

Lecture III: Leptonic Mixing Neutrinos in cosmology

Summer School on Particle Physics

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What will you learn from this lecture?

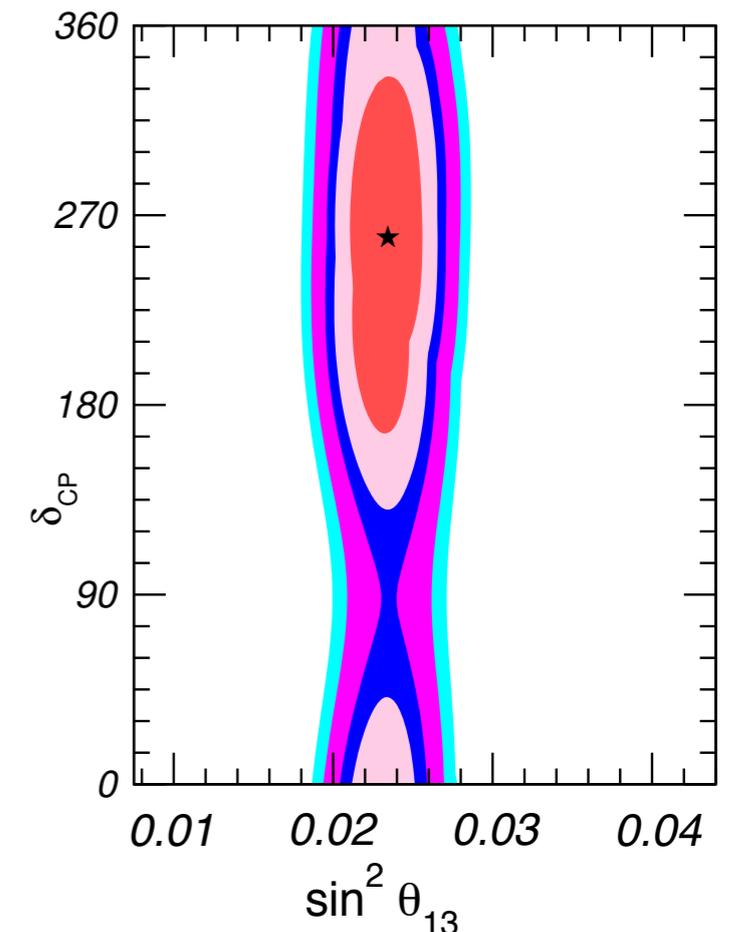
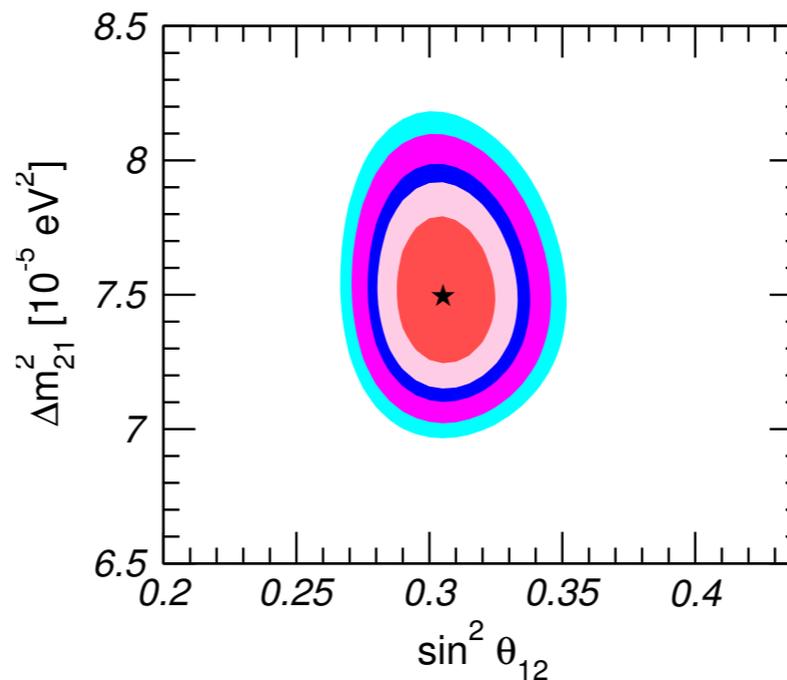
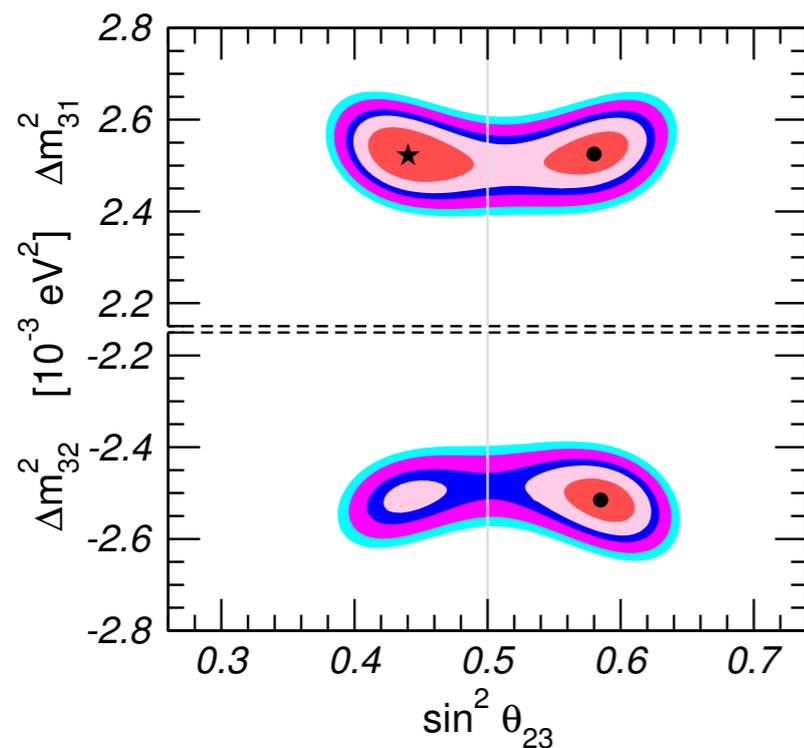
- **The problem of leptonic mixing**
 - Current status
 - Prospects to discover leptonic CPV and measure with precision the oscillation parameters
 - How to explain the observed mixing structure and Flavour symmetry models

- **Neutrinos in cosmology**
 - neutrinos in the Early Universe
 - sterile neutrinos as WDM
 - Leptogenesis and the baryon asymmetry

Plan of lecture III

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Recap of neutrino mixing



Important aspects:

- θ_{23} maximal or close to maximal
- θ_{12} significantly different from maximal
- θ_{13} quite large. This poses some challenges for understanding the origin of the flavour structure
- Mixings very different from quark sector

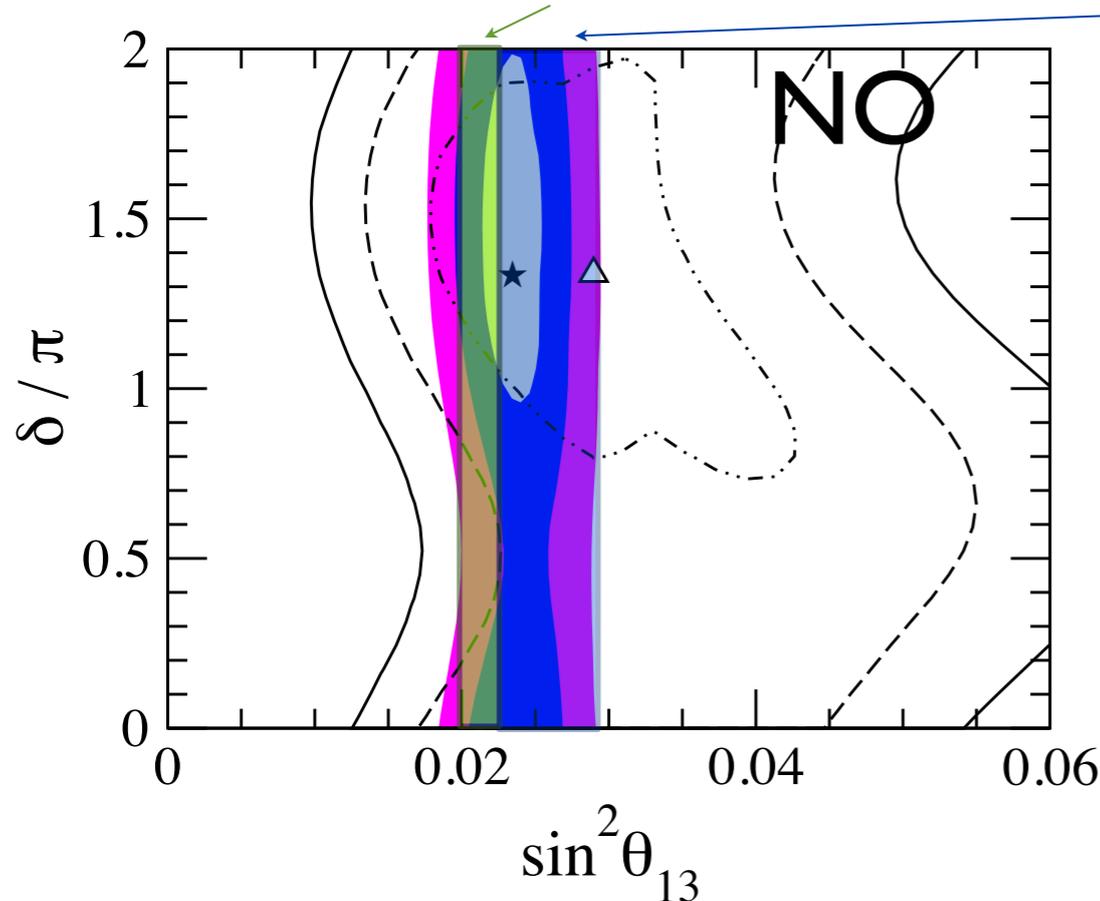
NuFit 3.0: M. C. Gonzalez-Garcia et al., 1611.01514

See also F. Capozzi et al., 1703.04471

Hints of CP-violation

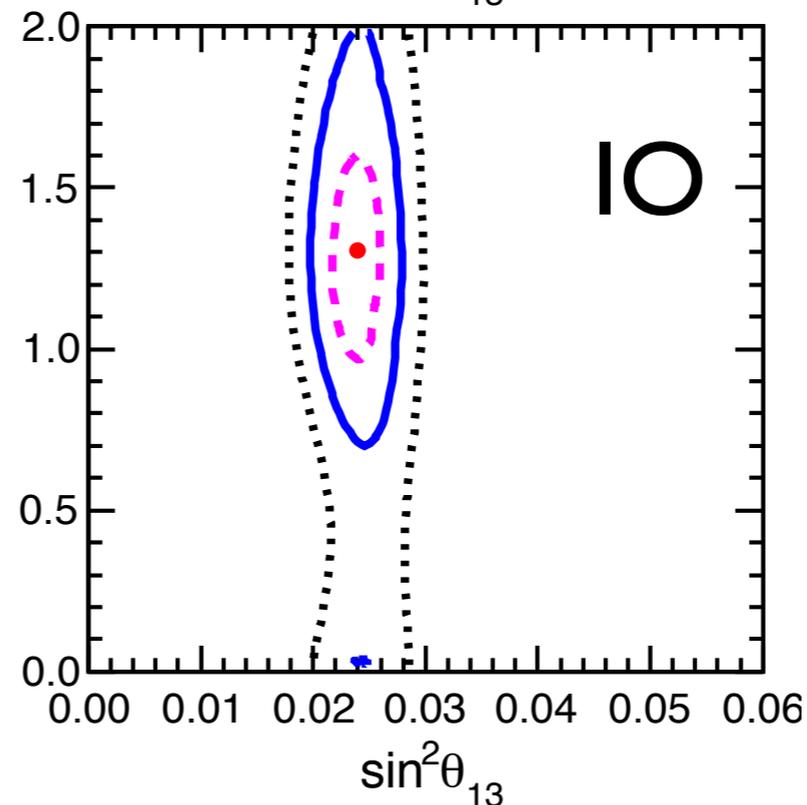
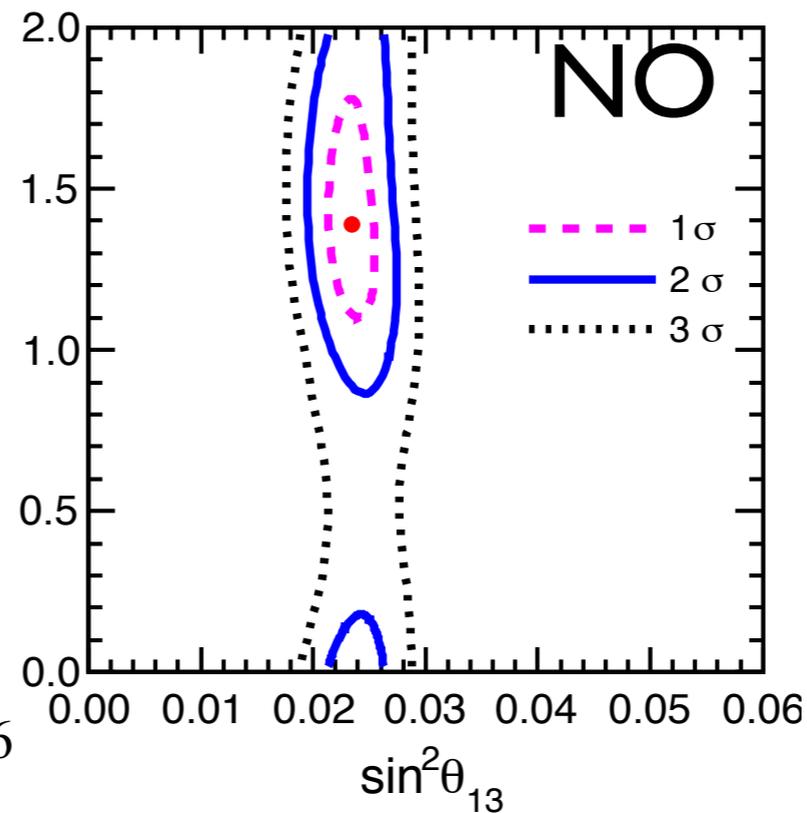
Neutrino 2014 Daya Bay results

Neutrino 2014 RENO results



D.V. Forero et al., 1405.7540

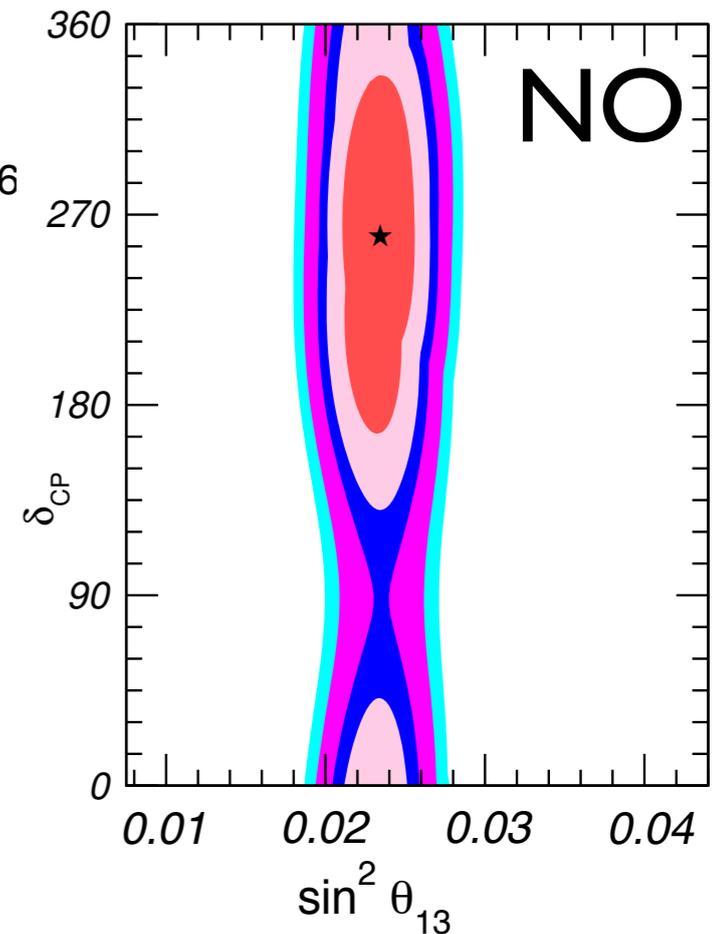
There is a slight preference for CP-violation, which is mainly due to the combination of T2K and reactor neutrino data.



F. Capozzi et al., 1312.2878

2016 results

V_{fit} Global fit to neutrino oscillation data



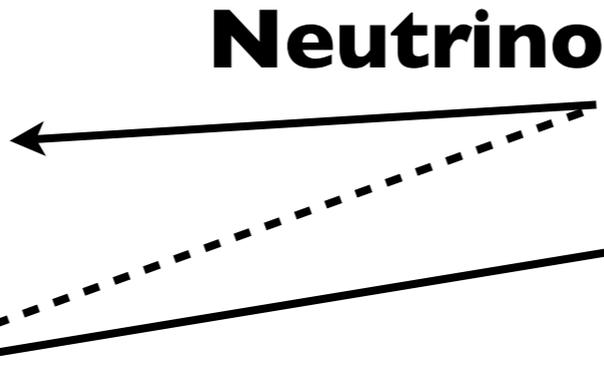
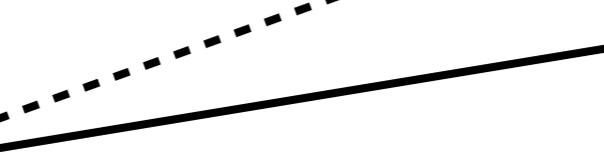
NuFit 3.0: M. C. Gonzalez-Garcia et al., 1611.01514

1. Different flavour models can lead to specific predictions for the value of the delta phase:

- Sum rules: $\sin \theta_{23} - \frac{1}{\sqrt{2}} = a_0 + \lambda \sin \theta_{13} \cos \delta + \text{higher orders}$
- discrete symmetries models
- charged lepton corrections to U_ν : $U_{\text{PMNS}} = U_e^\dagger U_\nu$

e.g. M.-C. Chen and Mahanthappa; Girardi et al.; Petcov; Alonso, Gavela, Isidori, Maiani; Ding et al.; Ma; Hernandez, Smirnov; Feruglio et al.; Mohapatra, Nishi; Holthausen, Lindner, Schmidt; and others

2. In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- B (or L) violation; 
- C, CP violation; 
- departure from thermal equilibrium. 

Leptogenesis in models of neutrino masses

CP-violation in LBL experiments

CP-violation will manifest itself in neutrino oscillations, due to the delta phase. The CP-asymmetry:

$$P(\nu_\mu \rightarrow \nu_e; t) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e; t) =$$
$$= 4s_{12}c_{12}s_{13}c_{13}^2s_{23}c_{23}\sin\delta \left[\sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{23}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{31}^2 L}{2E}\right) \right]$$

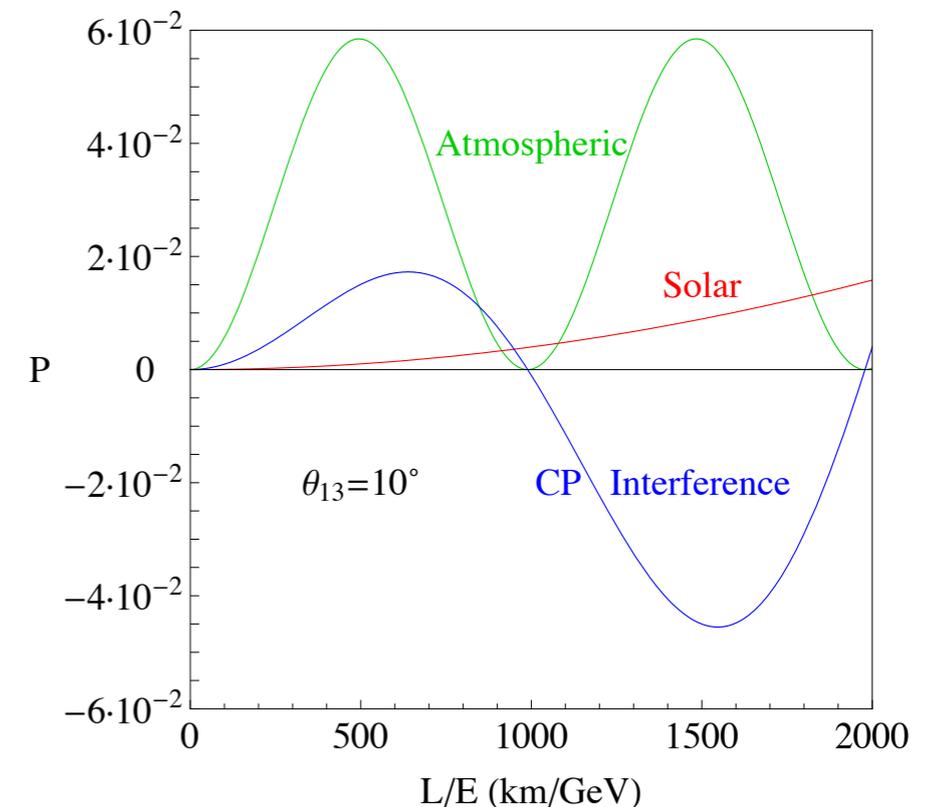
- CP-violation requires all angles to be nonzero.
- It is proportional to the **sin of the delta phase**.
- If one can neglects Δm_{21}^2 , the asymmetry goes to zero: effective 2-neutrino probabilities are CP-symmetric.

CPV needs to be searched for in **long baseline neutrino experiments** which have access to 3-neutrino oscillations.

$$\begin{aligned}
 P_{\mu e} \simeq & 4c_{23}^2 s_{13}^2 \frac{1}{(1-r_A)^2} \sin^2 \frac{(1-r_A)\Delta_{31}L}{4E} \\
 & + \sin 2\theta_{12} \sin 2\theta_{23} s_{13} \frac{\Delta_{21}L}{2E} \sin \frac{(1-r_A)\Delta_{31}L}{4E} \cos \left(\delta - \frac{\Delta_{31}L}{4E} \right) \\
 & + s_{23}^2 \sin^2 2\theta_{12} \frac{\Delta_{21}^2 L^2}{16E^2} - 4c_{23}^2 s_{13}^4 \sin^2 \frac{(1-r_A)\Delta_{31}L}{4E}
 \end{aligned}$$

A. Cervera et al., hep-ph/0002108;
 K. Asano, H. Minakata, I 103.4387;
 S. K. Agarwalla et al., I 302.6773...

- The CP asymmetry peaks for $\sin^2 2\theta_{13} \sim 0.001$. Large θ_{13} makes its searches possible but not ideal.
- Crucial to know mass ordering.
- CPV effects more pronounced at low energy.

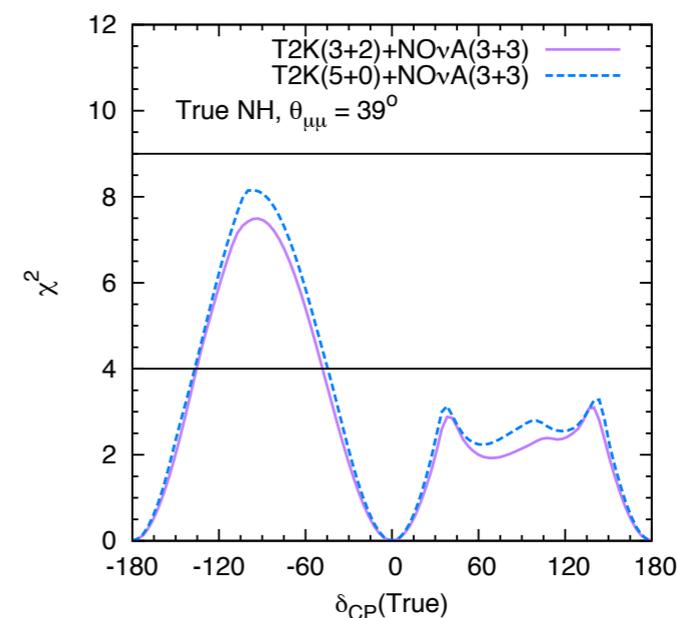
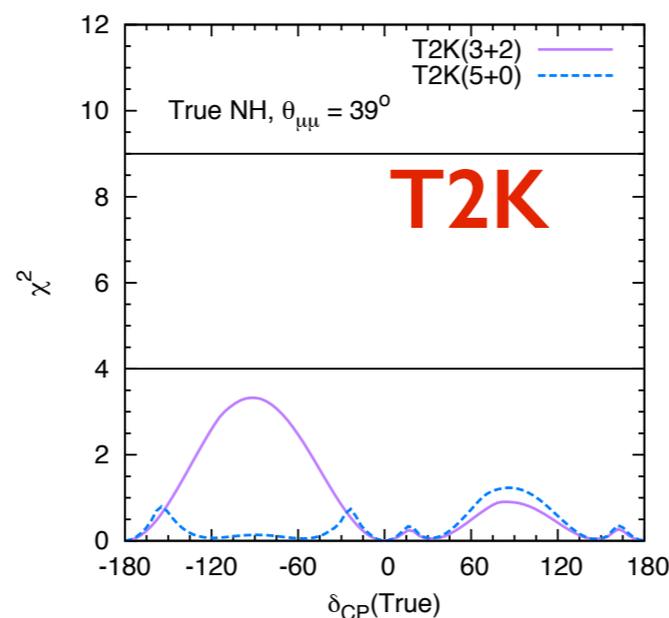
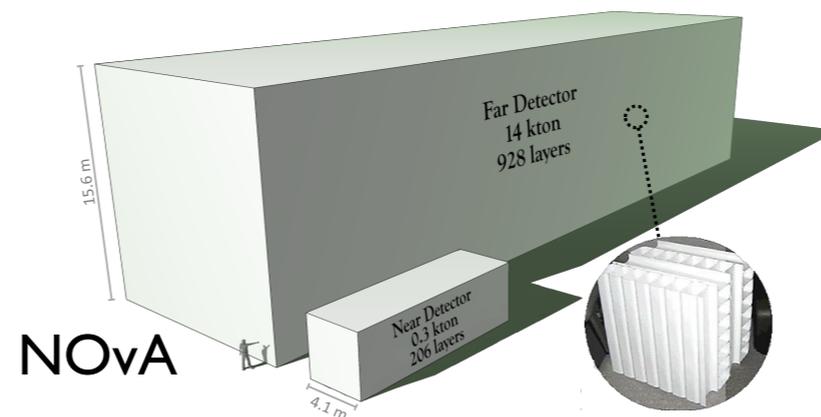
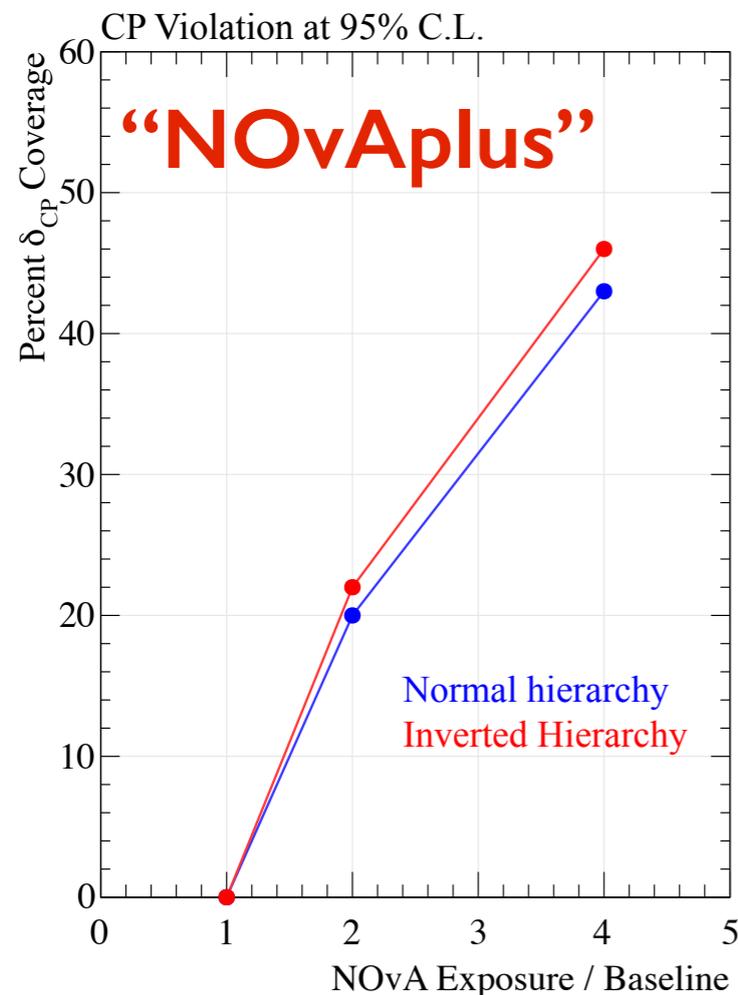
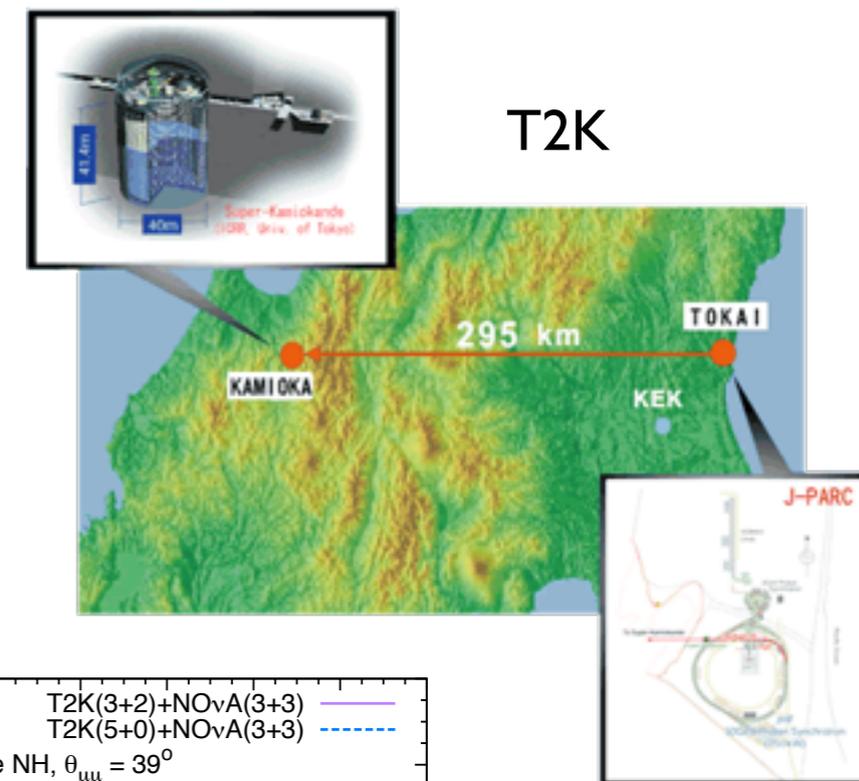


CPV Searches

Near future: T2K and NOvA. Some sensitivity to CPV

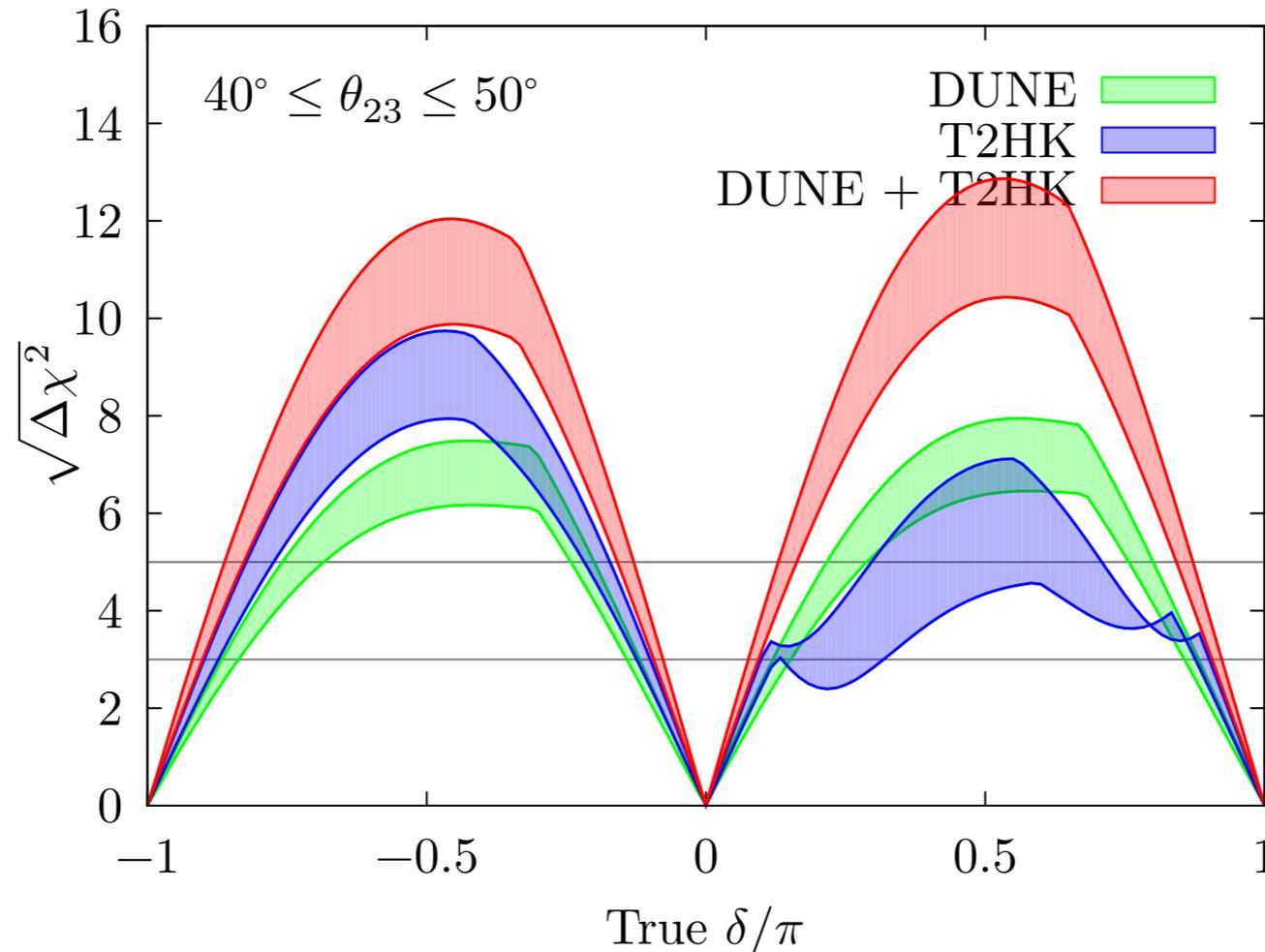
Category	Experiment	Status	Oscillation parameters
Accelerator	MINOS+ [74]	Data-taking	MH/CP/octant
Accelerator	T2K [21]	Data-taking	MH/CP/octant
Accelerator	NOvA [108]	Commissioning	MH/CP/octant
Accelerator	RADAR [76]	Design/ R&D	MH/CP/octant
Accelerator	CHIPS [75]	Design/ R&D	MH/CP/octant
Accelerator	LBNE [87]	Design/ R&D	MH/CP/octant
Accelerator	Hyper-K [97]	Design/ R&D	MH/CP/octant
Accelerator	LBNO [109]	Design/ R&D	MH/CP/octant
Accelerator	ESS ν SB [110]	Design/ R&D	MH/CP/octant
Accelerator	DAE δ ALUS [111]	Design/ R&D	CP

WG Report: Neutrinos, de Gouvea (Convener) et al., 1310.4340



M. Gosh et al., 1401.7243; see also Machado et al.; Huber et al.

NOvA Coll., 1308.0106



Comparisons should be made with great care as they critically depend on:

- setup assumed: detector and its performance, beam...
- values of oscillation parameters and their errors
- treatment of backgrounds and systematic errors.

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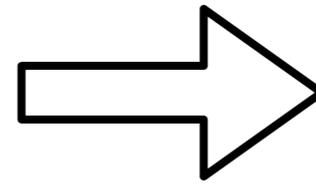
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Masses and mixing from the mass matrix

Neutrino masses and the mixing matrix arises from the diagonalisation of the mass matrix

$$M_M = (U^\dagger)^T m_{\text{diag}} U^\dagger$$

Theory



$$n_L = U^\dagger \nu_L$$

Experiments

Example. In the diagonal basis for the leptons

$$\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

the angle is $\tan 2\theta = \frac{2b}{a-c} \gg 1$ for $a \sim c$ and, or $a, c \ll b$

and masses $m_{1,2} \simeq \frac{a+c \pm 2b}{2}$

In a model of flavour, both the mass matrix for leptons and neutrinos will be predicted and need to be diagonalised:

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \mathcal{M}_\ell \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}$$

$$(\bar{\nu}_{eL}^c, \bar{\nu}_{\mu L}^c, \bar{\nu}_{\tau L}^c) \mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) V_L V_L^\dagger \mathcal{M}_\ell V_R V_R^\dagger \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \mathcal{M}_{\text{diag}} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

$$(\bar{\nu}_{eL}^c, \bar{\nu}_{\mu L}^c, \bar{\nu}_{\tau L}^c) U_\nu^\dagger U_\nu^T \mathcal{M}_\nu U_\nu U_\nu^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$(\bar{\nu}_{1L}^c, \bar{\nu}_{2L}^c, \bar{\nu}_{3L}^c) \mathcal{M}_{\text{diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

In a model of flavour, both the mass matrix for leptons and neutrinos will be predicted and need to be diagonalised:

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \mathcal{M}_\ell \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad (\bar{\nu}_{eL}^c, \bar{\nu}_{\mu L}^c, \bar{\nu}_{\tau L}^c) \mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) V_L V_L^\dagger \mathcal{M}_\ell V_R V_R^\dagger \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad (\bar{\nu}_{eL}^c, \bar{\nu}_{\mu L}^c, \bar{\nu}_{\tau L}^c) U_\nu^* U_\nu^T \mathcal{M}_\nu U_\nu U_\nu^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

$$(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \mathcal{M}_{\text{diag}} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \quad (\bar{\nu}_{1L}^c, \bar{\nu}_{2L}^c, \bar{\nu}_{3L}^c) \mathcal{M}_{\text{diag},\nu} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

in the CC interactions (and oscillations):

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} (\bar{e}'_L, \bar{\mu}'_L, \bar{\tau}'_L) \gamma^\mu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} W_\mu \Rightarrow \frac{g}{\sqrt{2}} (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \gamma^\mu U_{\text{osc}} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} W_\mu$$

$$U_{\text{osc}} = V_L^\dagger U_\nu$$

Phenomenological approaches

Various strategies and ideas can be employed to understand the observed pattern (many many models!).

- Mixing related to mass ratios

$$\theta_{12,23,13} = \text{function}\left(\frac{m_e}{m_\mu}, \dots, \frac{m_1}{m_2}\right)$$

too small

- Flavour symmetries

- Complementarity between quarks and leptons

$$\theta_{12} + \theta_C \simeq 45^\circ$$

- Anarchy (all elements of the matrix of the same order).

Symmetry approach

- Choose a leptonic symmetry (e.g. A4, S4, $\mu - \tau$)
- Use the fact that the see-saw mechanism leads to

$$U_\nu \neq V_L$$

- Obtain the zero-order matrix

$$U_0$$

- Add perturbations (coming from breaking of the symmetry or quantum corrections) to obtain the observed values.

$$U = U_0 + U_{\text{perturbations}}_{\text{small}}$$

θ_{13} poses new challenges as it is not very small.

What kind of leading matrices have been considered?

Example: Tribimaximal mixing

$$U_0 = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \mathcal{O}(0.001) & -\mathcal{O}(0.01) & \mathcal{O}(0.1) \\ \mathcal{O}(0.1) & \mathcal{O}(0.05) & -\mathcal{O}(0.01) \\ -\mathcal{O}(0.1) & -\mathcal{O}(0.05) & \mathcal{O}(0.01) \end{pmatrix}$$

Large corrections to θ_{13} are needed. Harrison, Perkins, Scott

Other possibilities: bimaximal mixing ($\theta_{12}|_0 = 45^\circ$),
golden ratio ($\tan \theta_{12}|_0 = \frac{2}{1 + \sqrt{5}}$), and hexagonal ($\theta_{12}|_0 = 30^\circ$).

Corrections to the basic pattern leads to **predictions for the parameters and relations among them:**

- Sum rules: $\sin \theta_{23} - \frac{1}{\sqrt{2}} = a_0 + \lambda \sin \theta_{13} \cos \delta + \text{higher orders}$
- charged lepton corrections to U_ν : $U_{\text{PMNS}} = U_e^\dagger U_\nu$

Example I: mu-tau symmetry

Large θ_{23} motivates to consider the mu-tau symmetry.

$$\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

The mixing is given by $\tan 2\theta = \frac{2b}{0} = \infty \Rightarrow \theta_{23} = 45^\circ$

For 3 generations, this mass matrix respects the symmetry

$$\mathcal{M}_\nu = \sqrt{\Delta m_A^2} \begin{pmatrix} \sim 0 & a\epsilon & a\epsilon \\ a\epsilon & 1 + \epsilon & 1 \\ a\epsilon & 1 & 1 + \epsilon \end{pmatrix}$$

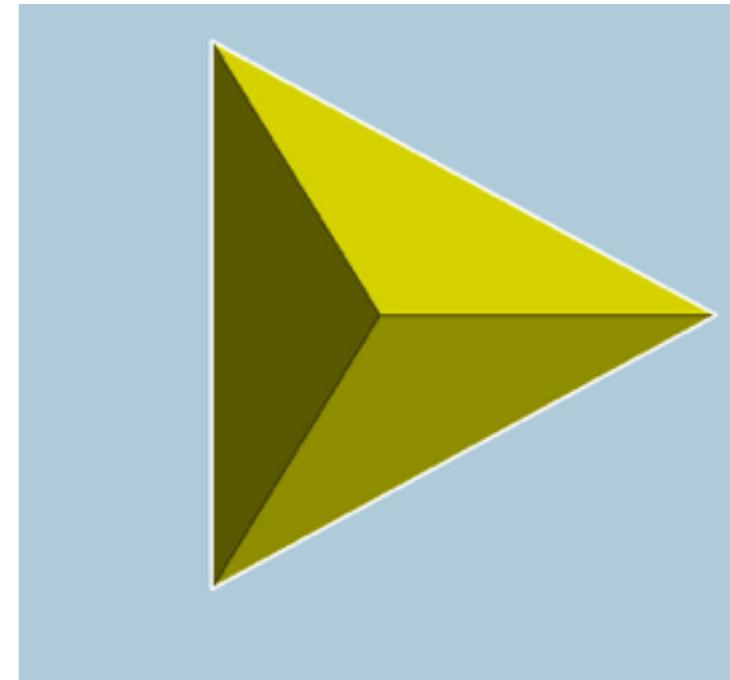
leading to $\theta_{23} = \frac{\pi}{4} - \frac{\Delta m_\odot^2}{\Delta m_A^2}$ $\theta_{13} \sim \epsilon^2 \sim \frac{\Delta m_\odot^2}{\Delta m_A^2} \sim 0.04$

The large value of θ_{13} needs more corrections.

Example 2: a discrete symmetry A4

An example of discrete symmetry: Z_2 (reflections).

A_4 is the group of even permutations of (1234) . This is a very studied example of discrete symmetry. It is the invariant group of a tetrahedron.



There are 12 elements:
 $I = 1234, T = 2314, S = 4321, ST, TS, STS...$
with $S^2 = I, T^3 = I, (ST^3) = I$.

It has the following representations: $1, 1', 1''$, and 3 , distinguished by how S and T behave on it.

We need to assign fermions to the representations:

$$L \rightarrow 3$$

$$e_R \rightarrow 1$$

$$\mu_R \rightarrow 1'$$

$$\tau_R \rightarrow 1''$$

As usual, masses require the “product” of two fermions:

$$1' \times 1' = 1''$$

$$1'' \times 1'' = 1'$$

$$1' \times 1'' = 1$$

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

In order to break the symmetry, scalars (called ‘flavons’) are needed: $\phi(3), \phi'(3), \xi(1)$

Requiring that the Lagrangian is invariant w.r.t. the flavour symmetry, the allowed interactions are fixed:

$$\mathcal{L} = y_e \bar{e}_R(\phi L) \frac{H_d}{\Lambda} + y_\mu \bar{\mu}_R(\phi L) \frac{H_d}{\Lambda} + y_\tau \bar{\tau}_R(\phi L) \frac{H_d}{\Lambda} + j_a \xi(LL) \frac{H_u H_u}{\Lambda^2} + j_b (\phi' LL) \frac{H_u H_u}{\Lambda^2}$$

$\quad | \quad (33)_1 \quad \quad |' \quad (33)_{1'}$
 $\quad \quad |'' \quad (33)_{1''}$
 $\quad \quad | \quad (33)_1$
 $\quad \quad (333)_1$

The flavons get a vev

$$\langle \phi \rangle = (v, v, v) \quad \langle \phi' \rangle = (v', 0, 0) \quad \langle \xi \rangle = u$$

and the resulting mass matrices are

$$M_l = v \frac{v_{Hd}}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu e^{i4\pi/3} & y_\mu e^{i2\pi/3} \\ y_\tau & y_\tau e^{i2\pi/3} & y_\tau e^{i4\pi/3} \end{pmatrix} \quad M_\nu = \frac{v_u^2}{\Lambda^2} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

Finally, the two matrices can be diagonalised and the resulting mixing matrix is the TBM one.

There are two major issues:

- **the vacuum alignment.** Without the specific choice of the vevs of the flavons, the required form of the mass matrix could not be achieved. Arranging for the potential to lead to such vevs is highly non trivial.

- **the value of θ_{13} .**

Due to the measured value of θ_{13} , large deviations from TBM are required and this poses some challenges to this approach. Extensions are being considered (e.g. Dirac neutrinos, additional flavons...)

Tests of flavour models

Two necessary ingredients for testing flavour models:

- Precision measurements of the oscillation parameters.
- The determination of the mass ordering and of the neutrino mass spectrum. Reactor neutrinos, LBL experiments (DUNE and T2HK), Atm nu experiments

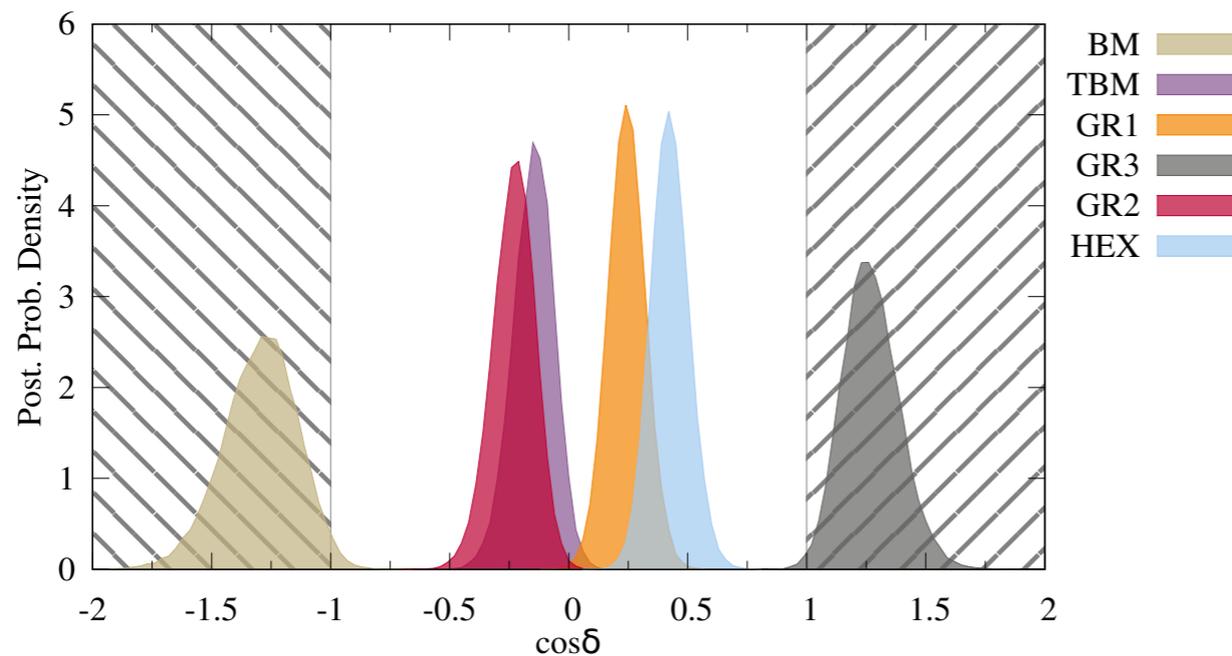
Reference	Hierarchy	$\sin^2 2\theta_{23}$	$\tan^2 \theta_{12}$	$\sin^2 \theta_{13}$
Anarchy Model:				
dGM [18]	Either			$\geq 0.011 @ 2\sigma$
$L_e - L_\mu - L_\tau$ Models:				
BM [35]	Inverted			0.00029
BCM [36]	Inverted			0.00063
GMN1 [37]	Inverted		≥ 0.52	≤ 0.01
GL [38]	Inverted			0
PR [39]	Inverted		≤ 0.58	≥ 0.007
S_3 and S_4 Models:				
CFM [40]	Normal			0.00006 - 0.001
HLM [41]	Normal	1.0	0.43	0.0044
	Normal	1.0	0.44	0.0034
KMM [42]	Inverted	1.0		0.000012
MN [43]	Normal			0.0024
MNY [44]	Normal			0.000004 - 0.000036
MPR [45]	Normal			0.006 - 0.01
RS [46]	Inverted	$\theta_{23} \geq 45^\circ$		≤ 0.02
	Normal	$\theta_{23} \leq 45^\circ$		0
TY [47]	Inverted	0.93	0.43	0.0025
T [48]	Normal			0.0016 - 0.0036
A_4 Tetrahedral Models:				
ABGMP [49]	Normal	0.997 - 1.0	0.365 - 0.438	0.00069 - 0.0037
AKKL [50]	Normal			0.006 - 0.04
Ma [51]	Normal	1.0	0.45	0
SO(3) Models:				
M [52]	Normal	0.87 - 1.0	0.46	0.00005
Texture Zero Models:				
CPP [53]	Normal			0.007 - 0.008
	Inverted			≥ 0.00005
	Inverted			≥ 0.032
WY [54]	Either			0.0006 - 0.003
	Either			0.002 - 0.02
	Either			0.02 - 0.15

Typically, the models considered have a reduced number of parameters, leading to **relations between the masses and/or mixing angles**.

Examples are the mixing-mass ratio relations and the so-called sumrules, e.g.:

Atmospheric sum rules: $\sin \theta_{23} - \frac{1}{\sqrt{2}} = \sin \theta_{13} \cos \delta$

Solar sum rules: $\cos \delta = \frac{t_{23} \sin^2 \theta_{12} + \sin^2 \theta_{13} \cos^2 \theta_{12} / t_{23} - \sin^2 \theta_{12}^\nu (t_{23} + \sin^2 \theta_{13} / t_{23})}{\sin 2\theta_{12} \sin \theta_{13}}$



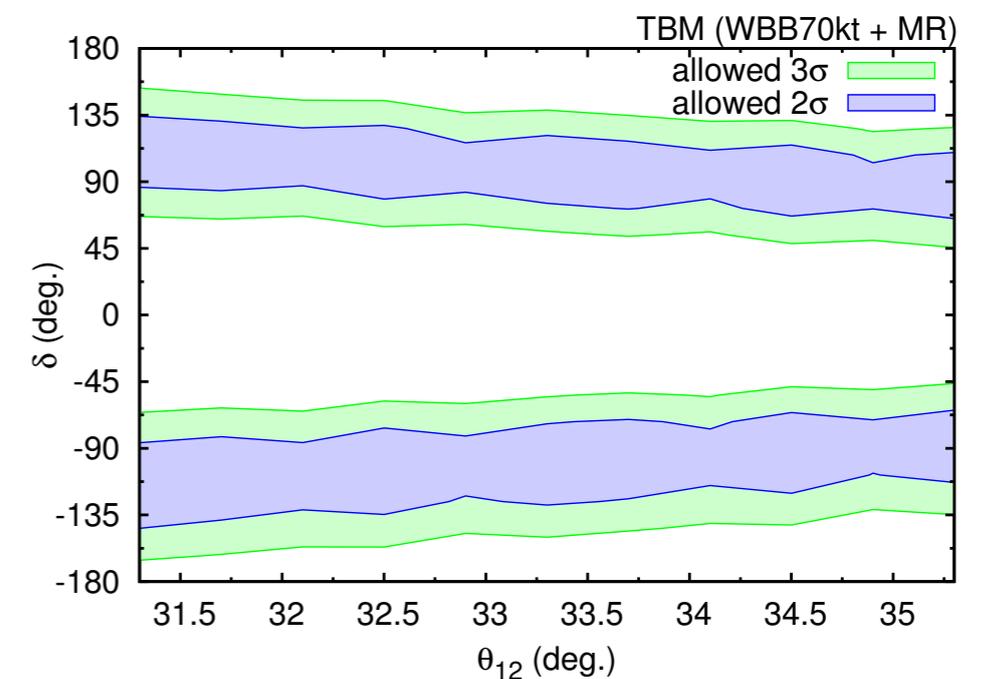
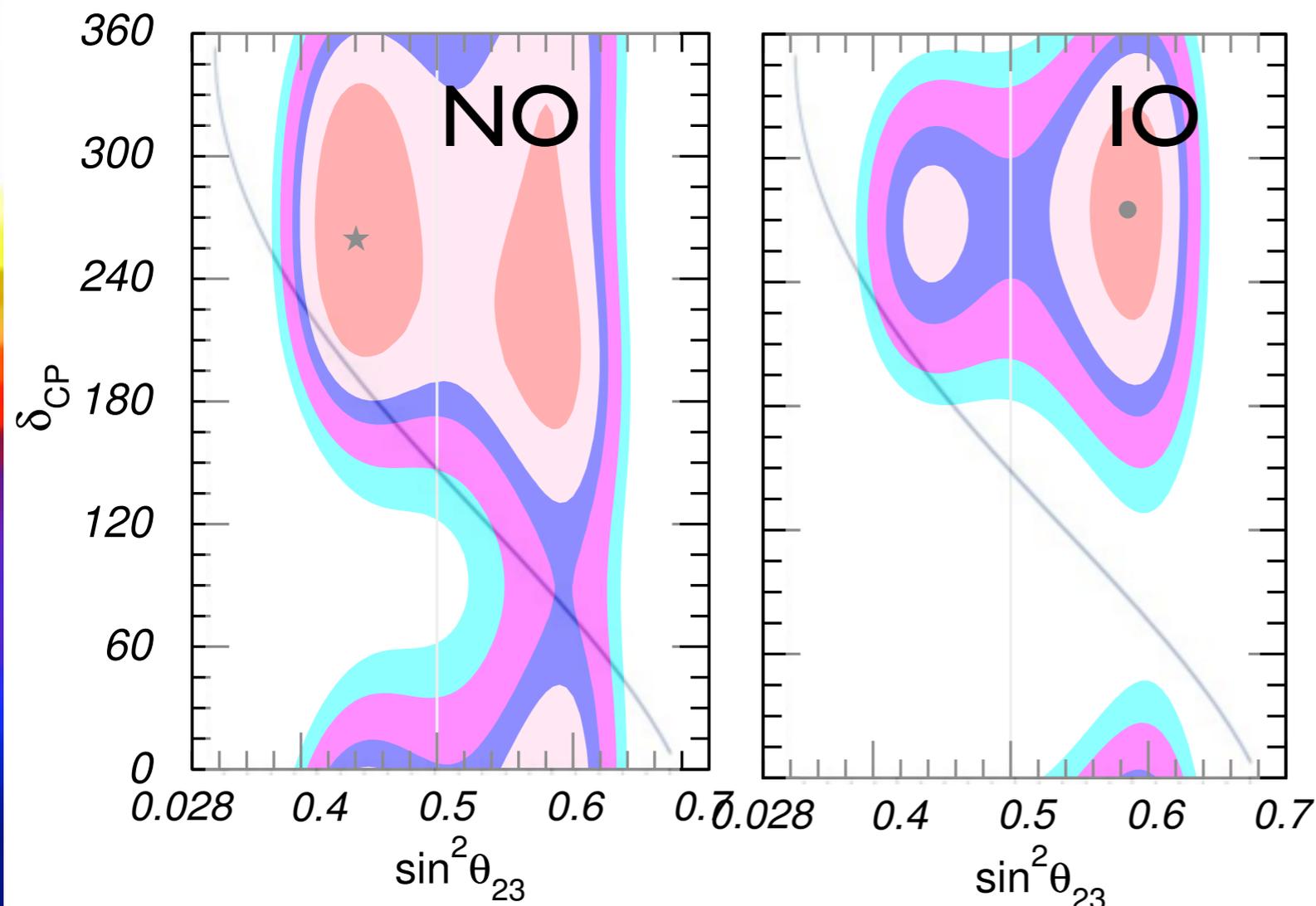
P. Ballett et al., 1410.7573

Future experimental strategy:

- theta23: LBL experiments
- theta13: reactor experiments
- theta12: reactor experiments
- delta: LBL experiments

	Current	Daya Bay II
Δm^2_{12}	3%	0.6%
Δm^2_{23}	5%	0.6%
$\sin^2\theta_{12}$	6%	0.7%
$\sin^2\theta_{23}$	20%	N/A
$\sin^2\theta_{13}$	14% \rightarrow 4%	$\sim 15\%$

Y.Wang, LPI3



P. Ballett et al., 1410.7573

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Useful formulae

Particles in a thermal bath are described by

$$f_{\text{eq}} = \frac{1}{\exp\left(\frac{p - \mu_\nu}{T}\right) \pm 1}$$

The number densities are given by

Internal d.o.f. \rightarrow

$$n_{\text{eq}} \simeq g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}} \quad \text{Non relativistic}$$
$$n_{\text{eq}} \simeq gT^3 \quad \text{Relativistic}$$

Entropy

$$s = \frac{2\pi^2}{45} g_* T^3$$

Relativistic d.o.f.

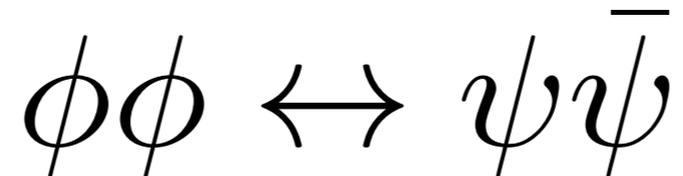
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Freeze-out

Typically, particles were in **thermal equilibrium** for T above their mass, if the interactions were fast enough.



As the Universe expands, the T drops and interactions slow down and the particles decouple. Then their number density is redshifted and a relic remains (ex., neutrinos, DM). The **condition for freezeout** is

$$\Gamma \sim H$$

where

$$\Gamma = \langle \sigma n \rangle$$

$$H = \sqrt{\frac{8\pi G_N}{3} \rho^2} \simeq \frac{T^2}{m_{\text{Pl}}}$$

interaction rate

expansion rate

For radiation domination

Hot relic

A cold relic is a particle with decouples when relativistic.

The typical example is neutrinos.

$$\sigma = G_F^2 T^2$$

$$n \sim g T^3$$

$$H \simeq \frac{T^2}{m_{\text{Pl}}}$$

Exercise
Compute T more precisely.

$$\Gamma \sim H \Rightarrow T \simeq \left(\frac{1}{G_F^2 m_{\text{Pl}}} \right)^{1/3} \sim 1 \text{ MeV}$$

In order to compute their contribution to the energy density of the Universe, let's consider the comoving number density (for entropy conservation)

$$Y \equiv \frac{n}{s} \quad \left. \begin{array}{l} \swarrow \\ \searrow \end{array} \right\} \text{both scale as } a^{-3}$$

So

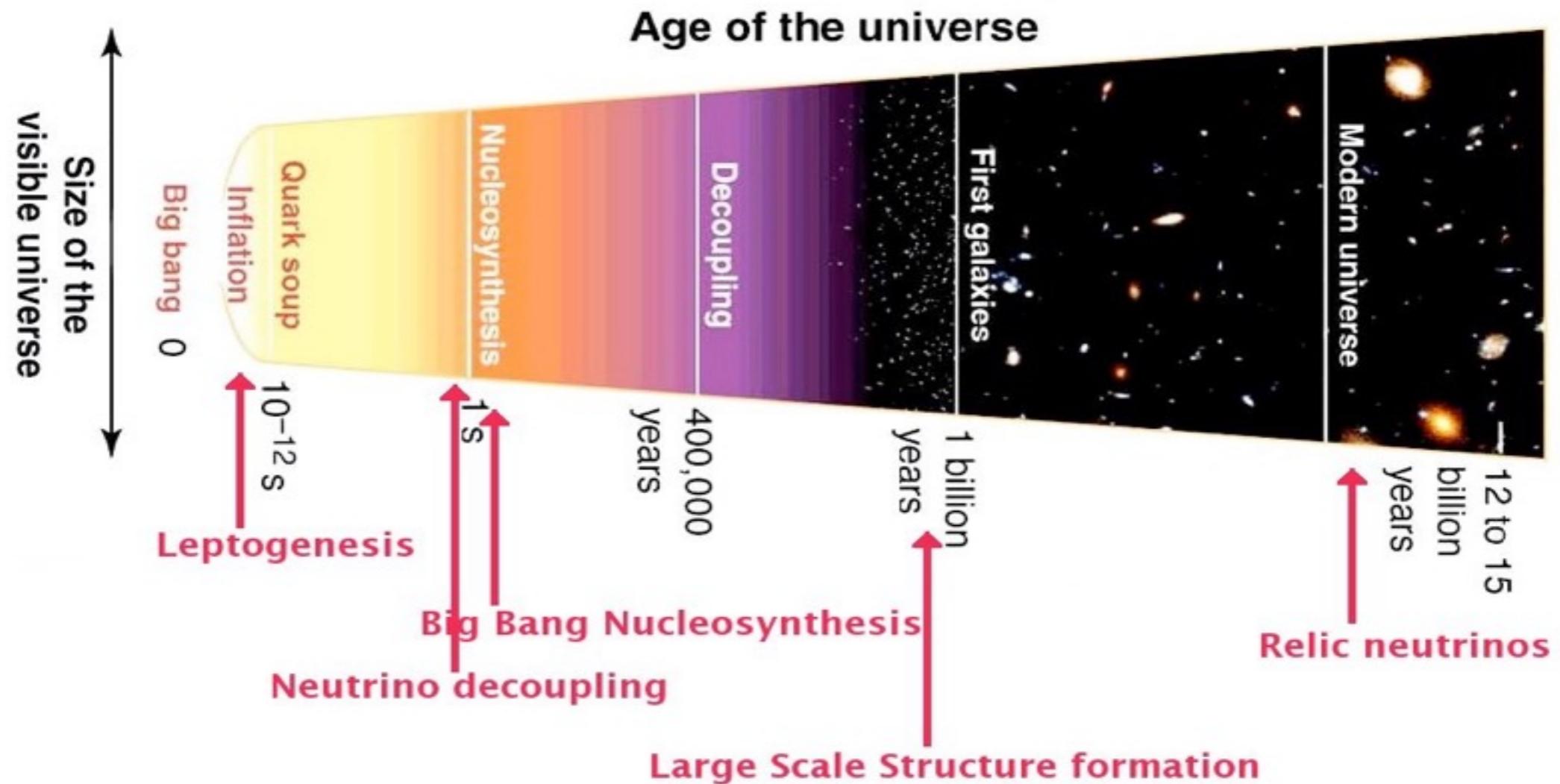
$$Y_{\text{today}} = Y_{\text{freeze-out}}$$

$$\Omega_{\nu} h^2 = \frac{\rho_{\nu}}{\rho_{\text{cr}}} h^2 = \frac{n_{\nu} m_{\nu}}{\rho_{\text{cr}}} h^2 = \boxed{\frac{m_{\nu}}{91.5 \text{ eV}}}$$

Exercise
Derive

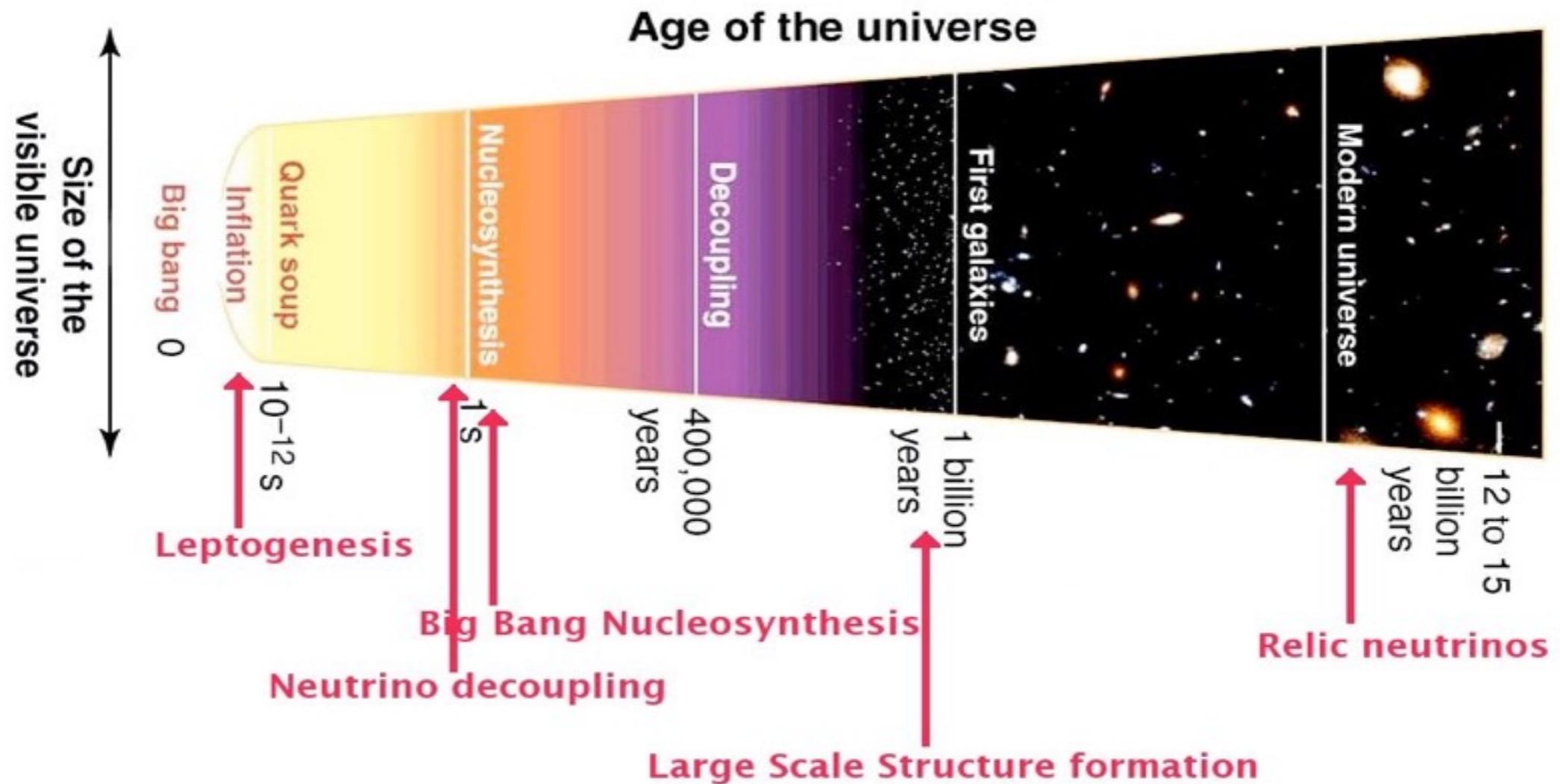
In general, the hot relic density abundance scales linearly with the mass.

Neutrinos have played an important role in shaping the Universe.



How many **relic neutrinos** are in a **cup of tea**?

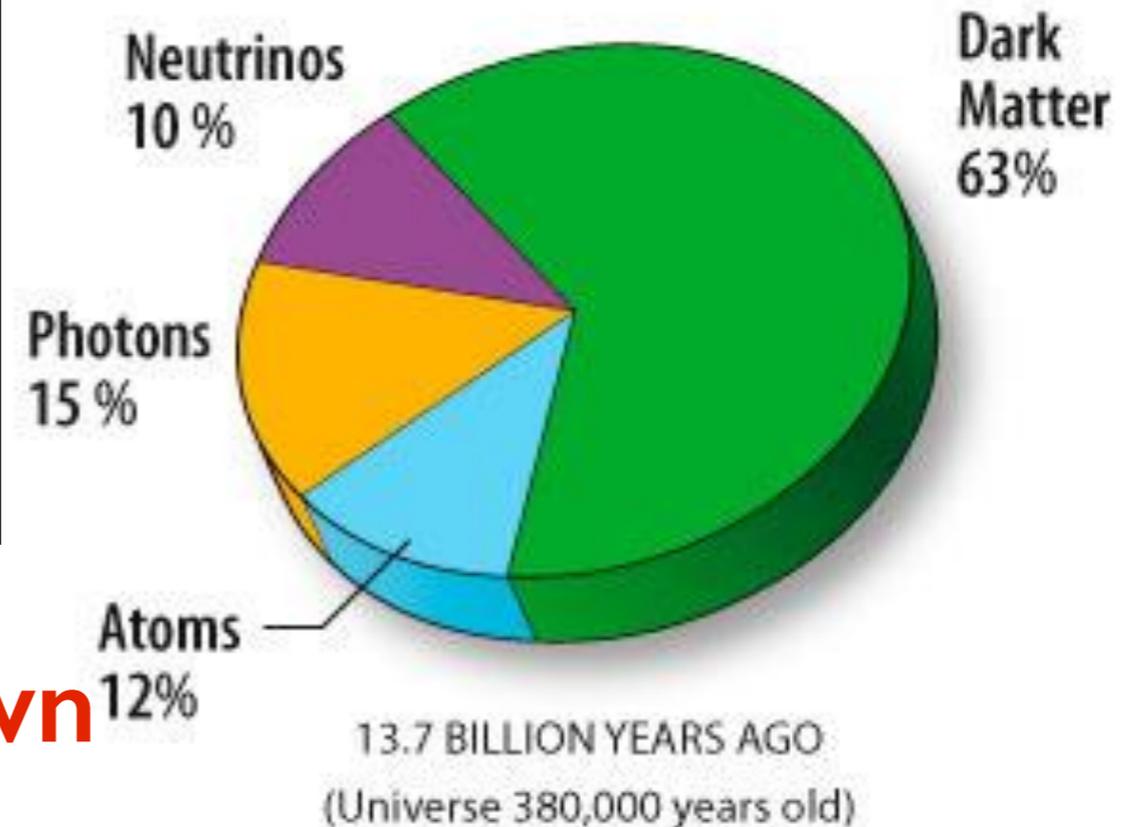
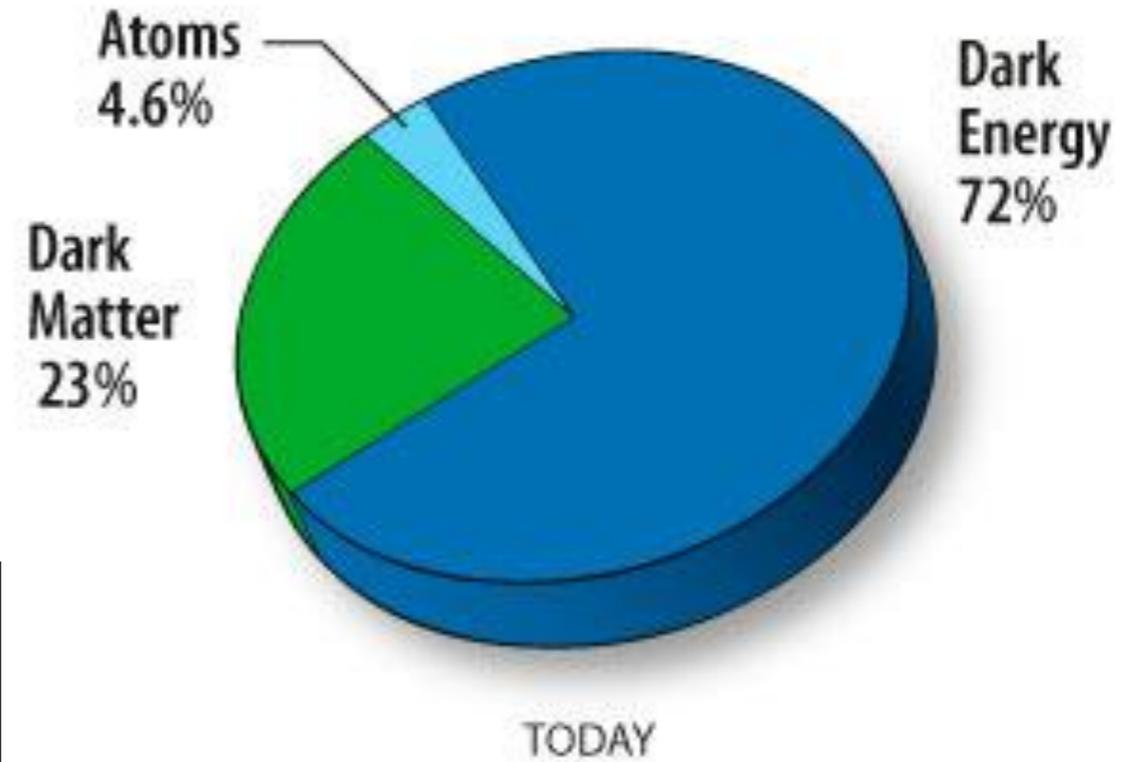
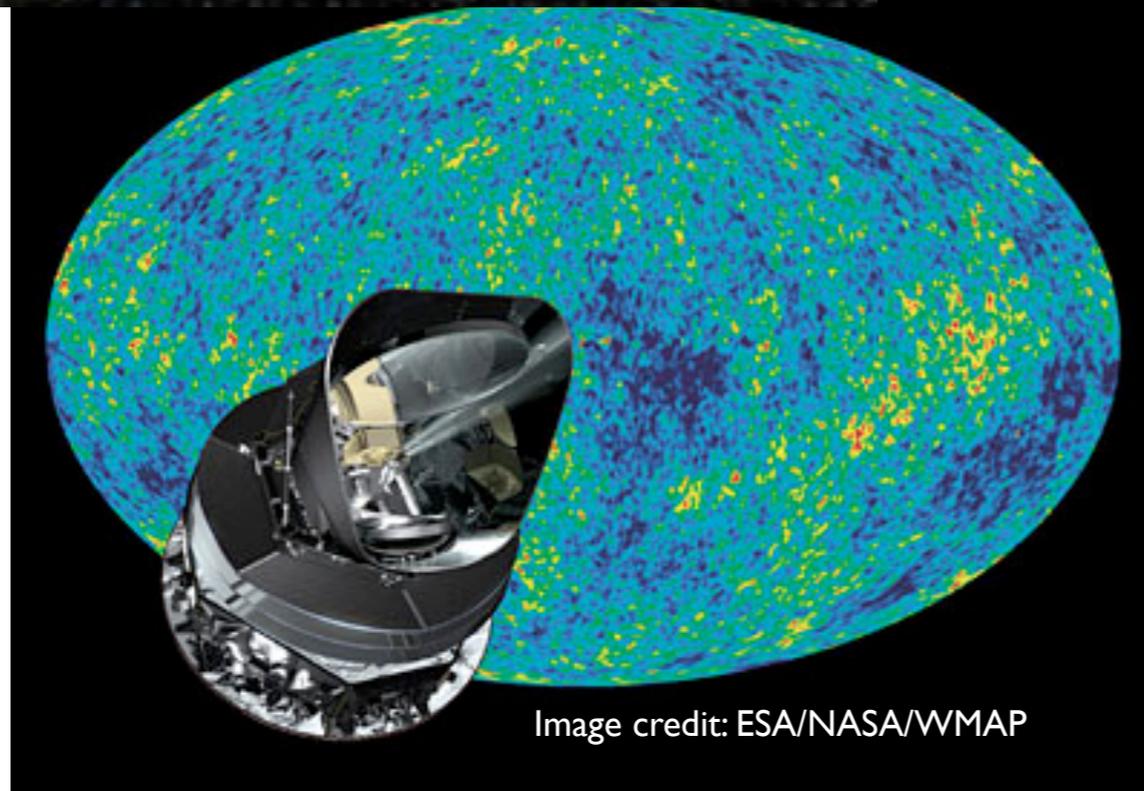
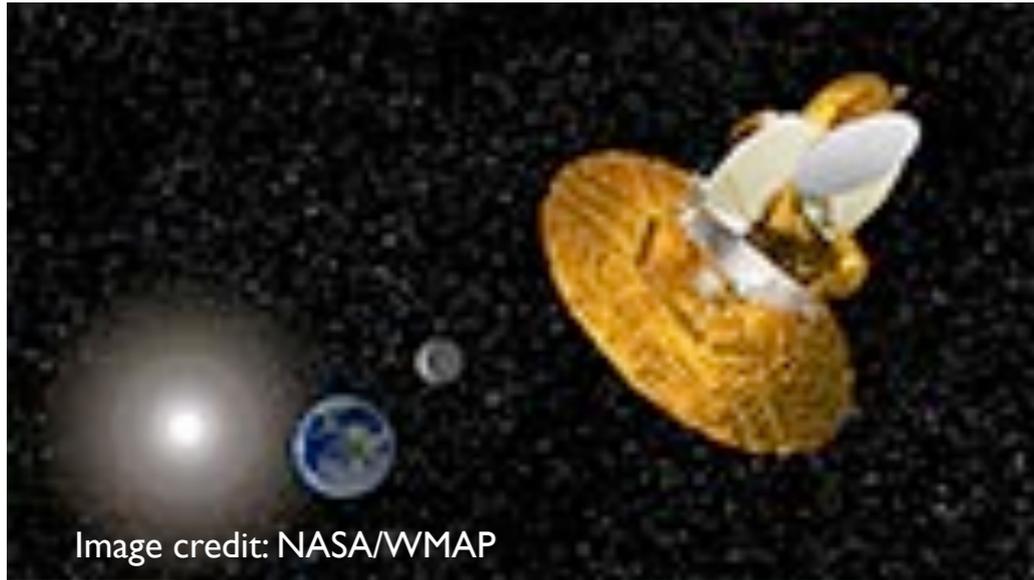
Neutrinos have played an important role in shaping the Universe.



How many **relic neutrinos** are in a **cup of tea?**

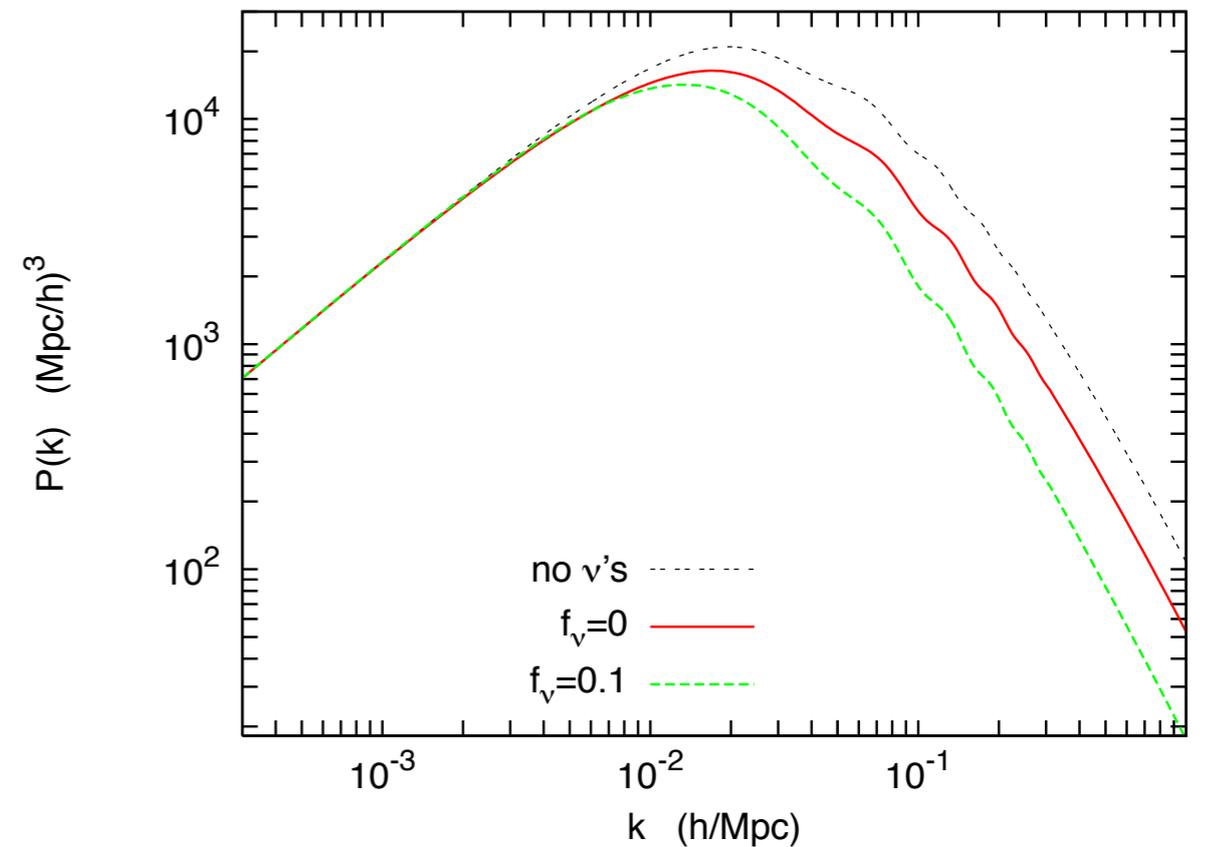
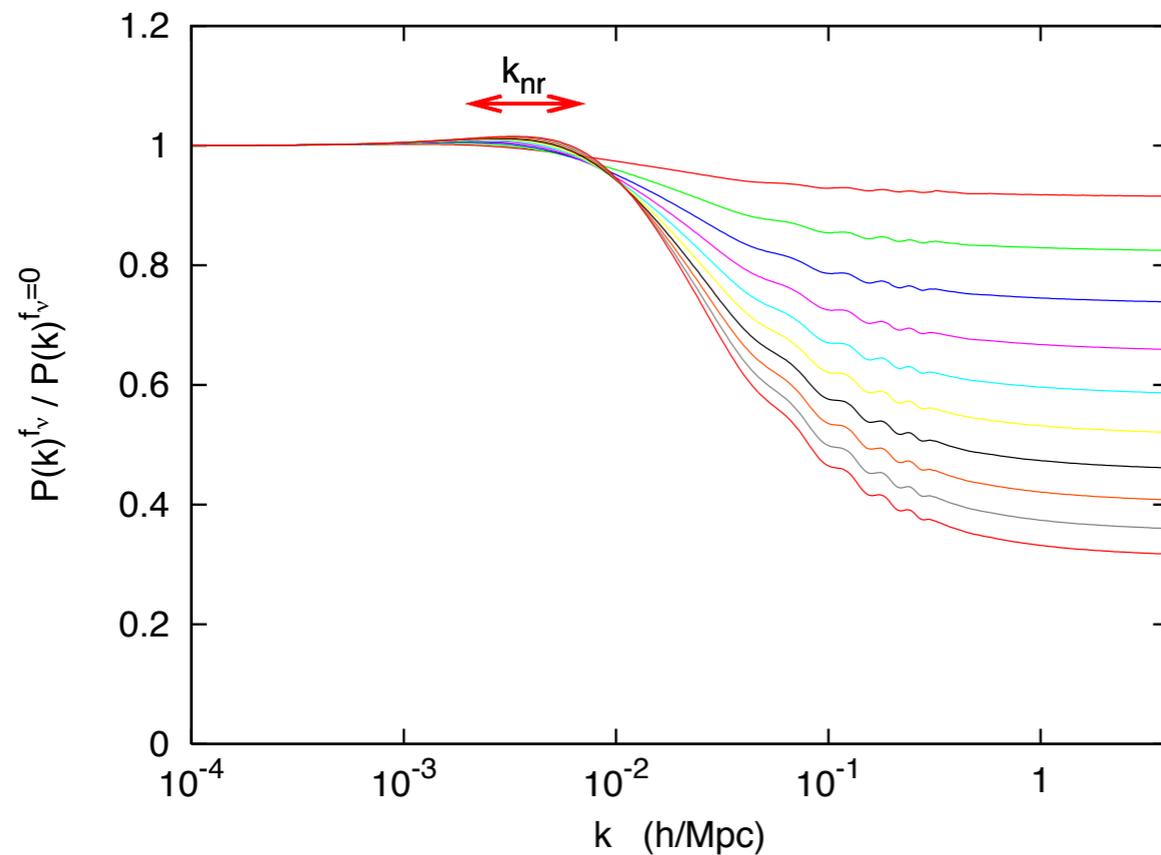
5600!

New Scientist 05 March 2008: Universe submerged in a sea of chilled neutrinos



Neutrinos are the only known component of Dark Matter.

Neutrino masses suppress the matter power spectrum at small scales due to their free-streaming.



J. Lesgourgues and S. Pastor, PRept 2006

Loss of power on scales: $k_{fs} = 0.11 \sqrt{\frac{\sum_i m_i}{1 \text{ eV}} \frac{5}{1+z}} \text{ Mpc}^{-1}$

Way to probe the matter power spectrum:

- **galaxy surveys, such as SDSS, BOSS, HETDEX...** U. Seljak et al., PRD 2005; F. De Bernardis et al., PRD 2008; S. Hannestad and Y.Y.Y. Wong, JCAP 2007; de Putter et al., 2012; G-B. Zhao et al., MNRAS 2013; ...

$$\sum_i m_i < 0.1 \text{ eV} - 0.2 \text{ eV}$$

- **Lyman alpha: this traces the intergalactic low density gas.** J. Lesgourgues and S. Pastor, PRept 2006; M. Viel et al., JCAP 2010; S. Gratton, A. Lewis, G. Efstathiou PRD 2008,...

$$\sum_i m_i < 0.11 \text{ eV} - 0.17 \text{ eV}$$

- **21 cm lines: MWA, SKA and FFTT.** Y. Mao et al., PRD 2008; M. McQuinn et al., AJ 2008; E. Visbal et al., JCAP 2009; J. R. Pritchard and E. Pierpaoli, PRD 2008.

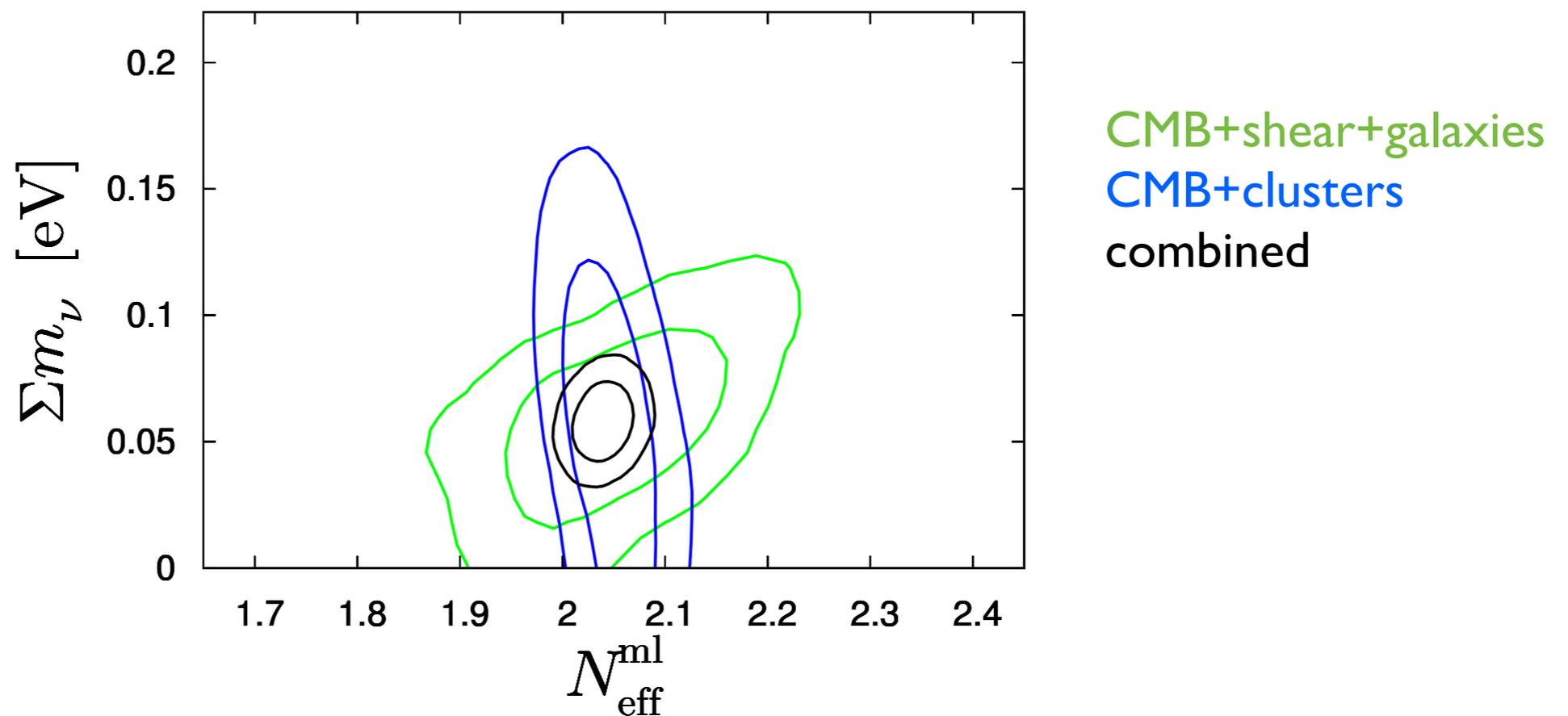
$$\sum_i m_i \sim 0.02 \text{ eV} - 0.003 \text{ eV}$$

- problem of non-linearity.

- problem of bias: $P_{\text{tracer}} = b^2(k) P_{\text{DM}}(k)$

- Lensing of galaxies

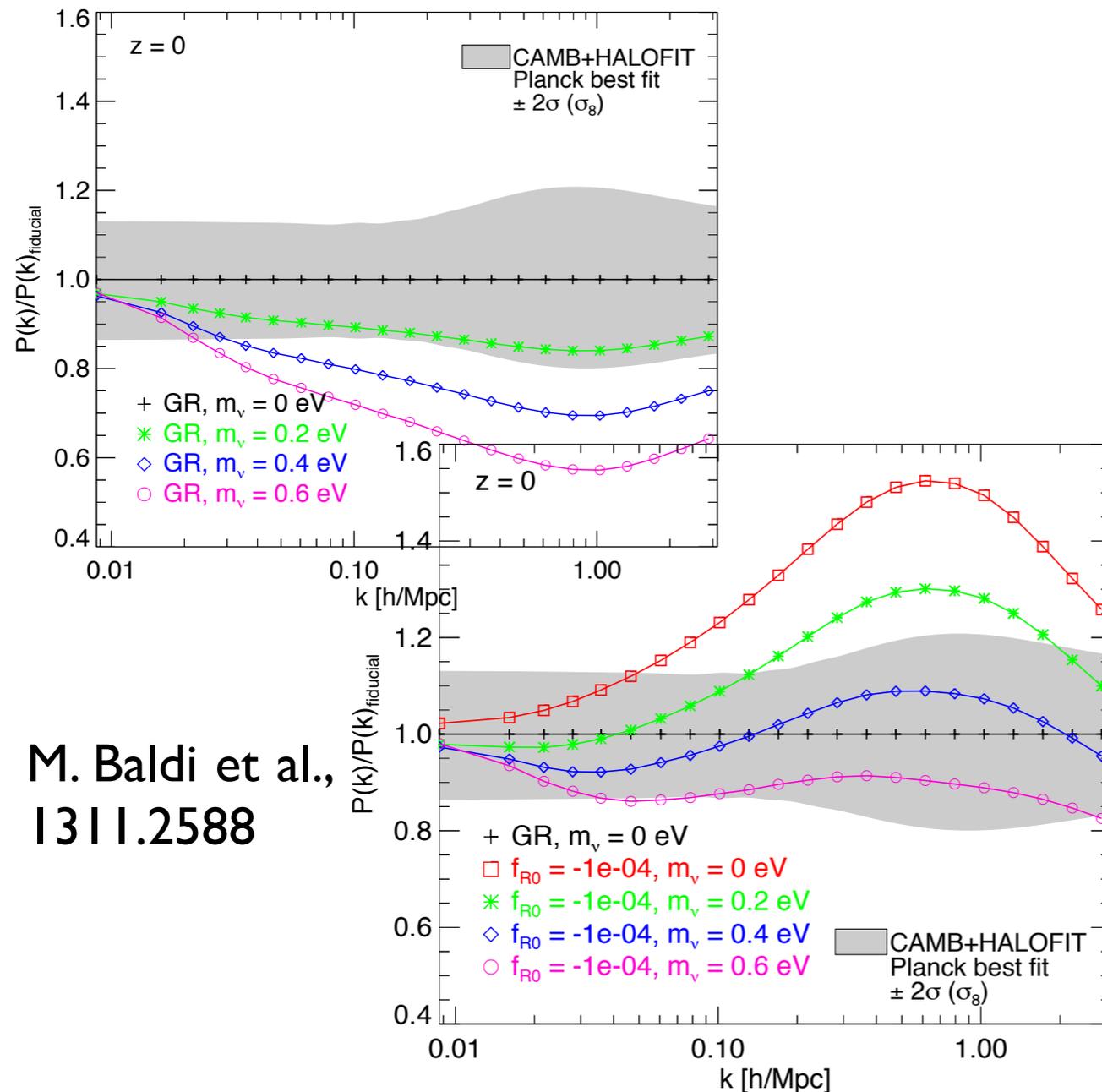
By using the cosmic shear, it is possible to reconstruct the matter distribution at different redshifts. A. Cooray, AA 1999; K. Ichiki et al., PRD 2009; Hamann et al., I209.1043; LSST; EUCLID... and many others



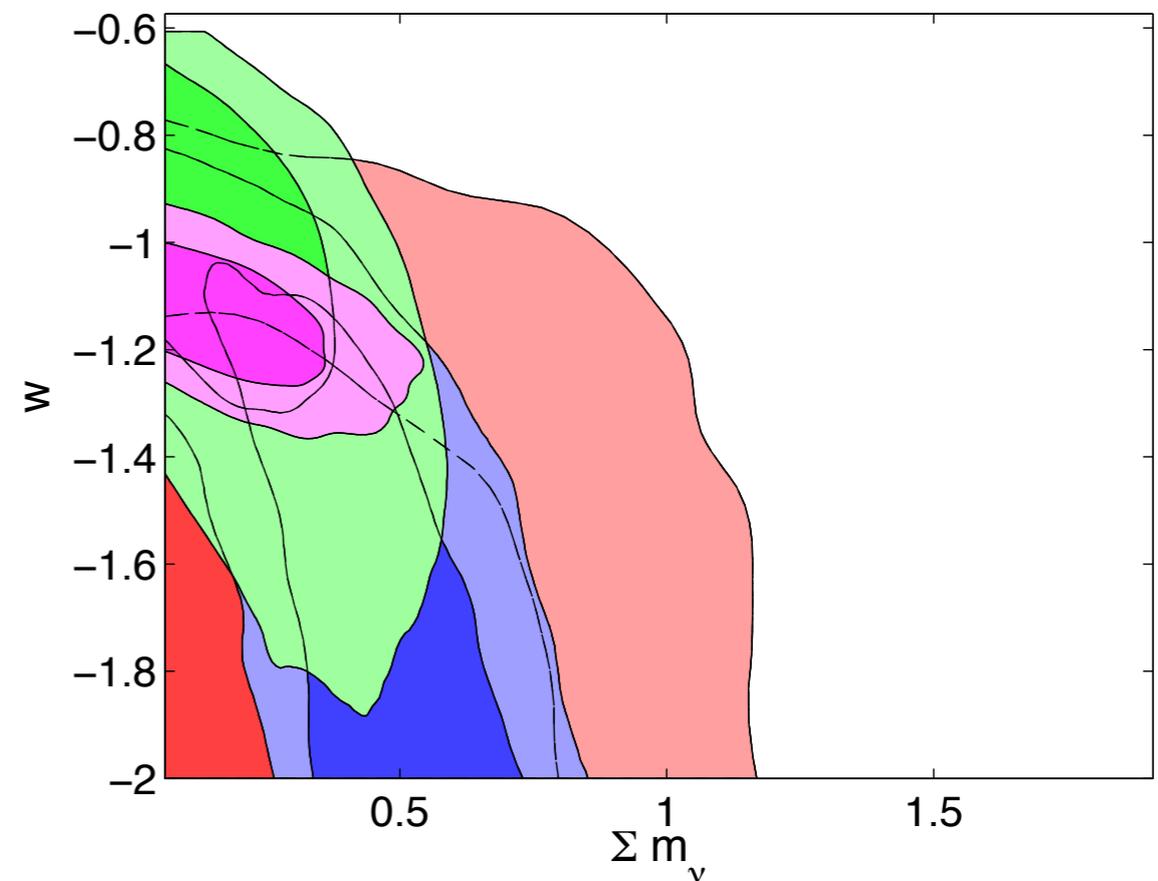
T. Basse et al, I304.2321

Theoretical modeling on non-linear scales ($k > 0.1$ Mpc^{-1}) at 1% level will be crucial.

Different effects can be degenerate with the measurement of neutrino masses, which therefore relies on assumptions on the cosmological model.



M. Baldi et al.,
1311.2588



E. Giusarma et al., 1306.5544

Combining different searches will play a crucial role.

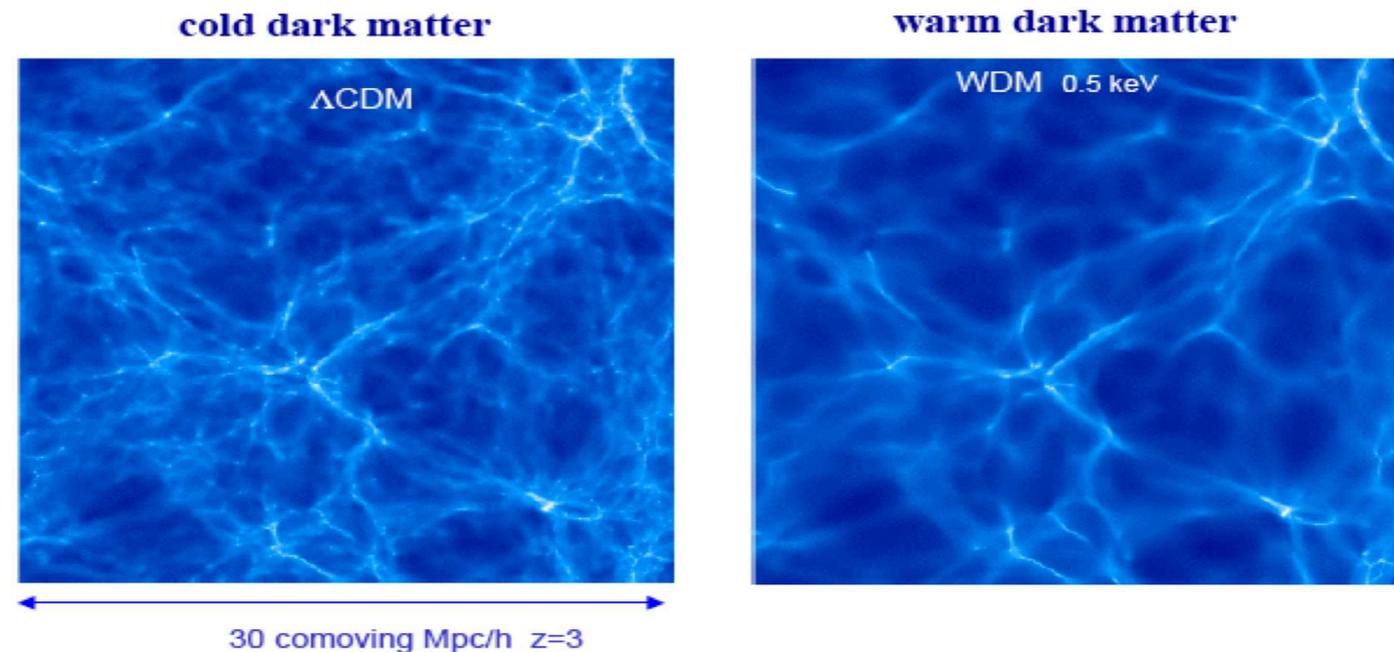
Plan of lecture III

- The problem of leptonic mixing
 - Current status
 - Prospects to discover leptonic CPV and measure with precision the oscillation parameters
 - How to explain the observed mixing structure and Flavour symmetry models

- **Neutrinos in cosmology**
 - neutrinos in the Early Universe
 - **sterile neutrinos as WDM**
 - Leptogenesis and the baryon asymmetry

Warm Dark Matter

DM candidates with clustering properties intermediate between hot dark matter and cold dark matter is named **warm dark matter**. For a standard distribution, the mass is in the keV range.

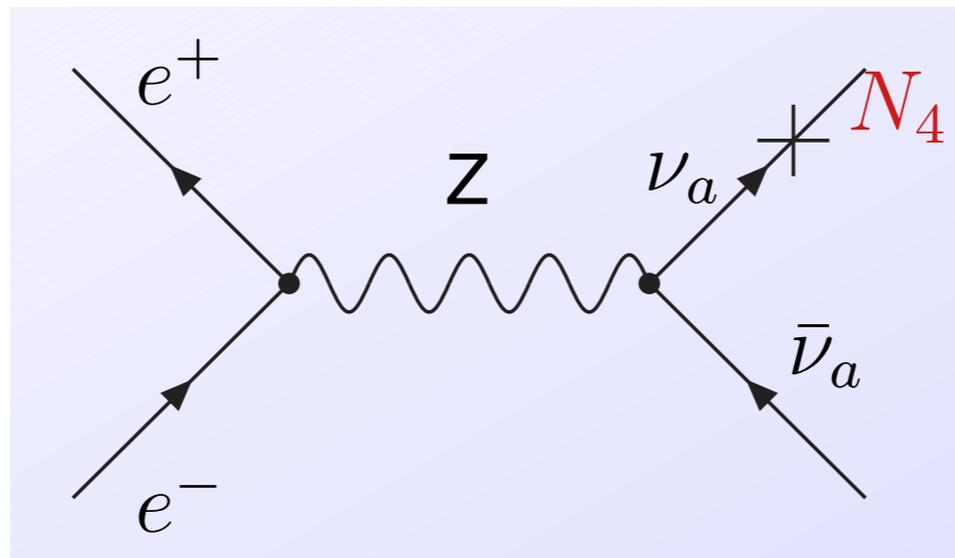


See, e.g.
Haehnelt, Frenk
et al., B. Moore
et al.....

A prime candidate are sterile neutrinos. In the right range of masses and mixing angles, sterile neutrinos can be “stable” on the cosmic timescales.

$$\Gamma_{3\nu} \simeq \sin^2 2\theta G_F^2 \frac{m_4^5}{768\pi^3} \sim 10^{-30} \text{S}^{-1} \frac{\sin^2 2\theta}{10^{-10}} \left(\frac{m_4}{\text{keV}} \right)^5$$

Their production is different from active neutrinos as they were **never in equilibrium** with the thermal plasma. In an interaction involving active neutrinos, a heavy neutrino would be produced via loss of coherence.



These oscillations happen in the thermal plasma, so the mixing angle will be in matter.

$$\sin^2 2\theta_m = \frac{\Delta^2(p) \sin^2 2\theta}{\Delta^2(p) \sin^2 2\theta + D^2 + (\Delta(p) \cos 2\theta - V_D + |V_T|)^2}$$

Analogue to matter effects in the earth and depend on the lepton asymmetry.

Genuine thermal effects. They always suppress the oscillations.

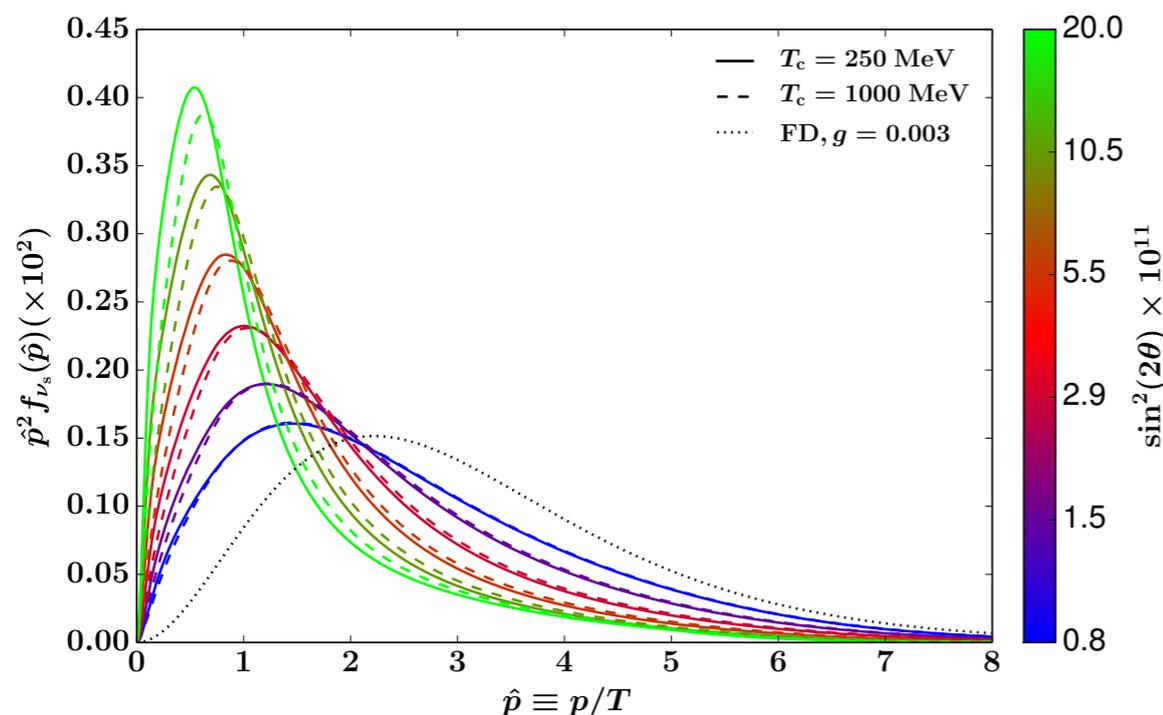
The production will depend on the mixing angle and on the interaction rate of the active neutrinos. A detailed computation requires to solve the associated Boltzmann equation for their distribution:

$$\frac{\partial}{\partial t} f_s(p, t) - Hp \frac{\partial}{\partial p} f_s(p, t) \simeq \frac{\Gamma_a}{2} \langle P(\nu_a \rightarrow \nu_s; p, t) \rangle (f_a(p, t) - f_s(p, t))$$

with $f_a(p, t) = (1 + e^{E/T})^{-1}$.

Exercise
It can be solved analytically

The final abundance is $\Omega_4 h^2 \simeq 0.3 \frac{\sin^2 2\theta}{10^{-8}} \left(\frac{m_4}{10\text{keV}} \right)^2$

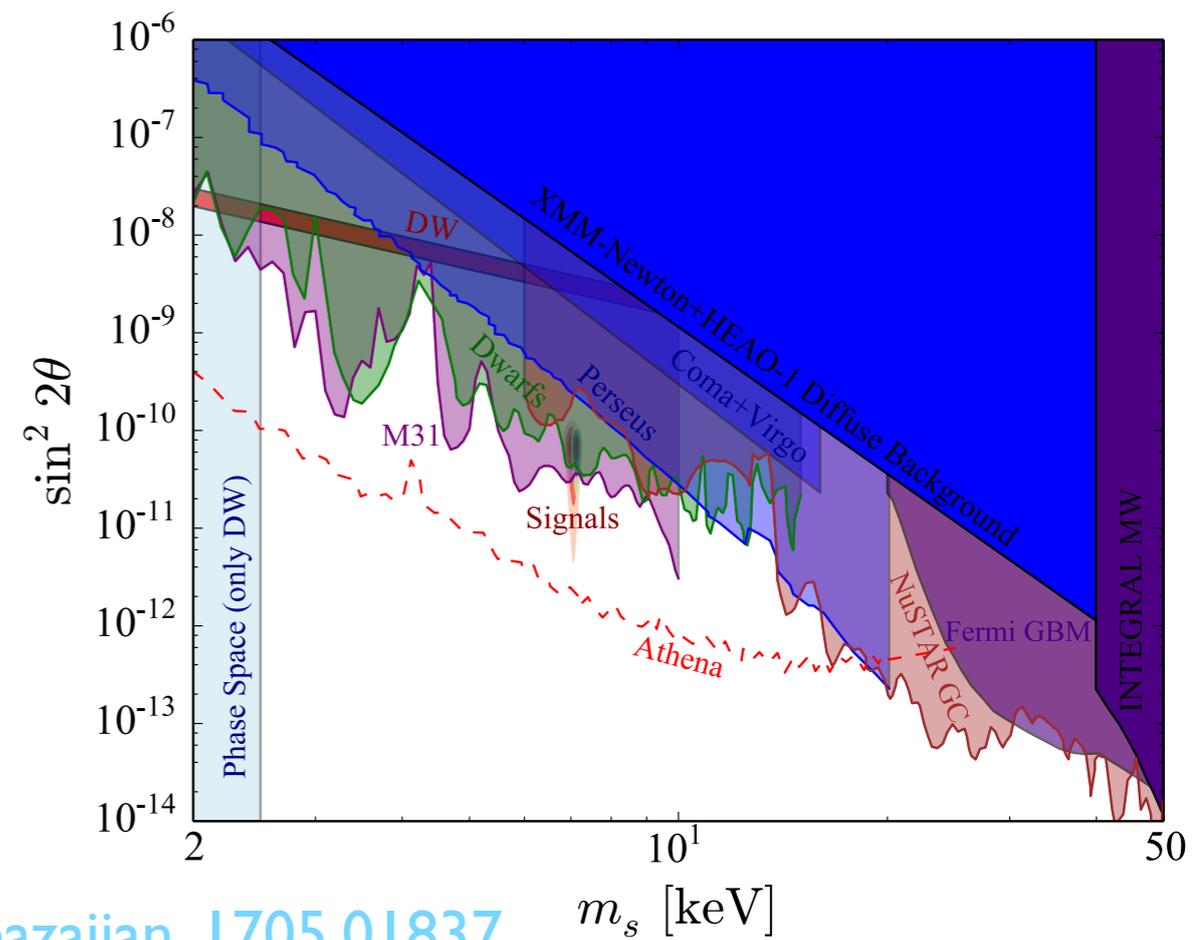


In presence of a large asymmetry, even smaller angles are required thanks to the resonant enhancement of the production.

Bounds on these DM candidates:

- **Structure formation.** If their mass is too low, they will behave too much as HDM erasing the structure at intermediate scales. This allows to put a bound in the several keV range.

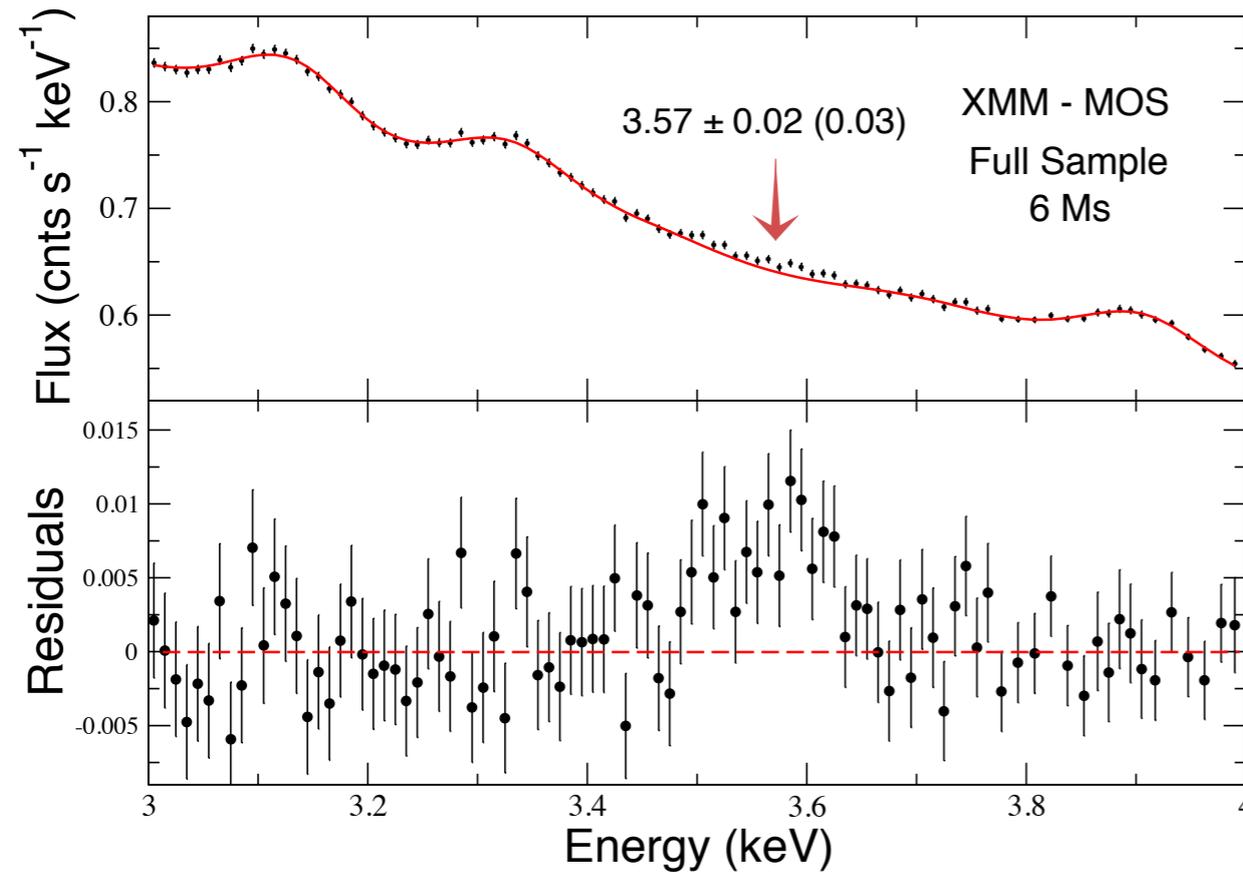
- **x-ray searches.** Although nearly sterile, their small mixing with active neutrinos make them decay in photons:



K. Abazajian, 1705.01837

$$\nu_4 \rightarrow \nu_a \gamma \quad \text{with} \quad E_\gamma = m_4/2 \quad \text{and} \quad Br(\nu\gamma) \sim 0.01$$

In 2014 two independent groups presented indications of a line around 7 keV.

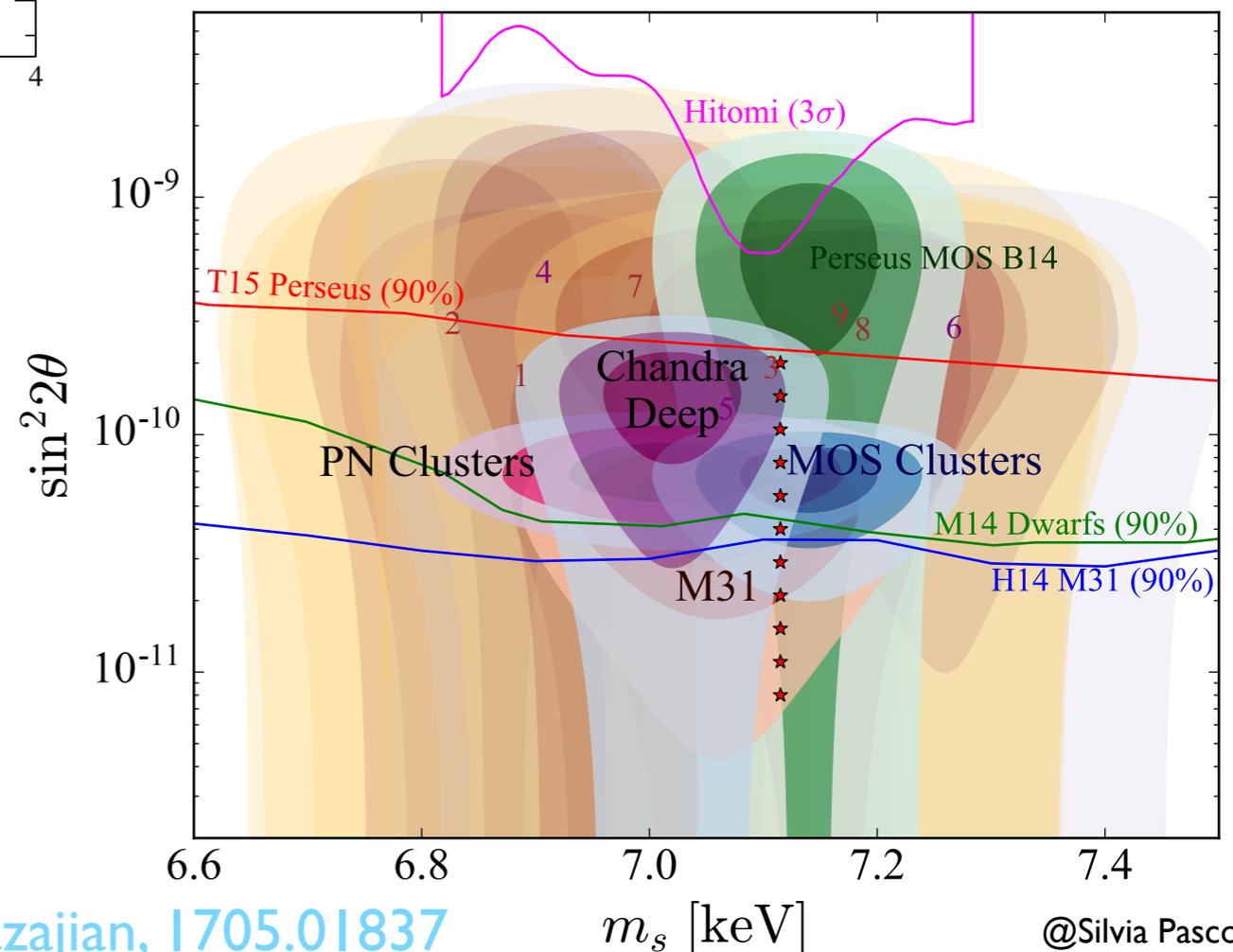


They analysed the emissions of several clusters.

E. Bulbul et al., 1402.2301. See also, A. Boyarsky et al., 1402.4119

If interpreted as sterile neutrinos, this would correspond to a 3.5 keV neutrino with a mixing

$$\sin^2 2\theta \simeq 7 \times 10^{-11}$$



K. Abazajian, 1705.01837

@Silvia Pascoli

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The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- B (or L) violation;
- C, CP violation;
- departure from thermal equilibrium.

The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- B (or L) violation;

In the **SM** also L is violated at the non-perturbative level. A lepton asymmetry is converted into a baryon asymmetry by sphaleron effects.

If neutrinos are Majorana particles, L is violated.

See-saw models require L violation (typically the Majorana mass of a heavy right-handed neutrino). In SUSY models without R-parity, L can be violated and neutrino masses generated.

The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- C, CP violation;

If C were conserved:

$$\Gamma(X^c \rightarrow Y^c + B^c) = \Gamma(X \rightarrow Y + B)$$

and no baryon asymmetry generated:

$$\frac{dB}{dt} \propto \Gamma(X^c \rightarrow Y^c + B^c) - \Gamma(X \rightarrow Y + B)$$

We have observed CPV in quark sector (too small) and we can search for it in the leptonic sector.

The baryon asymmetry. The theory

In order to generate dynamically a baryon asymmetry, the Sakharov's conditions need to be satisfied:

- out of equilibrium

In equilibrium

$$\Gamma(X \rightarrow Y + B) = \Gamma(Y + B \rightarrow X)$$

A generated baryon asymmetry is cancelled exactly by the antibaryon asymmetry.

When particles get out of equilibrium, this does not happen.

$$T < M_X$$

Baryogenesis

Let's consider a boson X , very heavy with BV couplings:

$$X \rightarrow lq \quad B_1 \quad Br(1) = r$$

$$X \rightarrow q\bar{q} \quad B_2 \quad Br(2) = 1 - r$$

The baryon number produced in the X and \bar{X} decays

$$B_x = B_1 r + B_2 (1 - r)$$

$$B_{\bar{X}} = -B_1 \bar{r} - B_2 (1 - \bar{r})$$

The total lepton number produced is then

$$\Delta B = (B_1 - B_2)(r - \bar{r})$$

Baryogenesis

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$$\begin{array}{lll} X \rightarrow lq & B_1 & Br(1) = r \\ X \rightarrow q\bar{q} & B_2 & Br(2) = 1 - r \end{array}$$

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The total lepton number produced is then

$$\Delta B = (B_1 - B_2)(r - \bar{r})$$

B violation

CP violation

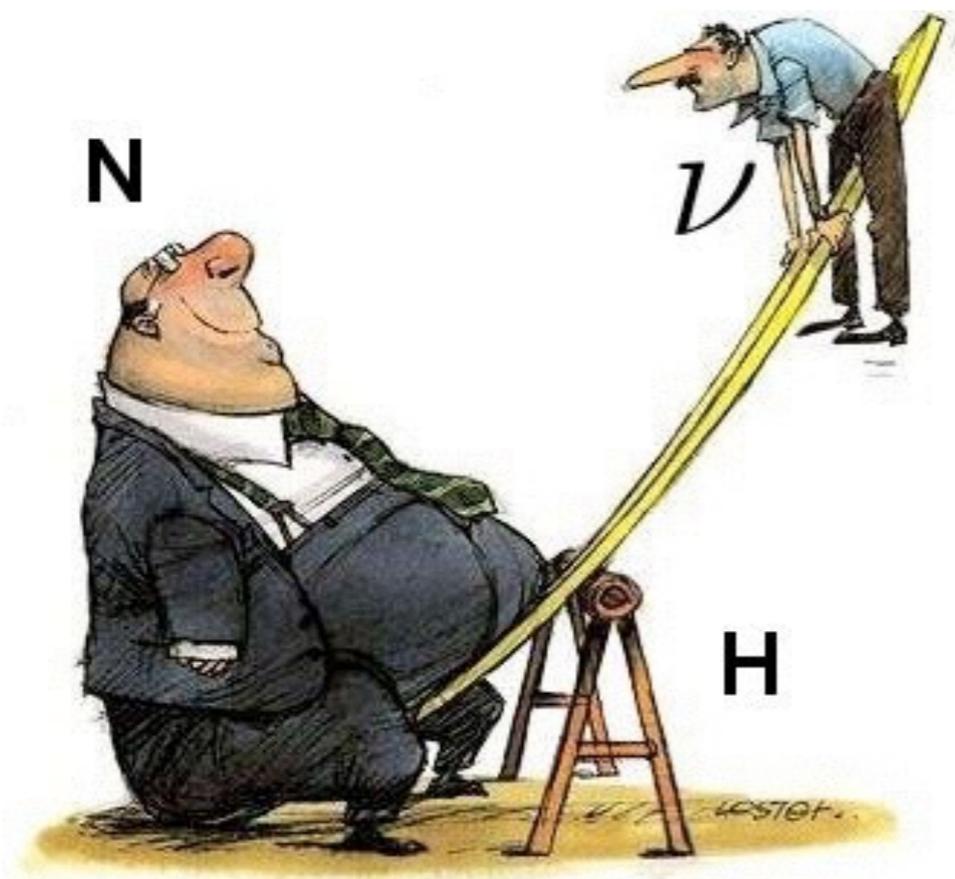
Out of equilibrium

Leptogenesis

The excess of quarks can be explained by **Leptogenesis** (Fukugita, Yanagida): the **heavy N** responsible for neutrino masses generate a **lepton asymmetry**.

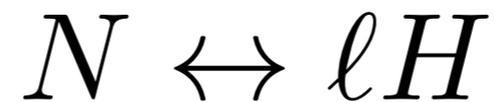
Recall: See saw mechanism type I

- Introduce a right handed neutrino **N**
- Couple it to the Higgs



$$\mathcal{L} = -Y_\nu \bar{N} L \cdot H - 1/2 \bar{N}^c M_R N$$

- At $T > M$, the right-handed neutrinos N are in equilibrium thanks to the processes which produce and destroy them:



- When $T < M$, N drops out of equilibrium



- A lepton asymmetry can be generated if

$$\Gamma(N \rightarrow \ell H) \neq \Gamma(N \rightarrow \ell^c H^c)$$

- Sphalerons convert it into a baryon asymmetry. $T=100$ GeV

$T=M$



In order to compute the baryon asymmetry:

1. evaluate the CP-asymmetry:

$$\epsilon_1 \equiv \frac{\Gamma(N_1 \rightarrow lH) - \Gamma(N_1 \rightarrow \bar{l}H^c)}{\Gamma(N_1 \rightarrow lH) + \Gamma(N_1 \rightarrow \bar{l}H^c)}$$

2. solve the Boltzmann equation to take into account the wash-out of the asymmetry with a k washout factor:

$$Y_L = k\epsilon_1$$

3. convert the lepton asymmetry into baryon asymmetry.

$$Y_B = \frac{k}{g^*} c_s \epsilon_1 \sim 10^{-3} - 10^{-4} \epsilon_1$$

[Fukugita, Yanagida; Covi, Roulet, Vissani; Buchmuller, Plumacher]

**Is there a connection
between low energy CPV
and the baryon
asymmetry?**

The general picture

ϵ depends on the CPV phases in Y_ν

$$\epsilon \propto \sum_j \Im(Y_\nu Y_\nu^\dagger)_{1j}^2 \frac{M_j}{M_1}$$

and in the U mixing matrix via the **see-saw formula**.

$$m_\nu = U^* m_i U^\dagger = -Y_\nu^T M_R^{-1} Y_\nu v^2$$

Let's consider see-saw type I with 3 NRs.

High energy		
M_R	3	0
Y_ν	9	6

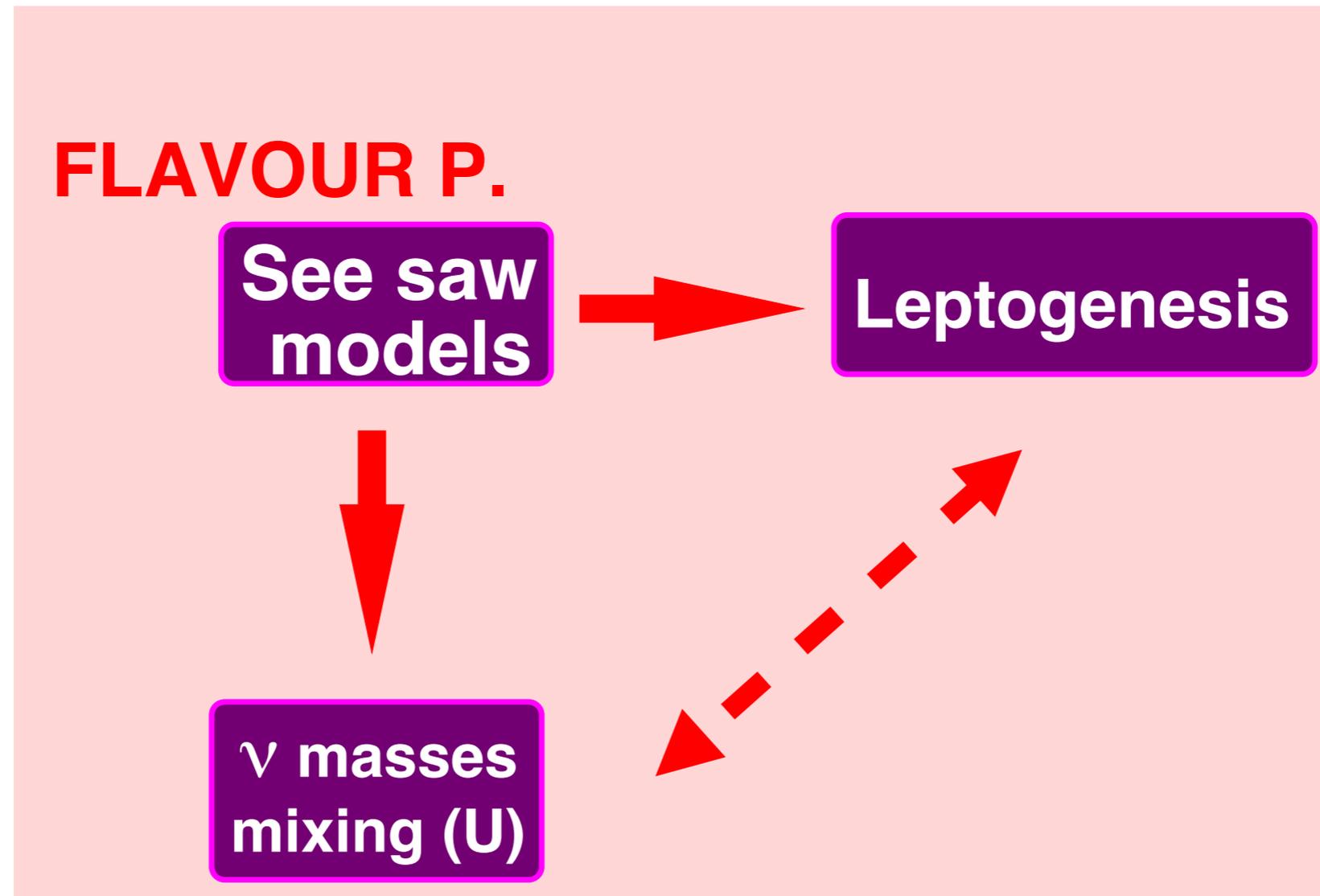
Low energy		
m_i	3	0
U	3	3

3 phases missing!

Specific flavour models

In understanding the origin of the flavour structure, the see-saw models have a reduced number of parameters.

It may be possible to predict the baryon asymmetry from the Dirac and Majorana phases.

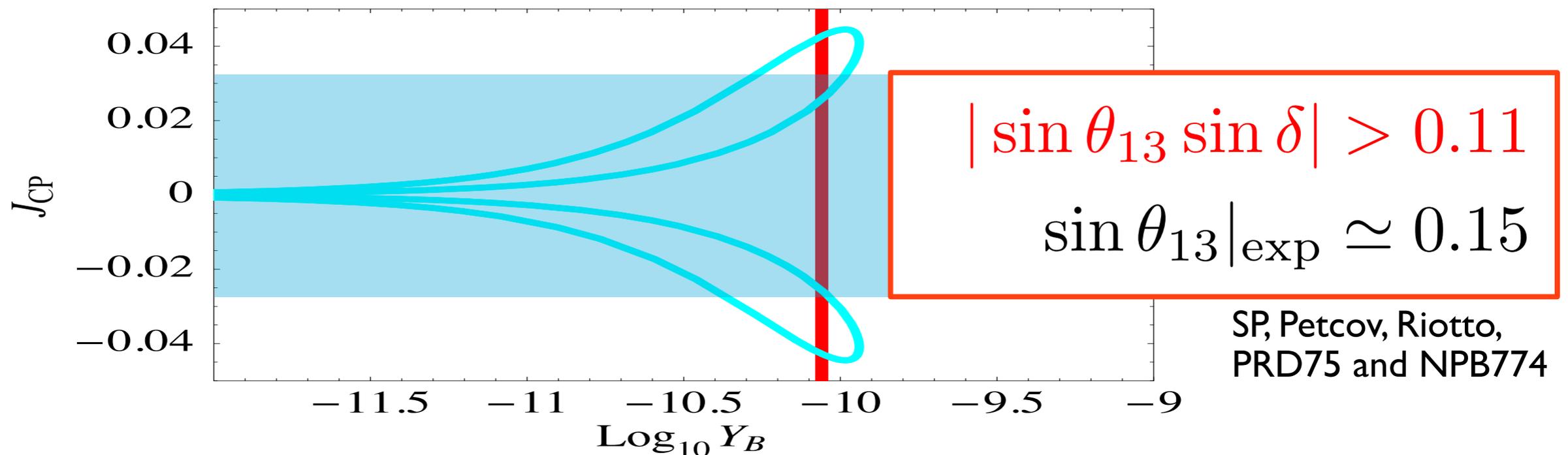


Does observing low energy CPV imply a baryon asymmetry?

It has been shown that, thanks to flavour effects, the low energy phases enter directly the baryon asymmetry.

Example in see-saw type I, with NH ($m_1 \ll m_2 \ll m_3$), $M_1 < M_2 < M_3$, $M_1 \sim 5 \cdot 10^{11}$ GeV:

$$\epsilon_\tau \propto M_1 f(R_{ij}) \left[c_{23} s_{23} c_{12} \sin \frac{\alpha_{32}}{2} - c_{23}^2 s_{12} s_{13} \sin \left(\delta - \frac{\alpha_{32}}{2} \right) \right]$$



Large θ_{13} implies that **delta can give an important (even dominant) contribution to the baryon asymmetry.** Large CPV is needed and a NH spectrum.

Conclusions (with some personal views)

1. Neutrinos have masses and mix and a wide experimental programme will measure their parameters with precision.
2. Neutrino masses cannot be accommodated in the Standard Model: extensions can lead to Dirac or Majorana neutrinos, with the latter the most studied cases. See-saw models are particularly favoured.
3. The main question concerns the energy scale of the new physics. Neutrino masses cannot pin it down by themselves and other signatures should be studied (leptogenesis, CLFV, collider LNV for TeV scale models, ...)
4. Models of flavour have typically a reduced number of parameters which can lead to relations testable in present and future experiments. Precision measurements will play a crucial role to disentangle various models.

A few references

Flavour models

S. F. King and C. Luhn, Neutrino Mass and Mixing with Discrete Symmetry, Rept.Prog.Phys. 76 (2013) 056201

Neutrinos in cosmology

J. Lesgourgues and S. Pastor, Massive neutrinos and cosmology, Phys.Rept. 429 (2006) 307-379 [astro-ph/0603494]

Sterile neutrinos in cosmology

M. Drewes (Munich, Tech. U.) et al., A White Paper on keV Sterile Neutrino Dark Matter, JCAP 1701 (2017) no. 01, 025 [arXiv:1602.04816]
K. Abazajian, 1705.01837

A few references

Leptogenesis

C. S. Fong, E. Nardi, A. Riotto, Leptogenesis in the Universe, *Adv. High Energy Phys.* 2012 (2012) 158303 [arXiv:1301.3062]