Role of the interaction core in the excitation spectrum of 1D gases and liquids

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Outline

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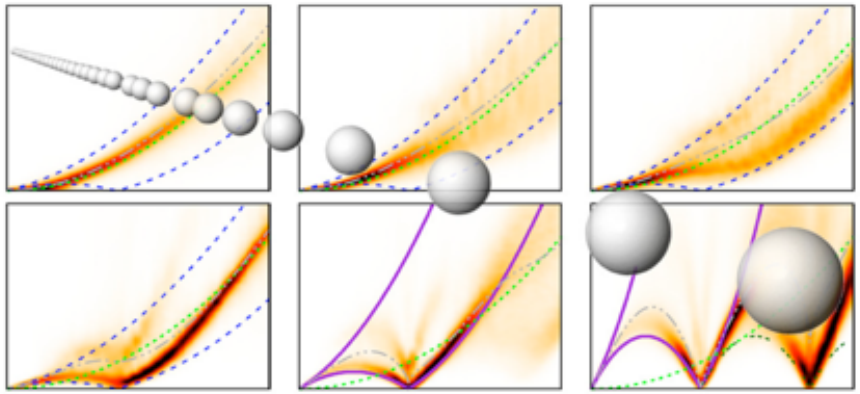
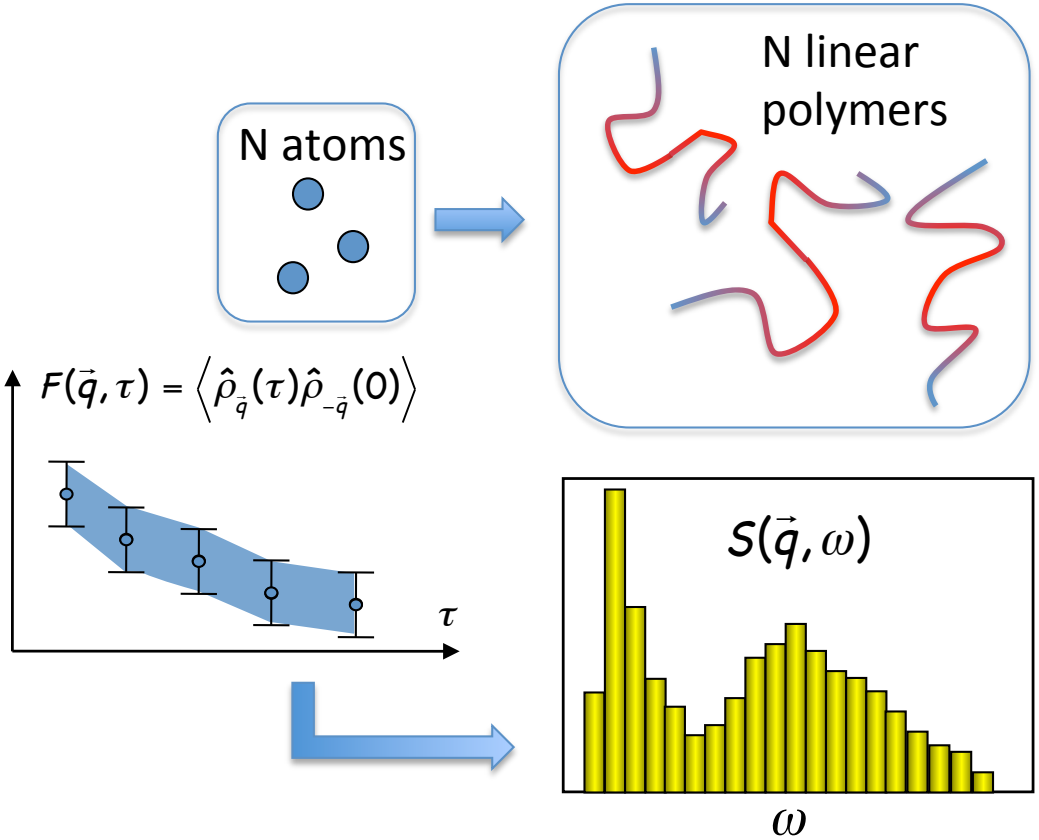
- Target systems:
  - 1D quantum systems in the continuum with different interaction cores: hard-rods, 1D ^{4}He, soft-rods
- How?
  - Mainly Path Integral projector methods (T=0) ⇒ imaginary time density-density correlation functions
  - Dynamic structure factor via "statistical" inversion of Laplace transform
- Results:
  - Mainly dynamic properties beyond Tomonaga Luttinger Liquid (TLL) theory





Collaborators

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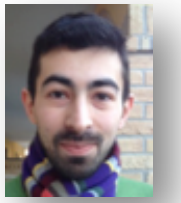
M. Motta now@ College William and Mary (USA)



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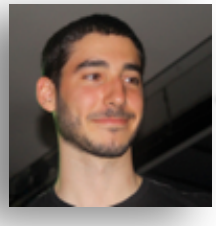

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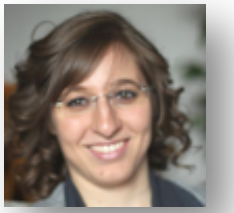

E. Vitali now@ College William and Mary (USA)

Dimensions are relevant!

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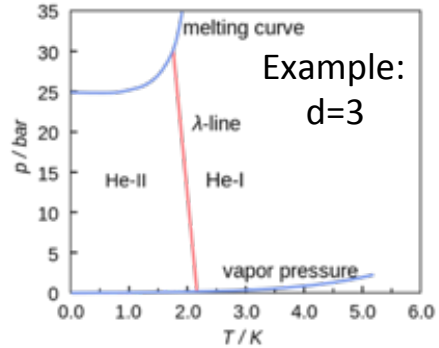
Critical behavior of d-dimensional superfluid

$$F_{GL} = \int d^d\vec{r}\left[\frac{1}{2m^*}\left|-i\hbar\vec{\nabla}\Psi(\vec{r})\right|^2 + a_2(T)\left|\Psi(\vec{r})\right|^2 + a_4\left|\Psi(\vec{r})\right|^4\right]$$

Complex order parameter



The effect of thermal and quantum fluctuations increase as we reduce the number of spatial dimensions (Mermin-Wagner theorem: no continuous symmetry breaking for d ≤ 2)

d=3
3D XY universality class
phase transition with
long range order at T > 0

d=2
Kosterlitz-Thouless (2D XY)
vortex unbinding &
no long range order at T > 0
long range order at T = 0

d=1

fluctuations destroy long range order, even at T = 0

1D is special

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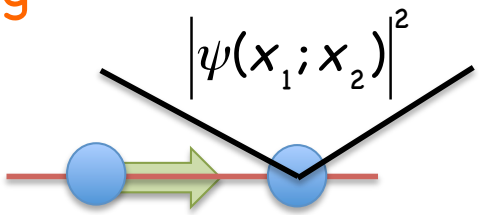
- Well-defined low energy collective modes for many 1D Fermi & Bose systems, in fact, ...

- Interacting 1D Bose systems with (short-range) repulsive interactions the reduced dimensionality of the space naturally induces presence of collective modes

- 1D Fermi statistics imposes a nodal structure which mimicks contact hard core interaction ⇒ Bose-Fermi mapping

  [Girardeau, J. Mat. Phys. (1960)]

  Same static and dynamic properties (except for momentum distribution)



⇒ Tomonaga Luttinger liquid (TLL) theory

[Tomonaga, Prog. Theor. Phys. (1950); Luttinger, J. Mat. Phys. (1963)]

universal field-theoretical/hydrodynamic description of interacting 1D Fermions or Bosons characterized by low-energy phonon modes

1D universal behaviour

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$$\hat{H}_{1D} = -\frac{\hbar^2}{2m}\int dx\, \hat{\Psi}^+(x)\,\partial^2\hat{\Psi}(x) + \frac{1}{2}\int dx \int dx'\, \hat{\Psi}^+(x)\hat{\Psi}^+(x')V(|\,x - x'\,|)\hat{\Psi}(x')\hat{\Psi}(x)$$

- Rewrite field operators in term of new fields: density and phase fluctuations [Haldane, Phys. Rev. Lett. (1980)]

$$\hat{\Psi}(x) = \sqrt{\rho + \frac{1}{\pi}\partial\hat{\phi}(x)}\; e^{i\hat{\theta}(x)}$$

⇒ effective harmonic Hamiltonian

Sound velocity:

$$\hat{H}_{TLL} = \frac{\hbar v_s}{2\pi}\int dx \left[K_L\left(\partial\theta(x)\right)^2 + \frac{1}{K_L}\left(\partial\phi(x)\right)^2\right]$$

Galileian invariant systems: one parameter

$$K_L = \frac{\hbar\pi\rho}{m v_s}$$

(ρ=N/L = density)

- H_{TLL} is exactly solvable; dynamic structure factor:

$$S(q,\omega) \underset{\substack{\omega\to 0\\ q\to 0}}{=} \frac{\hbar q}{2mv_s}\,\delta(\omega - v_s q)$$

The low-energy …

- … excitations are collective density fluctuations
- … physics is universal (independent of interaction details and statistics)
- … physics (Galileian invariant systems) is completely specified by K_L

Density fluctuation spectrum

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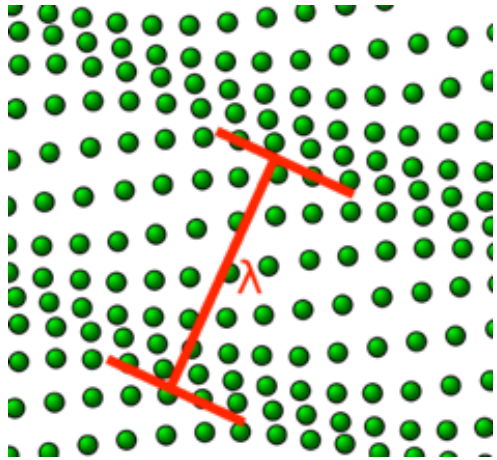
- Dynamic structure factor:

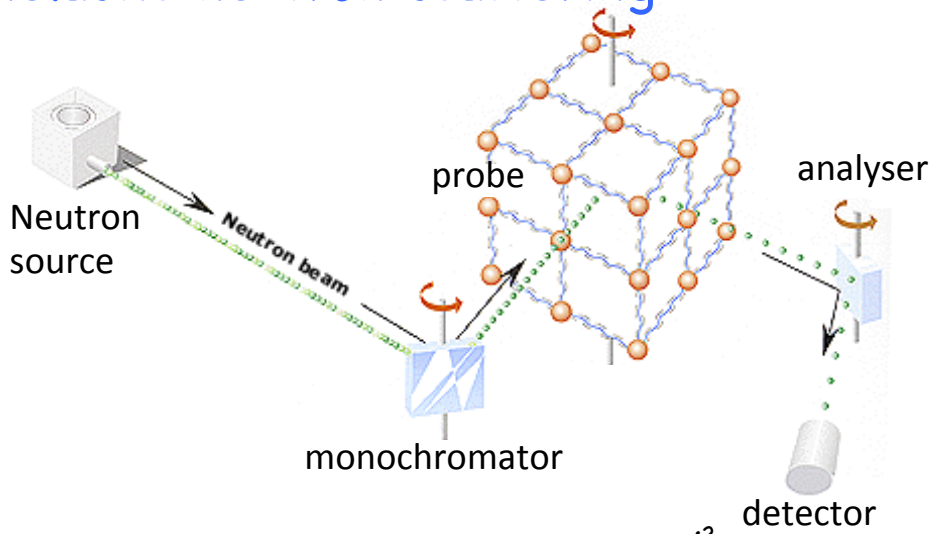
$$S(\vec{q},\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\omega t} \left\langle \hat{\rho}_{\vec{q}}(t)\hat{\rho}_{-\vec{q}}(0) \right\rangle$$

density fluctuation: $\rho_{\vec{q}}(t) = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-i\vec{q}\cdot\vec{r}_j(t)}$

Density fluctuation wave-length: $\lambda = 2\pi/|\vec{q}|$

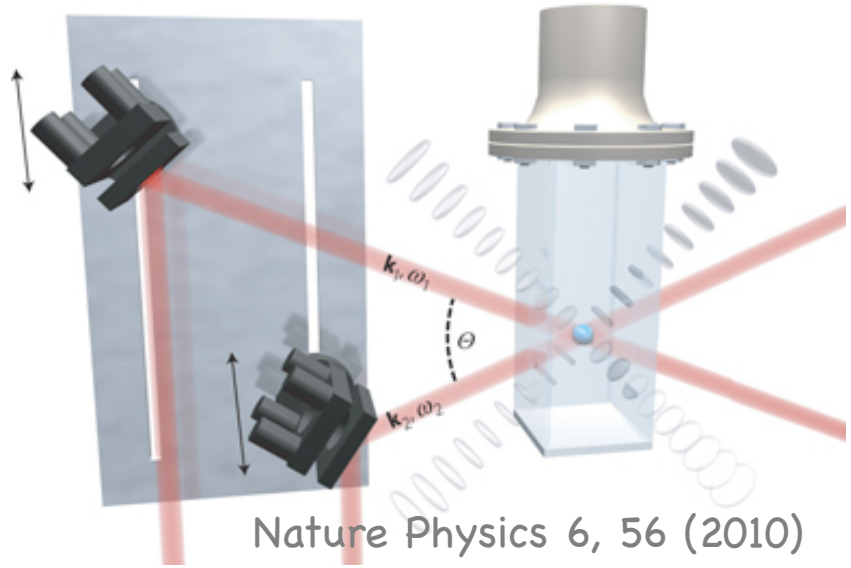


Inelastic neutron scattering



Partial differential scattering cross-section $\frac{d^2\sigma}{dEd\Omega} \propto S(\vec{q},\omega)$

Bragg spectroscopy



Nature Physics 6, 56 (2010)

Dispersion relation

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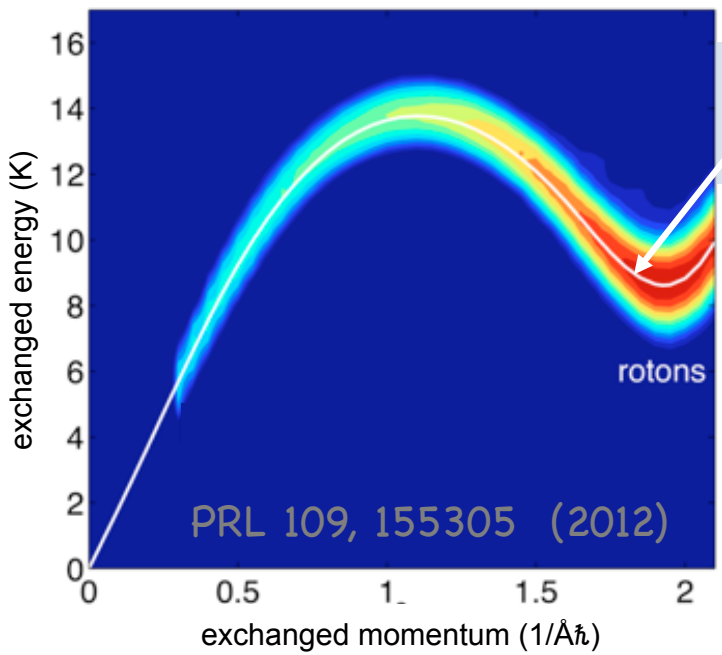
- Dynamic structure factor: One can read the dispersion relation of coherent modes

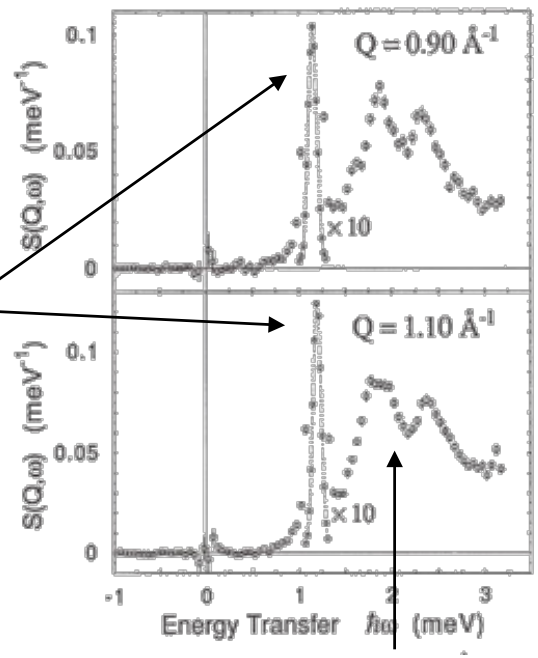
$$S(\vec{q},\omega)\Big|_{T=0} = \sum_{n=0}^{\infty} \left|\langle \psi_n | \hat{\rho}_{\vec{q}} | \psi_0 \rangle\right|^2 \delta[\omega - \underbrace{(E_n - E_0)}_{\text{excitations energies}} / \hbar]$$

peaks position → $\varepsilon_{\vec{q}}$ Energy-momentum dispersion curve

Example: 3D superfluid ^{4}He



single excitation peak



multi-excitations component

J. Phys.: Cond. Matt. 6, 821 (1994)

K_L matters!

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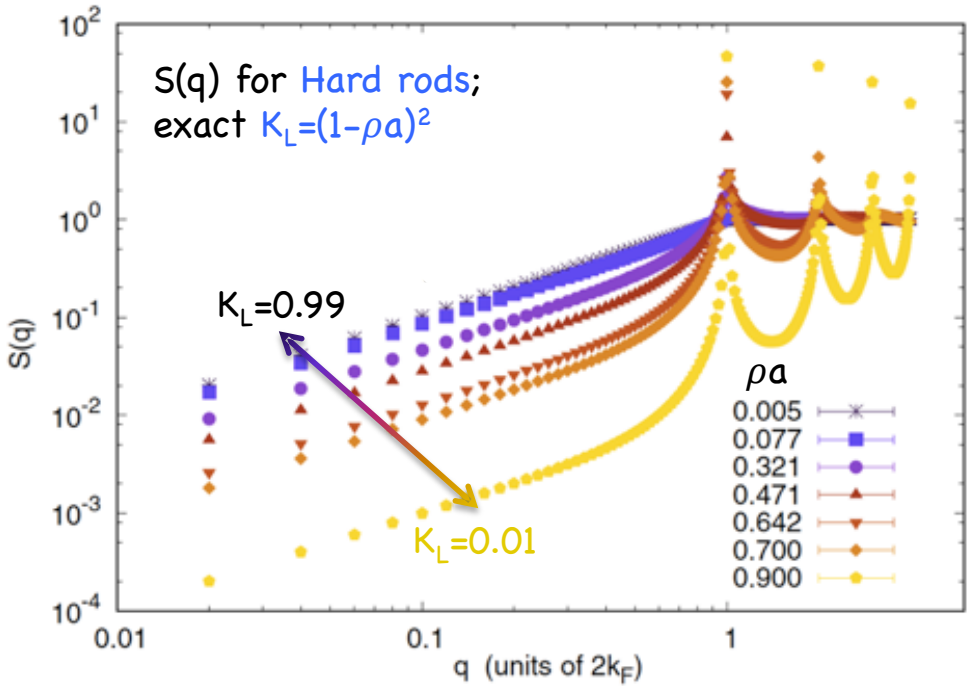
H_{TLL} fixes also the long range behaviour of spatial correlation functions in terms of K_L only [Haldane, Phys. Rev. Lett. (1981)]

- pair distribution function

$$g(|x-x'|) \underset{|x-x'|>>\frac{L}{N}}{\cong} 1 - \frac{2K_L}{\left(2\pi\rho\,|x-x'|\right)^2} + \sum_{l=1}^{\infty} A_l \frac{\cos\left(2\pi\rho\, l\,|x-x'|\right)}{\left(2\pi\rho\,|x-x'|\right)^{2l^2K_L}}$$

for K_L<1/2 slow power law decay of density oscillations ⇒ Bragg peaks in the static structure factor S(q)
⇒ quasi-solid (quasi long range order)

$$g(|x-x'|) - 1 \underset{|x-x'|\to\infty}{\propto} \frac{\cos(2\pi\rho\,|x-x'|)}{|x-x'|^{2K_L}}$$



The TLL parameter: K_L

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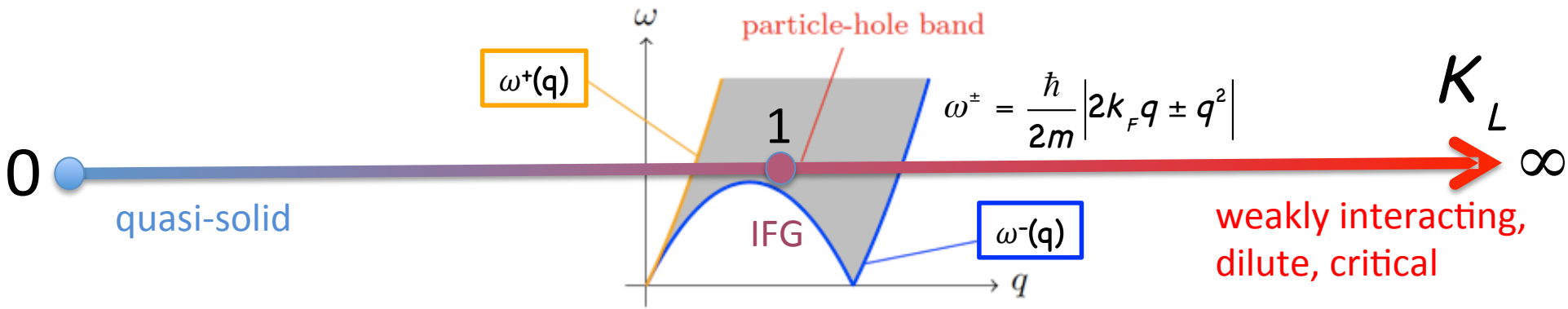
$$K_L = \frac{\hbar\pi}{m}\frac{\rho}{v_s}$$

$\Rightarrow$ K_L depends on the ratio ρ/v_s

Note also that: $K_L \propto \sqrt{\kappa}$ $\Rightarrow$ is a dimensionless compressibility

- Hard-core interactions with finite range a: when $\rho \approx 1/a$ by increasing n one should expect that $\rho/v_s \to 0$ $\Rightarrow$ $K_L \to 0$
- Interactions inducing a self-bound state $\Rightarrow$ sound velocity $v_s \to 0$ at the spinodal density (liquid/gas phase separation) $\Rightarrow$ $K_L \to +\infty$
- 1D ideal Fermi gas (IFG): $K_L^{IFG} = \frac{\hbar\pi\rho}{mv_F} = \frac{\hbar k_F}{mv_F} = 1$



What about higher energies?

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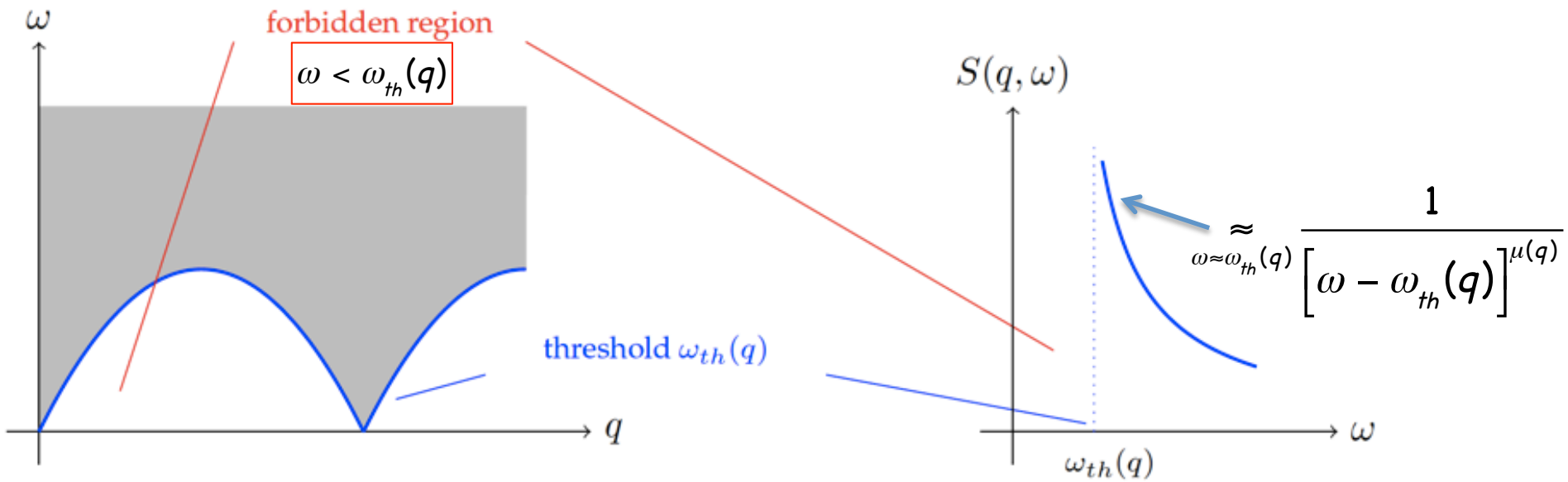
Non-linear Luttinger (NLL) theory [Imambekov et al. Rev. Mod. Phys. (2012)]

- Assumes a low energy threshold for $S(q,\omega)$: $\omega_{th}(q)$
- $\omega_{th}(q)$ is a renormalization of $\omega^{-}(q)$: the curvature determines an effective mass m^*

⇒ Exactly solvable Hamiltonian: $S(q,\omega)$ has a power-law behaviour near the edge with exponent $\mu(q)$



In general the physics is non-universal, numerical calculations or QMC are needed

Lieb Liniger model

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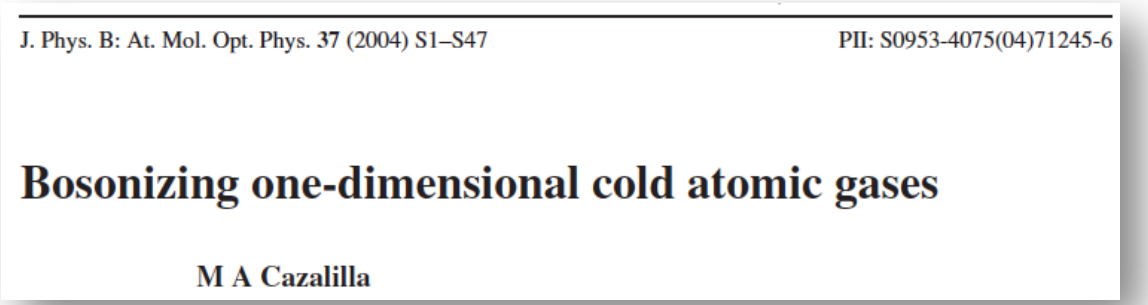
- Integrable 1D model [Lieb, Liniger, Phys.Rev. (1963)]

- Hamiltonian: contact interaction $$H = -\frac{\hbar^2}{2m}\left[\sum_{i=1}^{N}\partial_i^2 + c\sum_{i\neq j}\delta(x_i - x_j)\right]$$

dimensionless parameter $\gamma = c/\rho$ characterizes the interaction strength:

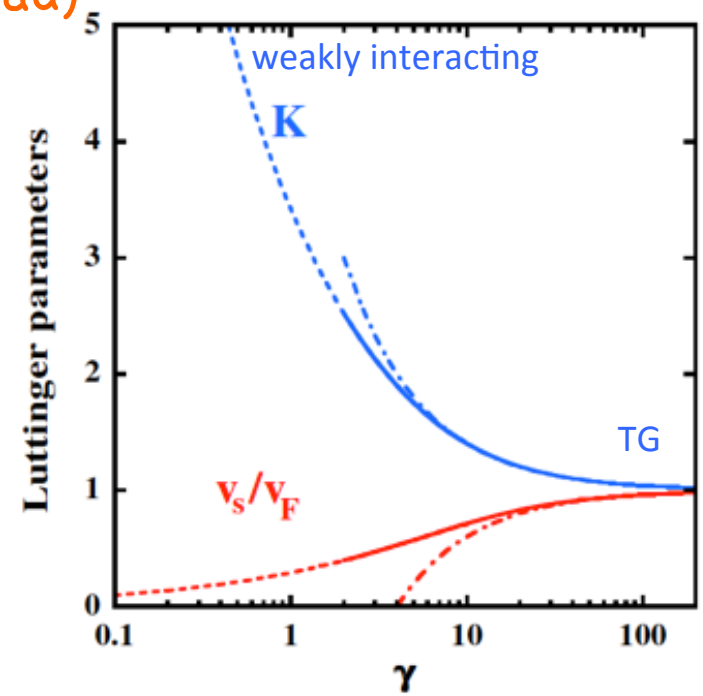
- $\gamma \ll 1$ weakly interacting regime
- $\gamma \gg 1$ strong repulsion regime (Tonks-Girardeau)


J. Phys. B: At. Mol. Opt. Phys. 37 (2004) S1–S47 PII: S0953-4075(04)71245-6

Bosonizing one-dimensional cold atomic gases

M A Cazalilla

- $\gamma \ll 1$ $K_L \to +\infty$
- $\gamma \gg 1$ $K_L \to 1$ (IFG)



Lieb Liniger model: S(q,ω)

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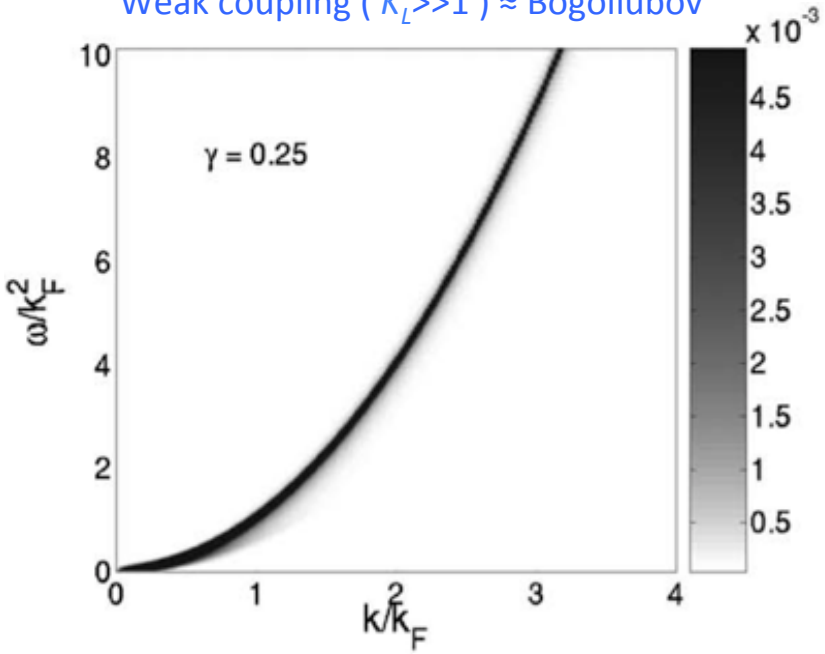
PHYSICAL REVIEW A 74, 031605(R) (2006)

Dynamical density-density correlations in the one-dimensional Bose gas

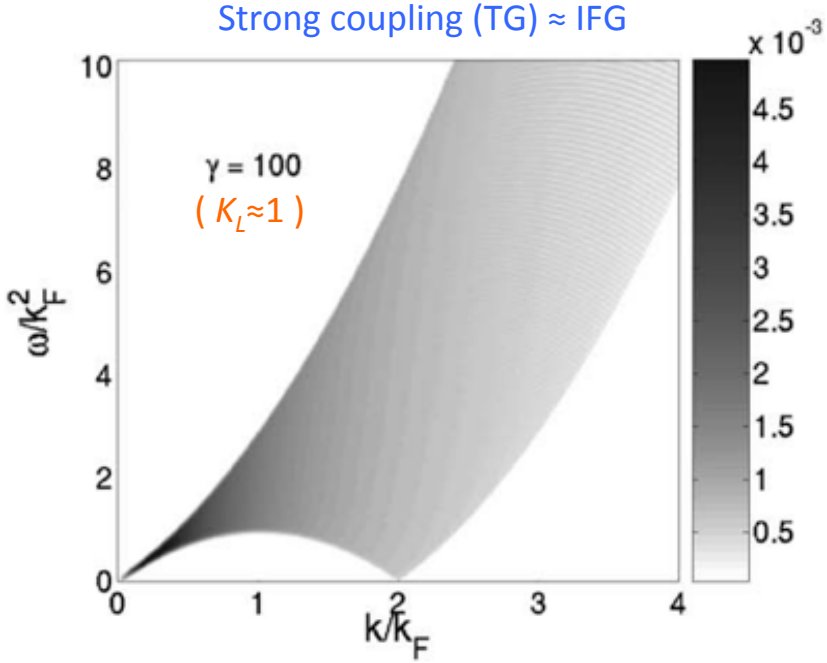
Jean-Sébastien Caux and Pasquale Calabrese
Institute for Theoretical Physics, University of Amsterdam, 1018 XE Amsterdam, The Netherlands
(Received 29 March 2006; published 29 September 2006)

- Calculation of $S(\vec{q},\omega)$ at T=0 for the Lieb-Liniger model using a hybrid theoretical/numerical method based on the exact Bethe-ansatz solution:

Weak coupling (K_L>>1) ≈ Bogoliubov



Strong coupling (TG) ≈ IFG



Models

$$H \cong -\sum_i \frac{\hbar^2 \partial_i^2}{2m} + \sum_{i<j} V\left(|x_i - x_j|\right)$$

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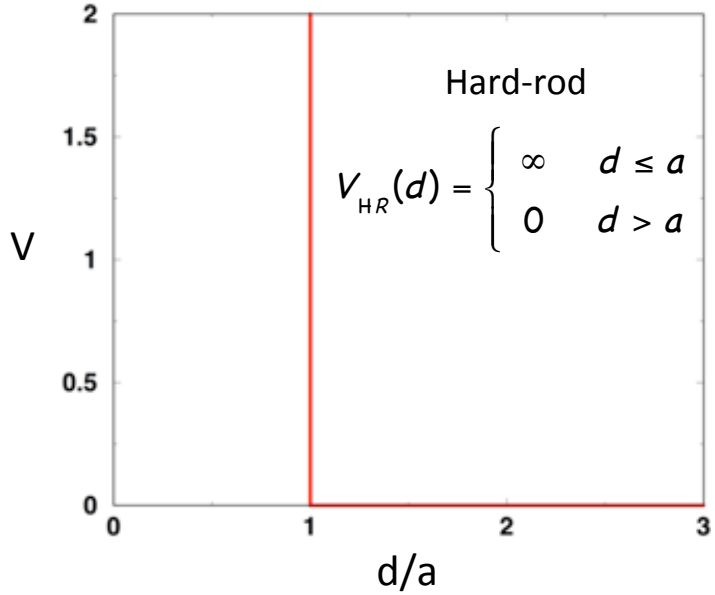
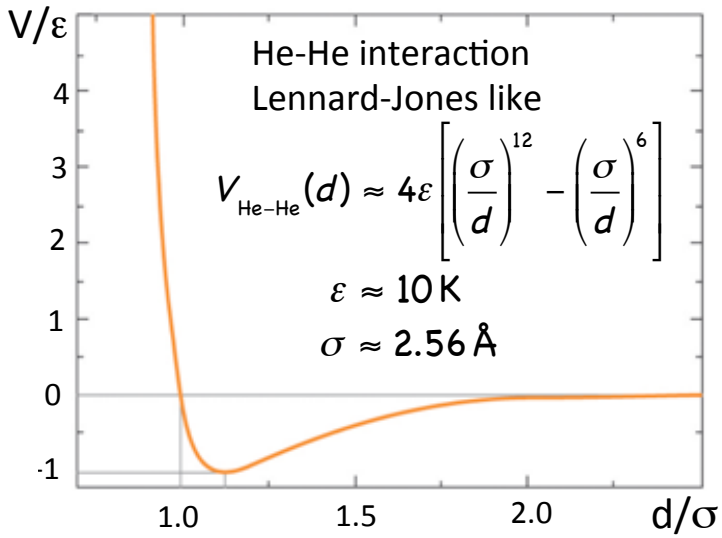
- Hard-rods

relevance:
quasi-1D quantum gases with volume exclusion effects



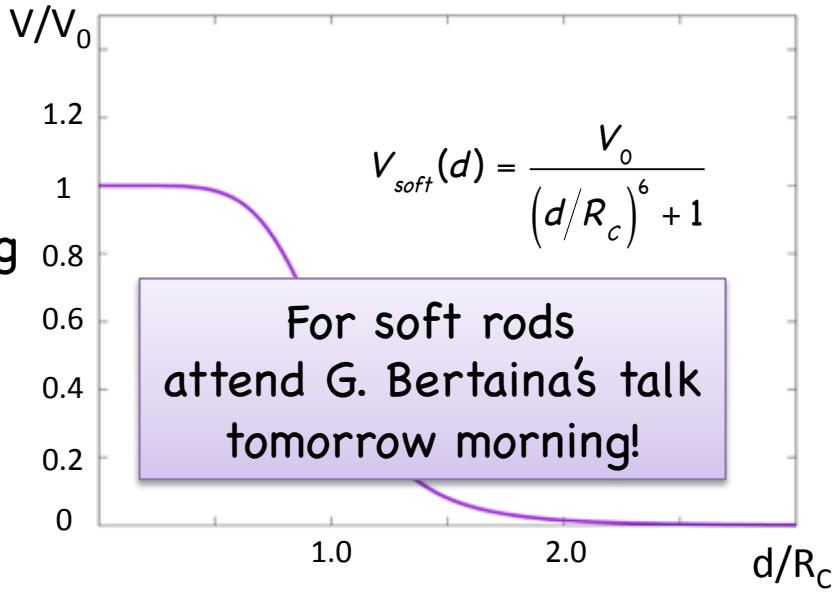
- 1D Helium

relevance:
cores of dislocations in quantum solid He,
He inside nanopores



- Soft-rods

relevance:
quasi-1D "dressed" Rydberg quantum gases



For soft rods attend G. Bertaina's talk tomorrow morning!

Our "exact" QMC tool

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We have used the "exact" T=0 K Path Integral Ground State (PIGS) projector method [Sarsa et al., J. Chem. Phys. (2000)]:

ground state ≈ discrete imaginary time evolved trial quantum state, ψ_T

Calculation of $\langle \psi_0 | \hat{O} | \psi_0 \rangle$ can be approximated,

<u>with arbitrary precision</u> (for Bose systems) by $\dfrac{\langle \psi_\tau | \hat{O} | \psi_\tau \rangle}{\langle \psi_\tau | \psi_\tau \rangle}$

with

$$\psi_\tau(R) = \int dR_1 \cdots dR_P \left\langle R \left| e^{-\frac{\tau}{P}\hat{H}} \right| R_P \right\rangle \times \cdots \times \left\langle R_2 \left| e^{-\frac{\tau}{P}\hat{H}} \right| R_1 \right\rangle \psi_T(R_1)$$

$R_i = \{\vec{r}_1^{\,i}, \cdots, \vec{r}_N^{\,i}\}$ many-body coordinates

ψ_T acts only at the beginning of the path ⇒ essentially unbiased like PIMC [Nava, Rossi, Reatto, Galli J. Chem. Phys. (2009)]

Notice: $\dfrac{\psi_\tau}{\sqrt{\langle \psi_\tau | \psi_\tau \rangle}} \xrightarrow[\tau \to \infty]{} \psi_0$ provided that $\langle \psi_0 | \psi_T \rangle \neq 0$

PIGS feature

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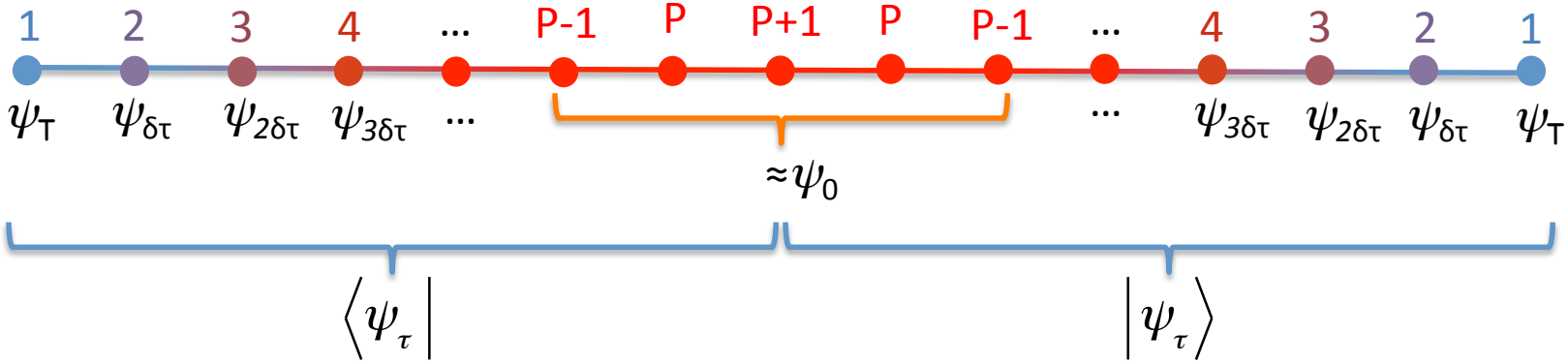
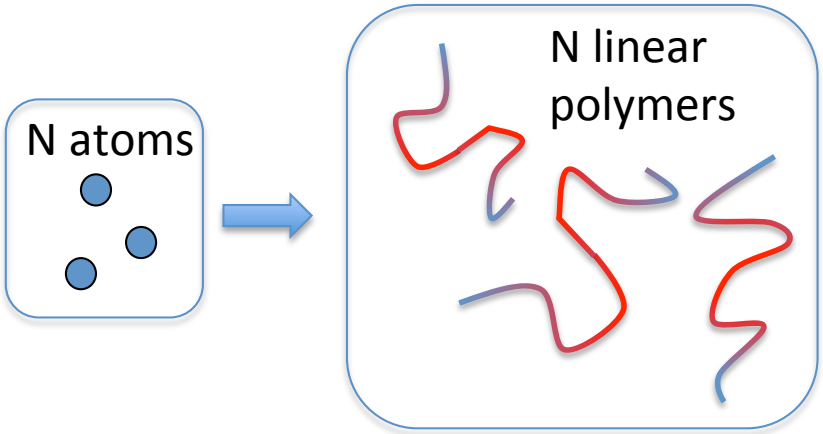
The whole imaginary time evolution is sampled, via a Metropolis algorithm, at each MC step

⇒ in computing $\langle \psi_\tau | \hat{O} | \psi_\tau \rangle$ PIGS gives rise to a discrete path $\{R_1,\ldots,R_{2P+1}\}$ of many-body coordinates



Quantum-Classical mapping: ground state averages equivalent to canonical averages of a classical system of special interacting linear polymers



Imaginary-time propagators

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- For HR we employed the pair-product approximation to express the propagator relative to a small time step $\delta\tau$ as

$$G(R,R';\delta\tau) = \prod_{i=1}^{N} G_0(r_i,r_i';\delta\tau) \prod_{i<j}^{N} G_{HR}(r_i,r_j;\delta\tau)$$

where G_0 is the free-particle propagator $\quad G_0(r,r';\delta\tau) = \dfrac{e^{-(r-r')^2/4\lambda\delta\tau}}{\sqrt{2\pi\lambda\delta\tau}} \qquad \lambda = \dfrac{\hbar^2}{2m}$

G_{HR} is obtained from the exactly known solution of the two-body scattering problem, similarly to a standard approach for hard spheres in 3D [Cao, Berne, J. Chem. Phys. 97 (1992)]:

$$G_{HR}(r,r';\delta\tau) = 1 - e^{-\frac{(r-a)(r'-a)}{2\lambda\delta\tau}}$$

ρa	K_L	$\delta\tau$ $(1/E_F(\rho))$
0.005	0.990	0.0025
0.077	0.852	0.0176
0.321	0.461	0.0509
0.471	0.280	0.0219
0.642	0.128	0.0142
0.700	0.090	0.0097
0.900	0.010	0.0012

For 1D ^{4}He: fourth-order pair-Suzuki approximation [for example see: Nava et al., J. Chem. Phys. (2009)] and observe convergence of ground state estimates with a typical projection time $\tau \simeq 0.8$ K^{-1} using a time step $\delta\tau = 1/160$ K^{-1}.

Trial state: SWF & SPIGS

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- HR: exact ψ_0 is known ⇒ the whole imaginary time propagation can be used to compute ground state properties like: $F(\vec{q},\tau) = \left\langle \hat{\rho}_{\vec{q}}(\tau)\hat{\rho}_{-\vec{q}}(0) \right\rangle$

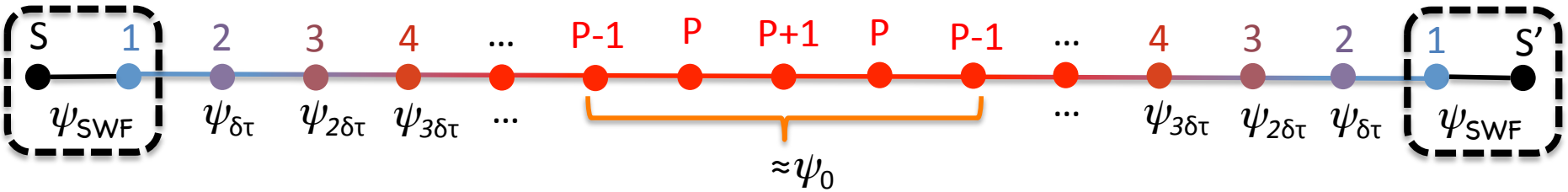
- When ψ_0 is not known we use a Shadow Wave function (SWF):

$$\psi_{SWF}(R) = \phi(R) \int dS\ K(R,S)\ \phi_s(S) \qquad S = \left\{\vec{s}_1, \cdots, \vec{s}_N\right\}$$

which corresponds to a variationally optimized (first) projection step in imaginary-time.

- PIGS+SWF ⇒ Shadow Path Integral Ground State (SPIGS)
  [Galli, Reatto, Mol. Phys. (2003)]:

shadow variables are integrated together with the other path-variables
⇒ add 2 more (many-body) variables at the tails of the path



Initial states

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- Hard rods (exact!):

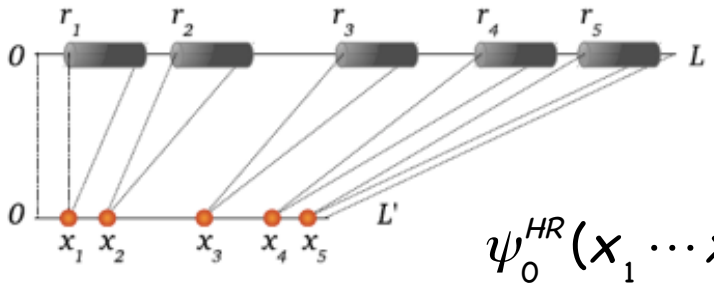
$$\psi_0^{HR}(x_1 \cdots x_N) = \frac{1}{\sqrt{N!}} \det\left(\frac{e^{ik_l x_j}}{\sqrt{L'}}\right)$$

$$L' = L - Na$$

$$k_l = \frac{2\pi}{L'} n_l$$

$$n_l = -\frac{N-1}{2} + (l-1)$$

$$l = 1, \cdots, N$$



$$x_1 < x_2 < \cdots < x_N$$

$$\psi_0^{HR}(x_1 \cdots x_N) \propto \prod_{i<j}^{N} \left| \sin\left[\frac{\pi(x_i - x_j)}{L'}\right] \right|$$

- 1D ^{4}He (SWF):

$$\psi_{SWF}(R) = \phi(R) \int dS\, K(R,S)\, \phi_s(S)$$

$$R = \{r_1, \cdots, r_N\}$$

$$S = \{s_1, \cdots, s_N\}$$

$$\phi(R) = \prod_{i<j}^{N} e^{-\frac{1}{2}\left(b_p/|r_i - r_j|\right)^{m_p}}$$

"kinetic" term

$$K(R,S) = \prod_{i=1}^{N} e^{-C(r_i - s_i)^2}$$

short range damping

Jastrow-McMillan terms

$$\phi_s(S) = \prod_{i<j}^{N} e^{-\frac{1}{2}\left(b_s/|s_i - s_j|\right)^{m_s} - \alpha \ln\left[\sin\left(\pi|s_i - s_j|/L\right)\right] \tanh\left(|s_i - s_j|/d\right)}$$

1D Reatto-Chester term [Phys. Rev. 155 (1967)] $\alpha = 1/K_L$

Imaginary time correlations functions

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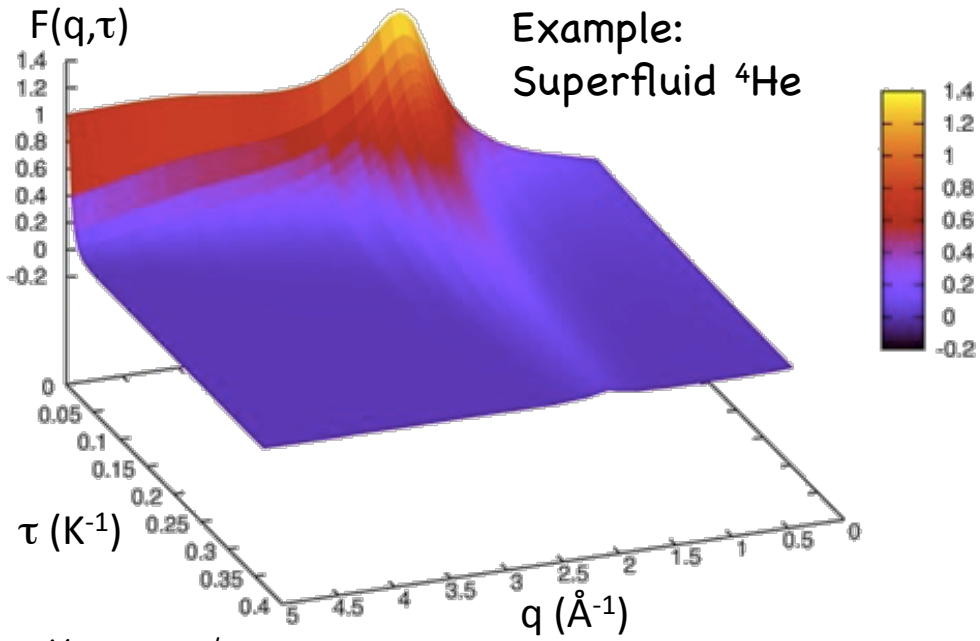
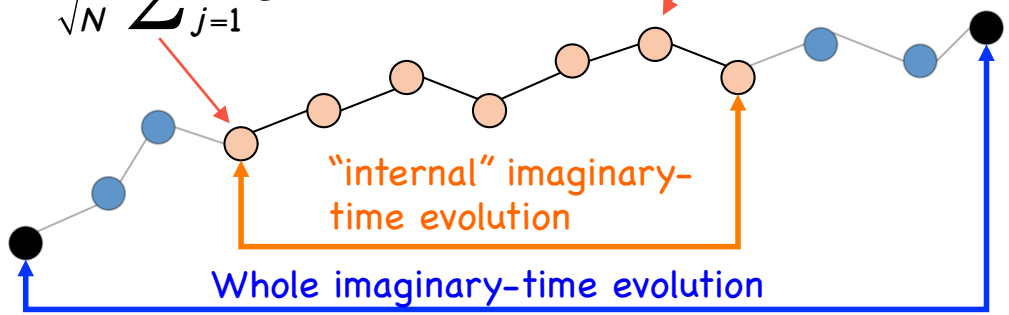
Accurate F(q,τ) can be computed with PIGS:
pure estimator!

$$F(\vec{q},\tau) = \left\langle \hat{\rho}_{\vec{q}}(\tau)\hat{\rho}_{-\vec{q}}(0) \right\rangle$$

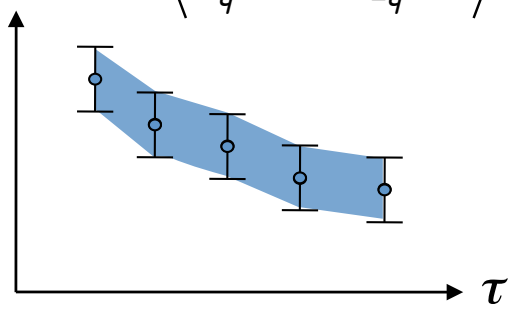


$$\rho_{-\vec{q}} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{i\vec{q}\cdot\vec{r}_j^{\,0}}$$

$$\rho_{\vec{q}}(\tau = l\delta\tau) = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-i\vec{q}\cdot\vec{r}_j^{\,l}}$$



$$F(\vec{q}, l\delta\tau) = \left\langle \hat{\rho}_{\vec{q}}(l\delta\tau)\hat{\rho}_{-\vec{q}}(0) \right\rangle$$



⇒ limited and noisy set of data

Our analytic continuation method

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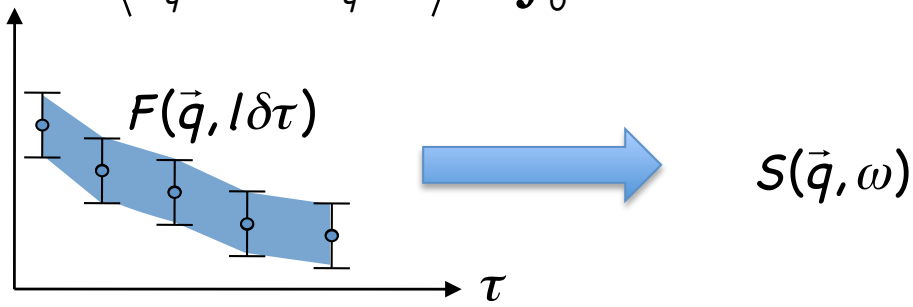
- QMC: only imaginary time correlations functions
- Analytic continuation task: extract $S(q,\omega)$ from limited and noisy $F(q,\tau)$

$$F(\vec{q}, l\delta\tau) = \left\langle \hat{\rho}_{\vec{q}}(l\delta\tau)\hat{\rho}_{-\vec{q}}(0) \right\rangle = \int_0^\infty d\omega\, e^{-\omega l \delta\tau} S(\vec{q},\omega) \qquad l = 0,1,2,\cdots$$



In these conditions, numerical inversion of the Laplace transform: ill-posed inverse problem

PHYSICAL REVIEW B **82**, 174510 (2010)

***Ab initio* low-energy dynamics of superfluid and solid ^{4}He**

E. Vitali, M. Rossi, L. Reatto, and D. E. Galli

GIFT ≡ Genetic Inversion via Falsification of Theories

Statistical inversion method: give up the idea to find *The Solution*, find (via a genetic algorithm) a wide class of spectral functions compatible with $F(q,\tau)$ and be confident only with properties shared by the large majority of them

GIFT: space of models

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1 Chose a wide space of models compatible with a priori information

$$S(q,\omega) \geq 0$$

$$S(q,\omega) = 0 \ if \ \omega \leq \omega_{th}(q) \ (\geq 0)$$

(T=0)

$$\int_0^{+\infty} d\omega \, S(q,\omega) = S(q)$$

$$\int_0^{+\infty} d\omega \, \omega \, S(q,\omega) = \frac{q^2}{2m}$$

$$\vdots$$

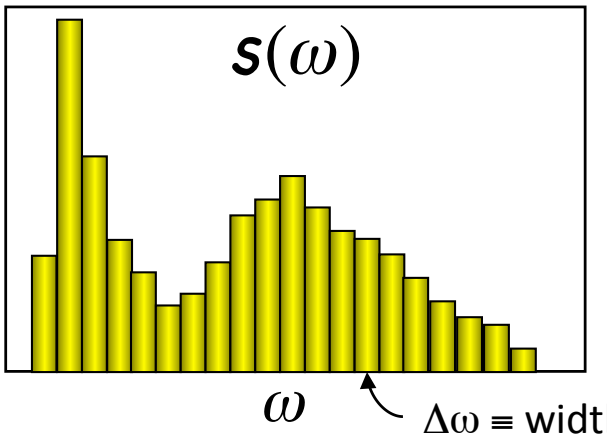
$$S(q,\omega) = \frac{S(q)}{M} \sum_{i=1}^{N_\omega} s_q^{(i)} \underbrace{\delta(\omega - \omega_i)}_{\text{Delta function}}$$

$$s_q^{(i)} \in \{0, 1, 2, \cdots, M\}$$

M ≡ maximum number of quanta of spectral weight

$$\omega_i = \omega_{th} + \Delta\omega \, (i - 1/2)$$

$$\frac{1}{M} \sum_{i=1}^{N_\omega} s_q^{(i)} = 1$$



$\Delta\omega$ ≡ width of the partition

Note that $S(q) = F(q,0)$ is part of QMC "observations"

How GIFT works

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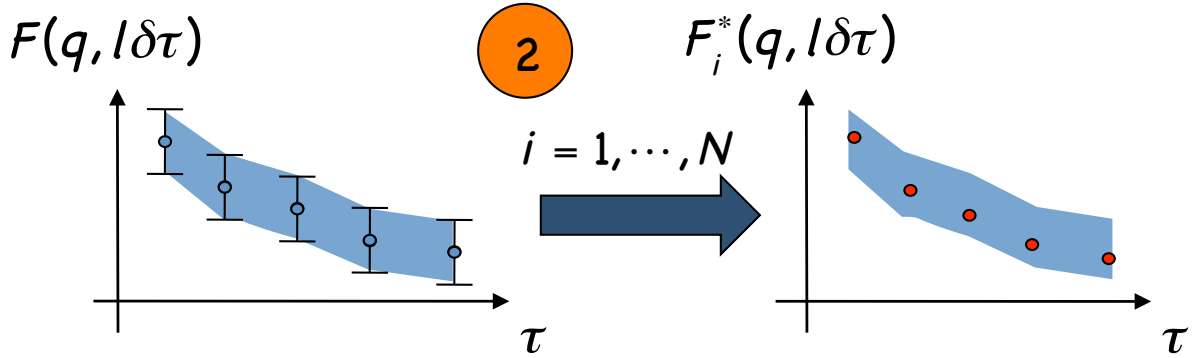
1 Chose a wide space of model spectral functions



Sample an equivalent F(q,τ) using QMC statistical information

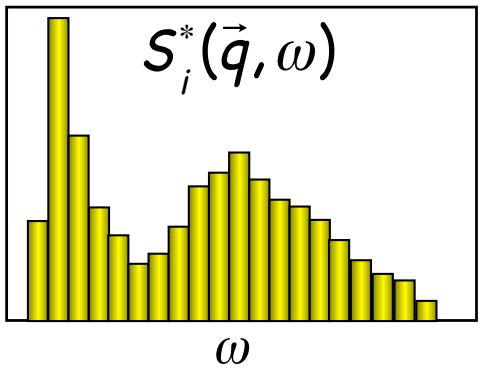
3

Fitness function

$$\Phi = -\sum_l \left[F_i^*(q, l\delta\tau) - \int_0^{+\infty} d\omega\, e^{-\omega l \delta\tau} S(q,\omega) \right]^2 - \gamma \left[\frac{q^2}{2m} - \int_0^{+\infty} d\omega\, \omega\, S(q,\omega) \right]^2$$

population of spectral functions: Genetic + Monte Carlo Stochastic dynamics

4



Repeat 2,3,4 $i = 1, \cdots, N$

5 Average procedure

$$S_{GIFT}(\vec{q}, \omega) = \frac{1}{N} \sum_i S_i^*(\vec{q}, \omega)$$

Hard Rods: exact properties

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- The hard-rods model is exactly solvable by Bethe-ansatz
  [Nagamiya, Proc. Phys. Math. Soc. Japan (1940); Mazzanti et al., Phys. Rev. Lett. (2008)]

PRL 116, 135302 (2016) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2016

One-Dimensional Liquid ^{4}He: Dynamical Properties beyond Luttinger-Liquid Theory

G. Bertaina,[1] M. Motta,[2] M. Rossi,[3,4,5] E. Vitali,[2] and D. E. Galli[1]

- Excited states are pseudo p-h excitations with renormalized mass $m^* = K_L\, m$
- Kowledge of excited states & m^* ⇒ exact ω_{th}

$$\omega_{th}(q) = \omega_*^-(q - 2k_F j)$$

$$2k_F j \le q \le 2k_F(j+1) \qquad j = 0,1,2,\cdots$$

- Exact ω_{th} ⇒ exact exponent $\mu(q)$ of the nonlinear LL theory:

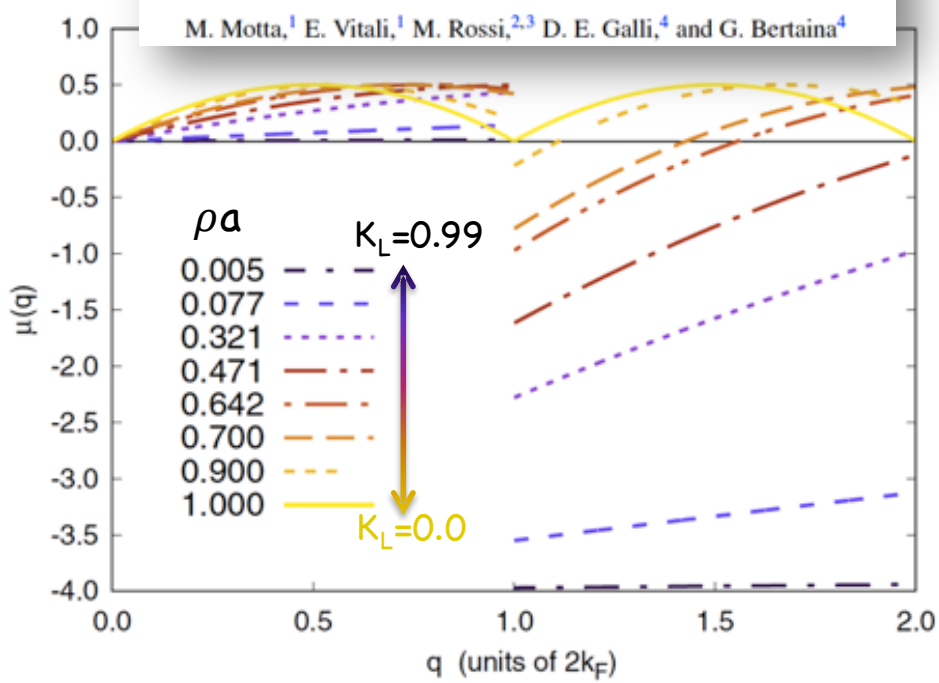
$$\mu(q) = -2\left(\frac{qa}{2\pi} - j\right)\left(\frac{qa}{2\pi} - j - 1\right)$$

$$2k_F j \le q \le 2k_F(j+1) \qquad j = 0,1,2,\cdots$$

PHYSICAL REVIEW A 94, 043627 (2016)

Dynamical structure factor of one-dimensional hard rods

M. Motta,[1] E. Vitali,[1] M. Rossi,[2,3] D. E. Galli,[4] and G. Bertaina[4]



Hard rods: $S_{GIFT}(q,\omega)$

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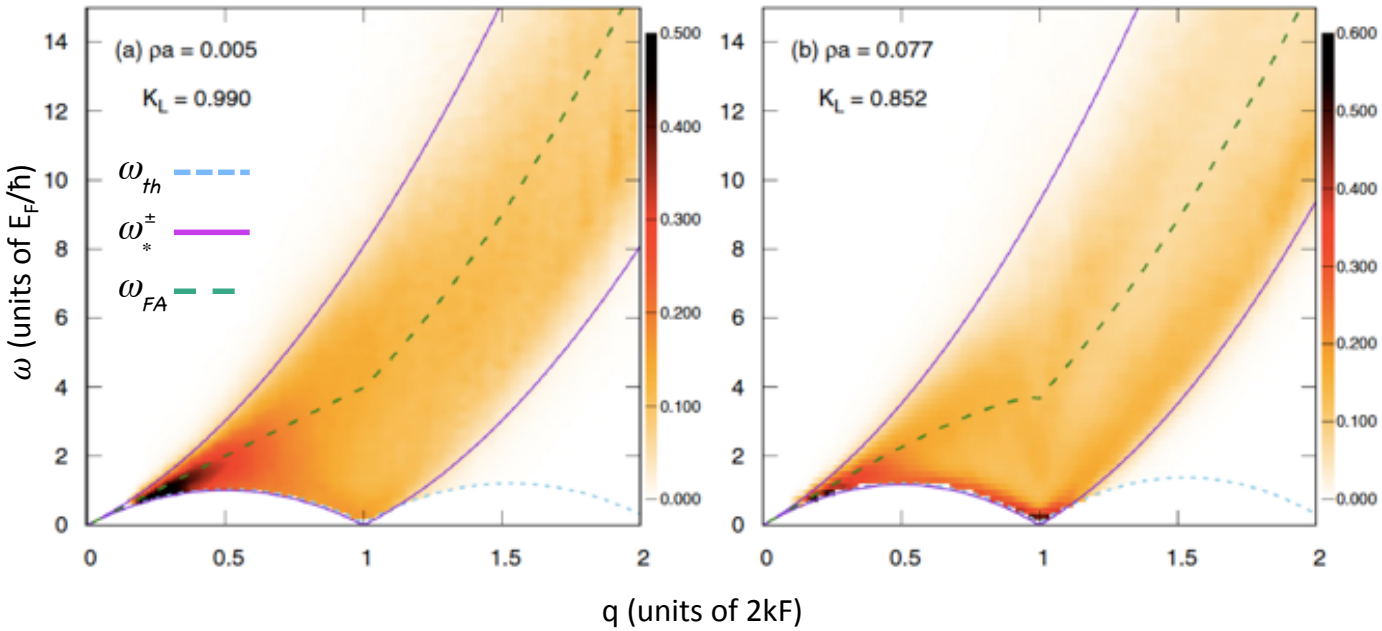
Low density regime: spectral weight is broadly distributed inside the p-h band, reminiscent of the Tonks-Girardeau model of impenetrable point like bosons (Lieb Liniger $\gamma\rightarrow\infty$), to which HR model reduces in the $\rho a\rightarrow 0$ limit.

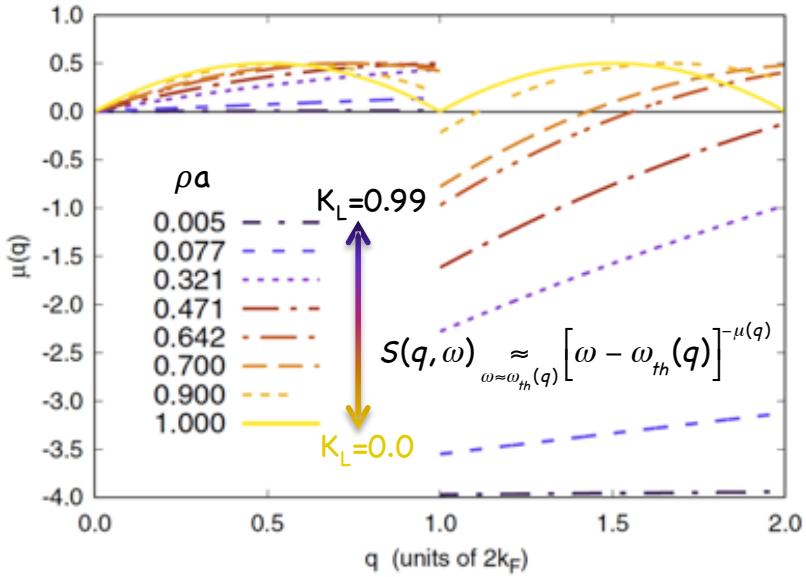


$$\omega_*^{\pm} = \frac{\hbar}{2m^*}\left|2k_F q \pm q^2\right|$$

$$m^* = K_L m$$

$$\omega_{FA} = \frac{\hbar q^2}{2mS(q)}$$

NLL & GIFT: For $q<2k_F$ $\mu(q)$ is slightly larger than zero ⇒ weak concentration of spectral weight close to ω_{th}. For $q>2k_F$, $\mu(q)<0$ and large ⇒ the support of $S(q,\omega)$ departs from ω_{th}.



$$S(q,\omega) \underset{\omega\approx\omega_{th}(q)}{\approx} \left[\omega-\omega_{th}(q)\right]^{-\mu(q)}$$

Hard rods: $S_{GIFT}(q,\omega)$

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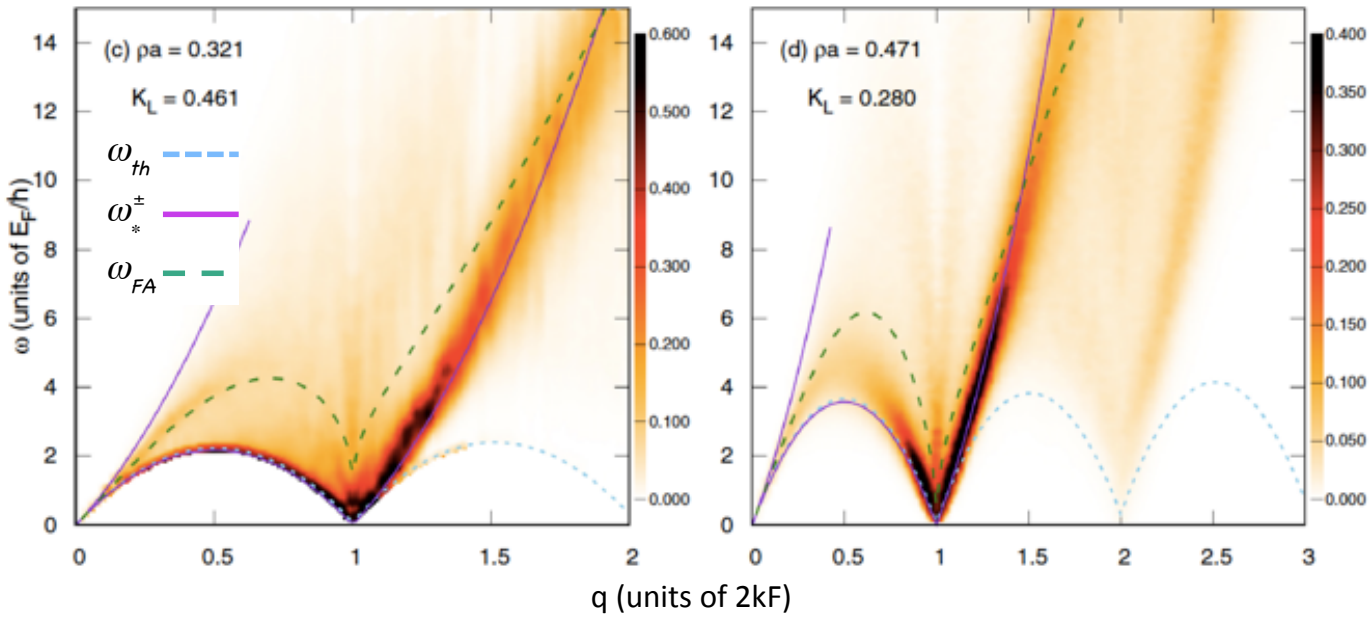
Higher densities:
When $K_L<1/2$, a peak manifests in S(q) at $q=2k_F$.
This change in behavior is also reflected in $S(q,\omega)$: the spectral weight concentrates close to the $\omega^-_*(q)$.

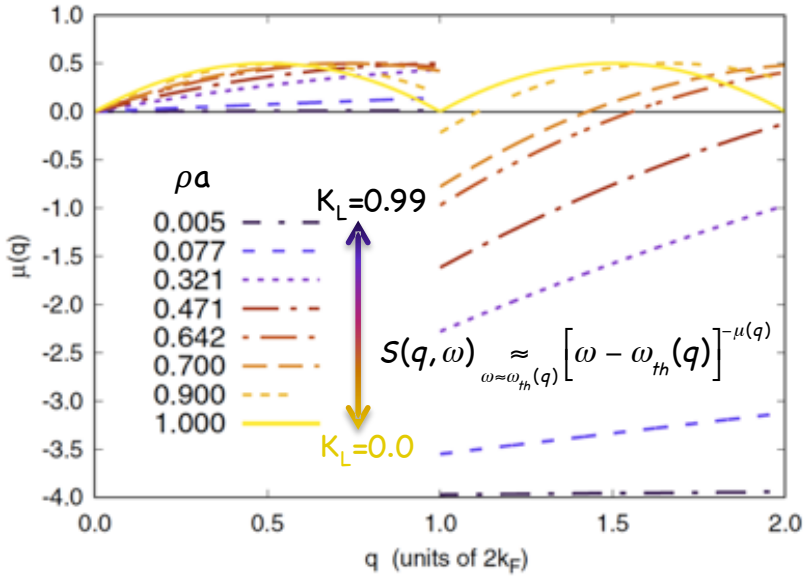


As K_L decreases below 1/2, the support of $S(q,\omega)$ extends below $\omega^-_*(q)$ for $q>2k_F$ ... still remaining above $\omega_{th}(q)$.

$$\omega^{\pm}_{*} = \frac{\hbar}{2m^*}\left|2k_F q \pm q^2\right|$$

$$m^* = K_L m$$

$$\omega_{FA} = \frac{\hbar q^2}{2mS(q)}$$



Hard rods: $S_{GIFT}(q,\omega)$

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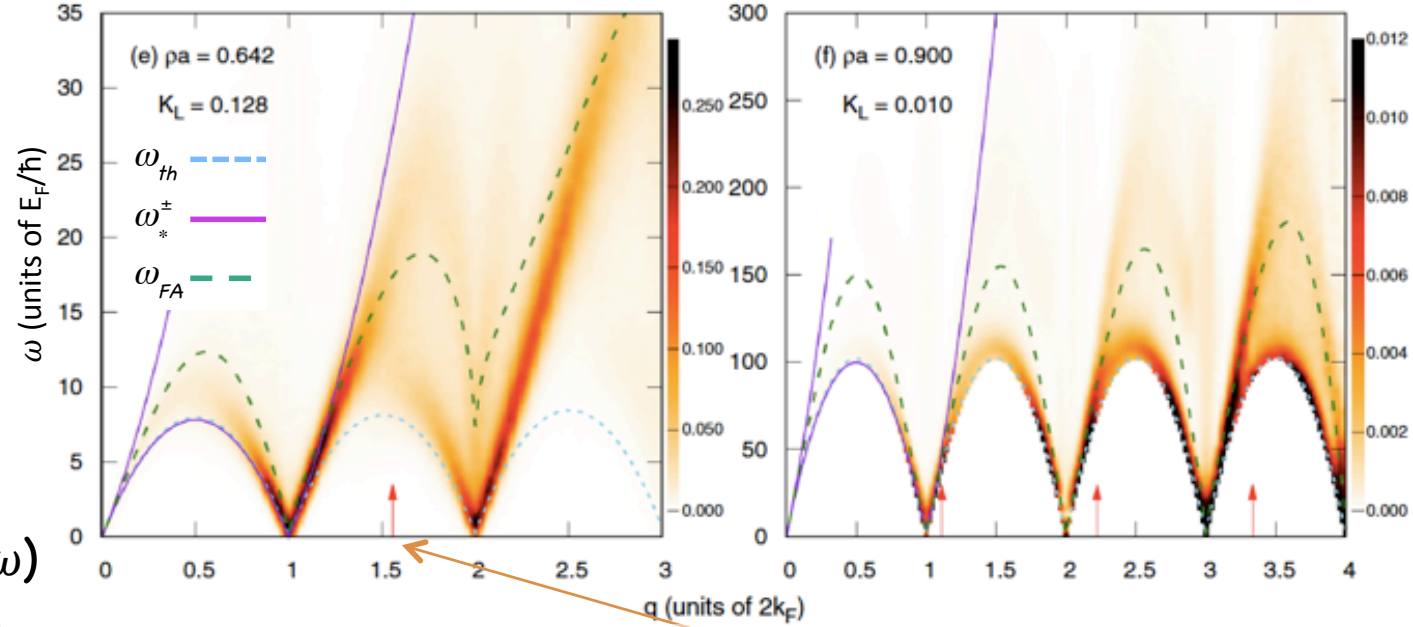
High-density regime: For $q/2k_F<1.4$ the spectral weight concentrates in a narrow region of the q-ω plane that gradually departs from $\omega_{th}(q)$.

For $1.4<q/2k_F<1.8$, $S(q,\omega)$ broadens and flattens. Finally, for $1.8<q/2k_F<2$, the spectral weight again concentrates close to $\omega_{th}(q)$.

Agreement with an exact prediction: $S(q,\omega)$ constant on a finite range of ω for special wave vectors $q_j=2\pi j/a$

[Mazzanti et al., PRL (2008)]

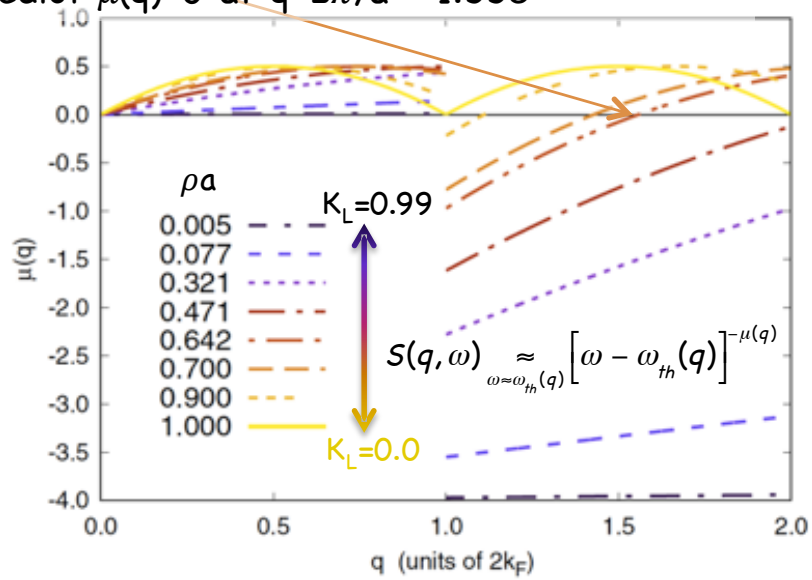


Note that NLL predict $\mu(q)=0$ at $q=2\pi/a = 1.558$

$$\omega_*^{\pm} = \frac{\hbar}{2m^*}\left|2k_F q \pm q^2\right|$$

$$m^* = K_L m$$

$$\omega_{FA} = \frac{\hbar q^2}{2mS(q)}$$



$$S(q,\omega) \underset{\omega\approx\omega_{th}(q)}{\approx} \left[\omega-\omega_{th}(q)\right]^{-\mu(q)}$$

Hard rods: $S_{GIFT}(q,\omega)$

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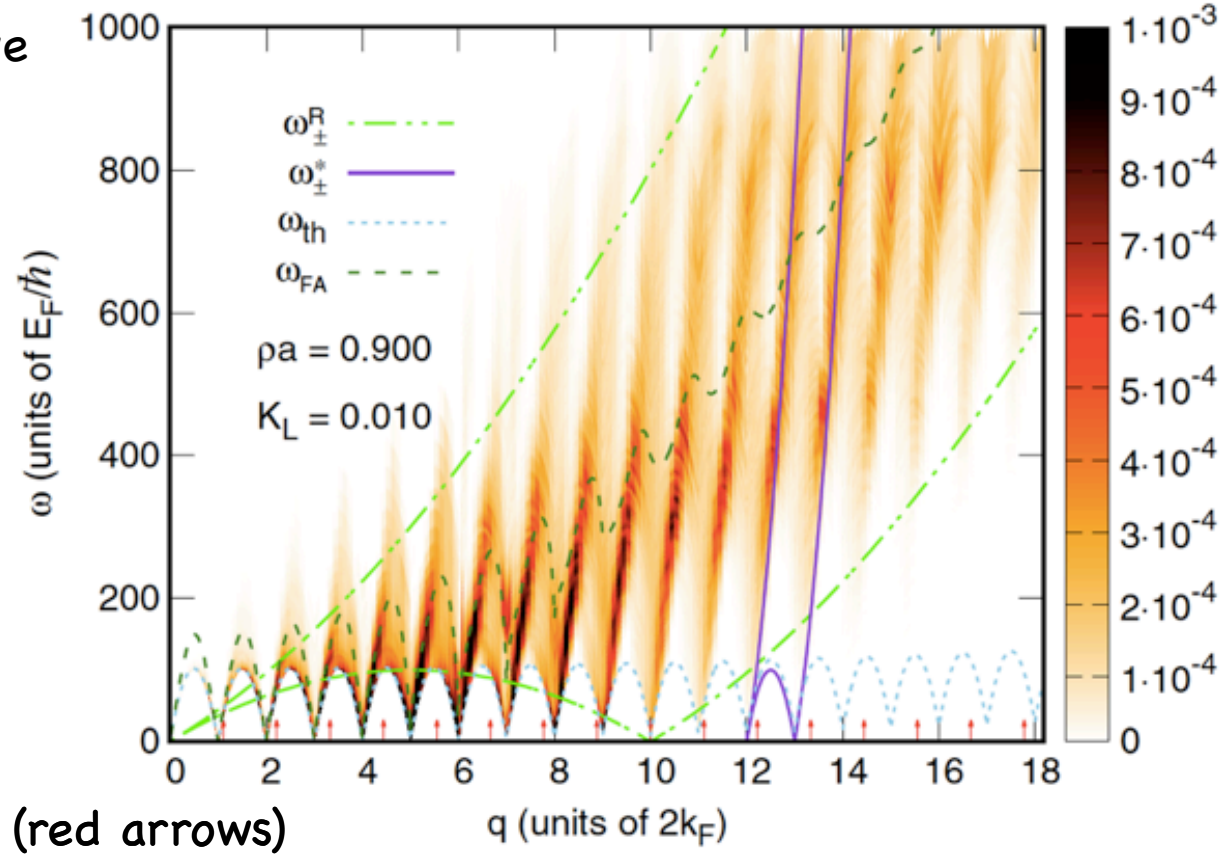
Università degli Studi di Milano

ρa=0.9: we observe that above $q/2k_F$=10, $S(q,\omega)$ is no more peaked along $\omega_{th}(q)$;

stripe structure: repeated p-h band (every $2k_F$); stripes are bounded by the p-h band of the rescaled IFG obtained defining $\rho'=\rho/(1-\rho a)$:

$$\omega_R^{\pm} = \frac{\hbar}{2m}\left|2\pi\rho' q \pm q^2\right|$$



$S(q,\omega)$ flat at special $q_j=2\pi j/a$ (red arrows)

HR spectrum can be described by the synergy of two IFGs:

- one with the same ρ, but renormalized mass $m^*=K_L m$
- the other with the same m, but renormalized density $\rho'=\rho/\sqrt{K_L}$

Only in the unphysical $\rho a \rightarrow 1$ limit, $S(q,\omega)$ would attain perfect periodicity

1D ^{4}He: K_L from S(q) & E(n)

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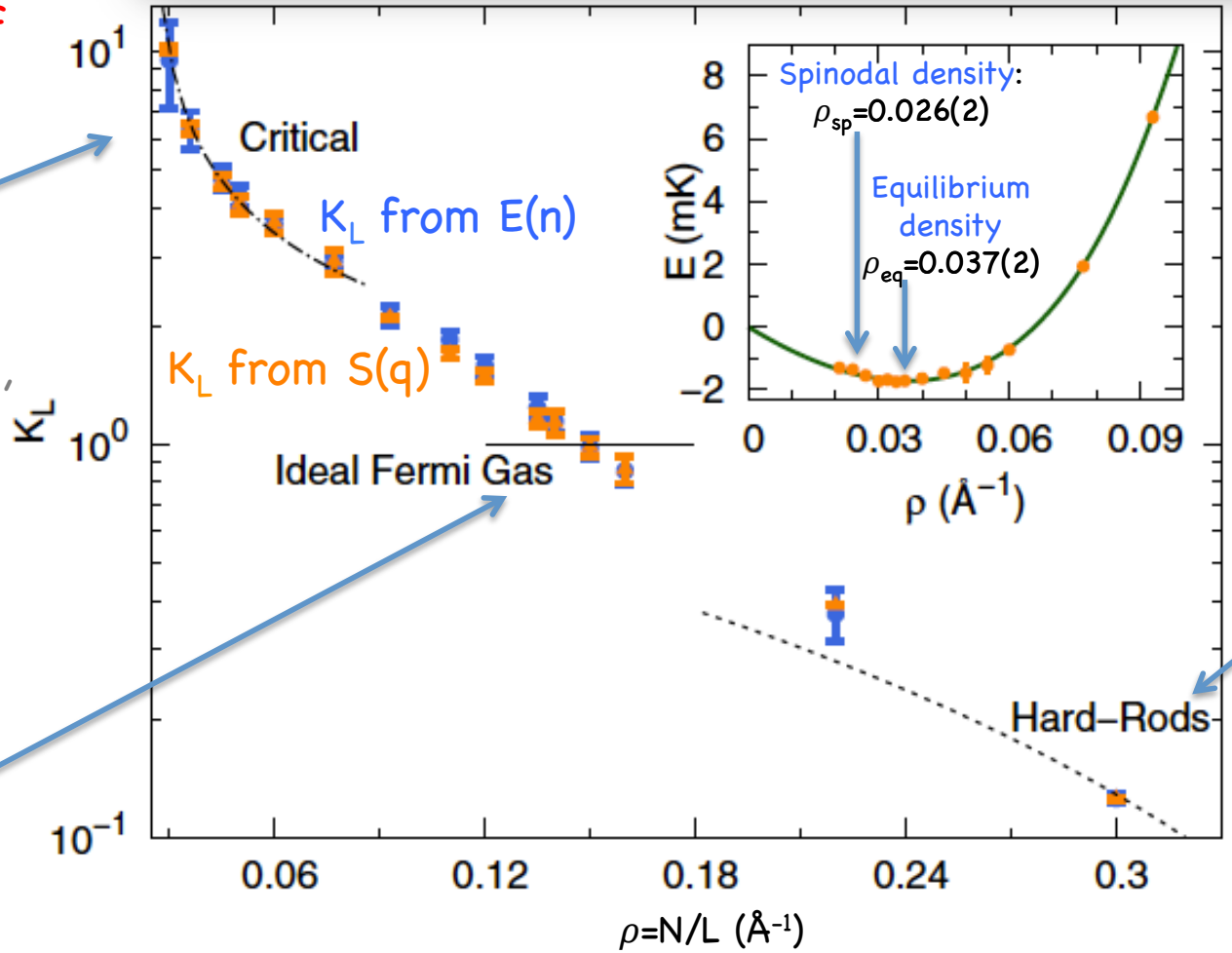
1D ^{4}He provides an intriguing scenario to observe the whole range of K_L : $0 < K_L < +\infty$

PRL 116, 135302 (2016) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2016

One-Dimensional Liquid ^{4}He: Dynamical Properties beyond Luttinger-Liquid Theory

G. Bertaina,[1] M. Motta,[2] M. Rossi,[3,4,5] E. Vitali,[2] and D. E. Galli[1]

Liquid/gas phase separation [Boninsegni, Moroni, JLTP (2000)] anticipated by high compressibility



Special density: K_L=1 like IFG

Hard-rods regime: very low compressibility

$K_L=(1-\rho a)^2$ with a=2.139 Å

1D ^{4}He: low density regime

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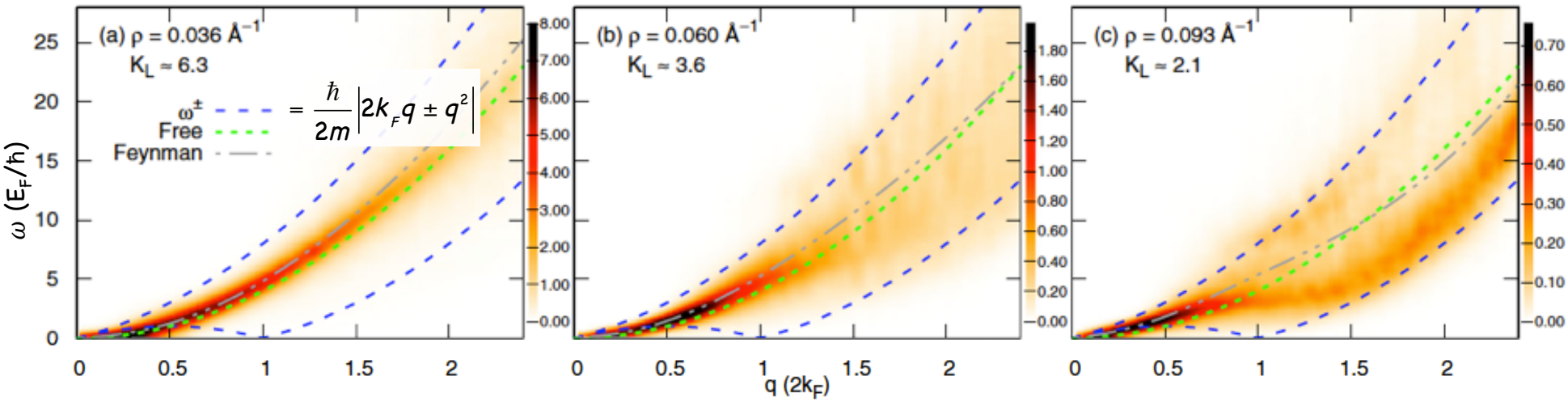

- Agreement with LL theory for every ρ: $S(q,\omega)$ always peaked around phonon dispersion $\omega = v_s q$ when $q \to 0$ and $\omega \to 0$
- Similar to Lieb-Liniger at large K_L (Bogoliubov-like) here due to critical behaviour not to weak coupling
- At large momentum ($q \gtrsim k_F$) and energy: broadening of spectral weight, filling of the 1D IFG particle-hole band
- By further increasing ρ, the spectral weight starts to concentrate again, emerging as a phonon and then bending downwards to approach $\omega^-(q)$: mixture of Bose & Fermi dynamical properties!

1D ^{4}He: high density regime

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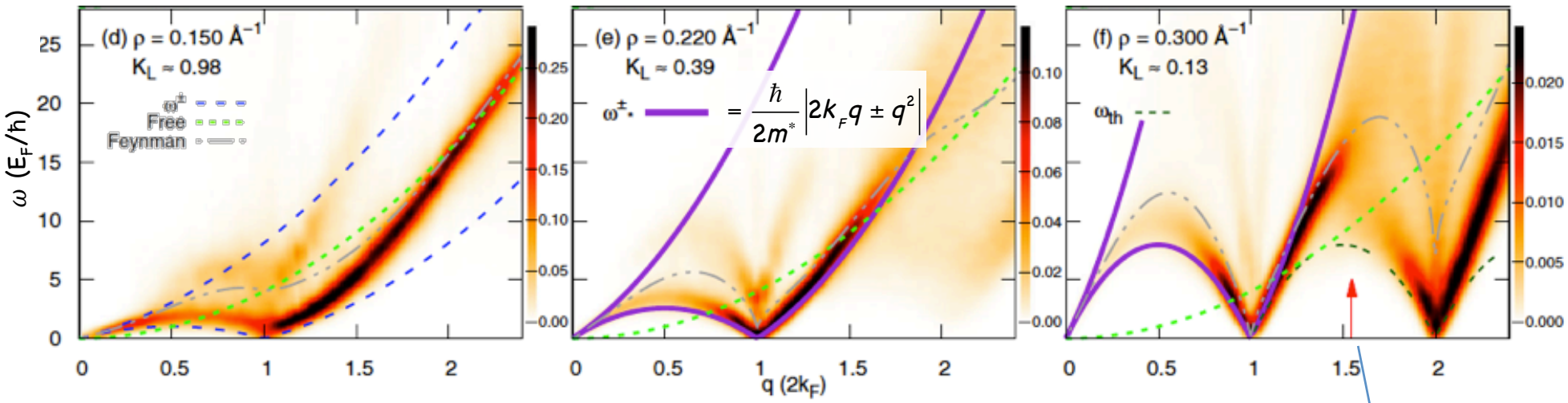
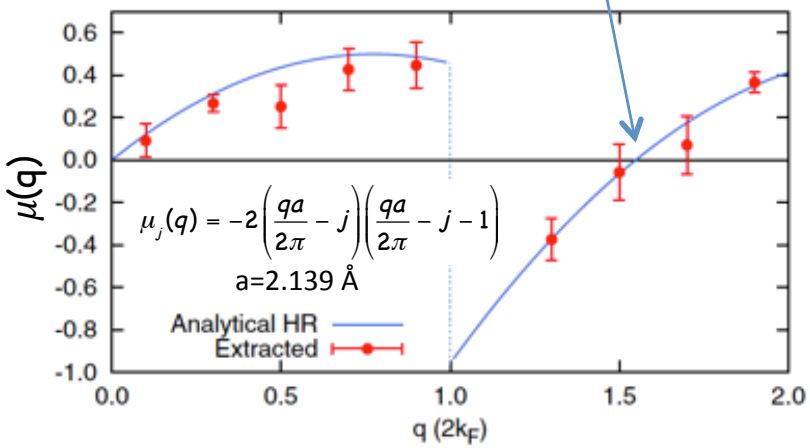


- For $K_L \simeq 1$ $S(2k_F,\omega)$ flat for $\omega \lesssim E_F/\hbar$ but, in contrast with IFG, HR & Lieb-Liniger models, $S(q,\omega)$ is not flat for $\omega > E_F/\hbar$ ⇒ strong non-universality.
- For $K_L < 1$, $S(q,\omega)$ is mostly distributed in a region with boundaries $\omega^{\pm}_*(q)$, which are modified like HRs: we notice that $\omega^-_*(q)=\omega^-(q)/K_L$ which again suggests that $m^*=K_L m$
- For $K_L \simeq 0.13$ using a=2.139 Å (scattering length of repulsive part of V_{He-He}) in the HR analytical expression for $\mu(q)$, we found agreement with NLL and the presence of a special wave vector where $S(q,\omega)$ flattens



1D ^{4}He: drag force

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- Energy loss per unit time due to an heavy impurity (Fermi's golden rule):

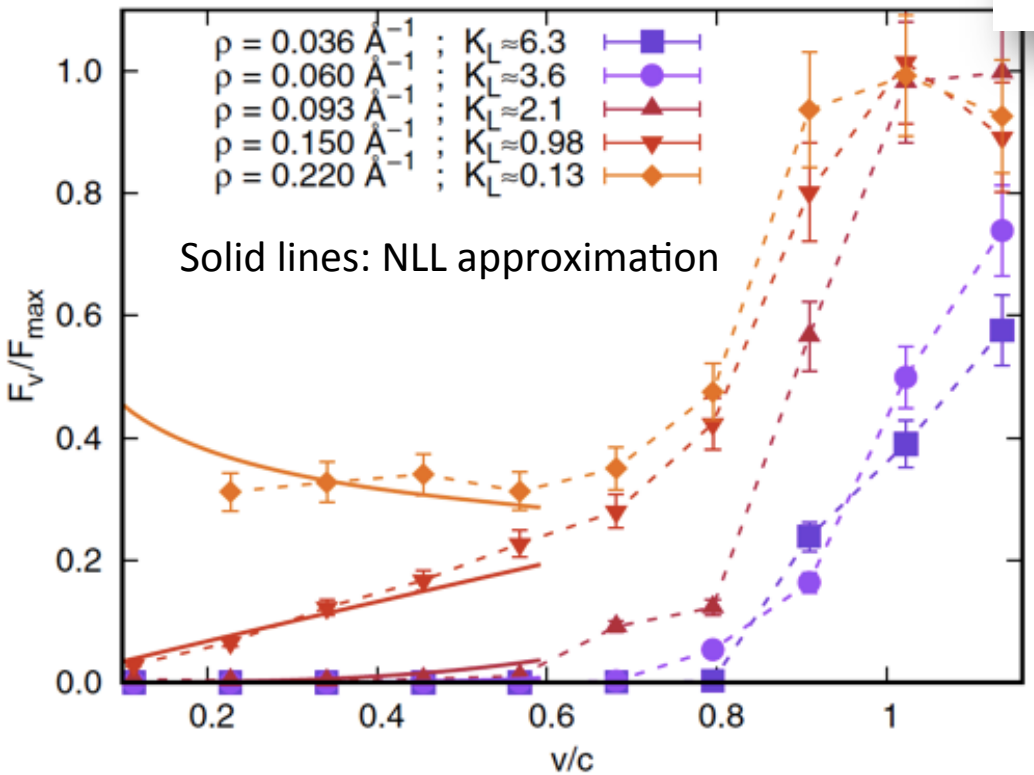
$$\frac{d\varepsilon}{dt} = -F_v\, v \approx -\left(\int_0^\infty dq \left|\tilde{V}_q\right|^2 \rho\hbar q\, S(q, qv)\right) v$$

$$\left(\hbar\omega_q = \hbar q v - \frac{\hbar^2 q^2}{2m} \cong \hbar q v\right)$$

J Low Temp Phys (2017) 187:419–426
DOI 10.1007/s10909-016-1704-8

Linear Response of One-Dimensional Liquid ^{4}He to External Perturbations

M. Motta[1] · G. Bertaina[2] · E. Vitali[1] · D. E. Galli[2] · M. Rossi[3,4]



Used contact potential with the same scattering length of the repulsive part of the ^{4}He interaction

This is a perturbative estimate relevant for "soft" impurities.
For hard-core impurities (like ^{3}He pinned to ^{4}He dislocations) superfluid response is suppressed.

Soft rods

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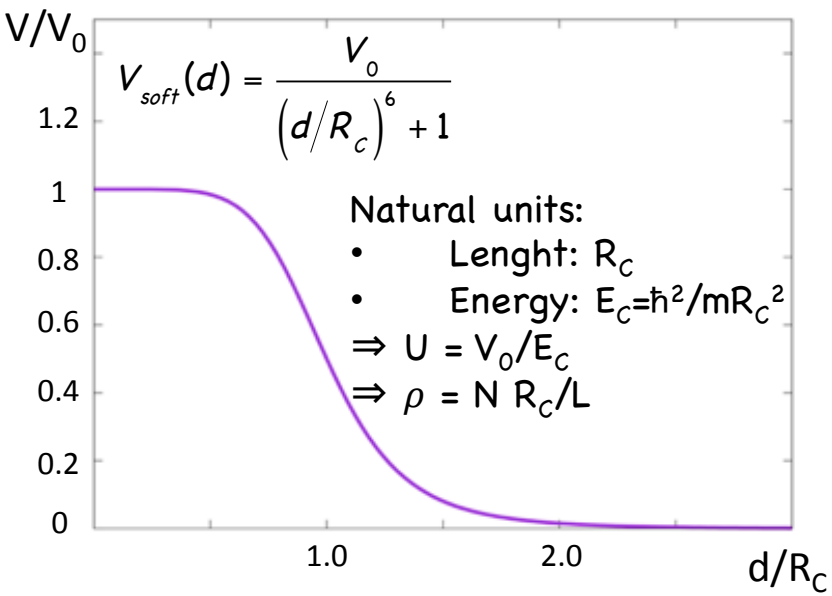
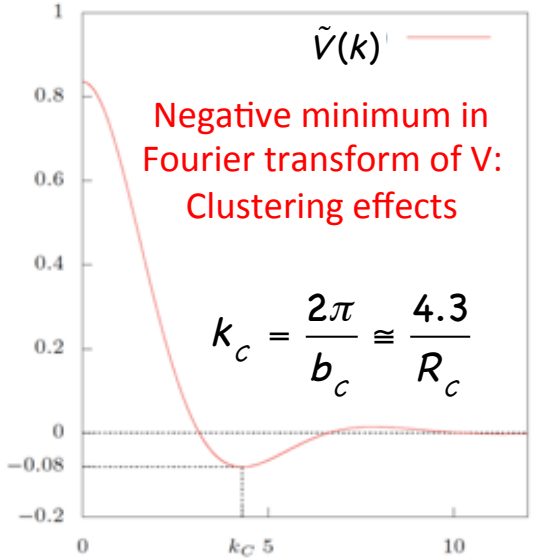
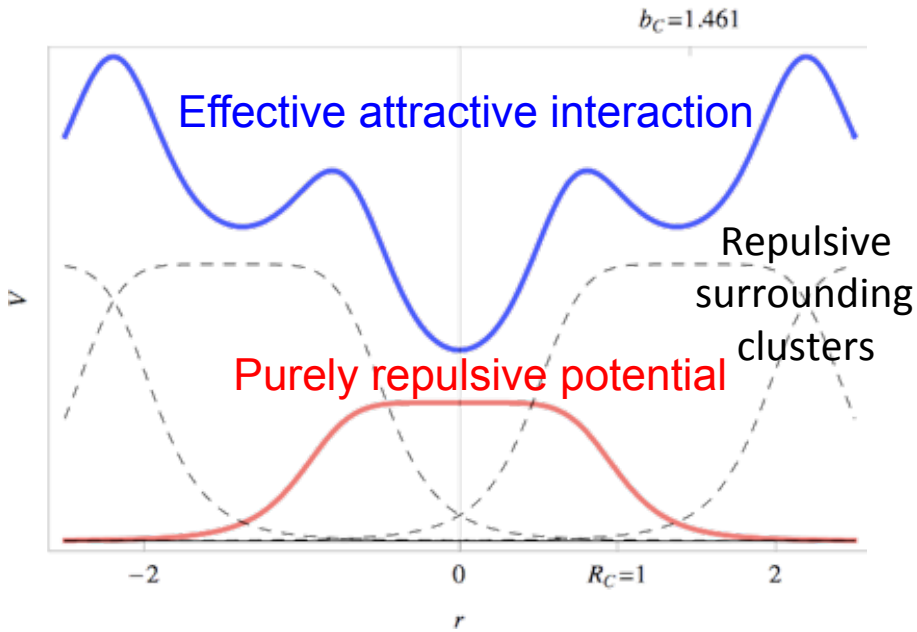


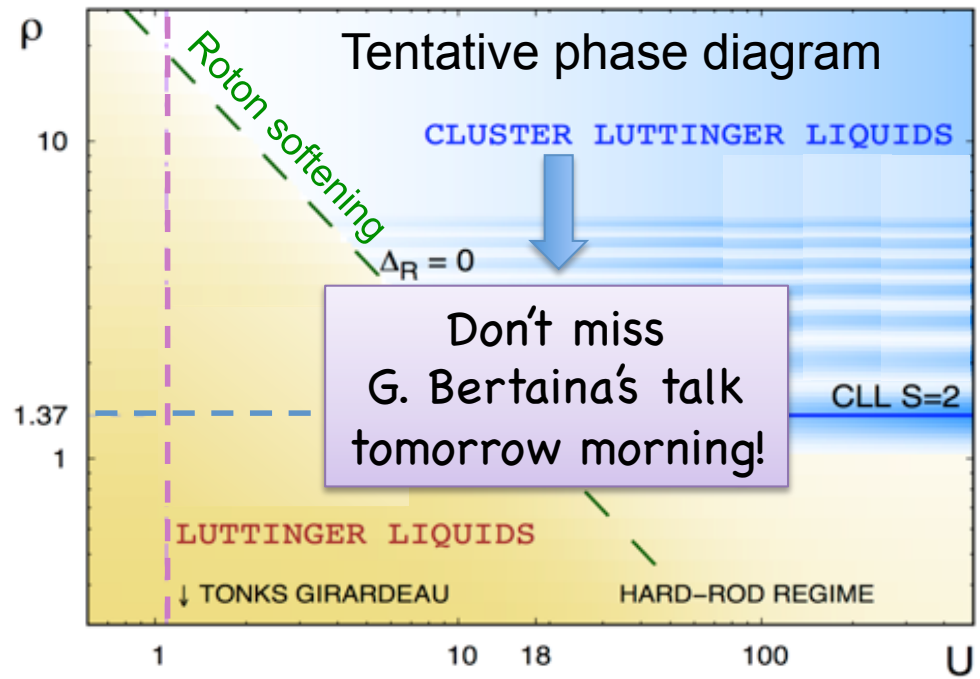


Bogoliubov spectrum:

$$\varepsilon_B(q) = \sqrt{\left(\frac{\hbar^2 q^2}{2m}\right)^2 + \frac{\hbar^2 q^2}{m}\rho\tilde{V}(q)}$$

1D lattice system:
M. Mattioli et al.
PRL 111 (2013);
Dalmonte et al.
Phys. Rev. B 92 (2015).





Conclusions (for hard-core interactions)

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- We have computed T=0 $S(q,\omega)$ of 1D quantum systems by means of state-of-the-art Path Integral projector and analytic continuation techniques.
- The low-energy properties of $S(q,\omega)$: agreement with the TLL and nonlinear LL theory and with new analytical relations derived for the HR model
- Quantitative estimation of $S(q,\omega)$ also in the high-energy regime, beyond the predictions of LL theories.
- Non-universality/similarities in HRs and 1D ^{4}He. In particular, at high densities both systems show a flat $S(q,\omega)$ at special wave vectors $q_j=2\pi j/a$, in agreement with a previous theoretical prediction for HR.
- Peculiar structure of the HR spectrum in the high-density and high-momentum regime, which can be described in terms of the p-h boundaries of two renormalized IFGs