tree level decay, $b \to c\bar{c}s$, beyond the SM $W$-mediated diagram. Thus, the expression $\bar{A}_{\psi K_S}/A_{\psi K_S} = (V_{cb}V_{cd}^*)/(V_{cb}^*V_{cd})$ remains valid, though the approximation of neglecting sub-dominant phases can be somewhat less accurate than Eq. (127). On the other hand,since $B^0 - \overline{B}^0$ mixing is an FCNC process, $M_{B\bar{B}}$ can in principle get large and even dominant contributions from new physics. We can parameterize the modification to the SM in terms of two parameters, $r_d^2$ signifying the change in magnitude, and $2\theta_d$ signifying the change in phase:

$$M_{B\bar{B}} = r_d^2\ e^{2i\theta_d}\ M_{B\bar{B}}^{\rm SM}(\rho,\eta). \tag{130}$$

This leads to the following generalization of Eq. (129):

$$S_{\psi K_S} = \sin(2\beta + 2\theta_d), \quad C_{\psi K_S} = 0\ . \tag{131}$$

The experimental measurements give the following ranges [11]:

$$S_{\psi K_S} = +0.69 \pm 0.02, \quad C_{\psi K_S} = +0.005 \pm 0.017\ . \tag{132}$$

### B. Is the CKM assumption Self-consistent?

The three generation standard model has room for CP violation, through the KM phase in the quark mixing matrix. Yet, one would like to make sure that indeed CP is violated by the SM interactions, namely that $\sin\delta_{\rm KM} \neq 0$. If we establish that this is the case, we would further like to know whether the SM contributions to CP violating observables are dominant. More quantitatively, we would like to put an upper bound on the ratio between the new physics and the SM contributions.

As a first step, one can assume that flavor changing processes are fully described by the SM, and check the consistency of the various measurements with this assumption. There are four relevant mixing parameters, which can be taken to be the Wolfenstein parameters $\lambda$, $A$, $\rho$ and $\eta$ defined in Eq. (A4). The values of $\lambda$ and $A$ are known rather accurately [1] from, respectively, $K \to \pi\ell\nu$ and $b \to c\ell\nu$ decays:

$$\lambda = 0.2251 \pm 0.0005, \quad A = 0.81 \pm 0.03. \tag{133}$$

Then, one can express all the relevant observables as a function of the two remaining parameters, $\rho$ and $\eta$, and check whether there is a range in the $\rho - \eta$ plane that is consistent with all measurements. The list of observables includes the following:

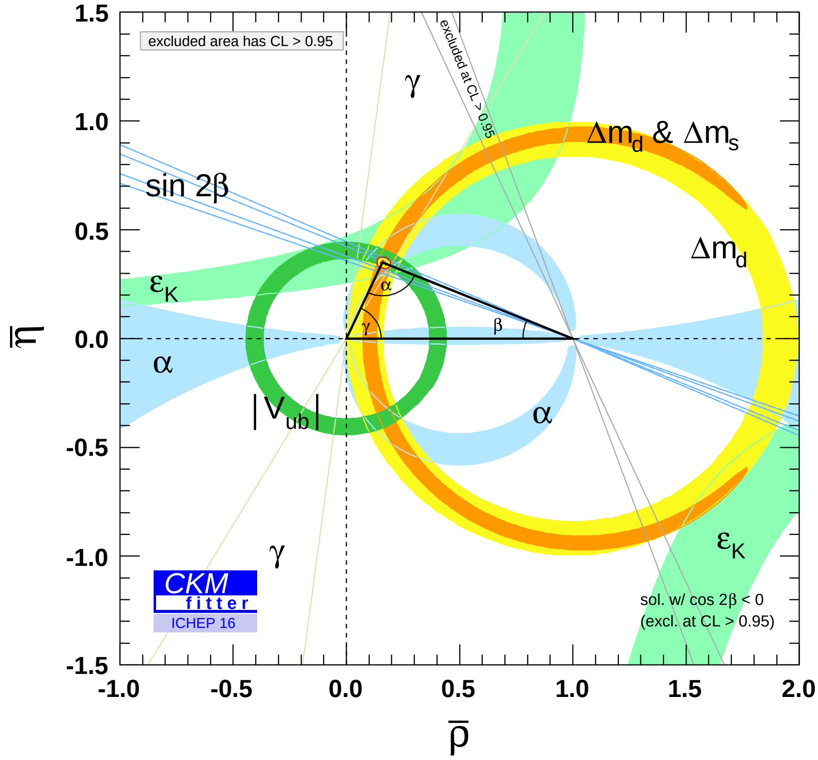



FIG. 2: Allowed region in the $\rho, \eta$ plane. Superimposed are the individual constraints from charmless semileptonic $B$ decays ($|V_{ub}|$), mass differences in the $B^0$ ($\Delta m_d$) and $B_s$ ($\Delta m_s$) neutral meson systems, and CP violation in $K \to \pi\pi$ ($\varepsilon_K$), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ ($\alpha$), and $B \to DK$ ($\gamma$). Taken from [12].

- The rates of inclusive and exclusive charmless semileptonic $B$ decays depend on $|V_{ub}|^2 \propto \rho^2 + \eta^2$;
- The CP asymmetry in $B \to \psi K_S$, $S_{\psi K_S} = \sin 2\beta = \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2}$;
- The rates of various $B \to DK$ decays depend on the phase $\gamma$, where $e^{i\gamma} = \frac{\rho+i\eta}{\sqrt{\rho^2+\eta^2}}$;
- The rates of various $B \to \pi\pi, \rho\pi, \rho\rho$ decays depend on the phase $\alpha = \pi - \beta - \gamma$;
- The ratio between the mass splittings in the neutral $B$ and $B_s$ systems is sensitive to $|V_{td}/V_{ts}|^2 = \lambda^2[(1-\rho)^2 + \eta^2]$;
- The CP violation in $K \to \pi\pi$ decays, $\epsilon_K$, depends in a complicated way on $\rho$ and $\eta$.

The resulting constraints are shown in Fig. 2. The consistency of the various constraints is impressive. In particular, the following ranges for $\rho$ and $\eta$ can account for all the measurements [1]:

$$\rho = +0.12 \pm 0.02, \quad \eta = +0.36 \pm 0.01. \tag{134}$$

One can make then the following statement [13]:

**Very likely, CP violation in flavor changing processes is dominated by the Kobayashi-Maskawa phase.**

In the next two subsections, we explain how we can remove the phrase "very likely" from this statement, and how we can quantify the KM-dominance.

### C. Is the KM mechanism at work?

In proving that the KM mechanism is at work, we assume that charged-current tree-level processes are dominated by the $W$-mediated SM diagrams (see, for example, [14]). This is a very plausible assumption. It is difficult to construct a model where new physics competes with the SM in flavor changing charged current processes, and does not violate the constraints from flavor changing neutral current processes. Thus we can use all tree level processes and fit them to $\rho$ and $\eta$, as we did before. The list of such processes includes the following:

1. Charmless semileptonic $B$-decays, $b \to u\ell\nu$, measure $R_u$ [see Eq. (A8)].
2. $B \to DK$ decays, which go through the quark transitions $b \to c\bar{u}s$ and $b \to u\bar{c}s$, measure the angle $\gamma$ [see Eq. (A9)].
3. $B \to \rho\rho$ decays (and, similarly, $B \to \pi\pi$ and $B \to \rho\pi$ decays) go through the quark transition $b \to u\bar{u}d$. With an isospin analysis, one can determine the relative phase between the tree decay amplitude and the mixing amplitude. By incorporating the measurement of $S_{\psi K_S}$, one can subtract the phase from the mixing amplitude, finally providing a measurement of the angle $\gamma$ [see Eq. (A9)].

In addition, we can use loop processes, but then we must allow for new physics contributions, in addition to the $(\rho, \eta)$-dependent SM contributions. Of course, if each such measurement adds a separate mode-dependent parameter, then we do not gain anything by using this information. However, there is a number of observables where the only relevant

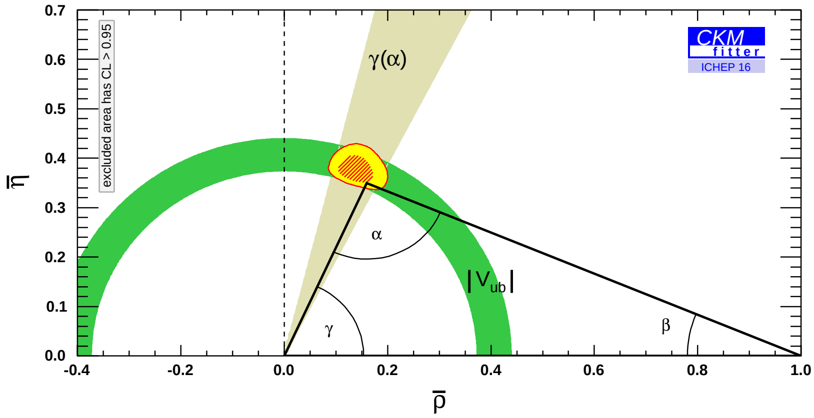



FIG. 3: The allowed region in the $\rho - \eta$ plane, assuming that tree diagrams are dominated by the Standard Model [12].

loop process is $B^0 - \overline{B}^0$ mixing. The list includes $S_{\psi K_S}$, $\Delta m_B$ and the CP asymmetry in semileptonic $B$ decays:

$$\begin{aligned} S_{\psi K_S} &= \sin(2\beta + 2\theta_d), \\ \Delta m_B &= r_d^2 (\Delta m_B)^{\rm SM}, \\ \mathcal{A}_{\rm SL} &= -\mathcal{R}e\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\sin 2\theta_d}{r_d^2} + \mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\cos 2\theta_d}{r_d^2}. \end{aligned} \tag{135}$$

As explained above, such processes involve two new parameters [see Eq. (130)]. Since there are three relevant observables, we can further tighten the constraints in the $(\rho, \eta)$-plane. Similarly, one can use measurements related to $B_s - \overline{B}_s$ mixing. One gains three new observables at the cost of two new parameters (see, for example, [15]).

The results of such fit, projected on the $\rho - \eta$ plane, can be seen in Fig. 3. It gives [12]

$$\eta = 0.38 \pm 0.03. \tag{136}$$

It is clear that $\eta \neq 0$ is well established:

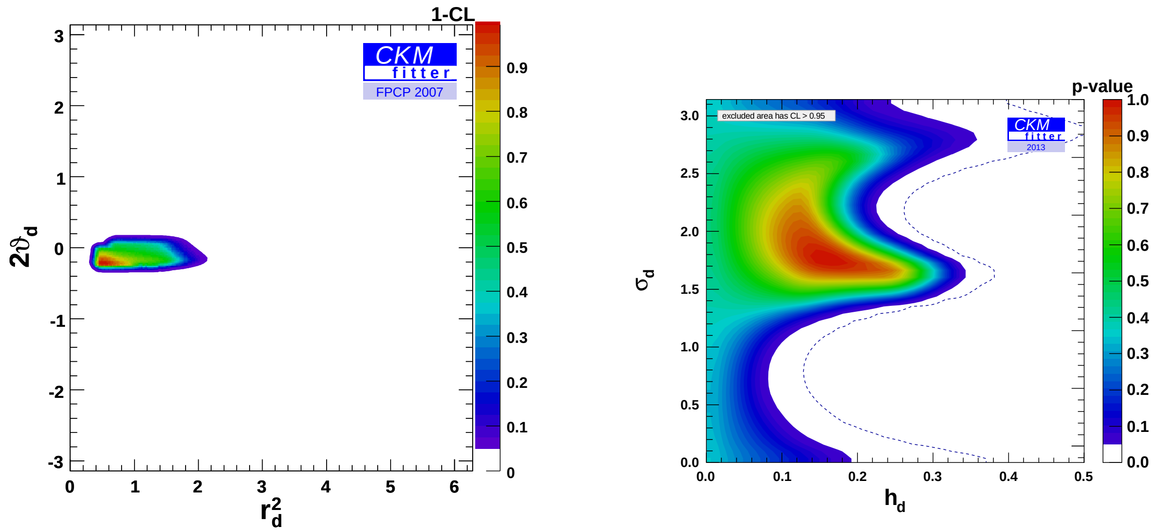
**The Kobayashi-Maskawa mechanism of CP violation is at work.**

The consistency of the experimental results (132) with the SM predictions (129) means that the KM mechanism of CP violation dominates the observed CP violation. In the next subsection, we make this statement more quantitative.

### D. How much can new physics contribute to $B^0 - \overline{B}^0$ mixing?

All that we need to do in order to establish whether the SM dominates the observed CP violation, and to put an upper bound on the new physics contribution to $B^0 - \overline{B}^0$ mixing, is to project the results of the fit performed in the previous subsection on the $r_d^2 - 2\theta_d$ plane. If we find that $\theta_d \ll \beta$, then the SM dominance in the observed CP violation will be established. The constraints are shown in Fig. 4(a). Indeed, $\theta_d \ll \beta$.

FIG. 4: Constraints in the (a) $r_d^2 - 2\theta_d$ plane, and (b) $h_d - \sigma_d$ plane, assuming that NP contributions to tree level processes are negligible [12].




An alternative way to present the data is to use the $h_d, \sigma_d$ parametrization,

$$r_d^2 e^{2i\theta_d} = 1 + h_d e^{2i\sigma_d}. \tag{137}$$

While the $r_d, \theta_d$ parameters give the relation between the full mixing amplitude and the SM one, and are convenient to apply to the measurements, the $h_d, \sigma_d$ parameters give the relation between the new physics and SM contributions, and are more convenient in testing theoretical models:

$$h_d e^{2i\sigma_d} = \frac{M_{B\bar{B}}^{\rm NP}}{M_{B\bar{B}}^{\rm SM}}. \tag{138}$$

The constraints in the $h_d - \sigma_d$ plane are shown in Fig. 4(b). We can make the following two statements:

1. A new physics contribution to $B^0-\overline{B}^0$ mixing amplitude that carries a phase that is significantly different from the KM phase is constrained to lie below the 10-20% level.

2. A new physics contribution to the $B^0-\overline{B}^0$ mixing amplitude which is aligned with the KM phase is constrained to lie below the 30-40% level.

One can reformulate these statements as follows:

1. The KM mechanism dominates CP violation in $B^0-\overline{B}^0$ mixing.

2. The CKM mechanism dominates the $B^0-\overline{B}^0$ mixing amplitude.

## XI. THE NEW PHYSICS FLAVOR PUZZLE

### A. A model independent discussion

It is clear that the Standard Model is not a complete theory of Nature:

1. It does not include gravity, and therefore it cannot be valid at energy scales above $m_{\rm Planck}\sim 10^{19}$ GeV:

2. It does not allow for neutrino masses, and therefore it cannot be valid at energy scales above $m_{\rm seesaw}\sim 10^{15}$ GeV;

3. The fine-tuning problem of the Higgs mass and the puzzle of the dark matter suggest that the scale where the SM is replaced with a more fundamental theory is actually much lower, $m_{\rm top-partners}, m_{\rm wimp}\lesssim$ a few TeV.

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\rm SM}$. These are terms of dimension higher than four in the fields which, therefore, have couplings that are inversely proportional to the scale of new physics $\Lambda_{\rm NP}$.

The lowest dimension non-renormalizable terms are dimension-five:

$$-\mathcal{L}^{\rm dim-5}_{\rm Seesaw}=\frac{Z^\nu_{ij}}{\Lambda_{\rm NP}}L_{Li}L_{Lj}\phi\phi+{\rm h.c.}. \qquad (139)$$

These are the seesaw terms, leading to neutrino masses.

**Exercise 5:** *How does the global symmetry breaking pattern (80) change when (139) is taken into account?*

**Exercise 6:** *What is the number of physical lepton flavor parameters in this case? Identify these parameters in the mass basis.*

As concerns quark flavor physics, consider, for example, the following dimension-six set of operators:

$$\mathcal{L}^{\rm dim-6}_{\Delta F=2} = \sum_{i\neq j} \frac{z_{ij}}{\Lambda^2} (\overline{Q_{Li}}\gamma_\mu Q_{Lj})^2, \tag{140}$$

where the $z_{ij}$ are dimensionless couplings. These terms contribute to the mass splittings between the corresponding two neutral mesons. For example, the term $\mathcal{L}_{\Delta B=2} \propto (\overline{d_L}\gamma_\mu b_L)^2$ contributes to $\Delta m_B$, the mass difference between the two neutral $B$-mesons. We use $M_{B\bar{B}} = \frac{1}{2m_B}\langle B^0|\mathcal{L}^{\rm dim-6}_{\Delta F=2}|\overline{B}^0\rangle$ and

$$\langle B^0|(\overline{d_{La}}\gamma^\mu b_{La})(\overline{d_{Lb}}\gamma_\mu b_{Lb})|\overline{B}^0\rangle = -\frac{1}{3} m_B^2 f_B^2 B_B. \tag{141}$$

Analogous expressions hold for the other neutral mesons.[4] This leads to $\Delta m_B/m_B = 2|M_{B\bar{B}}|/m_B \sim (|z_{bd}|/3)(f_B/\Lambda_{\rm NP})^2$.

The consistency of the experimental results with the SM predictions for neutral meson mixing, allows us to impose the condition $|M^{\rm NP}_{P\bar{P}}| < |M^{\rm SM}_{P\bar{P}}|$ for $P = K, B, B_s$, which implies that

$$\Lambda > \frac{4.4 \text{ TeV}}{|V^*_{ti}V_{tj}|/|z_{ij}|^{1/2}} \sim \begin{cases} 1.3\times 10^4 \text{ TeV} \times |z_{sd}|^{1/2} \\ 5.1\times 10^2 \text{ TeV} \times |z_{bd}|^{1/2} \\ 1.1\times 10^2 \text{ TeV} \times |z_{bs}|^{1/2} \end{cases} \tag{142}$$

A more detailed list of the bounds derived from the $\Delta F = 2$ observables in Table III is given in Table IV. The bounds refer to two representative sets of dimension-six operators: (i) left-left operators, that are also present in the SM, and (ii) operators with different chirality, where the bounds are strongest because of larger hadronic matrix elements.

The first lesson that we draw from these bounds on $\Lambda$ is that new physics can contribute to FCNC at a level comparable to the SM contributions even if it takes place at a scale that is six orders of magnitude above the electroweak scale. A second lesson is that if the new physics has a generic flavor structure, that is $z_{ij} = \mathcal{O}(1)$, then its scale must be above $10^4 - 10^5$ TeV (or, if the leading contributions involve electroweak loops, above $10^3 - 10^4$ TeV). *If indeed $\Lambda \gg TeV$, it means that we have misinterpreted the hints from the fine-tuning problem and the dark matter puzzle.*

[4] The PDG [1] quotes the following values, extracted from leptonic charged meson decays: $f_K \approx 0.16\ GeV$, $f_D \approx 0.23\ GeV$, $f_B \approx 0.18\ GeV$. We further use $f_{B_s} \approx 0.20\ GeV$.

TABLE IV: Lower bounds on the scale of new physics $\Lambda$, in units of TeV, for $|z_{ij}| = 1$, and upper bounds on $z_{ij}$, assuming $\Lambda = 1$ TeV [21].

| Operator | $\Lambda$ [TeV] CPC | $\Lambda$ [TeV] CPV | $\lvert z_{ij}\rvert$ | $\mathcal{I}m(z_{ij})$ | Observables |
|---|---|---|---|---|---|
| $(\bar{s}_L\gamma^\mu d_L)^2$ | $9.8 \times 10^2$ | $1.6 \times 10^4$ | $9.0 \times 10^{-7}$ | $3.4 \times 10^{-9}$ | $\Delta m_K;\ \epsilon_K$ |
| $(\bar{s}_R d_L)(\bar{s}_L d_R)$ | $1.8 \times 10^4$ | $3.2 \times 10^5$ | $6.9 \times 10^{-9}$ | $2.6 \times 10^{-11}$ | $\Delta m_K;\ \epsilon_K$ |
| $(\bar{c}_L\gamma^\mu u_L)^2$ | $1.2 \times 10^3$ | $2.9 \times 10^3$ | $5.6 \times 10^{-7}$ | $1.0 \times 10^{-7}$ | $\Delta m_D;\ A_\Gamma$ |
| $(\bar{c}_R u_L)(\bar{c}_L u_R)$ | $6.2 \times 10^3$ | $1.5 \times 10^4$ | $5.7 \times 10^{-8}$ | $1.1 \times 10^{-8}$ | $\Delta m_D;\ A_\Gamma$ |
| $(\bar{b}_L\gamma^\mu d_L)^2$ | $6.6 \times 10^2$ | $9.3 \times 10^2$ | $2.3 \times 10^{-6}$ | $1.1 \times 10^{-6}$ | $\Delta m_B;\ S_{\psi K}$ |
| $(\bar{b}_R d_L)(\bar{b}_L d_R)$ | $2.5 \times 10^3$ | $3.6 \times 10^3$ | $3.9 \times 10^{-7}$ | $1.9 \times 10^{-7}$ | $\Delta m_B;\ S_{\psi K}$ |
| $(\bar{b}_L\gamma^\mu s_L)^2$ | $1.4 \times 10^2$ | $2.5 \times 10^2$ | $5.0 \times 10^{-5}$ | $1.7 \times 10^{-5}$ | $\Delta m_{B_s};\ S_{\psi\phi}$ |
| $(\bar{b}_R s_L)(\bar{b}_L s_R)$ | $4.8 \times 10^2$ | $8.3 \times 10^2$ | $8.8 \times 10^{-6}$ | $2.9 \times 10^{-6}$ | $\Delta m_{B_s};\ S_{\psi\phi}$ |

A different lesson can be drawn from the bounds on $z_{ij}$. *It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic.* Specifically, if new particles at the TeV scale couple to the SM fermions, then there are two ways in which their contributions to FCNC processes, such as neutral meson mixing, can be suppressed: degeneracy and alignment. Either of these principles, or a combination of both, signifies non-generic structure.

One can use the language of effective operators also for the SM, integrating out all particles significantly heavier than the neutral mesons (that is, the top, the Higgs and the weak gauge bosons). Thus, the scale is $\Lambda_{\rm SM} \sim m_W$. Since the leading contributions to neutral meson mixings come from box diagrams, the $z_{ij}$ coefficients are suppressed by $\alpha_2^2$. To identify the relevant flavor suppression factor, one can employ the spurion formalism. For example, the flavor transition that is relevant to $B^0 - \overline{B}^0$ mixing involves $\overline{d_L} b_L$ which transforms as $(8,1,1)_{SU(3)^3_q}$. The leading contribution must then be proportional to $(Y^u Y^{u\dagger})_{13} \propto y_t^2 V_{tb} V_{td}^*$. Indeed, an explicit calculation (using VIA for the matrix element and neglecting QCD corrections) gives[5]

$$\frac{2M_{B\bar{B}}}{m_B} \approx -\frac{\alpha_2^2}{12}\frac{f_B^2}{m_W^2} S_0(x_t)(V_{tb}V_{td}^*)^2, \tag{143}$$

[5] A detailed derivation can be found in Appendix B of [16].

where $x_i = m_i^2/m_W^2$ and

$$S_0(x) = \frac{x}{(1-x)^2}\left[1 - \frac{11x}{4} + \frac{x^2}{4} - \frac{3x^2 \ln x}{2(1-x)}\right]. \tag{144}$$

Similar spurion analyses, or explicit calculations, allow us to extract the weak and flavor suppression factors that apply in the SM:

$$\begin{aligned}
\mathcal{I}m(z_{sd}^{\rm SM}) &\sim \alpha_2^2 y_t^2 |V_{td}V_{ts}|^2 \sim 1\times 10^{-10}, \\
z_{sd}^{\rm SM} &\sim \alpha_2^2 y_c^2 |V_{cd}V_{cs}|^2 \sim 5\times 10^{-9}, \\
\mathcal{I}m(z_{cu}^{\rm SM}) &\sim \alpha_2^2 y_b^2 |V_{ub}V_{cb}|^2 \sim 2\times 10^{-14}, \\
z_{bd}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{td}V_{tb}|^2 \sim 7\times 10^{-8}, \\
z_{bs}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{ts}V_{tb}|^2 \sim 2\times 10^{-6}.
\end{aligned} \tag{145}$$

(We did not include $z_{cu}^{\rm SM}$ in the list because it requires a more detailed consideration. The naively leading short distance contribution is $\propto \alpha_2^2 (y_s^4/y_c^2)|V_{cs}V_{us}|^2 \sim 5\times 10^{-13}$. However, higher dimension terms can replace a $y_s^2$ factor with $(\Lambda/m_D)^2$ [17]. Moreover, long distance contributions are expected to dominate. In particular, peculiar phase space effects [18, 19] have been identified which are expected to enhance $\Delta m_D$ to within an order of magnitude of the its measured value. The CP violating part, on the other hand, is dominated by short distance physics.)

It is clear then that contributions from new physics at $\Lambda_{\rm NP} \sim 1$ TeV should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

### B. Lessons for Supersymmetry from neutral meson mixing

We consider, as an example, the contributions from the box diagrams involving the squark doublets of the second and third generations, $\tilde{Q}_{L2,3}$, to the $B_s - \overline{B_s}$ mixing amplitude. The contributions are proportional to $K_{3i}^{d*}K_{2i}^{d}K_{3j}^{d*}K_{2j}^{d}$, where $K^d$ is the mixing matrix of the gluino couplings to a left-handed down quark and their supersymmetric squark partners ($\propto [(\delta^d_{LL})_{23}]^2$ in the mass insertion approximation, described in Appendix C 1). We work in the mass basis for both quarks and squarks. A detailed derivation [20] is given in Appendix C 2. It gives:

$$M_{B_s\overline{B_s}} = \frac{\alpha_s^2 m_{B_s} f_{B_s}^2 B_{B_s}\eta_{\rm QCD}}{108 m_{\tilde{d}}^2}[11\tilde{f}_6(x)+4x f_6(x)]\frac{(\Delta\tilde{m}^2_{\tilde{d}})^2}{\tilde{m}^4_{\tilde{d}}}(K^{d*}_{32}K^d_{22})^2. \tag{146}$$

Here $m_{\tilde{d}}$ is the average mass of the two squark generations, $\Delta m^2_{\tilde{d}}$ is the mass-squared difference, and $x = m^2_{\tilde{g}}/m^2_{\tilde{d}}$.

Eq. (146) can be translated into our generic language:

$$\begin{aligned}\Lambda_{\rm NP} &= m_{\tilde{q}}, \\ z_1^{bs} &= \frac{11\tilde{f}_6(x)+4xf_6(x)}{18}\alpha_s^2\left(\frac{\Delta\tilde{m}^2_{\tilde{d}}}{m^2_{\tilde{d}}}\right)^2(K^{d*}_{32}K^d_{22})^2 \approx 10^{-4}(\delta^{LL}_{23})^2,\end{aligned} \tag{147}$$

where, for the last approximation, we took the example of $x = 1$ [and used, correspondingly, $11\tilde{f}_6(1) + 4f_6(1) = 1/6$], and defined

$$\delta^{LL}_{23} = \left(\frac{\Delta\tilde{m}^2_{\tilde{d}}}{m^2_{\tilde{d}}}\right)(K^{d*}_{32}K^d_{22}). \tag{148}$$

Similar expressions can be derived for the dependence of $K^0 - \overline{K^0}$ on $(\delta^d_{MN})_{12}$, $B^0 - \overline{B^0}$ on $(\delta^d_{MN})_{13}$, and $D^0 - \overline{D^0}$ on $(\delta^u_{MN})_{12}$. Then we can use the constraints of Table IV to put upper bounds on $(\delta^q_{MN})_{ij}$. Some examples are given in Table V (see Ref. [21] for details and list of references).

We learn that, in most cases, we need $\delta^q_{ij}/m_{\tilde{q}} \ll 1/\text{TeV}$. One can immediately identify three generic ways in which supersymmetric contributions to neutral meson mixing can be suppressed:

1. Heaviness: $m_{\tilde{q}} \gg 1\ TeV$;
2. Degeneracy: $\Delta m^2_{\tilde{q}} \ll m^2_{\tilde{q}}$;
3. Alignment: $K^q_{ij} \ll 1$.

When heaviness is the only suppression mechanism, as in split supersymmetry [22], the squarks are very heavy and supersymmetry no longer solves the fine tuning problem.[6] If we

[6] When the first two squark generations are mildly heavy and the third generation is light, as in effective supersymmetry [23], the fine tuning problem is still solved, but additional suppression mechanisms are needed.

TABLE V: The phenomenological upper bounds on $(\delta^q_{LL})_{ij}$ and $\langle\delta^q_{ij}\rangle = \sqrt{(\delta^q_{LL})_{ij}(\delta^q_{RR})_{ij}}$. Here $q = u, d$ and $M = L, R$. The constraints are given for $m_{\tilde{q}} = 1$ TeV and $x = m^2_{\tilde{g}}/m^2_{\tilde{q}} = 1$. We assume that the phases could suppress the imaginary part by a factor of $\sim 0.3$. Taken from Ref. [21].

| $q$ $ij$ | $(\delta^q_{LL})_{ij}$ | $\langle\delta^q_{ij}\rangle$ |
|---|---|---|
| $d$ 12 | 0.03 | 0.002 |
| $d$ 13 | 0.2 | 0.07 |
| $d$ 23 | 0.2 | 0.07 |
| $u$ 12 | 0.1 | 0.008 |

want to maintain supersymmetry as a solution to the fine tuning problem, either degeneracy or alignment or a combination of both is needed. This means that the flavor structure of supersymmetry is not generic, as argued in the previous section.

Take, for example, $(\delta^d_{LL})_{12} \leq 0.03$. Naively, one might expect the alignment to be of order $(V_{cd}V^*_{cs}) \sim 0.2$, which is far from sufficient by itself. Barring a very precise alignment ($|K^d_{12}| \ll |V_{us}|$) and accidental cancelations, we are led to conclude that the first two squark generations must be quasi-degenerate. Actually, by combining the constraints from $K^0-\overline{K^0}$ mixing and $D^0-\overline{D^0}$ mixing, one can show that this is the case independently of assumptions about the alignment [24–26]. Analogous conclusions can be drawn for many TeV-scale new physics scenarios: a strong level of degeneracy is required (for definitions and detailed analysis, see [27]).

**Exercise 9:** *Does $K^d_{31} \sim |V_{ub}|$ suffice to satisfy the $\Delta m_B$ constraint with neither degeneracy nor heaviness? (Use the two generation approximation and ignore the second generation.)*

Is there a natural way to make the squarks degenerate? Degeneracy requires that the $3 \times 3$ matrix of soft supersymmetry breaking mass-squared terms $\tilde{m}^2_{Q_L} \simeq \tilde{m}^2_{\tilde{q}}\mathbf{1}$. We have mentioned already that flavor universality is a generic feature of gauge interactions. Thus, the requirement of degeneracy is perhaps a hint that supersymmetry breaking is *gauge mediated* to the MSSM fields.

### C. Minimal flavor violation (MFV)

If supersymmetry breaking is gauge mediated, the squark mass matrices for $SU(2)_L$-doublet and $SU(2)_L$-singlet squarks have the following form at the scale of mediation $m_M$:

$$\begin{aligned}
\tilde{M}^2_{U_L}(m_M) &= \left(m^2_{\tilde{Q}_L} + D_{U_L}\right)\mathbf{1} + M_u M_u^\dagger, \\
\tilde{M}^2_{D_L}(m_M) &= \left(m^2_{\tilde{Q}_L} + D_{D_L}\right)\mathbf{1} + M_d M_d^\dagger, \\
\tilde{M}^2_{U_R}(m_M) &= \left(m^2_{\tilde{U}_R} + D_{U_R}\right)\mathbf{1} + M_u^\dagger M_u, \\
\tilde{M}^2_{D_R}(m_M) &= \left(m^2_{\tilde{D}_R} + D_{D_R}\right)\mathbf{1} + M_d^\dagger M_d,
\end{aligned} \tag{149}$$

where $D_{q_A} = [(T_3)_{q_A} - (Q_{\mathrm{EM}})_{q_A} s_W^2] m_Z^2 \cos 2\beta$ are the $D$-term contributions. Here, the only source of the $SU(3)^3_q$ breaking are the SM Yukawa matrices.

This statement holds also when the renormalization group evolution is applied to find the form of these matrices at the weak scale. Taking the scale of the soft breaking terms $m_{\tilde{q}_A}$ to be somewhat higher than the electroweak breaking scale $m_Z$ allows us to neglect the $D_{q_A}$ and $M_q$ terms in (149). Then we obtain

$$\begin{aligned}
\tilde{M}^2_{Q_L}(m_Z) &\sim m^2_{\tilde{Q}_L}\left(r_3\mathbf{1} + c_u Y^u Y^{u\dagger} + c_d Y^d Y^{d\dagger}\right), \\
\tilde{M}^2_{U_R}(m_Z) &\sim m^2_{\tilde{U}_R}\left(r_3\mathbf{1} + c_{uR} Y^{u\dagger} Y^u\right), \\
\tilde{M}^2_{D_R}(m_Z) &\sim m^2_{\tilde{D}_R}\left(r_3\mathbf{1} + c_{dR} Y^{d\dagger} Y^d\right).
\end{aligned} \tag{150}$$

Here $r_3$ represents the universal RGE contribution that is proportional to the gluino mass ($r_3 = \mathcal{O}(6) \times (M_3(m_M)/m_{\tilde{q}}(m_M))$) and the $c$-coefficients depend logarithmically on $m_M/m_Z$ and can be of $\mathcal{O}(1)$ when $m_M$ is not far below the GUT scale.

Models of gauge mediated supersymmetry breaking (GMSB) provide a concrete example of a large class of models that obey a simple principle called *minimal flavor violation* (MFV) [28]. This principle guarantees that low energy flavor changing processes deviate only very little from the SM predictions. The basic idea can be described as follows. The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices, $Y^u$, $Y^d$ and $Y^e$. If this remains true in the presence of the new physics, namely $Y^u$, $Y^d$ and $Y^e$ are the only flavor non-universal parameters, then the model belongs to the MFV class.

Let us now formulate this principle in a more formal way, using the language of spurions that we presented in section V. The Standard Model with vanishing Yukawa couplings has a large global symmetry (77,78). In this section we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks is $SU(3)_q^3$ of Eq. (78) with the three generations of quark fields transforming as follows:

$$Q_L(3,1,1), \quad U_R(1,3,1), \quad D_R(1,1,3). \tag{151}$$

The Yukawa interactions,

$$\mathcal{L}_{\rm Yuk}^q = \overline{Q_L} Y^d D_R H + \overline{Q_L} Y^u U_R H_c, \tag{152}$$

($H_c = i\tau_2 H^*$) break this symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under $SU(3)_q^3$ [see Eq. (82)]:

$$Y^u \sim (3, \bar{3}, 1), \qquad Y^d \sim (3, 1, \bar{3}). \tag{153}$$

When we say "spurions", we mean that we pretend that the Yukawa matrices are fields which transform under the flavor symmetry, and then require that all the Lagrangian terms, constructed from the SM fields, $Y^d$ and $Y^u$, must be (formally) invariant under the flavor group $SU(3)_q^3$. Of course, in reality, $\mathcal{L}_{\rm Yuk}^q$ breaks $SU(3)_q^3$ precisely because $Y^{d,u}$ are *not* fields and do not transform under the symmetry.

The idea of minimal flavor violation is relevant to extensions of the SM, and can be applied in two ways:

1. If we consider the SM as a low energy effective theory, then all higher-dimension operators, constructed from SM-fields and $Y$-spurions, are formally invariant under $G_{\rm global}$.

2. If we consider a full high-energy theory that extends the SM, then all operators, constructed from SM and the new fields, and from $Y$-spurions, are formally invariant under $G_{\rm global}$.

**Exercise 10:** *Use the spurion formalism to argue that, in MFV models, the $K_L \to \pi^0 \nu\bar{\nu}$ decay amplitude is proportional to $y_t^2 V_{td} V_{ts}^*$.*

**Exercise 11:** *Find the flavor suppression factors in the $z_i^{bs}$ coefficients, if MFV is imposed, and compare to the bounds in Table IV.*

Examples of MFV models include models of supersymmetry with gauge-mediation or with anomaly-mediation of its breaking.

## XII. THE STANDARD MODEL FLAVOR PUZZLE

The SM has thirteen flavor parameters: six quark Yukawa couplings, four CKM parameters (three angles and a phase), and three charged lepton Yukawa couplings. (One can use fermions masses instead of the fermion Yukawa couplings, $y_f = \sqrt{2}m_f/v$.) The orders of magnitudes of these thirteen dimensionless parameters are as follows:

$$\begin{aligned}
y_t &\sim 1, \quad y_c \sim 10^{-2}, \quad y_u \sim 10^{-5}, \\
y_b &\sim 10^{-2}, \quad y_s \sim 10^{-3}, \quad y_d \sim 10^{-4}, \\
y_\tau &\sim 10^{-2}, \quad y_\mu \sim 10^{-3}, \quad y_e \sim 10^{-6}, \\
|V_{us}| &\sim 0.2, \quad |V_{cb}| \sim 0.04, \quad |V_{ub}| \sim 0.004, \quad \delta_{\rm KM} \sim 1.
\end{aligned} \tag{154}$$

Only two of these parameters are clearly of $\mathcal{O}(1)$, the top-Yukawa and the KM phase. The other flavor parameters exhibit smallness and hierarchy. Their values span six orders of magnitude. It may be that this set of numerical values are just accidental. More likely, the smallness and the hierarchy have a reason. The question of why there is smallness and hierarchy in the SM flavor parameters constitutes "The Standard Model flavor puzzle."

The motivation to think that there is indeed a structure in the flavor parameters is strengthened by considering the values of the four SM parameters that are not flavor parameters, namely the three gauge couplings and the Higgs self-coupling:

$$g_s \sim 1, \quad g \sim 0.6, \quad e \sim 0.3, \quad \lambda \sim 0.12. \tag{155}$$

This set of values does seem to be a random distribution of order-one numbers, as one would naively expect.

A few examples of mechanisms that were proposed to explain the observed structure of the flavor parameters are the following:

- An approximate Abelian symmetry ("The Froggatt-Nielsen mechanism" [29]);
- An approximate non-Abelian symmetry (see *e.g.* [30]);
- Conformal dynamics ("The Nelson-Strassler mechanism" [31]);
- Location in an extra dimension [32];
- Loop corrections (see *e.g.* [33]).

We take as an example the Froggatt-Nielsen mechanism.

## A. The Froggatt-Nielsen (FN) mechanism

Small numbers and hierarchies are often explained by approximate symmetries. For example, the small mass splitting between the charged and neural pions finds an explanation in the approximate isospin (global $SU(2)$) symmetry of the strong interactions.

Approximate symmetries lead to selection rules which account for the size of deviations from the symmetry limit. Spurion analysis is particularly convenient to derive such selection rules. The Froggatt-Nielsen mechanism postulates a $U(1)_H$ symmetry, that is broken by a small spurion $\epsilon_H$. Without loss of generality, we assign $\epsilon_H$ a $U(1)_H$ charge of $H(\epsilon_H) = -1$. Each SM field is assigned a $U(1)_H$ charge. In general, different fermion generations are assigned different charges, hence the term 'horizontal symmetry.' The rule is that each term in the Lagrangian, made of SM fields and the spurion, should be formally invariant under $U(1)_H$.

The approximate $U(1)_H$ symmetry thus leads to the following selection rules:

$$\begin{aligned}
Y^u_{ij} &\sim \epsilon_H^{|H(\bar{Q}_i)+H(U_j)+H(\phi_u)|}, \\
Y^d_{ij} &\sim \epsilon_H^{|H(\bar{Q}_i)+H(D_j)+H(\phi_d)|}, \\
Y^e_{ij} &\sim \epsilon_H^{|H(\bar{L}_i)+H(E_j)-H(\phi_d)|}.
\end{aligned} \tag{156}$$

As a concrete example, we take the following set of charges:

$$\begin{aligned}
H(\bar{Q}_i) &= H(U_i) = H(E_i) = (2,1,0), \\
H(\bar{L}_i) &= H(D_i) = (0,0,0), \\
H(\phi_u) &= H(\phi_d) = 0.
\end{aligned} \tag{157}$$

It leads to the following parametric suppressions of the Yukawa couplings:

$$Y^u \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \quad Y^d \sim (Y^e)^T \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix}. \tag{158}$$

We emphasize that for each entry we give the parametric suppression (that is the power of $\epsilon$), but each entry has an unknown (complex) coefficient of order one, and there are no relations between the order one coefficients of different entries.

The structure of the Yukawa matrices dictates the parametric suppression of the physical observables:

$$y_t \sim 1, \quad y_c \sim \epsilon^2, \quad y_u \sim \epsilon^4,$$

$$
\begin{aligned}
y_b &\sim 1, \quad y_s \sim \epsilon, \quad y_d \sim \epsilon^2, \\
y_\tau &\sim 1, \quad y_\mu \sim \epsilon, \quad y_e \sim \epsilon^2, \\
|V_{us}| &\sim \epsilon, \quad |V_{cb}| \sim \epsilon, \quad |V_{ub}| \sim \epsilon^2, \quad \delta_{\rm KM} \sim 1.
\end{aligned} \tag{159}
$$

For $\epsilon \sim 0.05$, the parametric suppressions are roughly consistent with the observed hierarchy. In particular, this set of charges predicts that the down and charged lepton mass hierarchies are similar, while the up hierarchy is the square of the down hierarchy. These features are roughly realized in Nature.

**Exercise 13:** *Derive the parametric suppression and approximate numerical values of $Y^u$, its eigenvalues, and the three angles of $V_L^u$, for $H(Q_i) = 4,2,0$, $H(U_i) = 3,2,0$ and $\epsilon_H = 0.2$*

Could we explain any set of observed values with such an approximate symmetry? If we could, then the FN mechanism cannot be really tested. The answer however is negative. Consider, for example, the quark sector. Naively, we have 11 $U(1)_H$ charges that we are free to choose. However, the $U(1)_Y \times U(1)_B \times U(1)_{\rm PQ}$ symmetry implies that there are only 8 independent choices that affect the structure of the Yukawa couplings. On the other hand, there are 9 physical parameters. Thus, there should be a single relation between the physical parameters that is independent of the choice of charges. Assuming that the sum of charges in the exponents of Eq. (156) is of the same sign for all 18 combinations, the relation is

$$
|V_{ub}| \sim |V_{us} V_{cb}|, \tag{160}
$$

which is fulfilled to within a factor of 2. There are also interesting inequalities (here $i < j$):

$$
|V_{ij}| \gtrsim m(U_i)/m(U_j), \ m(D_i)/m(D_j). \tag{161}
$$

All six inequalities are fulfilled. Finally, if we order the up and the down masses from light to heavy, then the CKM matrix is predicted to be $\sim \mathbf{1}$, namely the diagonal entries are not parametrically suppressed. This structure is also consistent with the observed CKM structure.

### B. The flavor of neutrinos

Five neutrino flavor parameters have been measured in recent years (see *e.g.* [34, 35]): two mass-squared differences,

$$\Delta m^2_{21} = (7.5 \pm 0.2) \times 10^{-5}\ \text{eV}^2, \quad |\Delta m^2_{32}| = (2.5 \pm 0.1) \times 10^{-3}\ \text{eV}^2, \tag{162}$$

and the three mixing angles,

$$|U_{e2}| = 0.55 \pm 0.01, \quad |U_{\mu 3}| = 0.67 \pm 0.03, \quad |U_{e3}| = 0.148 \pm 0.003. \tag{163}$$

These parameters constitute a significant addition to the thirteen SM flavor parameters and provide, in principle, tests of various ideas to explain the SM flavor puzzle.

The numerical values of the parameters show various surprising features:

- $|U_{\mu 3}| >$ any $|V_{ij}|$;
- $|U_{e2}| >$ any $|V_{ij}|$;
- $|U_{e3}|$ is not particularly small ($|U_{e3}| \not\ll |U_{e2}U_{\mu 3}|$);
- $m_2/m_3 \gtrsim 1/6 >$ any $m_i/m_j$ for charged fermions.

These features can be summarized by the statement that, in contrast to the charged fermions, neither smallness nor hierarchy have been observed so far in the neutrino related parameters.

One way of interpretation of the neutrino data comes under the name of neutrino mass anarchy [36–38]. It postulates that the neutrino mass matrix has no special structure, namely all entries are of the same order of magnitude. Normalized to an effective neutrino mass scale, $v^2/\Lambda_{\rm seesaw}$, the various entries are random numbers of order one. Note that anarchy means neither hierarchy nor degeneracy.

If true, the contrast between neutrino mass anarchy and quark and charged lepton mass hierarchy may be a deep hint for a difference between the flavor physics of Majorana and Dirac fermions. The source of both anarchy and hierarchy might, however, be explained by a much more mundane mechanism. In particular, neutrino mass anarchy could be a result of a FN mechanism, where the three left-handed lepton doublets carry the same FN charge. In that case, the FN mechanism predict parametric suppression of neither neutrino mass ratios nor leptonic mixing angles, which is quite consistent with (162) and (163). Indeed, the viable FN model presented in Section XII A belongs to this class.

Another possible interpretation of the neutrino data is to take $m_2/m_3 \sim |U_{e3}| \sim 0.15$ to be small, and require that they are parametrically suppressed (while the other two mixing angles are order one). Such a situation is impossible to accommodate in a large class of FN models [39].

The same data, and in particular the proximity of $(|U_{\mu 3}|, |U_{\tau 3}|)$ to $(1/\sqrt{2}, 1/\sqrt{2})$, and the proximity of $|U_{e2}|$ to $1/\sqrt{3} \simeq 0.58$, led to a very different interpretation. This interpretation, termed 'tribimaximal mixing' (TBM), postulates that the leptonic mixing matrix is parametrically close to the following special form [40]:

$$|U|_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{164}$$

Such a form is suggestive of discrete non-Abelian symmetries, and indeed numerous models based on an $A_4$ symmetry have been proposed [41, 42]. A significant feature of of TBM is that the third mixing angle should be close to $|U_{e3}| = 0$. Until 2012, there have been only upper bounds on $|U_{e3}|$, consistent with the models in the literature. In recent years, however, a value of $|U_{e3}|$ close to the previous upper bound has been established [43], see Eq. (163). Such a large value (and the consequent significant deviation of $|U_{\mu 3}|$ from maximal bimixing) puts in serious doubt the TBM idea. Indeed, it is difficult in this framework, if not impossible, to account for $\Delta m^2_{12}/\Delta m^2_{23} \sim |U_{e3}|^2$ without fine-tuning [44].

## XIII. HIGGS PHYSICS: THE NEW FLAVOR ARENA

The SM relates the Yukawa couplings to the corresponding mass matrices:

$$Y^f = \sqrt{2} M_f / v. \tag{165}$$

This simple equation implies three features:

1. *Proportionality:* $y_i \equiv Y^f_{ii} \propto m_i$;

2. *Factor* of proportionality: $y_i/m_i = \sqrt{2}/v$;

3. *Diagonality:* $Y^f_{ij} = 0$ for $i \neq j$.

In extensions of the SM, each of these three features might be violated. Thus, testing these features might provide a window to new physics and to allow progress in understanding the flavor puzzles.

A Higgs-like boson $h$ has been discovered by the ATLAS and CMS experiments at the LHC [45, 46]. The experiments normalize their results to the SM rates:

$$\mu_f \equiv \frac{\sigma(pp \to h)\mathrm{BR}(h \to f)}{[\sigma(pp \to h)\mathrm{BR}(h \to f)]^{\mathrm{SM}}}. \tag{166}$$

The measurements give [47]:

$$\begin{aligned}\mu_{\gamma\gamma} &= 1.14 \pm 0.14,\\ \mu_{ZZ^*} &= 1.17 \pm 0.23,\\ \mu_{WW^*} &= 0.99 \pm 0.15,\\ \mu_{b\bar{b}} &= 0.7 \pm 0.3,\\ \mu_{\tau\tau} &= 1.09 \pm 0.23.\end{aligned} \tag{167}$$

In addition, there are upper bounds on decays into the first two generation charged leptons [48, 49]:

$$\begin{aligned}\mu_{\mu\mu} &< 2.8,\\ \mu_{ee} &< 4 \times 10^5.\end{aligned} \tag{168}$$

As concerns quark flavor changing Higgs couplings, these have been searched for in $t \to qh$ decays ($q = c, u$), [50, 51]:

$$\begin{aligned}\mathrm{BR}(t \to ch) &< 4.0 \times 10^{-3},\\ \mathrm{BR}(t \to uh) &< 4.5 \times 10^{-3}.\end{aligned} \tag{169}$$

The first direct searches for the lepton-flavor violating (LFV) Higgs decays were carried out by the CMS and ATLAS collaborations [52–54] yielding the upper bounds:

$$\begin{aligned}\mathrm{BR}(h \to \tau\mu) &< 2.5 \times 10^{-3},\\ \mathrm{BR}(h \to \tau e) &< 6.1 \times 10^{-3},\\ \mathrm{BR}(h \to \mu e) &< 3.4 \times 10^{-4}.\end{aligned} \tag{170}$$

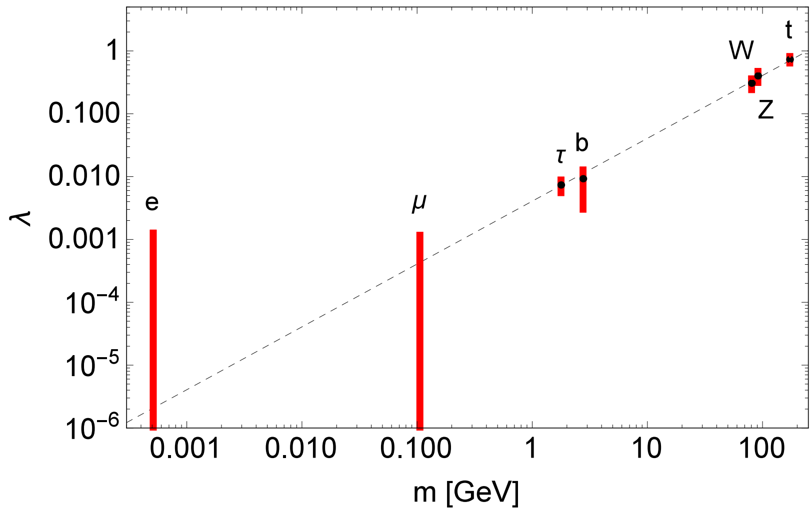



FIG. 5: The allowed ranges for the Higgs couplings. The SM prediction is presented by the dashed line. (Aielet Efrati, private communication.)

and the ranges:

$$\begin{aligned} \mathrm{BR}(h \to \tau\mu) &= (0.0 \pm 1.2) \times 10^{-3} \quad \text{(CMS)}, \\ \mathrm{BR}(h \to \tau\mu) &= (5.3 \pm 5.1) \times 10^{-3} \quad \text{(ATLAS)}. \end{aligned} \tag{171}$$

The measurements quoted in Eqs. (167) and (168) can be presented in the $y_i - m_i$ plane. We do so in Fig. 5. The first two features quoted above are already being tested. The upper bounds on flavor violating decays quoted in Eqs. (169) and (170) test the third feature. We can make the following statements:

- $y_e, y_\mu < y_\tau$. This goes in the direction of proportionality.
- The third generation Yukawa couplings, $y_t, y_b, y_\tau$, obey $y_3/m_3 \approx \sqrt{2}/v$. This is in agreement with the predicted factor of proportionality.
- There are strong upper bounds on violation of diagonality:

$$Y_{tq} \lesssim 0.1, \quad Y_{\tau\ell} \lesssim 0.002, \quad Y_{\mu e} \lesssim 0.0005. \tag{172}$$

- The era of Higgs flavor physics has begun.

Beyond the search for new physics via Higgs decays, it is interesting to ask whether the measurements of the Higgs couplings to quarks and leptons can shed light on the standard model and/or new physics flavor puzzles. If eventually the values of $y_b$ and/or $y_\tau$ deviate from their SM values, the most likely explanation of such deviations will be that there are more than one Higgs doublets, and that the doublet(s) that couple to the down and charged lepton sectors are not the same as the one that couples to the up sector. A more significant test of our understanding of flavor physics, which might provide a window into new flavor physics, will come further in the future, when $\mu_{\mu^+\mu^-}$ is measured. The ratio

$$X_{\mu^+\mu^-} \equiv \frac{\mathrm{BR}(h \to \mu^+\mu^-)}{\mathrm{BR}(h \to \tau^+\tau^-)}, \tag{173}$$

is predicted within the SM with impressive theoretical cleanliness. To leading order, it is given by $X_{\mu^+\mu^-} = m_\mu^2/m_\tau^2$, and the corrections of order $\alpha_W$ and of order $m_\mu^2/m_\tau^2$ to this leading result are known. It is an interesting question to understand what can be learned from a test of this relation [55]. In fact, as mentioned above, the bound (168) already shows that $X_{\mu^+\mu^-} < 1$ (in fact, $X_{\mu^+\mu^-} \lesssim 0.02$), namely that second generation Yukawa couplings are smaller than third generation ones. It is also interesting to test diagonality via the search for the SM-forbidden decay modes, $h \to \mu^\pm\tau^\mp$ [56–59]. A measurement of, or an upper bound on

$$X_{\mu\tau} \equiv \frac{\mathrm{BR}(h \to \mu^+\tau^-) + \mathrm{BR}(h \to \mu^-\tau^+)}{\mathrm{BR}(h \to \tau^+\tau^-)}, \tag{174}$$

would provide additional information relevant to flavor physics. We demonstrate below the potential power of Higgs flavor physics to lead to progress in our understanding of the flavor puzzles by focussing on the measurements of $\mu_{\tau^+\tau^-}$, $X_{\mu^+\mu^-}$ and $X_{\mu\tau}$ [55].

Let us take as an example how we can use the set of these three measurements if there is a single light Higgs boson. A violation of the SM relation $Y_{ij}^{\rm SM} = \frac{\sqrt{2}m_i}{v}\delta_{ij}$, is a consequence of nonrenormalizable terms. The leading ones are the $d = 6$ terms. In the interaction basis, we have

$$\begin{aligned} \mathcal{L}_Y^{d=4} &= -\lambda_{ij}\bar{f}_L^i f_R^j \phi + \text{h.c.}, \\ \mathcal{L}_Y^{d=6} &= -\frac{\lambda'_{ij}}{\Lambda^2}\bar{f}_L^i f_R^j \phi(\phi^\dagger\phi) + \text{h.c.}\,, \end{aligned} \tag{175}$$

where expanding around the vacuum we have $\phi = (v + h)/\sqrt{2}$. Defining $V_{L,R}$ via

$$\sqrt{2}m = V_L\left(\lambda + \frac{v^2}{2\Lambda^2}\lambda'\right)V_R^\dagger v, \tag{176}$$

where $m = \mathrm{diag}(m_e, m_\mu, m_\tau)$, and defining $\hat{\lambda}$ via

$$\hat{\lambda} = V_L\lambda' V_R^\dagger, \tag{177}$$

we obtain

$$Y_{ij} = \frac{\sqrt{2}m_i}{v}\delta_{ij} + \frac{v^2}{\Lambda^2}\hat{\lambda}_{ij}. \tag{178}$$

To proceed, one has to make assumptions about the structure of $\hat{\lambda}$. In what follows, we consider first the assumption of minimal flavor violation (MFV) and then a Froggatt-Nielsen (FN) symmetry.

**Exercise 14:** *Find the predictions of models with Natural Flavor Conservation (NFC) for* $\mu_{\tau^+\tau^-}$, $X_{\mu^+\mu^-}$ *and* $X_{\tau\mu}$.

### A. MFV

MFV requires that the leptonic part of the Lagrangian is invariant under an $SU(3)_L \times SU(3)_E$ global symmetry, with the left-handed lepton doublets transforming as $(3,1)$, the right-handed charged lepton singlets transforming as $(1,3)$ and the charged lepton Yukawa matrix $Y$ is a spurion transforming as $(3,\bar{3})$.

Specifically, MFV means that, in Eq. (175),

$$\lambda' = a\lambda + b\lambda\lambda^\dagger\lambda + \mathcal{O}(\lambda^5), \tag{179}$$

where $a$ and $b$ are numbers. Note that, if $V_L$ and $V_R$ are the diagonalizing matrices for $\lambda$, $V_L\lambda V_R^\dagger = \lambda^{\rm diag}$, then they are also the diagonalizing matrices for $\lambda\lambda^\dagger\lambda$, $V_L\lambda\lambda^\dagger\lambda V_R^\dagger = (\lambda^{\rm diag})^3$. Then, Eqs. (176), (177) and (178) become

$$\begin{aligned}
\frac{\sqrt{2}m}{v} &= \left(1+\frac{av^2}{2\Lambda^2}\right)\lambda^{\rm diag} + \frac{bv^2}{2\Lambda^2}(\lambda^{\rm diag})^3, \\
\hat{\lambda} &= a\lambda^{\rm diag} + b(\lambda^{\rm diag})^3 = a\frac{\sqrt{2}m}{v} + \frac{2\sqrt{2}bm^3}{v^3}, \\
Y_{ij} &= \frac{\sqrt{2}m_i}{v}\delta_{ij}\left[1+\frac{av^2}{\Lambda^2}+\frac{2bm_i^2}{\Lambda^2}\right],
\end{aligned} \tag{180}$$

where, in the expressions for $\hat{\lambda}$ and $Y$, we included only the leading universal and leading non-universal corrections to the SM relations.

We learn the following points about the Higgs-related lepton flavor parameters in this class of models:

1. $h$ has no flavor off-diagonal couplings:

$$Y_{\mu\tau}, Y_{\tau\mu} = 0. \tag{181}$$

2. The values of the diagonal couplings deviate from their SM values. The deviation is small, of order $v^2/\Lambda^2$:

$$y_\tau \approx \left(1+\frac{av^2}{\Lambda^2}\right)\ \frac{\sqrt{2}m_\tau}{v}. \tag{182}$$

3. The ratio between the Yukawa couplings to different charged lepton flavors deviates from its SM value. The deviation is, however, very small, of order $m_\ell^2/\Lambda^2$:

$$\frac{y_\mu}{y_\tau} = \frac{m_\mu}{m_\tau}\left(1-\frac{2b(m_\tau^2-m_\mu^2)}{\Lambda^2}\right). \tag{183}$$

The predictions of the SM with MFV non-renormalizable terms are then the following:

$$\begin{aligned}
\mu_{\tau^+\tau^-} &= 1+2av^2/\Lambda^2,\\
X_{\mu^+\mu^-} &= (m_\mu/m_\tau)^2(1-4bm_\tau^2/\Lambda^2),\\
X_{\tau\mu} &= 0.
\end{aligned} \tag{184}$$

Thus, MFV will be excluded if experiments observe the $h\to\mu\tau$ decay. On the other hand, MFV allows for a universal deviation of $\mathcal{O}(v^2/\Lambda^2)$ of the flavor-diagonal dilepton rates, and a smaller non-universal deviation of $\mathcal{O}(m_\tau^2/\Lambda^2)$.

## B. FN

The FN mechanism, with $H(\phi)=0$, dictates the following parametric suppression for the leptonic Yukawa matrix:

$$\lambda_{ij} \sim \epsilon_H^{H(E_j)-H(L_i)}. \tag{185}$$

We emphasize that the FN mechanism dictates only the parametric suppression. Each entry has an arbitrary order one coefficient. The resulting parametric suppression of the masses and leptonic mixing angles is given by [60]

$$m_{\ell_i}/v \sim \epsilon_H^{H(E_i)-H(L_i)}, \quad |U_{ij}| \sim \epsilon_H^{H(L_j)-H(L_i)}. \tag{186}$$

Since $H(\phi^\dagger\phi)=0$, the entries of the matrix $\lambda'$ have the same parametric suppression as the corresponding entries in $\lambda$ [61], though the order one coefficients are different:

$$\lambda'_{ij} = \mathcal{O}(1)\times\lambda_{ij}. \tag{187}$$

This structure allows us to estimate the entries of $\hat{\lambda}_{ij}$ in terms of physical observables:

$$\begin{aligned}
\hat{\lambda}_{33} &\sim m_\tau/v, \\
\hat{\lambda}_{22} &\sim m_\mu/v, \\
\hat{\lambda}_{23} &\sim |U_{23}|(m_\tau/v), \\
\hat{\lambda}_{32} &\sim (m_\mu/v)/|U_{23}|.
\end{aligned} \tag{188}$$

We learn the following points about the Higgs-related lepton flavor parameters in this class of models:

1. $h$ has flavor off-diagonal couplings:

$$\begin{aligned}
Y_{\mu\tau} &= \mathcal{O}\left(\frac{|U_{23}|vm_\tau}{\Lambda^2}\right), \\
Y_{\tau\mu} &= \mathcal{O}\left(\frac{vm_\mu}{|U_{23}|\Lambda^2}\right).
\end{aligned} \tag{189}$$

2. The values of the diagonal couplings deviate from their SM values:

$$y_\tau \approx \frac{\sqrt{2}m_\tau}{v}\left[1+\mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)\right]. \tag{190}$$

3. The ratio between the Yukawa couplings to different charged lepton flavors deviates from its SM value:

$$\frac{y_\mu}{y_\tau} = \frac{m_\mu}{m_\tau}\left[1+\mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)\right]. \tag{191}$$

The predictions of the SM with FN-suppressed non-renormalizable terms are then the following:

$$\begin{aligned}
\mu_{\tau^+\tau^-} &= 1+\mathcal{O}(v^2/\Lambda^2), \\
X_{\mu^+\mu^-} &= (m_\mu/m_\tau)^2(1+\mathcal{O}(v^2/\Lambda^2)), \\
X_{\tau\mu} &= \mathcal{O}(v^4/\Lambda^4).
\end{aligned} \tag{192}$$

Thus, FN will be excluded if experiments observe deviations from the SM of the same size in both flavor-diagonal and flavor-changing $h$ decays. On the other hand, FN allows non-universal deviations of $\mathcal{O}(v^2/\Lambda^2)$ in the flavor-diagonal dilepton rates, and a smaller deviation of $\mathcal{O}(v^4/\Lambda^4)$ in the off-diagonal rate.

## XIV. NEW PHYSICS?

In this section we discuss two sets of recent measurements of flavor changing processes that arouse much interest: $h \to \tau\mu$ and $B \to D^{(*)}\tau\nu$.

### A. $h \to \tau\mu$

In this section we entertain the idea that $\mathrm{BR}(h \to \tau\mu) \sim 0.001$, that is, close to the present upper bound, will be established by the experiments, and explore the implications of such a hypothetical discovery.

The $h \to \tau\mu$ decay has several aspects that are worth emphasizing:

- It violates the lepton-flavor symmetry $U(1)_\mu \times U(1)_\tau$, which is an accidental symmetry of the SM.
- It is an FCNC process.
- It violates the prediction that $Y^e \propto M_e$, which applies at tree level in all models of NFC.

Due to these three aspects, an observation of $h \to \tau\mu$ at the permil level will have far-reaching implications. The accidental symmetry of the SM was presented in Eq. (80). This symmetry forbids the $h \to \tau\mu$ decay, since it violates muon number and tau number. Accidental symmetries are, however, broken by higher dimension terms, so that their breaking confirms that the SM is not a full theory of Nature but only a low energy effective theory.

In fact, violation of $U(1)_e \times U(1)_\mu \times U(1)_\tau$ has already been experimentally discovered by neutrino oscillation experiments (atmospheric, accelerator and reactor) and by the MSW effect (solar neutrinos). To explain these previous observations, one has to consider the dimension-five terms of Eq. (139):

$$\mathcal{L}_{d=5} = \frac{Z^\nu_{ij}}{\Lambda_{\rm LNV}} L_{Li} L_{Lj} \phi\phi + \mathrm{h.c.}. \tag{193}$$

These terms break lepton flavor, as well as total lepton number. The measurement of $|\Delta m^2_{32}| \sim 0.0025\ \mathrm{eV}^2$ implies a new physics scale,

$$\Lambda_{\rm LNV} \sim 10^{15}\ \mathrm{GeV}\ Z_{\rm max}, \tag{194}$$

where $Z_{\rm max}$ is the largest eigenvalue of the matrix $Z$. Since $U(1)_\mu \times U(1)_\tau$ is broken by (195), the $h \to \tau\mu$ is induced by these terms. However, the decay rate suffers from the FCNC suppression factors:

- Loop suppression $\sim \alpha_2^2$;
- Mixing suppression $\sim |U_{\mu 3}U_{\tau 3}|^2$;
- GIM suppression $\sim (\Delta m_{32}^2/m_W^2)^2$.

The GIM suppression is particularly effective: Given the smallness of $\Delta m_{32}^2/m_W^2$, the predicted branching ratio is unobservably tiny, ${\rm BR}(h \to \tau\mu) \sim 10^{-50}$.

Thus, to explain a branching ratio as high as 0.001, another source of lepton flavor violation has to be invoked. Indeed, it can be achieved by dimension-six terms:

$$\mathcal{L}_{d=6} = \frac{Z^e_{ij}}{\Lambda^2_{\rm LFV}}(\phi^\dagger\phi)\phi\overline{L_{Li}}E_{Rj} + {\rm h.c.}. \tag{195}$$

With these terms we have

$$\begin{aligned} M_e &= \frac{v}{\sqrt{2}}\left(Y^e + \frac{v^2}{2\Lambda^2_{\rm LFV}}Z^e\right), \\ Y_h^E &= Y^e + \frac{3v^2}{2\Lambda^2_{\rm LFV}}Z^e, \end{aligned} \tag{196}$$

so that

$$Y_h^E = (\sqrt{2}M_e/v) + \frac{v^2}{2\Lambda^2_{\rm LFV}}Z^e. \tag{197}$$

Branching ratio of order 0.001 is induced if

$$\Lambda_{\rm LFV} \sim 5 \text{ TeV } \sqrt{Z^e_{\mu\tau}}. \tag{198}$$

Thus, the new physics is likely to be directly accessible at the LHC.

A branching ratio of order 0.001 ($X_{\mu\tau} \sim 0.02$) would further imply that the decay rate is not many orders of magnitude smaller than the $h \to \tau\tau$ decay rate. The $h \to \tau\mu$ decay is, however, an FCNC process which, within the SM and many of its extensions (such as the minimal supersymmetric SM (MSSM)), is loop suppressed. With loop suppression, what is required is that

$$(v^2/\Lambda^2_{\rm LFV})(\alpha_W/4\pi)X_{\mu\tau} \not\ll y_\tau \sim 10^{-2}. \tag{199}$$

Here $X_{\mu\tau}$ is some flavor changing coupling. Note that the electroweak loop factor by itself provides a suppression of order $10^{-3}$. Thus, Eq. (199) is very challenging for model building. For example, it cannot be satisfied in the MSSM [62].

Thus, $\mathrm{BR}(h \to \tau\mu) \not\ll \mathrm{BR}(h \to \tau\tau)$ is suggestive that $h \to \tau\mu$ proceeds at tree level. We remind you that tree level flavor changing Higgs couplings arise if there are bare mass terms or several Higgs doublets. The two most relevant extensions of the SM are then vector-like leptons and multi-Higgs doublets. The former framework leads, generally, to unacceptably large $Z \to \tau\mu$ and $\tau \to \mu\gamma$ decay rates, leaving the two Higgs doublet model (2HDM) as the simplest model that can account for $\mathrm{BR}(h \to \tau\mu) \sim 0.001$.

For a 2HDM, we can use, without loss of generality, the basis $\{\phi_v, \phi_A\}$ where

$$\langle \phi_v^0 \rangle = v/\sqrt{2}, \quad \langle \phi_A^0 \rangle = 0. \tag{200}$$

The rotation angle from this basis to the basis of doublets that contain the light ($h$) and heavy ($H$) neutral CP-even scalar mass eigenstates is usually denoted by $\alpha - \beta$ (not to be confused with $\alpha$ and $\beta$ of the unitarity triangle). In particular,

$$h = s_{\alpha-\beta}\mathrm{Re}(\phi_v^0) + c_{\alpha-\beta}\mathrm{Re}(\phi_A^0), \tag{201}$$

leading to

$$Y_h^e = s_{\alpha-\beta}(\sqrt{2}M_e/v) + c_{\alpha-\beta}Y_A^e. \tag{202}$$

Note that $Y_A^e$ is an arbitrary, unknown Yukawa matrix. With

$$c_{\alpha-\beta}(Y_A^e)_{\mu\tau} \not\ll s_{\alpha-\beta}(\sqrt{2}m_\tau/v), \tag{203}$$

$\mathrm{BR}(h \to \tau\mu) \not\ll \mathrm{BR}(h \to \tau\tau)$ is generated. If one sets all other $(Y_A^f)_{ij} = 0$, no phenomenological problems arise. Thus, the 2HDM is the simplest framework to explain a hypothetical measurement of large rate of $h \to \tau\mu$. Yet, such a large rate is inconsistent with all motivated flavor models.

**Exercise 15:** *Estimate the ratio* $(|Y_{e\mu}^h|^2 + |Y_{\mu e}^h|^2)/(|Y_{\mu\tau}^h|^2 + |Y_{\tau\mu}^h|^2)$ *in the FN framework. The upper bound on* $\mathrm{BR}(\mu \to e\gamma)$ *requires* $\sqrt{|Y_{e\mu}^h|^2 + |Y_{\mu e}^h|^2} \leq 1.2 \times 10^{-6}$. *Estimate the corresponding upper bound on* $\mathrm{BR}(h \to \tau\mu)$ *in the FN framework.*

### B. $B \to D^{(*)}\tau\nu$

Babar [63, 64], Belle [65, 66] and LHCb [67] have measured the following ratios, which test lepton flavor universality:

$$R(D^{(*)}) \equiv \frac{\Gamma(B \to D^{(*)}\tau\nu)}{\Gamma(B \to D^{(*)}\ell\nu)}, \tag{204}$$

where $\ell = e, \mu$. The HFAG average of these measurements gives [11]

$$\begin{aligned} R(D^*) &= 0.310 \pm 0.017, \\ R(D) &= 0.403 \pm 0.047, \end{aligned} \tag{205}$$

to be compared with the SM predictions [68, 69]:

$$\begin{aligned} R(D^*)_{\rm SM} &= 0.252 \pm 0.003, \\ R(D)_{\rm SM} &= 0.300 \pm 0.008. \end{aligned} \tag{206}$$

The combined significance of these excesses is $4.0\sigma$. In this section we entertain the idea that a deviation from the SM will indeed be established. We follow mainly the analyses of Refs. [70, 71].

We assume that the new physics that affects the $b \to c\tau\nu$ transition takes place at an energy scale larger than the electroweak breaking scale, in which case it can be represented by higher dimensional operators. The most general $d = 6$ terms are [70]

$$\begin{aligned} \mathcal{L}_{\rm eff} &= c^{ijkl}_{QQLL}(\bar{Q}_i\gamma_\mu\sigma^a Q_j)(\bar{L}_k\gamma^\mu\sigma_a L_l) + c^{ijkl}_{QuLe}(\bar{Q}_i U_j)i\sigma^2(\bar{L}_k E_l) \\ &+ c^{ijkl}_{dQLe}(\bar{D}_i Q_j)(\bar{L}_k E_l) + c^{ijkl}_{dQLe'}(\bar{D}_i\sigma_{\mu\nu}Q_j)(\bar{L}_k\sigma^{\mu\nu}E_l) + \text{h.c.}. \end{aligned} \tag{207}$$

As concerns the quark and lepton doublets, we assume that only the third generation ones are involved, and we take them to be $Q_3 = (V_{ib}u_{iL}, b_L)^T$ and $L_3 = (\nu_\tau, \tau_L)^T$. (In this way we avoid strong constraints from *e.g.* $b \to s\nu\bar{\nu}$ and $b \to s\tau^+\tau^-$.) The various operators unavoidably contribute to other processes. We list some of these processes, and the corresponding flavor-related suppression factor, in Table VI.

Given that the deviation from the SM is large, of order 30%, and that within the SM the semileptonic decay $b \to c\tau\nu$ is a $W$-mediated tree-level decay, it is likely that the new physics contribution is also a tree-level one. The mediator can be a Lorentz scalar or vector, and a color singlet or triplet:

TABLE VI: Processes affected by operators that modify $B \to D^{(*)}\tau\nu$ and the flavor-related suppression factors of their contributions. In the last line we list the simplified models that are excluded by the various processes. (The $V_\mu$ model is strongly disfavored by the $R(D^{(*)}$ measurement but not excluded. The $\phi$ model is excluded by the $\tau_{B_c}$ constraint unless there is a fine-tuned cancelation between independent Yukawa couplings.)

| Operator | Models | $b \to c\tau\nu$ [73] | $\tau_{B_c}$ [72] | $t \to b\tau\nu$ | $t \to c\tau\tau$ | $b\bar{b}/\Upsilon(c\bar{c}/\psi) \to \tau\tau$ [71, 74] |
|---|---|---|---|---|---|---|
| $QQLL$ | $W'_\mu, X_\mu, S, T$ | $V_{cb}$ | $\sim 0$ | $V_{tb}$ | $V_{tb}V_{cb}^*$ | $1(\lvert V_{cb}\rvert^2)$ |
| $QULE$ | $S, \phi, D$ | 1 | 1 | 0 | $V_{tb}$ | $0(V_{cb})$ |
| $DQLE$ | $U_\mu, V_\mu, \phi$ | $V_{cb}$ | $\lvert V_{cb}\rvert$ | $V_{tb}$ | 0 | 1(0) |
| (Excluded) | | $X_\mu, T, D, (V_\mu)$ | $(\phi)$ | | | $W'_\mu$ |

- A vector $W'^a_\mu(1,3)_0$:

$$\mathcal{L}_W = g_1 \bar{Q}_3 \tau_a \not{W}^a Q_3 + g_2 \bar{L}_3 \tau_a \not{W}^a L_3. \tag{208}$$

Integrating out $W'_\mu$, we obtain the following EFT Lagrangian:

$$\mathcal{L}_W^{\rm EFT} = -\frac{g_1 g_2}{M_{W'}^2}\left(\bar{Q}_3 \tau^a \gamma_\mu Q_3\right)\left(\bar{L}_3 \tau_a \gamma^\mu L_3\right). \tag{209}$$

The relevant CC interactions are given by

$$\mathcal{L}_{\rm CC} = -\frac{g_1 g_2 V_{cb}}{2M_{W'}^2}\left(\bar{\tau}_L \gamma^\mu \nu_L\right)\left(\bar{c}_L \gamma_\mu b_L\right) + \text{h.c.}. \tag{210}$$

- A vector $U_\mu(3,1)_{+2/3}$:

$$\mathcal{L}_U = g_1 \bar{Q}_3 \not{U} L_3 + g_2 \bar{d}_3 \not{U} e_3 + \text{h.c.}. \tag{211}$$

Integrating out $U_\mu$, we obtain the following EFT Lagrangian:

$$\begin{aligned}\mathcal{L}_U^{\rm EFT} &= -\frac{|g_1|^2}{M_U^2}\left(\bar{Q}_3 \gamma_\mu L_3\right)\left(\bar{L}_3 \gamma^\mu Q_3\right) - \frac{|g_2|^2}{M_U^2}\left(\bar{e}_3 \gamma_\mu d_3\right)\left(\bar{d}_3 \gamma^\mu e_3\right) \\ &\quad - \left[\frac{g_1 g_2^*}{M_U^2}\left(\bar{Q}_3 \gamma^\mu L_3\right)\left(\bar{e}_3 \gamma_\mu d_3\right) + \text{h.c.}\right]. \end{aligned} \tag{212}$$

The relevant CC interactions are given by

$$\mathcal{L}_{\rm CC} = -\frac{|g_1|^2 V_{cb}}{M_U^2}\left(\bar{\tau}_L \gamma^\mu \nu_L\right)\left(\bar{c}_L \gamma_\mu b_L\right) + \frac{2V_{cb} g_1 g_2^*}{M_U^2}\left(\bar{\tau}_R \nu_L\right)\left(\bar{c}_L b_R\right) + \text{h.c.}. \tag{213}$$

- A vector $X_\mu(3,3)_{+2/3}$:

$$\mathcal{L}_X \quad = g\bar{Q}_3\tau_a \not{X}^a L_3 + \text{h.c.}. \tag{214}$$

Integrating out $X_\mu$, we obtain the following EFT Lagrangian:

$$\mathcal{L}_X^{\text{EFT}} \quad = -\frac{|g|^2}{M_X^2}\left(\bar{Q}_3\tau^a\gamma_\mu L_3\right)\left(\bar{L}_3\tau_a\gamma^\mu Q_3\right). \tag{215}$$

The relevant CC interactions are given by

$$\mathcal{L}_{\text{CC}} \quad = \frac{V_{cb}|g|^2}{4M_X^2}\left(\bar{\tau}_L\gamma^\mu\nu_L\right)\left(\bar{c}_L\gamma_\mu b_L\right) + \text{h.c.}. \tag{216}$$

- A vector $V_\mu(3,2)_{-5/3}$:

$$\mathcal{L}_V \quad = g_1\bar{Q}_3\not{V}e_3^c + g_2\bar{L}_3\not{V}d_3^c + \text{h.c.}. \tag{217}$$

Integrating out $V_\mu$, we obtain the following EFT Lagrangian:

$$\begin{aligned}\mathcal{L}_V^{\text{EFT}} \quad &= -\frac{|g_1|^2}{M_V^2}\left(\bar{Q}_3\gamma_\mu e_3^c\right)\left(\bar{e}_3^c\gamma^\mu Q_3\right) - \frac{|g_2|^2}{M_V^2}\left(\bar{L}_3\gamma_\mu d_3^c\right)\left(\bar{d}_3^c\gamma^\mu L_3\right) \\ &\quad - \left[\frac{g_1g_2^*}{M_V^2}\left(\bar{Q}_3\gamma^\mu e_3^c\right)\left(\bar{d}_3^c\gamma_\mu L_3\right) + \text{h.c.}\right].\end{aligned} \tag{218}$$

The relevant CC interactions are given by

$$\mathcal{L}_{\text{CC}} \quad = -\frac{2V_{cb}g_1g_2^*}{M_V^2}\left(\bar{\tau}_R\nu_L\right)\left(\bar{c}_L b_R\right) + \text{h.c.}. \tag{219}$$

- A scalar $S(3,1)_{-1/3}$:

$$\mathcal{L}_S = \quad \lambda_1 S\bar{L}_3\epsilon^T Q_3^c + \lambda_2 S\bar{e}_3 u_2^c + \text{h.c.}. \tag{220}$$

We impose a global $3B-L$ symmetry, which prevent an additional Yukawa couplings of the form $Sdu$ and $SQQ$. Integrating out $S$, we obtain the following EFT Lagrangian:

$$\mathcal{L}_S^{\text{EFT}} = \quad \frac{|\lambda_1|^2}{M_S^2}|\bar{L}_3\epsilon^T Q_3^c|^2 + \frac{|\lambda_2|^{2\,2}}{M_S^2}|\bar{e}_3u_2^c|^2 - \left[\frac{\lambda_1^*\lambda_2}{M_S^2}\left(\overline{L_3^c}\epsilon Q_3\right)\left(\bar{e}_3u_2^c\right) + \text{h.c.}\right]. \tag{221}$$

The relevant CC interactions are given by

$$\mathcal{L}_{\text{CC}} \quad = -\frac{|\lambda_1|^2V_{cb}}{2M_S^2}\left(\bar{\tau}_L\gamma^\mu\nu_L\right)\left(\bar{c}_L\gamma_\mu b_L\right) + \frac{\lambda_1^*\lambda_2}{2M_S^2}\left(\bar{\tau}_R\nu_L\right)\left(\bar{c}_R b_L\right) - \frac{\lambda_1^*\lambda_2}{8M_S^2}\left(\bar{\tau}_R\sigma^{\mu\nu}\nu_L\right)\left(\bar{c}_R\sigma_{\mu\nu}b_L\right) + \text{h.c.} \tag{222}$$

- A scalar $T(3,3)_{-1/3}$:

$$\mathcal{L}_T = \quad \lambda T^a \bar{L}_3 \tau_a \epsilon^T Q_3^c + \lambda^* T^{*a} \overline{Q_3^c} \epsilon \tau_a L_3. \tag{223}$$

We impose global $3B-L$ symmetry to forbid $TQQ$ terms. Integrating out $T$, we obtain the following EFT Lagrangian:

$$\mathcal{L}_T^{\rm EFT} = \quad \frac{1}{M_T^2}|\lambda|^2 \left(\bar{L}_3 \tau_a \epsilon^T Q_3^c\right)\left(\overline{Q_3^c}\epsilon\tau_a L_3\right). \tag{224}$$

The relevant CC interactions are given by

$$\mathcal{L}_{\rm CC} \quad = \frac{|\lambda|^2 V_{cb}}{8M_T^2}\left(\bar{\tau}_L\gamma^\mu\nu_L\right)\left(\bar{c}_L\gamma_\mu b_L\right). \tag{225}$$

- A scalar $\phi(1,2)_{+1/2}$:

$$\mathcal{L}_{H'} = -Y_b\bar{Q}_3\phi b_R - Y_c\bar{Q}_3\tilde{\phi}c_R - Y_\tau\bar{L}_3\phi\tau_R + \text{h.c.} \tag{226}$$

generates

$$c_{dQLe}^{3333} = Y_b Y_\tau^*/M_{H^+}^2, \quad c_{Qule}^{3233} = Y_c Y_\tau/M_{H^+}^2. \tag{227}$$

It can account for (205) with $c_{dQLe}^{3333} = (50\pm14)\text{TeV}^{-2}$ and $c_{QuLe}^{3233} = (-1.6\pm0.5)\text{TeV}^{-2}$.

- A scalar $D(3,2)_{+7/6}$:

$$\mathcal{L}_D = \quad \lambda_1 D\bar{Q}_3 e_3 + \lambda_2 D\epsilon\bar{u}_2 L_3 + \text{h.c.}. \tag{228}$$

Integrating out $D$, we obtain the following EFT Lagrangian:

$$\mathcal{L}_D^{\rm EFT} \quad = \frac{|\lambda_1|^2}{M_D^2}\left|\bar{Q}_3 e_3\right|^2 - \frac{|\lambda_2|^2}{M_D^2}\left|\bar{u}_2 L_3\right|^2 - \left[\frac{\lambda_1^*\lambda_2}{M_D^2}\left(\bar{u}_2 L_3\right)\epsilon\left(\bar{e}_3 Q_3\right)\right]. \tag{229}$$

The relevant CC interactions are given by

$$\mathcal{L}_{\rm CC} \quad = \frac{\lambda_1^*\lambda_2}{2M_D^2}\left(\bar{\tau}_R\nu_L\right)\left(\bar{c}_R b_L\right) + \frac{\lambda_1^*\lambda_2}{8M_D^2}\left(\bar{\tau}_R\sigma^{\mu\nu}\nu_L\right)\left(\bar{c}_R\sigma_{\mu\nu}b_L\right) + \text{h.c.}. \tag{230}$$

### C. $B \to K^{(*)}\mu^+\mu^-$

The LHCb experiment has measured the following ratios, which test lepton flavor universality [75, 76]:

$$R_{K^{(*)}} \equiv \frac{\Gamma(B\to K^{(*)}\mu^+\mu^-)}{\Gamma(B\to K^{(*)}e^+e^-)}. \tag{231}$$

The experimental values are

$$\begin{aligned} R_{K,[1,6\ \mathrm{GeV}^2]} &= 0.745 \pm 0.090, \\ R_{K^*,[1.1,6.0\ \mathrm{GeV}^2]} &= 0.69^{+0.11}_{-0.07} \pm 0.05, \\ R_{K^*,[0.045,1.1\ \mathrm{GeV}^2]} &= 0.66^{+0.11}_{-0.07} \pm 0.03, \end{aligned} \tag{232}$$

to be compared with the SM predictions [77]:

$$\begin{aligned} R^{\mathrm{SM}}_{K,[1,6\ \mathrm{GeV}^2]} &= 1.00 \pm 0.01, \\ R^{\mathrm{SM}}_{K^*,[1.1,6.0\ \mathrm{GeV}^2]} &= 1.00 \pm 0.01, \\ R^{\mathrm{SM}}_{K^*,[0.045,1.1\ \mathrm{GeV}^2]} &= 0.983 \pm 0.014. \end{aligned} \tag{233}$$

The significance of these deficits is about $2.2 - 2.5\sigma$ in each mode.

**Exercise 16:** *The deviation from the SM suggests that the new physics couples to left-handed quarks and leptons. Find the list of intermediate bosons that can contribute to the decay at tree level.*

## XV. FLAVORED CONCLUSIONS

(i) Measurements of CP violating $B$-meson decays have established that the Kobayashi-Maskawa mechanism is the dominant source of the observed CP violation.

(ii) Measurements of flavor changing $B$-meson decays have established the the Cabibbo-Kobayashi-Maskawa mechanism is a major player in flavor violation.

(iii) The consistency of all these measurements with the CKM predictions sharpens the new physics flavor puzzle: If there is new physics at, or below, the TeV scale, then its flavor structure must be highly non-generic.

(iv) Measurements of neutrino flavor parameters have not only not clarified the standard model flavor puzzle, but actually deepened it. Whether they imply an anarchical structure, or a tribimaximal mixing, it seems that the neutrino flavor structure is very different from that of quarks.

(v) If the LHC experiments, ATLAS and CMS, discover new particles that couple to the Standard Model fermions, then, in principle, they will be able to measure new flavor parameters. Consequently, the new physics flavor puzzle is likely to be understood.

(vi) If the flavor structure of such new particles is affected by the same physics that sets the flavor structure of the Yukawa couplings, then the LHC experiments (and future flavor factories) may be able to shed light also on the standard model flavor puzzle.

(vii) The recently discovered Higgs-like boson provides an opportunity to make progress in our understanding of the flavor puzzle(s).

(viii) Extensions of the SM where new particles couple to quark- and/or lepton-pairs are constrained by flavor.

(ix) Experiments show violation of $\tau - \ell$ universality in $B$ decays at a level close to $4\sigma$.

The huge progress in flavor physics in recent years has provided answers to many questions. At the same time, new questions arise. The LHC era is likely to provide more answers and more questions.

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## APPENDIX A: THE CKM MATRIX

The CKM matrix $V$ is a $3 \times 3$ unitary matrix. Its form, however, is not unique:

($i$) There is freedom in defining $V$ in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of $V$ are written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{A1}$$

($ii$) There is further freedom in the phase structure of $V$. This means that the number of physical parameters in $V$ is smaller than the number of parameters in a general unitary $3 \times 3$ matrix which is nine (three real angles and six phases). Let us define $P_q$ ($q = u, d$)

to be diagonal unitary (phase) matrices. Then, if instead of using $V_{qL}$ and $V_{qR}$ for the rotations (16) and (19) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, $V$ does change:

$$V \to P_u V P_d^*. \tag{A2}$$

This freedom is fixed by demanding that $V$ has the minimal number of phases. In the three generation case $V$ has a single phase. (There are five phase differences between the elements of $P_u$ and $P_d$ and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [2].

The fact that $V$ is unitary and depends on only four independent physical parameters can be made manifest by choosing a specific parametrization. The standard choice is [78]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{A3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The $\theta_{ij}$'s are the three real mixing parameters while $\delta$ is the Kobayashi-Maskawa phase. It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$. It is convenient to choose an approximate expression where this hierarchy is manifest. This is the Wolfenstein parametrization, where the four mixing parameters are $(\lambda, A, \rho, \eta)$ with $\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter and $\eta$ representing the CP violating phase [79, 80]:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{A4}$$

A very useful concept is that of the *unitarity triangles*. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{A5}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{A6}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{A7}$$

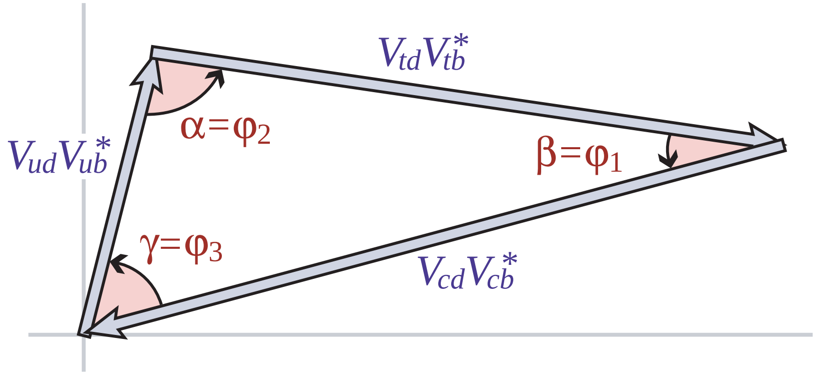


FIG. 6: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (A7) only. The unitarity triangle related to Eq. (A7) is depicted in Fig. 6.

The rescaled unitarity triangle is derived from (A7) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters $(\rho, \eta)$. The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the $(\rho, \eta)$ plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{A8}$$

The three angles of the unitarity triangle are defined as follows [81, 82]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{A9}$$

They are physical quantities and can be independently measured by CP asymmetries in $B$ decays. It is also useful to define the two small angles of the unitarity triangles (A6,A5):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{A10}$$

The $\lambda$ and $A$ parameters are very well determined at present, see Eq. (133). The main effort in CKM measurements is thus aimed at improving our knowledge of $\rho$ and $\eta$:

$$\rho = 0.12 \pm 0.02, \quad \eta = 0.36 \pm 0.01. \tag{A11}$$

The present status of our knowledge is best seen in a plot of the various constraints and the final allowed region in the $\rho - \eta$ plane. This is shown in Fig. 2. The present status of our knowledge of the absolute values of the various entries in the CKM matrix can be summarized as follows:

$$|V| = \begin{pmatrix} 0.97434 \pm 0.00012 & 0.2251 \pm 0.0005 & (3.57 \pm 0.15) \times 10^{-3} \\ 0.2249 \pm 0.0005 & 0.9735 \pm 0.0001 & (4.11 \pm 0.13) \times 10^{-2} \\ (8.7 \pm 0.3) \times 10^{-3} & (4.03 \pm 0.13) \times 10^{-2} & 0.99915 \pm 0.00005 \end{pmatrix}. \tag{A12}$$

## APPENDIX B: CPV IN $B$ DECAYS TO FINAL CP EIGENSTATES

We define decay amplitudes of $B$ (which could be charged or neutral) and its CP conjugate $\overline{B}$ to a multi-particle final state $f$ and its CP conjugate $\overline{f}$ as

$$A_f = \langle f|\mathcal{H}|B\rangle \quad , \quad \overline{A}_f = \langle f|\mathcal{H}|\overline{B}\rangle \quad , \quad A_{\overline{f}} = \langle \overline{f}|\mathcal{H}|B\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle \ , \tag{B1}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases $\xi_B$ and $\xi_f$ according to

$$\begin{aligned} CP\ |B\rangle \ &= \ e^{+i\xi_B}\,|\overline{B}\rangle \quad , \quad CP\ |f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \ , \\ CP\ |\overline{B}\rangle \ &= \ e^{-i\xi_B}\,|B\rangle \quad , \quad CP\ |\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \ , \end{aligned} \tag{B2}$$

so that $(CP\ )^2 = 1$. The phases $\xi_B$ and $\xi_f$ are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP\ , \mathcal{H}] = 0$, then $A_f$ and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_B)}\,A_f \ . \tag{B3}$$

A state that is initially a superposition of $B^0$ and $\overline{B}^0$, say

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\overline{B}^0\rangle \ , \tag{B4}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{B5}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times $t$ in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [83]. The simplified time evolution is determined by a $2\times 2$ effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices $M$ and $\Gamma$ as

$$\mathcal{H} = M - \frac{i}{2}\Gamma\,. \tag{B6}$$

$M$ and $\Gamma$ are associated with $(B^0,\overline{B}^0) \leftrightarrow (B^0,\overline{B}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of $M$ and $\Gamma$ are associated with the flavor-conserving transitions $B^0 \to B^0$ and $\overline{B}^0 \to \overline{B}^0$ while off-diagonal elements are associated with flavor-changing transitions $B^0 \leftrightarrow \overline{B}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters $p$ and $q$ to specify the components of the strong interaction eigenstates, $B^0$ and $\overline{B}^0$, in the light ($B_L$) and heavy ($B_H$) mass eigenstates:

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\overline{B}^0\rangle \tag{B7}$$

with the normalization $|p|^2+|q|^2=1$. The special form of Eq. (B7) is related to the fact that CPT imposes $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$. Solving the eigenvalue problem gives

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12}-(i/2)\Gamma_{12}}\,. \tag{B8}$$

If either CP or T is a symmetry of $\mathcal{H}$, then $M_{12}$ and $\Gamma_{12}$ are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_B} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1\,, \tag{B9}$$

where $\xi_B$ is the arbitrary unphysical phase introduced in Eq. (B2).

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|B_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference $\Delta m_B$ and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma_B \equiv \Gamma_H - \Gamma_L. \tag{B10}$$

Note that here $\Delta m_B$ is positive by definition, while the sign of $\Delta\Gamma_B$ is to be experimentally determined. The average mass and width are given by

$$m_B \equiv \frac{M_H+M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H+\Gamma_L}{2}. \tag{B11}$$

It is useful to define dimensionless ratios $x$ and $y$:

$$x \equiv \frac{\Delta m_B}{\Gamma_B}, \quad y \equiv \frac{\Delta\Gamma_B}{2\Gamma_B}. \tag{B12}$$

Solving the eigenvalue equation gives

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \qquad \Delta m_B \Delta\Gamma_B = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{B13}$$

All CP-violating observables in $B$ and $\overline{B}$ decays to final states $f$ and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of $A_f$, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, $q/p$. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $B^0 \leftrightarrow \overline{B}^0$ oscillations and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

For neutral $D$, $B$, and $B_s$ mesons, $\Delta\Gamma/\Gamma \ll 1$ and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ after an elapsed proper time $t$ as $|B^0{}_{\rm phys}(t)\rangle$ or $|\overline{B}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$\begin{aligned} |B^0{}_{\rm phys}(t)\rangle &= g_+(t)\,|B^0\rangle - \frac{q}{p}\, g_-(t)|\overline{B}^0\rangle, \\ |\overline{B}^0_{\rm phys}(t)\rangle &= g_+(t)\,|\overline{B}^0\rangle - \frac{p}{q}\, g_-(t)|B^0\rangle\,, \end{aligned} \tag{B14}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{B15}$$

One obtains the following time-dependent decay rates:

$$\begin{aligned} \frac{d\Gamma[B^0{}_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t) \\ &+ 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\,, \quad \text{(B16)} \\ \frac{d\Gamma[\overline{B}^0_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|(p/q)A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t) \\ &+ 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\,, \quad \text{(B17)} \end{aligned}$$

where $\mathcal{N}_f$ is a common normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (B16,B17). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $B \leftrightarrow \overline{B}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$

are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (B16,B17) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

One possible manifestation of CP-violating effects in meson decays [84] is in the interference between a decay without mixing, $B^0 \to f$, and a decay with mixing, $B^0 \to \overline{B}^0 \to f$ (such an effect occurs only in decays to final states that are common to $B^0$ and $\overline{B}^0$, including all CP eigenstates). It is defined by

$$\mathcal{I}m(\lambda_f) \neq 0 \,, \tag{B18}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \,. \tag{B19}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates $f_{CP}$

$$\mathcal{A}_{f_{CP}}\,(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]} \,. \tag{B20}$$

For $\Delta\Gamma = 0$ and $|q/p| = 1$ (which is a good approximation for $B$ mesons), $\mathcal{A}_{f_{CP}}$ has a particularly simple form [85–87]:

$$\begin{aligned} \mathcal{A}_f(t) &= S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \\ S_f &\equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \,, \end{aligned} \tag{B21}$$

Consider the $B \to f$ decay amplitude $A_f$, and the CP conjugate process, $\overline{B} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in $A_f$ and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^{\pm}$ bosons and hence are often called "weak phases". The weak phase of any single term is convention dependent. However, the difference between the weak phases in two different terms in $A_f$ is convention independent. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in $A_f$ and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (B3). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution $a_i$ to $A_f$ in three parts: its magnitude $|a_i|$, its weak phase $\phi_i$, and its strong phase $\delta_i$. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f &= |a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} &= |a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{B22}$$

Similarly, for neutral meson decays, it is useful to write

$$M_{12} = |M_{12}|e^{i\phi_M} \quad , \quad \Gamma_{12} = |\Gamma_{12}|e^{i\phi_\Gamma} \ . \tag{B23}$$

Each of the phases appearing in Eqs. (B22,B23) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$ and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

In the approximations that only a single weak phase contributes to decay, $A_f = |a_f|e^{i(\delta_f+\phi_f)}$, and that $|\Gamma_{12}/M_{12}| = 0$, we obtain $|\lambda_f| = 1$ and the CP asymmetries in decays to a final CP eigenstate $f$ [Eq. (B20)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}\ (t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f). \tag{B24}$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

## APPENDIX C: SUPERSYMMETRIC FLAVOR VIOLATION

### 1. Mass insertions

Supersymmetric models provide, in general, new sources of flavor violation. We here present the formalism of mass insertions. We do that for the charged sleptons, but the formalism is straightforwardly adapted for squarks.

The supersymmetric lepton flavor violation is most commonly analyzed in the basis in which the charged lepton mass matrix and the gaugino vertices are diagonal. In this basis, the slepton masses are not necessarily flavor-diagonal, and have the form

$$\widetilde{\ell}^*_{Mi}(M^2_{\widetilde{\ell}})^{MN}_{ij}\widetilde{\ell}_{Nj} = (\widetilde{\ell}^*_{Li}\ \widetilde{\ell}^*_{Rk}) \begin{pmatrix} M^2_{Lij} & A_{il}v_d \\ A_{jk}v_d & M^2_{Rkl} \end{pmatrix} \begin{pmatrix} \widetilde{\ell}_{Lj} \\ \widetilde{\ell}_{Rl} \end{pmatrix}, \tag{C1}$$

where $M, N = L, R$ label chirality, and $i, j, k, l = 1, 2, 3$ are generational indices. $M^2_L$ and $M^2_R$ are the supersymmetry breaking slepton masses-squared. The $A$ parameters enter in the trilinear scalar couplings $A_{ij}\phi_d\widetilde{\ell}_{Li}\widetilde{\ell}^*_{Rj}$, where $\phi_d$ is the down-type Higgs boson, and $v_d = \langle\phi_d\rangle$. We neglect small flavor-conserving terms involving $\tan\beta = v_u/v_d$.

In this basis, charged LFV takes place through one or more slepton mass insertion. Each mass insertion brings with it a factor of

$$\delta^{MN}_{ij} \equiv (M^2_{\widetilde{\ell}})^{MN}_{ij}/\tilde{m}^2, \tag{C2}$$

where $\tilde{m}^2$ is the representative slepton mass scale. Physical processes therefore constrain

$$(\delta^{MN}_{ij})_{\text{eff}} \sim \max\left[\delta^{MN}_{ij}, \delta^{MP}_{ik}\delta^{PN}_{kj}, \ldots, (i \leftrightarrow j)\right]. \tag{C3}$$

For example,

$$(\delta^{LR}_{12})_{\text{eff}} \sim \max\left[A_{12}v_d/\tilde{m}^2, M^2_{L1k}A_{k2}v_d/\tilde{m}^4, A_{1k}v_dM^2_{Rk2}/\tilde{m}^4, \ldots, (1 \leftrightarrow 2)\right]. \tag{C4}$$

Note that contributions with two or more insertions may be less suppressed than those with only one.

It is useful to express the $\delta^{MN}_{ij}$ mass insertions in terms of parameters in the mass basis. We can write, for example,

$$\delta^{LL}_{ij} = \frac{1}{\tilde{m}^2}\sum_\alpha K^L_{i\alpha}K^{L*}_{j\alpha}\Delta\tilde{m}^2_{L\alpha}. \tag{C5}$$

Here, we ignore $L-R$ mixing, so that $K^L_{i\alpha}$ is the mixing angle in the coupling of a neutralino to $\ell_{Li} - \widetilde{\ell}_{L\alpha}$ (with $\ell_i = e, \mu, \tau$ denoting charged lepton mass eigenstates and $\widetilde{\ell}_\alpha = \widetilde{\ell}_1, \widetilde{\ell}_2, \widetilde{\ell}_3$ denoting charged slepton mass eigenstates), and $\Delta\tilde{m}^2_{L\alpha} = m^2_{\widetilde{\ell}_{L\alpha}} - \tilde{m}^2$. Using the unitarity of the mixing matrix $K^L$, we can write

$$\tilde{m}^2\delta^{LL}_{ij} = \sum_\alpha K^L_{i\alpha}K^{L*}_{j\alpha}(\Delta\tilde{m}^2_{L\alpha} + \tilde{m}^2) = (M^2_{\widetilde{\ell}})^{LL}_{ij}, \tag{C6}$$

thus reproducing the definition (C2).

In many cases, a two generation effective framework is useful. To understand that, consider a case where (no summation over $i,j,k$)

$$\begin{aligned} |K^L_{ik}K^{L*}_{jk}| &\ll |K^L_{ij}K^{L*}_{j}|, \\ |K^L_{ik}K^{L*}_{jk}\Delta m^2_{\widetilde{\ell}_{Lk}\widetilde{\ell}_{Li}}| &\ll |K^L_{ij}K^{L*}_{j}\Delta m^2_{\widetilde{\ell}_{Lj}\widetilde{\ell}_{Li}}|, \end{aligned} \tag{C7}$$

where $\Delta m^2_{\widetilde{\ell}_j\widetilde{\ell}_i} = m^2_{\widetilde{\ell}_{Lj}} - m^2_{\widetilde{\ell}_{Li}}$. Then, the contribution of the intermediate $\widetilde{\ell}_k$ can be neglected and, furthermore, to a good approximation $K^L_{ii}K^{L*}_{ji} + K^L_{ij}K^{L*}_{jj} = 0$. For these cases, we obtain

$$\delta^{LL}_{ij} = \frac{\Delta m^2_{\widetilde{\ell}_{Lj}\widetilde{\ell}_{Li}}}{\tilde{m}^2} K^L_{ij}K^{L*}_{jj}. \tag{C8}$$

### 2. Neutral meson mixing

We consider the squark-gluino box diagram contribution to $D^0 - \overline{D}^0$ mixing amplitude that is proportional to $K^u_{2i}K^{u*}_{1i}K^u_{2j}K^{u*}_{1j}$, where $K^u$ is the mixing matrix of the gluino couplings to left-handed up quarks and their up squark partners. (In the language of the mass insertion approximation, we calculate here the contribution that is $\propto [(\delta^u_{LL})_{12}]^2$.) We work in the mass basis for both quarks and squarks.

The contribution is given by

$$M^D_{12} = -i\frac{4\pi^2}{27}\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD} \sum_{i,j}(K^u_{2i}K^{u*}_{1i}K^u_{2j}K^{u*}_{1j})(11\tilde{I}_{4ij} + 4\tilde{m}_g^2 I_{4ij}). \tag{C9}$$

where

$$\begin{aligned} \tilde{I}_{4ij} &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{p^2}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)} \\ &= \frac{i}{(4\pi)^2}\left[\frac{\tilde{m}_g^2}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)}\right. \\ &\quad \left. + \frac{\tilde{m}_i^4}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^4}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right], \end{aligned} \tag{C10}$$

$$\begin{aligned} I_{4ij} &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)} \\ &= \frac{i}{(4\pi)^2}\left[\frac{1}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)}\right. \\ &\quad \left. + \frac{\tilde{m}_i^2}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^2}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right]. \end{aligned} \tag{C11}$$

We now follow the discussion in refs. [20, 88]. To see the consequences of the super-GIM mechanism, let us expand the expression for the box integral around some value $\tilde{m}_q^2$ for the squark masses-squared:

$$
\begin{aligned}
I_4(\tilde{m}_g^2, \tilde{m}_i^2, \tilde{m}_j^2) &= I_4(\tilde{m}_g^2, \tilde{m}_q^2 + \delta\tilde{m}_i^2, \tilde{m}_q^2 + \delta\tilde{m}_j^2) \\
&= I_4(\tilde{m}_g^2, \tilde{m}_q^2, \tilde{m}_q^2) + (\delta\tilde{m}_i^2 + \delta\tilde{m}_j^2) I_5(\tilde{m}_g^2, \tilde{m}_q^2, \tilde{m}_q^2, \tilde{m}_q^2) \\
&+ \frac{1}{2}\left[(\delta\tilde{m}_i^2)^2 + (\delta\tilde{m}_j^2)^2 + 2(\delta\tilde{m}_i^2)(\delta\tilde{m}_j^2)\right] I_6(\tilde{m}_g^2, \tilde{m}_q^2, \tilde{m}_q^2, \tilde{m}_q^2, \tilde{m}_q^2) + \cdots
\end{aligned}
\tag{C12}
$$

where

$$
I_n(\tilde{m}_g^2, \tilde{m}_q^2, \ldots, \tilde{m}_q^2) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - \tilde{m}_g^2)^2 (p^2 - \tilde{m}_q^2)^{n-2}}, \tag{C13}
$$

and similarly for $\tilde{I}_{4ij}$. Note that $I_n \propto (\tilde{m}_q^2)^{n-2}$ and $\tilde{I}_n \propto (\tilde{m}_q^2)^{n-3}$. Thus, using $x \equiv \tilde{m}_g^2/\tilde{m}_q^2$, it is customary to define

$$
I_n \equiv \frac{i}{(4\pi)^2 (\tilde{m}_q^2)^{n-2}} f_n(x), \qquad \tilde{I}_n \equiv \frac{i}{(4\pi)^2 (\tilde{m}_q^2)^{n-3}} \tilde{f}_n(x). \tag{C14}
$$

The unitarity of the mixing matrix implies that

$$
\sum_i (K^u_{2i} K^{u*}_{1i} K^u_{2j} K^{u*}_{1j}) = \sum_j (K^u_{2i} K^{u*}_{1i} K^u_{2j} K^{u*}_{1j}) = 0. \tag{C15}
$$

Consequently, the terms that are proportional $f_4, \tilde{f}_4, f_5$ and $\tilde{f}_5$ vanish in their contribution to $M_{12}$. When $\delta\tilde{m}_i^2 \ll \tilde{m}_q^2$ for all $i$, the leading contributions to $M_{12}$ come from $f_6$ and $\tilde{f}_6$. We learn that for quasi-degenerate squarks, the leading contribution is quadratic in the small mass-squared difference. The functions $f_6(x)$ and $\tilde{f}_6(x)$ are given by

$$
\begin{aligned}
f_6(x) &= \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(1-x)^5}, \\
\tilde{f}_6(x) &= \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(1-x)^5}.
\end{aligned}
\tag{C16}
$$

For example, with $x = 1$, $f_6(1) = -1/20$ and $\tilde{f}_6 = +1/30$; with $x = 2.33$, $f_6(2.33) = -0.015$ and $\tilde{f}_6 = +0.013$.

To further simplify things, let us consider a two generation case. Then

$$
\begin{aligned}
M_{12}^D &\propto 2(K^u_{21}K^{u*}_{11})^2(\delta\tilde{m}_1^2)^2 + 2(K^u_{22}K^{u*}_{12})^2(\delta\tilde{m}_2^2)^2 + (K^u_{21}K^{u*}_{11}K^u_{22}K^{u*}_{12})(\delta\tilde{m}_1^2 + \delta\tilde{m}_2^2)^2 \\
&= (K^u_{21}K^{u*}_{11})^2(\tilde{m}_2^2 - \tilde{m}_1^2)^2.
\end{aligned}
\tag{C17}
$$

We thus rewrite Eq. (C9) for the case of quasi-degenerate squarks:

$$
M_{12}^D = \frac{\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD}}{108\tilde{m}_q^2}[11\tilde{f}_6(x) + 4x f_6(x)] \frac{(\Delta\tilde{m}_{21}^2)^2}{\tilde{m}_q^4}(K^u_{21}K^{u*}_{11})^2. \tag{C18}
$$

For example, for $x = 1$, $11\tilde{f}_6(x) + 4xf_6(x) = +0.17$. For $x = 2.33$, $11\tilde{f}_6(x) + 4xf_6(x) = +0.003$.

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