Non-equilibrium thermodynamics of quantum processes: three

or an invitation to study stochastic thermodynamics of quantum processes

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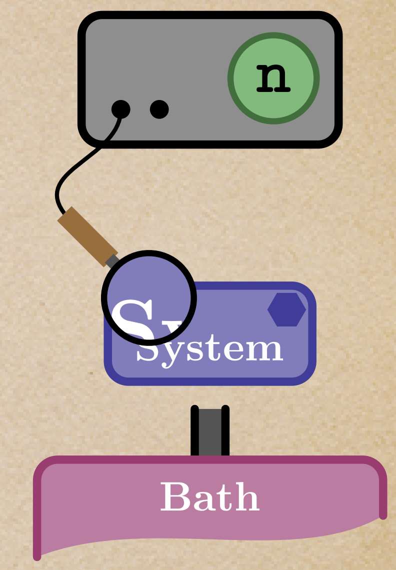
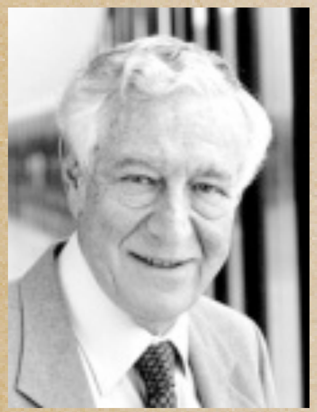
(ICTP, Trieste, 4 September 2017)

Content & structure

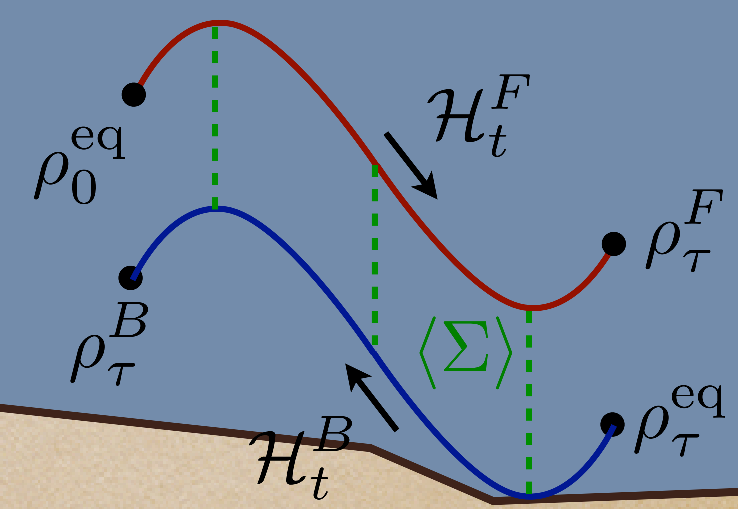


Non-equilibrium definition of thermodynamic work: fluctuation theorems



Landauer principle & quantum (open-system) dynamics

Irreversibility & entropy production in closed q-systems



Quantum correlations, coherences and thermodynamics

QTEQ QUANTUM TECHNOLOGY at QUEEN'S

Entropy production

Jarzynski equality $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$

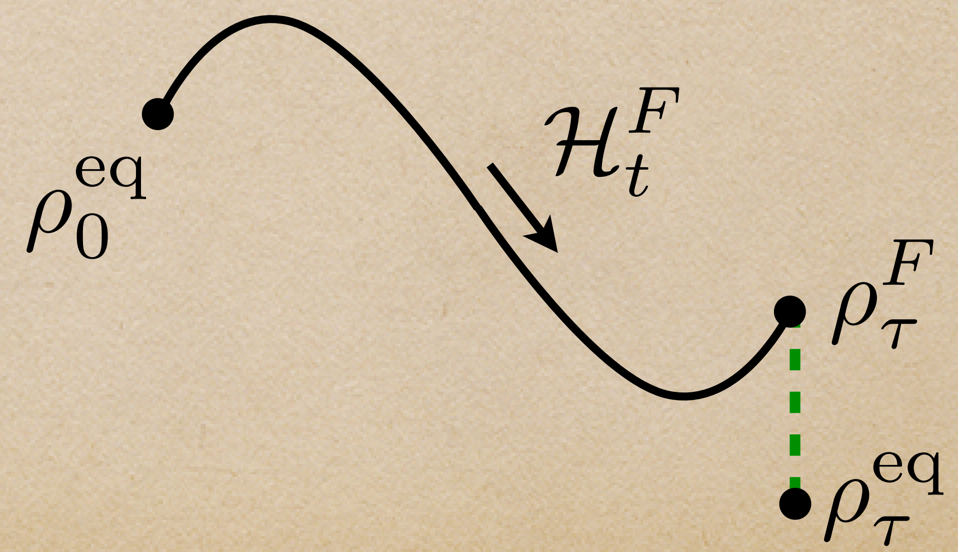
$$\langle W \rangle \geq \Delta F \longrightarrow \boxed{\langle \Sigma \rangle = \beta(\langle W \rangle - \Delta F) \geq 0}$$

(Clausius law)

$$\Sigma = \beta(W - \Delta F)$$

Entropy production

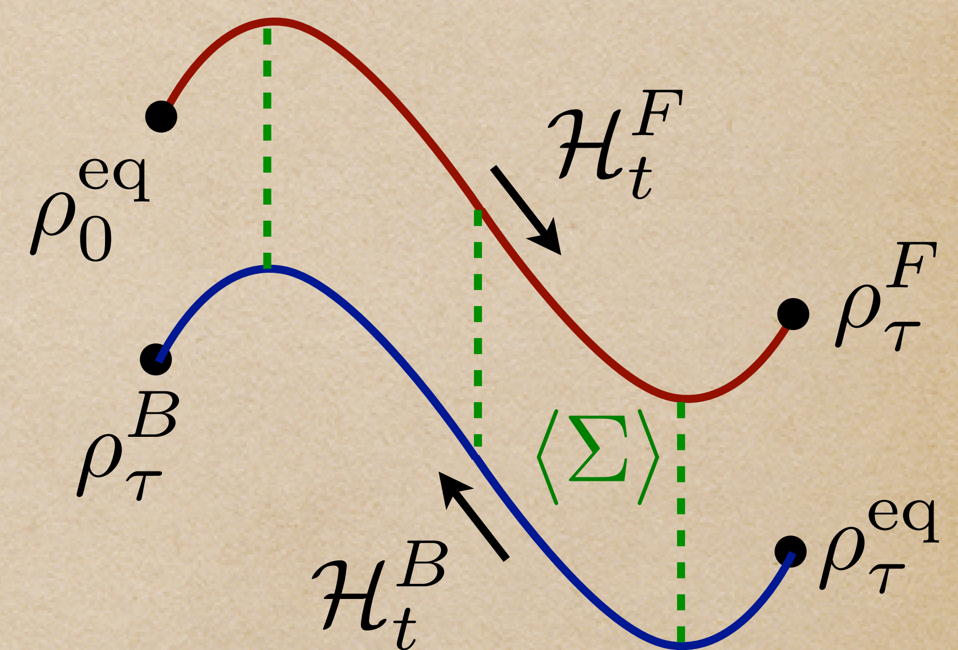
(Prigogine, de Groot)



Entropy production

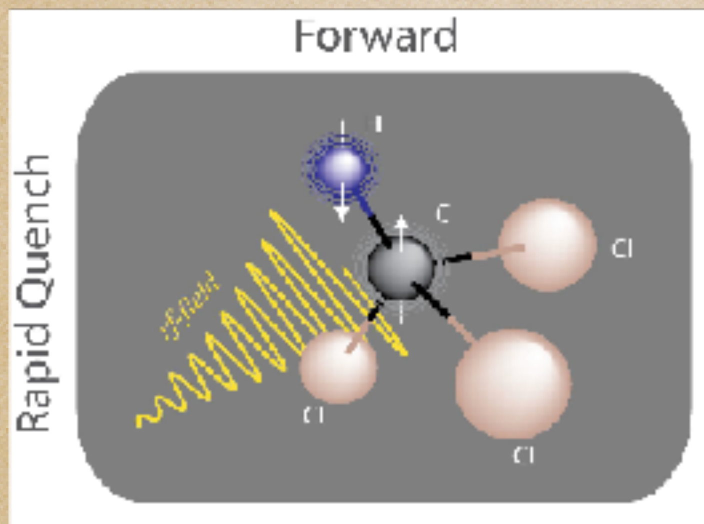
$$\frac{P_F(W)}{P_B(-W)} = e^{\beta(W-\Delta F)}$$ Tasaki-Crooks

$$\boxed{\begin{aligned}\langle\Sigma\rangle &= S\left(\rho_t^{\mathrm{F}} \| \rho_{\tau-t}^{\mathrm{B}}\right)\\ &= \mathrm{tr}\left[\rho_t^{\mathrm{F}}\left(\ln\rho_t^{\mathrm{F}} - \ln\rho_{\tau-t}^{\mathrm{B}}\right)\right]\end{aligned}}$$



T. B. Batalhao, et al. Phys. Rev. Lett. 115, 190601 (2015)

Experimental assessment of entropy production

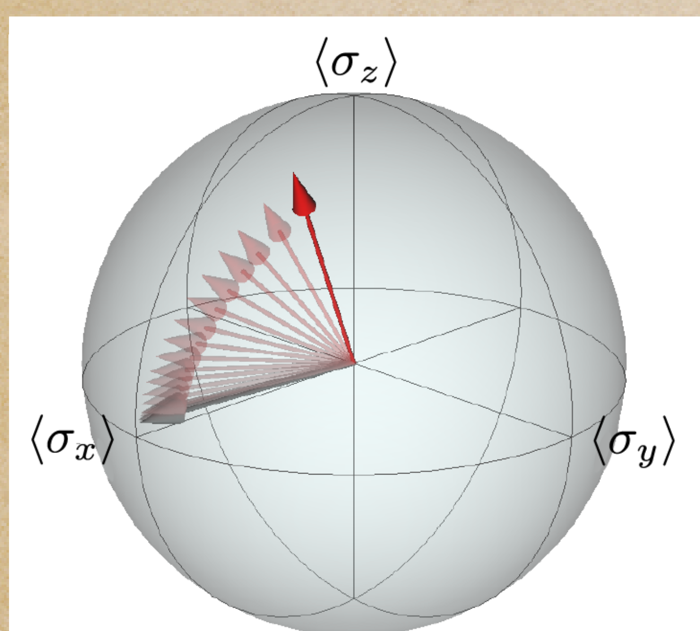


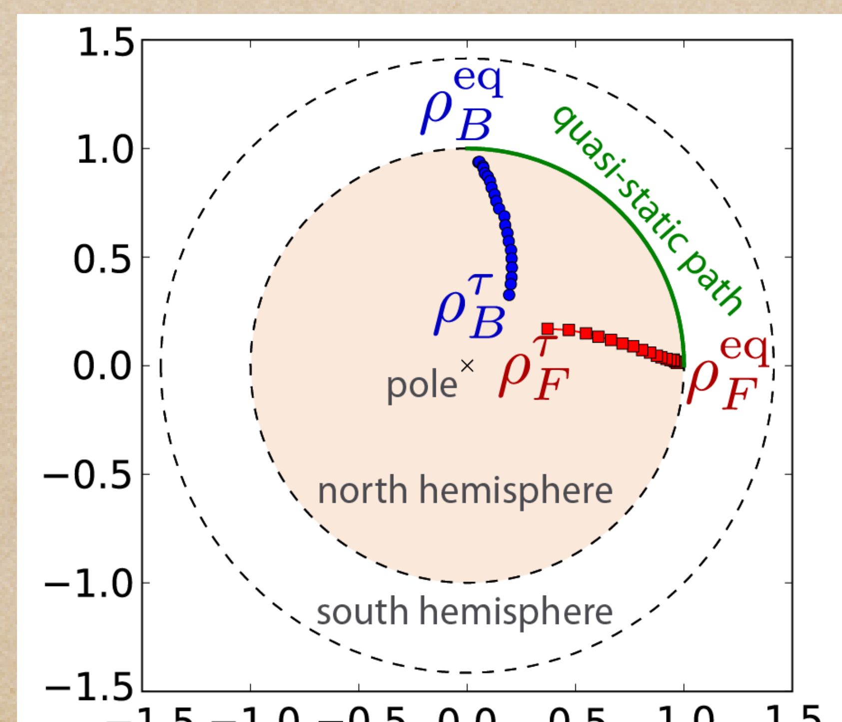
$$\mathcal{H}_t^{\mathrm{F}} = 2\pi\hbar\nu(t)\left(\sigma_x^{\mathrm{C}}\cos\phi(t) + \sigma_y^{\mathrm{C}}\sin\phi(t)\right)$$

$$\phi(t) = \pi t/(2\tau)$$

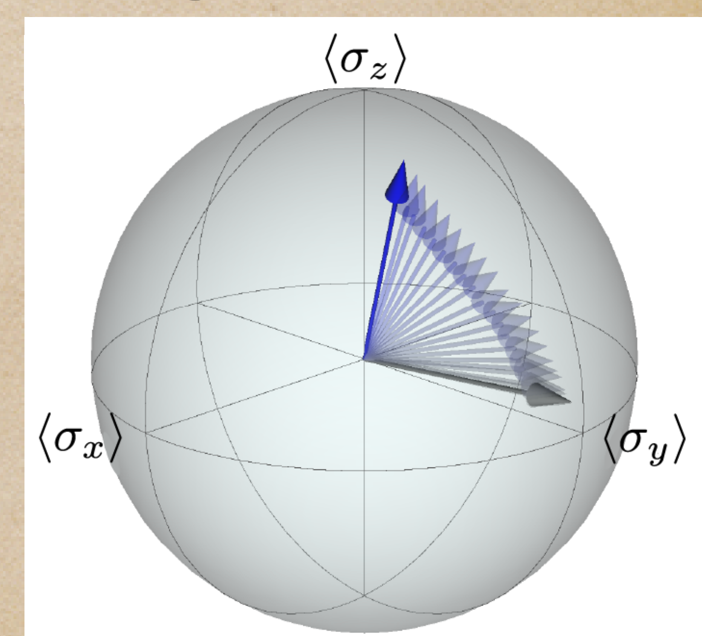
$$\nu(t) = \nu_0\,(1 - t/\tau) + \nu_\tau t/\tau$$

Forward





Backward



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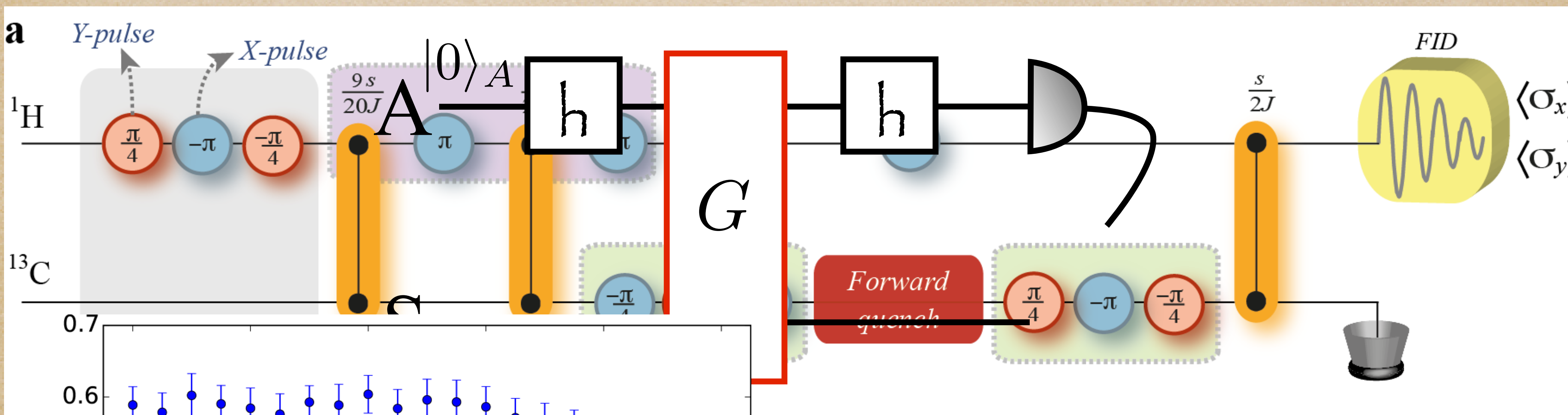
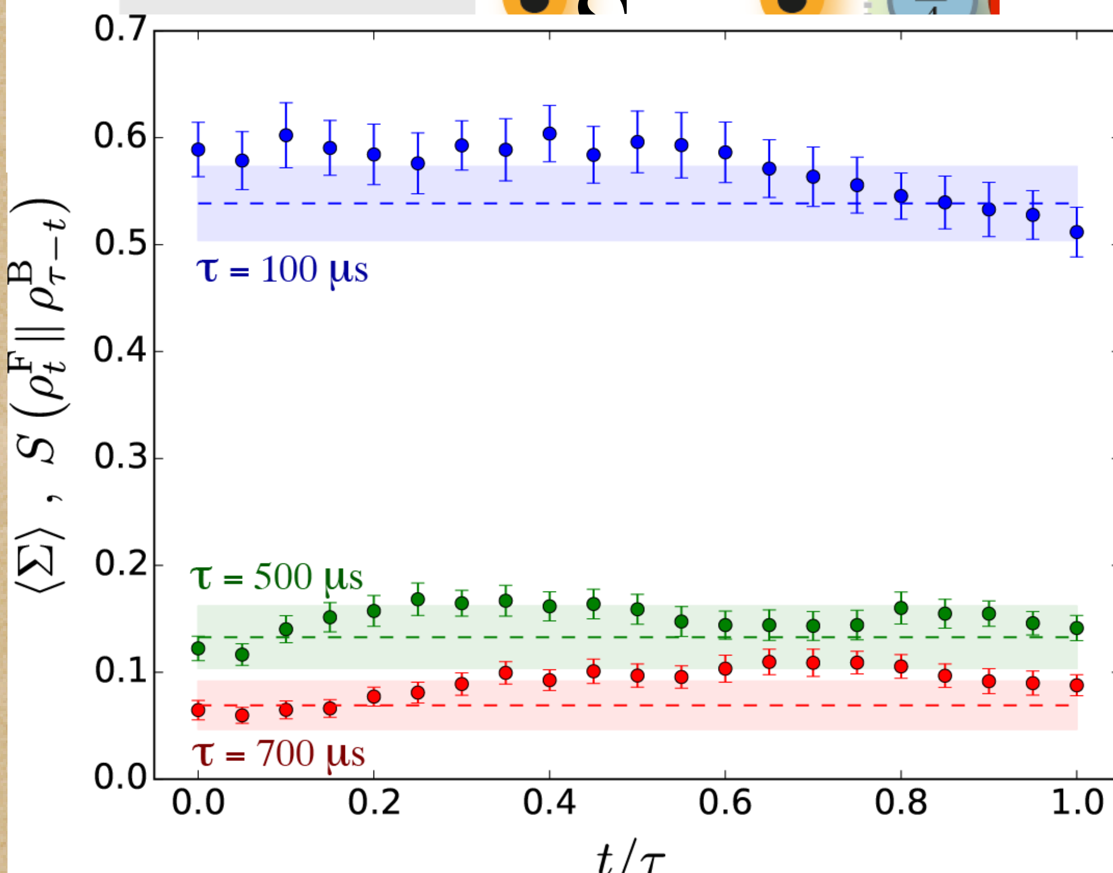
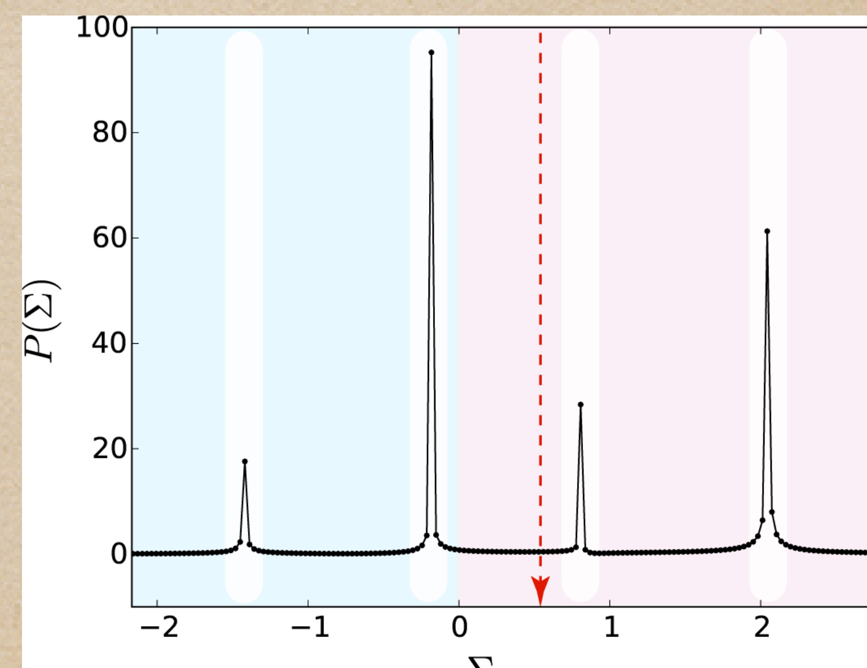




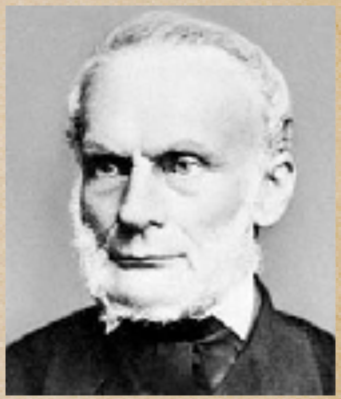


T. B. Batalhao, Phys. Rev. Lett. 115, 190601 (2015)

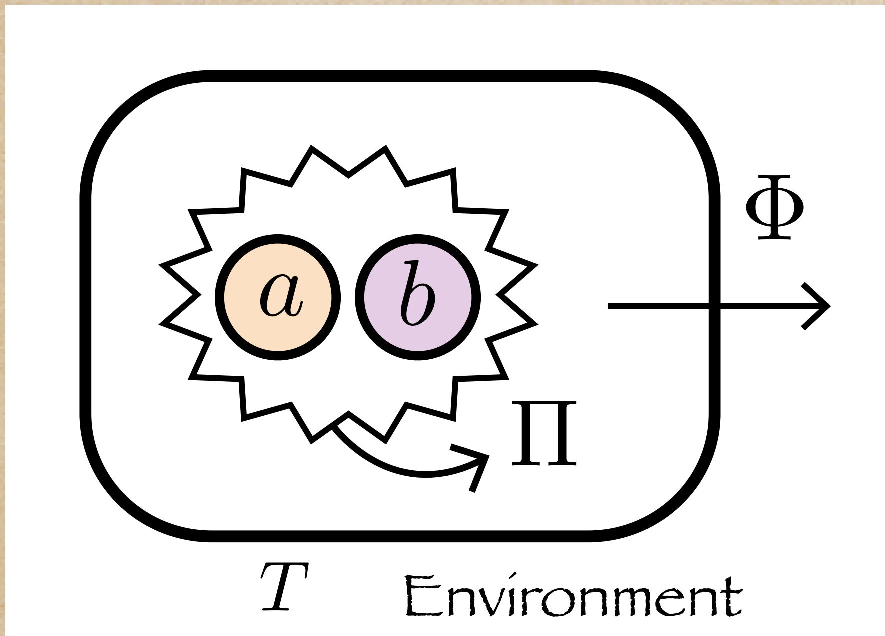
Entropy production in open systems

Second Law: $\Delta S \geq \int \frac{\delta Q}{T}$ $\qquad$ $\Delta S = \Sigma + \int \frac{\delta Q}{T}$

Clausius: "Uncompensated transformation" $\qquad$ Entropy production



Rudolf Clausius



$$\boxed{\frac{\mathrm{d}S}{\mathrm{d}t} = \Phi(t) + \Pi(t)}$$

$\Pi(t)$ Entropy production rate

$\Phi(t)$ Entropy flux rate

Stationary state $\Pi_{\mathrm{s}} = -\Phi_{\mathrm{s}}$

Π, Φ are not observable. No continuity equation for entropy

QTEQ
QUANTUM TECHNOLOGY at QUEEN'S

Entropy production which entropy to use?

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}(\rho)$$

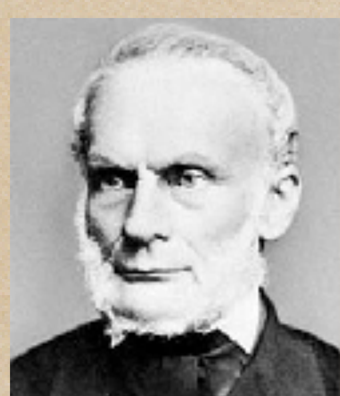
$$\Pi_{vN}(t) = -\partial_t S_{vN}(\rho|\rho_t^*)$$

H Spohn, J Lebowitz
S Deffner, E Lutz
H-P Breuer

For thermal bath: $\Pi_{vN}(t) = \frac{dS_{vN}}{dt} + \Phi_{vN}(t)$

$$= \frac{dS_{vN}}{dt} - \frac{1}{T}\mathrm{Tr}[H\mathcal{D}(\rho)]$$

$$= \frac{dS_{vN}}{dt} + \frac{\Phi_E(t)}{T}$$



Rudolf Clausius

Energy flux from system to environment

Entropy production

Π, Φ are not observable. No continuity equation for entropy

No unifying theory of entropy production, to date

$$\Pi(t)$$

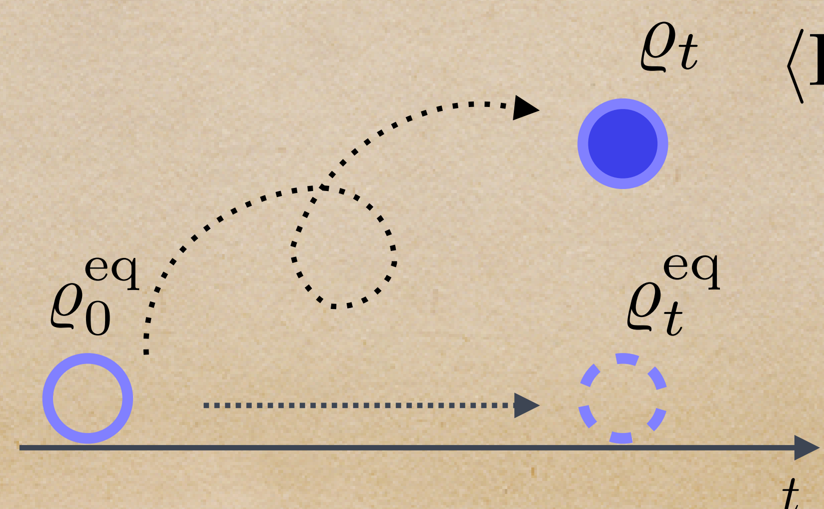
L Onsager
(theory of chemical kinetics)
related to energy currents

J Schnakenberger
transition rates of a system
governed by a master equation

G Gallavotti & E Cohen
integral fluctuation theorem
$\langle e^{-\Sigma} \rangle = 1$

Ch. Jarzynski
& Tasaki-Crooks

$$\frac{P_{\mathrm{f}}(+\Sigma)}{P_{\mathrm{b}}(-\Sigma)} = e^{\Sigma}$$



$$\langle \Pi(t) \rangle = -\partial_t S(\varrho_t || \varrho_t^{\mathrm{eq}}) - \mathrm{Tr}[\varrho_t \partial_t \ln \varrho_t^{\mathrm{eq}}]$$

H Spohn, J Lebowitz

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Entropy production which entropy to use?

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}(\rho)$$

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$$= \frac{dS_{vN}}{dt} - \frac{1}{T}\mathrm{Tr}[H\mathcal{D}(\rho)]$$

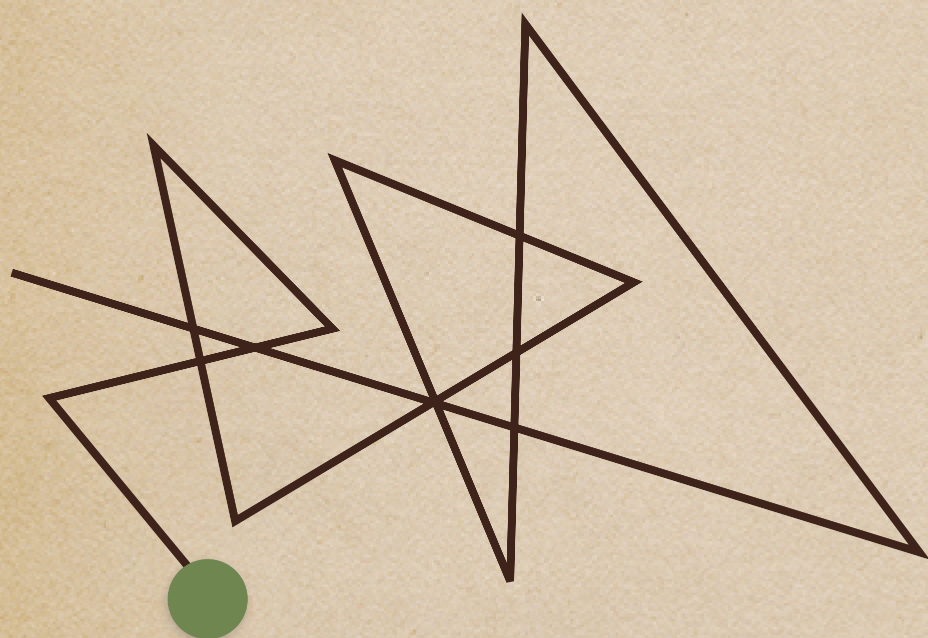
$$= \frac{dS_{vN}}{dt} + \frac{\Phi_E(t)}{T}$$

$\Pi(t), \Phi(t)$ diverge as $T \to 0$

Idealised large heat reservoirs

Several attempts at fixing it....

Entropy production which entropy to use?



$$\Sigma(t) \equiv D[\rho(t)||\rho_s(t)\prod_r \rho_r^{\mathrm{eq}}] \geq 0$$

Nice physical interpretation:
how far is the state of the compound
a factorised system-environment state?

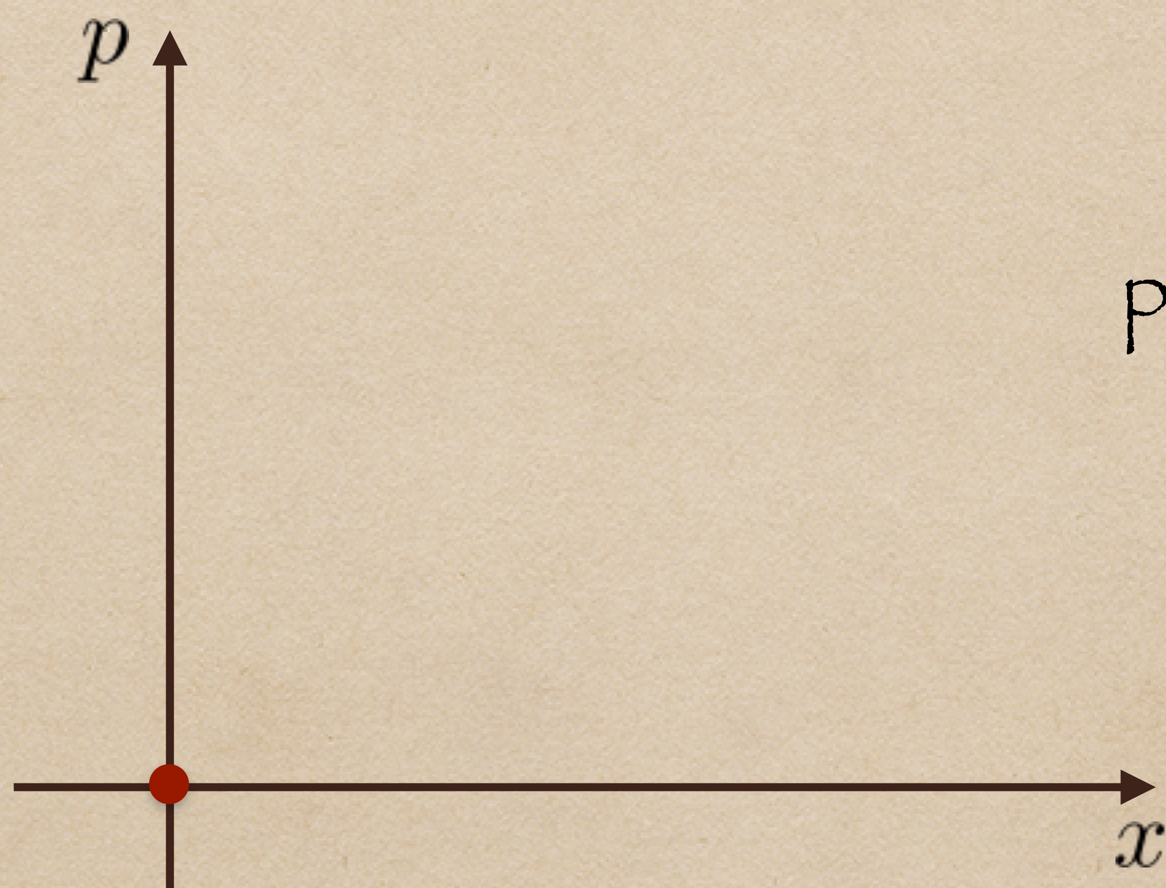
However: it does not increase monotonically in time (signature of recurrence?). Monotonicity only for large environments

Esposito et al. NJP 12, 013013 ('10); Pucci et al. J Stat P04005 ('13)

QTEQ QUANTUM TECHNOLOGY at QUEEN'S

Phase space

Classical



x p

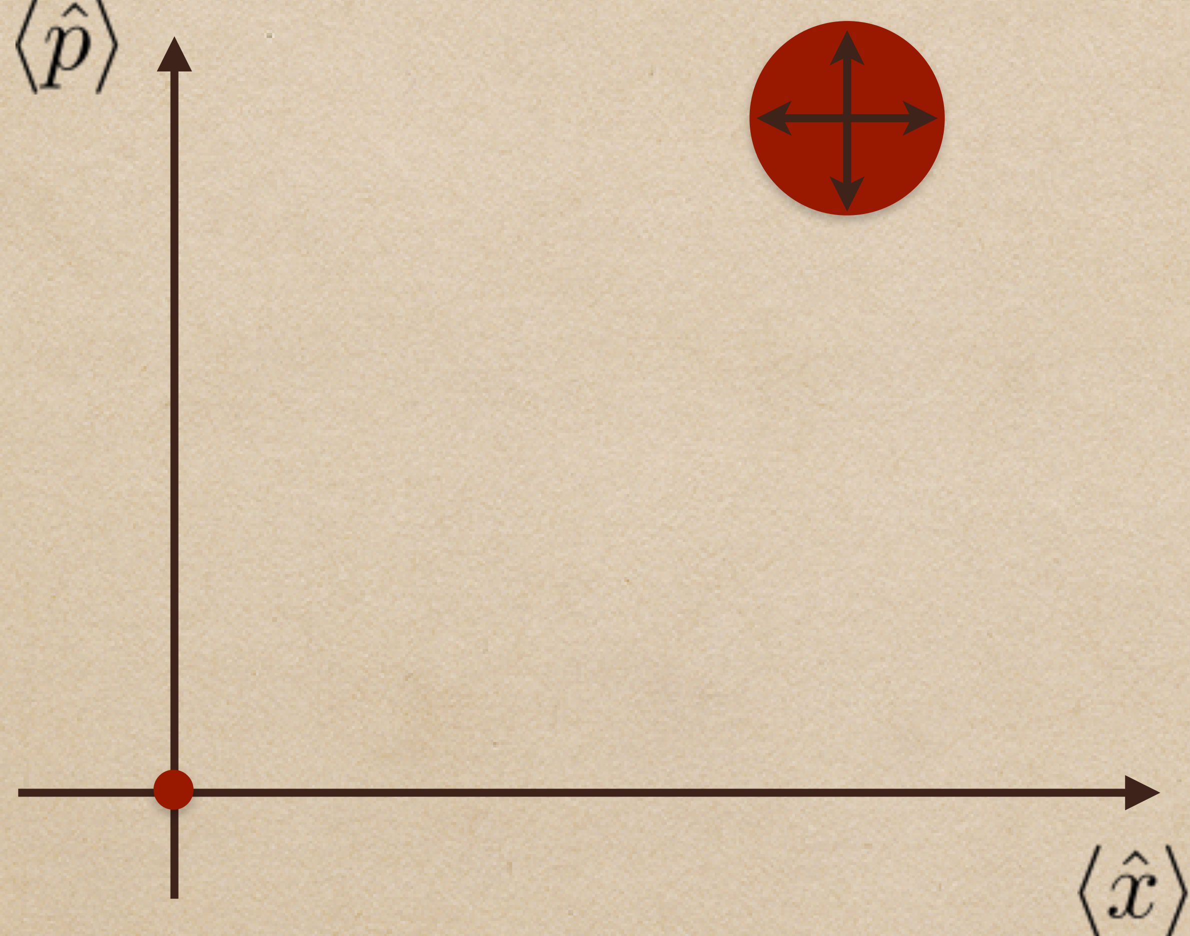
position & momentum

Fully deterministic motion: no uncertainty

QTEQ QUANTUM TECHNOLOGY at QUEEN'S

Phase space

Quantum



$$\hat{x} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$$

$$\hat{p} = i(\hat{a}^\dagger - \hat{a})/\sqrt{2}$$

quadrature operators

$$[\hat{x}, \hat{p}] = i$$

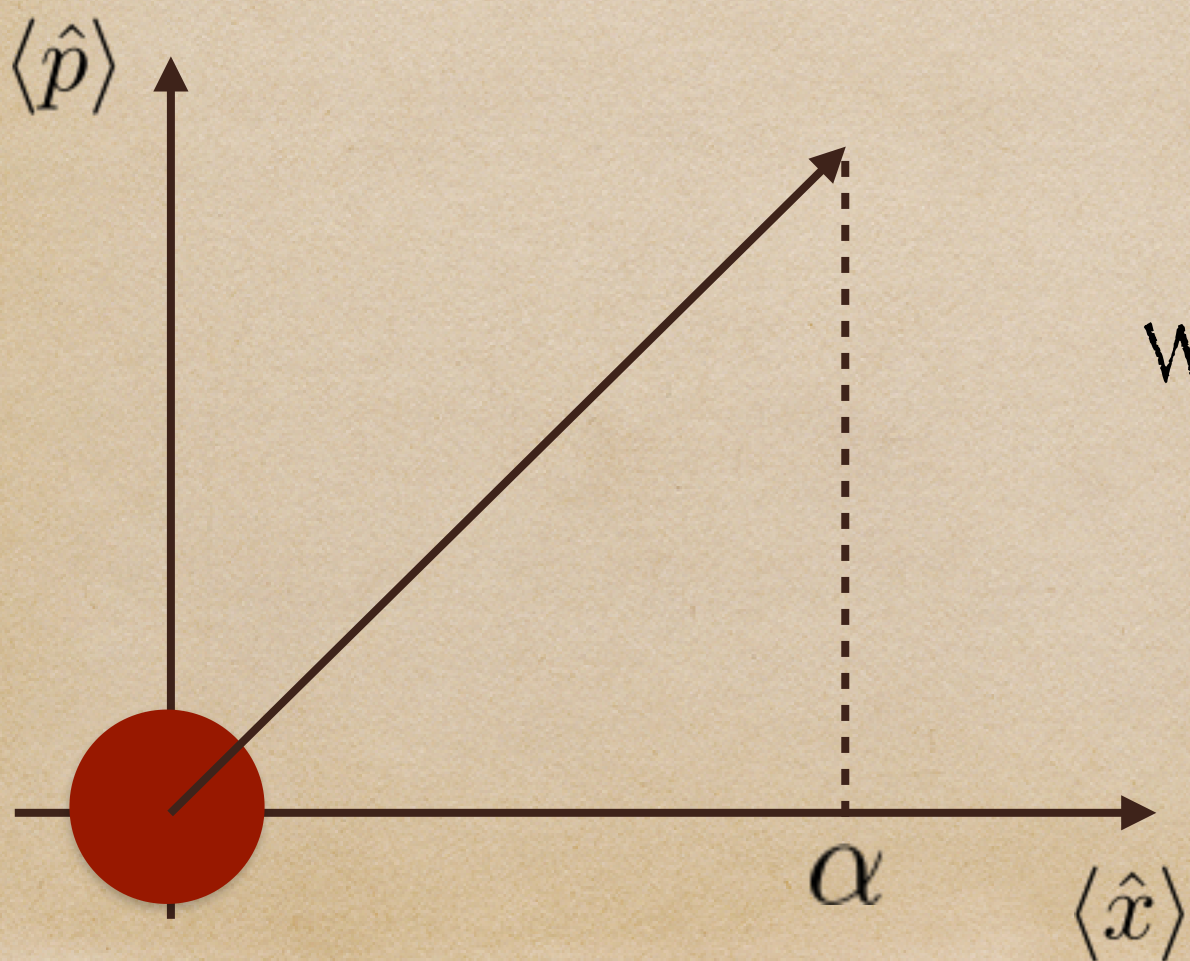
$$(\Delta x)(\Delta p) \geq 1/2$$

Determinism is lost: strictly speaking, no phase space!

Yet, very useful...

1-to-1 correspondence between states and phase-space description

$$\rho = \frac{1}{\pi^N}\int \cdots \int \chi(\alpha_1, \dots, \alpha_N)\left(\otimes_{j=1}^{N} \hat{D}_j^\dagger(\alpha_j)\right) d^2\alpha_1 \cdots d^2\alpha_N$$



$$\hat{D}(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}$$

Weyl displacement operator

Yet, very useful...

1-to-1 correspondence between states and phase-space description

$$\rho = \frac{1}{\pi^N} \int \cdots \int \chi(\alpha_1, \ldots, \alpha_N) \left(\otimes_{j=1}^{N} \hat{D}_j^\dagger(\alpha_j) \right) d^2\alpha_1 \cdots d^2\alpha_N$$

$$\mathrm{Tr}[\hat{D}(\alpha)\hat{D}(\beta)] = \pi\delta^2(\alpha - \beta)$$

$$\chi(\alpha_1, \ldots, \alpha_N) = \mathrm{Tr}[\rho(\otimes_{j=1}^{N} \hat{D}_j(\alpha_j))]$$

Weyl characteristic function

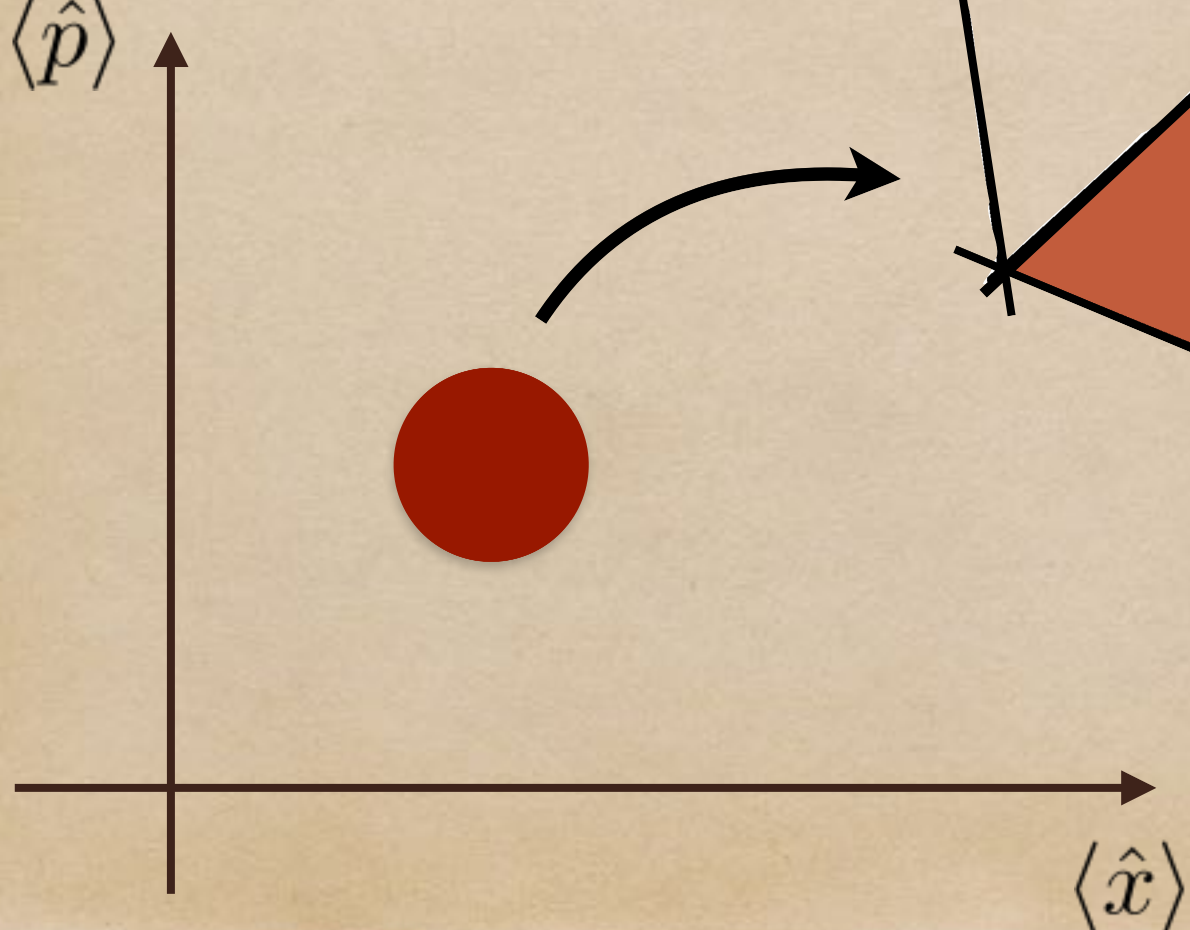
$$W_\rho(\xi_1 \cdots \xi_N) = \mathcal{F}^{\otimes N}_{\xi_1 \cdots \xi_N}[\chi(\alpha_1, \cdots, \alpha_N)]$$

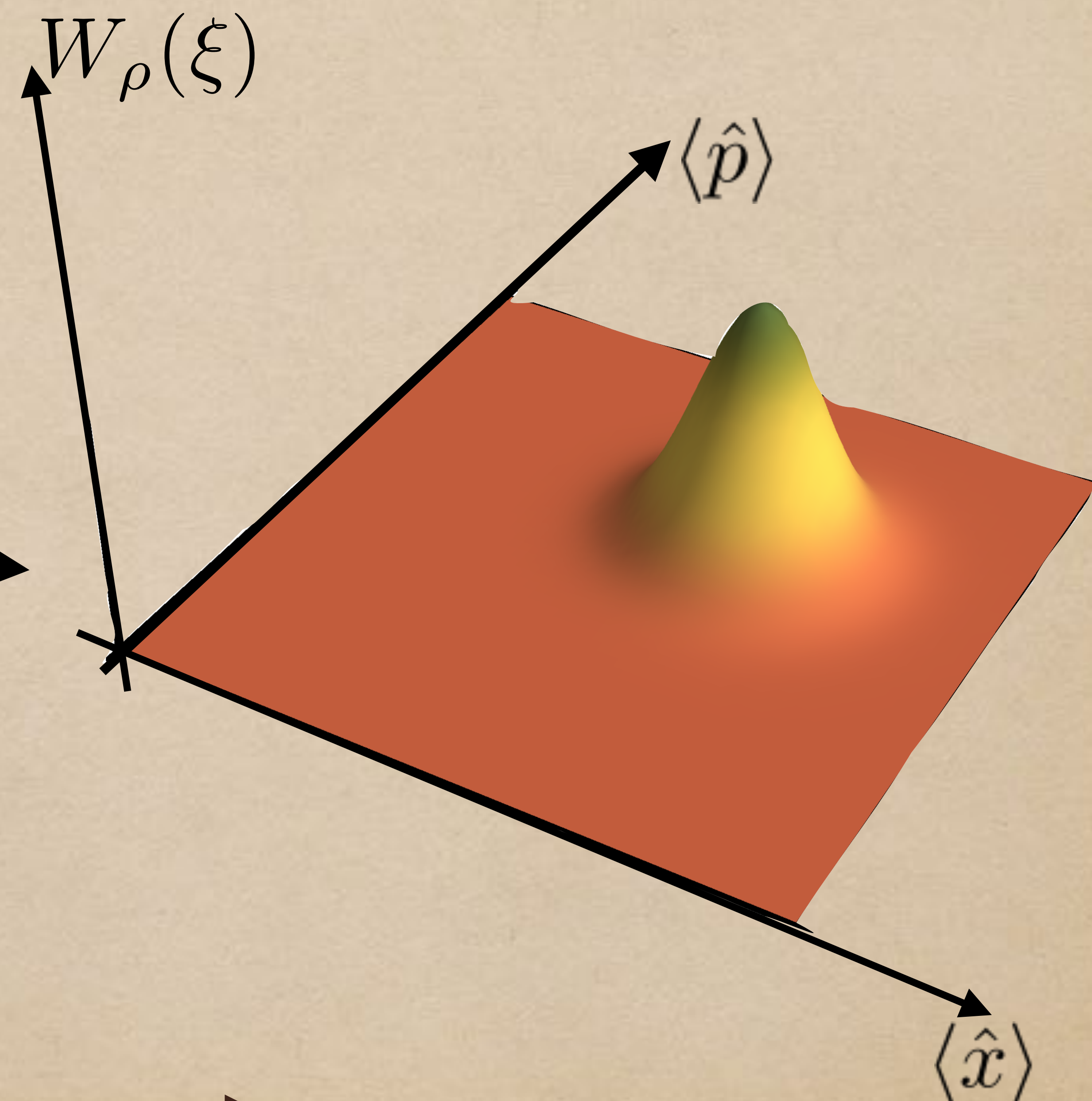
Wigner function

Yet, very useful...

$$\langle \hat{x} \rangle = \sqrt{2}\xi_r$$

$$\langle \hat{p} \rangle = \sqrt{2}\xi_i$$





QTEQ QUANTUM TECHNOLOGY at QUEEN'S

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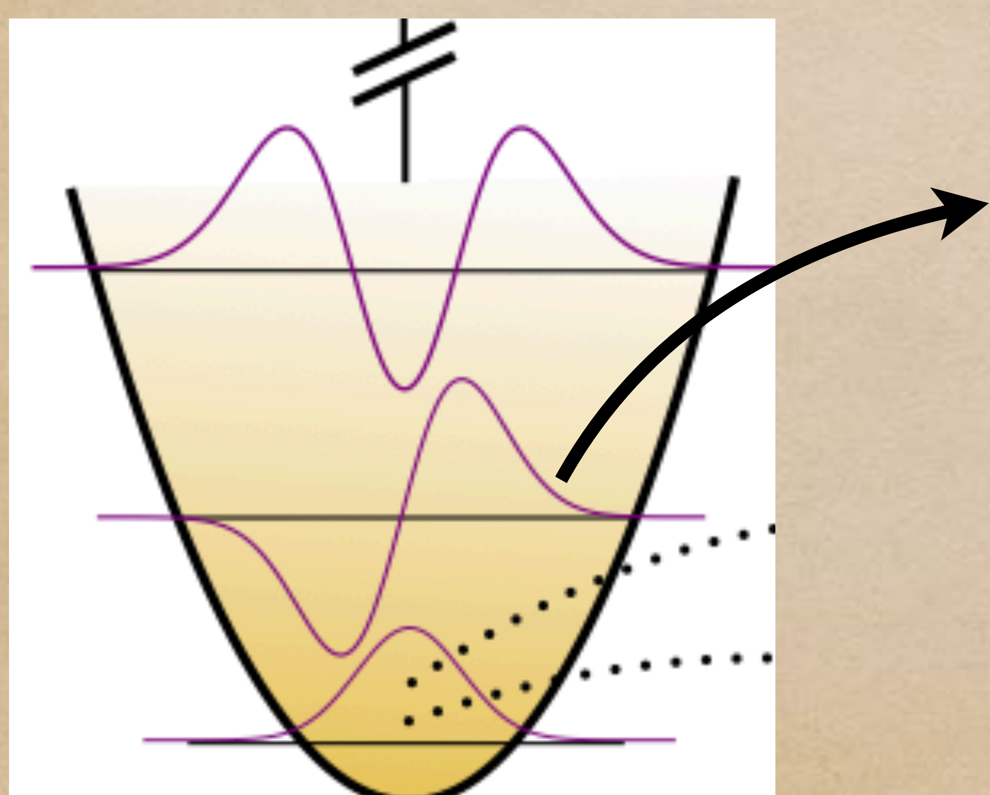

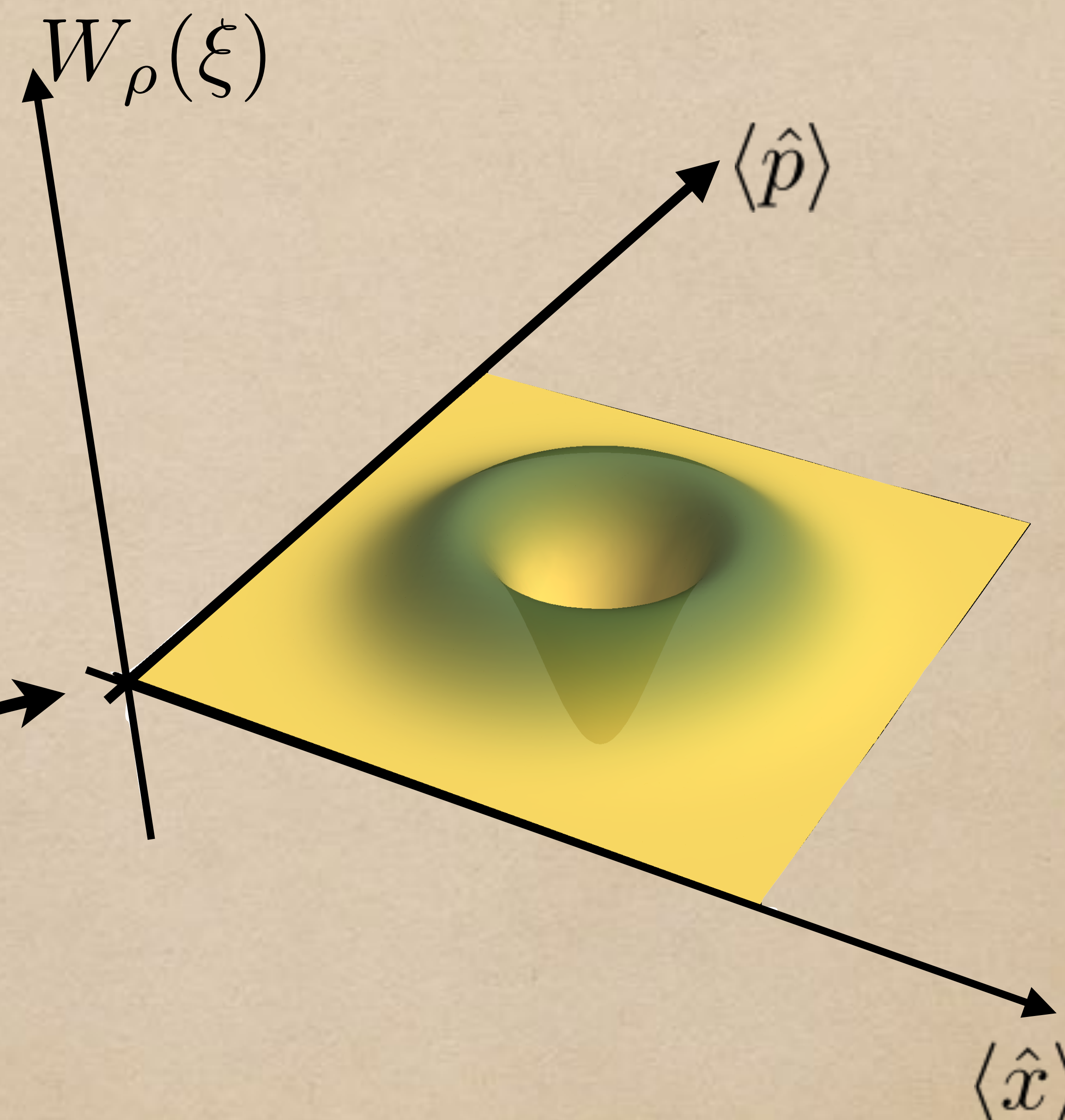

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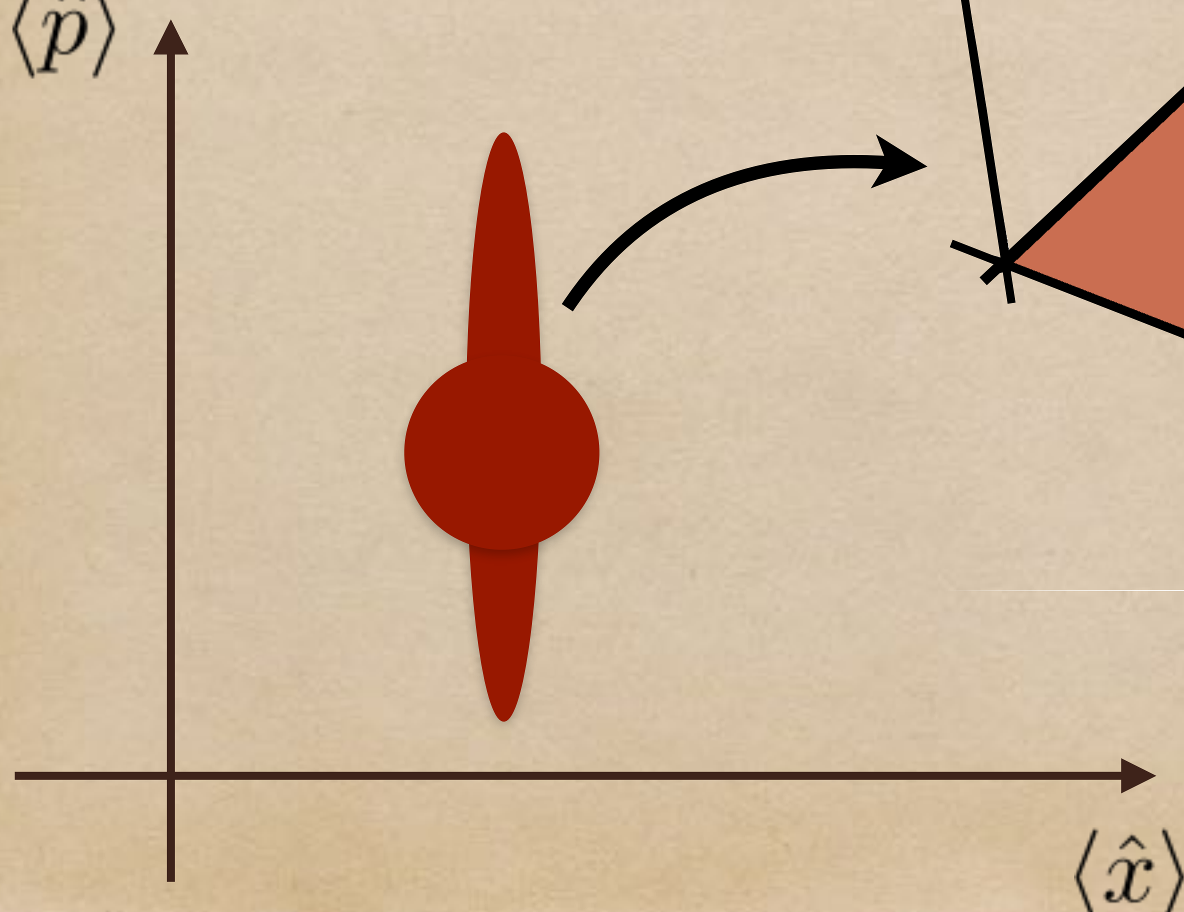
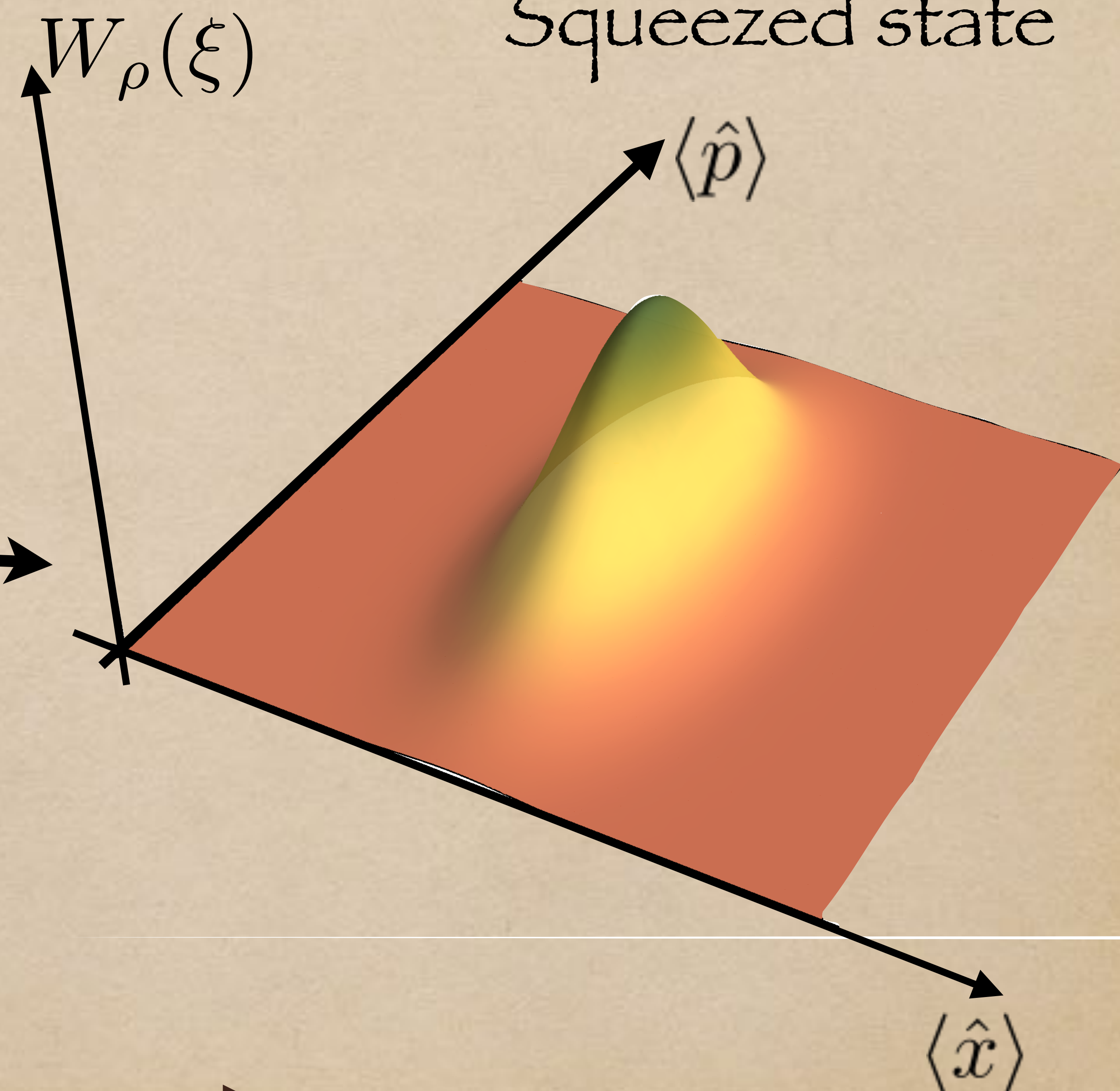
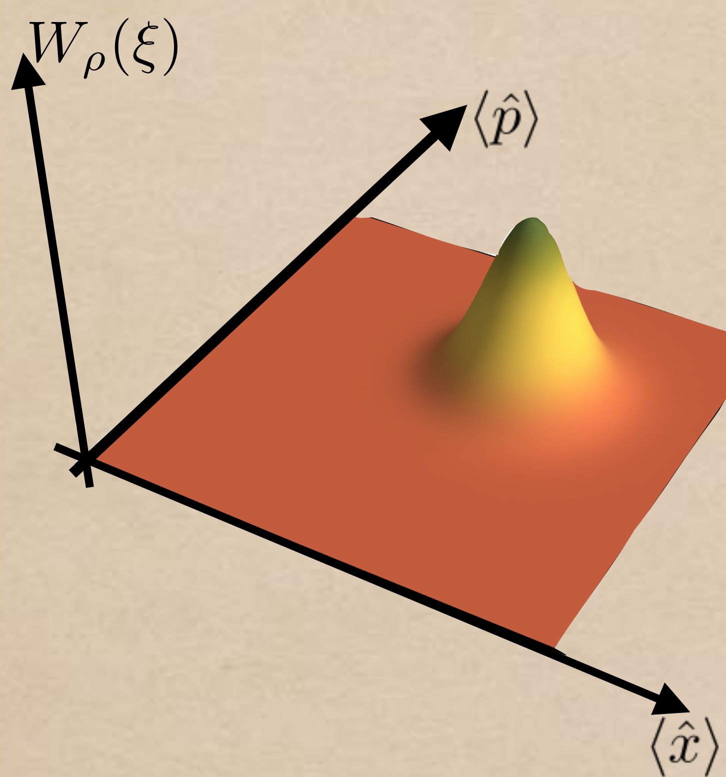
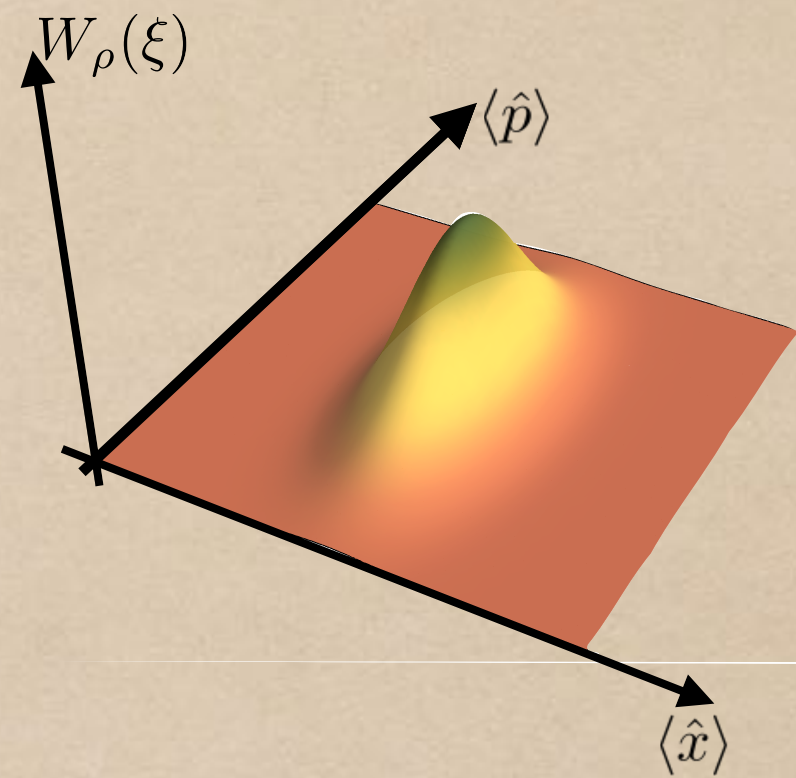


Squeezed state



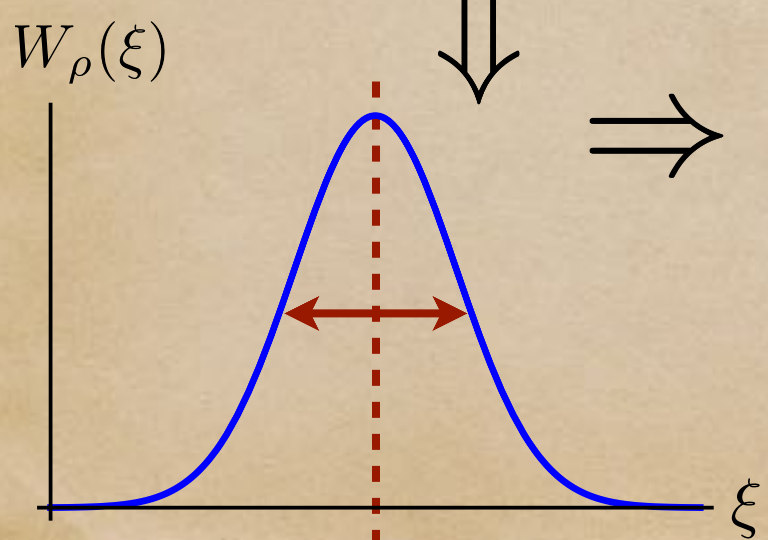
Gaussian states





Gaussian states:= states with a Gaussian Wigner function

$\Downarrow$



$\Rightarrow$ 1st and 2nd moments of phase-space variables

$$\mathbf{v} = (x_1, p_1, x_2, p_2, \ldots, x_N, p_N)$$

$$\sigma_{ij} = \langle \{v_i, v_j\} \rangle - 2\langle v_i \rangle \langle v_j \rangle$$

Our proposal for q-harmonic systems

$$S = -\int \mathrm{d}^2\alpha \, W(\alpha) \ln W(\alpha)$$

Entropy of the Wigner function

For Gaussian states:

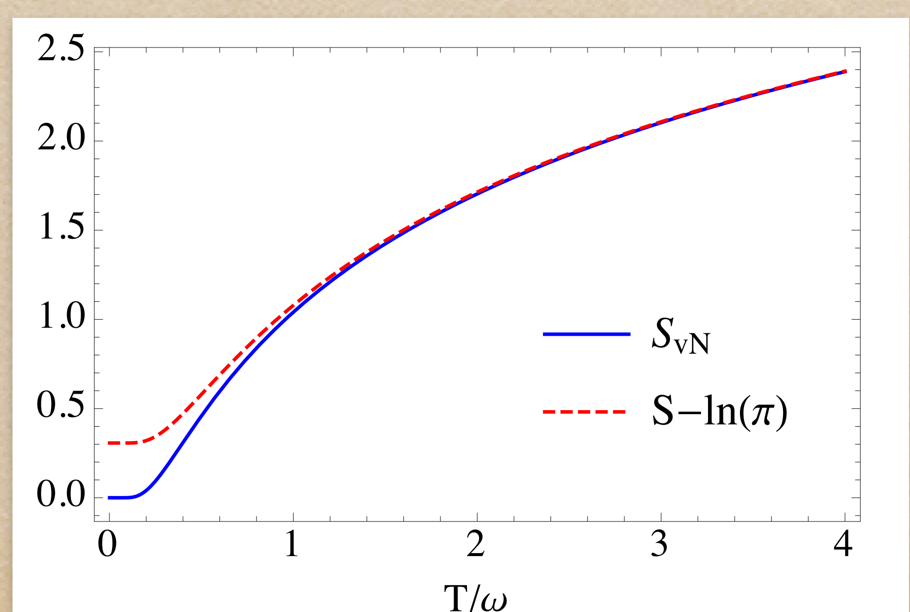
- coincides with Rènyi-2 entropy $S_2(\varrho) = -\ln \mathrm{Tr}\varrho^2$ → can be directly related to free energy difference

J C Baez, arXiv 1182.2098 (2011)

- satisfies the strong sub-additivity inequality → can be used to construct correlation measures $\mathcal{I}_2(\varrho_{a:b})$

G Adesso, et al., PRL 109, 190502 (2012)

- for thermal states:



$$\boxed{\Pi(t) = -\partial_t S(W(t)|W_{\mathrm{eq}}) \geq 0 \quad \text{(Gaussian states)}}$$

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

Why it makes sense

$$\Pi(t) = -\partial_t S(W(t)|W_{\mathrm{eq}})$$
$$= -\int \mathrm{d}^2\alpha\, \mathcal{D}(W) \ln(W/W_{\mathrm{eq}})$$

$$\Phi(t) = \int \mathrm{d}^2\alpha\, \mathcal{D}(W) \ln W_{\mathrm{eq}}$$

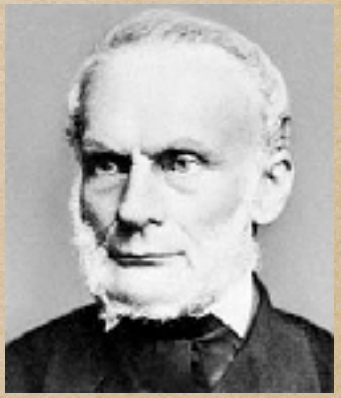
For a single harmonic oscillator in a thermal bath:

$$\Phi(t) = \frac{\gamma}{\bar{n}+1/2}(\langle a^\dagger a\rangle - \bar{n})$$

Observable!!

$$= \frac{\Phi_E}{\omega(\bar{n}+1/2)} \simeq \frac{\Phi_E}{T}$$

but no divergence at zero-temperature



Rudolf Clausius

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

Why it makes sense

$$\mathcal{D}(\rho) = -i\frac{\gamma}{2}[\hat{q}, \{\hat{p}, \rho\}] - \gamma m\omega(\bar{n} + 1/2)[\hat{q}, [\hat{q}, \rho]]$$

$$\Phi(t) = \frac{\gamma}{\bar{n}+1/2}\left[\frac{\langle \hat{p}^2\rangle}{m\omega} - (\bar{n}+1/2)\right]$$

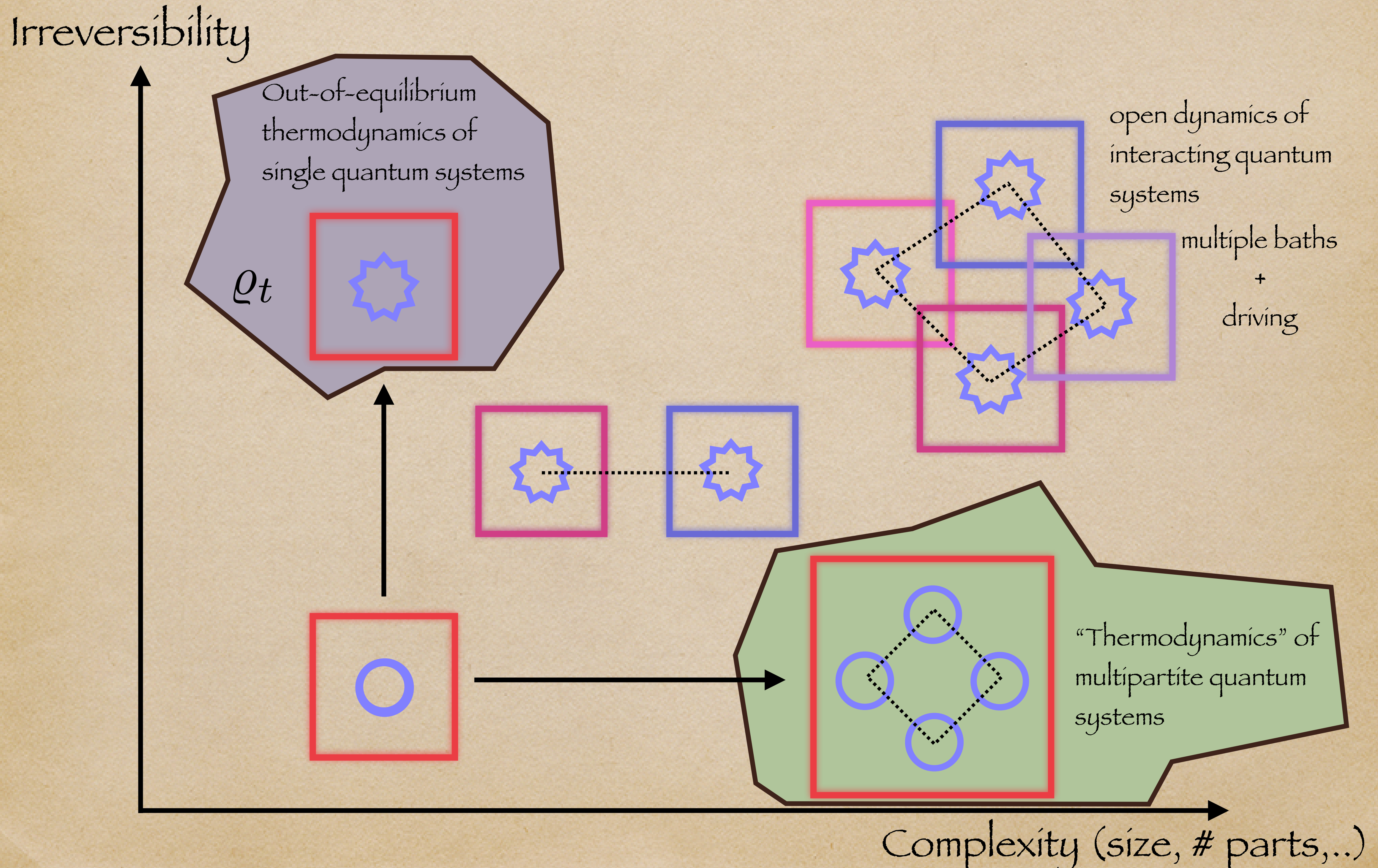
$$= \frac{\gamma}{\bar{n}+1/2}\left[\langle a^\dagger a\rangle - \bar{n} - \frac{\langle aa + a^\dagger a^\dagger\rangle}{2}\right]$$

$$\simeq \frac{\gamma}{\bar{n}+1/2}\left[\langle a^\dagger a\rangle - \bar{n}\right]$$

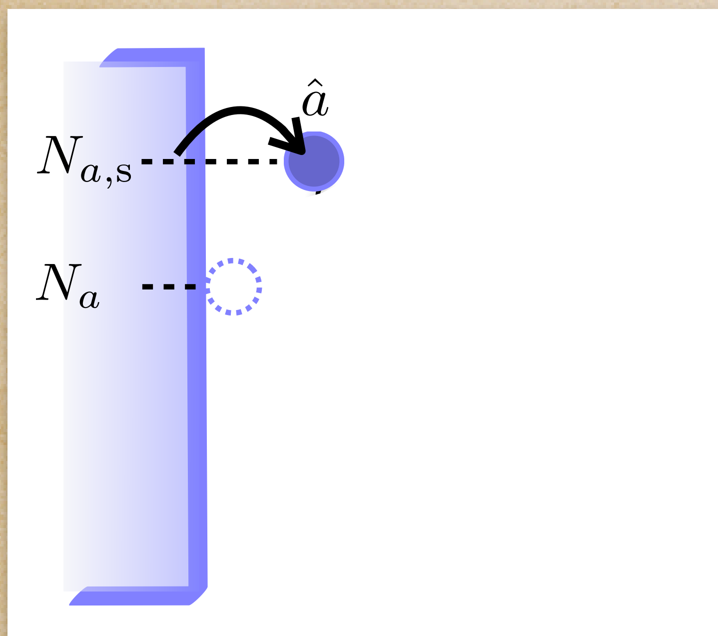
Observable!!

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

Entropy production: open questions



Entropy production in open systems



For a single harmonic oscillator in a thermal bath:

$$\Pi_s = 2\kappa_a \left(\frac{\langle \hat{q}_a^2 \rangle_s + \langle \hat{p}_a^2 \rangle_s}{2N_a + 1} - 1 \right)$$

$$\Pi_\mathrm{s} = 2\kappa_a \left(\frac{\langle \hat{q}_a^2 \rangle_\mathrm{s} + \langle \hat{p}_a^2 \rangle_\mathrm{s}}{2N_a + 1} - 1 \right) + 2\kappa_b \left(\frac{\langle \hat{q}_b^2 \rangle_\mathrm{s} + \langle \hat{p}_b^2 \rangle_\mathrm{s}}{2N_b + 1} - 1 \right)$$

Experimentally testable (and indeed tested!)

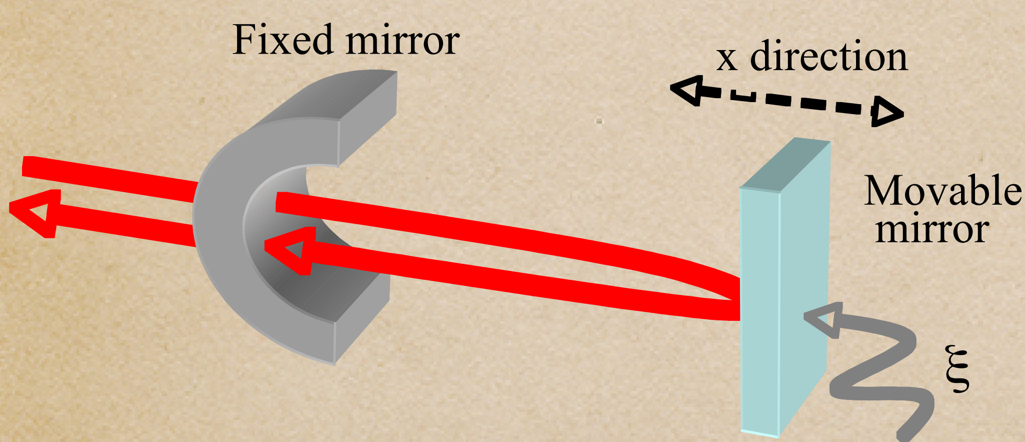
M Brunelli et al., arXiv:1602.06958 (2016)

M Brunelli and MP, arXiv:1610.01172 (2016)

J. Santos, G. Landi, and MP, Phys. Rev. Lett. 118, 220601 2017)

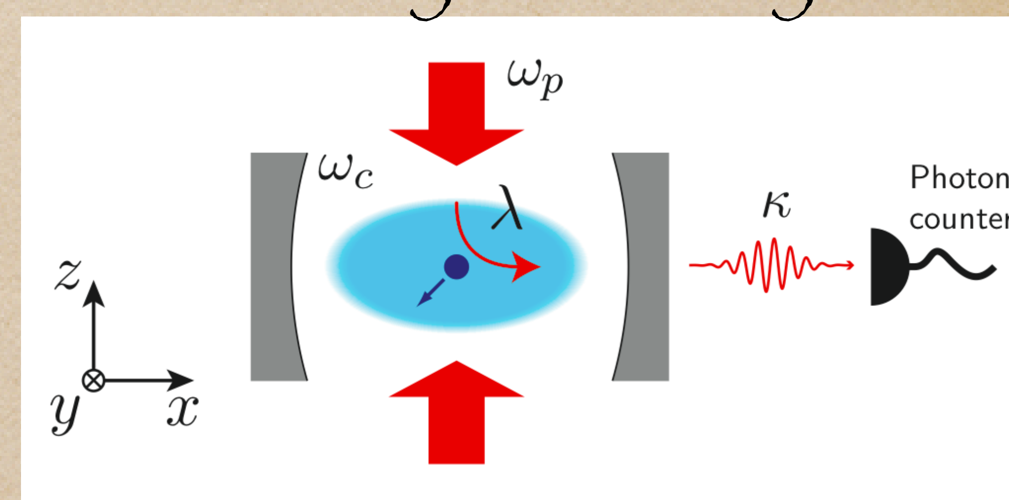
Entropy production & mesoscopics

Optomechanics



$$H = \frac{\hbar\omega}{2}(p^2 + q^2) + \hbar(\omega_c - gq)a^\dagger a$$
$$+ i\hbar\mathcal{E}(a^\dagger e^{-i\omega_0 t} - a e^{i\omega_0 t})$$

Intra-cavity atomic systems

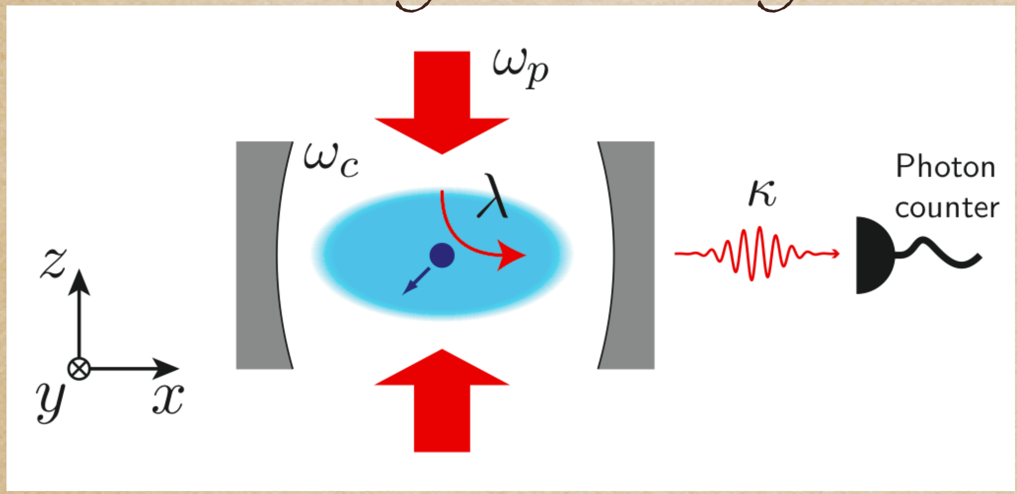


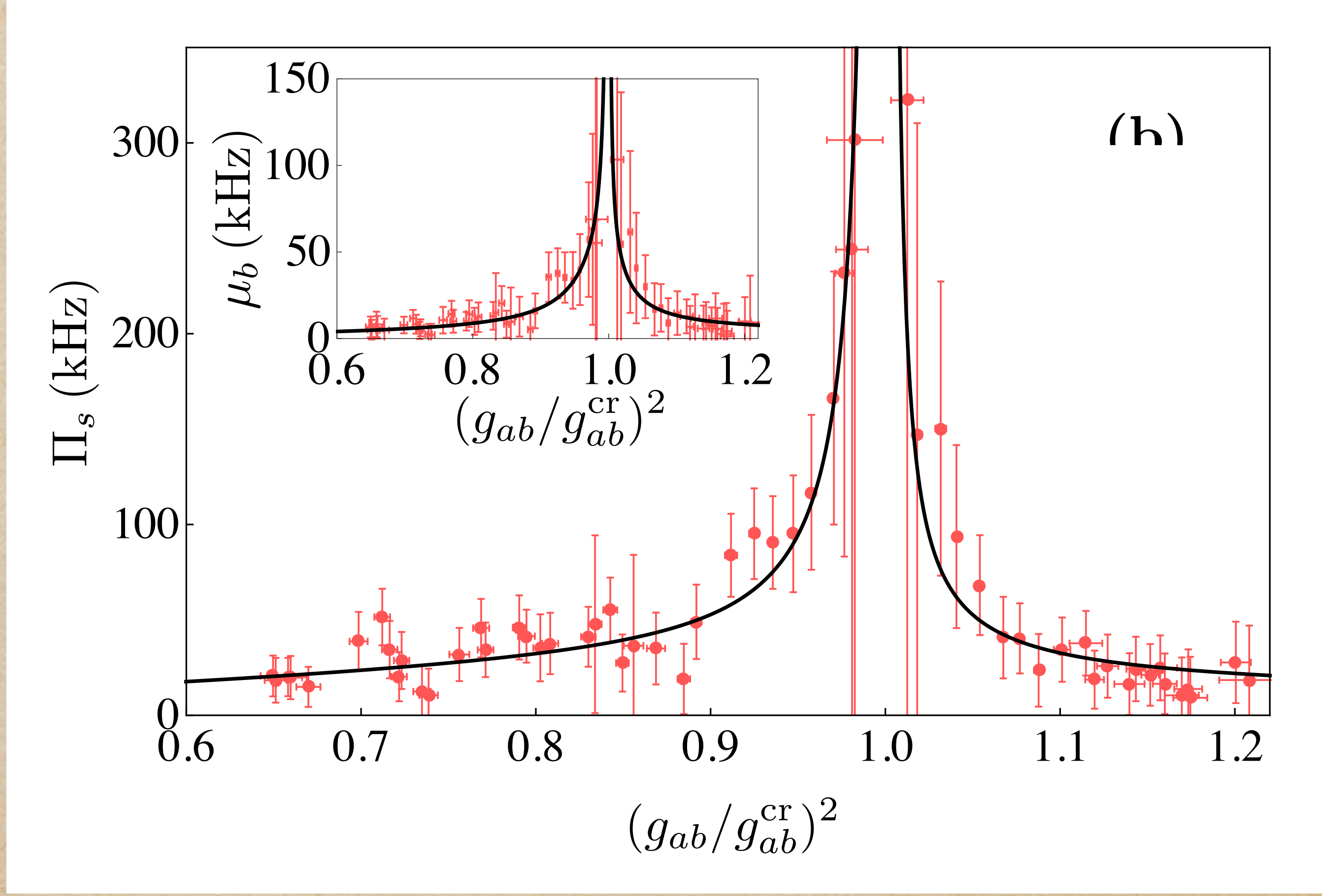
$$\hat{H} = \omega_0 \hat{J}_z + \omega \hat{a}^\dagger \hat{a} + \frac{2\lambda}{\sqrt{N}}\left(\hat{a} + \hat{a}^\dagger\right)\hat{J}_x$$

M Brunelli et al. arXiv:1602.06958 (2016)

Entropy production & mesoscopics

Intra-cavity atomic systems





J. Santos, G. Landi, and MP, Phys. Rev. Lett.118, 220601 2017)