# Lecture II: Neutrino masses, from phenomenology to theory

Summer School on Particle Physics

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@Silvia Pascoli

# *What will you learn from this lecture?*

- What do we know about neutrino parameters?
- Dirac vs Majorana neutrinos
- How to test the nature of neutrinos and measure their masses
- What type of masses neutrinos can have
- What extensions of the SM can lead to neutrino masses

# *Plan of lecture II*

- What do we know about neutrino parameters?

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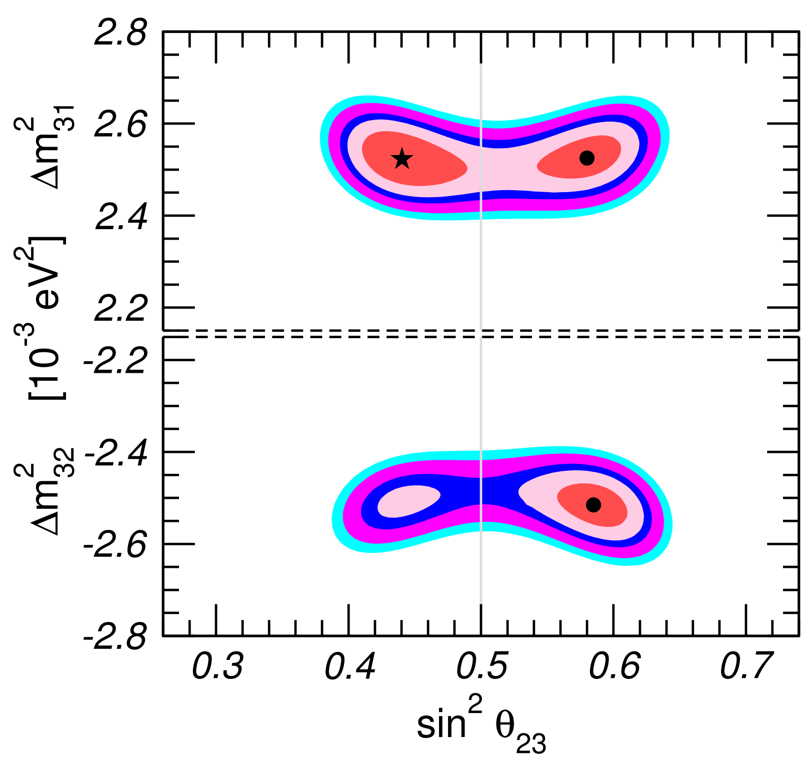
# *Current status of neutrino parameters*

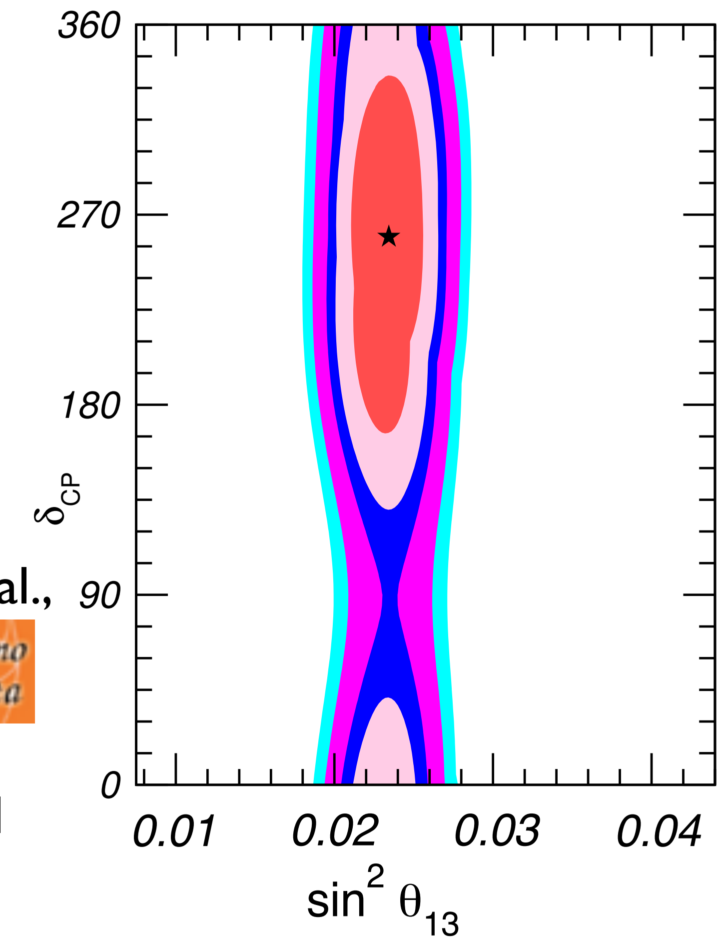


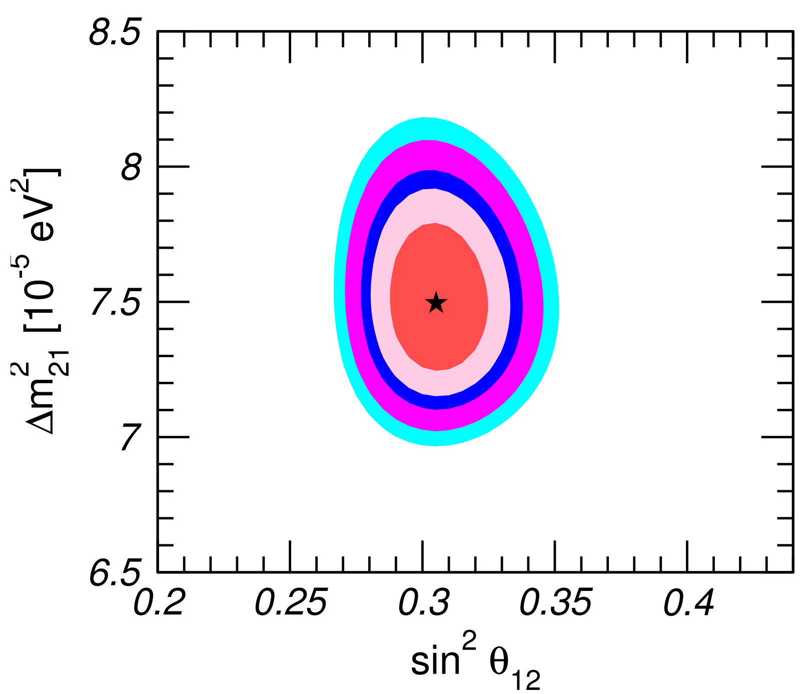

3 sizable mixing angles

2 mass squared differences

NuFit 3.0: M. C. Gonzalez-Garcia et al., 1611.01514

See also F. Capozzi et al., 1703.04471



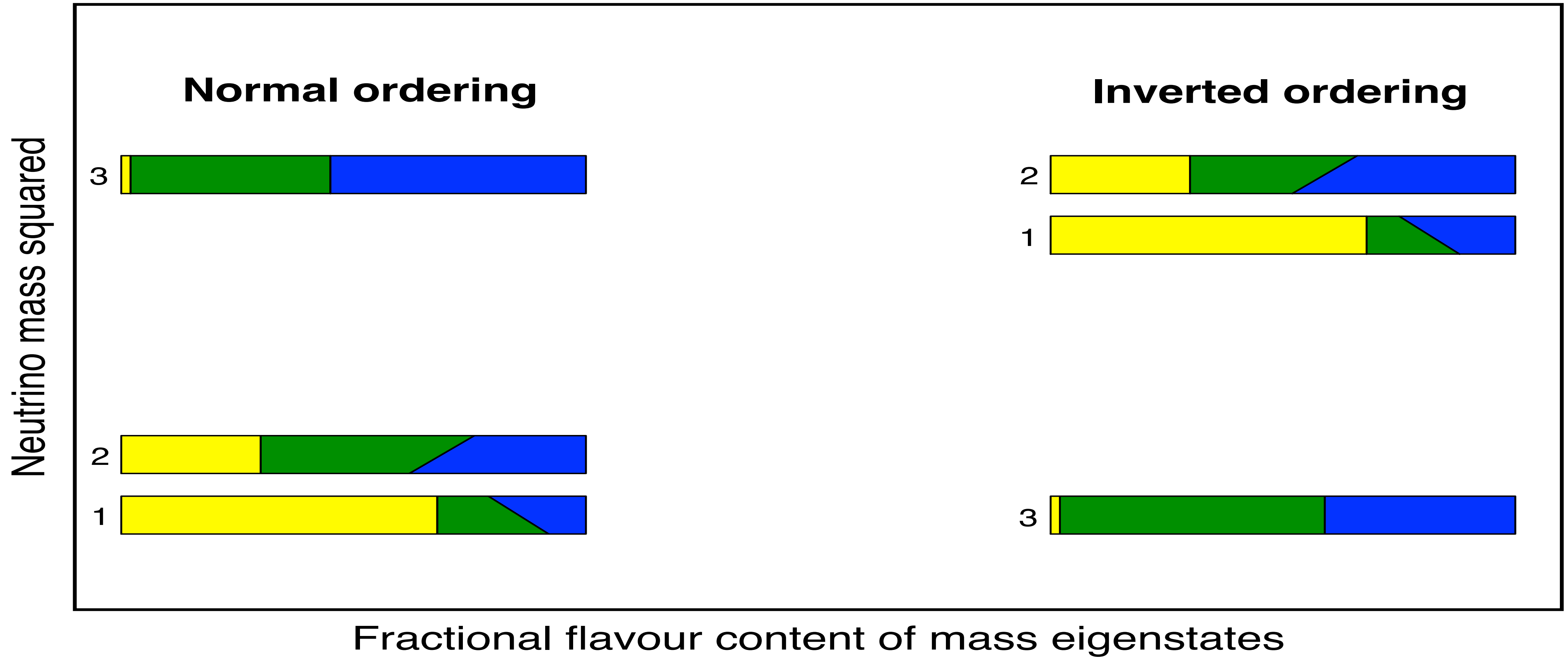




- **neutrinos have mass**
- **neutrinos mix** (Misaligned flavour and massive states)

**First evidence of physics beyond the Standard Model.**

# Neutrino masses

$\Delta m_s^2 \ll \Delta m_A^2$ implies at least 3 massive neutrinos.




$$m_1 = m_{\rm min}$$
$$m_2 = \sqrt{m_{\rm min}^2 + \Delta m_{\rm sol}^2}$$
$$m_3 = \sqrt{m_{\rm min}^2 + \Delta m_{\rm A}^2 + \Delta m_{\rm sol}^2/2}$$

$$m_3 = m_{\rm min}$$
$$m_1 = \sqrt{m_{\rm min}^2 + |\Delta m_{\rm A}^2| - \Delta m_{\rm sol}^2/2}$$
$$m_2 = \sqrt{m_{\rm min}^2 + |\Delta m_{\rm A}^2| + \Delta m_{\rm sol}^2/2}$$

Measuring the masses requires:

- the mass scale: $m_{\rm min}$
- the mass ordering. There is a hint in favour of NO based mainly on atmospheric events.

F. Capozzi et al., 1703.04471; See also SK, talks at ICHEP 2016 and NOW 2016

# *Phenomenology questions for the future*

- **What is the nature of neutrinos? Dirac vs Majorana?**
- **What are the values of the masses?** Absolute scale (KATRIN, ...?) and the ordering.
- **Is there CP-violation?** Its discovery in the next generation of LBL depends on the value of delta.
- **What are the precise values of mixing angles?** Do they suggest an underlying pattern?
- **Is the standard picture correct?** Are there NSI? Sterile neutrinos? Other effects?

# Phenomenology questions for the future

- **What is the nature of neutrinos? Dirac vs Majorana?**
  Neutrinoless dbeta decay

- **What are the values of the masses?** Absolute scale (KATRIN, ...?) and the ordering.

- **Is there CP-violation?** Its discovery in the next generation of LBL depends on the value of delta.

  LBL: T2K, NOvA, DUNE, T2HK, ESSnuSB, Daedalus, nuFACT..., PINGU, ORCA, INO, JUNO

- **What are the precise values of mixing angles?** Do they suggest an underlying pattern?

  reactor SBL and MBL, atm, LBL, ...

- **Is the standard picture correct?** Are there NSI? Sterile neutrinos? Other effects? MINOS+, MicroBooNE, SoLid, ...

# Phenomenology questions for the future

- 1. What is the nature of neutrinos?
- 2. What are the values of the masses? Absolute scale (KATRIN, ...?) and the ordering.
- 3. Is there CP-violation? Its discovery in the next generation of LBL depends on the value of delta.
- 4. What are the precise values of mixing angles? Do they suggest a underlying pattern?
- 5. Is the standard picture correct? Are there NSI? Sterile neutrinos? Other effects?

# *Plan of lecture II*

- What do we know about neutrino parameters?

- The nature of neutrinos: Dirac vs Majorana neutrinos

- How to test the nature of neutrinos and measure their masses

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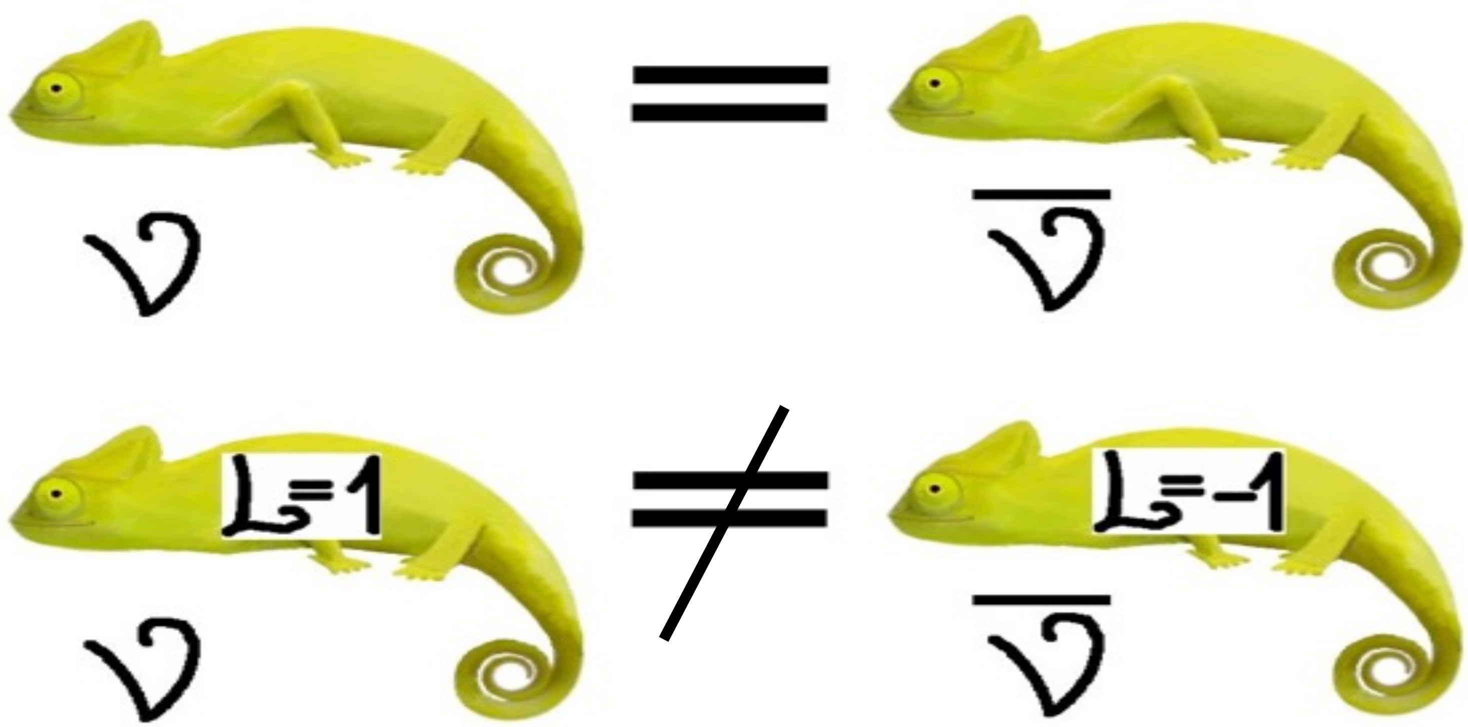
# Nature of neutrinos: Dirac vs Majorana

Neutrinos can be Majorana or Dirac particles. In the SM only neutrinos can be Majorana because they are neutral.

Majorana particles are indistinguishable from antiparticles.

Dirac neutrinos are labelled by the lepton number.




The nature of neutrinos is linked to the conservation of the Lepton number (L). This information is crucial in understanding the Physics BSM: with or without L-conservation? and it can be linked to the existence of matter in the Universe.

# Charge conjugation

This operation changes a field in its charge-conjugate (opposite quantum numbers):

$$\psi^c = C\bar{\psi}^T = i\gamma^2\psi^*$$

Properties: $C\gamma^{\alpha T}C^\dagger = -\gamma^\alpha \quad , \quad CC^\dagger = 1 \quad , \quad C^T = -C$

In Weyl representation: $C = i\gamma^2\gamma^0$

Let's apply it to a left-handed field

$$(\psi_L)^c = i\gamma^2\psi_L^* = i\begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \eta^* \end{pmatrix} = \begin{pmatrix} i\sigma^2\eta^* \\ 0 \end{pmatrix}$$

We find that it behaves as a right-handed field!

$$(\psi_L)^c = (\psi^c)_R$$

**Exercise**
using the properties of C, show that this equation is true independently of the representation of the gamma matrices.

# Majorana fields

A Majorana field satisfies the Majorana condition

$$\boxed{\psi = \psi^c}$$

Majorana particles have 2 degrees of freedom:

$$\psi = \frac{1}{(2\pi)^{3/2}} \int \frac{1}{2E} \left(u_s(p) a_s(p) e^{ipx} + \xi v_s(p) a_s^\dagger(p) e^{-ipx}\right) d^3p$$

and, with respect to Dirac particles, the propagators

$$\overline{\psi(x_1)\psi^T}(x_2) = -S(x_1 - x_2) C \qquad \overline{\bar{\psi}^T(x_1)\bar{\psi}}(x_2) = C^\dagger S(x_1 - x_2)$$

# *Plan of lecture II*

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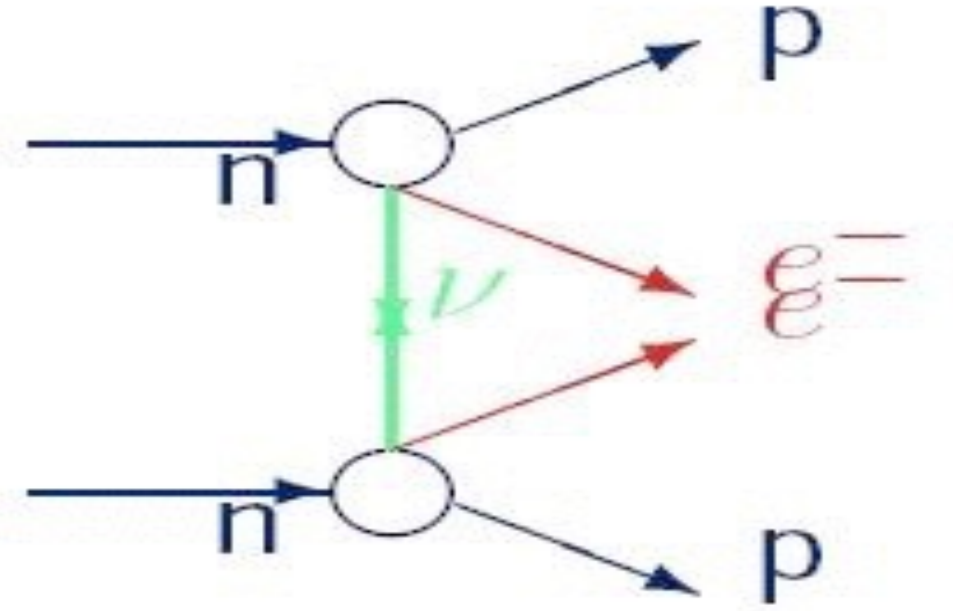
# *Neutrinoless double beta decay*

Neutrinoless double beta decay, (A, Z) → (A, Z+2) + 2 e, tests the nature of neutrinos. It violates L by 2 units.




The half-life time depends on neutrino properties

$$\left[T_{0\nu}^{1/2}(0^+ \to 0^+)\right]^{-1} \propto |M_F - g_A^2 M_{GT}|^2 \, |<m>|^2$$

$$|<m>| \equiv |\, m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}} \,|,$$

Mixing angles (known)

CPV phases (unknown)

# *Predictions for betabeta decay*

The predictions for |<m>| depend on the neutrino mass spectrum

**Exercise**
Choose one of the neutrino mass spectra and compute |<m>|, using the latest values of the oscillation parameters

- NH (m1<<m2<<m3): |<m>| ~ 1-5 meV

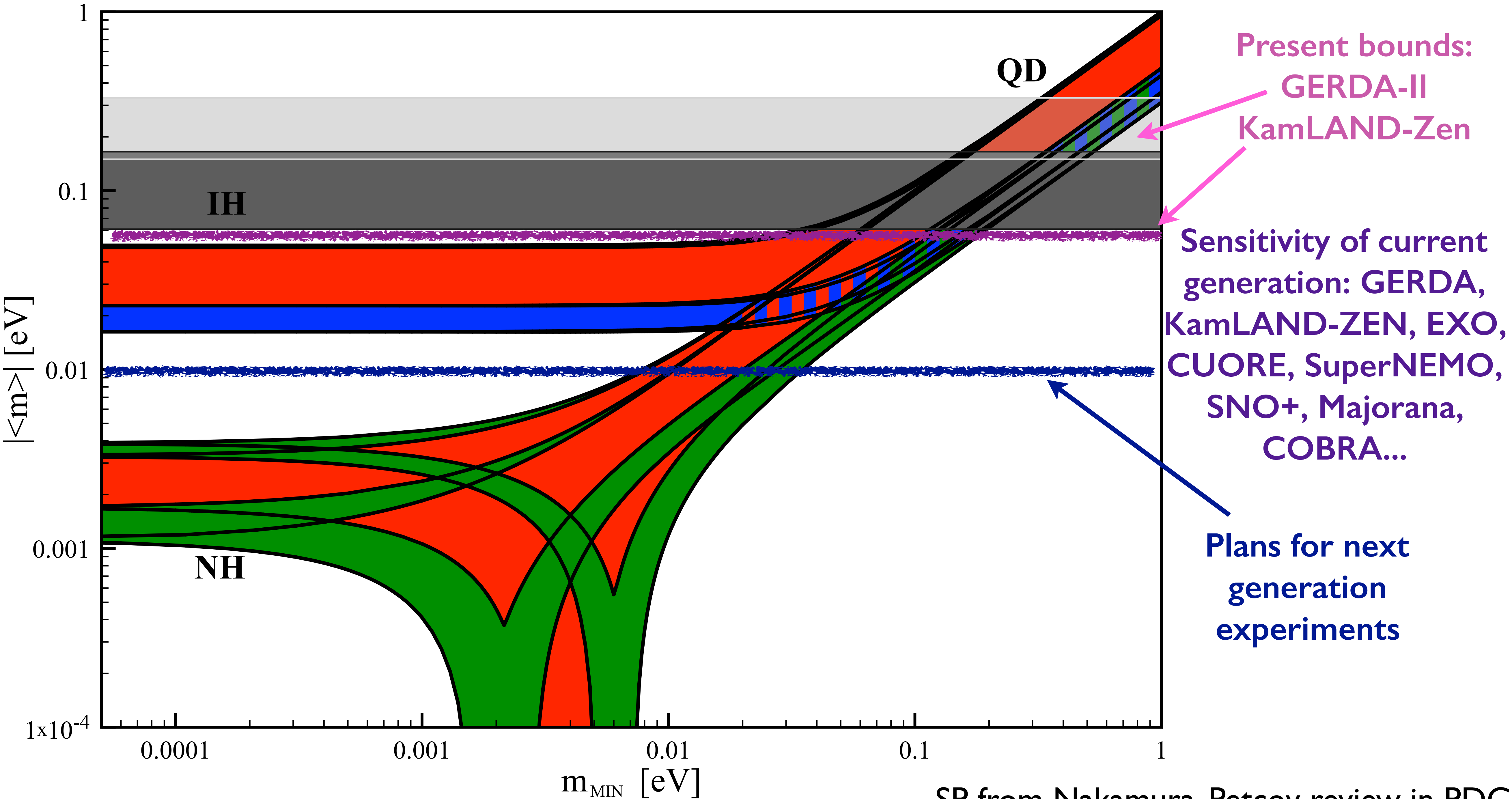
$$|<m>| \simeq \left| \sqrt{\Delta m^2_{\odot}} \cos^2\theta_{13} \sin^2\theta_{\odot} + \sqrt{\Delta m^2_{\rm atm}} \sin^2\theta_{13} e^{i\alpha_{32}} \right|$$

- IH (m3<<m1~m2): 10 meV < |<m>| < 50 meV

$$\sqrt{\Delta m^2_{\rm atm}} \cos 2\theta_{\odot} \leq |<m>| \simeq \sqrt{\left(1 - \sin^2 2\theta_{\odot} \sin^2 \frac{\alpha_{21}}{2}\right) \Delta m^2_{\rm atm}} \leq \sqrt{\Delta m^2_{\rm atm}}$$

- QD (m1~m2~m3): 44 meV < |<m>| < m1

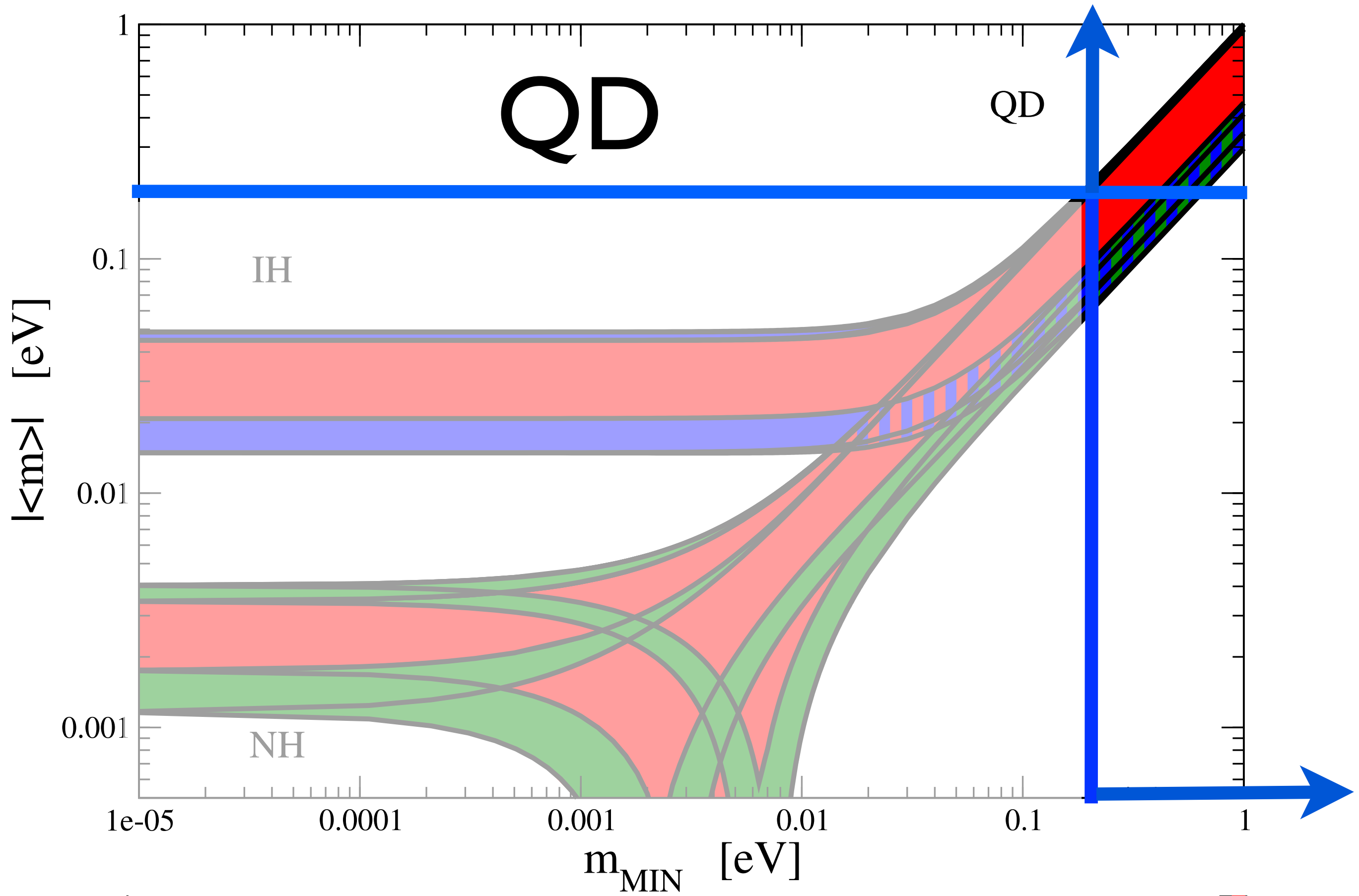
$$|<m>| \simeq m_{\bar{\nu}_e} \left| \left( \cos^2\theta_{\odot} + \sin^2\theta_{\odot} e^{i\alpha_{21}} \right) \cos^2\theta_{13} + \sin^2\theta_{13} e^{i\alpha_{31}} \right|$$



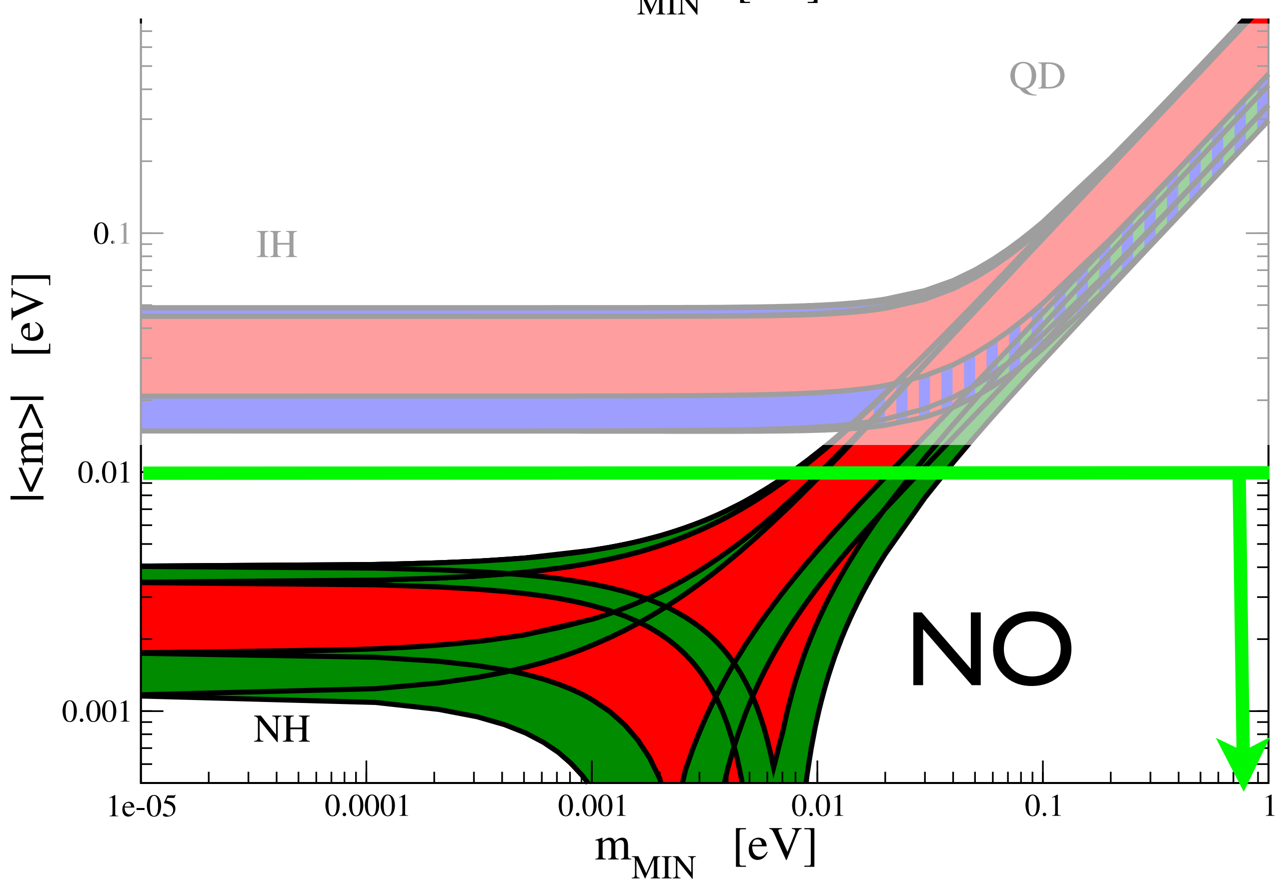

SP from Nakamura, Petcov review in PDG

**Wide experimental program** for the future: **a positive signal would indicate that L is violated!**

# Determining neutrino masses with neutrinoless dbeta decay



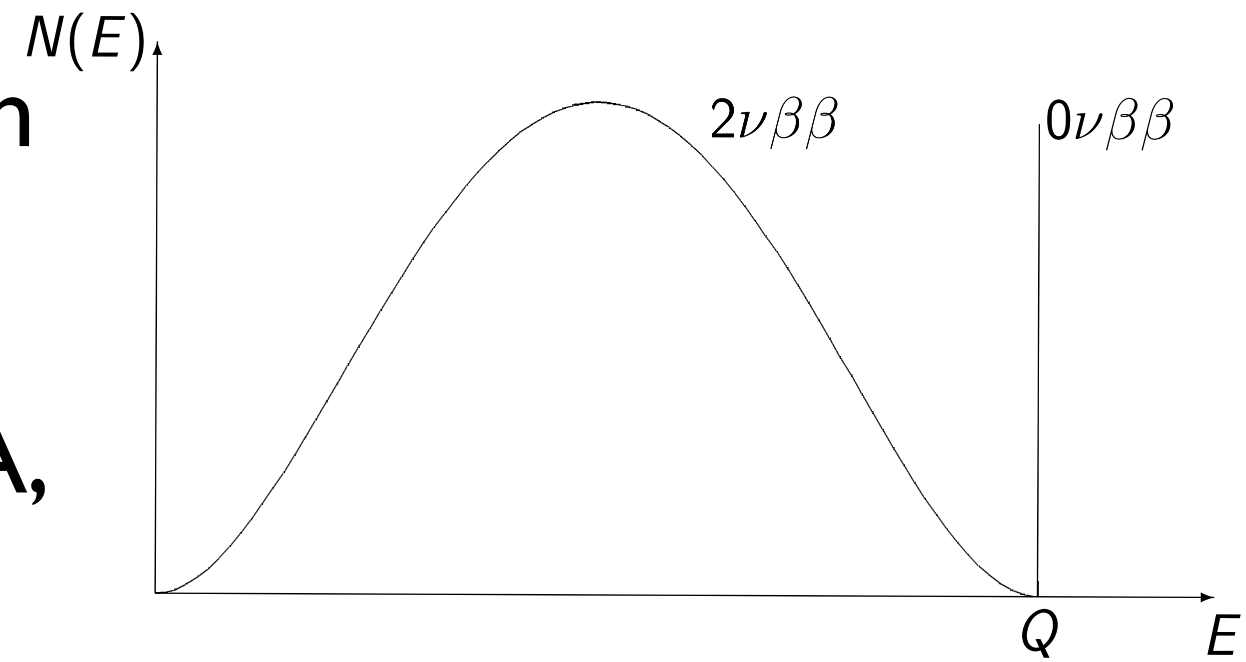

- If |<m>| > 0.2 eV, then the neutrino spectrum is QD. The measurement of m1 is entangled with the value of the Majorana phase.



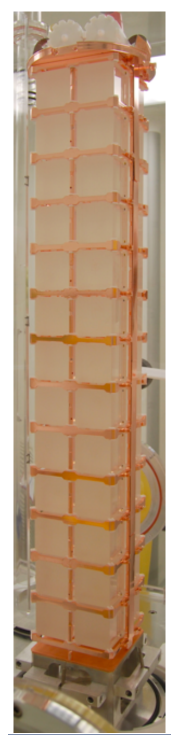

- • If no signal for |<m>| ~10 meV, then only NO is allowed.
  • If LBL experiments find IO, neutrino are Dirac particles (without fine-tuned cancellations).

# Experimental searches of betabeta decay

Neutrinoless double beta decay can be tested in nuclei in which single beta decay is kinematically forbidden but double beta decay (A, Z) → (A, Z+2) + 2 e + 2 ν is allowed.

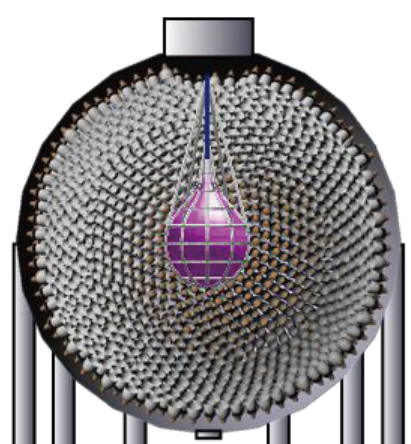


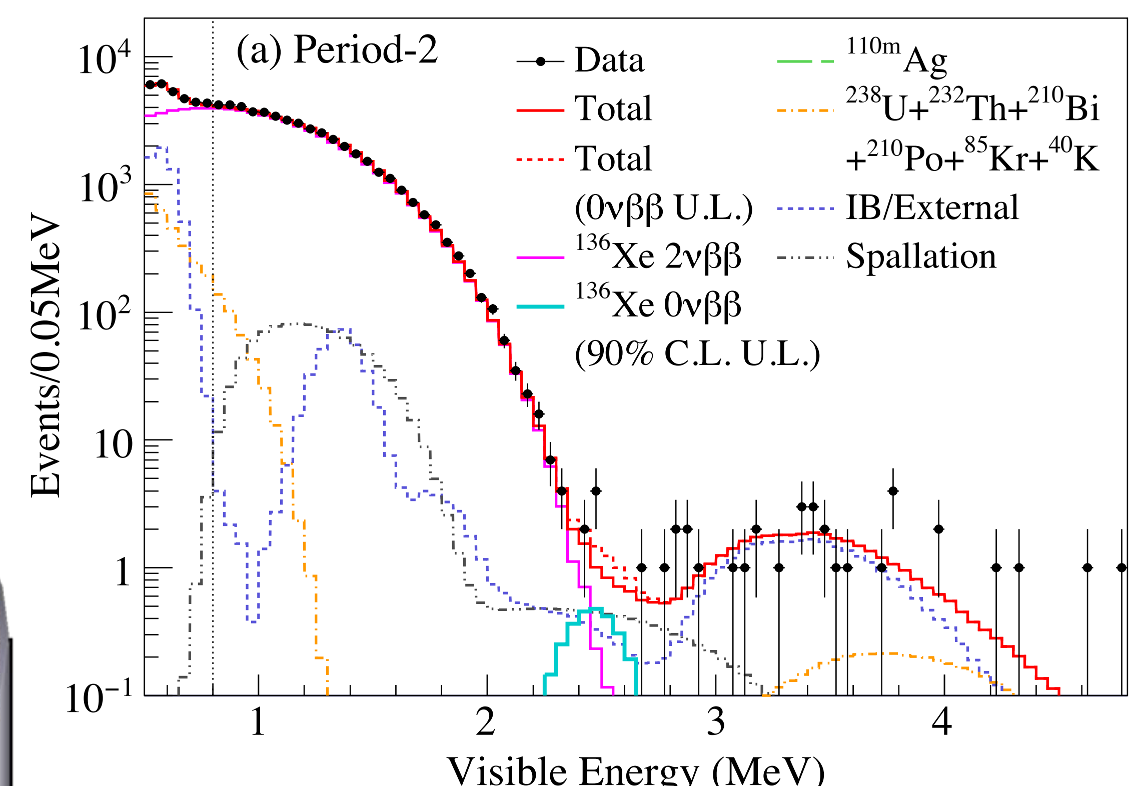

GERDA



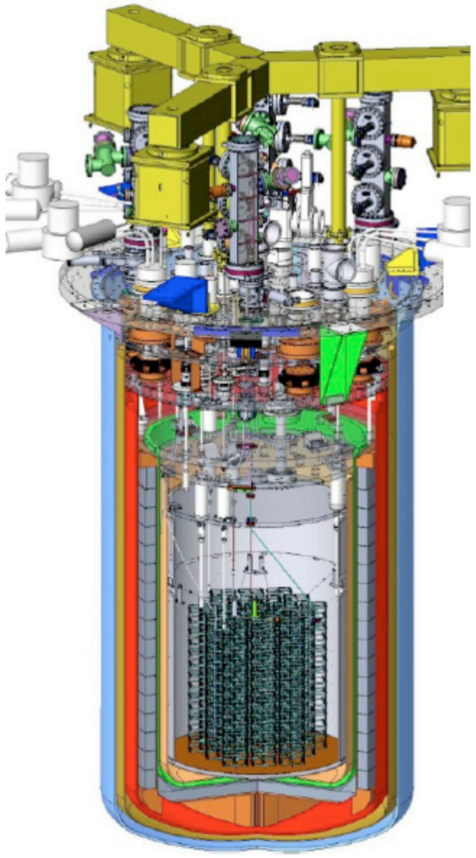
CUORE-0

EXO-200 location, at the WIPP Site, USA



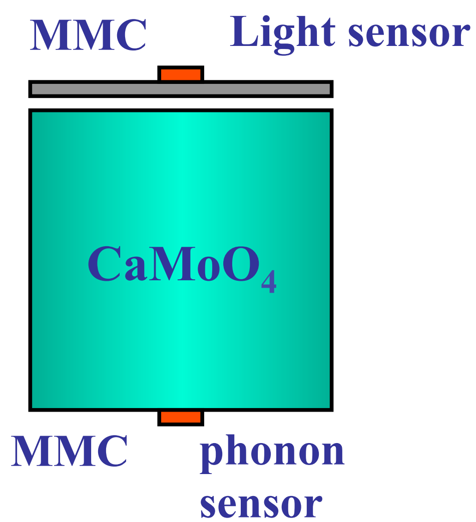


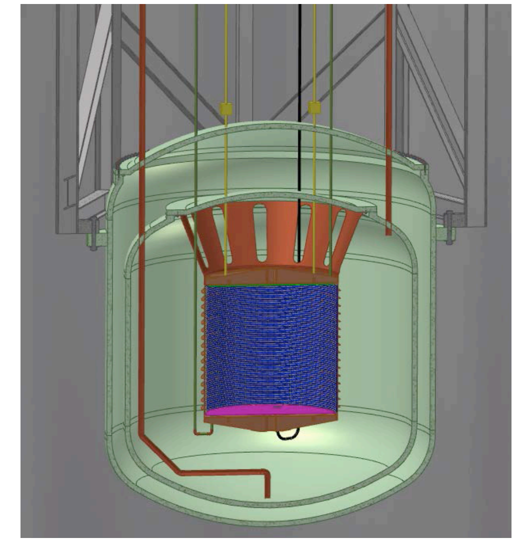

KamLAND-Zen, PRL 117 (2016)



**CUORE**
bolometer
with cc

**AMoRE**

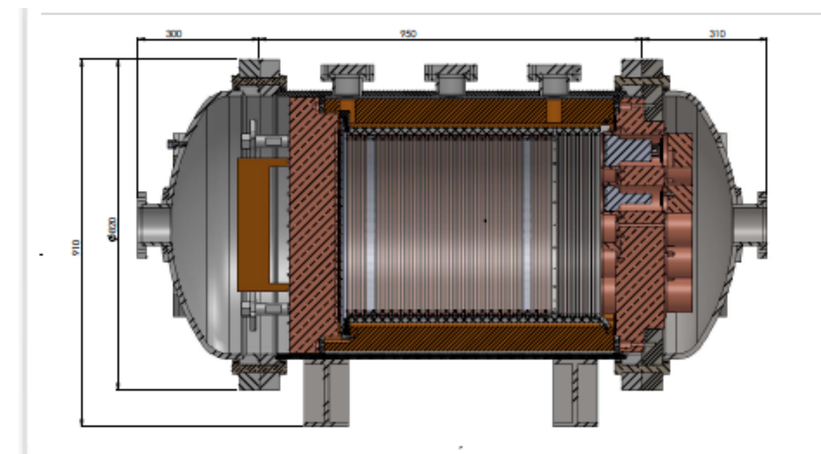


**nEXO**
5ton of Xe

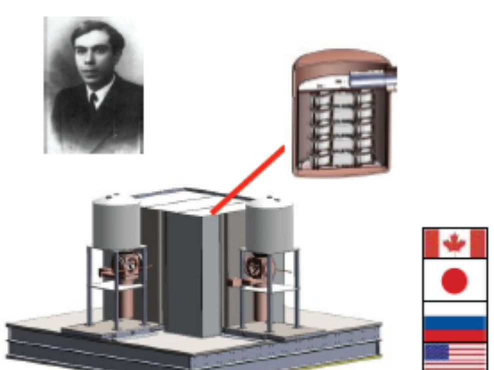
**NEXT**
5ton of Xe

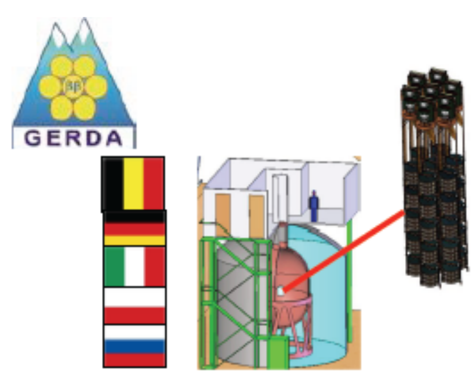
**MAGIX**
5ton of Xe

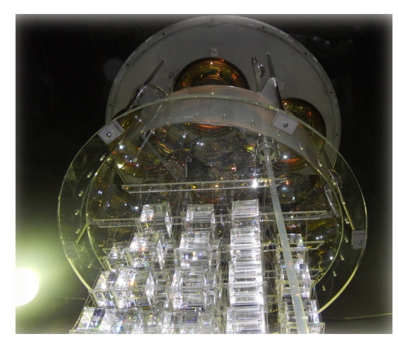


**Majorana**
uses Ge

**GERDA**
uses Ge

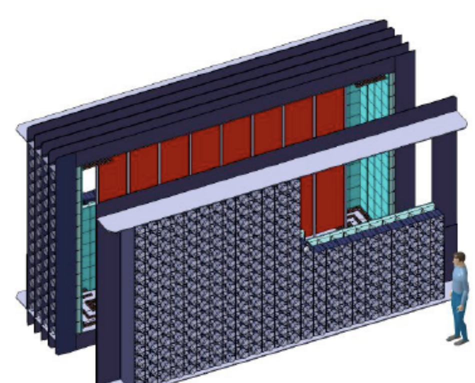




**CANDLES**
uses Ca

SNO+



**SuperNEMO**
and **DCBA**

L.Winslow at Neutrino 2014

# Determination of neutrino masses

As we know only mass squared differences, we need to establish the mass ordering and the mass scale.

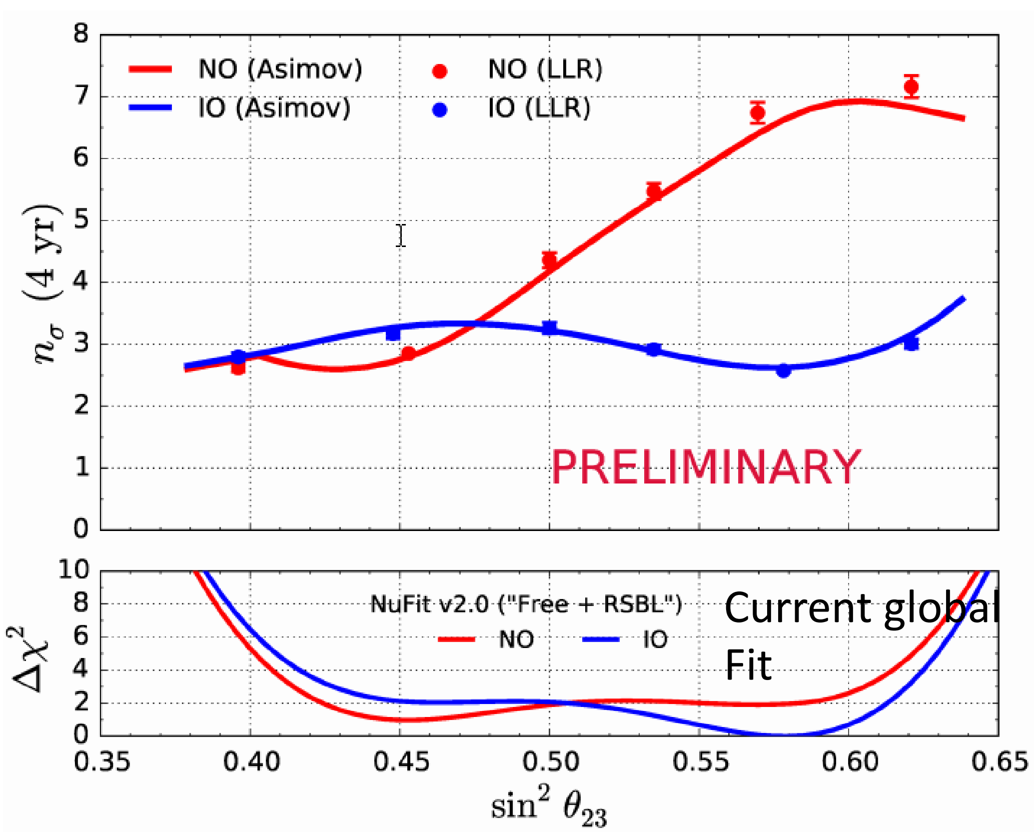
- Mass ordering:
  - Neutrinoless double beta decay (with some caveats).
  - Neutrino oscillations relying on matter effects (atmospheric neutrinos, long baseline neutrino oscillations)
  - Neutrino oscillations in vacuum (reactor neutrinos)

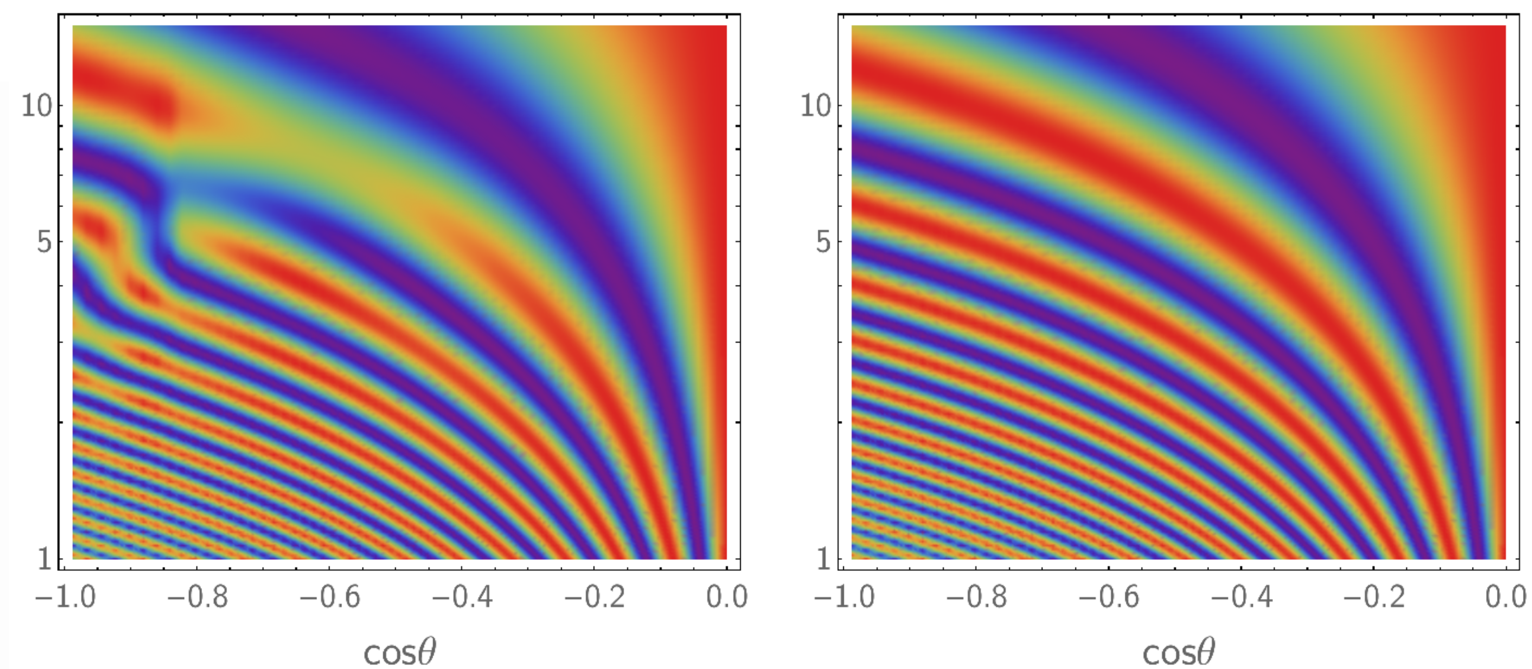
- Value of masses:
  - beta decay
  - neutrino cosmology

# *Atmospheric neutrinos and the ordering*

Atmospheric neutrino oscillations are sensitive to the mass ordering. This requires large number of events, good energy and angular resolution and, possibly, charge discrimination.

Petcov et al.; Akhmedov, Smirnov et al.; Gandhi et al.; Mena et al.; Schwetz et al.; Koskinen; Gonzalez-Garcia et al.; Barger et al.; .........

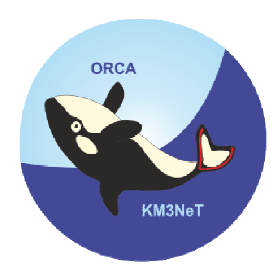


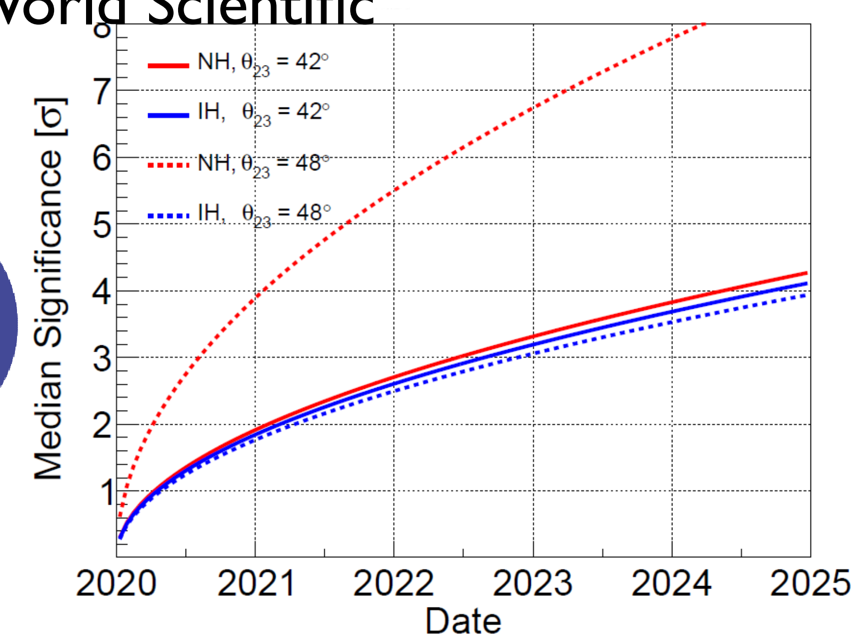




P. Coloma and SP, World Scientific


PINGU

PINGU in IceCube,
ORCA in KM3Net,
ICAL at INO



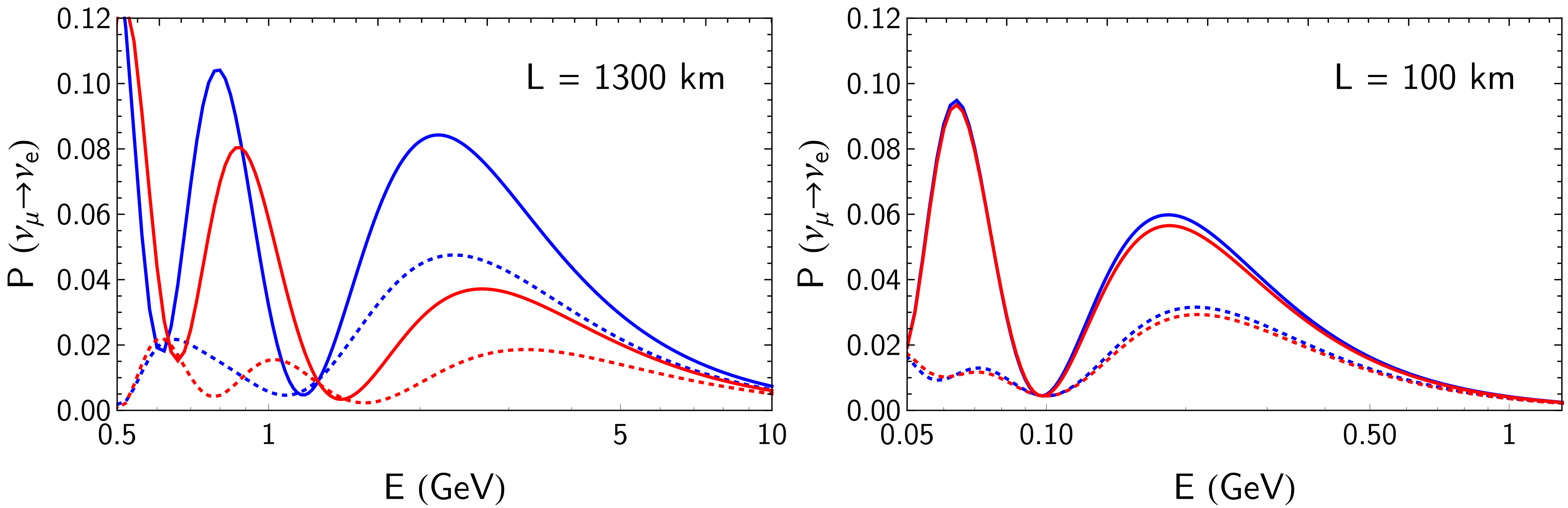

ORCA

# Long baseline oscillations and the ordering

Long baseline neutrino oscillation experiments (T2K, NOvA, DUNE, T2HK) study the subdominant channels

$$P_{\mu e} \simeq 4 c_{23}^2 s_{13}^2 \frac{1}{(1-r_A)^2} \sin^2 \frac{(1-r_A)\Delta_{31} L}{4E}$$
$$+ \sin 2\theta_{12} \sin 2\theta_{23} s_{13} \frac{\Delta_{21} L}{2E} \sin \frac{(1-r_A)\Delta_{31} L}{4E} \cos\left(\delta - \frac{\Delta_{31} L}{4E}\right)$$
$$+ s_{23}^2 \sin^2 2\theta_{12} \frac{\Delta_{21}^2 L^2}{16 E^2} - 4 c_{23}^2 s_{13}^4 \sin^2 \frac{(1-r_A)\Delta_{31} L}{4E}$$

A. Cervera et al., hep-ph/0002108; K. Asano, H. Minakata, 1103.4387; S. K. Agarwalla et al., 1302.6773...



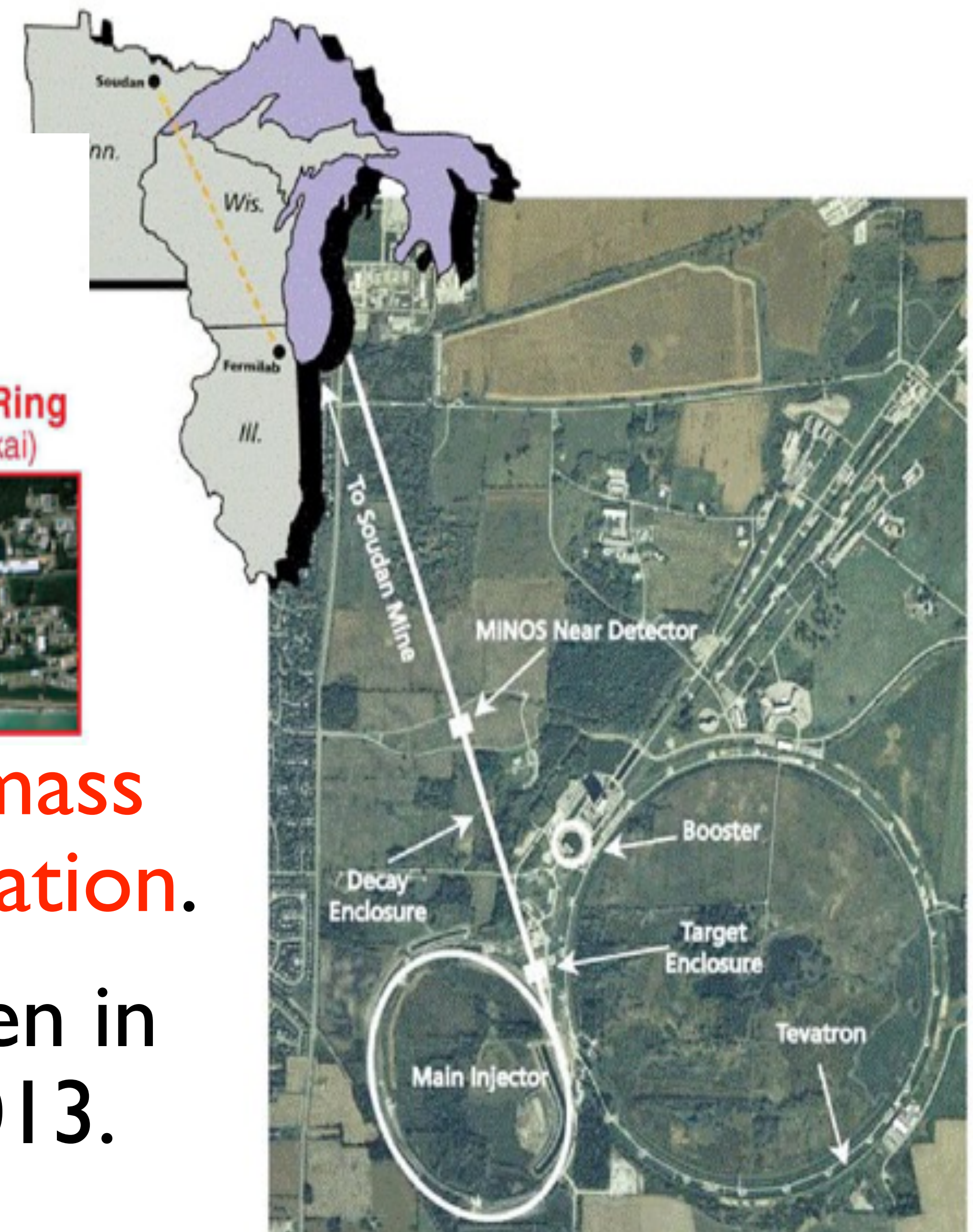
# Future long baseline neutrino experiments

The probability is sensitive to

- **CP-violation (U complex)**
- **Matter effects**

Currently running are Superbeams:

NOvA: off-axis
L=810 km

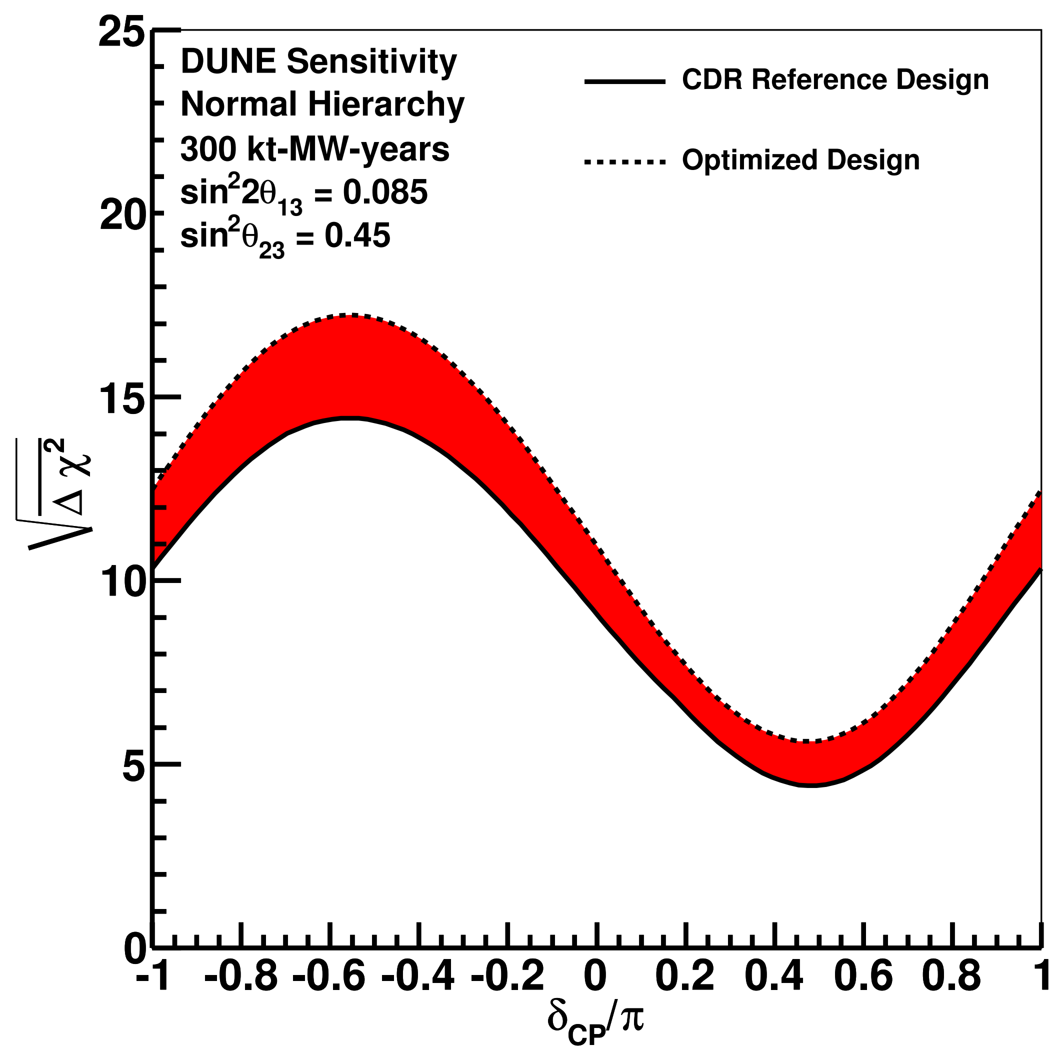



T2K: off-axis
L= 295 km

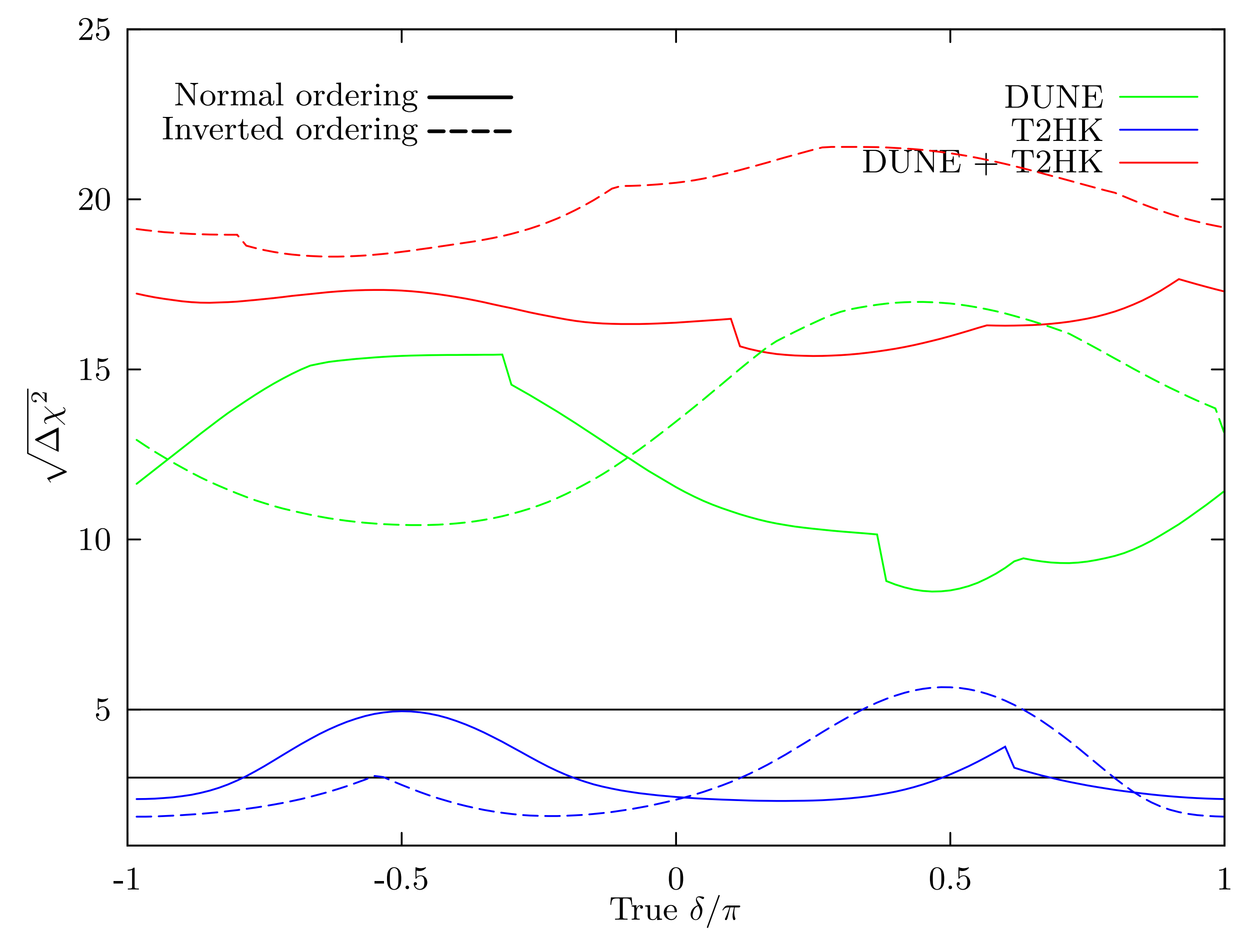

Goals: get some information about the mass hierarchy and open the hunt for CP-violation.

- In July 24 2010, first T2K event was seen in SK! 7 sigma evidence for theta13 July 2013.
- NOvA has also presented data.

- Matter effects modify the oscillation probability as discussed and are stronger the longer the baseline.
- The determination of CPV and of the mass ordering are entangled (problem of degeneracies).




DUNE CDR

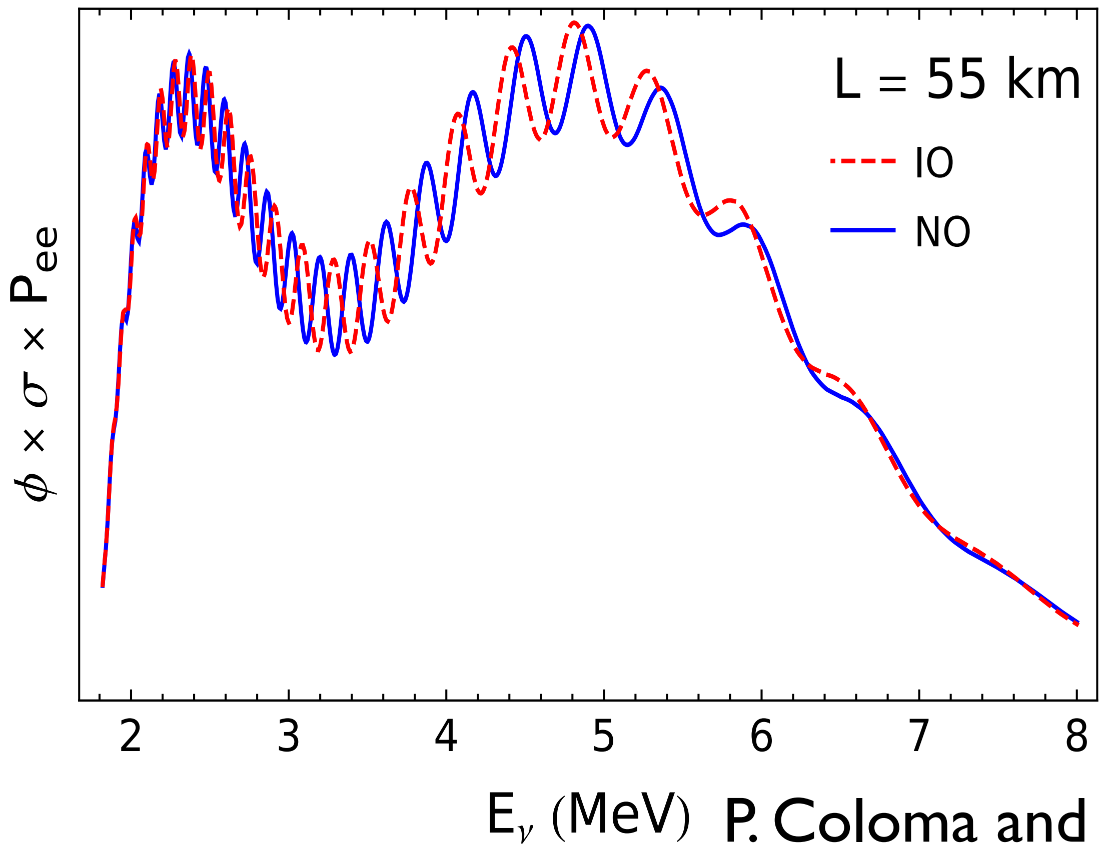



P. Ballett et al., 1612.07275

# Reactor neutrinos and the ordering

Thanks to the "unexpectedly large" value of theta13, it might be possible to establish the neutrino mass ordering from neutrino oscillations within this decade at some confidence level.

$$P_{\nu_e \to \nu_e} = 1 - \cos^4\theta_{13}\,\sin^2 2\theta_{12}\,\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right) - \sin^2 2\theta_{13}\left[\cos^2\theta_{12}\,\sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) + \sin^2\theta_{12}\,\sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)\right]$$




P. Coloma and SP, World Scientific

Petcov, Piai, hep-ph/0112074, Choubey, Petcov, Piai, hep-ph/0306017, Goshal, Petcov, 1208.6473; see also Ciuffoli et al.; Qian et al.

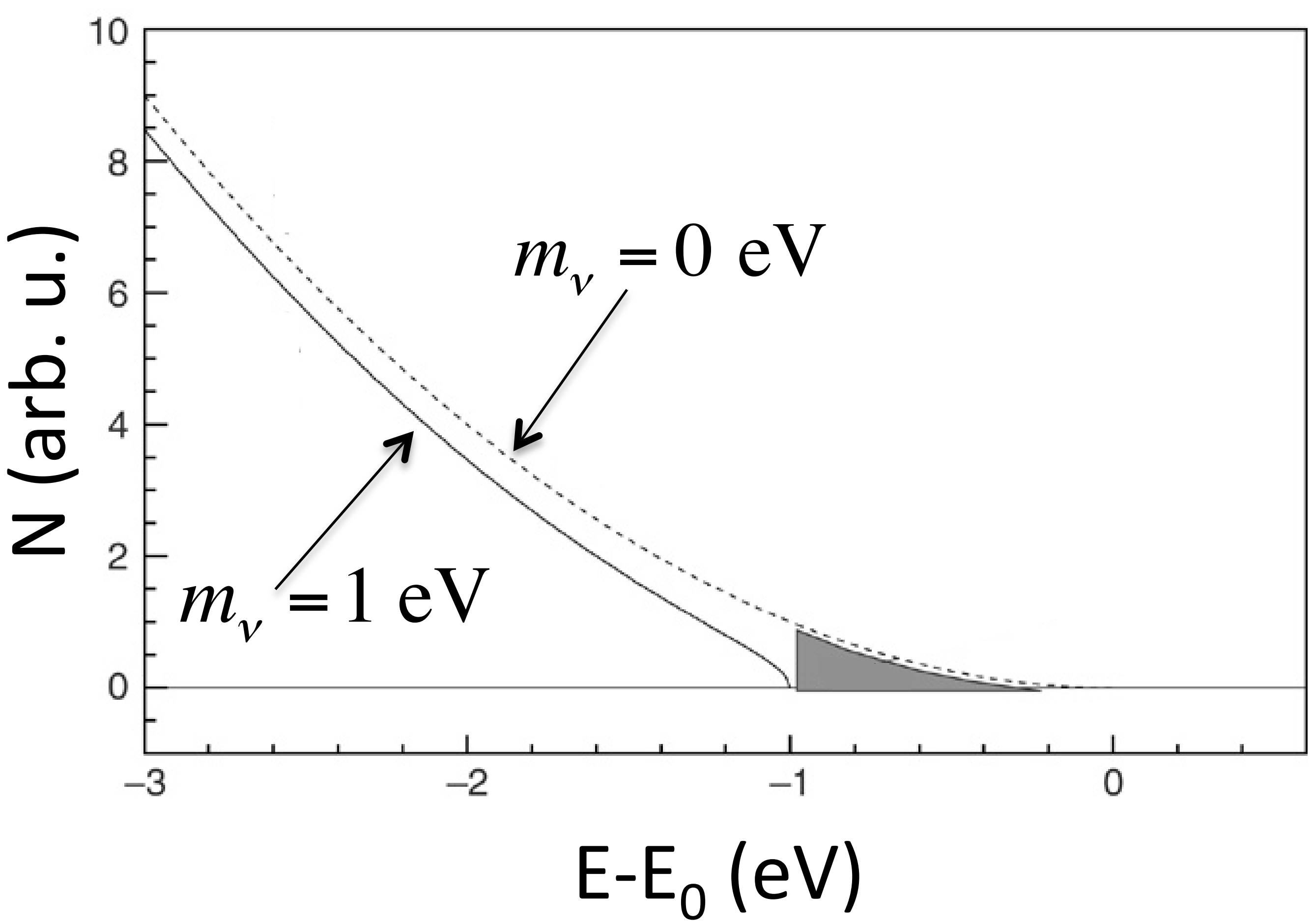
The JUNO reactor experiment is considering detectors at ~60 km to perform this measurement.
Excellent energy resolution is needed.

# Direct mass measurements

The electron spectrum in beta decays depends on neutrino masses as

$$\frac{d\Gamma}{dE_e} = \sum_i |U_{ei}|^2 \frac{d\Gamma_i}{dE_e}(m_i) \qquad \text{with}$$

$$\frac{d\Gamma_i}{dE_e} = \mathcal{C}|M|^2 p_e(E_e + m_e)(E_e - E_0)\sqrt{(E_e - E_0)^2 - m_i^2}\,F(E_e)$$




$$m_\beta \sim \sqrt{|U_{ei}|^2 m_i^2} \sim m_0$$

C. Weinheimer, PNPP 2006

- $^3H$ beta decay experiments: Troitsk and Mainz
  They provide the most stringent limit (95% CL):

  $m_0 < 2.3$ eV Kraus et al., EPIC

  $m_0 < 2.05$ eV Aseev et al.,

- Searches with cryogenic bolometers using $^{187}Re$

  MIBETA (Milano/Como): $m_0 < 15.6$ eV at 90% C.L. Sisti et al., NIMA 520

  MANU: $m_0 < 26$ eV Gatti, NPB91 ; MARE-1 and MARE-2

- KATRIN started operations in Oct 2016. It will reach a sensitivity down to m<0.2 eV and a 5-sigma discovery of m=0.35 eV.

- Project 8 aims at measuring the beta spectrum by cyclotron radiation emission spectroscopy. It can reach a sensitivity to m= 40 meV.

# Neutrino masses from cosmology

Two main techniques to probe the matter density:

- observing the distribution of biased tracers
- gravitational lensing

| Probe | Current $\sum m_\nu$ (eV) | Forecast $\sum m_\nu$ (eV) | Key Systematics | Current Surveys | Future Surveys |
|---|---|---|---|---|---|
| CMB Primordial | 1.3 | 0.6 | Recombination | WMAP, Planck | None |
| CMB Primordial + Distance | 0.58 | 0.35 | Distance measurements | WMAP, Planck | None |
| Lensing of CMB | $\infty$ | $0.2 - 0.05$ | NG of Secondary anisotropies | Planck, ACT [39], SPT [96] | EBEX [57], ACTPol, SPTPol, POLARBEAR [5], CMBPol [6] |
| Galaxy Distribution | 0.6 | 0.1 | Nonlinearities, Bias | SDSS [58, 59], BOSS [82] | DES [84], BigBOSS [81], DESpec [85], LSST [92], Subaru PFS [97], HETDEX [35] |
| Lensing of Galaxies | 0.6 | 0.07 | Baryons, NL, Photometric redshifts | CFHT-LS [23], COSMOS [50] | DES [84], Hyper SuprimeCam, LSST [92], Euclid [88], WFIRST[100] |
| Lyman $\alpha$ | 0.2 | 0.1 | Bias, Metals, QSO continuum | SDSS, BOSS, Keck | BigBOSS[81], TMT[99], GMT[89] |
| 21 cm | $\infty$ | $0.1 - 0.006$ | Foregrounds, Astrophysical modeling | GBT [11], LOFAR [91], PAPER [53], GMRT [86] | MWA [93], SKA [95], FFTT [49] |
| Galaxy Clusters | 0.3 | 0.1 | Mass Function, Mass Calibration | SDSS, SPT, ACT, XMM [101] Chandra [83] | DES, eRosita [87], LSST |

K.N. Abazajian et al., 1103.5083

$$\sum_i m_i < 0.66 \text{ eV}$$

Planck Coll., 1303.5076

Most precise determination of masses in future. Problem of underlying cosmological model and systematic errors.

# *Plan of lecture II*

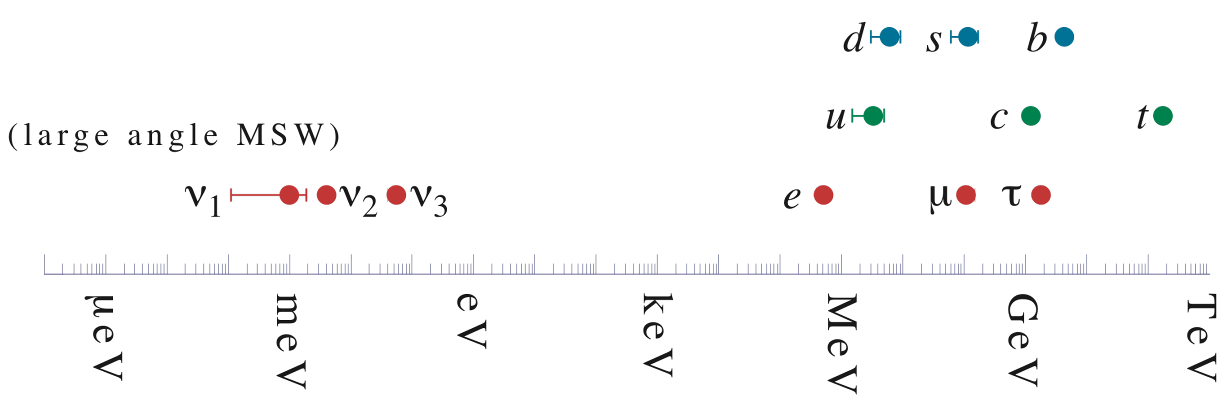
- What do we know about neutrino parameters?
- Dirac vs Majorana neutrinos
- How to test the nature of neutrinos and measure their masses
- What type of masses neutrinos can have
- What extensions of the SM can lead to neutrino masses

**The ultimate goal is to understand**
**- where do neutrino masses come from?**
**- why there is leptonic mixing? and what is at the origin of the observed structure?**

# Open window on Physics beyond the SM

Neutrinos give a new perspective on physics BSM.

## 1. Origin of masses




Why neutrinos have mass?
and why are they so much lighter?
and why their hierarchy is at most mild?

## 2. Problem of flavour

$$\begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

$$\begin{pmatrix} 0.8 & 0.5 & 0.16 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Why leptonic mixing is so different from quark mixing?

This information is **complementary** with the one from **flavour physics experiments and from colliders**.

@Silvia Pascoli

**What kind of masses can neutrinos have?**

# Neutrino masses in the nuSM lagrangian

A mass term for a fermion connects a left-handed field with a right-handed one. For example the "usual" Dirac mass

$$m_\psi(\bar{\psi}_R\psi_L + \text{h.c.}) = m_\psi\bar{\psi}\psi$$

**Exercise**
check this formula

## Dirac masses

This is the simplest case. We assume that we have two independent Weyl fields: $\nu_L$ , $\nu_R$
and we can write down the term as above.

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R\nu_L + \text{h.c.})$$

**Does it conserve lepton number?**

$$\nu_L \to e^{i(+1)\alpha}\nu_L$$
$$\nu_R \to e^{i(?)\alpha}\nu_R$$

# *Neutrino masses in the nuSM lagrangian*

A mass term for a fermion connects a left-handed field with a right-handed one. For example the "usual" Dirac mass

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**Exercise**
check this formula

## *Dirac masses*

This is the simplest case. We assume that we have two independent Weyl fields: $\nu_L$ , $\nu_R$
and we can write down the term as above.

$$\mathcal{L}_{mD} = -m_\nu(\bar{\nu}_R\nu_L + \mathrm{h.c.})$$

**This conserves lepton number!**

$$\nu_L \to e^{i\alpha}\nu_L$$
$$\nu_R \to e^{i\alpha}\nu_R$$

$$\mathcal{L}_{mD} \to \mathcal{L}_{mD}$$

# Diagonalize a Dirac mass term

If there are several fields, there will be a Dirac mass matrix.

$$\mathcal{L}_{mD} = -\bar{\nu}_{Ra} \, (m_D)_{ab} \, \nu_{Lb} + \text{h.c.}$$

This requires two unitary mixing matrices to diagonalise it

$$m_D = V m_{\text{diag}} U^\dagger$$

and the massive states are

$$n_L = U^\dagger \nu_L \quad n_R = V^\dagger \nu_R$$

This is the mixing matrix which enters in neutrino oscillations. So the form of the mass matrix determines the mixing pattern.

# *Majorana masses*

If we have only the left-handed field, we can still write down a mass term, called Majorana mass term. We use the fact that

$$(\psi_L)^c = (\psi^c)_R$$

**then the mass term is**

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

**Exercise**
Show that these two formulations are equivalent.

Hint:
$$\bar{\nu_L^c} \nu_L = (C \bar{\nu}_L^T)^\dagger \gamma^0 \nu_L = \bar{\nu}_L^* C^\dagger \gamma^0 \nu_L$$
$$= \nu_L^T \gamma^{0*} C^\dagger \gamma^0 \nu_L = -\nu_L^T C^{-1} \nu_L$$

**This breaks lepton number!**

$$\nu_L \to e^{i\alpha} \nu_L \qquad \mathcal{L}_{mM} \to e^{2i\alpha} \mathcal{L}_{mM}$$

# Diagonalize a Majorana mass term

If there are several fields, there will be a Majorana mass matrix. We can show that it is symmetric.

$$M_M = M_M^T$$

In fact:

$$\nu_L^T M_M C^{-1} \nu_L = (\nu_L^T M_M C^{-1} \nu_L)^T$$
$$= -\nu_L^T M_M^T C^{-1,T} \nu_L = \nu_L^T M_M^T C^{-1} \nu_L$$

This implies that only one unitary mixing matrix is required to diagonalise it

$$M_M = (U^\dagger)^T m_{\rm diag} U^\dagger$$

The massive fields are related to the flavour ones as

$$n_L = U^\dagger \nu_L$$

and the Lagrangian can be rewritten in terms of a Majorana field

$$\mathcal{L}_M = -\frac{1}{2}\overline{n_L^c} m_{\text{diag}} n_L - \frac{1}{2}\overline{n_L} m_{\text{diag}} n_L^c = -\frac{1}{2}\bar{\chi} m_{\text{diag}} \chi$$

with

$$\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c$$

A Majorana mass term (breaks L) leads to Majorana neutrinos (breaks L).

# Dirac + Majorana masses

If we have both the left-handed and right-handed fields, we can write down three mass terms:

- a Dirac mass term
- a Majorana mass term for the left-handed field and
- a Majorana mass term for the right-handed field.

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \mathrm{h.c.}$$

**What do we expect the massive neutrinos to be? Dirac, Majorana, both?**

# Dirac + Majorana masses

If we have both the left-handed and right-handed fields, we can write down three mass terms:

- a Dirac mass term
- a Majorana mass term for the left-handed field and
- a Majorana mass term for the right-handed field.

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2}\nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2}\nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

**This breaks lepton number, in both the Majorana mass terms.**

The expectation is that, as lepton number is not conserved, neutrinos will be Majorana particles. Let's prove it.

We start by rewriting $\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\psi^c_L}\mathcal{M}\psi_L + \text{h.c.}$

with $\psi_L \equiv \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $\mathcal{M} \equiv \begin{pmatrix} M_{M,L} & m_D^T \\ m_D & M_{M,R} \end{pmatrix}$

In fact

$$\mathcal{L}_{mD+M} = -\frac{1}{2}\bar{\nu^c_L} M_{M,L}\nu_L - \frac{1}{2}\bar{\nu_R} M_{M,R}\nu_R - \bar{\nu}_R m_D \nu_L + \text{h.c.}$$

and one can use $\bar{\nu^c_L} m_D^T \nu_R^c = \bar{\nu}_R m_D \nu_L$

**Exercise**
Show that these two formulations are equivalent.

Then, we need to diagonalise the full mass matrix, and we find the Majorana massive states, in analogy to what we have done for the Majorana mass case.

$$\chi \equiv n_L + n_L^c \Rightarrow \chi = \chi^c$$

The difference is that $n_L = U_j \nu_L + U_k \nu_R^c$

Not unitary

Mixing between mass states and sterile neutrinos

# Summary of neutrino mass terms

## Dirac masses

$$\mathcal{L}_{mD} = -m_\nu (\bar{\nu}_R \nu_L + \text{h.c.})$$

This term conserves lepton number.

## Majorana masses

$$\mathcal{L}_{mM} \propto -M_M \bar{\nu_L^c} \nu_L + \text{h.c.} = M_M \nu_L^T C^{-1} \nu_L$$

This term breaks lepton number.

## Dirac + Majorana masses

$$\mathcal{L}_{mD+M} = -m_\nu \bar{\nu}_R \nu_L - \frac{1}{2} \nu_L^T M_{M,L} C^{-1} \nu_L - \frac{1}{2} \nu_R^T M_{M,R} C^{-1} \nu_R + \text{h.c.}$$

Lepton number is broken -> Majorana neutrinos.

# *Plan of lecture II*

- What do we know about neutrino parameters?
- Dirac vs Majorana neutrinos
- How to test the nature of neutrinos and measure their masses
- What type of masses neutrinos can have
- What extensions of the SM can lead to neutrino masses

**Can neutrino masses arise in the SM? and if not, how can we extend the SM to generate them?**

# Neutrino masses in the SM and beyond

In the SM, neutrinos do not acquire mass and mixing:

- like the other fermions as there are no right-handed neutrinos.

$$m_e \bar{e}_L e_R \qquad\qquad m_\nu \bar{\nu}_L \nu_R$$

*Solution*: Introduce $\nu_R$ for Dirac masses

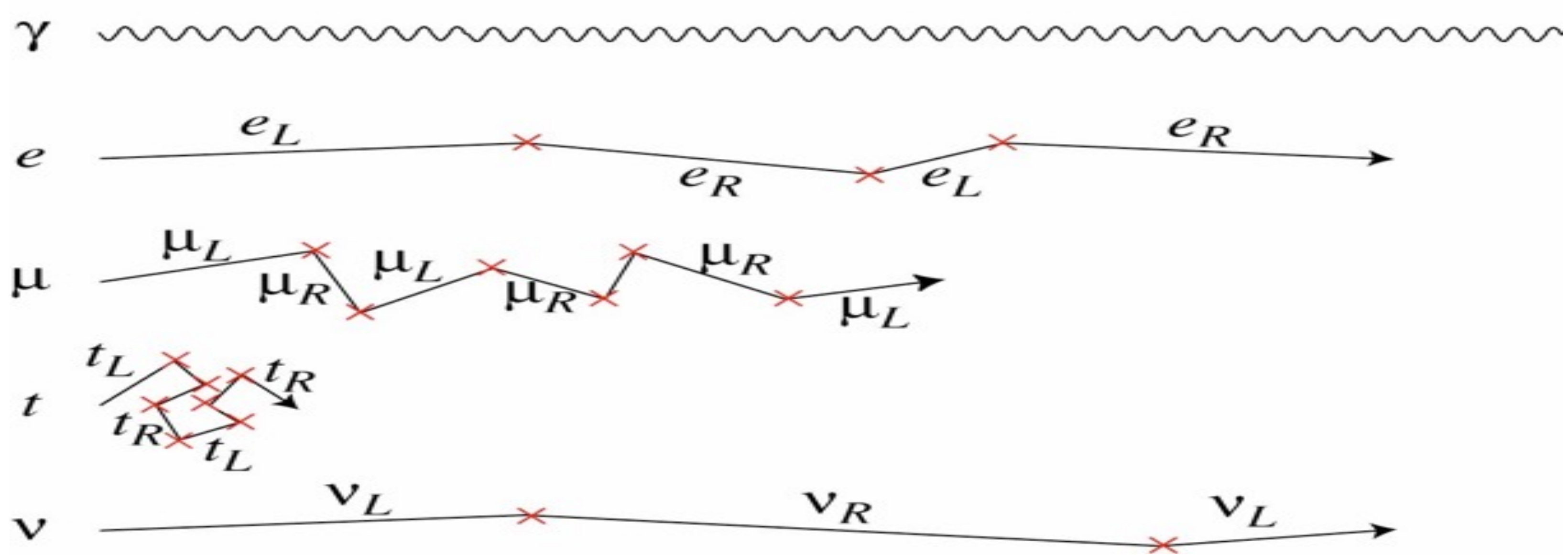
- they do not have a Majorana mass term

$$M \nu_L^T C \nu_L$$

as this term breaks the SU(2) gauge symmetry.
*Solution*: Introduce an SU(2) scalar triplet or gauge invariant non-renormalisable terms (D>4). This term breaks Lepton Number.

# *Dirac Masses*

If we introduce a right-handed neutrino, then a lepton-number conserving interaction with the Higgs boson emerges.




Thanks to
H. Murayama

$$\mathcal{L} = -y_\nu \bar{L} \cdot \tilde{H} \nu_R + \text{h.c.}$$

with $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ and $\tilde{H} = \begin{pmatrix} H^{0,*} \\ -H^- \end{pmatrix}$

This term is
- SU(2) invariant and
- respects lepton number

## When the neutral component of the Higgs field gets a vev, a Dirac mass term for neutrinos is generated.

$$\mathcal{L}_{\nu H} = -y_\nu (\bar{\nu}_L, \bar{\ell}_L) \cdot \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} \nu_R + \text{h.c.}$$

$$= -y_\nu (\bar{\nu}_L H^{0*} - \bar{\ell}_L H^-) \nu_R + \text{h.c.}$$

$$= -y_\nu \frac{v_H}{\sqrt{2}} \bar{\nu}_L \nu_R + \text{h.c.} + \dots$$

$H^0 \to \frac{v_H}{\sqrt{2}} + h^0$

It follows that

$$y_\nu \sim \frac{\sqrt{2} m_\nu}{v_H} \sim \frac{0.2 \text{ eV}}{200 \text{ GeV}} \sim 10^{-12}$$

Tiny couplings!

Many theorists consider this explanation of neutrino masses not satisfactory. We would expect this Yukawa couplings to be similar to the ones in the quark sector:

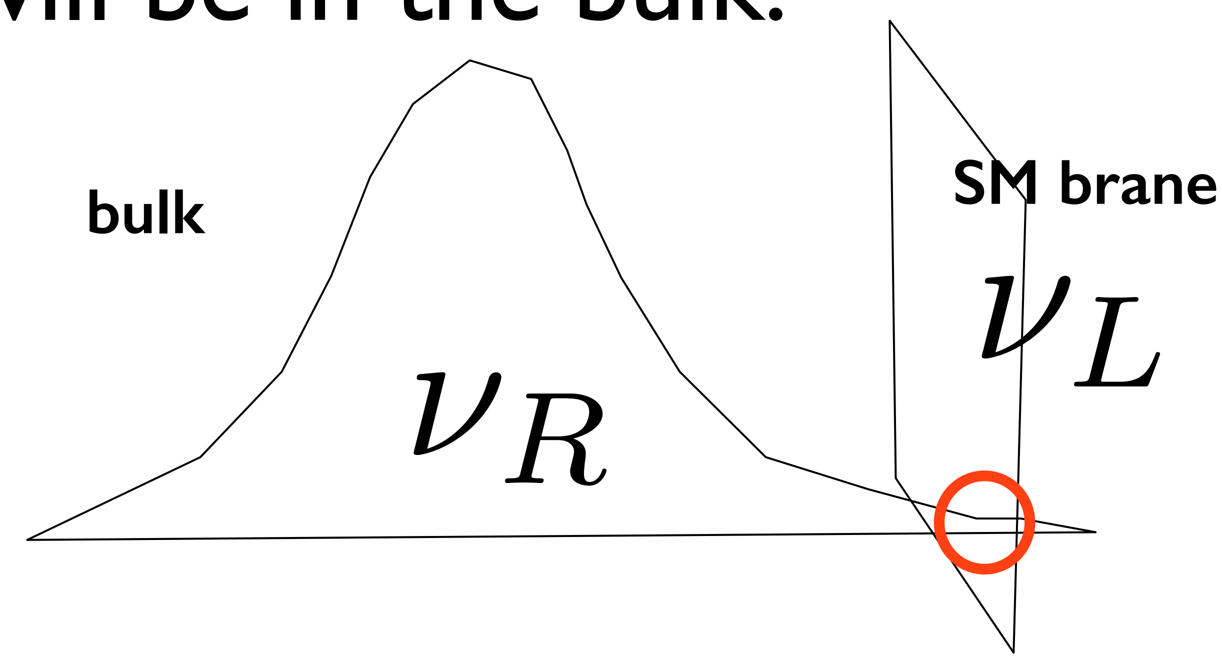
1. why the coupling is so small????
2. why the mixings are large? (instead of small as in the quark sector)
3. why neutrino masses have at most a mild hierarchy if they are not quasi-degenerate? instead of what happens to quarks?

Dirac masses are strictly linked to lepton number conservation. But this is an accidental global symmetry. Should it be conserved at high scales?

There are models which address the problem of the smallness of the couplings.

## Extra-D models

In these models all gauge-interacting fields are in the SM brane. Right-handed neutrinos are singlets and therefore will be in the bulk.




The overlap of the wavefunctions (which are normalised) of the left-handed and right-handed neutrinos leads to a small Yukawa coupling.

See e.g. Arkani-Hamed et al., 2002; Grossman and Neubert, 2000. Models with warped extra-D....

# *Majorana Masses*

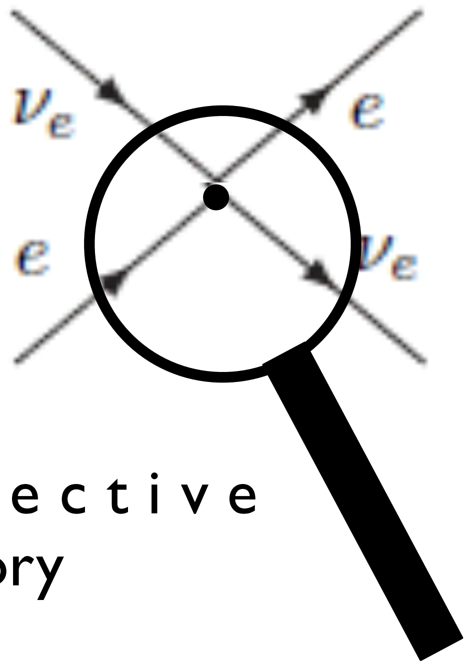
In order to have an SU(2) invariant mass term for neutrinos, it is necessary to introduce a Dimension 5 operator (or to allow for new scalar fields, e.g. a scalar triplet):

$$-\mathcal{L} = \lambda \frac{\nu_L H \, \nu_L H}{M} = \frac{\lambda v^2}{M} \nu_L^T C \nu_L$$

D=5 term

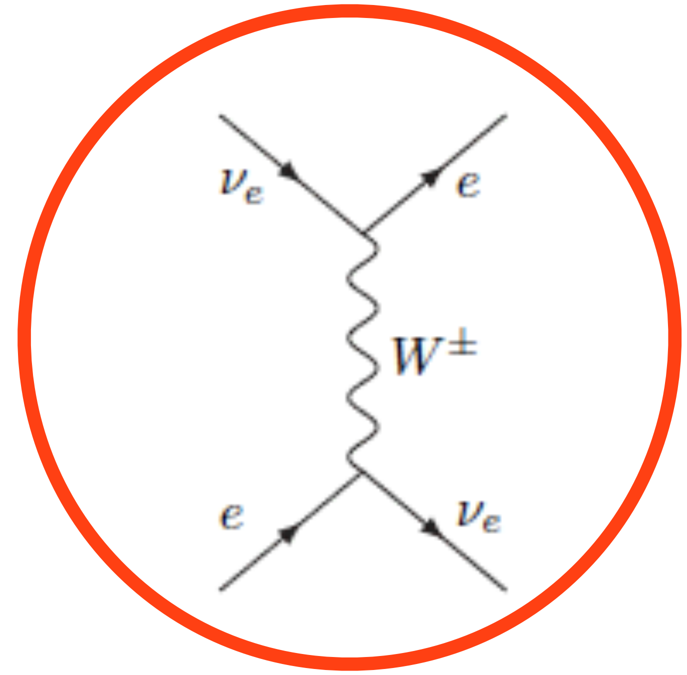
**Lepton number violation!**

If neutrino are Majorana particles, a Majorana mass can arise as the low energy realisation of a higher energy theory (new mass scale!).



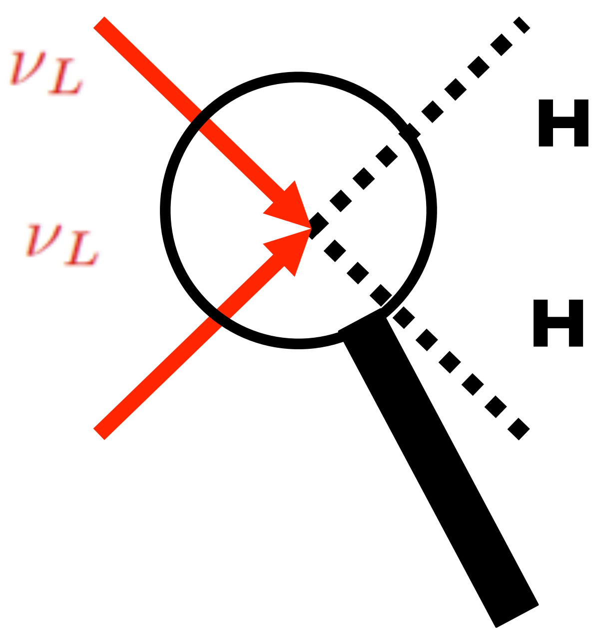

effective theory

$$\mathcal{L} \propto G_F(\bar{e}_L\gamma_\mu\nu_L)(\bar{\nu}_L\gamma^\mu e_L)$$




Standard Model: W exchange

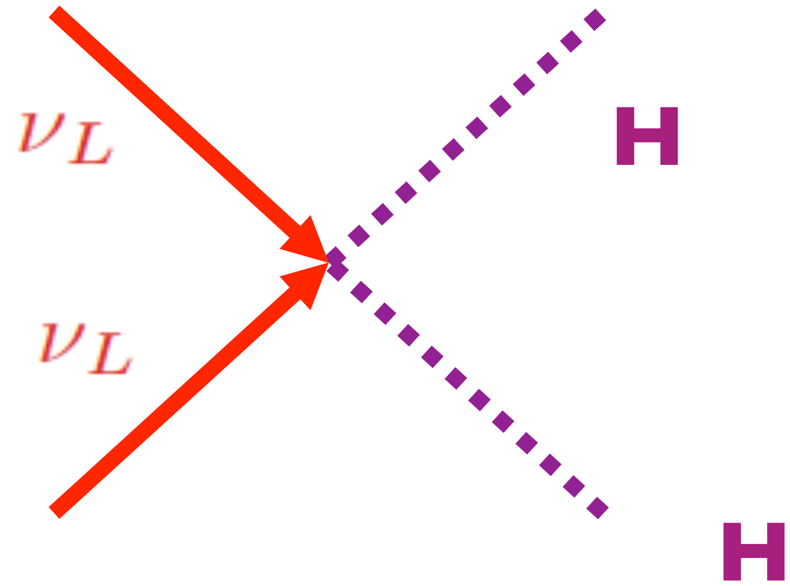
$$\mathcal{L}_{SM} \propto g\bar{\nu}_L\gamma^\mu e_L W_\mu \Rightarrow G_F \propto \frac{g^2}{m_W^2}$$




Neutrino mass

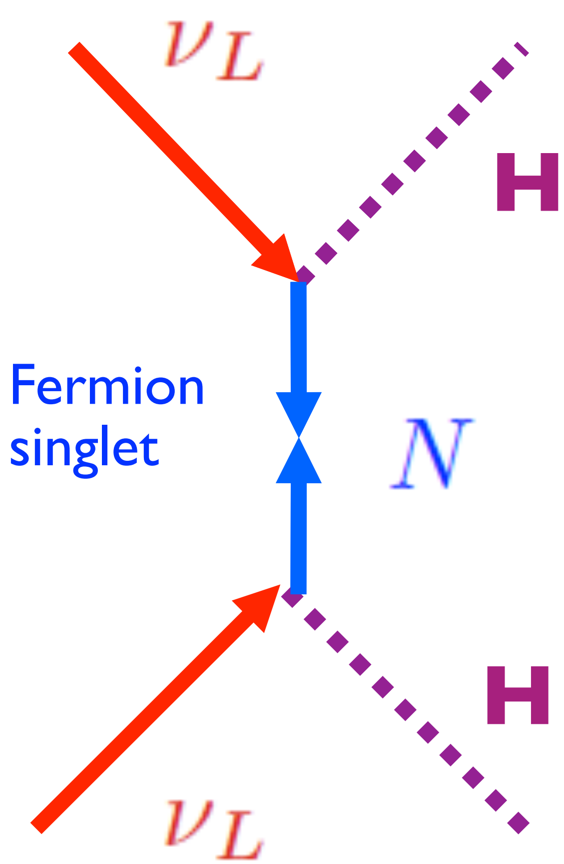
$$-\mathcal{L} = \lambda\frac{\nu_L H\,\nu_L H}{M} = \frac{\lambda v^2}{M}\nu_L^T C\nu_L$$


New theory: new particle exchange with mass M



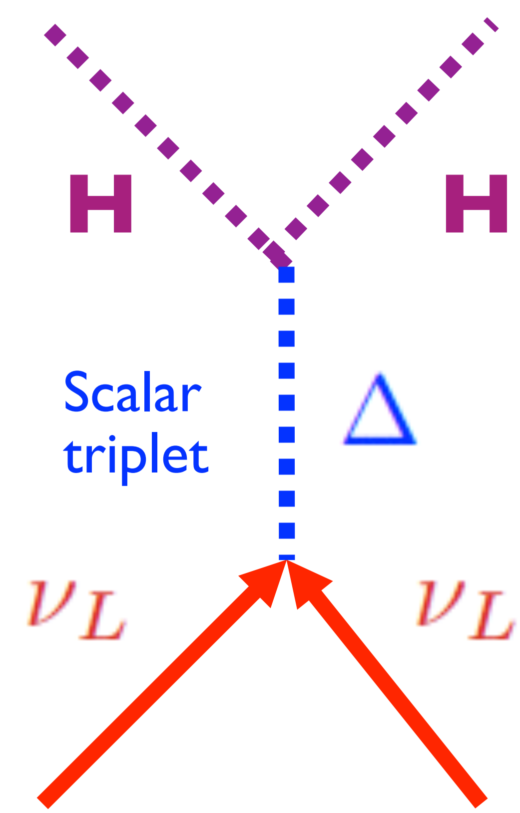

$$\frac{\nu_L H \, \nu_L H}{\Lambda}$$

## See-saw Type I



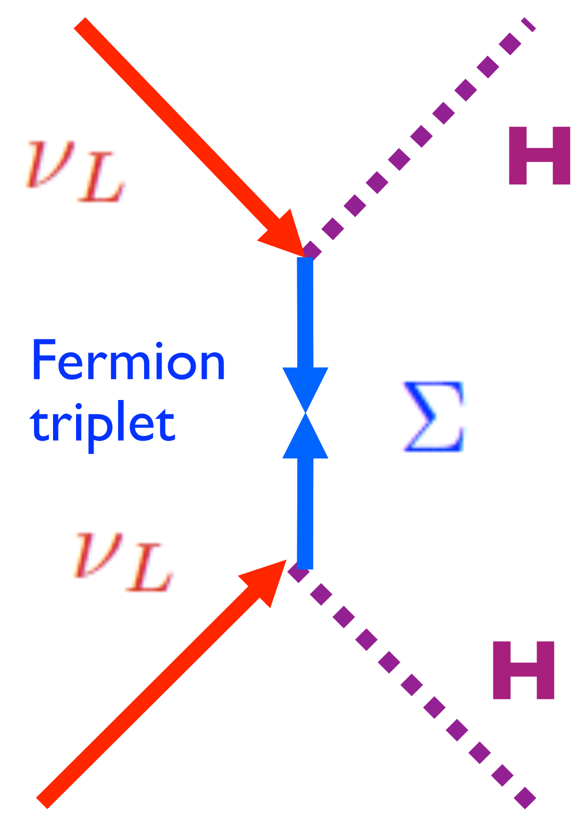

Minkowski, Yanagida, Glashow, Gell-Mann, Ramond, Slansky, Mohapatra, Senjanovic

## See-saw Type II



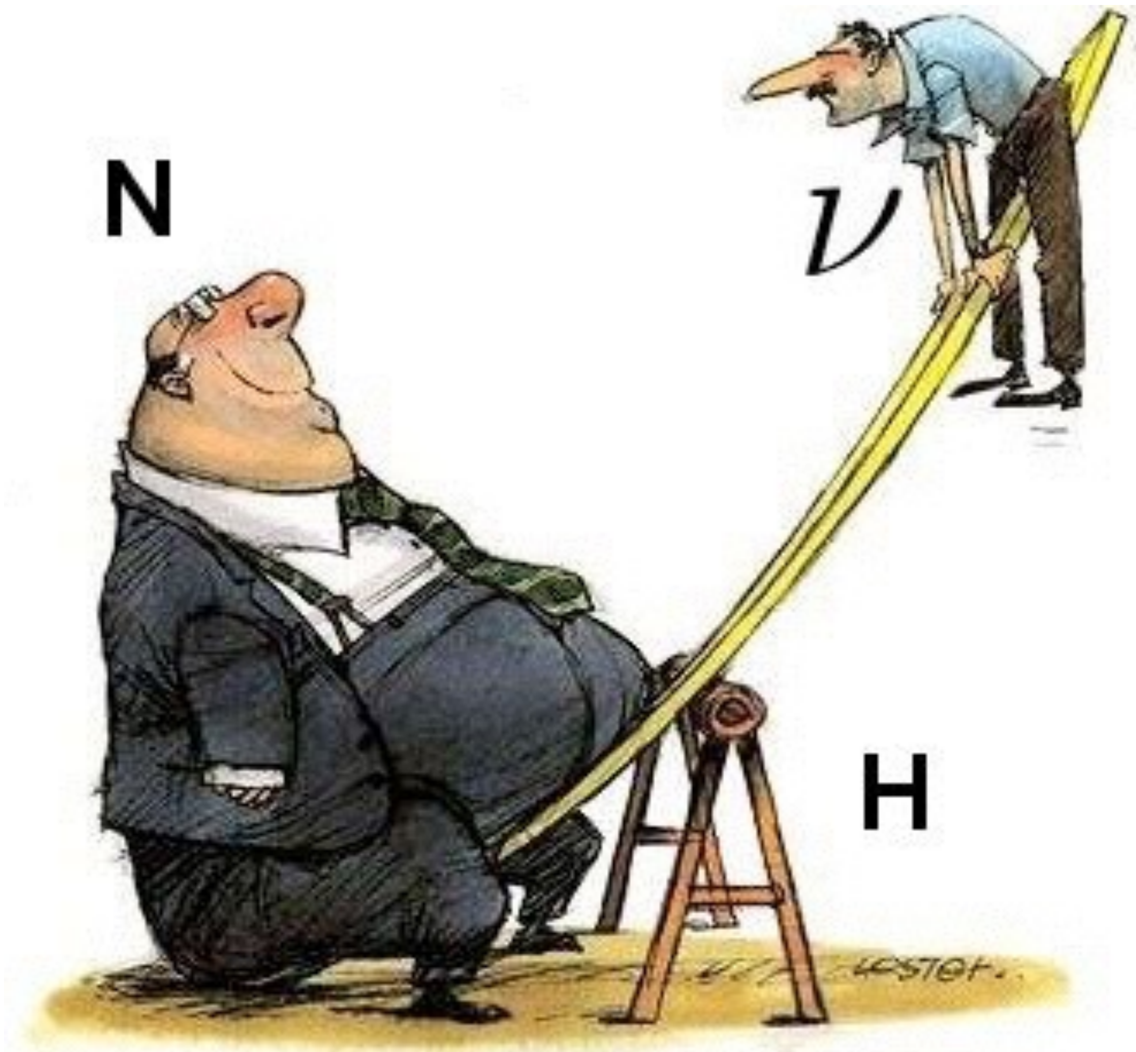

Magg, Wetterich, Lazarides, Shafi. Mohapatra, Senjanovic, Schecter, Valle

## See-saw Type III




Ma, Roy, Senjanovic, Hambye

# *Models of neutrino masses BSM*




*See-saw type I*

- Introduce a right handed neutrino **N (sterile neutrino)**
- Couple it to the Higgs and left handed neutrinos

The Lagrangian is

breaks lepton number

$$\mathcal{L} = -Y_\nu \bar{N} L \cdot H - 1/2 \bar{N}^c M_R N$$

When the Higgs boson gets a vev, Dirac masses will be generated. The mass matrix will be (for one generation)

$$\mathcal{L} = \left(\nu_L^T N^T\right) \left( \begin{array}{cc} 0 & m_D \\ m_D^T & M \end{array} \right) \left( \begin{array}{c} \nu_L \\ N \end{array} \right)$$

This is of the Dirac+Majorana type we discussed earlier. So we know that the massive states are found by diagonalising the mass matrix and the massive states will be Majorana neutrinos.

$$\left| \begin{array}{cc} -\lambda & m_D \\ m_D & M-\lambda \end{array} \right| = 0$$

$$\lambda^2 - M\lambda - m_D^2 = 0$$

$$\lambda_{1,2} = \frac{M \pm \sqrt{M^2 + 4m_D^2}}{2} \simeq \frac{\overset{M}{M - M}}{2} - \frac{4m_D^2}{4M} = -\frac{m_D^2}{M}$$

One massive state remains very heavy, the light neutrino masses acquires a tiny mass!

$$m_\nu \simeq \frac{m_D^2}{M} \sim \frac{1\,\mathrm{GeV}^2}{10^{10}\,\mathrm{GeV}} \sim 0.1\,\mathrm{eV}$$

Mixing between active neutrinos and heavy neutrinos will emerge but it will be typically very small

$$\tan 2\theta = \frac{2m_D}{M}$$

and can be related to neutrino masses $m_\nu \simeq \frac{m_D^2}{M} \simeq \sin^2\theta M$

# Pros and cons of type I see-saw models

## Pros:

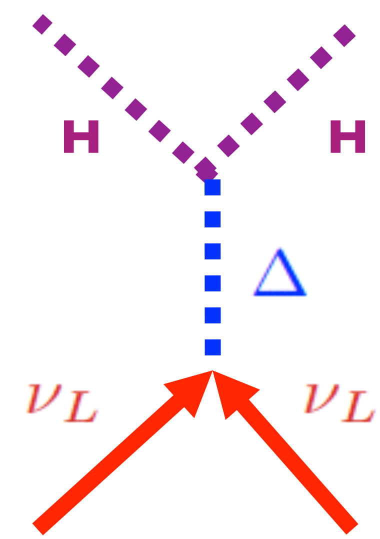
- they explain "naturally" the smallness of neutrino masses.
- can be embedded in GUT theories!
- neutrino masses are a indirect test of GUT theories
- have several phenomenological consequences (depending on the mass scale), e.g. leptogenesis, LFV

## Cons:

- the new particles are typically too heavy to be produced at colliders (but TeV scale see-saws)
- the mixing with the new states are tiny
- in general: difficult to test

# See-saw type II

We introduce a Higgs triplet which couples to the Higgs and left handed neutrinos. It has hypercharge 2.




$$\mathcal{L}_\Delta \propto y_\Delta L^T C^{-1} \sigma_i \Delta_i L + \text{h.c.}$$

with $$\Delta_i = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix}$$

Once the Higgs triplet gets a vev, Majorana neutrino masses arise:

$$m_\nu \sim y_\Delta v_\Delta$$

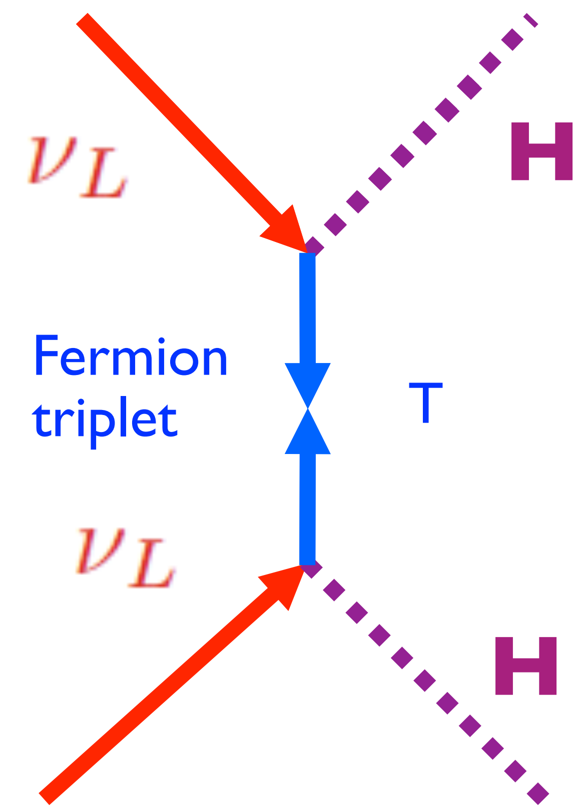
Cons: why the vev is very small?
Pros: the component of the Higgs triplet could tested directly at the LHC.

# See-saw type III

We introduce a fermionic triplet which has hypercharge 0.

$$\mathcal{L}_T \propto y_T \bar{L} \sigma H \cdot T + \text{h.c.}$$

with $T = \begin{pmatrix} T^0 & T^+ \\ T^- & -T^0 \end{pmatrix}$




Majorana neutrino masses are generated as in see-saw type I:

$$m_\nu \simeq -y_T^T M_T^{-1} y_T v_H^2$$

Pros: the component of the fermionic triplet have gauge interactions and can be produced at the LHC
Cons: why the mass of T is very large?

# Extensions of the see saw mechanism

Models in which it is possible to lower the mass scale (e.g. TeV or below), keeping large Yukawa couplings have been studied. Examples: inverse and extended see-saw.

Let's introduce two right-handed singlet neutrinos.

$$\mathcal{L} = Y\bar{L}\cdot HN_1 + Y_2\bar{L}\cdot HN_2^c + \Lambda\bar{N}_1 N_2 + \mu' N_1^T C N_1 + \mu N_2^T C N_2$$

$$\begin{pmatrix} 0 & Yv & Y_2 v \\ Yv & \mu' & \Lambda \\ Y_2 v & \Lambda & \mu \end{pmatrix}$$

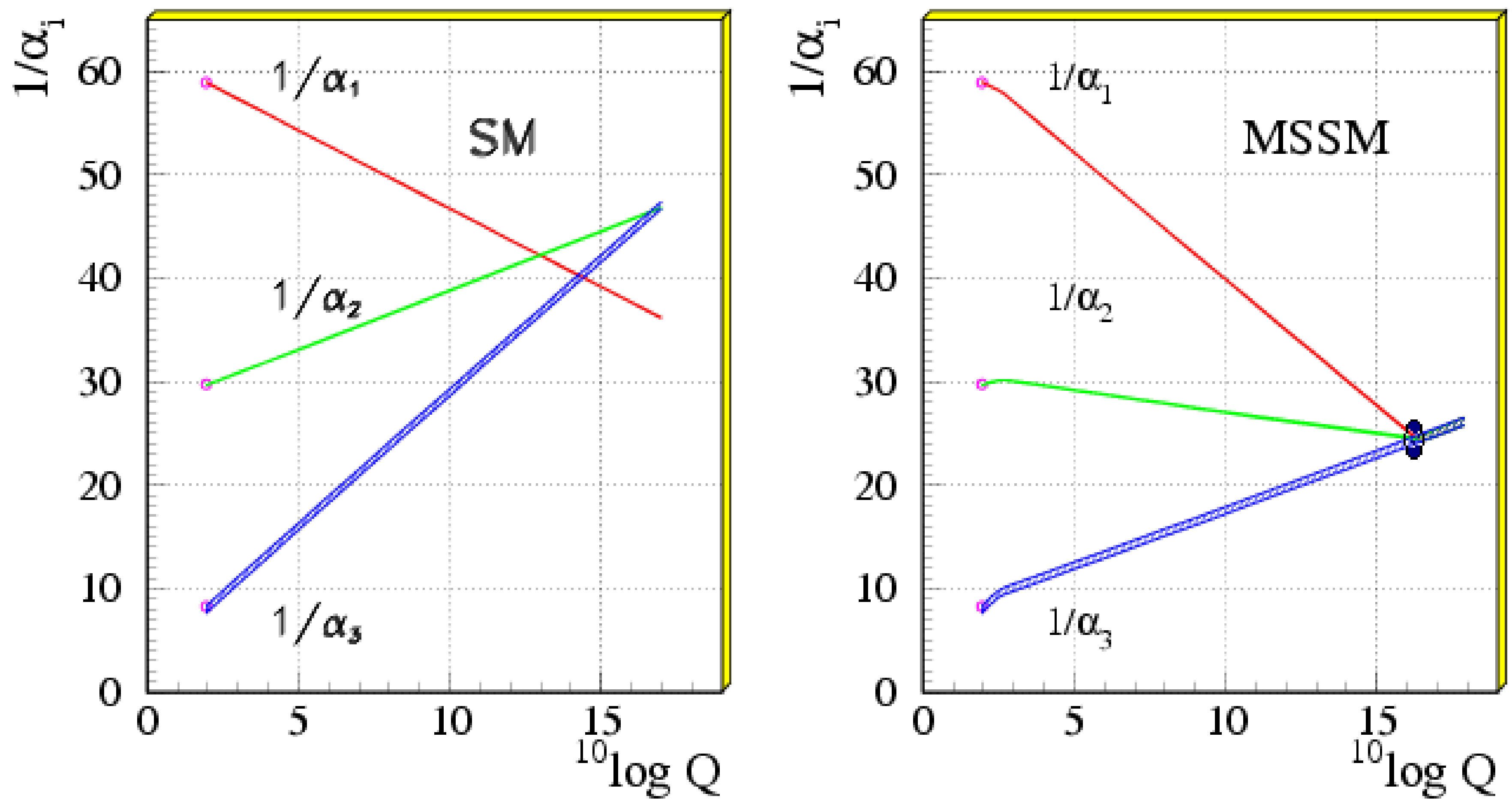
See e.g. Gavela et al., 0906.1461; Ibarra, Molinaro, Petcov, 1103.6217; Kang, Kim, 2007; Majee et al., 2008; Mitra, Senjanovic, Vissani, 1108.0004; Malinsky, Romao, Valle, 2005

$$m_{tree} \simeq -m_D^T M^{-1} m_D \simeq \frac{v^2}{2(\Lambda^2 - \mu'\mu)} \left(\mu Y_1^T Y_1 + \epsilon^2 \mu' Y_2^T Y_2 - \Lambda\epsilon(Y_2^T Y_1 + Y_1^T Y_2)\right)$$

Small neutrino masses emerge due to cancellations between the contributions of the two sterile neutrinos (typically associated to small breaking of some L).

# *GUT theories and the see-saw mechanism*

The SM has a very complex gauge structure (3 gauge couplings) and charge assignments for the fields. GUT aim at providing a unified picture.




S. Rabi, PDG

Due to the renormalisation of the couplings, they "run" and unify at a very high energy scale, typically $10^{16}$ GeV. Ingredients: gauge group (only 1 group and 1 coupling), fermion reps, Higgs sector, symmetry breaking.

Let's make a parallel with the SM.

1. Gauge group: SU(2)L x U(1)Y

2. Choose representations of the group and assign the fermions to it.

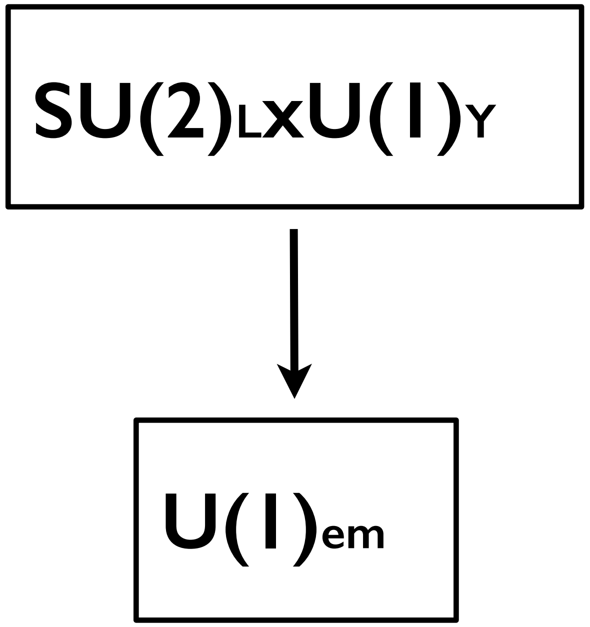
SU(2) singlet: e.g. $e_R, \quad u_R, \quad d_R$

SU(2) doublet: e.g. $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

3. Introduce a scalar Higgs sector. This breaks the symmetry to a subgroup. To break SU(2)L, the scalar needs to be a doublet and to preserve U(1)em it needs to have a neutral component.

$$\begin{pmatrix} H^0 \\ H^- \end{pmatrix}$$

4. $H_0$ gets a vev and the symmetry is broken




5. Invariance w.r.t. the gauge group dictates the type of terms in the Lagrangian: both the gauge interactions and the Yukawa ones.

E.g.

$$\mathcal{L}_{\nu H} = -y_\nu (\bar{\nu}_L, \bar{\ell}_L) \cdot \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} \nu_R + \text{h.c.}$$

6. Masses for the gauge bosons, the Higgs field and the fermions result from it and depend on $v_H$.

# Left-right models

This is a very simple model in which the see-saw can be naturally embedded.

1. Gauge group: SU(2)L x SU(2)R x U(1)B-L

2. Fermion assignment:

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$ Doublet, singlet, -1

$$\begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$ Singlet, doublet, -1

and so on for the quarks.

3. Introduce a scalar Higgs sector.

As we want to break the symmetry from SU(2)L x SU(2)R x U(1)B-L to SU(2)L x U(1)Y, the Higgs needs to be a singlet of SU(2)L and transform non-trivially w.r.t. SU(2)R. We take a triplet of SU(2)R.

$$\begin{pmatrix} \xi^{+}/\sqrt{2} & \xi^{++} \\ \xi^{0} & \xi + /\sqrt{2} \end{pmatrix}$$

4. The symmetry is broken

$SU(2)_L x SU(2)_R x U(1)_{B-L}$

↓

$SU(2)_L x U(1)_Y$

The EW breaking is a achieved by a Higgs boson, doublet of SU(2)L and SU(2)R.

## 5. Invariance of the Yukawa couplings

$$\mathcal{L} \propto y_1 \bar{L}_R H L_L + \ldots + y_\xi \bar{L}_R^c i\sigma_2 \xi L_R + \text{h.c.}$$

## 6. Masses for neutrinos

$$\mathcal{L} \propto y_1 v_H \bar{\nu}_R \nu_L + \ldots + y_\xi v_\xi \bar{\nu}_R^c \nu_R + \text{h.c.}$$

Usual Dirac mass term

Majorana mass term for N

Remembering that $v_{xi} >> v_H$, the usual see-saw structure has emerged and neutrino mass will be given by

$$m_\nu \simeq \frac{(y_1 v_H)^2}{2 y_\xi v_\xi}$$

# SO(10) GUT models

SO(10) contains SU(2)L x SU(2)R x U(1)B-L and a right-handed neutrino and can easily implement the see-saw mechanism.

The leptons and quarks belong to the same representation, their masses come from the same source and will be related.

The scale of breaking (and consequently the mass for the right-handed neutrino) is at a very high energy scale: see-saw naturally implemented.

1. Gauge group: SO(10)
only one gauge coupling g!!!

## 2. Fermion assignment:

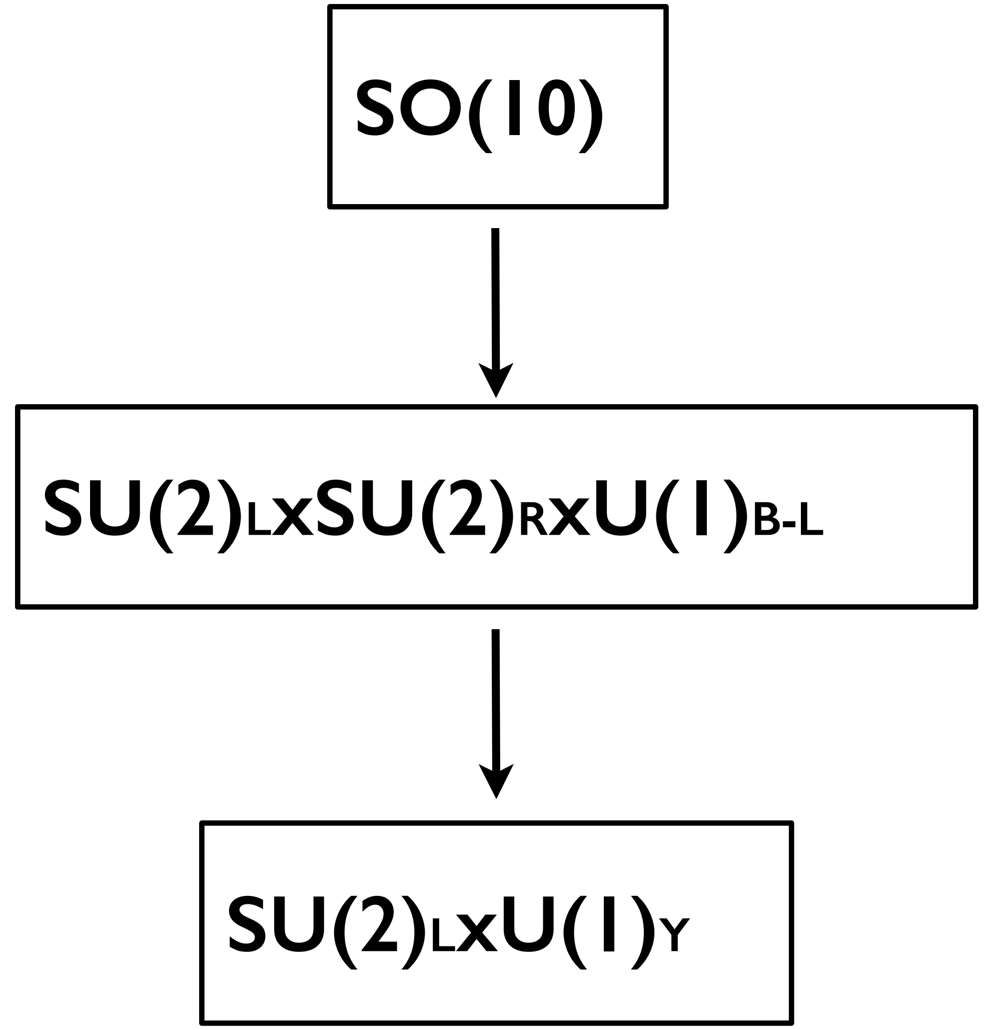
$$f(16)_L = \left( \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} \nu_R^c \\ e_R^c \end{pmatrix}, \begin{pmatrix} u_R^c \\ d_R^c \end{pmatrix} \right)$$

Quarks and leptons belong to the same representation! Their behaviour is related.
The right-handed neutrino is present and belongs also to this representation.

## 3. Introduce a scalar Higgs sector.

We want to break the symmetry from SO(10) to SU(2)L x SU(2)R x U(1)B-L. This is achieved using a Higgs in the 45-representation.
There are also other useful scalar representations: H(10), H(120), H(126)... Some of their components can also get vevs.

## 4. The symmetry is broken.




## 5. Invariance of the Yukawa couplings

Neutrino masses require two fermions (so 2 f(16)).

$$f(16) \otimes f(16) = f(10) + f(120) + f(126)$$

$$\mathcal{L} \propto g_{10} f(16) f(16) H(10) + g_{126} f(16) f(16) H(126)$$

## 6. Masses for neutrinos

Once the H(10) gets a vev, Dirac masses emerge for the quarks and leptons. They are related

$$M_u(GUT) = M_d(GUT) = M_l(GUT) = M_\nu(GUT)$$

Usual Dirac mass term

This relation is in conflict with data. So we need to introduce also H(126) to give a large mass to the right-handed neutrino:

$$M_N = g_{126} v_{126}$$

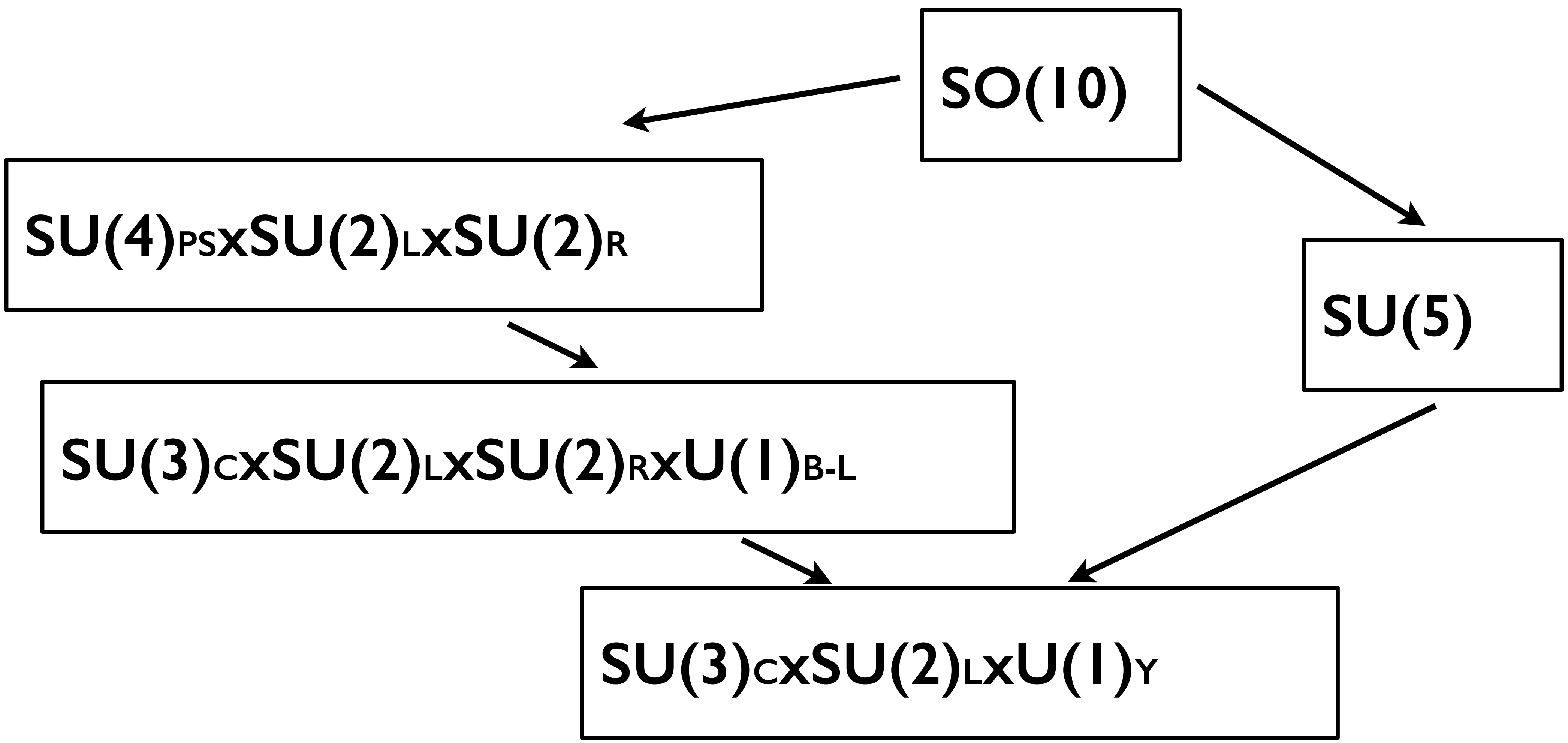
Majorana mass term for N

The usual see-saw structure is present.

$$m_\nu \sim \frac{g_{10}^2 v_H^2}{g_{126} v_{126}} \propto m_q^2 \quad \Rightarrow \quad m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_u^2 : m_c^2 : m_t^2$$

This relation can be relaxed via H(10)+H(126), a direct Majorana mass and/or a specific structure for MN.

The see-saw can emerge naturally in GUTheories: e.g. SO(10). They provide the necessary elements: N, large M and L violation.



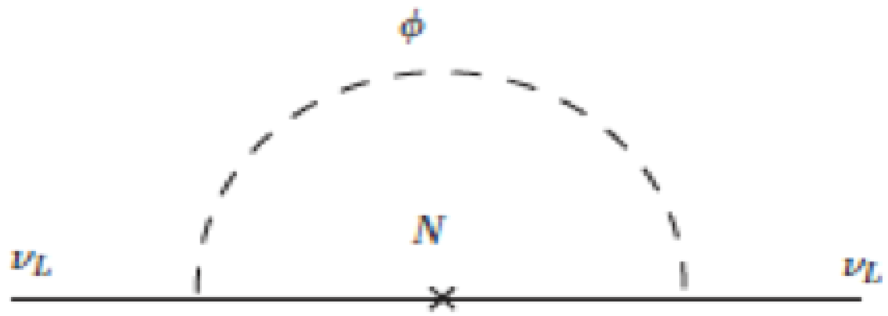

They typically lead to proton decay and to relations between quark and lepton masses.

# Other models of neutrino masses

## Radiative masses

If neutrino masses emerge via loops, in models in which Dirac masses are forbidden, there is an additional suppression.

Some of these models have also dark matter candidates.




$$m_\nu \propto \frac{g^2}{16\pi^2} f(M, \mu_\phi^2)$$

See Ma, PRL81; also e.g. Boehm et al., PRD77; ...

## R-parity violating SUSY

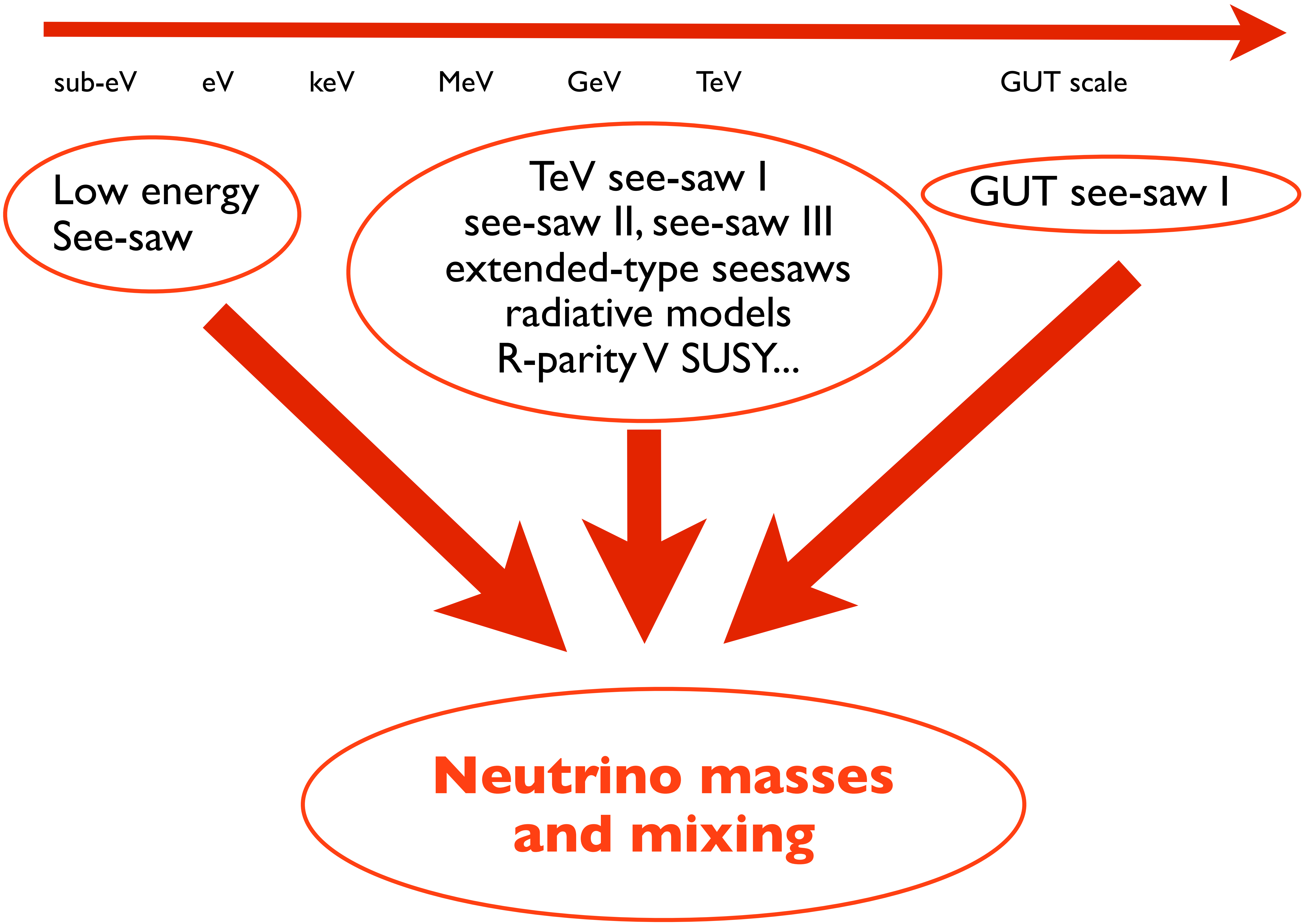
In the MSSM, there are no neutrino masses. But it is possible to introduce terms which violate R (and L).

$$V = \ldots - \mu H_1 H_2 + \epsilon_i \tilde{L}_i H_2 + \lambda'_{ijk} \tilde{L}_i \tilde{L}_j \tilde{E}_k + \ldots$$

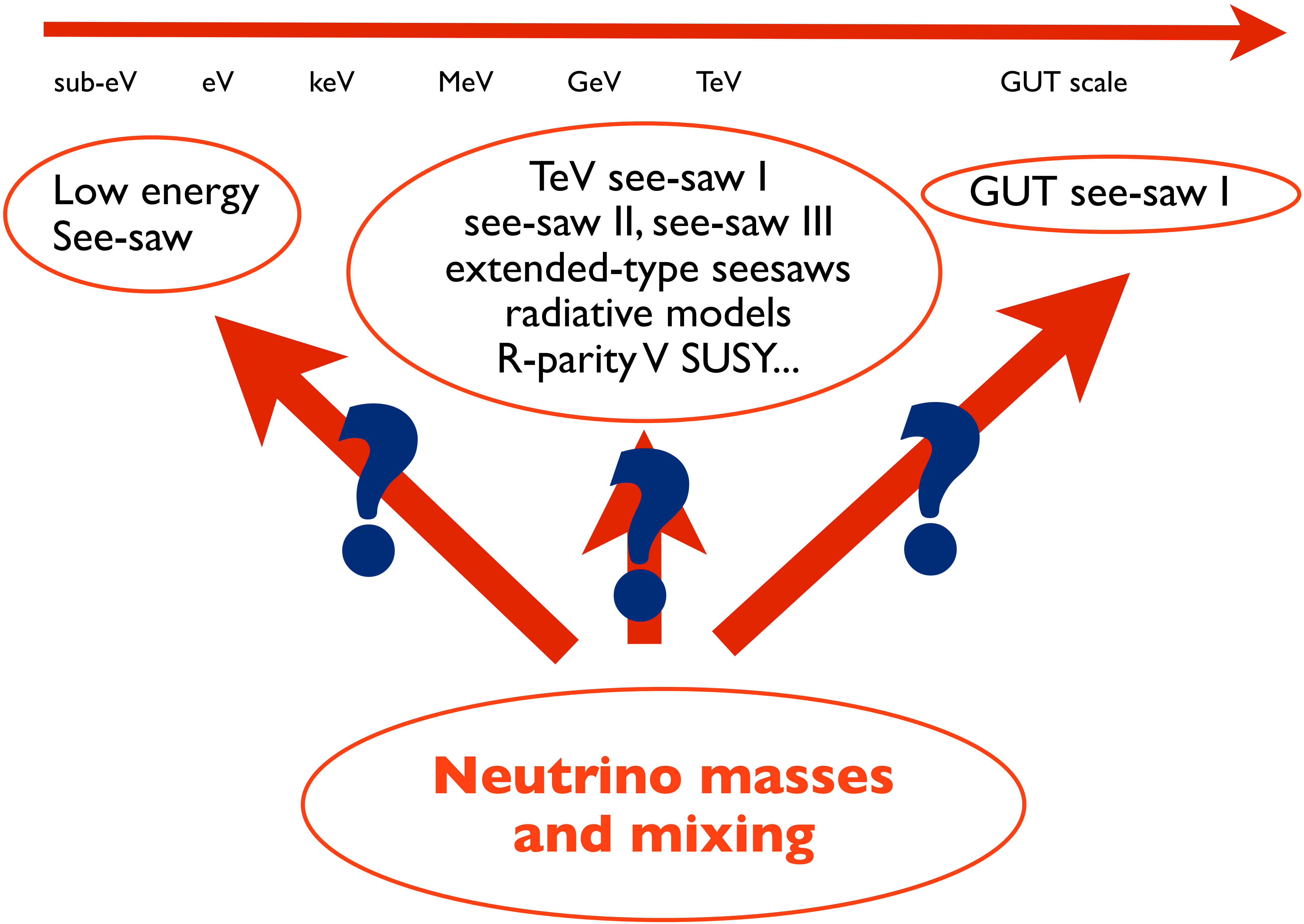
See e.g. Aulakh, Mohapatra, PLB119; Hall, Suzuki, NPB231; Ross, Valle, PLB151; Ellis et al., NPB261; Dawson, PRD57, ...

The bilinear term induces mixing between neutrinos and higgsino, the trilinear term masses at loop-level.

# What is the new physics?



# What is the new physics?



# Complementarity with other searches

There are many (direct and indirect) signatures of these extensions of the SM.

Neutrino masses

**Signatures**

- Indirect signals (proton decay)
- Peak searches
- Direct signals in colliders
- Charged lepton flavour violation
- Kinks in beta decay
- Nuless 2beta decay
- Leptogenesis

Establishing the origin of neutrino masses requires to have as much information as possible about the masses and to combine it with other signatures of the models.

# Summary

1. Neutrinos have masses and a wide experimental programme aims at measuring them.

2. Neutrinos can be Dirac or Majorana particles. Neutrinoless double beta decay is the most sensitive test.

3. Neutrino masses beyond the Standard Model: Dirac, Majorana and Dirac+Majorana masses

4. We have looked at models of masses BSM:
   Dirac masses
   see saw type I
   see-saw type II
   see-saw type III

# A few references


## Absolute mass measurements

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## Neutrinoless double deta decay

S. M. Bilenky and C. Giunti, Double beta decay, Ann. Rev. Int.J.Mod.Phys. A30 (2015) no.04n05, 1530001 [arXiv:1411.4791]

## Models of neutrino masses:

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