Neutrinos
Lecture I: theory and phenomenology of neutrino oscillations

Summer School on Particle Physics

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What will you learn from these lectures?

- The basics of neutrinos: a bit of history and the basic concepts
- Neutrino oscillations: in vacuum, in matter, experiments
- Nature of neutrinos, neutrino less double beta decay
- Neutrino masses and mixing BSM
- Neutrinos in cosmology (if we have time)

Today, we look at

- A bit of history: from the initial idea of the *neutrino* to the solar and atmospheric neutrino anomalies
- The basic picture of neutrino oscillations (mixing of states and coherence)
- The formal details: how to derive the probabilities
- Neutrino oscillations both in vacuum and in matter
- Their relevance in present and future experiments

Useful references

- C. Giunti, C. W. Kim, Fundamentals of Neutrino Physics and Astrophysics, Oxford University Press, USA (May 17, 2007)
- M. Fukugita, T. Yanagida, Physics of Neutrinos and applications to astrophysics, Springer 2003
- Z.-Z. Xing, S. Zhou, Neutrinos in Particle Physics, Astronomy and Cosmology, Springer 2011
- A. De Gouvea, TASI lectures, hep-ph/0411274
- A. Strumia and F. Vissani, hep-ph/0606054.

Plan of lecture I

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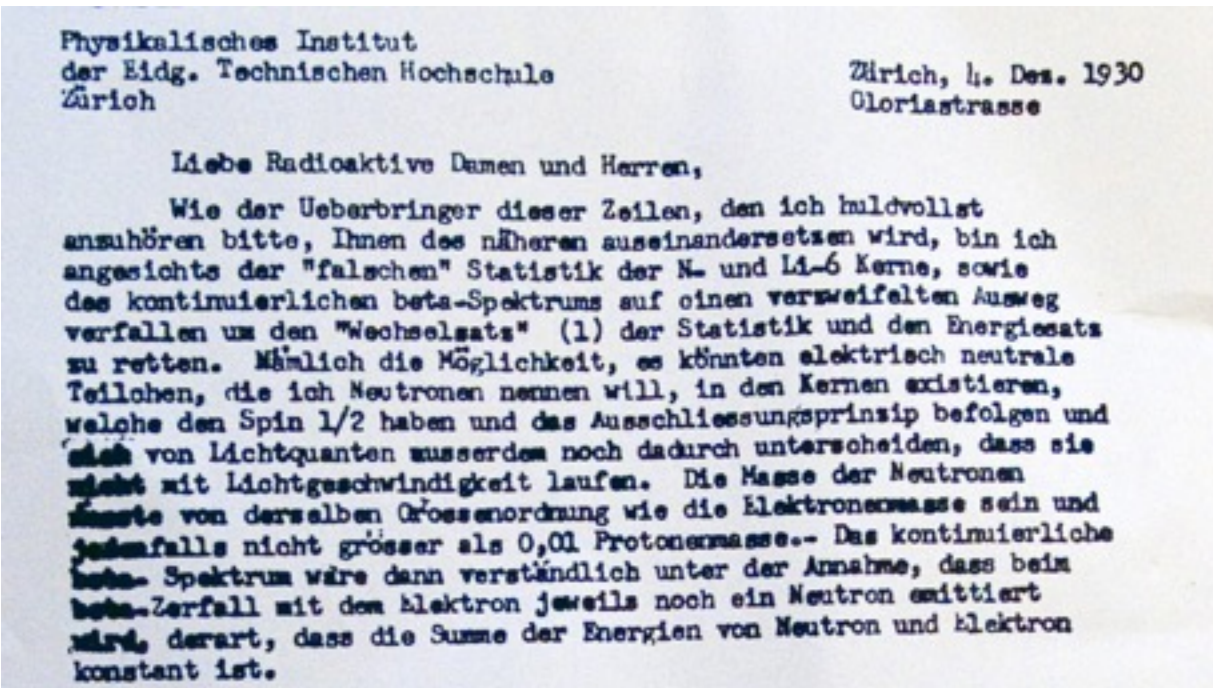
A brief history of neutrinos

- The proposal of the "neutrino" was put forward by W. Pauli in 1930. [Pauli Letter Collection, CERN]



Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

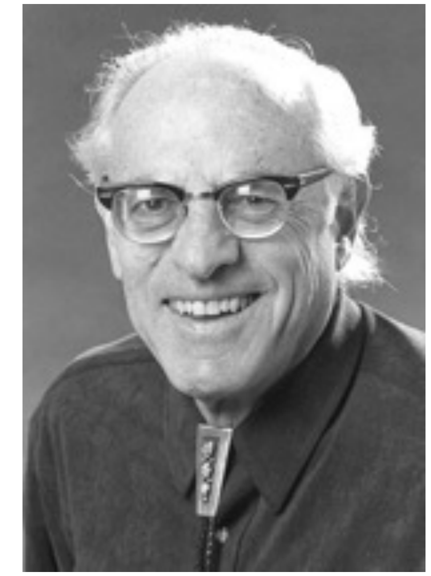
Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Dear radioactive ladies and gentlemen,

…I have hit upon a desperate remedy to save the … energy theorem. Namely the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin 1/2 … The mass of the neutron must be … not larger than 0.01 proton mass. …in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant.

- Since the neutron was discovered two years later by J. Chadwick, Fermi, following the proposal by E. Amaldi, used the name "neutrino" (little neutron) in 1932 and later proposed the Fermi theory of beta decay.

- Reines and Cowan discovered the neutrino in 1956 using inverse beta decay. [Science 124, 3212:103]

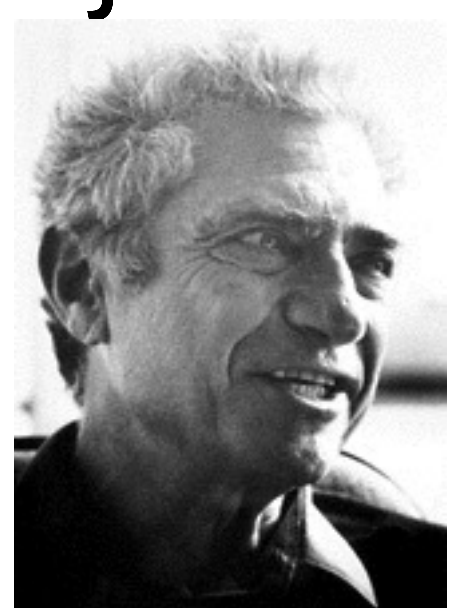


The Nobel Prize in Physics 1995

- Madame Wu in 1956 demonstrated that P is violated in weak interactions.

- Muon neutrinos were discovered in 1962 by L. Lederman, M. Schwartz and J. Steinberger.

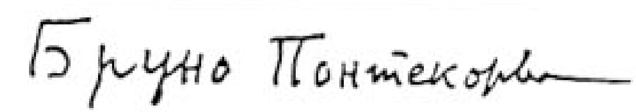




The Nobel Prize in Physics 1988

- The first idea of neutrino oscillations was considered by B. Pontecorvo in 1957.

[B. Pontecorvo, J. Exp. Theor. Phys. 33 (1957)549.
B. Pontecorvo, J. Exp. Theor. Phys. 34 (1958) 247.]

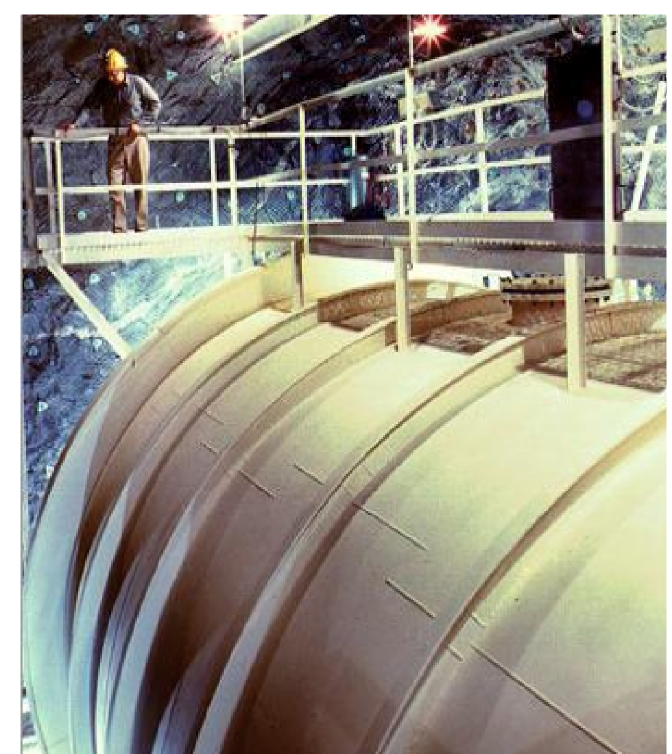


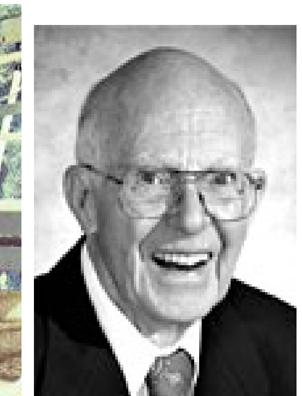
- Mixing was introduced at the beginning of the '60 by Z. Maki, M. Nakagawa, S. Sakata,

Prog. Theor. Phys. 28 (1962) 870, Y. Katayama, K. Matumoto, S. Tanaka, E. Yamada, Prog. Theor. Phys. 28 (1962) 675 and M. Nakagawa, et. al., Prog. Theor. Phys. 30 (1963)727.

- First indications of ν oscillations came from solar ν.

- R. Davis built the Homestake experiment to detect solar ν, based on an experimental technique by Pontecorvo.

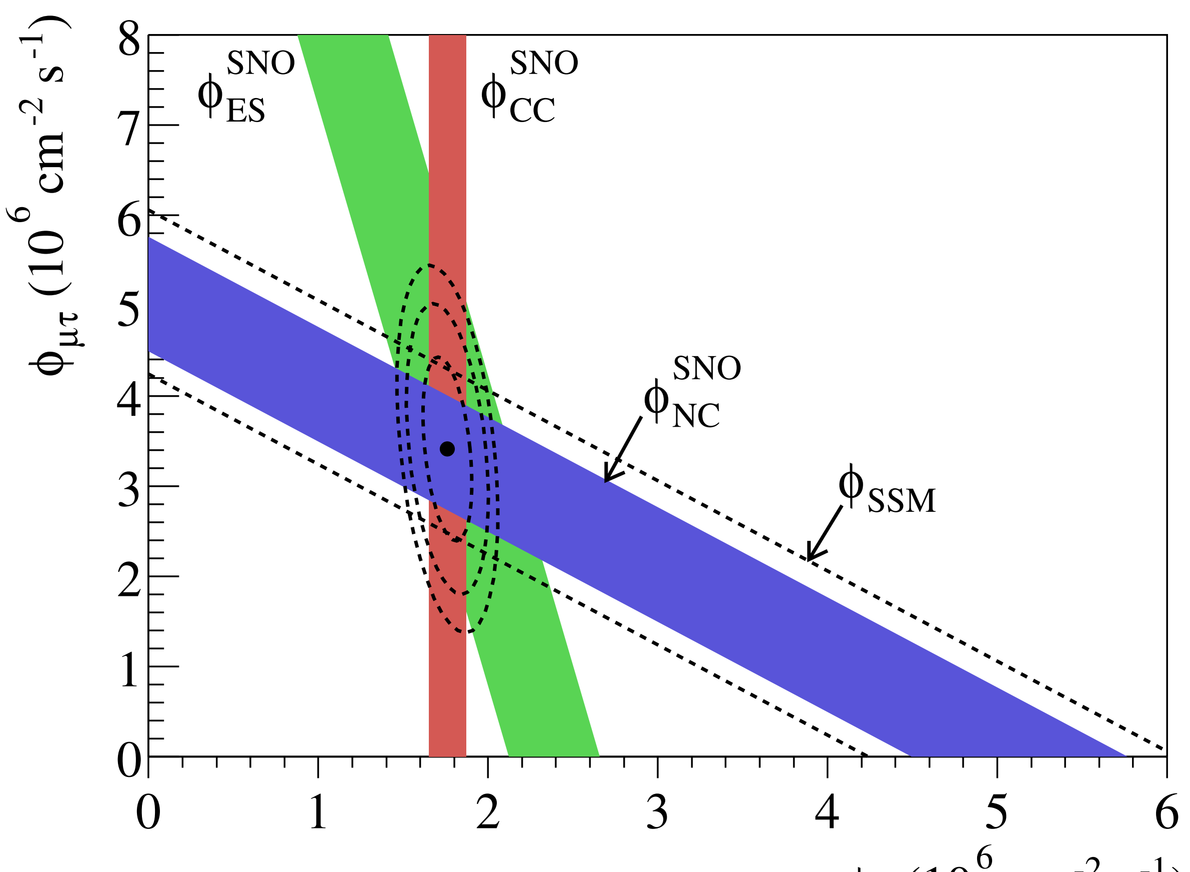




Raymond Davis Jr.
1/4 of the prize
USA
University of Pennsylvania
Philadelphia, PA, USA
b. 1914

- Compared with the predicted solar neutrino fluxes (J. Bahcall et al.), a significant deficit was found. First results were announced [R. Davis, Phys. Rev. Lett. 12 (1964)302 and R. Davis et al., Phys. Rev. Lett. 20 (1968) 1205].

- This anomaly received further confirmation (SAGE, GALLEX, SuperKamiokande, SNO...) and was finally interpreted as neutrino oscillations.



SNO, PRL 89 2002

The Nobel Prize in Physics 2015

An anomaly was also found in atmospheric neutrinos.

- Atmospheric neutrinos had been observed by various experiments but the first relevant indication of an anomaly was presented in 1988 [Kamiokande Coll., Phys. Lett. B205 (1988) 416], subsequently confirmed by MACRO.

- Strong evidence was presented in 1998 by SuperKamiokande (corroborated by Soudan2 and MACRO) [SuperKamiokande Coll., Phys. Rev. Lett. 81 (1998) 1562]. This is considered the start of "modern neutrino physics"!

The Nobel Prize in Physics 2015

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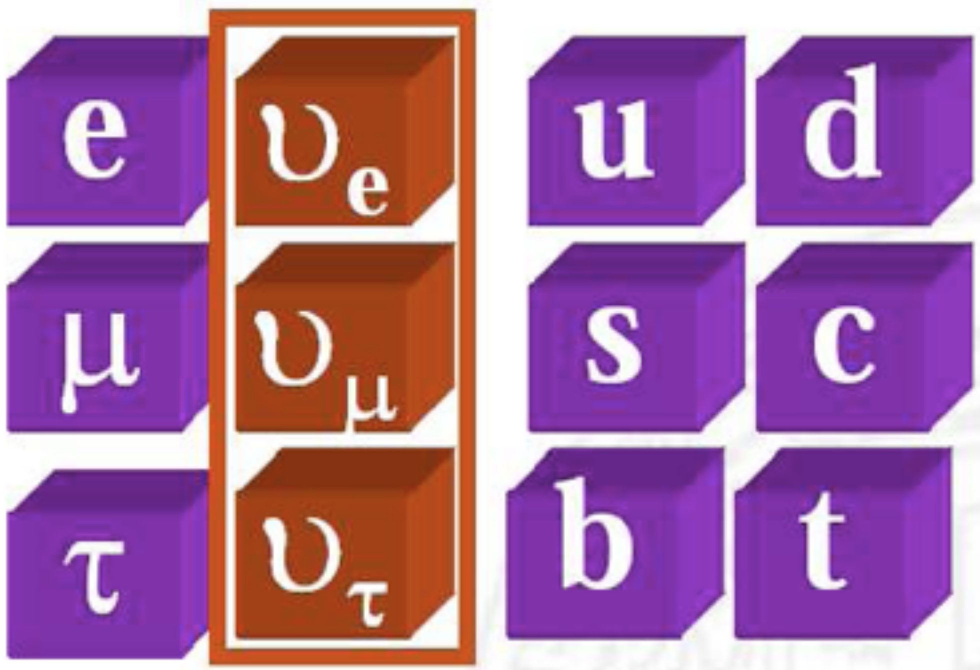
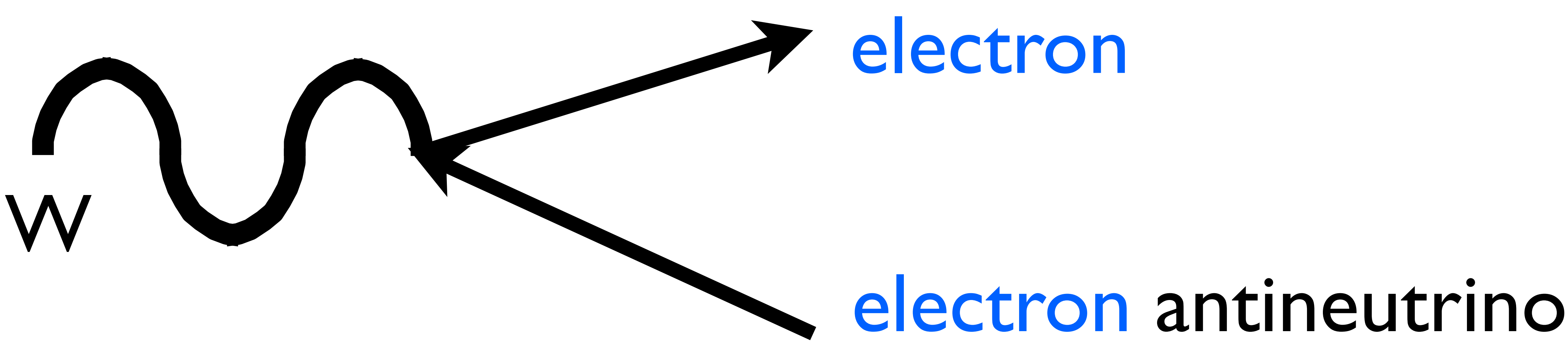
Neutrinos in the SM



- Neutrinos come in 3 flavours, corresponding to the charged lepton.

- They belong to SU(2) doublets:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$



Neutrino mixing

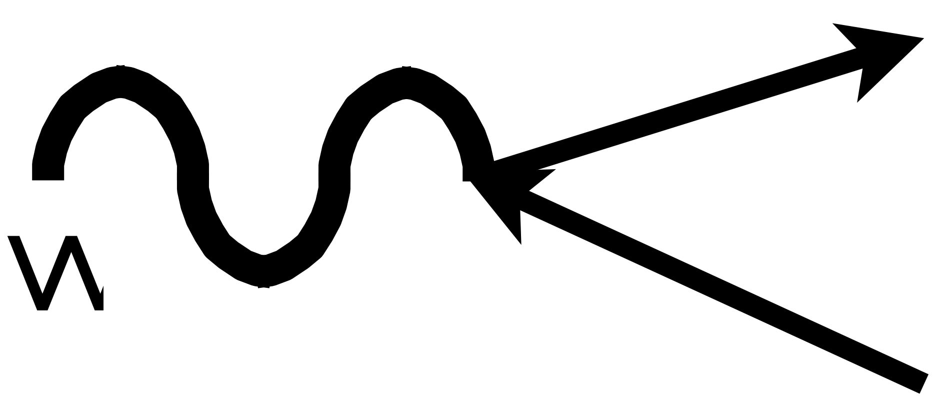
Mixing is described by the *Pontecorvo-Maki-Nakagawa-Sakata* matrix: $|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$

Flavour states (α) — Mass states (i)

which enters in the CC interactions

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{k\alpha} (U^*_{\alpha k} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + \text{h.c.})$$

This implies that in an interaction with an electron, the corresponding (anti-)neutrino will be produced, as a superposition of different mass eigenstates.



Positron

electron neutrino $= \sum_i U_{ei} \nu_i$

Neutrino mixing

Mixing is described by the *Pontecorvo-Maki-Nakagawa-Sakata* matrix: $|\nu_\alpha\rangle = \sum U \ldots |\nu_i\rangle$ ← Mass states

?

Do charged leptons mix?

which ent…rs i… …tions

$$\mathcal{L}_{CC} = - \frac{\ldots}{\sqrt{2}} \sum_{k\alpha} (U_{\alpha k}^* \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + \text{h.c.})$$

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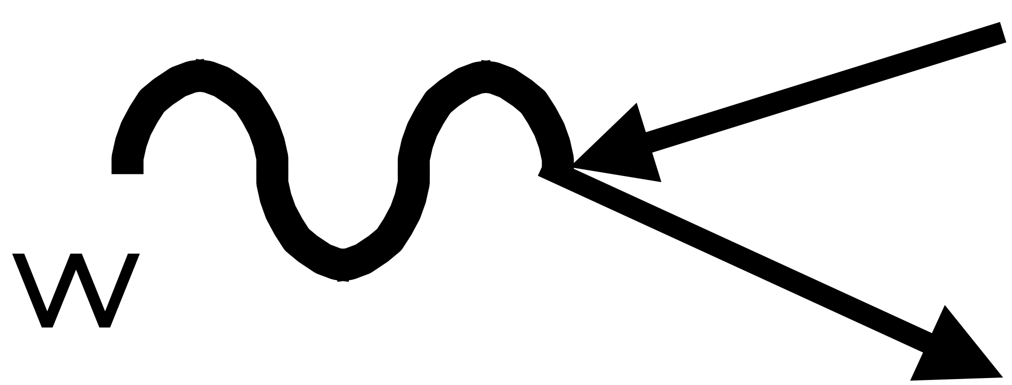

$$= \sum_i U_{ei} \nu_i$$

• 2-neutrino mixing matrix depends on 1 angle only. The phases get absorbed in a redefinition of the leptonic fields (a part from 1 Majorana phase).

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

• 3-neutrino mixing matrix has 3 angles and 1(+2) CPV phases.

$$\begin{pmatrix} \bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3 \end{pmatrix} e^{i\psi} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{CKM-} \\ \text{type} \end{pmatrix} \begin{pmatrix} e^{i\rho_e} & 0 & 0 \\ 0 & e^{i\rho_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

Rephasing

$$\begin{aligned} e &\to e^{-i(\rho_e+\psi)} e \\ \mu &\to e^{-i(\rho_\mu+\psi)} \mu \\ \tau &\to e^{-i\psi} \tau \end{aligned}$$

the kinetic, NC and mass terms are not modified: these phases are unphysical.

For Dirac neutrinos, the same rephasing can be done. For Majorana neutrinos, the Majorana condition forbids such rephasing: 2 physical CP-violating phases.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

For antineutrinos, $U \to U^*$

CP-conservation requires U is real $\Rightarrow \delta = 0, \pi$

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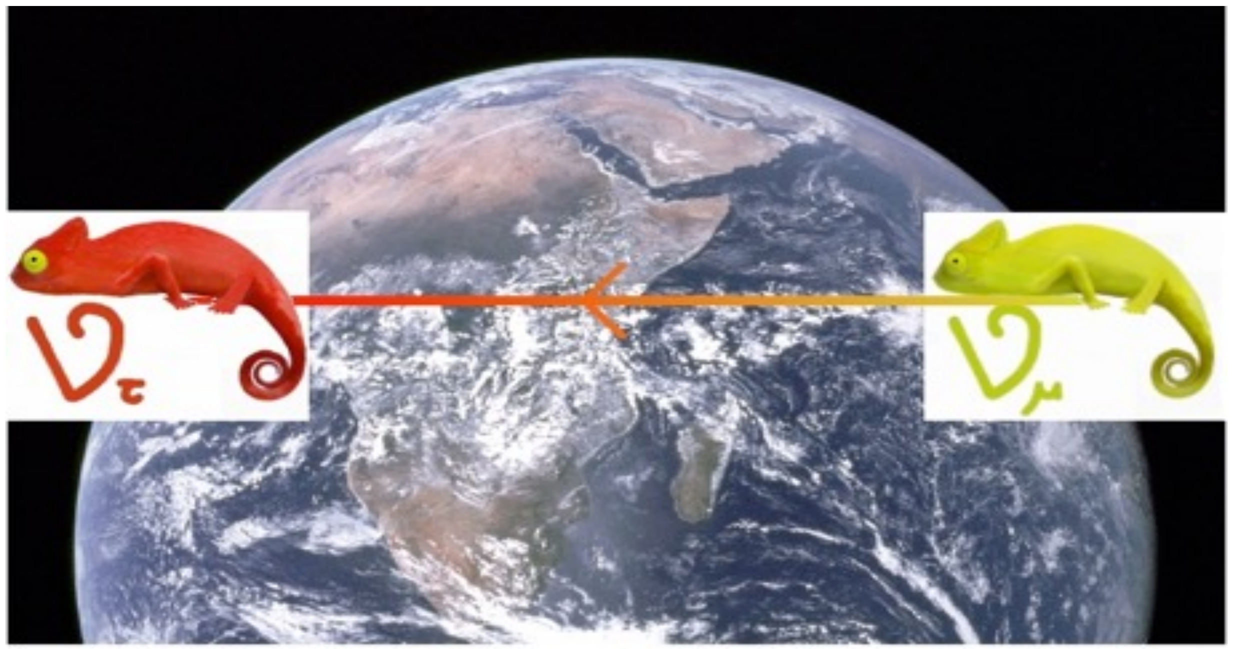
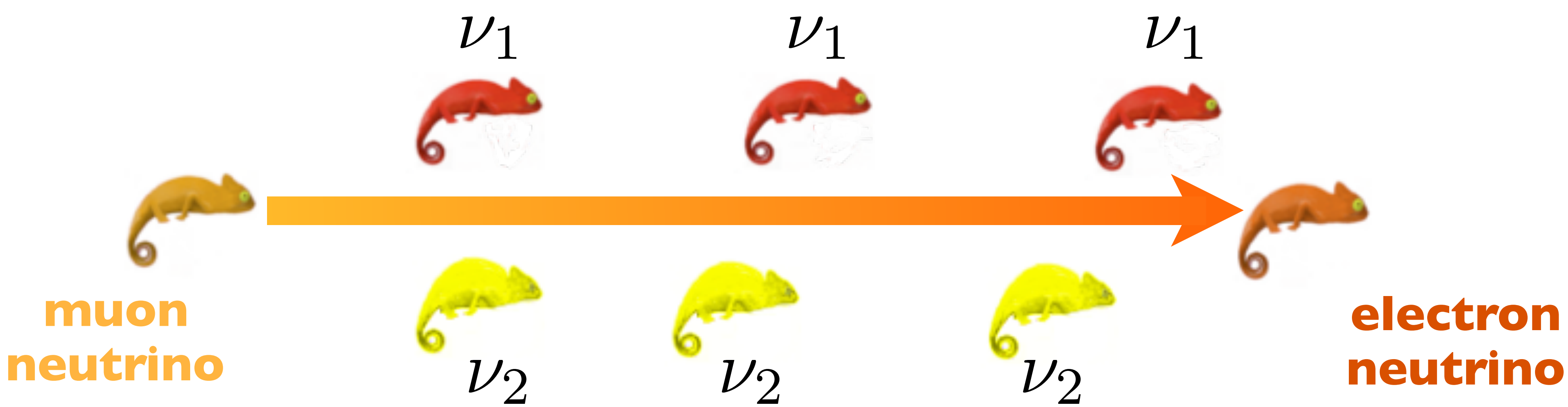
Neutrinos oscillations: the basic picture



Contrary to what expected in the SM, neutrinos oscillate: after being produced, they can **change their flavour**.



Neutrino oscillations imply that neutrinos have mass and they mix.
First evidence of physics beyond the SM.

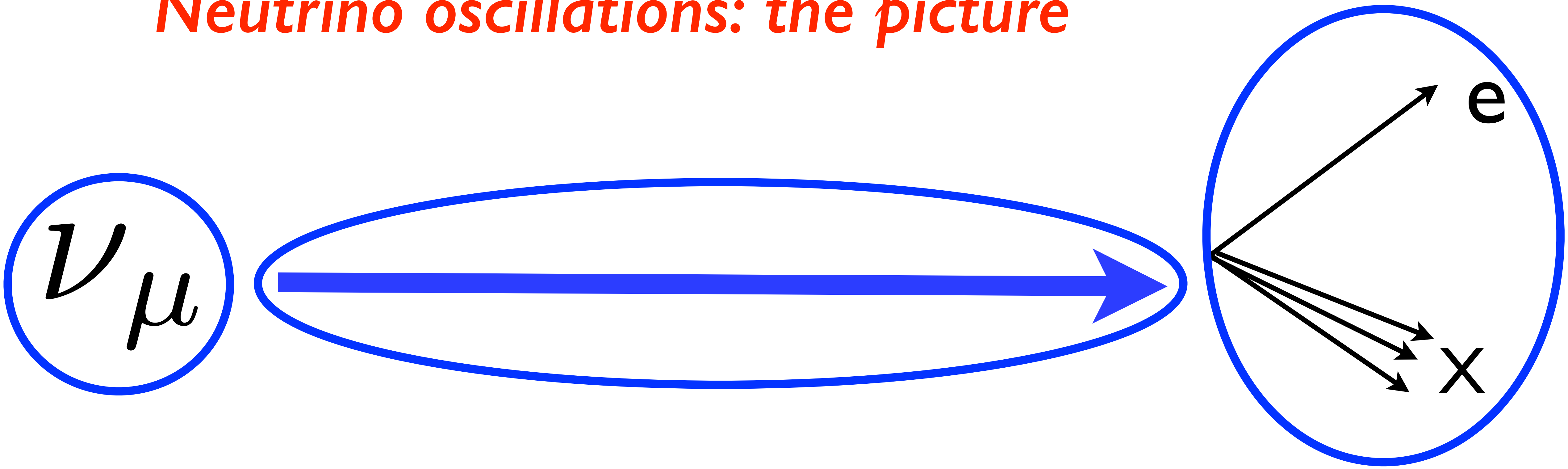
Neutrino oscillations and Quantum Mechanics analogs

Neutrino oscillations are analogous to many other systems in QM, in which the initial state is a **coherent superposition of eigenstates of the Hamiltonian**:

- NH3 molecule: produced in a superposition of "up" and "down" states

- Spin states: for example a state with spin up in the z-direction in a magnetic field aligned in the x-direction B=(B,0,0). This gives raise to spin-precession, i.e. the state changes the spin orientation with a typical oscillatory behaviour.

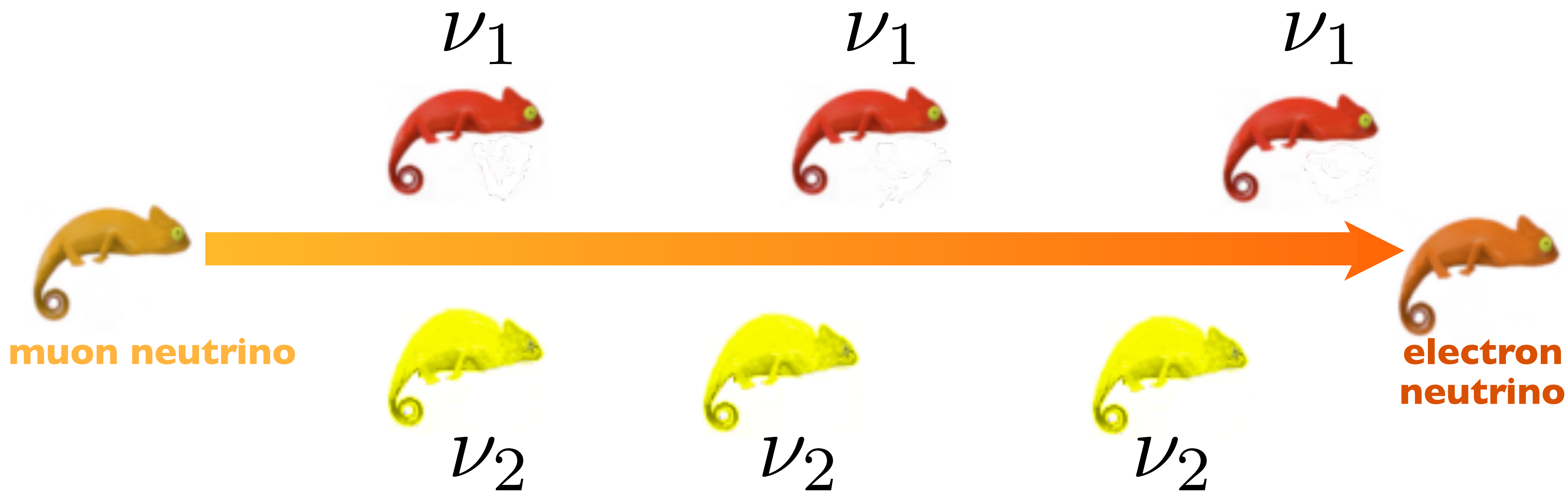
Neutrino oscillations: the picture



Production	Propagation	Detection
Flavour states	Massive states (eigenstates of the Hamiltonian)	Flavour states

At production, coherent superposition of massive states:

$$|\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle$$



Production

$$|\nu_\mu\rangle = \sum_i U_{\mu i} |\nu_i\rangle$$

Propagation

$$\nu_1 : \quad e^{-iE_1 t}$$
$$\nu_2 : \quad e^{-iE_2 t}$$
$$\nu_3 : \quad e^{-iE_3 t}$$

Detection: projection over

$$\langle \nu_e |$$

As the propagation phases are different, the state evolves with time and can change to other flavours.

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Neutrinos oscillations in vacuum: the theory

Let's assume that at t=0 a muon neutrino is produced

$$|\nu, t=0\rangle = |\nu_\mu\rangle = \sum_i U_{\mu i} |\nu_i\rangle$$

The time-evolution is given by the solution of the Schroedinger equation with free Hamiltonian:

$$|\nu, t\rangle = \sum_i U_{\mu i} e^{-iE_i t} |\nu_i\rangle$$

In the same-momentum approximation:

$$E_1 = \sqrt{p^2 + m_1^2} \quad E_2 = \sqrt{p^2 + m_2^2} \quad E_3 = \sqrt{p^2 + m_3^2}$$

Note: other derivations are also valid (same E formalism, etc).

At detection one projects over the flavour state as these are the states which are involved in the interactions. The probability of oscillation is

$$P(\nu_\mu \to \nu_\tau) = |\langle \nu_\tau | \nu, t\rangle|^2$$

$$= \left| \sum_{ij} U_{\mu i} U^*_{\tau j} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \right|^2$$

$$= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-iE_i t} \right|^2$$

Typically, neutrinos are very relativistic: $E_i \simeq p + \frac{m_i^2}{2p}$

$$= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-i\frac{m_i^2}{2E} t} \right|^2$$

Δm^2_{i1}

$$= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-i\frac{m_i^2 - m_1^2}{2E} t} \right|^2$$

Exercise
Derive

Implications of the existence of neutrino oscillations

The oscillation probability implies that

$$P(\nu_\alpha \to \nu_\beta) \quad = \quad \left| \sum_i U_{\alpha 1} U^*_{\beta 1} e^{-i \frac{\Delta m^2_{i1}}{2E} L} \right|^2$$

- **neutrinos have mass** (as the different components of the initial state need to propagate with different phases)

- **neutrinos mix** (as U needs not be the identity. If they do not mix the flavour eigenstates are also eigenstates of the propagation Hamiltonian and they do not evolve)

General properties of neutrino oscillations

- Neutrino oscillations conserve the total lepton number: a neutrino is produced and evolves with times

- They violate the flavour lepton number as expected due to mixing.

- Neutrino oscillations do not depend on the overall mass scale and on the Majorana phases.

- CPT invariance: $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$

- CP-violation:

$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ requires $U \neq U^*(\delta \neq 0, \pi)$

2-neutrino case

Let's recall that the mixing is

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

We compute the probability of oscillation

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} \right|^2$$

$$= \left| \cos\theta \sin\theta - \cos\theta \sin\theta e^{-i \frac{\Delta m^2_{21}}{2E} L} \right|^2$$

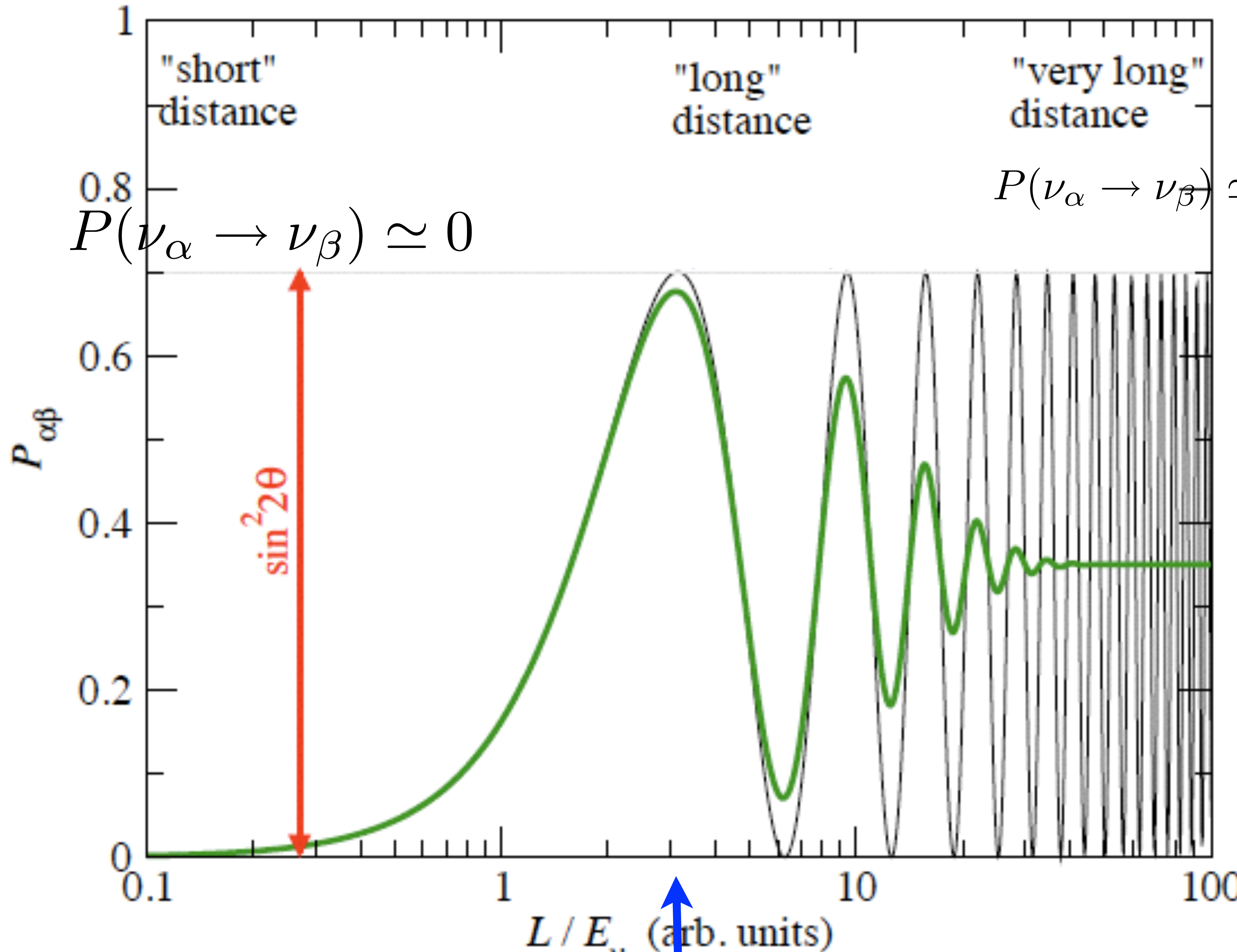
$$= \sin^2(2\theta) \sin^2(\frac{\Delta m^2_{21}}{4E} L)$$

$$\frac{\Delta m^2_{21}}{4E} L = 1.27 \, \frac{\Delta m^2_{21} [\mathrm{eV}^2]}{4 \; E[\mathrm{GeV}]} L[\mathrm{km}]$$

Exercise
Derive

$$P(\nu_\alpha \to \nu_\beta) \simeq 0$$

$$P(\nu_\alpha \to \nu_\beta) \simeq \frac{1}{2}\sin^2(2\theta)$$



First oscillation maximum

Thanks to T. Schwetz

Properties of 2-neutrino oscillations

- Appearance probability:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- Disappearance probability:

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- No CP-violation as there is no Dirac phase in the mixing matrix

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$$

- Consequently, no T-violation (using CPT):

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\nu_\beta \rightarrow \nu_\alpha)$$

3-neutrino oscillations

They depend on two mass squared-differences

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

In general the formula is quite complex

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

Interesting 2-neutrino limits

For a given L, the neutrino energy determines the impact of a mass squared difference. Various limits are of interest in concrete experimental situations.

- $\boxed{\frac{\Delta m^2_{21}}{4E} L \ll 1}$, applies to atmospheric, reactor (Daya Bay...), current accelerator neutrino experiments...

The oscillation probability reduces to a 2-neutrino limit:

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1}U_{\beta 1}^* + U_{\alpha 2}U_{\beta 2}^* + U_{\alpha 3}U_{\beta 3}^* e^{-i\frac{\Delta m_{31}^2}{2E}L} \right|^2$$

We use the fact that $U_{\alpha 1}U_{\beta 1}^* + U_{\alpha 2}U_{\beta 2}^* + U_{\alpha 3}U_{\beta 3}^* = \delta_{\alpha\beta}$

$$= \left| -U_{\alpha 3}U_{\beta 3}^* + U_{\alpha 3}U_{\beta 3}^* e^{-i\frac{\Delta m_{31}^2}{2E}L} \right|^2$$

$$= \left| U_{\alpha 3}U_{\beta 3}^* \right|^2 \left| -1 + e^{-i\frac{\Delta m_{31}^2}{2E}L} \right|^2$$

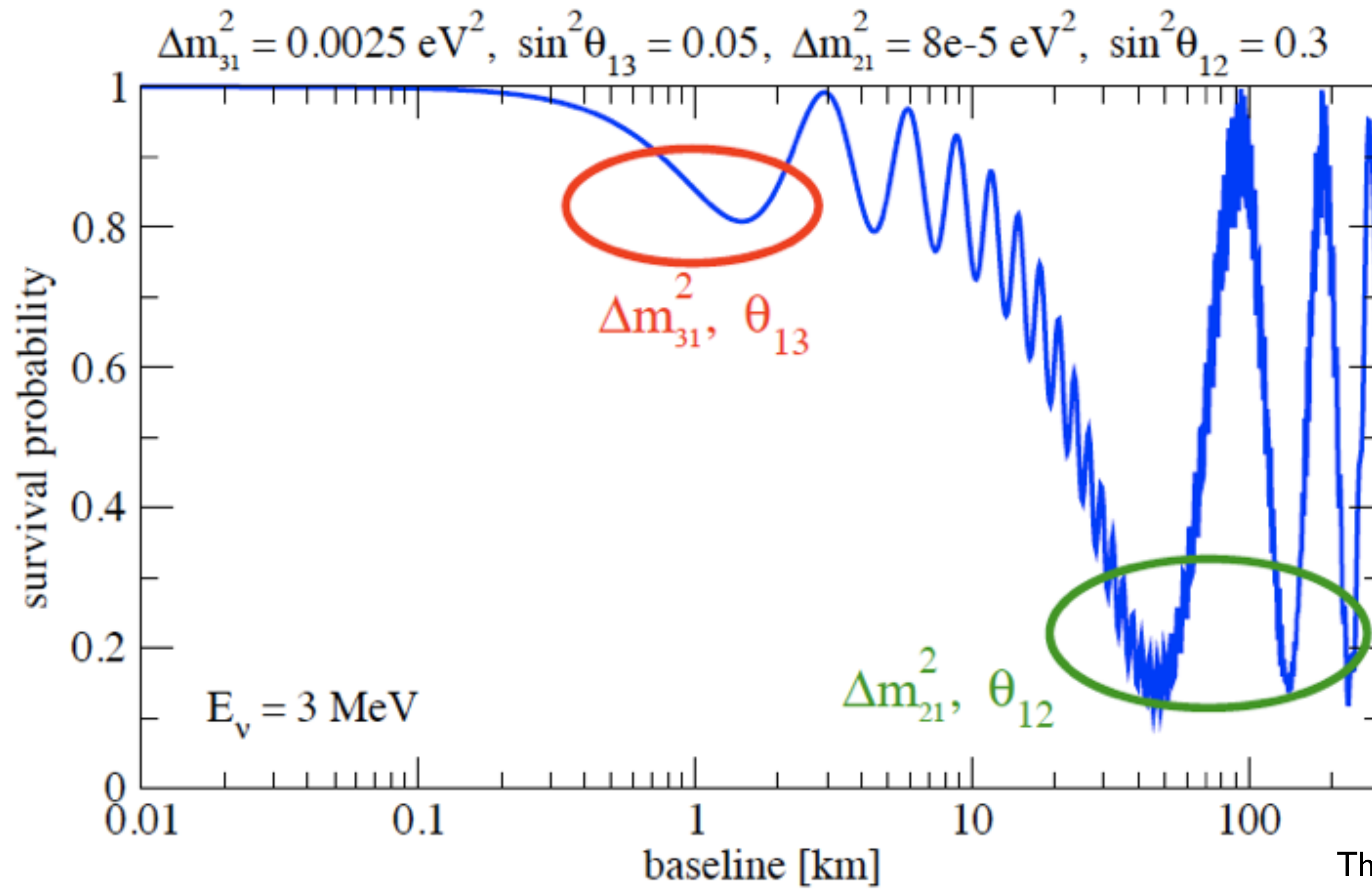
The same we have encountered in the 2-neutrino case

$$= 2\left| U_{\alpha 3}U_{\beta 3} \right|^2 \sin^2\left(\frac{\Delta m_{31}^2}{4E}L\right)$$

Exercise
Derive

- $\frac{\Delta m_{31}^2}{4E} L \gg 1$: for reactor neutrinos (KamLAND). The oscillations due to the atmospheric mass squared differences get averaged out.

$$P(\bar{\nu}_e \to \bar{\nu}_e; t) \simeq c_{13}^4 \left(1 - \sin^2(2\theta_{12}) \sin^2 \frac{\Delta m_{21}^2 L}{4E}\right) + s_{13}^4$$



Thanks to T. Schwetz

CP-violation will manifest itself in neutrino oscillations, due to the delta phase. Let's consider the CP-asymmetry:

$$P(\nu_\alpha \rightarrow \nu_\beta; t) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; t) =$$

$$= \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i\frac{\Delta m^2_{21} L}{2E}} + U_{\alpha 3} U^*_{\beta 3} e^{-i\frac{\Delta m^2_{31} L}{2E}} \right|^2 - (U \rightarrow U^*)$$

$$= U_{\alpha 1} U^*_{\beta 1} U^*_{\alpha 2} U_{\beta 2} e^{i\frac{\Delta m^2_{21} L}{2E}} + U^*_{\alpha 1} U_{\beta 1} U_{\alpha 2} U^*_{\beta 2} e^{-i\frac{\Delta m^2_{21} L}{2E}} - (U \rightarrow U^*) + \cdots$$

$$= 4 s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin\delta \left[\sin\left(\frac{\Delta m^2_{21} L}{2E}\right) + \left(\frac{\Delta m^2_{23} L}{2E}\right) + \left(\frac{\Delta m^2_{31} L}{2E}\right) \right]$$

Exercise**
Derive

- CP-violation requires all angles to be nonzero.
- It is proportional to the sine of the delta phase.
- If one can neglect Δm^2_{21}, the asymmetry goes to zero as we have seen that effective 2-neutrino probabilities are CP-symmetric.

Further theoretical issues on neutrino oscillations

Energy-momentum conservation

Let's consider for simplicity a 2-body decay: $\pi \to \mu \, \bar{\nu}_\mu$.

Energy-momentum conservation seems to require:

$$E_\pi = E_\mu + E_1 \quad \text{with } E_1 = \sqrt{p^2 + m_1^2}$$
$$E_\pi = E_\mu + E_2 \quad \text{with } E_2 = \sqrt{p^2 + m_2^2}$$

?

How can the picture be consistent?

Further theoretical issues on neutrino oscillations

Energy-momentum conservation

Let’s consider for simplicity a 2-body decay: $\pi \to \mu\, \bar{\nu}_\mu$.

Energy-momentum conservation seems to require:

$$E_\pi = E_\mu + E_1 \quad \text{with } E_1 = \sqrt{p^2 + m_1^2}$$
$$E_\pi = E_\mu + E_2 \quad \text{with } E_2 = \sqrt{p^2 + m_2^2}$$

These two requirements seems to be incompatible. Intrinsic quantum uncertainty, localisation of the initial pion lead to an uncertainty in the energy-momentum and allow coherence of the initial neutrino state.

- If the energy and/or momentum of the muon is measured with great precision, then coherence is lost and only neutrino $\nu 1$ (or $\nu 2$) is produced.

- In any typical experimental situation, this is not the case and neutrino oscillations take place.

- However for large mass differences, e.g. in presence of heavy sterile neutrinos, this situation could arise.

For a detailed discussion see, Akhmedov, Smirnov, 1008.2077.

The need for wavepackets

- In deriving the oscillation formulas we have implicitly assumed that neutrinos can be described by plane-waves, with definite momentum.

- However, production and detection are well localised and very distant from each other. This leads to a momentum spread which can be described by a wave-packet formalism.

Typical sizes:
- e.g. production in decay: the relevant timescale is the pion lifetime (or the time travelled in the decay pipe),

$$\Delta t \sim \tau_\pi \Rightarrow \Delta E \Rightarrow \Delta p \quad \Delta x$$

For details see, Akhmedov, Smirnov, 1008.2077; Giunti and Kim, Neutrino Physics and Astrophysics.

Decoherence and the size of a wave-packet

- The different components of the wavepacket, $\nu 1$, $\nu 2$ and $\nu 3$, travel with slightly different velocities (as their mass is different).

- If the neutrinos travel extremely long distances, these components stop to overlap, destroying coherence and oscillations.

- In terrestrial experimental situation this is not relevant. But this can happen for example for supernovae neutrinos.

Plan of lecture I

- A bit of history: from the initial idea to the solar and atmospheric neutrino anomalies

- The basic picture of neutrino oscillations (mixing of states and coherence)

- The formal details: how to derive the probabilities

- Neutrino oscillations both in vacuum and in matter

- Their relevance in present and future experiments

Neutrinos oscillations in matter

- When neutrinos travel through a medium, they interact with the background of electron, proton and neutrons and acquire an effective mass.

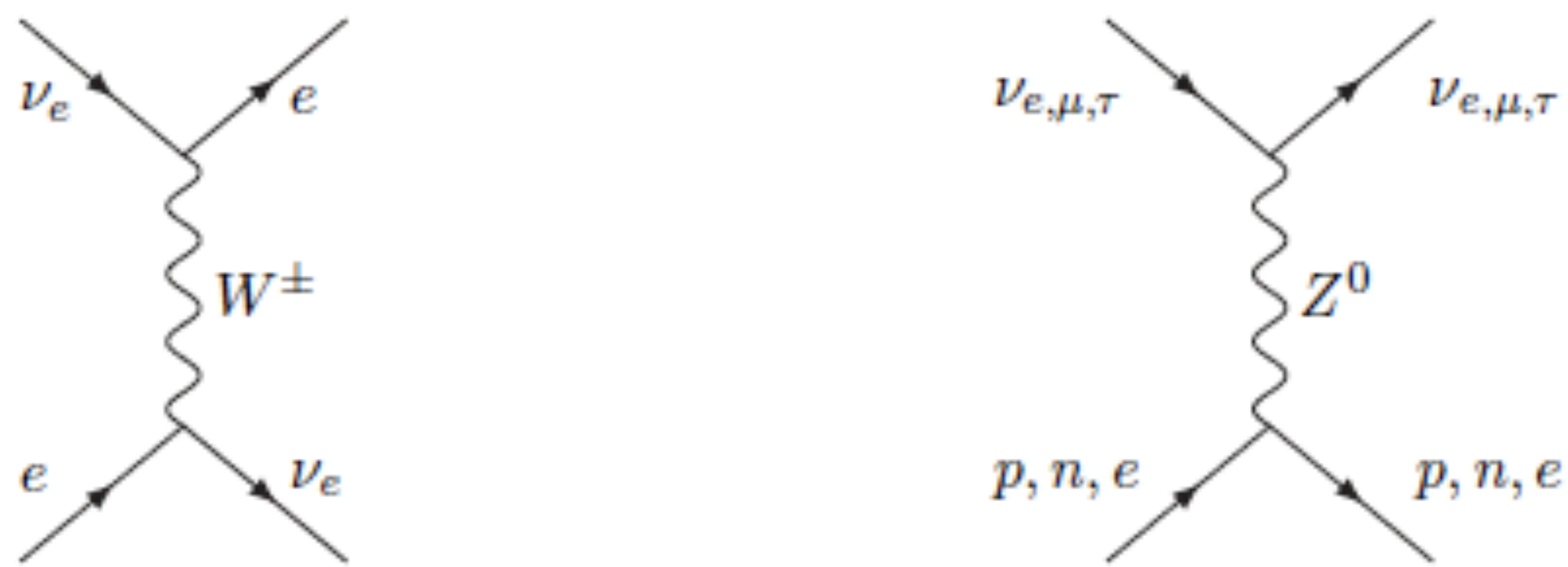
- This modifies the mixing between flavour states and propagation states and the eigenvalues of the Hamiltonian, leading to a different oscillation probability w.r.t. vacuum.

- Typically the background is CP and CPT violating, e.g. the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations are CP and CPT violating.

Effective potentials

Inelastic scattering and absorption processes go as G_F^2 and are typically negligible. Neutrinos undergo also forward elastic scattering, in which they do not change momentum. [L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); ibid. D 20, 2634 (1979), S. P. Mikheyev, A. Yu Smirnov, Sov. J. Nucl. Phys. 42 (1986) 913.]

Electron neutrinos have CC and NC interactions, while muon and tau neutrinos only the latter.



For a useful discussion, see E. Akhmedov, hep-ph/0001264; A. de Gouvea, hep-ph/0411274.

We treat the electrons as a background, averaging over it and we take into account that neutrinos see only the left-handed component of the electrons.

$$\langle \bar{e}\gamma_0 e\rangle = N_e \quad \langle \bar{e}\vec{\gamma} e\rangle = \langle \vec{v}_e\rangle \quad \langle \bar{e}\gamma_0\gamma_5 e\rangle = \langle \frac{\vec{\sigma}_e \cdot \vec{p}_e}{E_e}\rangle \quad \langle \bar{e}\vec{\gamma}\gamma_5 e\rangle = \langle \vec{\sigma}_e\rangle$$

For an unpolarised at rest background, the only term is the first one. N_e is the electron density.

The neutrino dispersion relation can be found by solving the Dirac eq with plane waves, in the ultrarelativistic limit

$$E \simeq p \pm \sqrt{2} G_F N_e$$

medium	A_{CC} for $\nu_e, \bar{\nu}_e$ only	A_{NC} for $\nu_{e,\mu,\tau}, \bar{\nu}_{e,\mu,\tau}$
$e, \bar{e}$	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\sqrt{2}G_F(N_e - N_{\bar{e}})(1-4s_W^2)/2$
$p, \bar{p}$	0	$\pm\sqrt{2}G_F(N_p - N_{\bar{p}})(1-4s_W^2)/2$
$n, \bar{n}$	0	$\mp\sqrt{2}G_F(N_n - N_{\bar{n}})/2$
ordinary matter	$\pm\sqrt{2}G_F N_e$	$\mp\sqrt{2}G_F N_n/2$

Strumia and Vissani

The Hamiltonian

Let's start with the vacuum Hamiltonian for 2-neutrinos

$$i\frac{d}{dt}\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

Recalling that $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$, one can go into the flavour basis

$$\begin{aligned} i\frac{d}{dt}\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} &= U\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}U^\dagger\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \\ &= \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} \end{aligned}$$

We have neglected common terms on the diagonal as they amount to an overall phase in the evolution.

The full Hamiltonian in matter can then be obtained by adding the potential terms, diagonal in the flavour basis. For electron and muon neutrinos

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

For antineutrinos the potential has the opposite sign.

In general the evolution is a complex problem but there are few cases in which analytical or semi-analytical results can be obtained.

2-neutrino case in constant density

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.

- Eigenvalues:

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin(2\theta)\right)^2}$$

- The diagonal basis and the flavour basis are related by a unitary matrix with angle in matter

Exercise
Derive

$$\tan(2\theta_m) = \frac{\frac{\Delta m^2}{2E}\sin(2\theta)}{\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e}$$

- If $\sqrt{2}G_F N_e \ll \frac{\Delta m^2}{2E}\cos 2\theta$, we recover the vacuum case and

$$\theta_m \simeq \theta$$

- If $\sqrt{2}G_F N_e \gg \frac{\Delta m^2}{2E}\cos(2\theta)$, matter effects dominate and oscillations are suppressed.

- If $\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta$: resonance and maximal mixing

$$\theta_m = \pi/4$$

- The resonance condition can be satisfied for
  - neutrinos if $\Delta m^2 > 0$
  - antineutrinos if $\Delta m^2 < 0$

$$P(\nu_e \to \nu_\mu; t) = \sin^2(2\theta_m)\sin^2\frac{(E_A - E_B)L}{2}$$

2-neutrino oscillations with varying density

Let's consider the case in which N_e depends on time. This happens, e.g., if a beam of neutrinos is produced and then propagates through a medium of varying density (e.g. Sun, supernovae).

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e(t) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

At a given instant of time t, the Hamiltonian can be diagonalised by a unitary transformation as before. We find the instantaneous matter basis and the instantaneous values of the energy. The expressions are exactly as before but with the angle which depends on time, $\theta(t)$.

We have

$$|\nu_\alpha\rangle = U(t)|\nu_I\rangle, \qquad U^\dagger(t) H_{m,fl} U(t) = \mathrm{diag}(E_A(t), E_B(t))$$

Starting from the Schroedinger equation, we can express it in the instantaneous basis

$$i\frac{d}{dt} U_m(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2} G_F N_e(t) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} U_m(t) \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} E_A(t) & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & E_B(t) \end{pmatrix} \begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

The evolution of ν_A and ν_B are not decoupled. In general, it is very difficult to find an analytical solution to this problem.

Adiabatic case

In the adiabatic case, each component evolves independently. In the non adiabatic one, the state can "jump" from one to the other.

If the evolution is sufficiently slow (adiabatic case):

$$|\dot{\theta}(t)| \ll | E_A - E_B|$$

we can follow the evolution of each component independently.

Adiabaticity condition

$$\gamma^{-1} \equiv \frac{2|\dot{\theta}|}{|E_A - E_B|} = \frac{\sin(2\theta)\frac{\Delta m^2}{2E}}{|E_A - E_B|^3}|\dot{V}_{CC}| \ll 1$$

In the Sun, typically we have $\gamma \sim \frac{\Delta m^2}{10^{-9}\text{eV}^2}\frac{\text{MeV}}{E_\nu}$

Solar neutrinos: MSW effect

The oscillations in matter were first discussed by L. Wolfenstein, S. P. Mikheyev, A. Yu Smirnov.

- Production in the centre of the Sun: matter effects dominate at high energy, negligible at low energy.

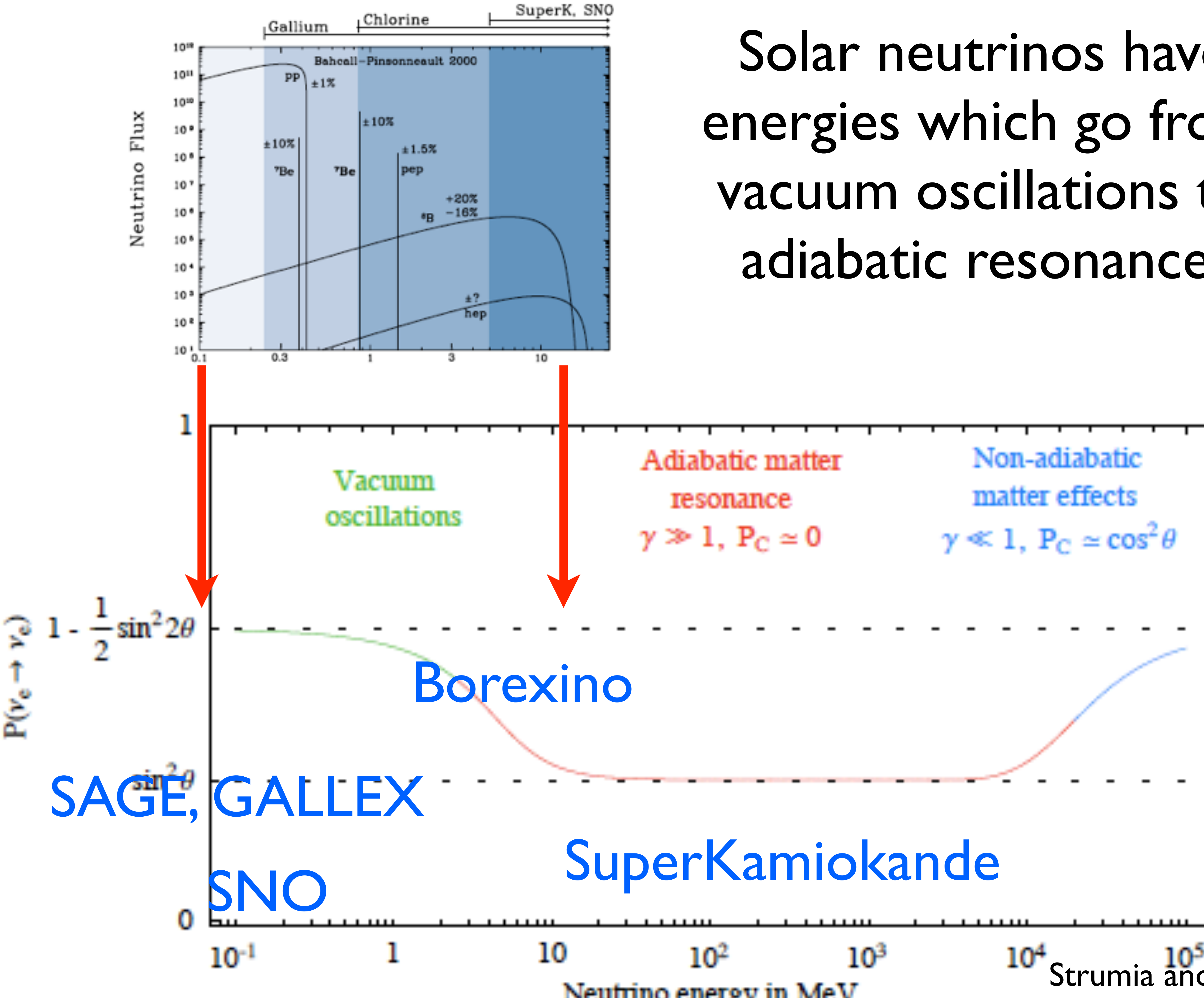
The probability of ν_e to be

ν_A	is	$\cos^2\theta_m$
ν_B	is	$\sin^2\theta_m$

If matter effects dominate, $\sin^2\theta_m \simeq 1$

- $P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2(2\theta)$ (averaged vacuum oscillations), when matter effects are negligible (low energies)
- $P(\nu_e \to \nu_e) = \sin^2\theta$ (dominant matter effects and adiabaticity) (high energies)

Solar neutrinos have energies which go from vacuum oscillations to adiabatic resonance.

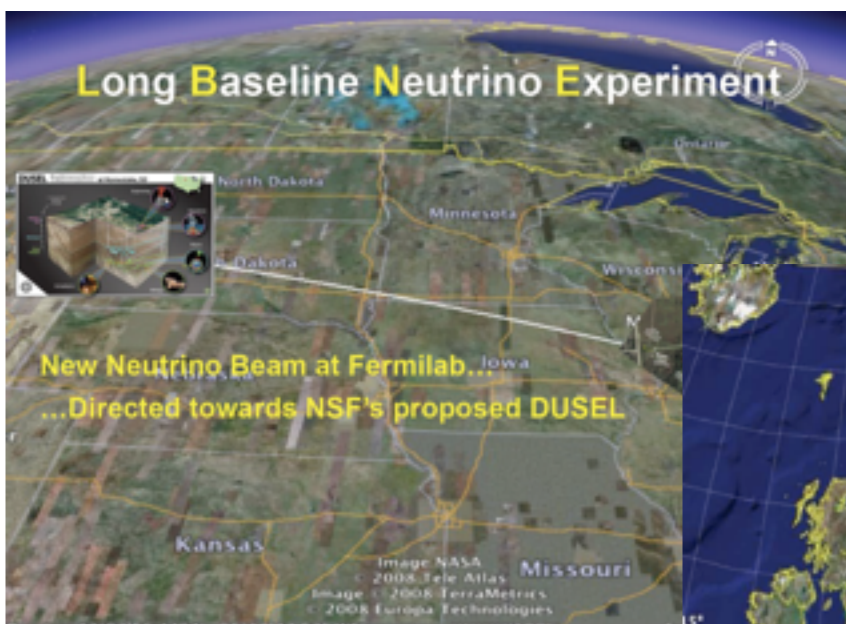


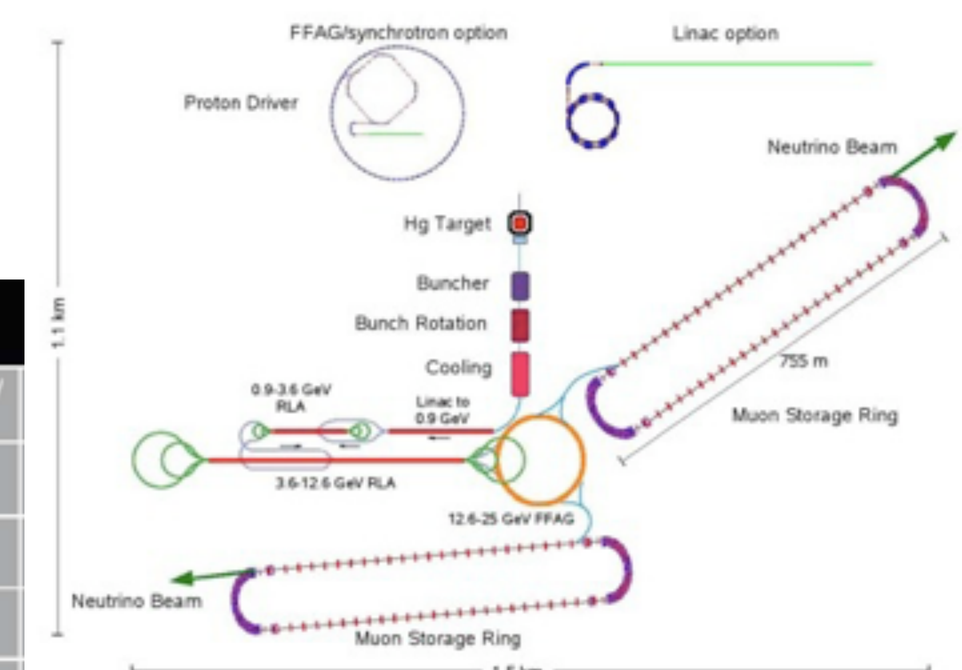
Strumia and Vissani

3-neutrino oscillations in the crust

There are long-baseline neutrino experiments which look for oscillations $\nu_\mu \Rightarrow \nu_e$ both for CPV and matter effects.

For distances, 100-3000 km, we can assume that the Earth has constant density, but we need to take into account 3-nu effects. For longer distances more complex matter effects.



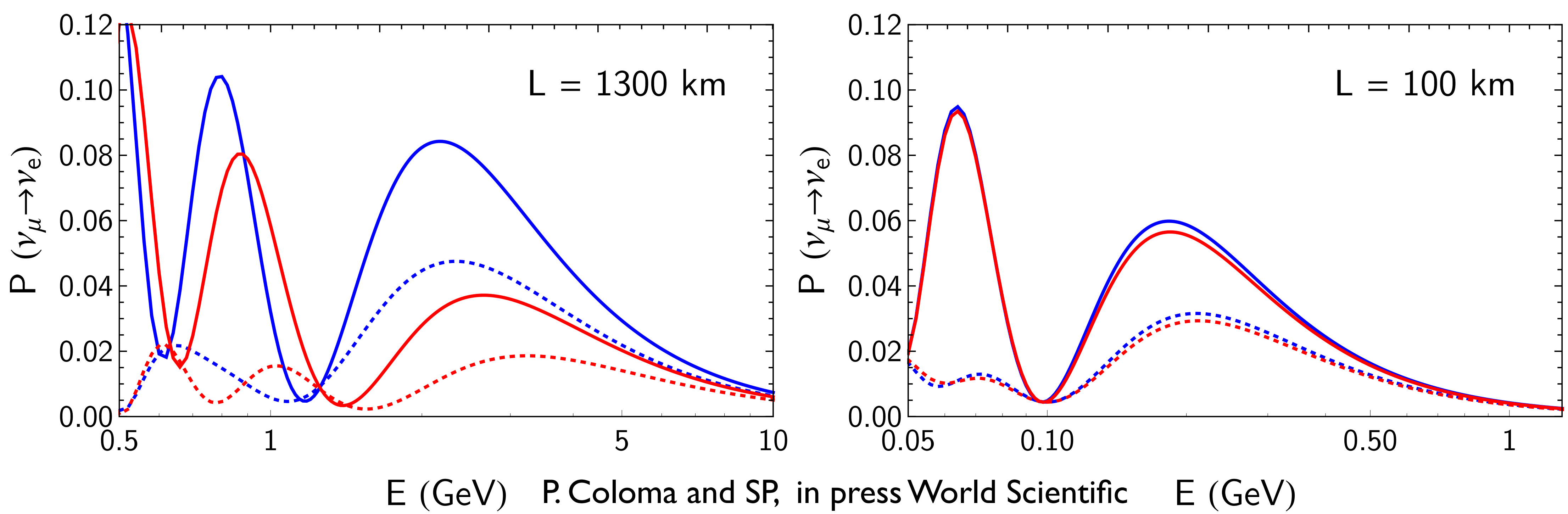


One can compute the probability by expanding the full 3-neutrino oscillation probability in the small parameters $\theta_{13}, \Delta m^2_{\rm sol}/\Delta m^2_A$.

$$P_{\mu e} \simeq 4 c_{23}^2 s_{13}^2 \frac{1}{(1-r_A)^2} \sin^2 \frac{(1-r_A)\Delta_{31} L}{4E}$$
$$+ \sin 2\theta_{12} \sin 2\theta_{23} s_{13} \frac{\Delta_{21} L}{2E} \sin \frac{(1-r_A)\Delta_{31} L}{4E} \cos\left(\delta - \frac{\Delta_{31} L}{4E}\right)$$
$$+ s_{23}^2 \sin^2 2\theta_{12} \frac{\Delta_{21}^2 L^2}{16 E^2} - 4 c_{23}^2 s_{13}^4 \sin^2 \frac{(1-r_A)\Delta_{31} L}{4E}$$

$$r_A \equiv \frac{2E}{\Delta m^2_{31}} \sqrt{2} G_F N_e$$

A. Cervera et al., hep-ph/0002108; K. Asano, H. Minakata, 1103.4387; S. K. Agarwalla et al., 1302.6773...



P. Coloma and SP, in press World Scientific

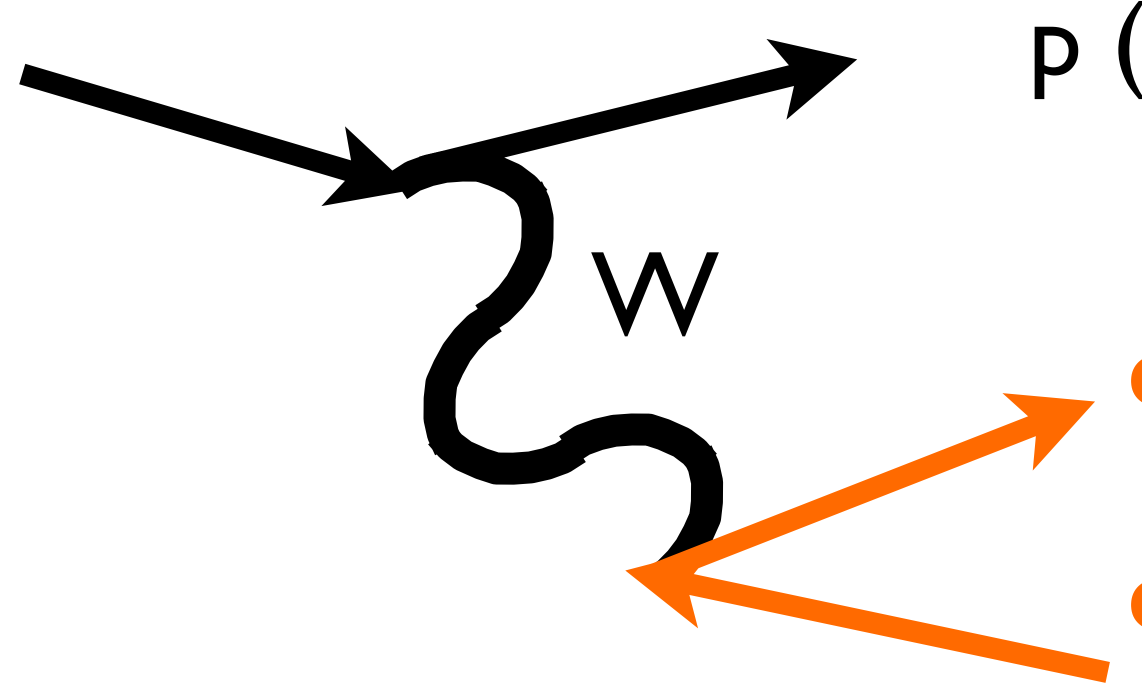
Plan of lecture I

- A bit of history: from the initial idea to the solar and atmospheric neutrino anomalies
- The basic picture of neutrino oscillations (mixing of states and coherence)
- The formal details: how to derive the probabilities
- Neutrino oscillations both in vacuum and in matter
- Their relevance in present and future experiments

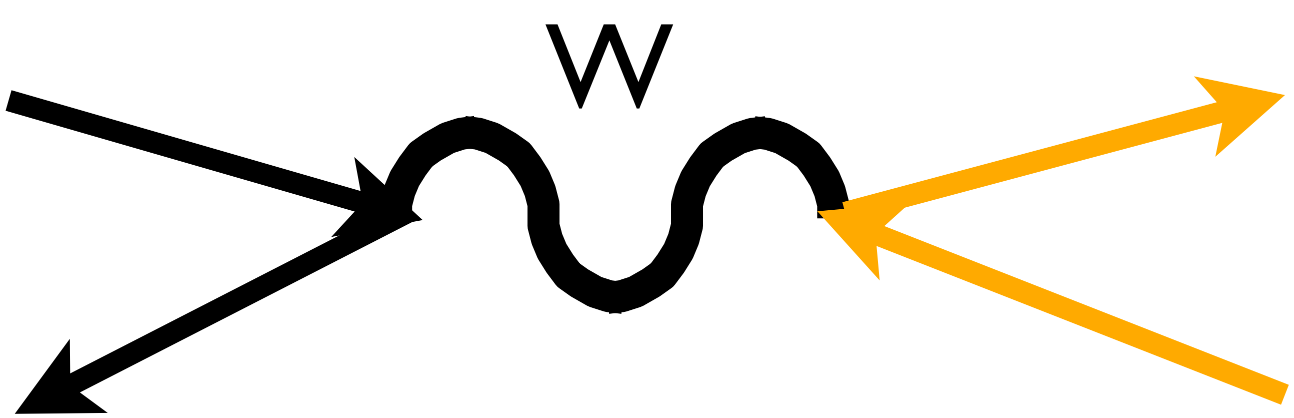
Neutrinos oscillations in experiments

Neutrino production

In CC (NC) SU(2) interactions, the W boson (Z boson) will be exchanged leading to the production of neutrinos.



Beta decay.

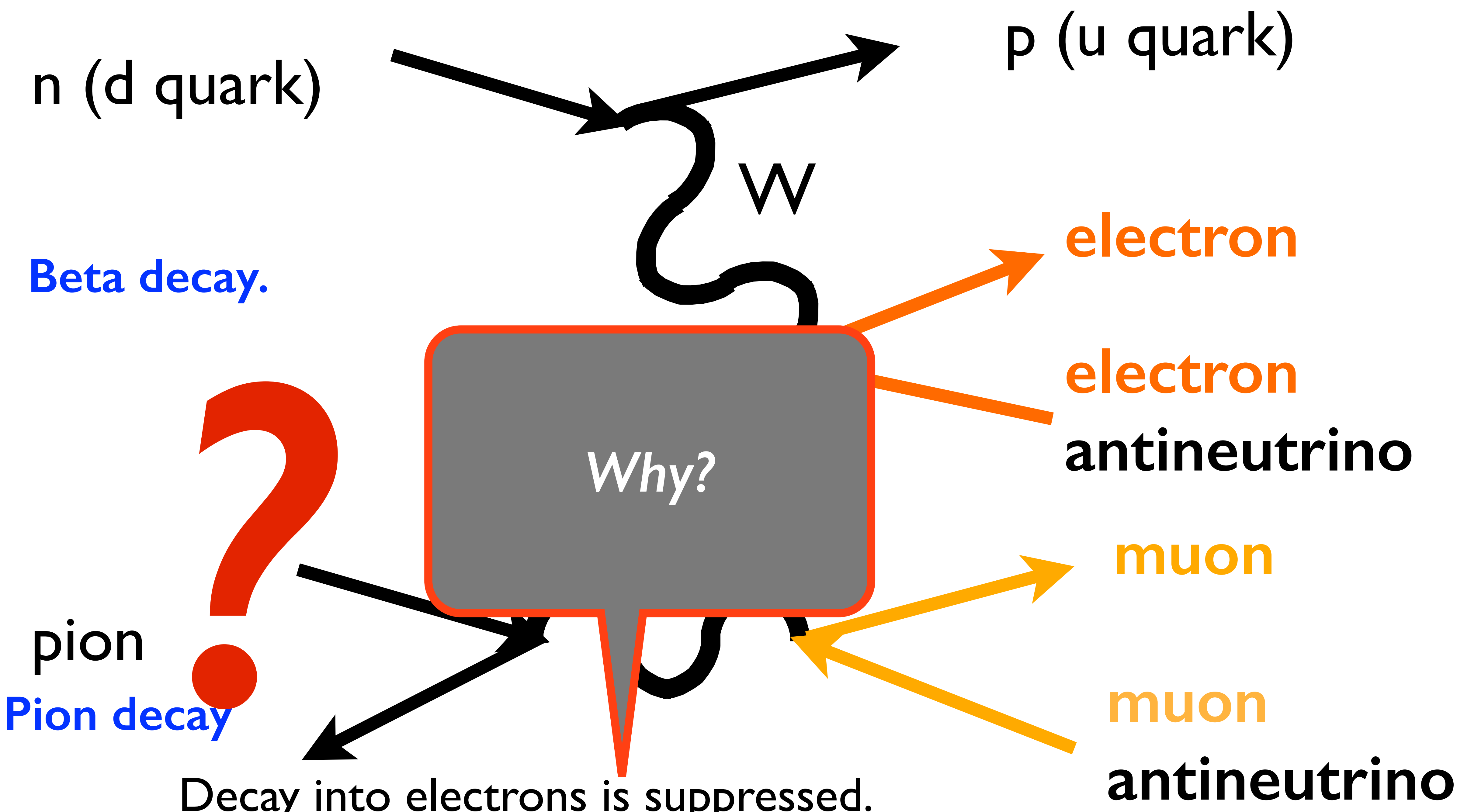


Pion decay

Decay into electrons is suppressed.

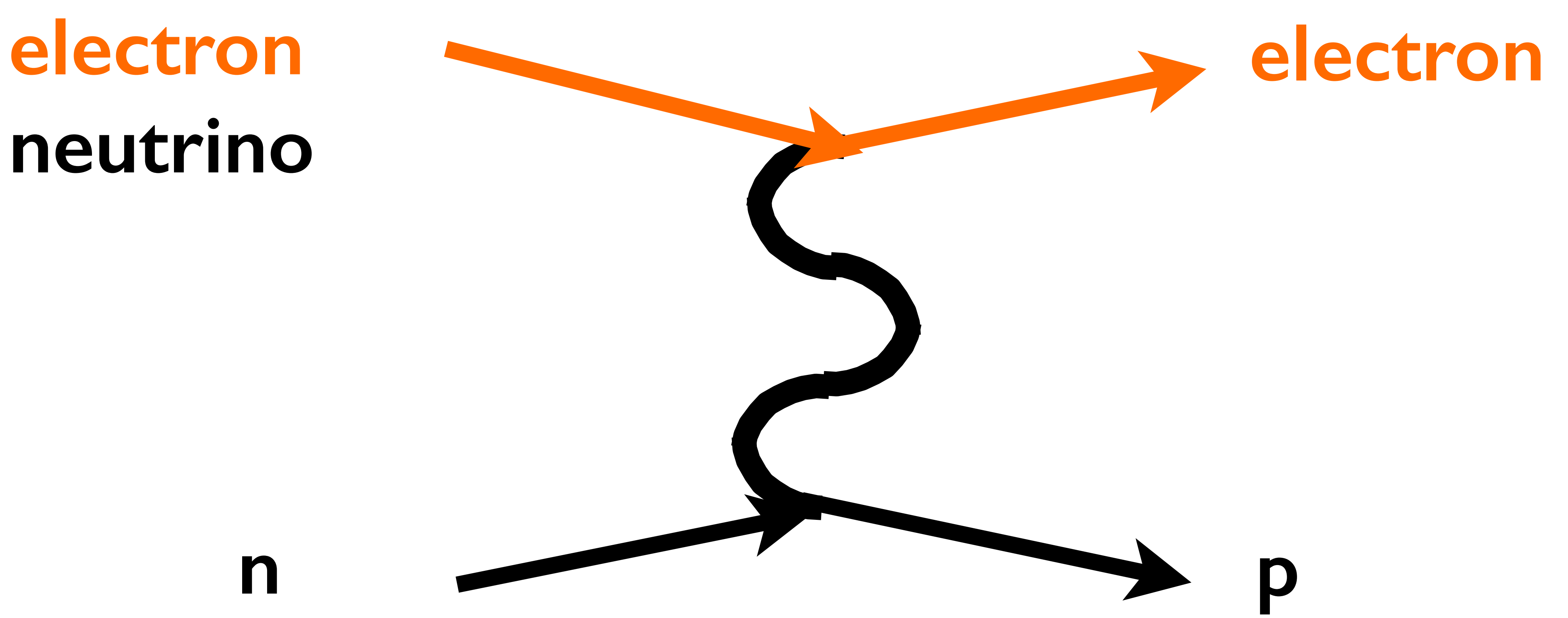
Neutrino production

In CC (NC) SU(2) interactions, the W boson (Z boson) will be exchanged leading to the production of neutrinos.



Neutrino detection

Neutrino detection proceeds via CC (and NC) SU(2) interactions. Example:



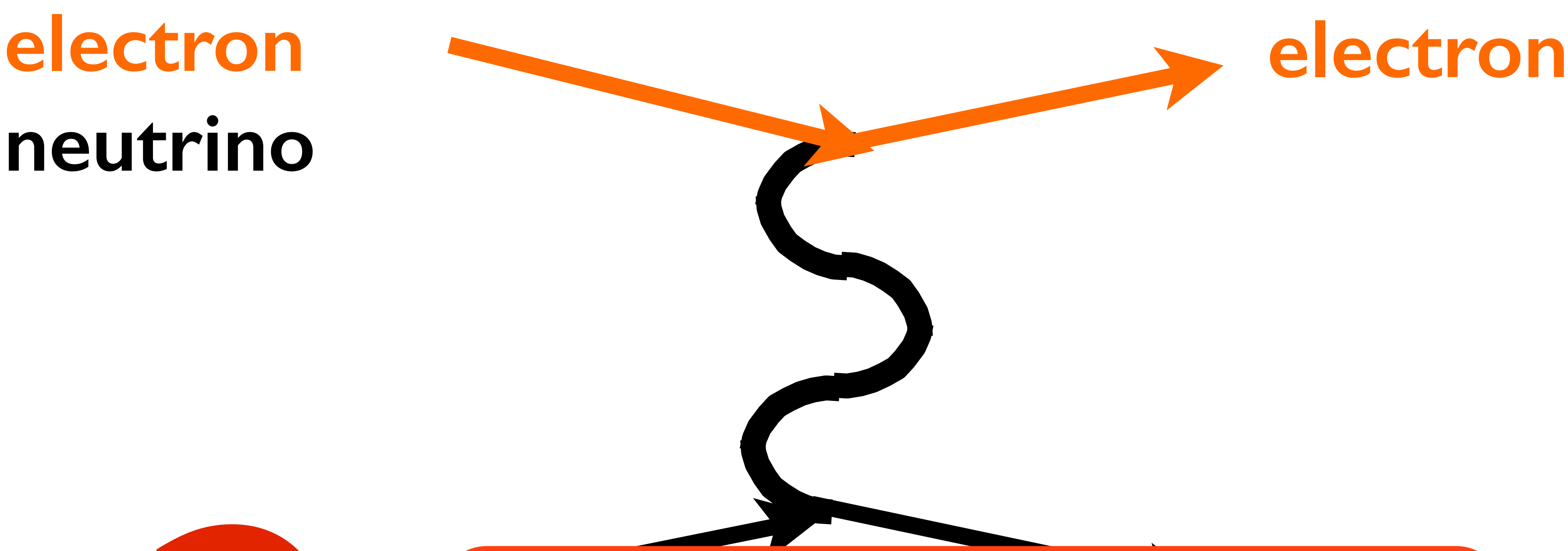
Notice that the leptons have different masses:

m_e = 0.5 MeV < m_{mu} = 105 MeV < m_{tau}= 1700 MeV

A certain lepton will be produced in a CC only if the neutrino has sufficient energy.

Neutrino detection

Neutrino detection proceeds via CC (and NC) SU(2) interactions. Example:

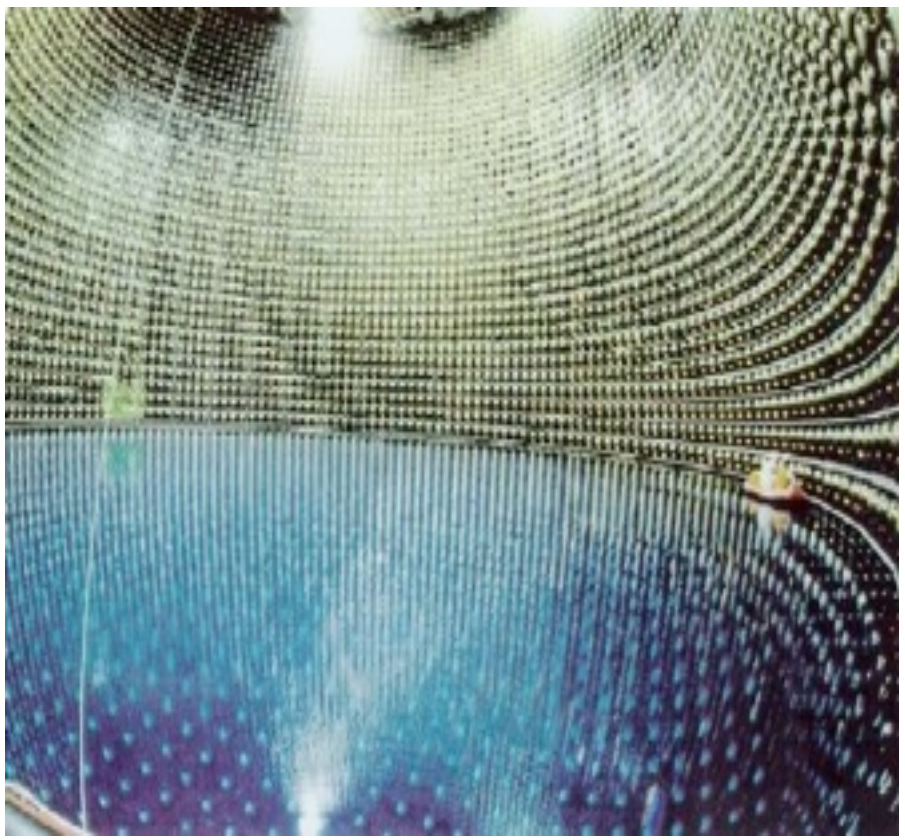


Notice that the ... m_e = 0.5 MeV < ... MeV

Can a 3 MeV reactor neutrino produce a muon in a CC interaction?

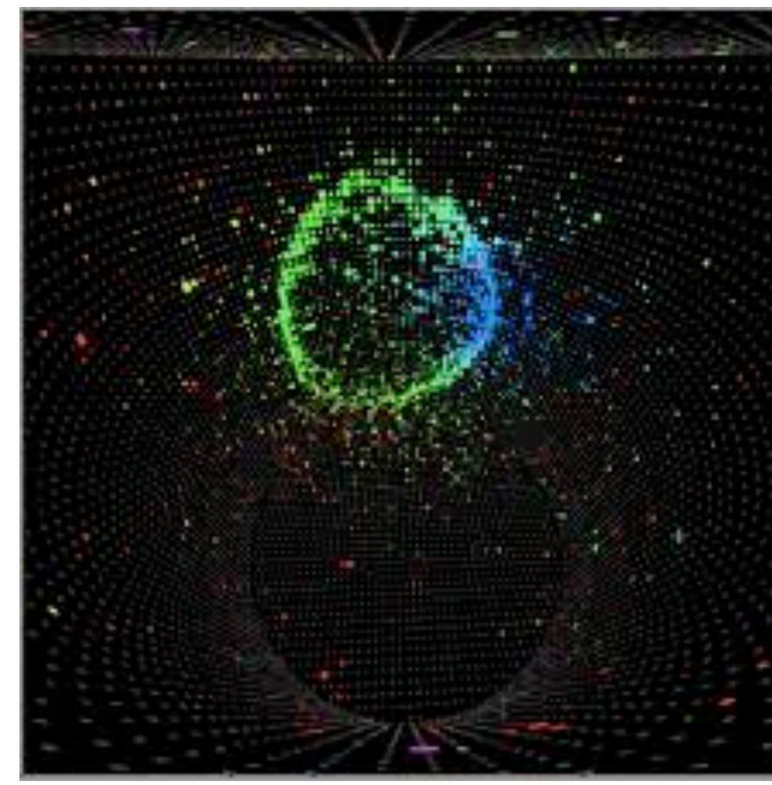
A certain lepton will be produced in a CC only if the neutrino has sufficient energy.

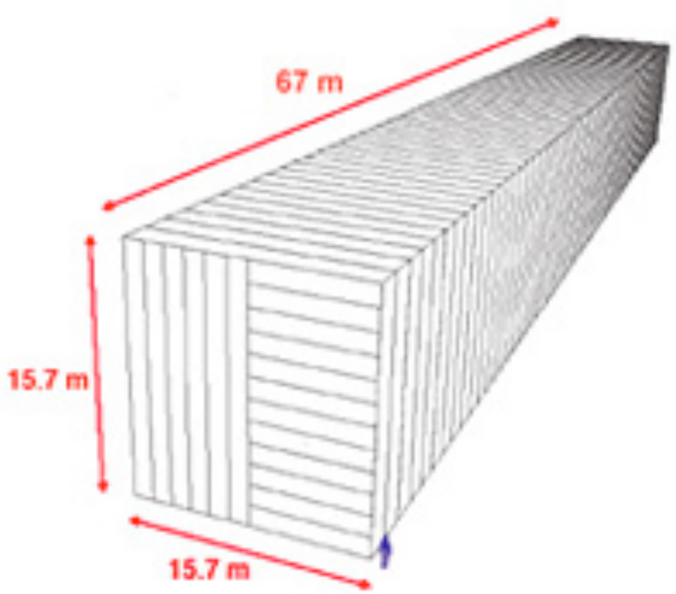
We are interested mainly in produced charged particles as these can emit light and/or leave tracks in segmented detectors (magnetisation -> charge reconstruction).



Super-Kamiokande detector

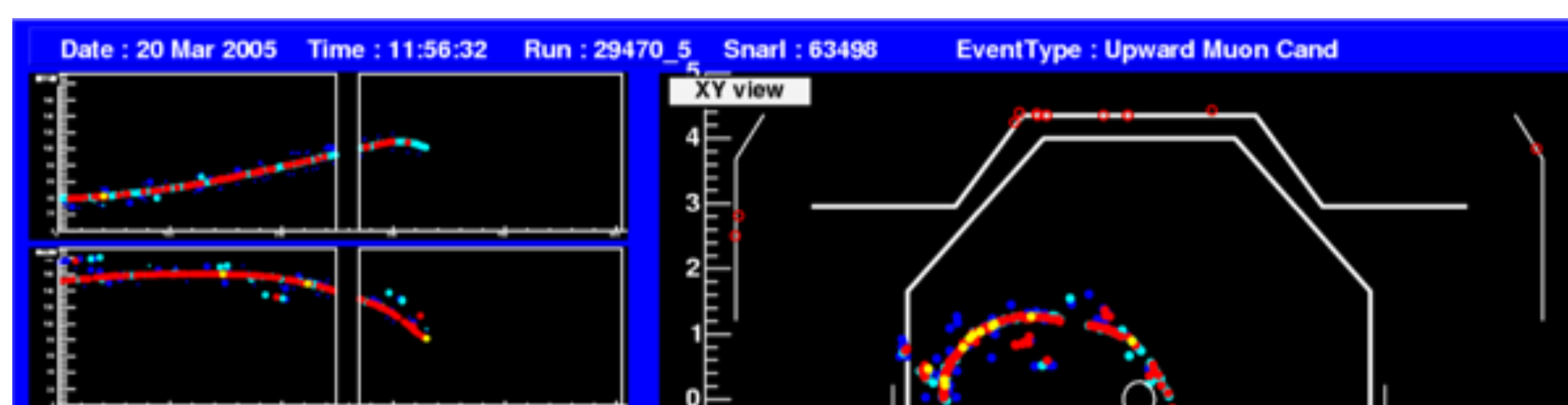
T2K experiment



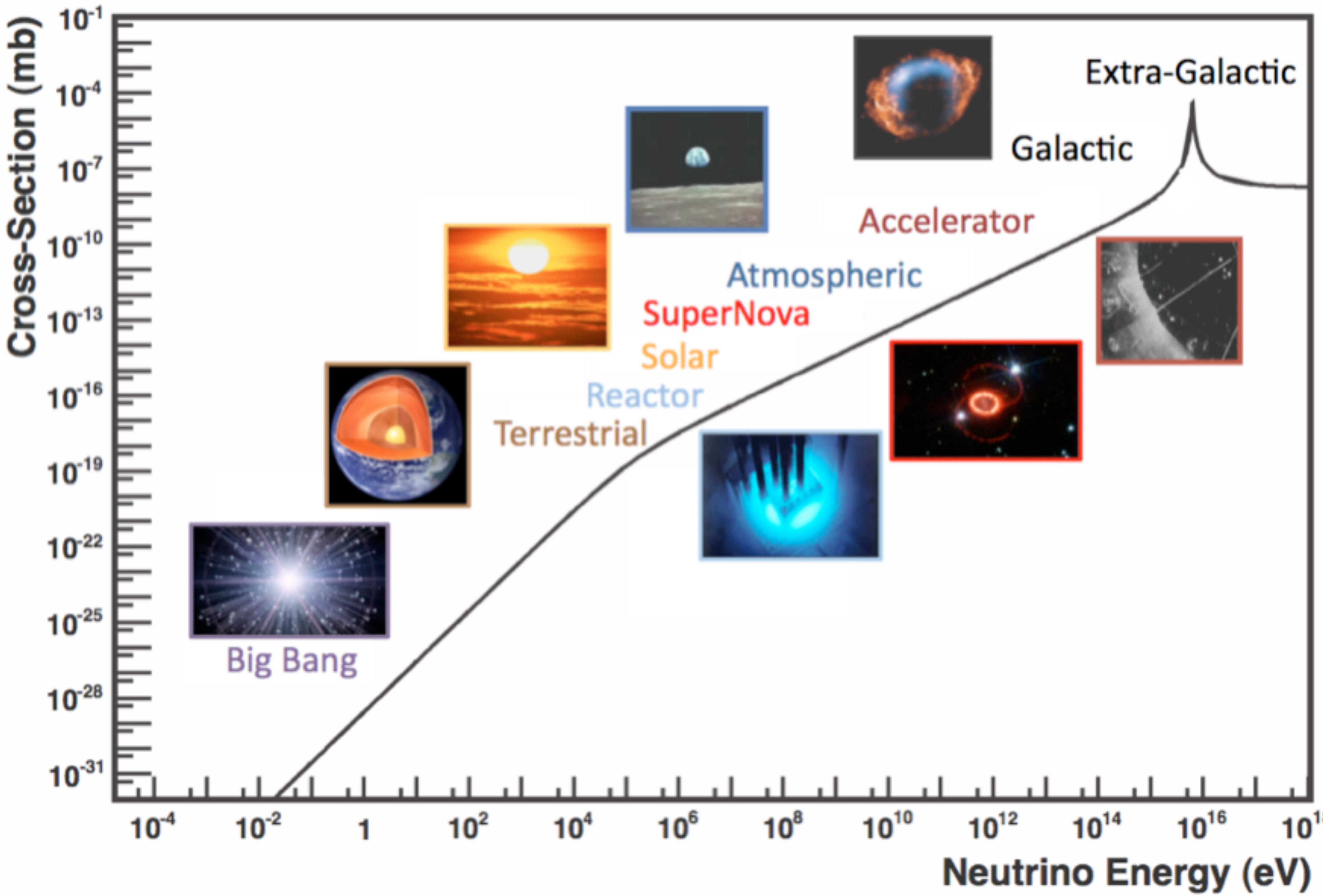


NOvA detector

MINOS experiment



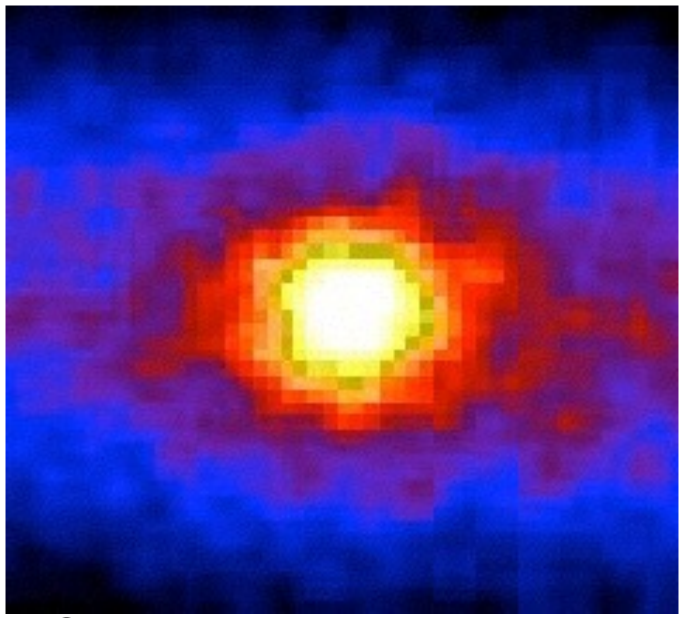
Neutrino sources



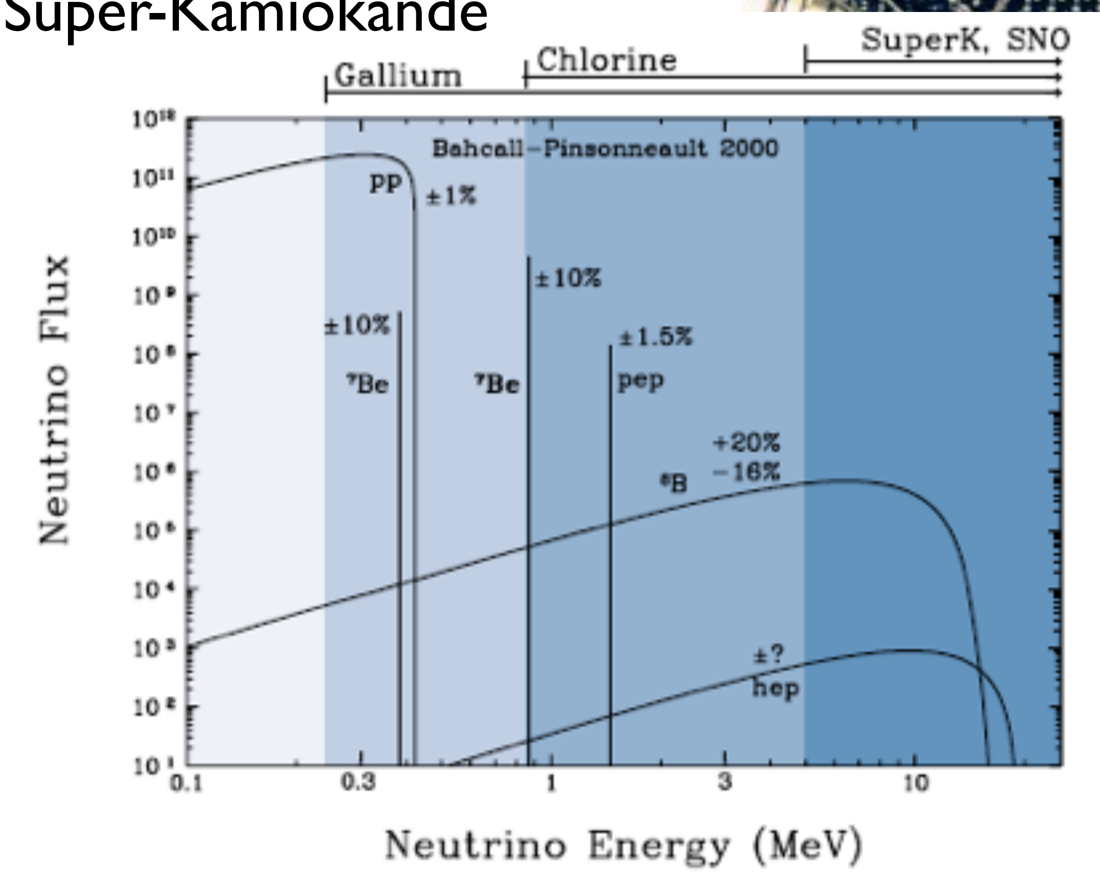
J. Formaggio and S. Zeller, 1305.7513

Solar neutrinos

Electron neutrinos are copiously produced in the Sun, at very high electron densities.



Super-Kamiokande



http://www.sns.ias.edu/~jnb/

- Typical energies: 0.1-10 MeV.
- MSW effect at high energies, vacuum oscillations at low energy (see previous discussion).
- One can observed CC ν_e and NC: measuring the oscillation disappearance and the overall flux.

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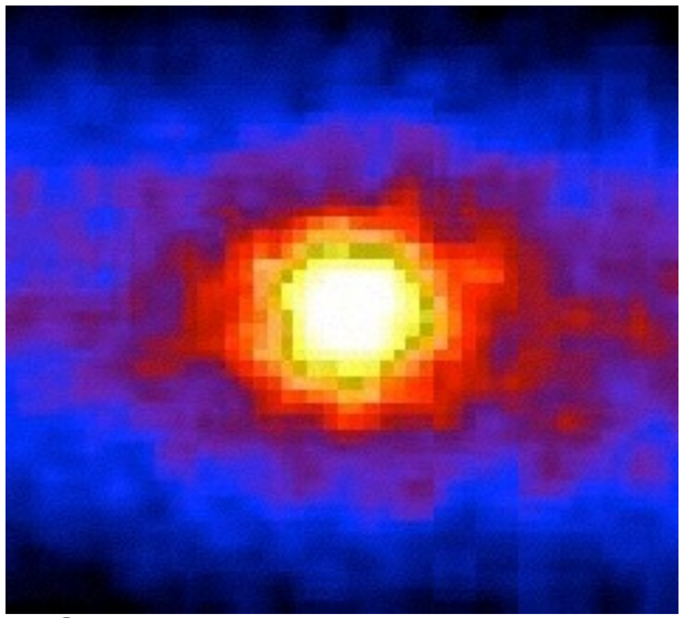
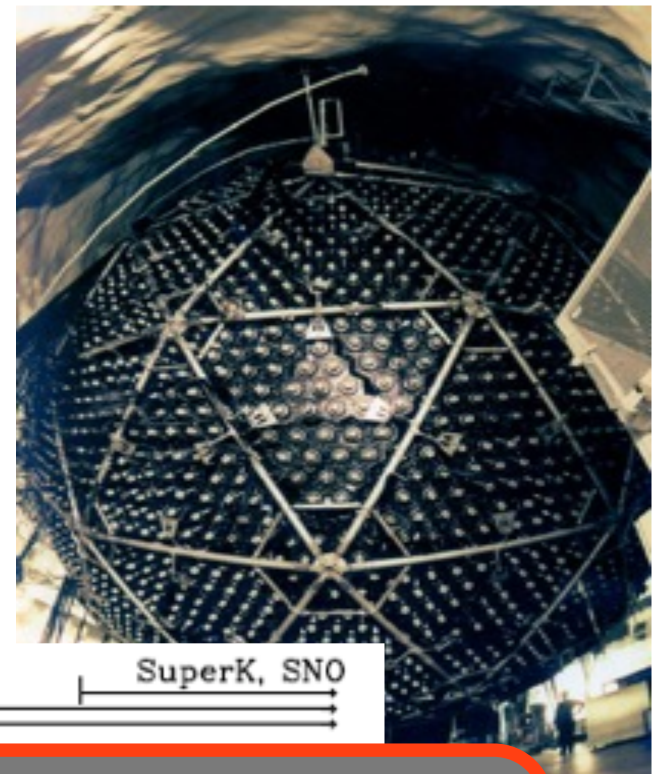
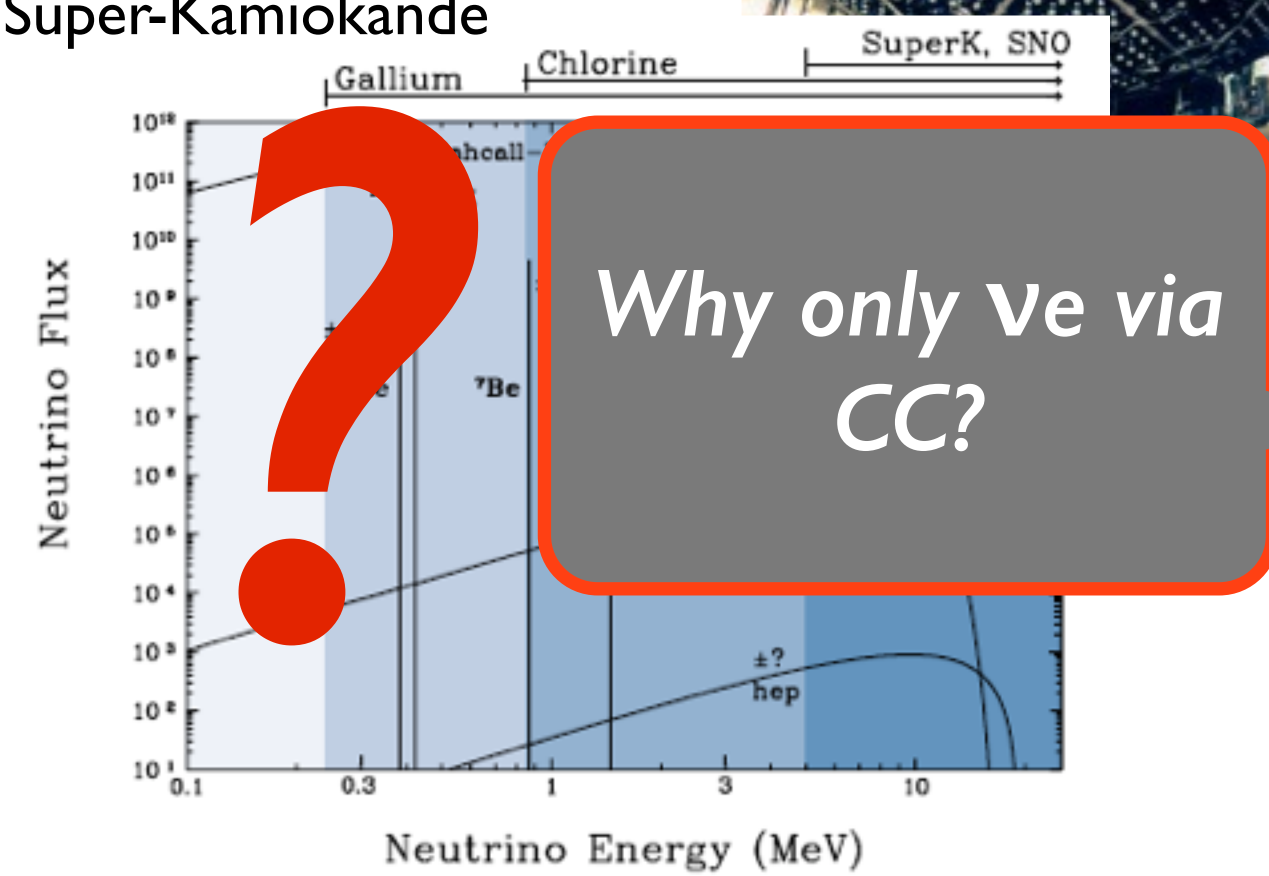


Super-Kamiokande





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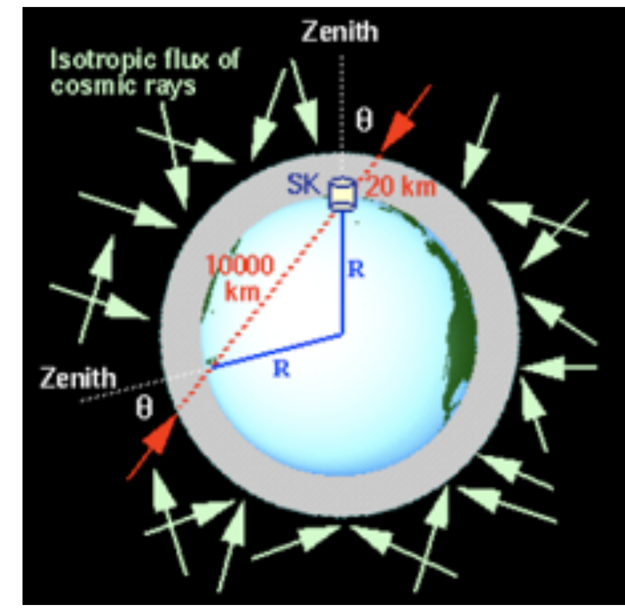
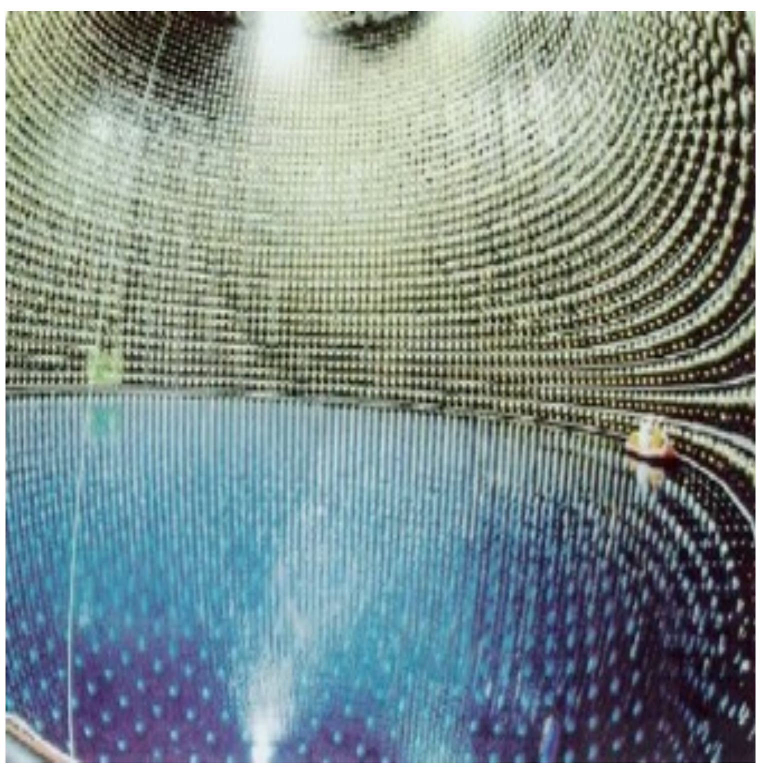
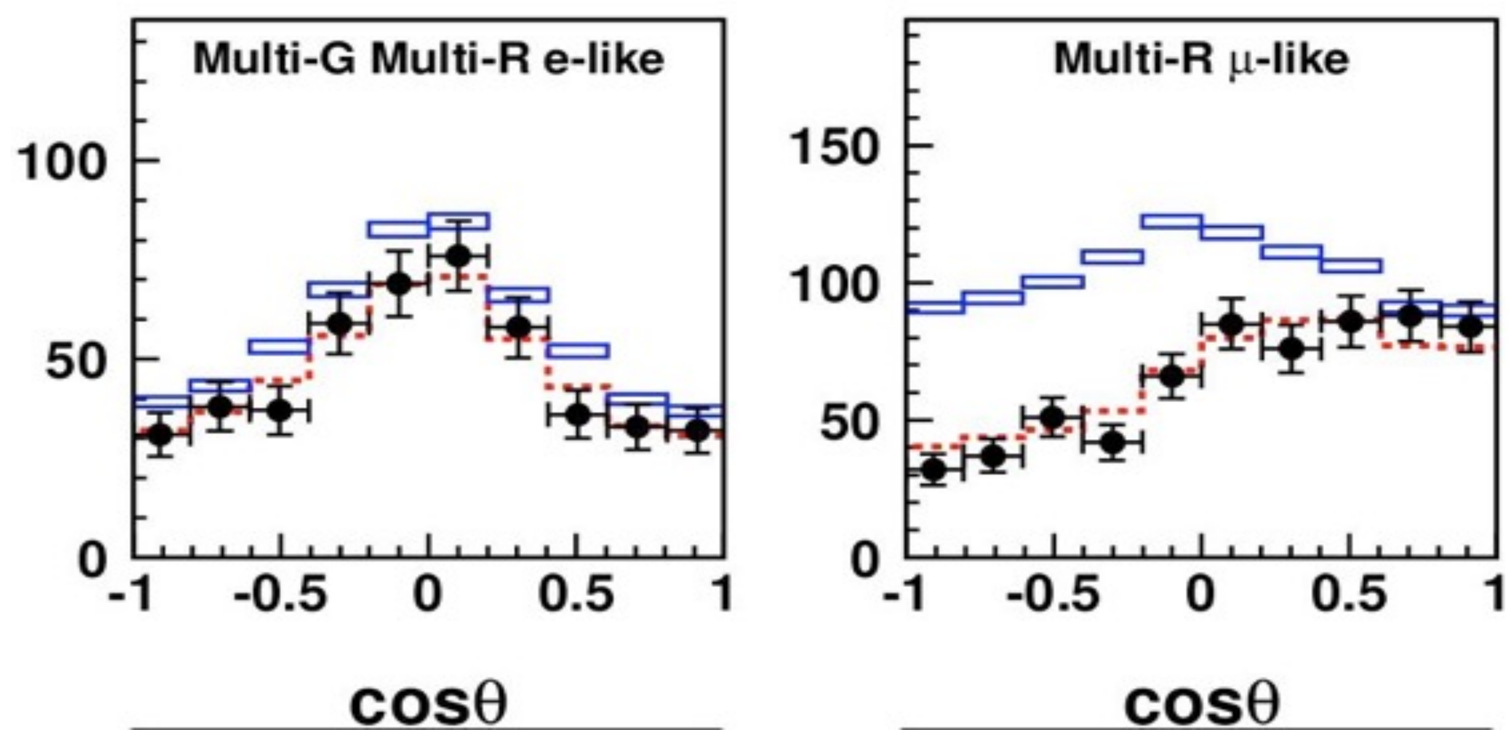
Atmospheric neutrinos

Cosmic rays hit the atmosphere and produce pions (and kaons) which decay producing lots of muon and electron (anti-) neutrinos.

- Typical energies: 100 MeV - 100 GeV
- Typical distances: 100-10000 km.







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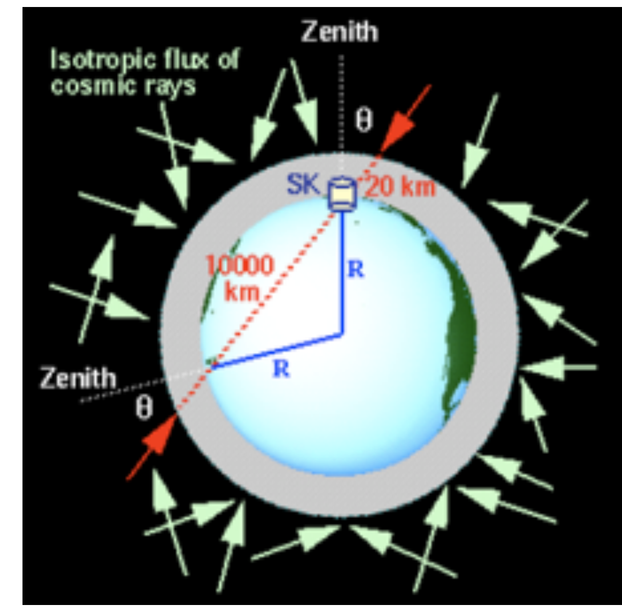
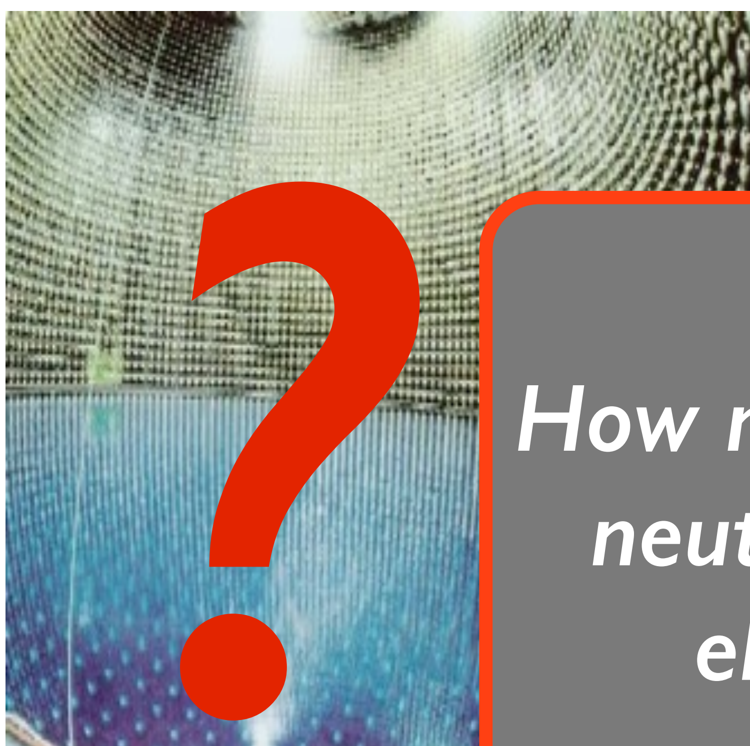
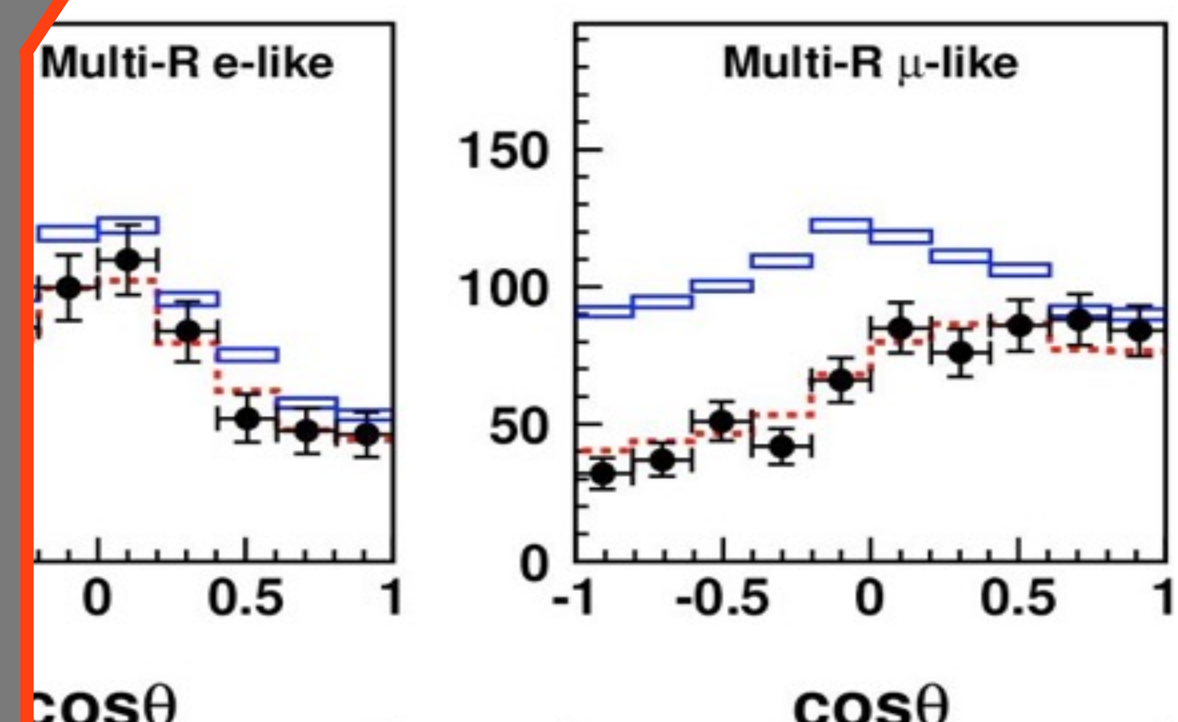




How many muon neutrinos per electron neutrino?



Reactor neutrinos

Copious amounts of electron antineutrinos are produced from reactors.

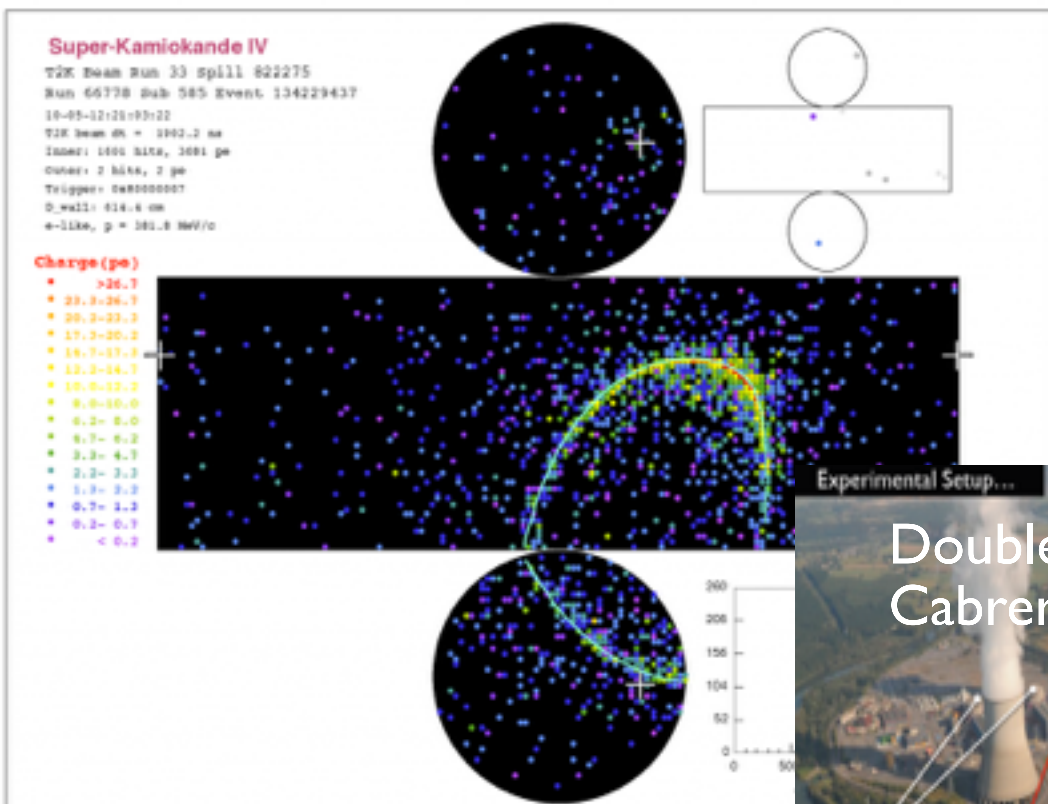
- Typical energy: 1-3 MeV;
- Typical distances: 1-100 km.

- At these energies inverse beta decay interactions dominate and the disappearance probability is

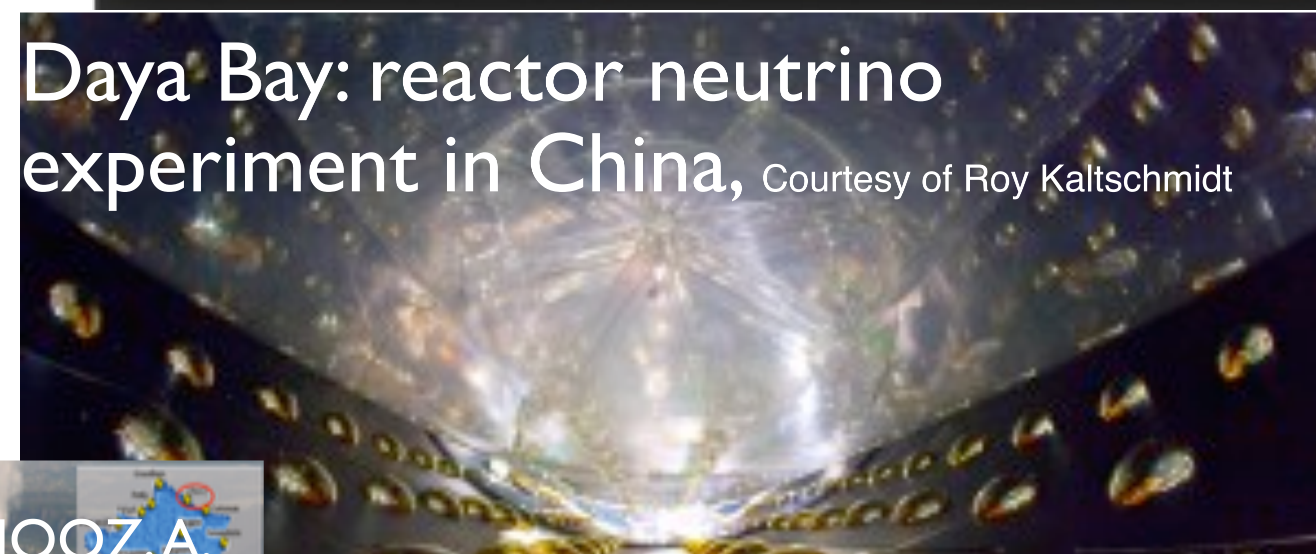
$$P(\bar{\nu}_e \to \bar{\nu}_e; t) \quad = \quad 1 - \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Sensitivity to θ_{13}. Reactors played an important role in the discovery of θ_{13} and in its precise measurement.

In 2012, previous hints (DoubleCHOOZ, T2K, MINOS) for a nonzero third mixing angle were confirmed by Daya Bay and RENO: important discovery.

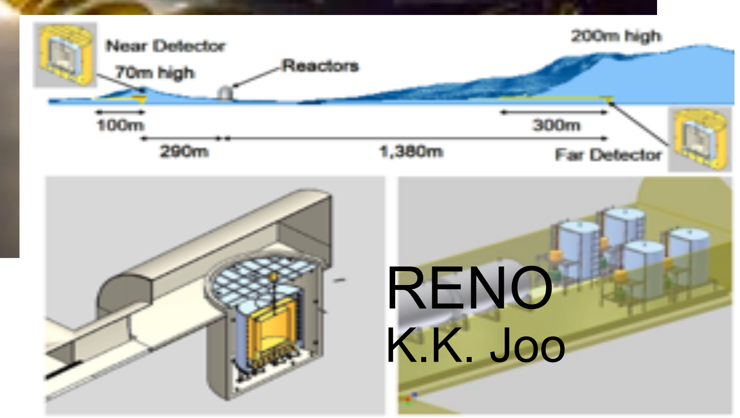


T2K event in 2011



Daya Bay: reactor neutrino experiment in China, Courtesy of Roy Kaltschmidt

Double-CHOOZ, A. Cabrera

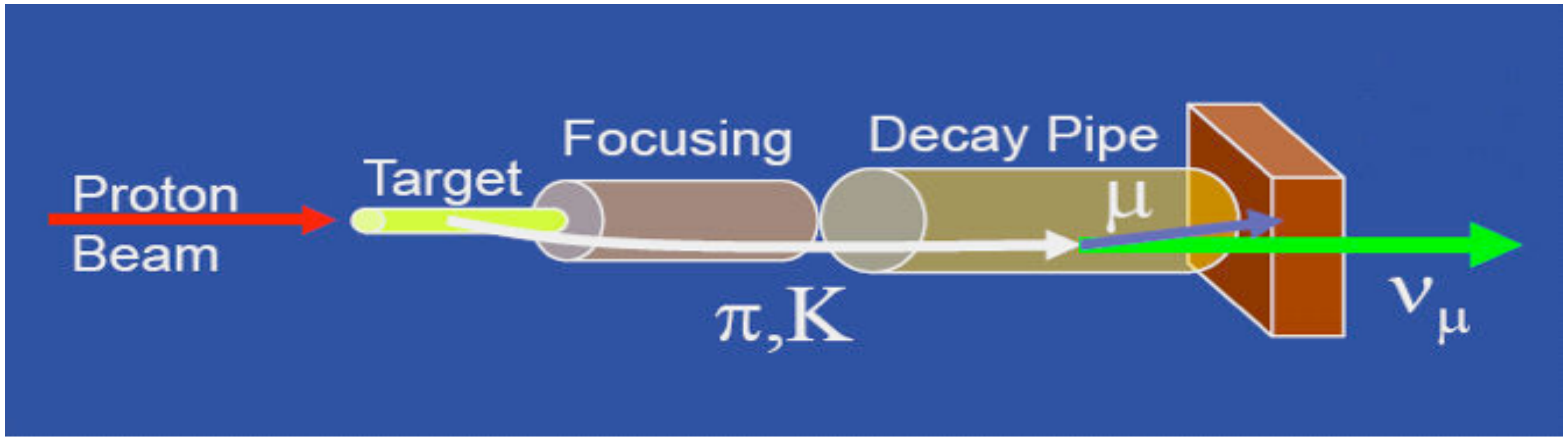


RENO
K.K. Joo

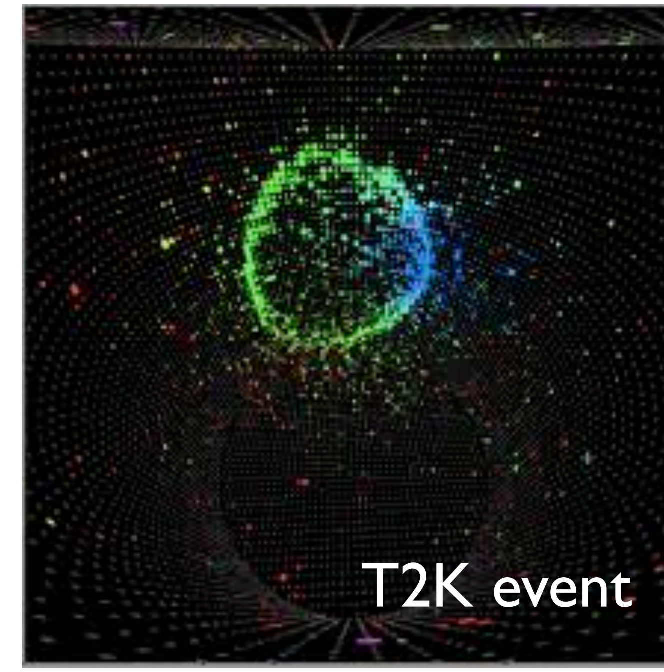
This discovery has very important implications for the future neutrino programme and understanding of the origin of mixing.

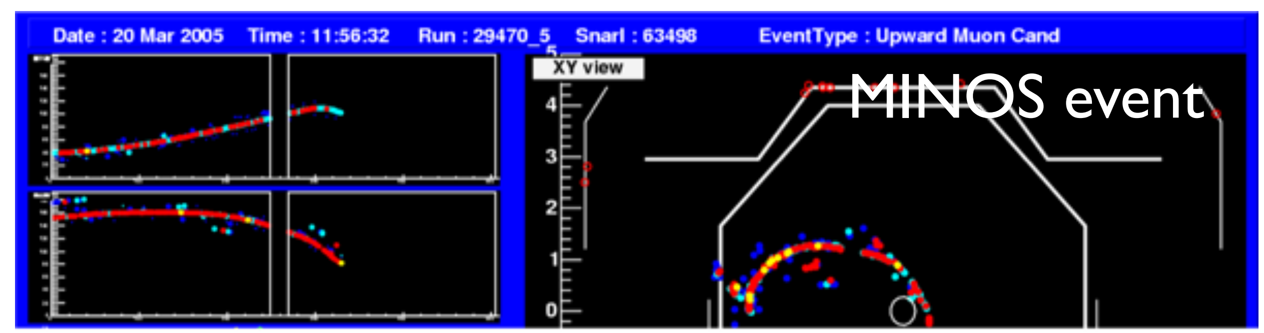
Accelerator neutrinos

Conventional beams: muon neutrinos from pion decays



Neutrino production.
Credit: Fermilab





- Typical energies:

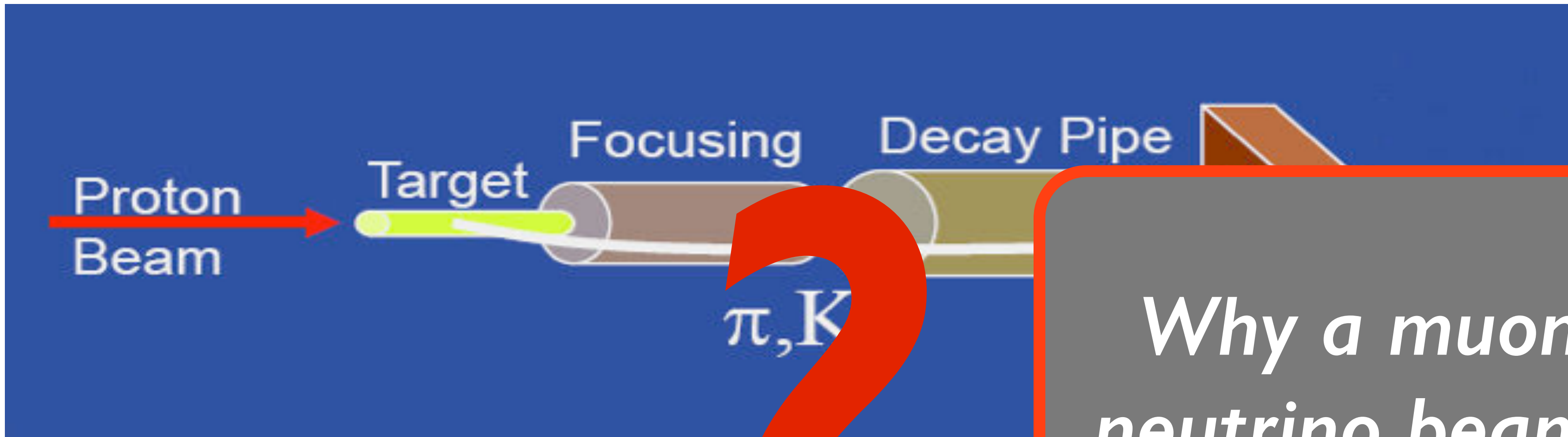
MINOS: E~4 GeV; T2K: E~700 MeV; NOvA: E~2 GeV.
OPERA and ICARUS: E~20 GeV.

- Typical distances: 100 km - 2000 km.

MINOS: L=735 km; T2K: L=295 km; NOvA: L=810 km.
OPERA and ICARUS: L=700 km.

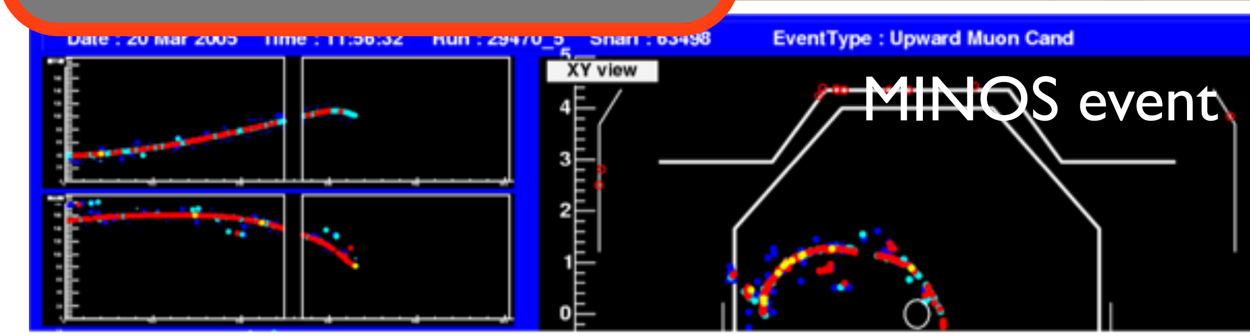
Accelerator neutrinos

Conventional beams: muon neutrinos from pion decays



Neutrino production.
Credit: Fermilab

Why a muon neutrino beam?



- Typical energies:

MINOS: E~4 GeV; T2K: E~700 MeV; NOvA: E~2 GeV.
OPERA and ICARUS: E~20 GeV.

- Typical distances: 100 km - 2000 km.

MINOS: L=735 km; T2K: L=295 km; NOvA: L=810 km.
OPERA and ICARUS: L=700 km.

At these energies, one can detect electron, muon (and tau) ν via CC interactions.

MINOS: $$P(\nu_\mu \to \nu_\mu; t) = 1 - 4s^2_{23}c^2_{13}(1 - s^2_{23}c^2_{13}) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

T2K, NOvA: $$P(\nu_\mu \to \nu_e; t) = s^2_{23} \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

OPERA (and ICARUS): $$P(\nu_\mu \to \nu_\tau; t) = c^4_{13} \sin^2(2\theta_{23}) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Sensitivity to $\Delta m^2_{31}, \quad \theta_{23}, \quad \theta_{13}$

Conclusions

- Neutrino oscillations have played a major role in the study of neutrino properties:
their discovery implies that neutrinos have mass and mix.

- They will continue to provide critical information as they are sensitive to the mixing angles, the mass hierarchy and CP-violation.

- A wide-experimental program is underway. Stay tuned!