Can meteorological ensembles be usefully used in water resources management?

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Rational decision-making in water resources management should be based on Bayesian prediction and decision approaches, thus requiring the assessment of the so-called "predictive probability distributions"

This is why hydrological model forecasts are important tools to support decision-making in the area of water resources management, provided they allow assessing the predictive probability distributions to be used within the decision making process.

Why Predictive Uncertainty ?

In many fields of hydrology (flood warning and evacuation management; flood diversion and detention; real-time reservoir management; etc.), **Decision Makers have to take important decisions without perfect knowledge of future events**.

Since decisions frequently have heavy social, economical and environmental consequences, simulation and forecasting models are generally used to complement all available data and information and to predict the future outcomes.

Why Predictive Uncertainty ?

Predictive models cannot forecast “exactly” what will happen, but allow the Decision Makers to improve their prior belief on what will actually occur.

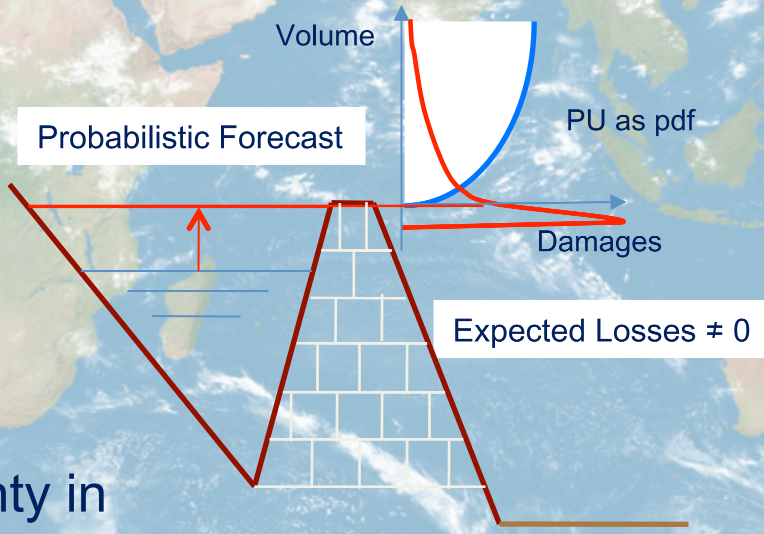
Given that predictions are not exact, it is essential to assess Predictive Uncertainty in order to correctly estimate the “expected consequences” of decisions in order to increase their reliability and reduce the possibility of wrong decisions.

Deterministic Vs Uncertain Forecasts

The Reservoir Management Case

In the Reservoir Management Problem it is easy to show that Deterministic Forecasts lead to wrong estimates of losses.

In this simple example losses occur if the reservoir is overtopped. If the Deterministic Forecast predicts that the maximum level will not exceed the dam top, the estimated losses are equal to zero.



This is obviously wrong because the uncertainty in the forecast implies that the “expected value” of losses is not null. The “expected value” of losses can be estimated if and when an assessment of Predictive Uncertainty will be available.

Definition of Predictive Uncertainty (PU)

Bearing in mind that damages are caused by a real future event and not by our model(s) forecasts, PU can be defined as **our assessment of the probability of occurrence of a future (real) event conditional upon all available knowledge**, generally based on observations and models forecasts.

$$f(y|D, M_1, \ldots, M_m)$$

WARNING: Predictive Uncertainty should not be confused with Validation Uncertainty.

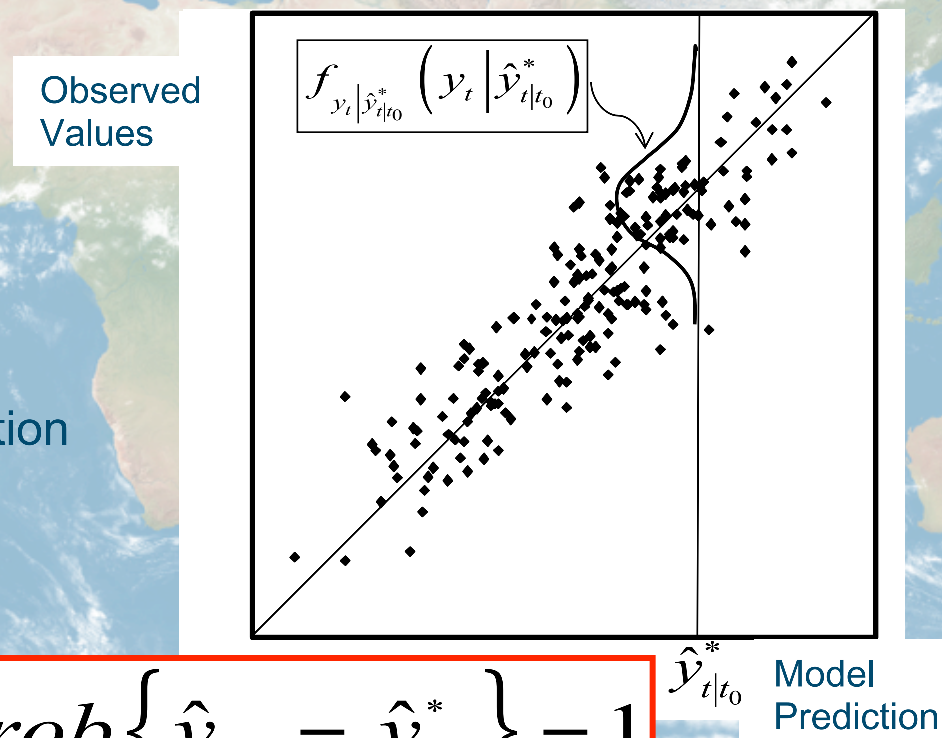
Validation vs Predictive Uncertainty

Validation Uncertainty
(in hindcast mode $t \leq t_0$)

Uncertainty of model predictions knowing (conditional on) the observations

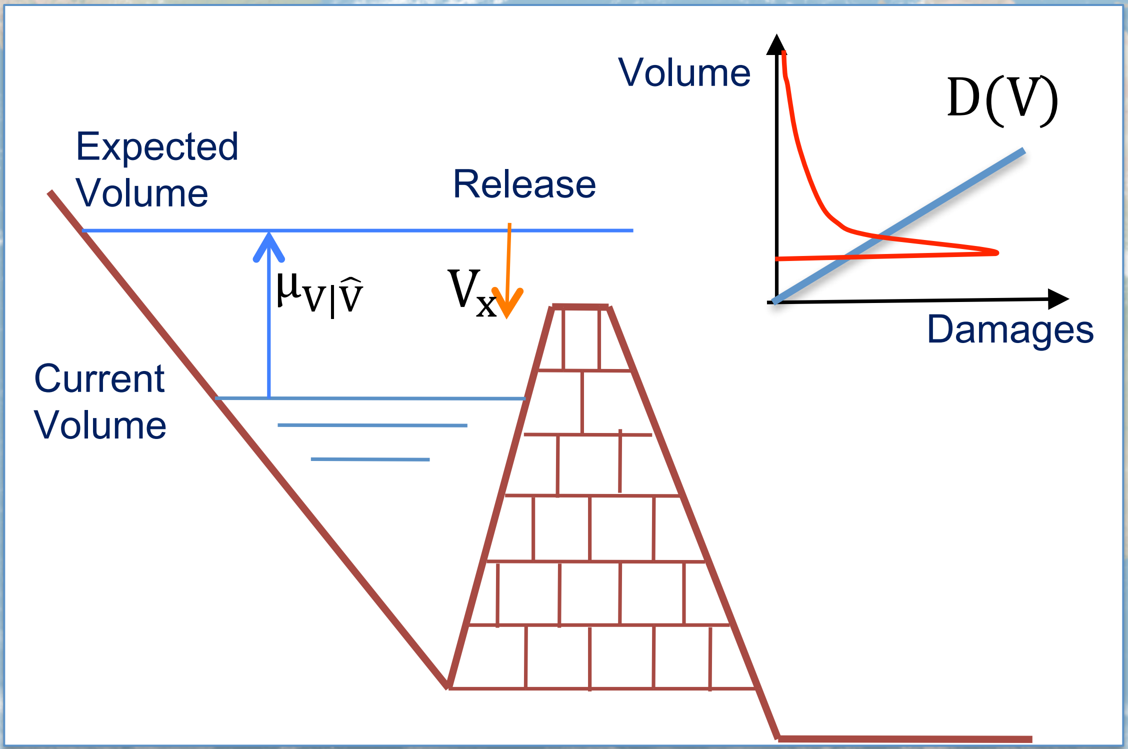
Predictive Uncertainty
(in operation mode $t \geq t_0$)

Uncertainty of future occurrences knowing (conditional on) the model prediction



$$Prob\left\{\hat{y}_{t|t_0} = \hat{y}^*_{t|t_0}\right\} = 1$$

Benefits for using predictive distributions: an example



A model forecasts that in the next few hours the volume stored in the reservoir will reach the value of $\hat{V}$

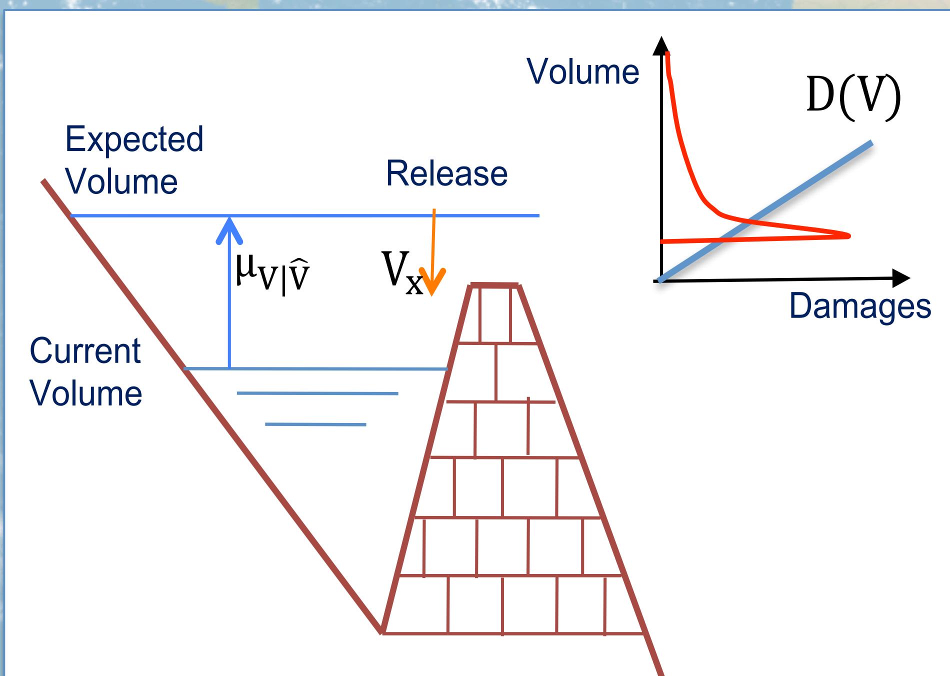
The forecast is uncertain. Therefore, the objective is to find an optimal release V_x by minimizing the **expected damages** as a function of the decision variable V_x

$$\min_{V_x} L(V_x) = c_1 V_x + \begin{cases} 0 & \forall\, \hat{V} - V_x \leq V_{max} \\ c_2\left(\hat{V} - V_{max} - V_x\right) & \forall\, \hat{V} - V_x > V_{max} \end{cases}$$

Benefits for using predictive distributions: an example

Using specific techniques (MCP, QR, BMA, etc.) one can assess the "predictive density"

$$f(V|\hat{V}) = \frac{e^{-\frac{\left(V-\mu_{V|\hat{V}}\right)^2}{2\,\sigma^2_{V|\hat{V}}}}}{\sqrt{2\pi\,\sigma^2_{V|\hat{V}}}}$$

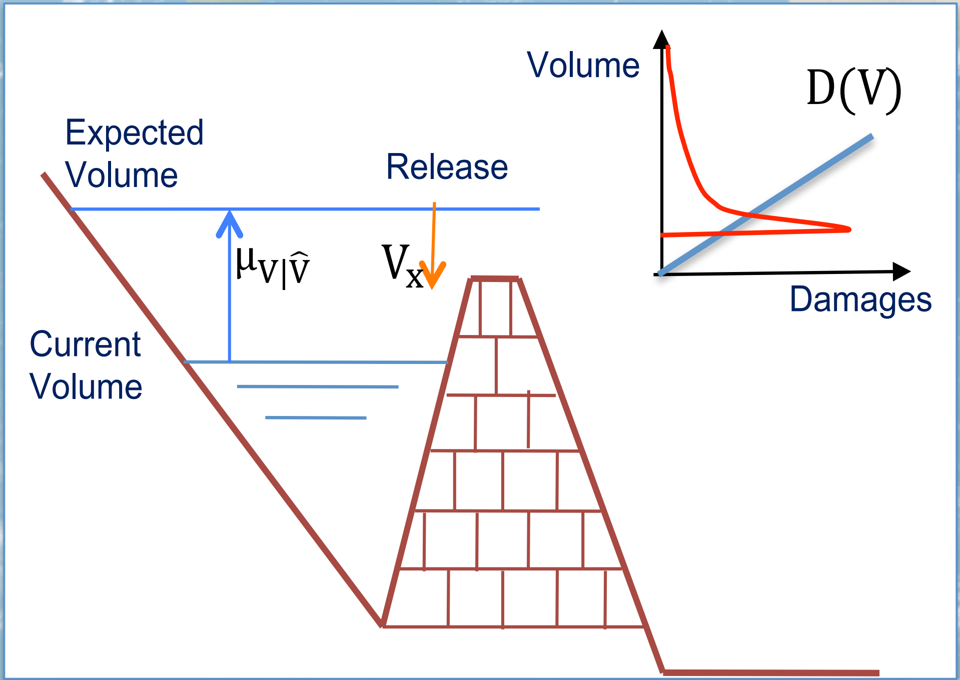


… and write it as a function of the decision variable V_x

$$f(V|\hat{V},V_x) = \frac{e^{-\frac{\left[V-\left(\mu_{V|\hat{V}}-V_x\right)\right]^2}{2\,\sigma^2_{V|\hat{V}}}}}{\sqrt{2\pi\,\sigma^2_{V|\hat{V}}}}$$

Benefits for using predictive distributions: an example

If the model forecast would have been "perfect" then $\hat{V} = V$ and the optimal release would be $V_x = \hat{V} - V_{max}$



Since the forecast is not "perfect", minimization of the expected losses leads to the following interesting result

$$V_x = \mu_{V|\hat{V}} - V_{max} + \sigma_{V|\hat{V}} N^{-1}\left\{1 - \frac{c_1}{c_2}\right\}$$

In other words, the more one is uncertain, the more he has to release on the basis of the principle of precaution

Predictive Uncertainty or Predictive Knowledge

Until the present times limited benefits have been gained by the use of Predictive Uncertainty, mostly because it has always been perceived by the end-users in the negative meaning of "**lack of knowledge**".

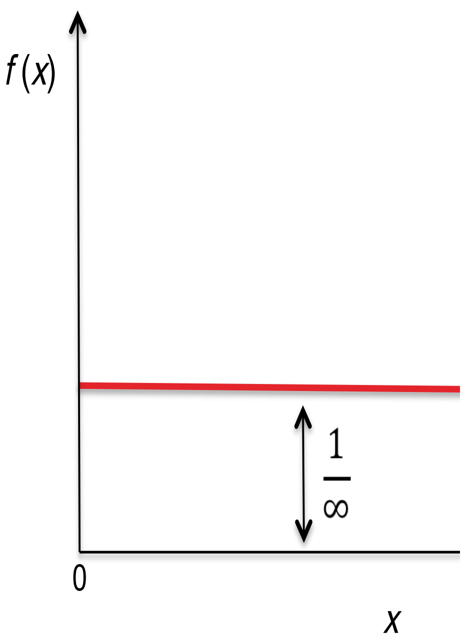
On the contrary, as in the classical case of the half-empty - half-full glass, what is generally referred-to as a measure of Predictive Uncertainty should be better communicated, perceived and interpreted as a measure of **"Predictive Knowledge" (PK)**, aimed at supporting rational decision making.

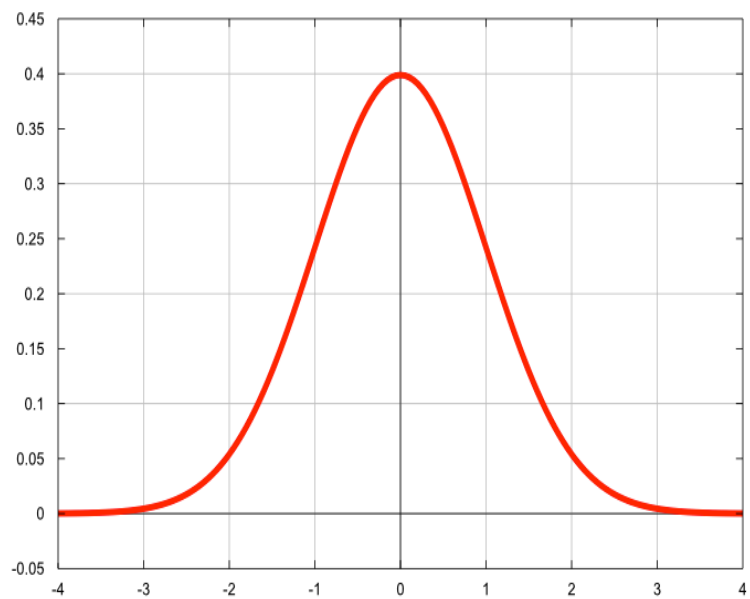
Is the glass half empty or half full?

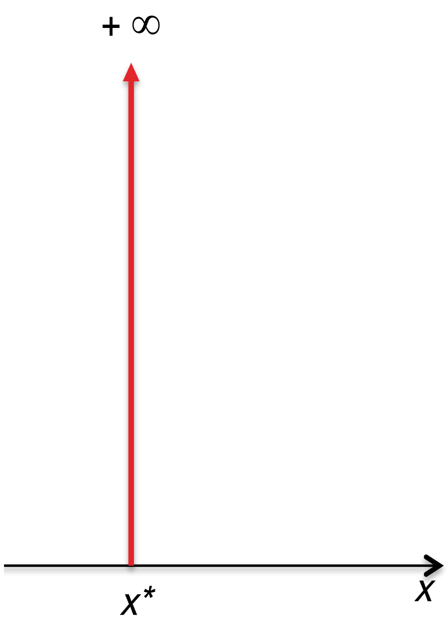
Total Uncertainty

Predictive Uncertainty

No Uncertainty







The Role of Flood Forecasting Models

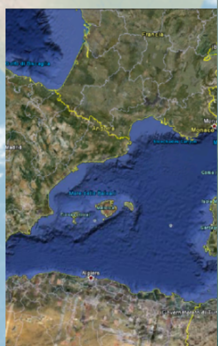
Coherently, flood forecasting models are not the end of the prediction-decision process, but rather “tools” aimed at increasing the DMs predictive knowledge on future events.

Models are allowed to be imperfect, provided that they increase DMs Predictive Knowledge, no more in terms of actual forecasts, but rather in terms of denser predictive densities, the measures of PK, to be operationally used in the rational decision making process.

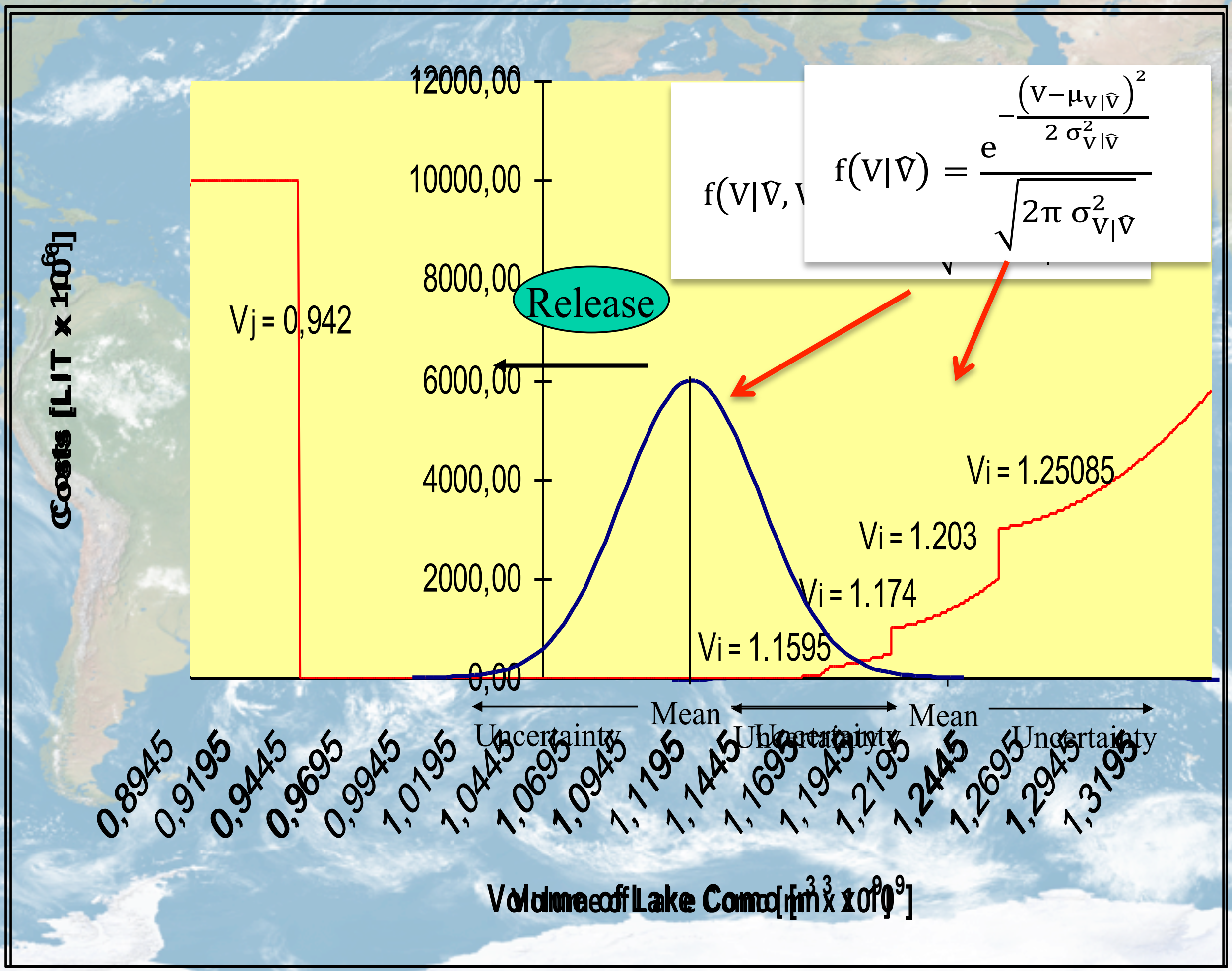
Forecasting models are no-more the essential component of a flood forecasting system, but just imperfect virtual reality tools.

The use of Predictive Uncertainty

To improve management of the Lake Como in Italy



The full use of Predictive Knowledge



Managing the Lake Como by using Predictive Knowledge approach

Results obtained by simulating 15 years of operations from January 1st, 1981 to December 31st, 1995

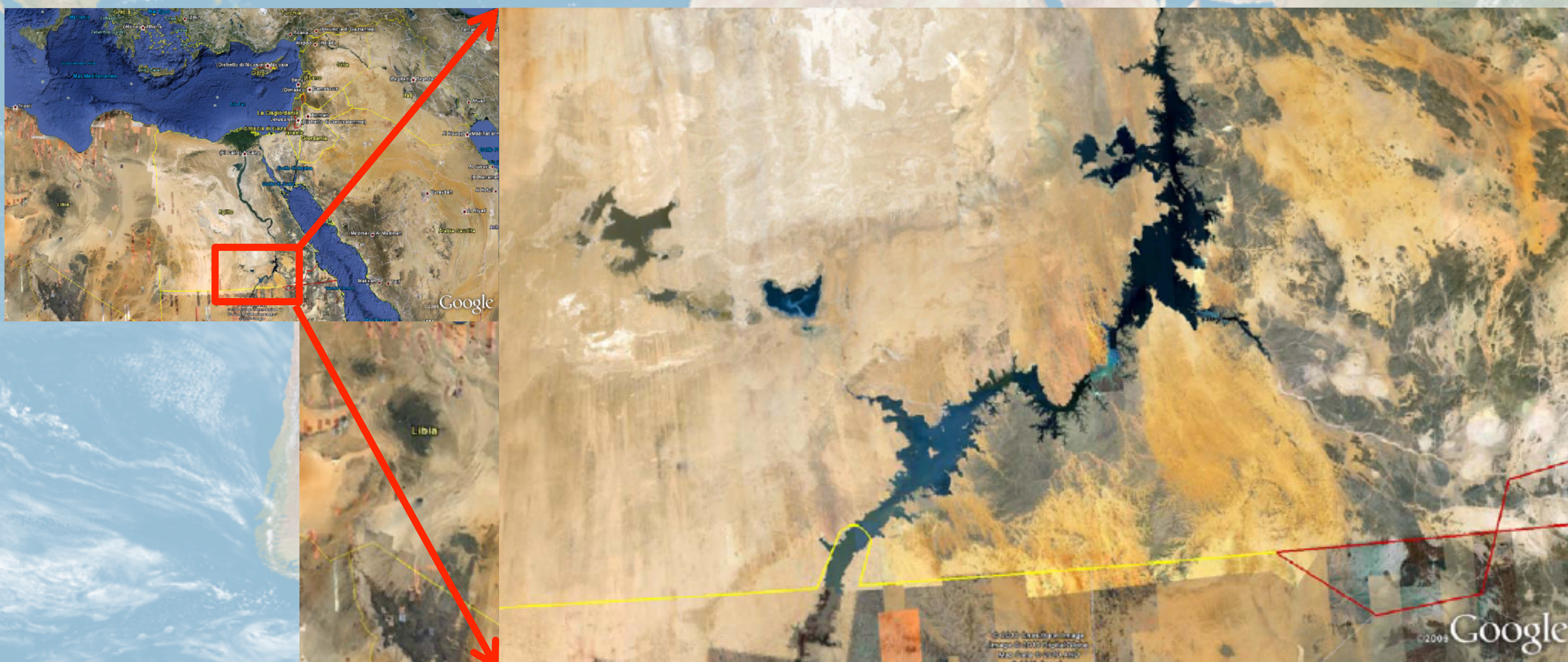
Water Level	Number of Days: Historical	Number of Days: Optimized
<-40 cm	214	0
≥ 120 cm	133	54
≥ 140 cm	71	32
≥ 173 cm	35	11

Water Deficit decreased from 890.27 Mm^3 to 694.49 Mm^3

Energy Production increased by 3%

The use of Predictive Uncertainty

To improve management of the Lake Nasser in Egypt



Managing the Aswan Reservoir by using PK

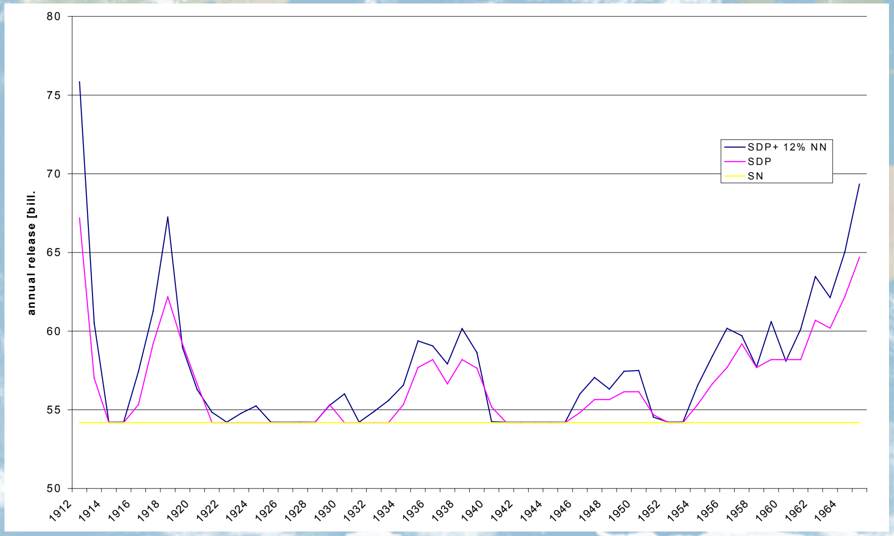
Predictive Knowledge can be used to improve management of a large reservoir such as the Aswan Reservoir. Results obtained by minimizing the sum of losses due to unprofitable releases of water + future expected losses, estimated using the predictive density conditional to a model forecast, resulted into an average additional water availability of

2.7 Billion m^3/year

of water. A large volume at no-cost if compared to the expected benefit of the **Jonglei canal**, estimated in

3.5 - 4.8 Billion m^3/year

but at enormous economical, social and environmental costs.



To meet the aim of correctly describing the predictive density, increasing interest has been directed towards ensemble hydrological forecasts of flow, water volume or level forecasts at predefined river sections, to be derived through model prediction chains where precipitation meteorological ensembles (Tracton and Kalnay, 1993; Molteni et al., 1996) are used as forcing.

However, it can be proven that ensembles do not provide a correct description of the predictive probability densities to be used to estimate expected damages or expected benefits within the frame of a Bayesian decision-making process. They provide biased and lesser-dispersed estimates (Hamill and Colucci, 1997; Eckel and Walters, 1998; Schwanenberg et al, 2015).

In parallel to the use of ensembles, with the aim of obtaining a correct predictive probability density, a number of uncertainty post processors were also developed such as the Hydrological Uncertainty Processor (HUP, Krzysztofowicz, 1999), the Bayesian Model Averaging (BMA, Raftery, 1993; Raftery et al., 2005) or the Model Conditional Processor (MCP, Todini 2008). However, not all these uncertainty post processors allow accounting for the time variability of the meteorological and hydrological ensembles spread.

The two approaches, the ensembles and the post processors, can be combined in order to meet the goal of properly describing the predictive probability densities, and, consequently improving decision-making.

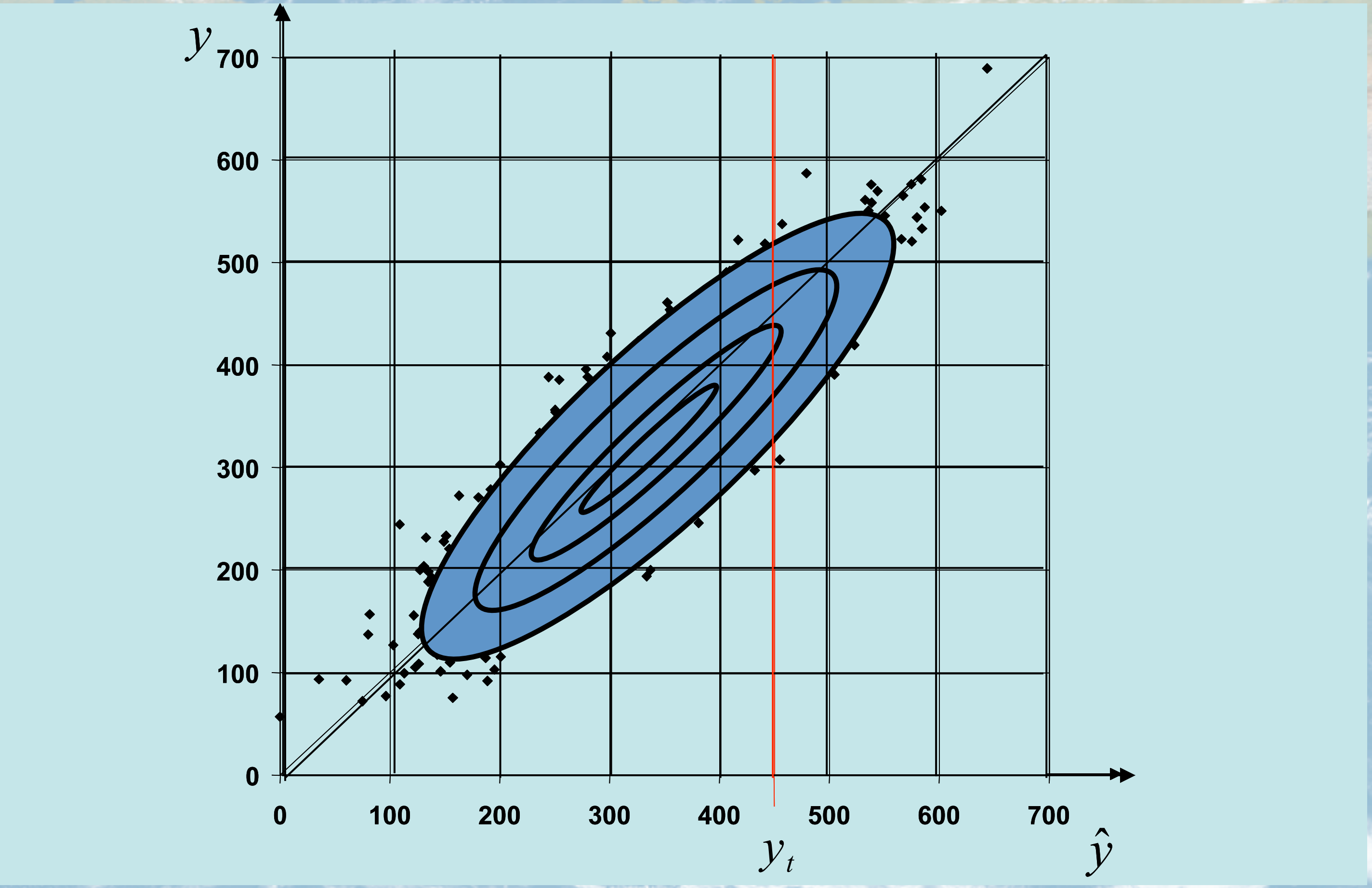
MODEL AND PARAMETER UNCERTAINTY

When the behaviour of a set of conditions such as errors deriving from the different sources varies at random in time in an “unpredictable manner” then one can use the “mixture of models” concept.

Please bear in mind that if the conditions ARE predictable then one is better off by using the Model which best fits the observations under the relevant conditions.

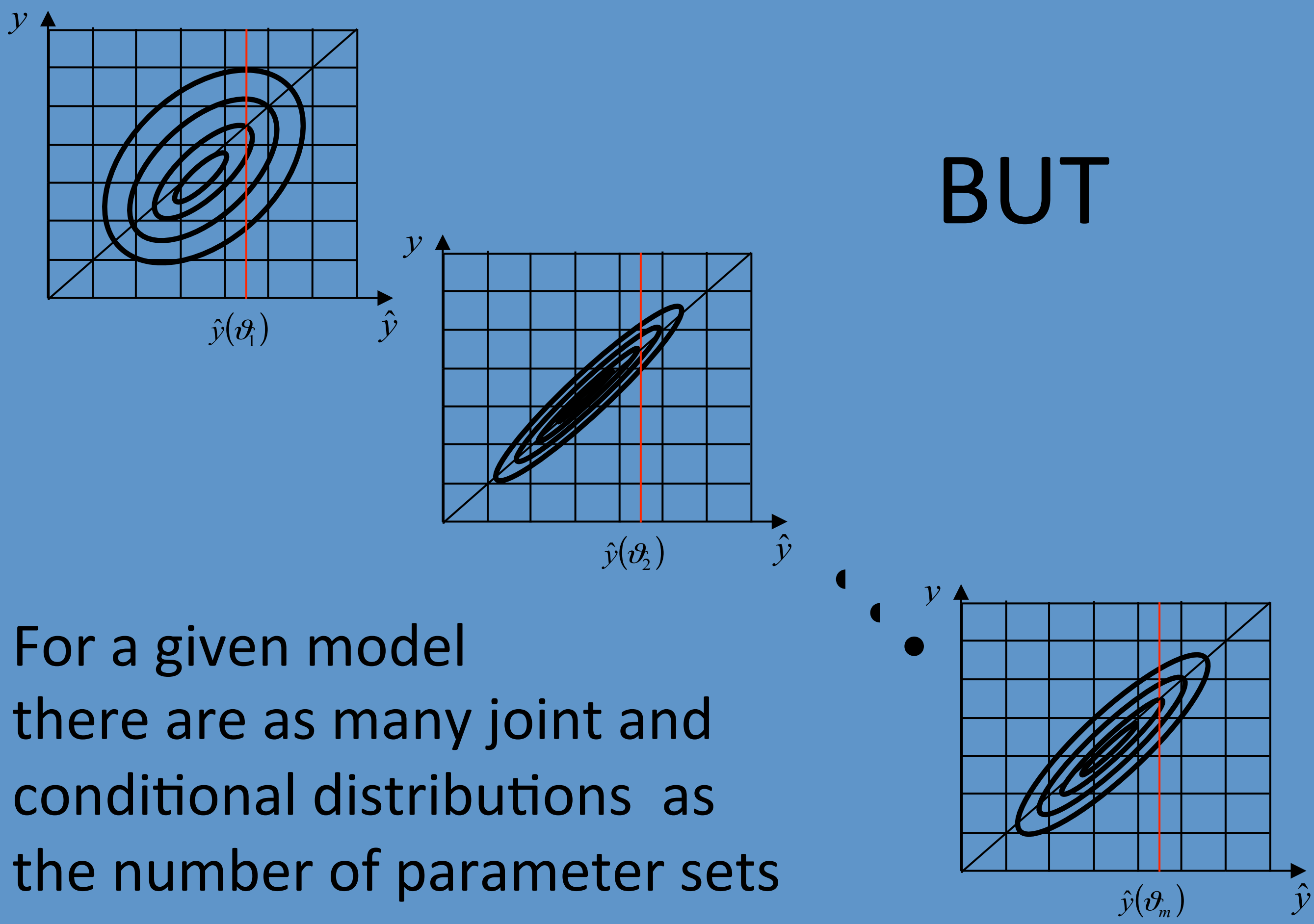
MODEL AND PARAMETER UNCERTAINTY

MODEL AND PARAMETER UNCERTAINTY



For a given model and a set of parameters one can derive predictand and model joint/conditional probability densities

MODEL AND PARAMETER UNCERTAINTY



BUT

For a given model
there are as many joint and
conditional distributions as
the number of parameter sets

$$Prob\left(y_{t+k\Delta t} \leq y^{*} \,\middle|\, \hat{y}_{t+k\Delta t}\left(\vartheta_i\right), M, \mathscr{D}_{hist} \right)$$

MODEL AND PARAMETER UNCERTAINTY

Therefore one must derive the "Posterior Density" of parameters $g_B\left(\vartheta \middle| M, \mathscr{D}_{hist}\right)$ using the classical Bayesian Inference. This PD is then used to marginalise, namely to integrate out, the effect of parameters.

In a continuous domain:

$$F\left(y_{t+k\Delta t} \middle| M, \mathscr{D}_{hist}\right) = \int_{\Omega_\vartheta} F\left(y_{t+k\Delta t} \middle| \hat{y}_{t+k\Delta t}\left(\vartheta\right), M, \mathscr{D}_{hist}\right) g_B\left(\vartheta \middle| M, \mathscr{D}_{hist}\right) d\vartheta$$

or in discrete mode:

$$F\left(y_{t+k\Delta t} \middle| M, \mathscr{D}_{hist}\right) \cong \sum_i F_i\left(y_{t+k\Delta t} \middle| \hat{y}_{t+k\Delta t}\left(\vartheta_i\right), M, \mathscr{D}_{hist}\right) g_B\left(\vartheta_i \middle| M, \mathscr{D}_{hist}\right)$$

MODEL AND PARAMETER UNCERTAINTY

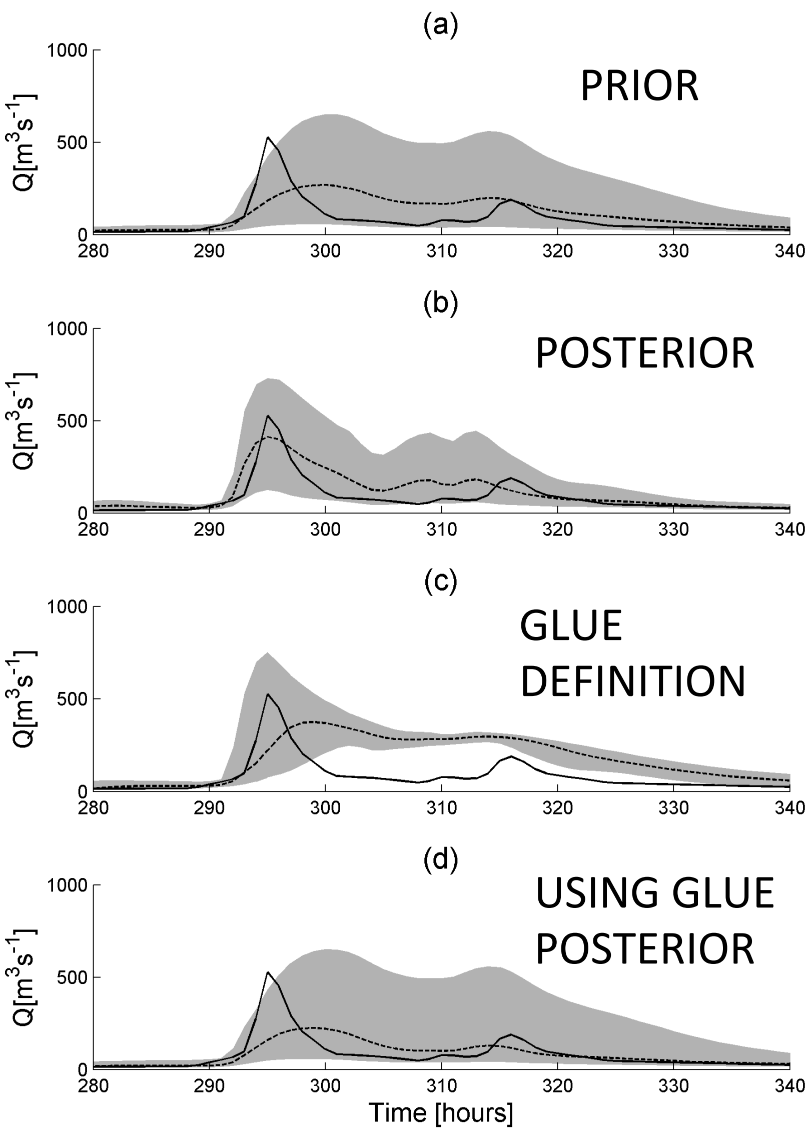
Please note that this is TOTALLY different from what is proposed in GLUE, where the definition of PU is given as:

$$P\left(\hat{Z}_t < z\right) = \sum_{i=1}^{B} L\left[M\left(\vartheta_i\right)\middle|\hat{Z}_{t,i} < z\right]$$

where $L = g_G\left(\vartheta_i \middle| M, \mathcal{D}_{hist}\right)$ is nothing else than the posterior parameter density.

The conditional predictive density (???) and the marginalisation of parameters uncertainty (???) are not present in this definition.

MODEL AND PARAMETER UNCERTAINTY



$$F\left\{y\middle|M,\mathcal{D}_{hist}\right\} = \sum_{i=1}^{B} F\left\{y\middle|\hat{y}(\vartheta_i),M\right\} g_0\left\{\vartheta_i\middle|M,\mathcal{D}_{hist}\right\}$$

$$F\left\{y\middle|M,\mathcal{D}_{hist}\right\} = \sum_{i=1}^{B} F\left\{y\middle|\hat{y}(\vartheta_i),M\right\} g_B\left\{\vartheta_i\middle|M,\mathcal{D}_{hist}\right\}$$

$$F\left(\hat{y}_t\middle|M,\mathcal{D}_{hist}\right) = \sum_{i=1}^{B} L\left[M(\vartheta_i)\middle|\hat{y}_{t,i} \le \hat{y}_t\right]$$
$$= \sum_{i=1}^{B} g_G\left\{\vartheta_i\middle|M,\mathcal{D}_{hist}\right\} \quad \forall \vartheta_i\middle|\hat{y}_{t,i} \le \hat{y}_t$$

$$F\left\{y\middle|M,\mathcal{D}_{hist}\right\} = \sum_{i=1}^{B} F\left\{y\middle|\hat{y}(\vartheta_i),M\right\} g_G\left\{\vartheta_i\middle|M,\mathcal{D}_{hist}\right\}$$

MODEL AND PARAMETER UNCERTAINTY

From the presently available experiences, Marginalisation of parameters uncertainty, although statistically correct, does not produce substantial differences from using a best fit parameter set.

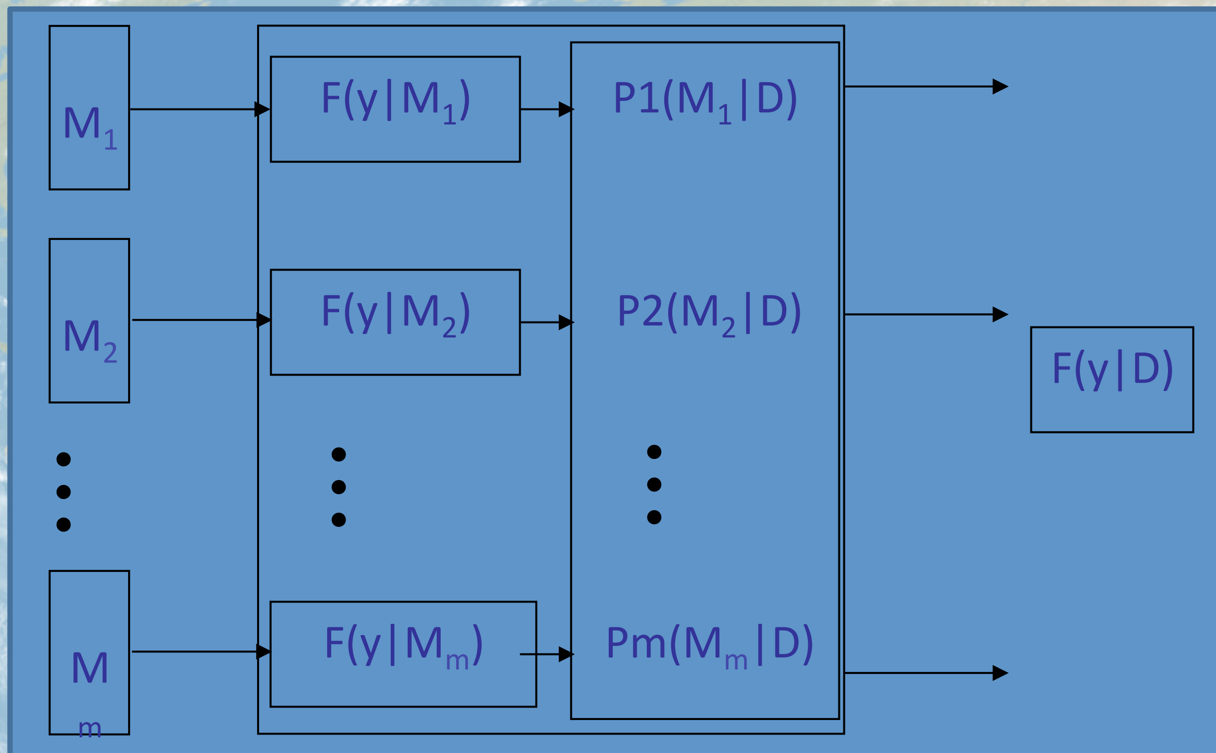
This is mostly due to the fact that the nearly best parameters produce predictions that are closely related among them, while the posterior probability of the worst parameters is obviously very low.

MODEL AND PARAMETER UNCERTAINTY

This is why it is more interesting to approach the problem in terms of few alternative models of widely different nature. For instance a physically based model, a conceptual model and a data driven ANN model.

This has given rise to the development of several multi-model
Predictive Uncertainty Processors.

MODEL AND PARAMETER UNCERTAINTY



$$F\left(y_t \middle| M_i, \mathscr{D}_{hist}\right) = \sum_{i=1}^{n} F_i\left(y_t \middle| M_i\right) Prob\left\{M_i \middle| \mathscr{D}_{hist}\right\}$$

Available Single or Multi-model Predictive Uncertainty Processors

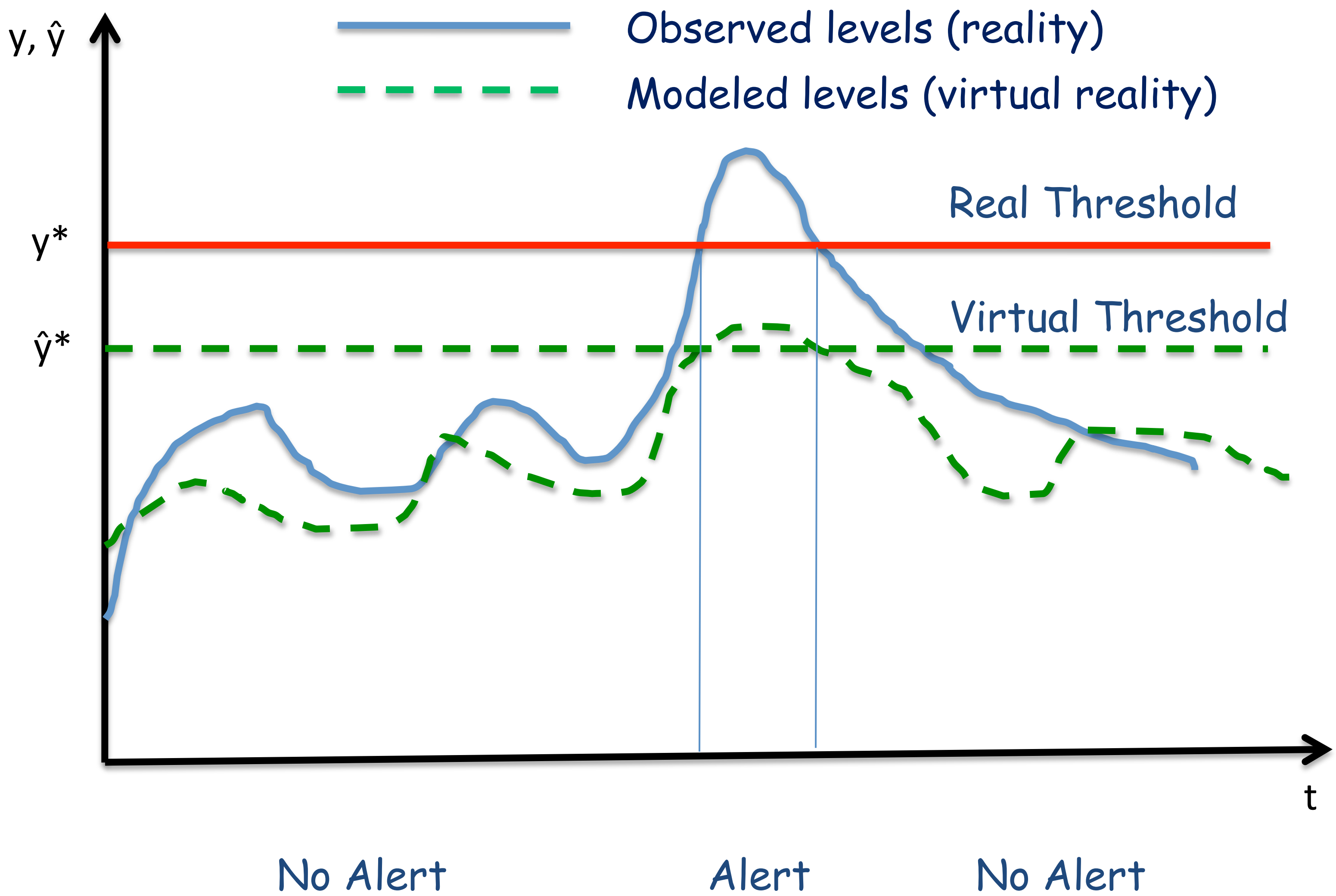
BINARY RESPONSE

Logistic Regression – Hosmer and Lemeshow,1989;
Bayesian Nonparametric Binary Response - Qian et al., 1998;
Binary Prediction Trees – Pittman et al., 2003;
Bayesian Multivariate Binary Predictor – Todini et al., 2008.
……………………….

FULL DENSITY

Hydrological Uncertainty Processor – Krzysztofowicz, 1999;
Krzysztofowicz and Kelly, 2000
Bayesian Model Averaging – Raftery et al., 2003;
Model Conditional Processor – Todini, 2008.
……………………….

The Binary Processors



MERGING MODELS THROUGH UNCERTAINTY POST-PROCESSORS

Continuous Processors

Today the possibility exists of merging the different types of models through Uncertainty Post-Processors, using the so called Bayesian Model Averaging (BMA) proposed by Raftery or to generalise the Krzysztofowicz results to multi-models via the Model Conditional Processor (MCP) proposed by Todini.

AVAILABLE PREDICTIVE UNCERTAINTY PROCESSORS

Krzysztofowicz Bayesian **Processor**

Krzysztofowicz approach has many limitations:

- It uses an auto-regressive model as the a priori model (for instance, this type of model is not suitable for flood routing)
- It has a scalar formulation; it is rathefr complicated to extend it to the multi-model case

AVAILABLE PREDICTIVE UNCERTAINTY PROCESSORS

Raftery Bayesian Model Averaging

BMA aims at assessing the unconditional mean and variance of any future value of a predictand on the basis of several model forecasts.

$$E\left\{y \middle| \mathscr{D}, \mathscr{M}\right\} = \sum_{k=1}^{K} w_k E\left\{y \middle| \hat{y}_k\right\}$$

$$Var\left\{y \middle| \mathscr{D}, \mathscr{M}\right\} \cong \sum_{k=1}^{K} w_k Var\left\{y \middle| \hat{y}_k\right\} + \sum_{k=1}^{K} w_k \left(\hat{y}_k - \sum_{k=1}^{K} w_k E\left\{y \middle| \hat{y}_k\right\} \right)^2$$

The approach provides a good approximation of the Conditional Predictive Density

AVAILABLE PREDICTIVE UNCERTAINTY PROCESSORS

The BMA weights are estimated by solving the following non-linear optimization problem

$$\begin{cases} \max\limits_{w_k} \log \mathscr{L} = \sum_{s=1}^{S} \sum_{t=1}^{T} \log\left(\sum_{k=1}^{K} w_k p_k \left(y_{st} \middle| \hat{y}_{kst} \right) \right) \\ s.t. \quad \sum_{k=1}^{K} w_k = 1 \end{cases}$$

on the assumption that the probability densities of the observations as well as of the model forecasts are all approximately Gaussian, which is correct if using NQT

AVAILABLE PREDICTIVE UNCERTAINTY PROCESSORS

Bayesian Model Averaging Limitations

- The probability density of observations and model forecasts may be far to be Gaussian, but this can be fixed by converting observations and forecasts in the Gaussian space.
- The Estimation-Maximization approach proposed to estimate the BMA weights does not necessarily lead to the optimal values.

AVAILABLE PREDICTIVE UNCERTAINTY PROCESSORS

The Model Conditional Processor

If one can make the hypothesis that all the transformed variables follow a multi-Gaussian joint probability density, a more natural approach would be to:

- Develop a set of models in the real untransformed space (one or more than one)
- Build the joint probability density in the Gaussian space (Predictand, a priori model, deterministic model, etc.)
- Directlyly compute the probability of the predictand conditional on ALL the model predictions

A Useful Property of the Multivariate Normal Distribution

Given a vector of random variables $\mathbf{x} = \begin{bmatrix} \mathbf{y} \\ \hat{\mathbf{y}} \end{bmatrix}$ Normally distributed with

Mean $\boldsymbol{\mu}_{\mathbf{x}} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{y}} \\ \boldsymbol{\mu}_{\hat{\mathbf{y}}} \end{bmatrix}$ and Variance $\boldsymbol{\Sigma}_{\mathbf{xx}} = \begin{bmatrix} \boldsymbol{\Sigma}_{\mathbf{yy}} & \boldsymbol{\Sigma}_{\mathbf{y}\hat{\mathbf{y}}} \\ \boldsymbol{\Sigma}_{\hat{\mathbf{y}}\mathbf{y}} & \boldsymbol{\Sigma}_{\hat{\mathbf{y}}\hat{\mathbf{y}}} \end{bmatrix}$ this implies that also

$\mathbf{y} \approx N\left(\boldsymbol{\mu}_{\mathbf{y}}, \boldsymbol{\Sigma}_{\mathbf{yy}}\right)$ and $\hat{\mathbf{y}} \approx N\left(\boldsymbol{\mu}_{\hat{\mathbf{y}}}, \boldsymbol{\Sigma}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}\right)$ are Normally distributed

The conditional distribution of y given yhat is also Normal

$$N\left(\boldsymbol{\mu}_{\mathbf{y}|\hat{\mathbf{y}}}, \boldsymbol{\Sigma}_{\mathbf{yy}|\hat{\mathbf{y}}}\right)$$

with conditional Mean $\boldsymbol{\mu}_{\mathbf{y}|\hat{\mathbf{y}}} = \boldsymbol{\mu}_{\mathbf{y}} + \boldsymbol{\Sigma}_{\mathbf{y}\hat{\mathbf{y}}}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}^{-1}\left(\hat{\mathbf{y}} - \boldsymbol{\mu}_{\hat{\mathbf{y}}}\right)$

and conditional Variance $\boldsymbol{\Sigma}_{\mathbf{yy}|\hat{\mathbf{y}}} = \boldsymbol{\Sigma}_{\mathbf{yy}} - \boldsymbol{\Sigma}_{\mathbf{y}\hat{\mathbf{y}}}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}^{-1}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}\mathbf{y}}$

AVAILABLE PREDICTIVE UNCERTAINTY PROCESSORS

The Model Conditional Processor

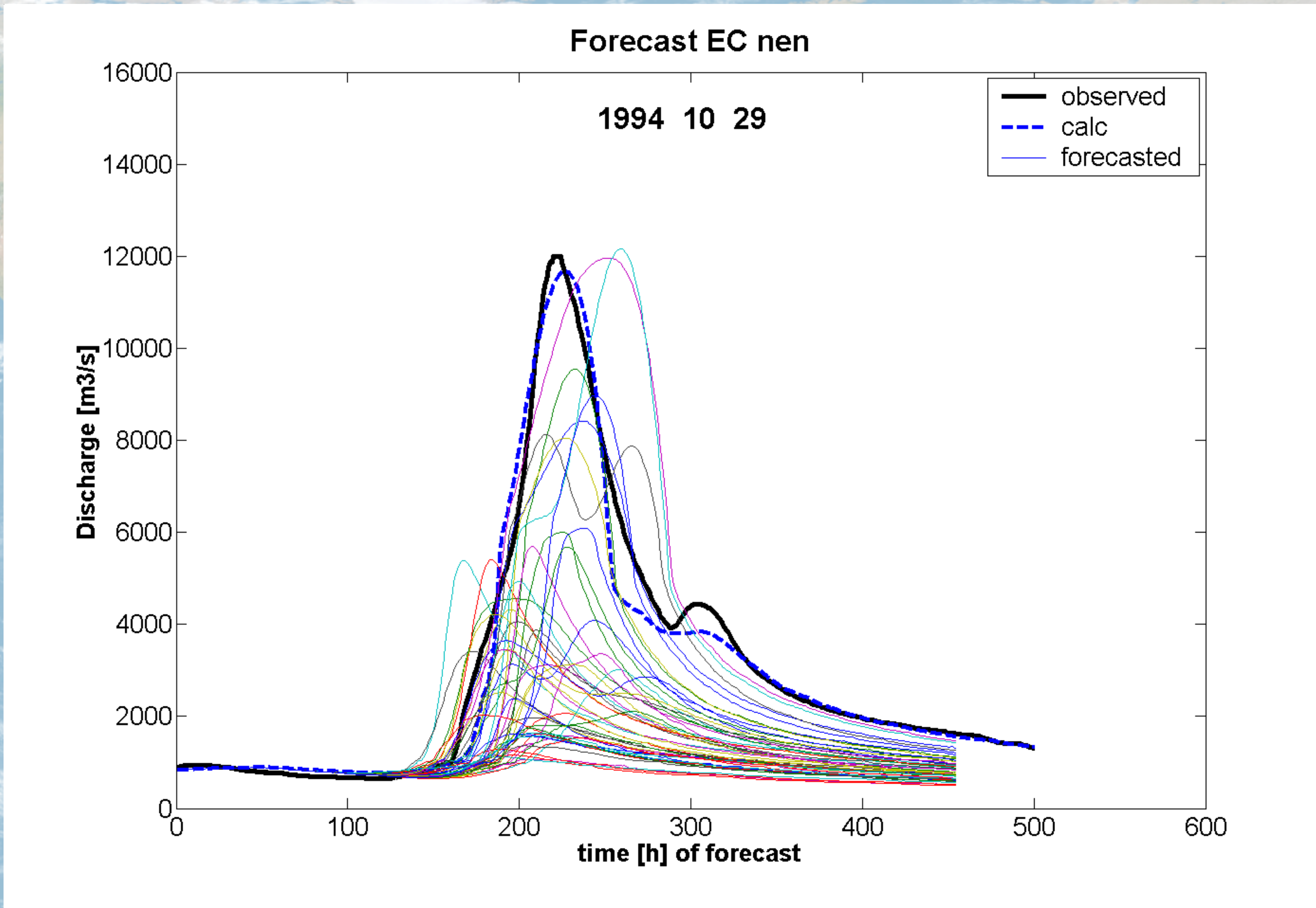
Therefore, the MCP was developed by directly applying this definition of conditional density after converting observations and forecasts into the multi-dimensional Gaussian space. As previously shown the predictive density is the Normal distribution $N\left(\boldsymbol{\mu}_{\mathbf{y}|\hat{\mathbf{y}}}, \boldsymbol{\Sigma}_{\mathbf{yy}|\hat{\mathbf{y}}}\right)$

with conditional Mean $\boldsymbol{\mu}_{\mathbf{y}|\hat{\mathbf{y}}} = \boldsymbol{\mu}_{\mathbf{y}} + \boldsymbol{\Sigma}_{\mathbf{y}\hat{\mathbf{y}}}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}^{-1}\left(\hat{\mathbf{y}} - \boldsymbol{\mu}_{\hat{\mathbf{y}}}\right)$

and conditional Variance $\boldsymbol{\Sigma}_{\mathbf{yy}|\hat{\mathbf{y}}} = \boldsymbol{\Sigma}_{\mathbf{yy}} - \boldsymbol{\Sigma}_{\mathbf{y}\hat{\mathbf{y}}}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}\hat{\mathbf{y}}}^{-1}\boldsymbol{\Sigma}_{\hat{\mathbf{y}}\mathbf{y}}$

IN ALL THIS DISCUSSION WHERE DO ENSEMBLES FIT IN?

Whoever used hydrological ensemble forecasting based on meteorological ensembles, experienced what shown in the following slides



Forecast EC nen
1994 10 29
observed
calc
forecasted
Discharge [m3/s]
16000
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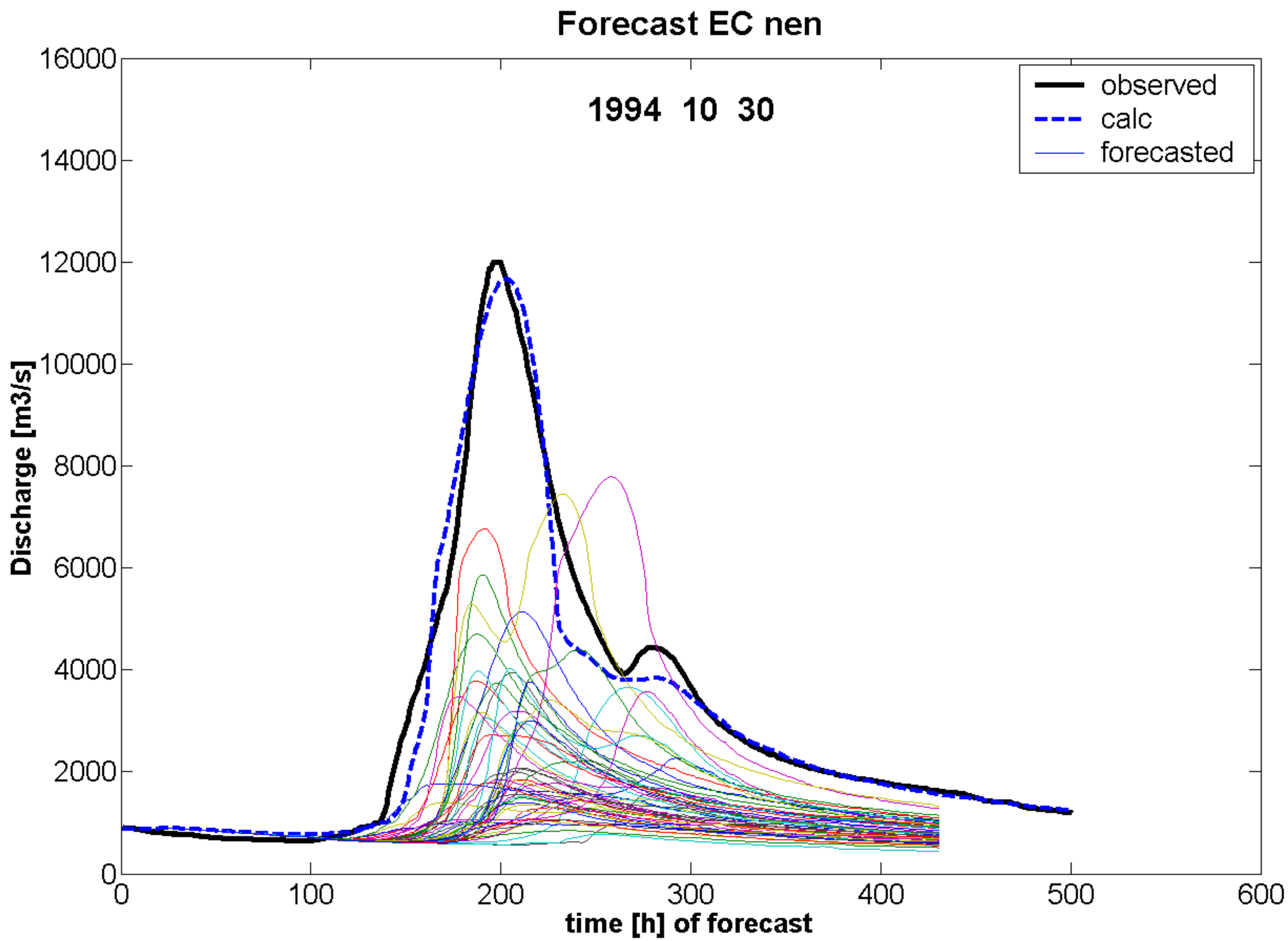
time [h] of forecast



Forecast EC nen
1994 10 30
observed
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Discharge [m3/s]
time [h] of forecast
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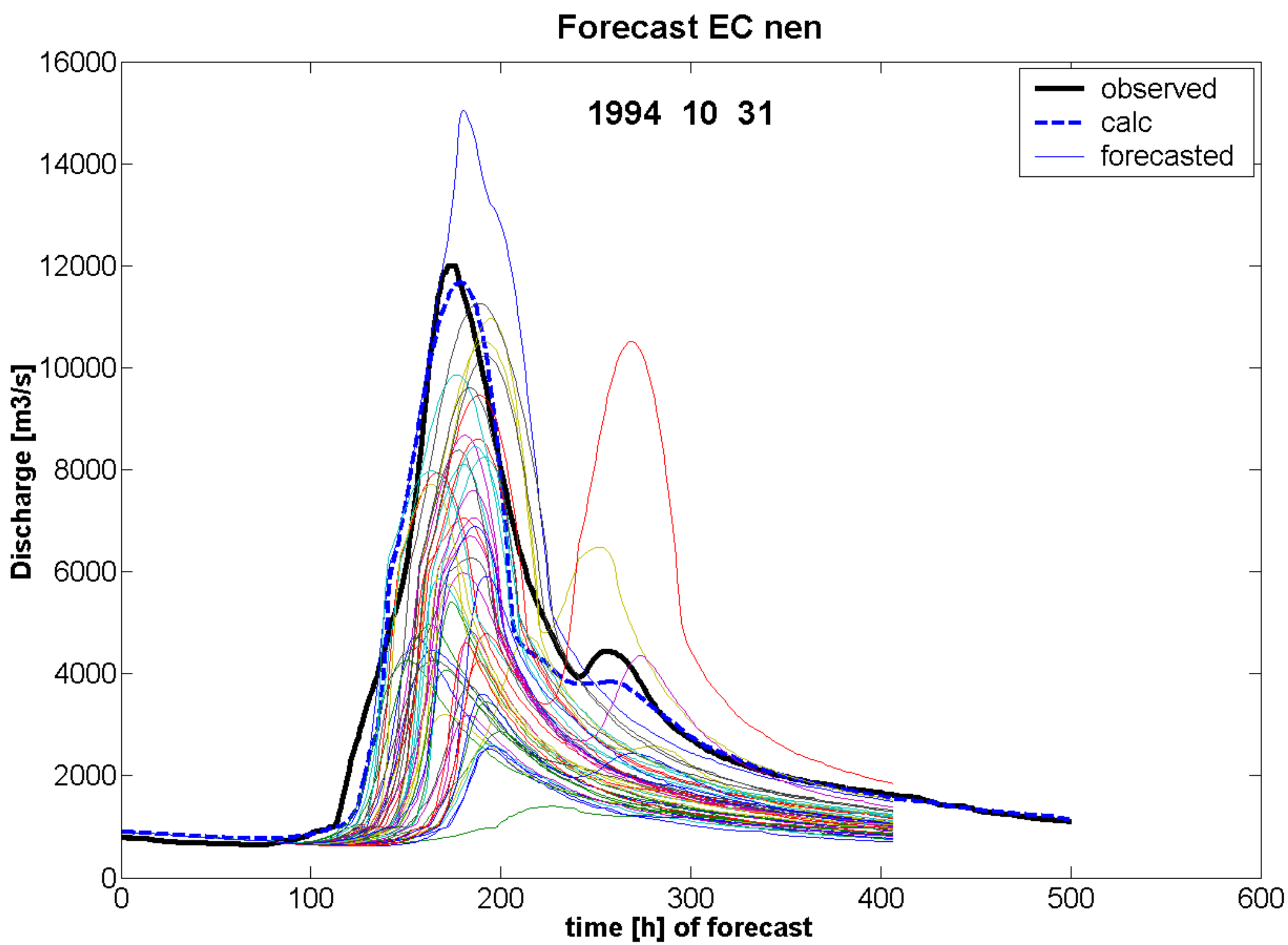

Forecast EC nen
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Discharge [m3/s]
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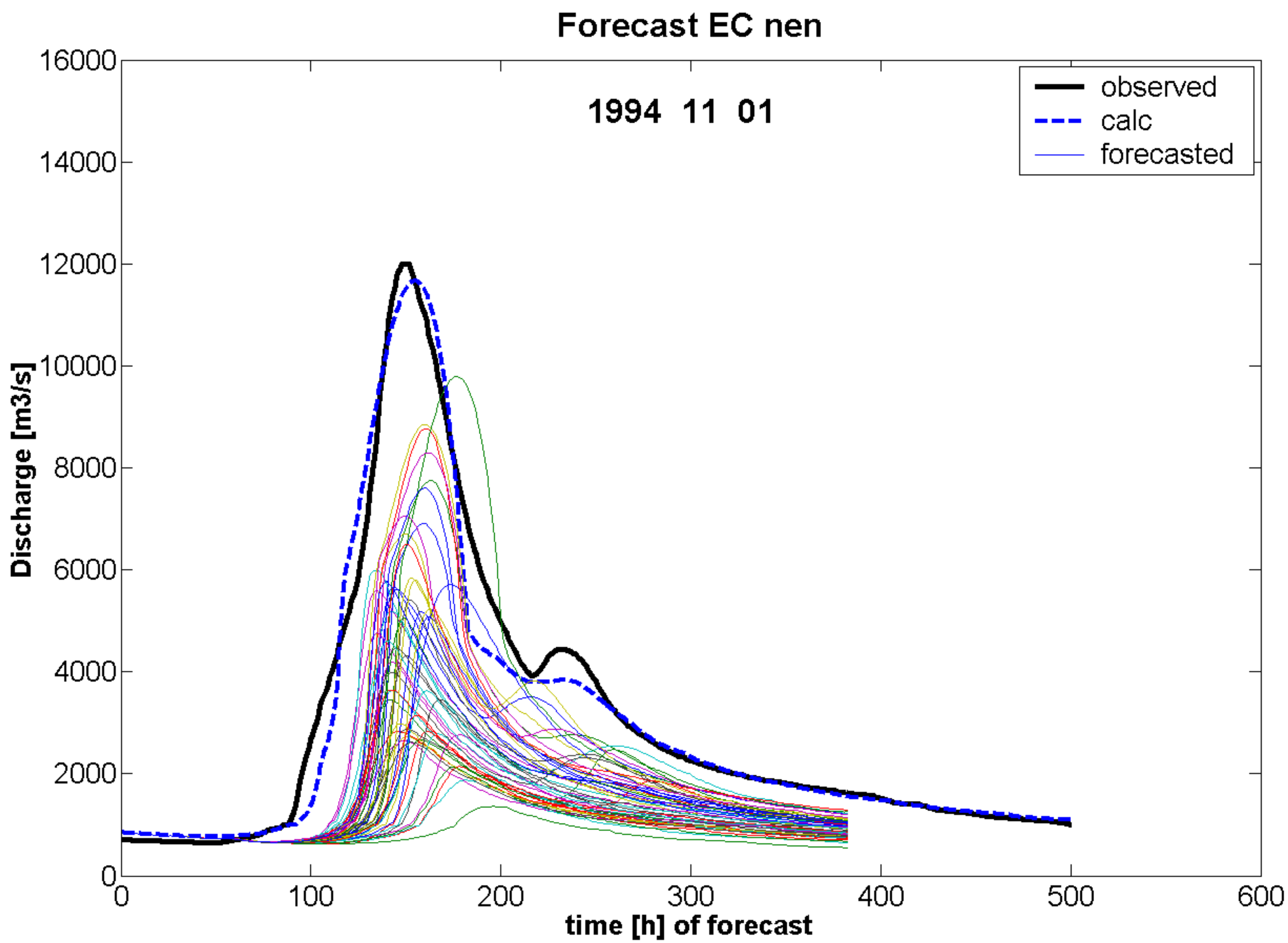

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Discharge [m3/s]
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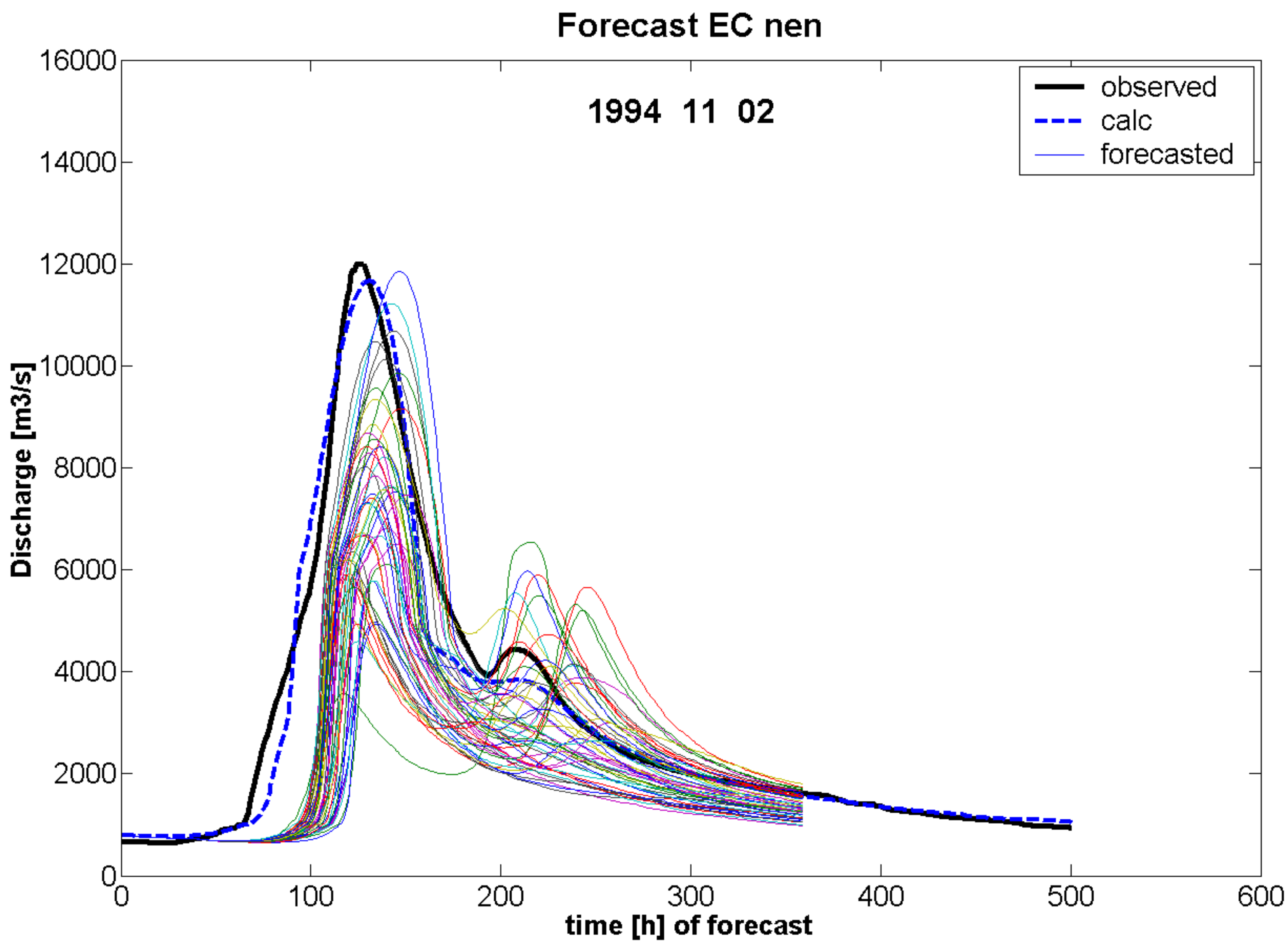

Forecast EC nen
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observed
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Discharge [m3/s]
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time [h] of forecast
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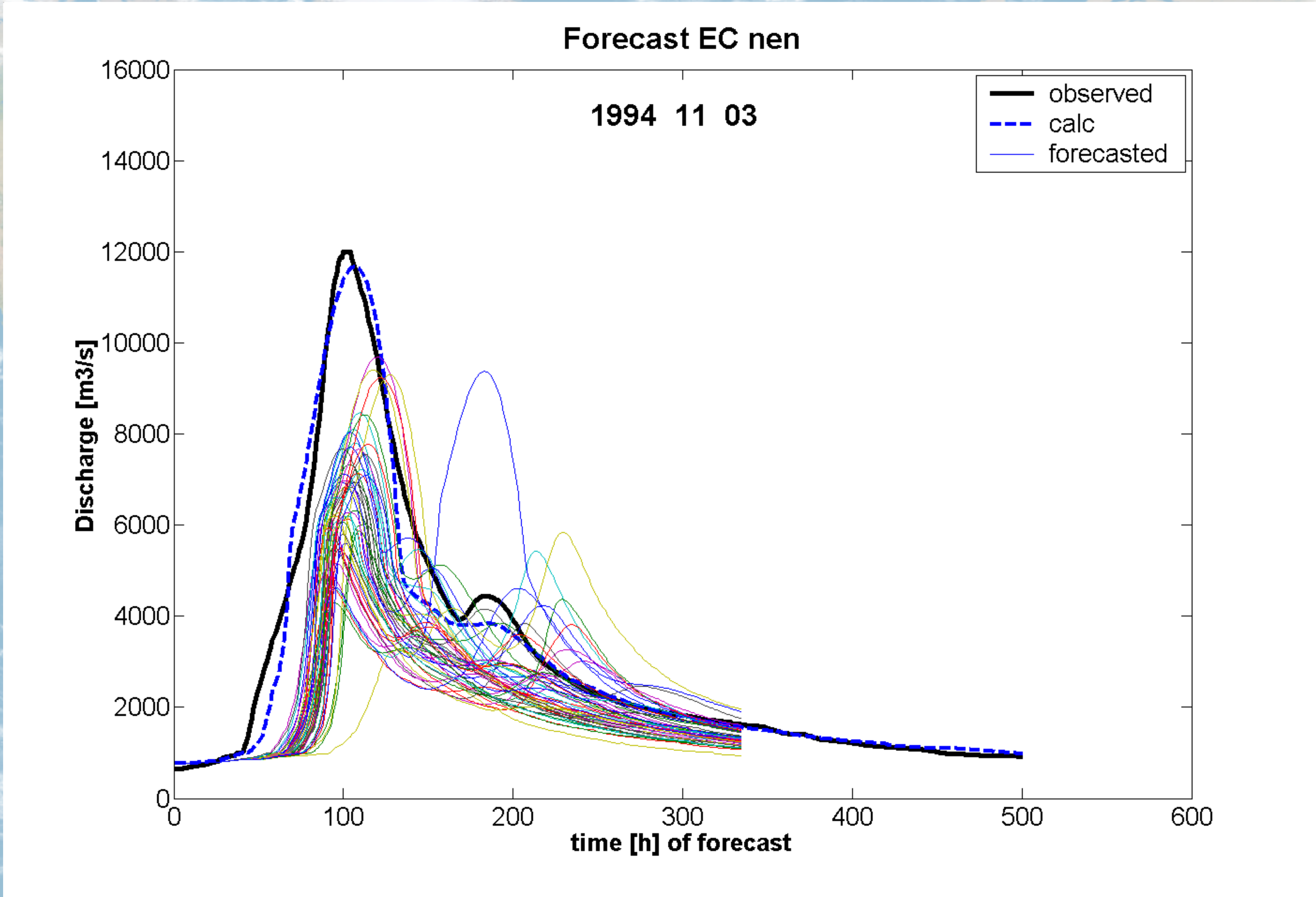

Forecast EC nen
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observed
calc
forecasted
Discharge [m3/s]
time [h] of forecast
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14000
16000
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100
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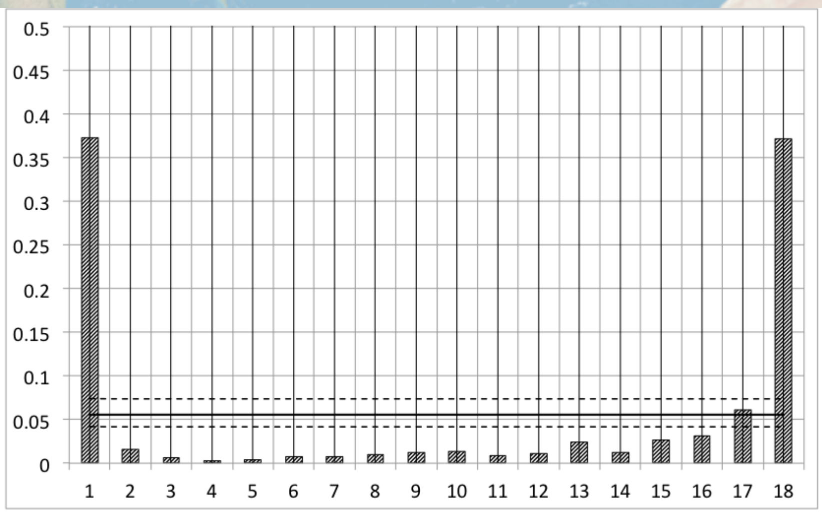
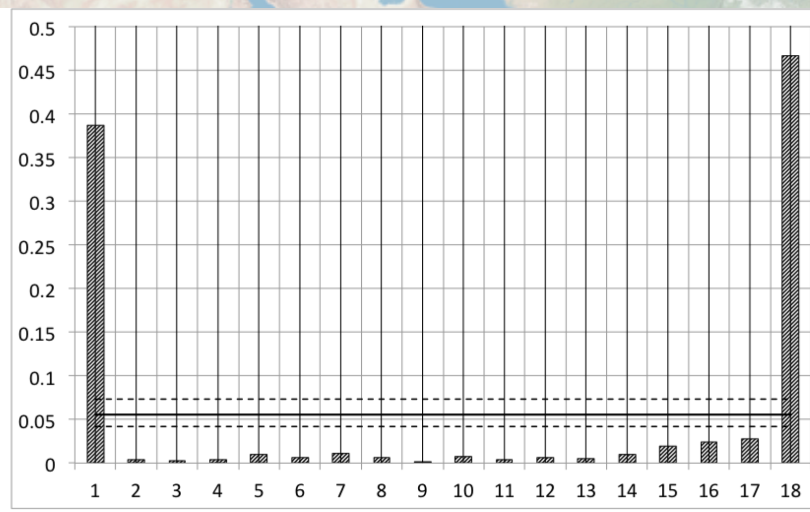
500
600

The previous figure, although indicative, can be criticized on the grounds that it is not a fully objective demonstration that ensembles do not represent Predictive Probability Densities. Therefore we found a more objective approach to confirm that ensembles provide biased and lesser-dispersed estimates (Hamill and Colucci, 1997; Eckel and Walters, 1998; Schwanenberg et al, 2015).

By assuming that the ranked ensemble members represent the quantiles of the predictive distribution, then one expects that observations will fall evenly distributed betwee quantiles.





This is not so; both in calibration (left) and in validation (right) most of observations fall out of the ensemble range and far from the Wilson (1927) bounds

SO HOW CAN WE PROPERLY ACCOUNT FOR THE INFORMATION CONTAINED IN THE ENSEMBLES SPREAD?

There are two possibilities..

- The first one does not requires ordering of the ensemble members

- The second one requires ordering of the ensemble members

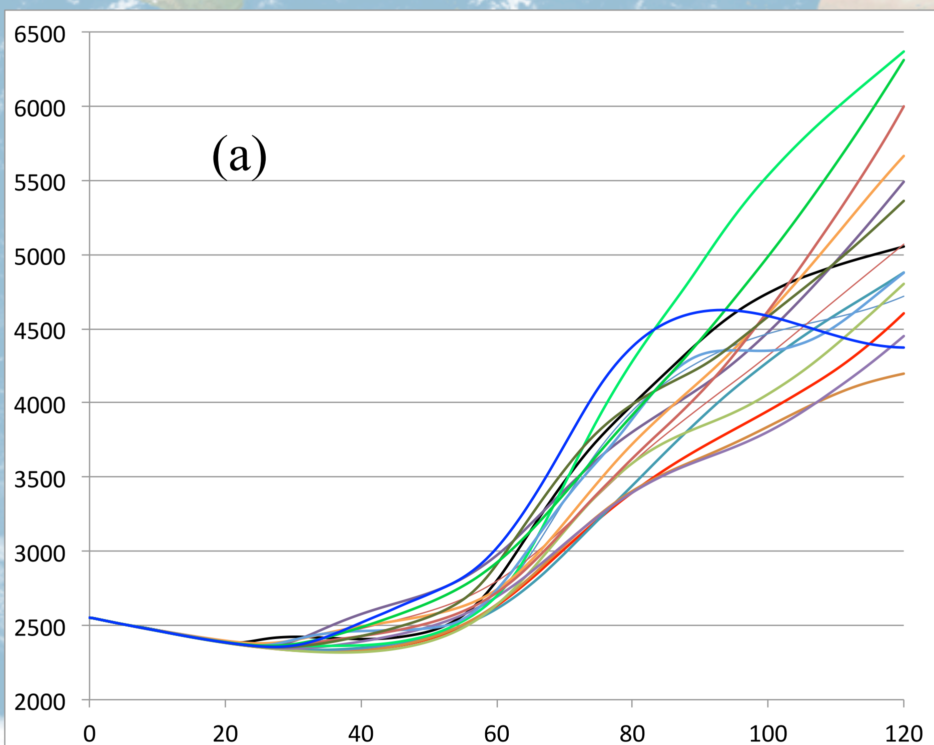
FIRST POSSIBILITY

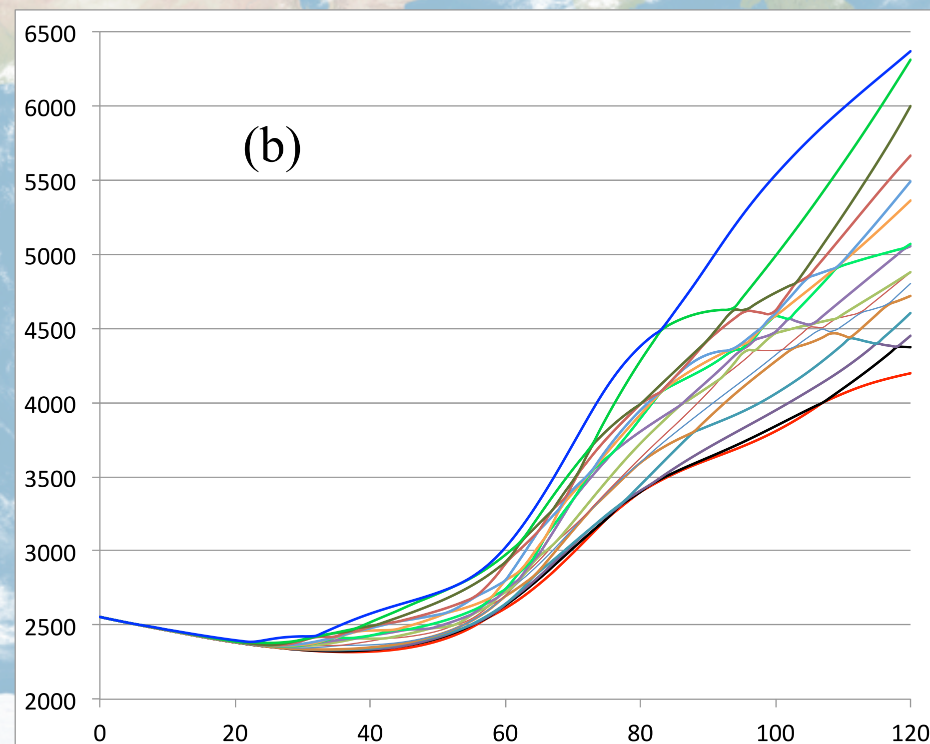
You might think of using the post-processors in a multi-variable context: virtually every member of the set would be a different "model".

However, this requires a link between the same member at step t and the same member at step $t + \Delta t$, which actually does not exist.

However, it is possible to create a kind of temporal connection by shuffling members of an ensemble.

By advocating exchangeability, we can associate members with quantiles and create a fictitious succession that allows members to be treated as different "models" each of which representing a different quantile.





SECOND POSSIBILITY

Alternatively, the problem can be addressed using MCP using the ensemble mean, after transforming into the Gaussian space the observations and the predictions to derive predictive distribution, that is, the distribution of future conditioned values to the models.

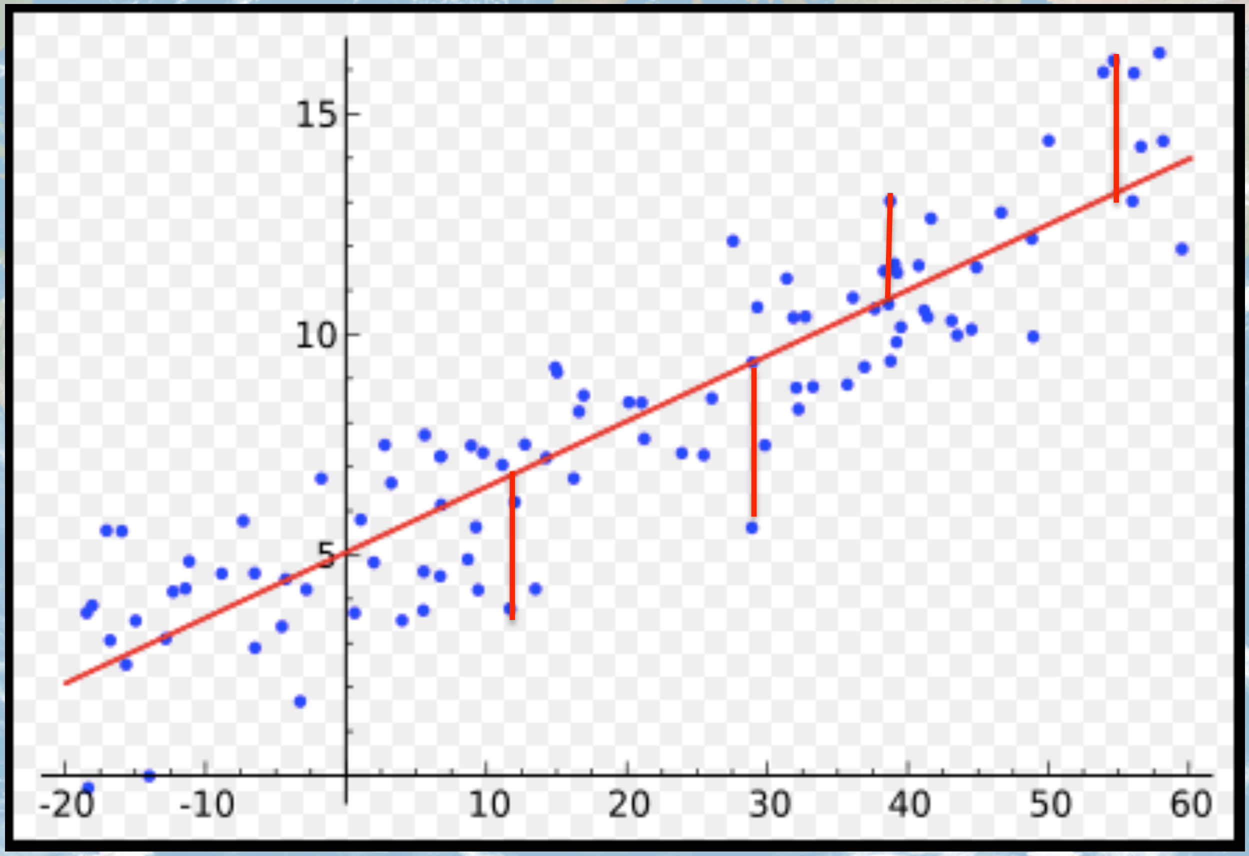
$$\begin{cases} \mu_{\eta_t|\hat{\boldsymbol{\eta}}_{m,t}} = \boldsymbol{\Sigma}_{\eta\hat{\boldsymbol{\eta}}_m} \boldsymbol{\Sigma}_{\hat{\boldsymbol{\eta}}_m\hat{\boldsymbol{\eta}}_m}^{-1} \hat{\boldsymbol{\eta}}_{m,t} \\ \sigma_{\eta_t|\hat{\boldsymbol{\eta}}_{m,t}}^2 = 1 - \boldsymbol{\Sigma}_{\eta\hat{\boldsymbol{\eta}}_m} \boldsymbol{\Sigma}_{\hat{\boldsymbol{\eta}}_m\hat{\boldsymbol{\eta}}_m}^{-1} \boldsymbol{\Sigma}_{\eta\hat{\boldsymbol{\eta}}_m}^T \end{cases}$$

Please note that this in nothing else than a Univariate Linear Regression, which does not take into acocunt the ensemble spread.

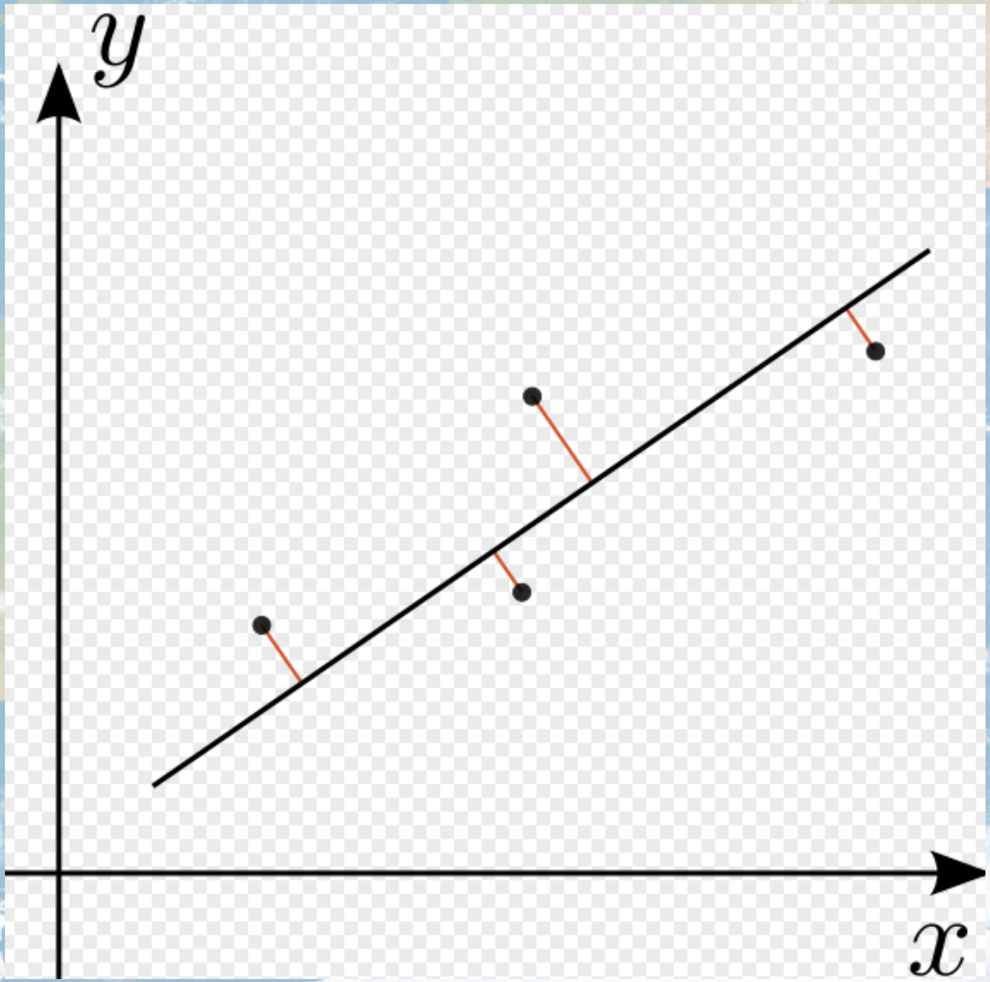
Therefore, in the presence of predicion errors described by an ensemble of m members, the previous equations can be modified, as in the Deming regression, with the introduction of the variance ensemble mean, used asthe regressor and its variance can be estimated as the variance of the ensemble divided by the ensemble numerosity

$$\begin{cases} \mu_{\eta_t|\hat{\boldsymbol{\eta}}_{m,t}} = \boldsymbol{\Sigma}_{\eta\hat{\boldsymbol{\eta}}_m} \left(\boldsymbol{\Sigma}_{\hat{\boldsymbol{\eta}}_m\hat{\boldsymbol{\eta}}_m} + \frac{1}{m}\mathbf{R}_t\right)^{-1} \hat{\boldsymbol{\eta}}_{m,t} \\ \sigma^2_{\eta_t|\hat{\boldsymbol{\eta}}_{m,t}} = 1 - \boldsymbol{\Sigma}_{\eta\hat{\boldsymbol{\eta}}_m} \left(\boldsymbol{\Sigma}_{\hat{\boldsymbol{\eta}}_m\hat{\boldsymbol{\eta}}_m} + \frac{1}{m}\mathbf{R}_t\right)^{-1} \boldsymbol{\Sigma}^T_{\eta\hat{\boldsymbol{\eta}}_m} \end{cases}$$

Linear Regression



Deming Regression



EXAMPLE: THE PO RIVER

01/05/2000 30/06/2004 20/01/2009

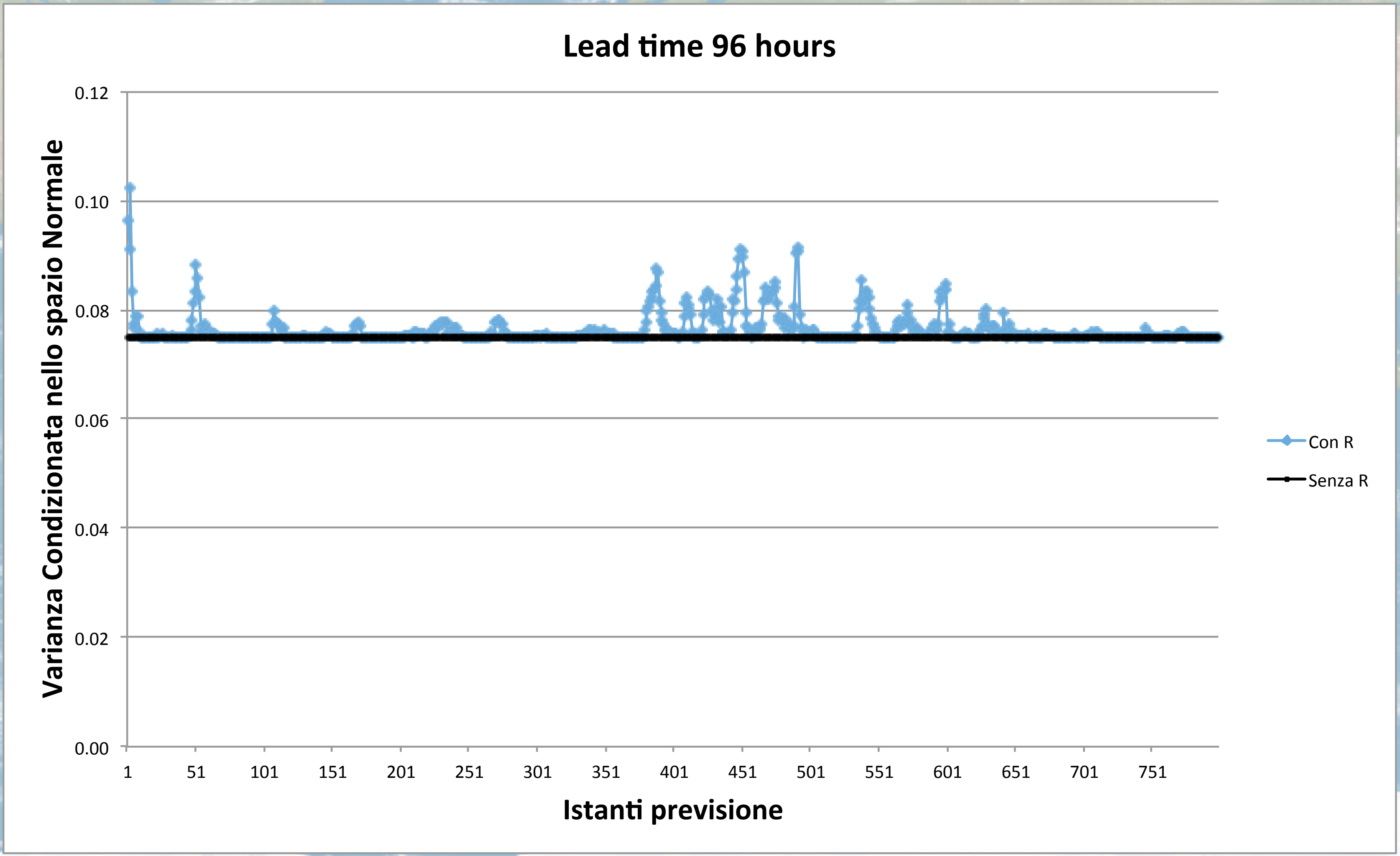
MCP: VALIDATION CALIBRATION

Dati forniti dalla Protezione Civile Regione Emilia Romagna

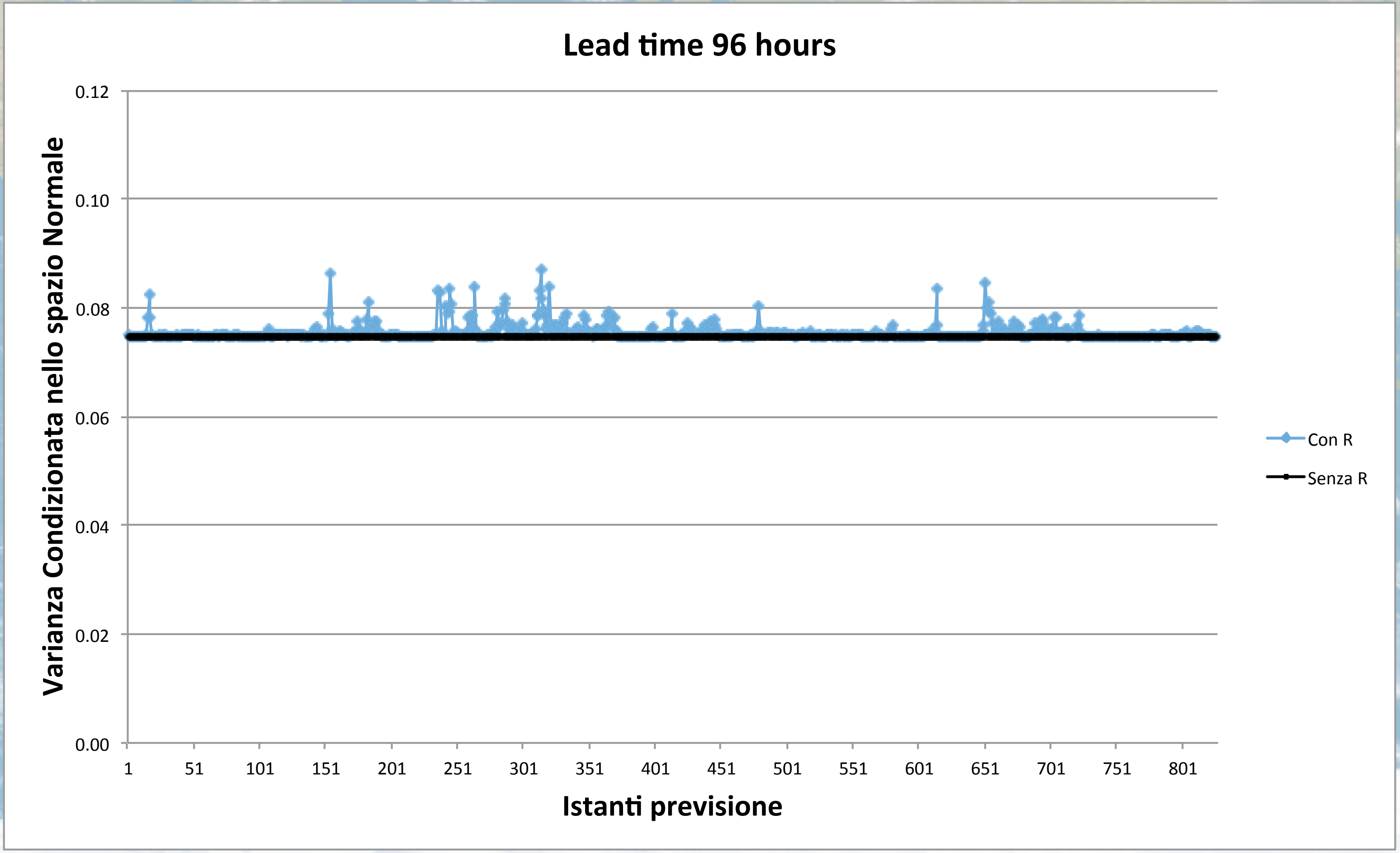
Valori orari tiranti idrici

Previsioni orarie delle portate fino a 92 h di anticipo

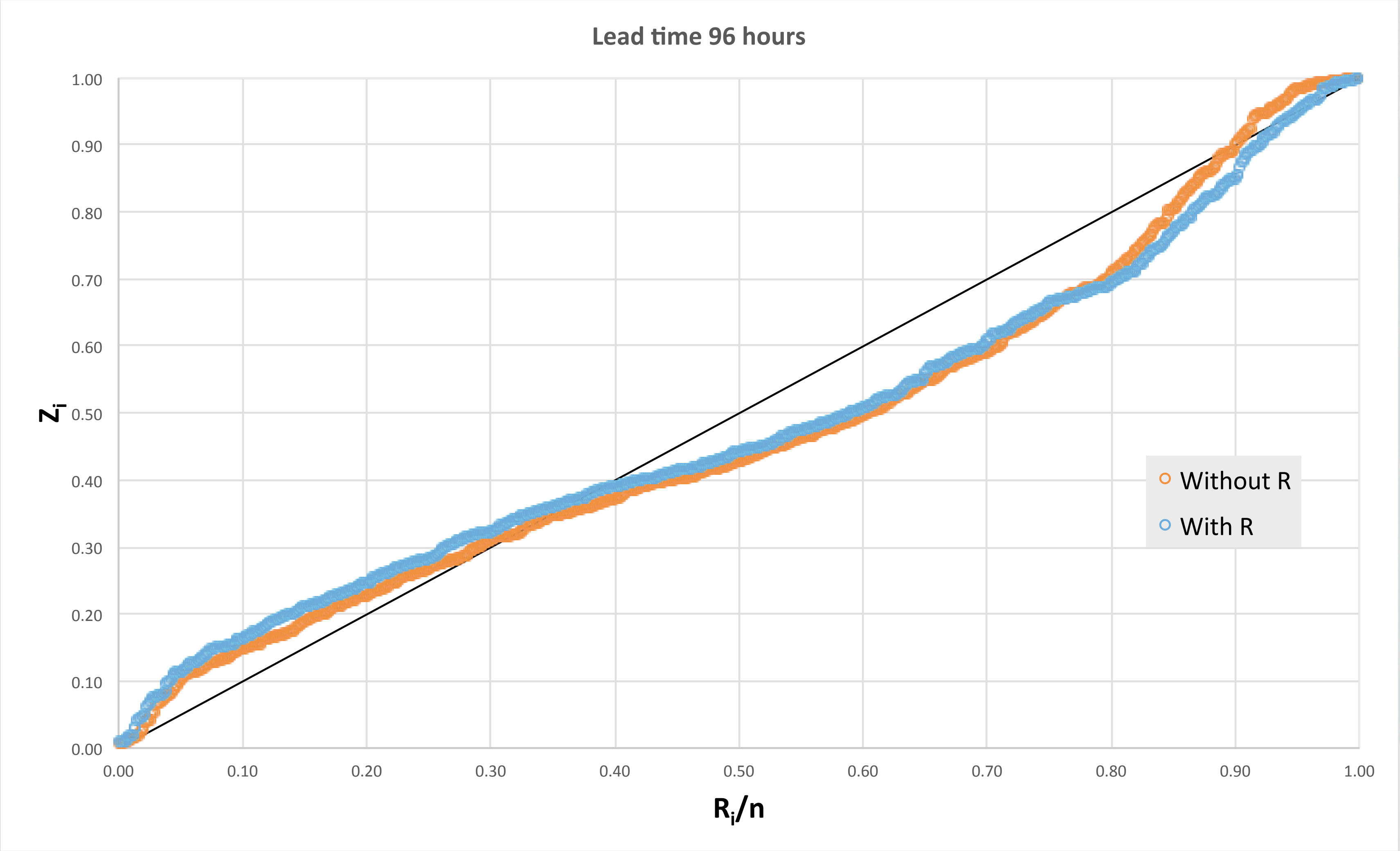
Predictive Variance in the Gaussian space Calibration



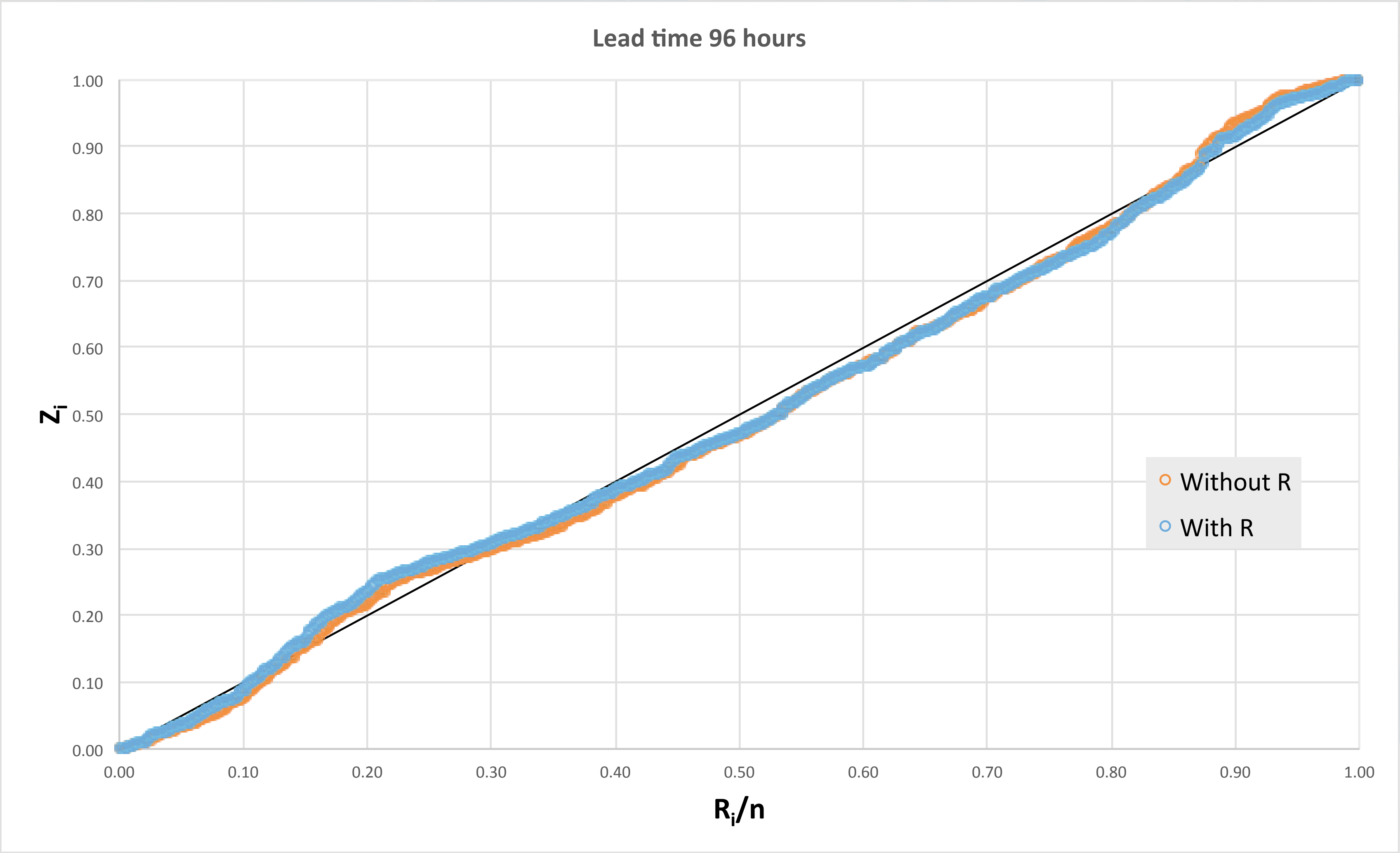
Predictive Variance in the Gaussian space
Validation



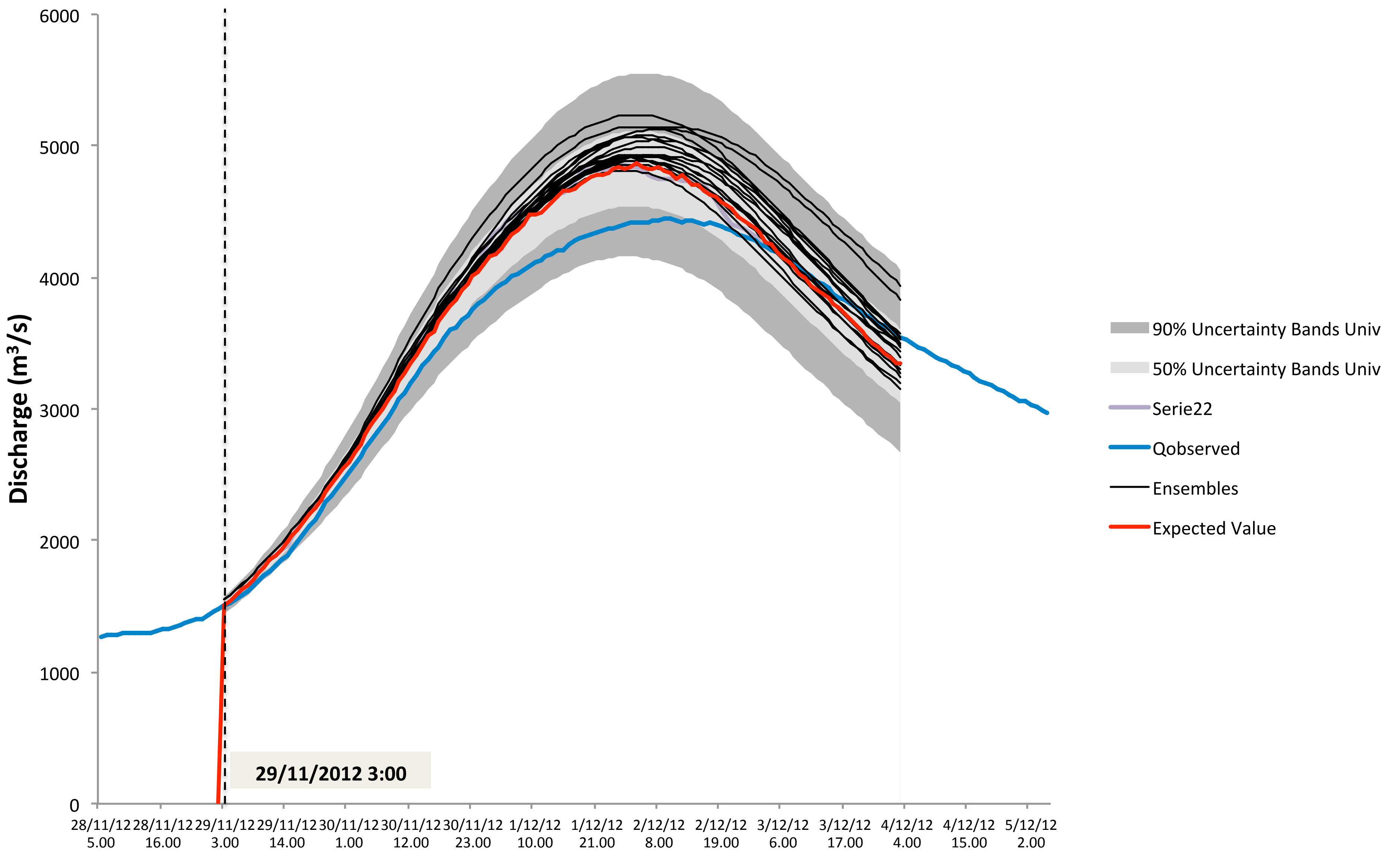
Relaiability Diagram
Calibration



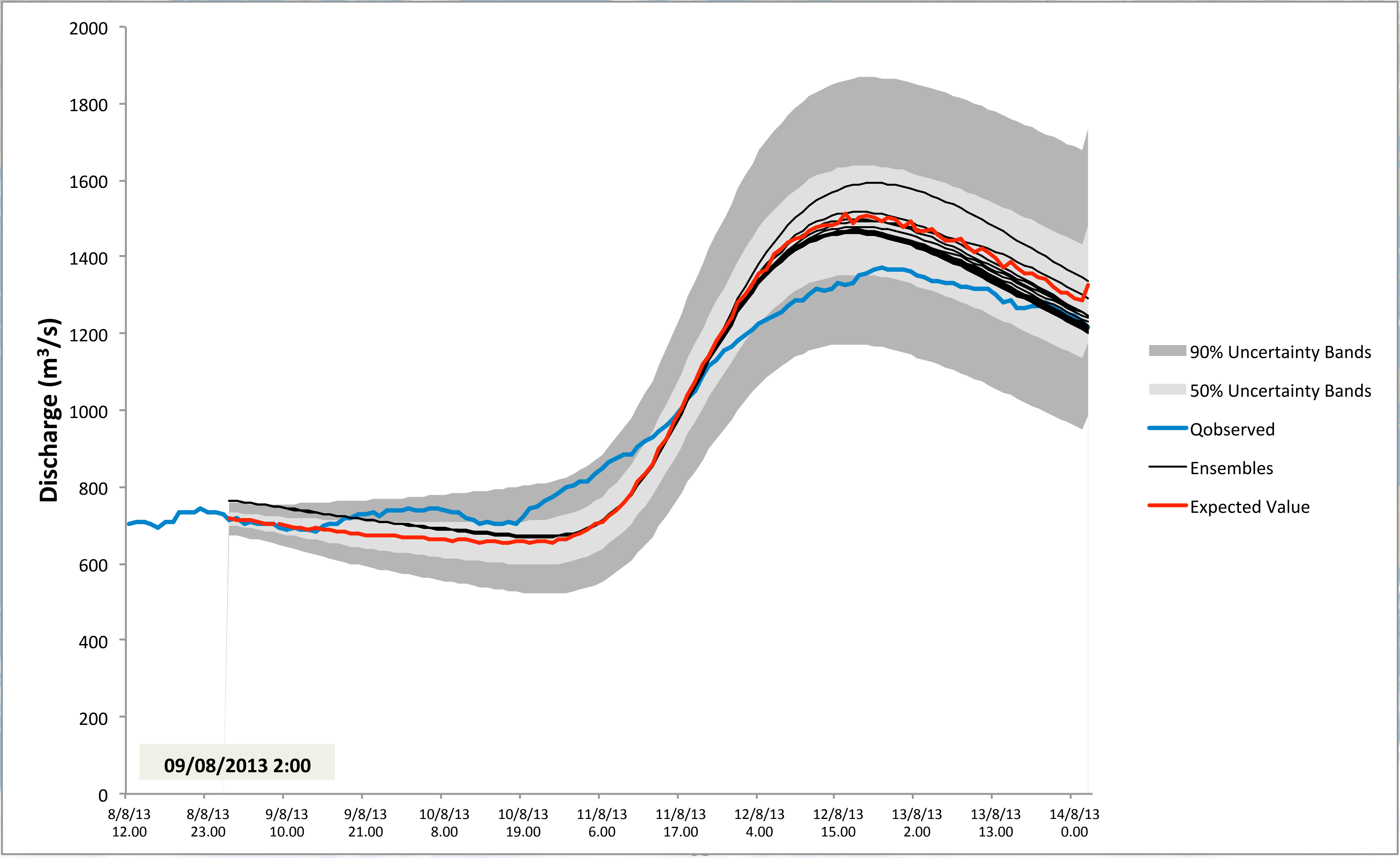
Relaiability Diagram Validation



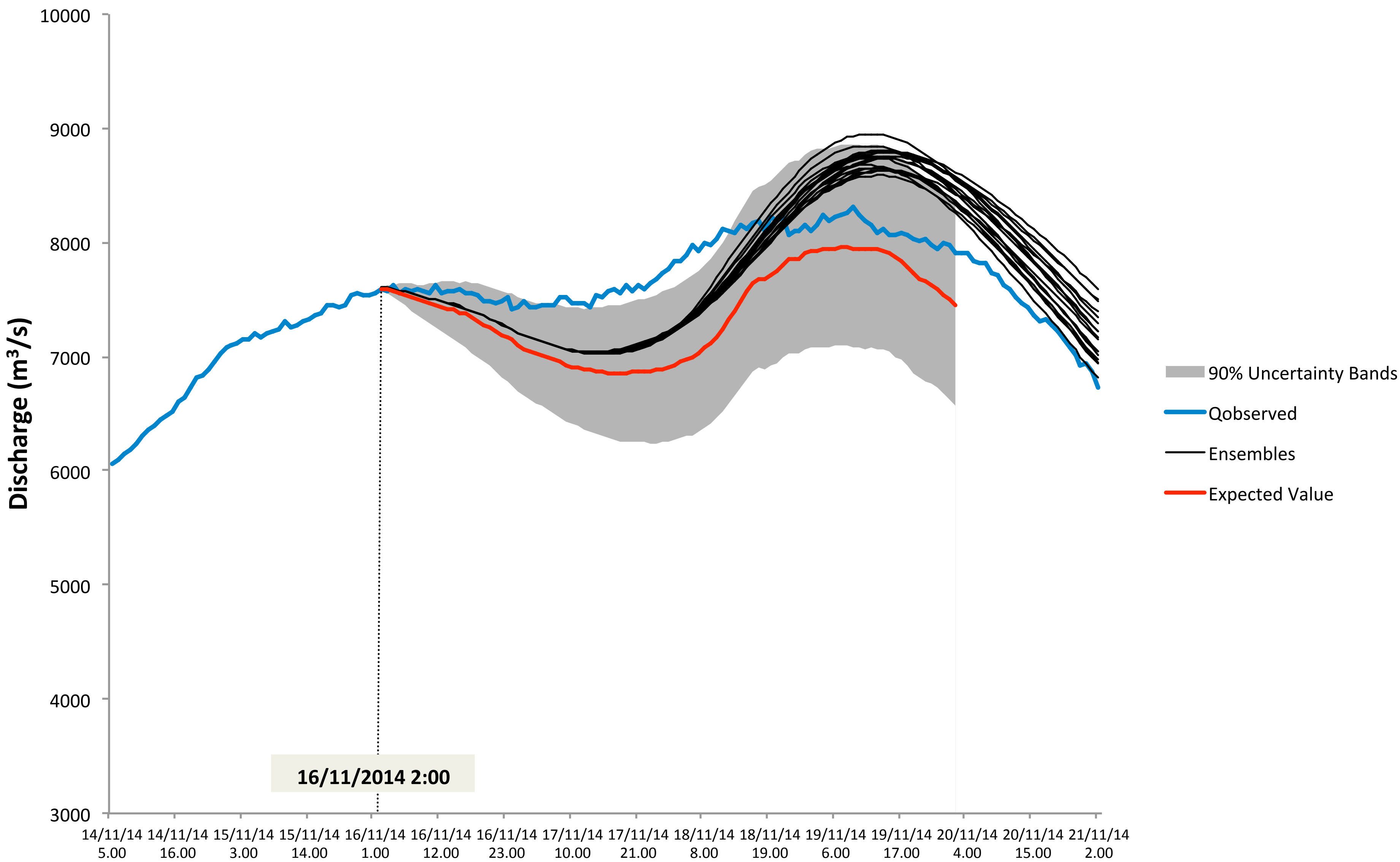
Forecasting example
Calibration



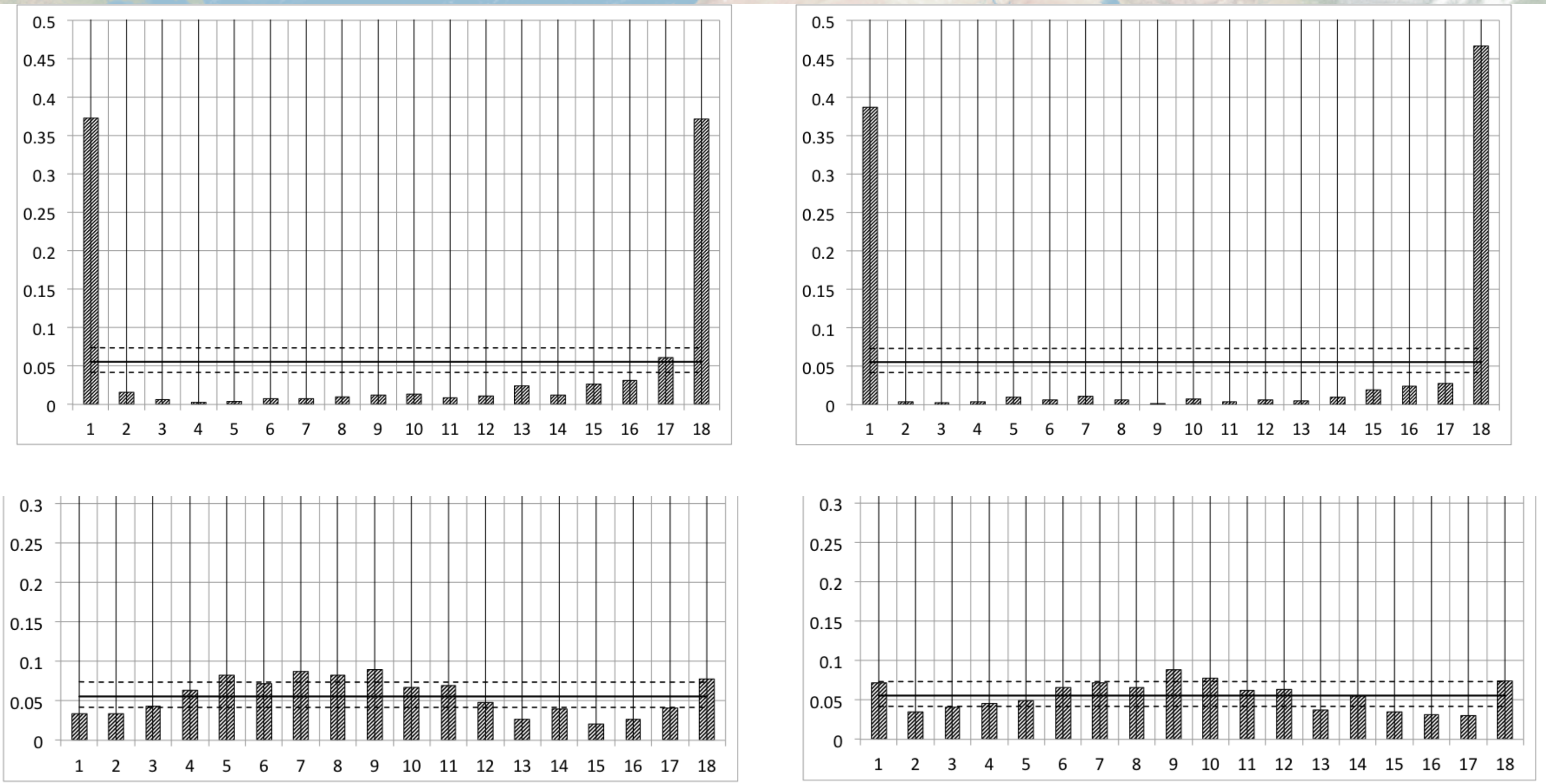
Forecasting example
Calibration



Forecasting example
Validation



Comparison of the resulting predictive density with the one directly produced by the ensemble clearly shows that the new density, although not Perfect, is definitely closer to the Wislon bounds.



Where do we go from here?

There are several recent interesting predictive approaches such as for instance the multi-temporal approach which allow answering important questions such as:

- Which is the probability that flooding will occur in the next 24 hours
- Which is the most likely time of occurrence of the event?

In my opinion the future goes into these approaches rather than using the ensembles.

Thank you
for your attention

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