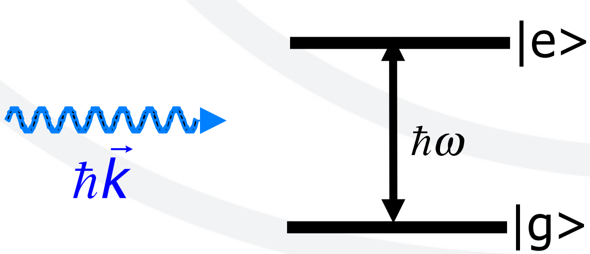
INTERACTING IONS: BLACKBOARD

Trapped Atom-Light Interaction

2-level atom trapped in harmonic potential



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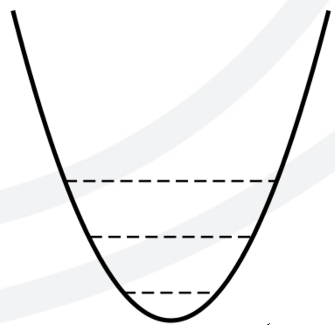

$$H_{int} = \tfrac{1}{2}\hbar\omega\sigma_z \qquad H_{ext} = \hbar\nu\left(a^{\dagger}a + \frac{1}{2}\right)$$

Interaction with near resonant lin.pol. travelling wave; lowest order in multipole expansion

$$H_L = \hbar\Omega_R\sigma_x \cos(kz - \omega_L t + \phi)$$

Rabi frequency $\Omega_R \equiv d_{eg} \cdot F_0 / \hbar$

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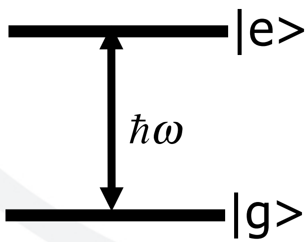
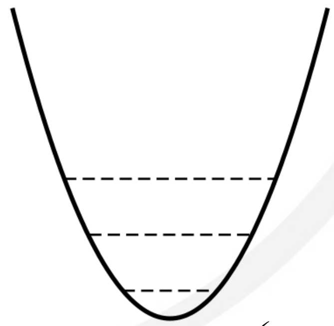


$\hbar\vec{k}$; $\hbar\omega$

$\otimes$



$$H_{\text{int}} = \tfrac{1}{2}\hbar\omega\sigma_z \qquad H_{\text{ext}} = \hbar\nu\left(a^\dagger a + \frac{1}{2}\right)$$

Interaction with near resonant lin.pol. travelling wave; lowest order in multipole expansion

$$H_L = \hbar\Omega_R\sigma_x\cos(kz - \omega_L t + \phi)$$

Rabi frequency $\Omega_R \equiv d_{eg}\cdot F_0 / \hbar$

$$= \frac{1}{2}\hbar\Omega_R(\sigma_+ + \sigma_-)\left(e^{i(kz-\omega_L t+\phi)} + e^{-i(kz-\omega_L t+\phi)}\right)$$

With position operator $\hat{z} = \sqrt{\frac{\hbar}{2m\nu}}(a^\dagger + a) = \Delta z(a^\dagger + a)$

and Lamb-Dicke parameter $\eta \equiv \Delta z\, k = 2\pi\frac{\Delta z}{\lambda} = \sqrt{\frac{(\hbar k)^2}{2m} / \hbar\nu}$

$$\Rightarrow H_L = \frac{1}{2}\hbar\Omega_R(\sigma_+ + \sigma_-)\left(e^{i\left[\eta(a^\dagger+a)-\omega_L t+\phi\right]} + H.c.\right)$$

Trapped Atom-Light Interaction

Unitary transformation $\tilde{H}_L = e^{\frac{i}{\hbar}H_0 t} H_L e^{-\frac{i}{\hbar}H_0 t}$

with $H_o = H_{ext} + H_{\text{int}} = \hbar\nu(a^\dagger a + \frac{1}{2}) + \frac{1}{2}\hbar\omega\sigma_z$

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left[e^{i\left[(\omega-\omega_L)t+\phi\right]}\sigma_+ e^{i\eta\left[a^\dagger(t)+a(t)\right]} + H.c.\right]$$

where $a^\dagger(t) = a^\dagger e^{i\nu t}$ and $a(t) = a e^{-i\nu t}$

Expansion in η :

$$\tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left[e^{i\left[(\omega-\omega_L)t+\phi\right]}\sigma_+\left[1 + i\eta(a^+ e^{i\nu t} + a e^{-i\nu t}) + ...\right] + H.c\right]$$

Lowest order in η :

$$\tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left[e^{i\left[(\omega-\omega_L)t+\phi\right]}\sigma_+ + i\eta\left[e^{i(\omega-\omega_L+\nu)t}\sigma_+ a^+ + e^{i(\omega-\omega_L-\nu)t}\sigma_+ a\right] + H.c\right]$$

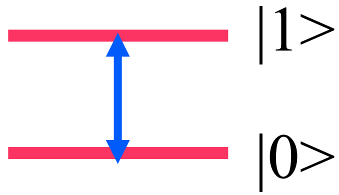
$\omega_L = \omega$, "Carrier"

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi}\right)$$

$\omega_L = \omega - \nu, \phi = 0$, "red sideband"

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\eta\left[\sigma_+ a + \sigma_- a^+\right]$$

Single Qubit Gate



$$\omega_L = \omega$$

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\left(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi}\right)$$

Time evolution operator (interaction picture) $U(t) = \exp\left(-\frac{i}{\hbar}\tilde{H}_L t\right)$

With $\phi = 0$: $U(\vartheta) = \exp(-i\frac{\vartheta}{2}\sigma_x) = \begin{pmatrix} \cos\vartheta/2 & -i\sin\vartheta/2 \\ -i\sin\vartheta/2 & \cos\vartheta/2 \end{pmatrix}$ where $\vartheta \equiv \Omega t$

Trapped Atom-Light Interaction

$\omega_L = \omega, \phi = 0,$ "Carrier"

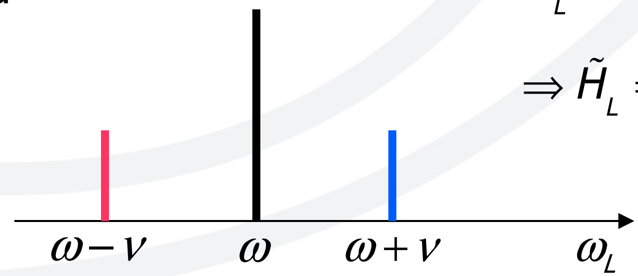
$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\sigma_x$$

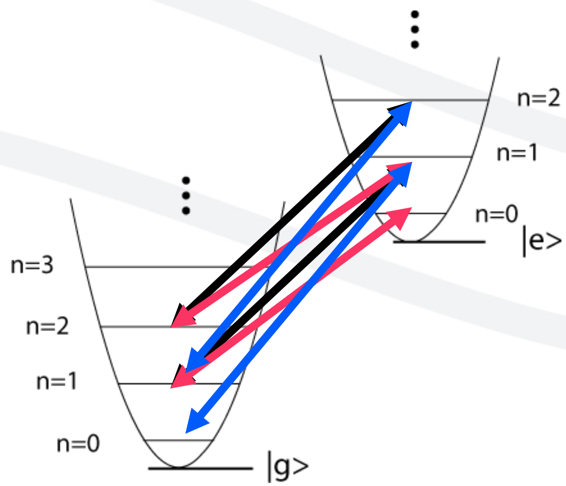
$\omega_L = \omega - \nu,$ (ϕ=0) "red sideband"

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\eta\left[\sigma_+ a + \sigma_- a^+\right]$$

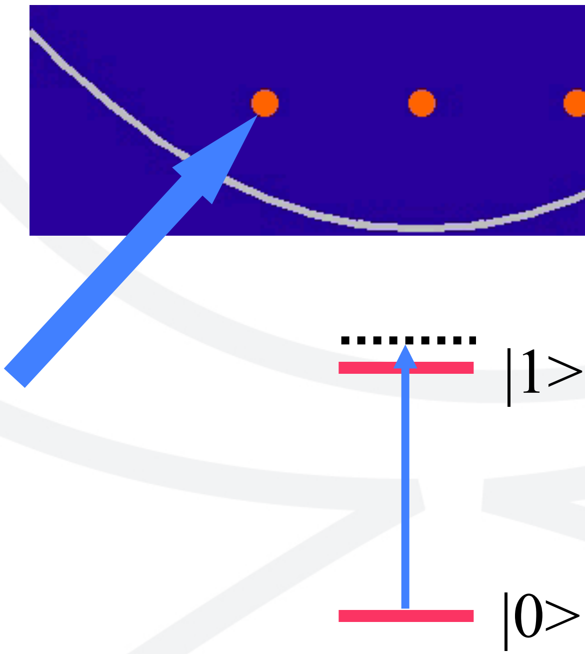
$\omega_L = \omega + \nu,$ (ϕ=0) "blue sideband"

$$\Rightarrow \tilde{H}_L = \frac{1}{2}\hbar\Omega_R\eta\left[\sigma_+ a^\dagger + \sigma_- a\right]$$





Two-Qubit Gate



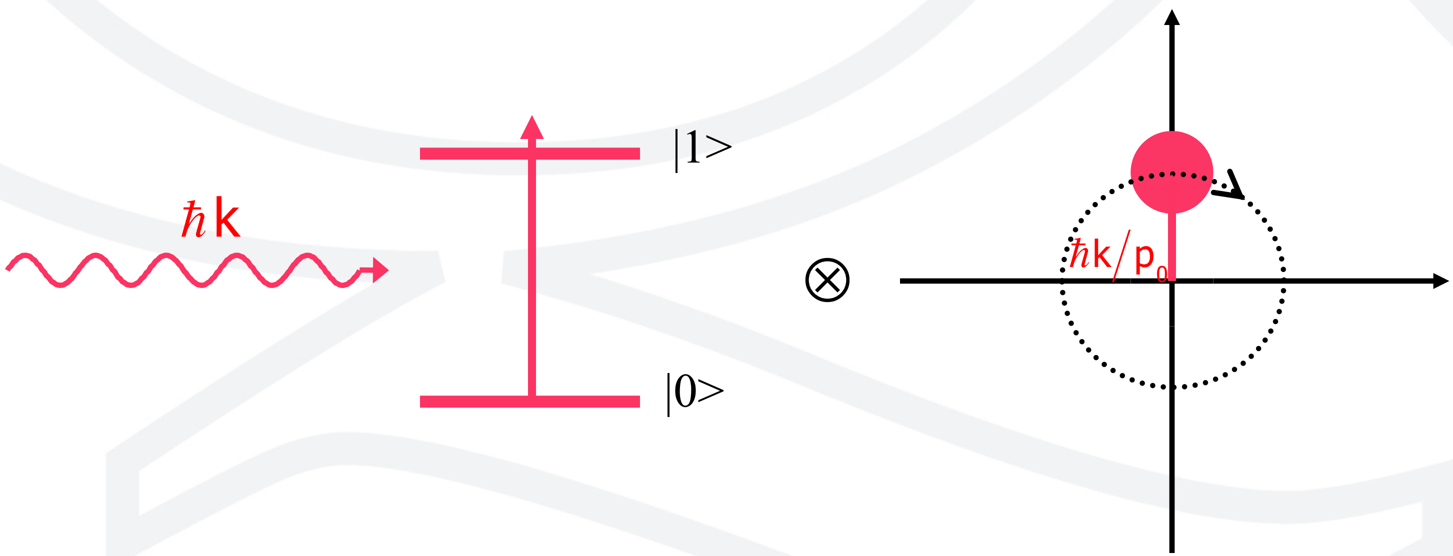
Electromagnetic radiation used to

- couple internal and external degrees of freedom

$$\eta \equiv \frac{\hbar k}{2p_0} = \frac{\Delta z}{\lambda} 2\pi$$

Coupling internal and motional states

Semi-classical illustration. QM calculation



$$H_I \propto \sigma_+ \exp\left[i\eta\left(a + a^+\right)\right] + \text{ H.c.}$$