Topological Phases of Matter with Ultracold Atoms and Photons

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European Commission

MARIE CURIE ACTIONS

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Overview

Lectures 1 & 2

Introduction to Topological Phases of Matter

Lecture 3

Topological Phases of Matter with Ultracold Atoms

Lecture 4

Topological Phases of Matter with Photons

Review of quantum fluids of light:

Carusotto & Ciuti, RMP **85**, 299 (2013)

Reviews of topological photonics:

Lu, Joannopoulos, & Soljačić, Nature Physics **12**, 626 (2016)

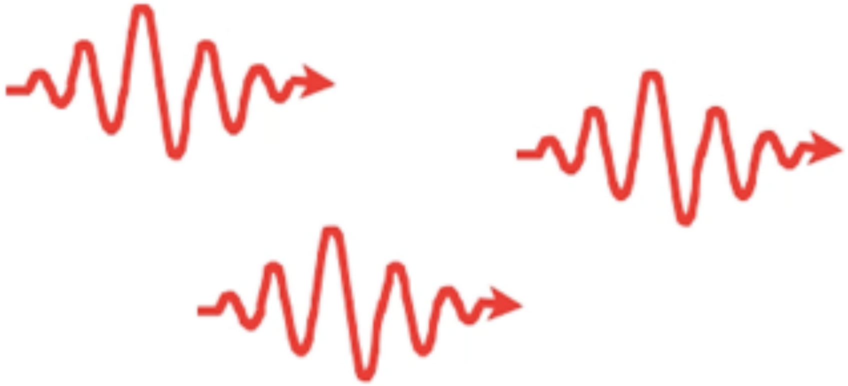
Lu, Joannopoulos, & Soljačić, Nature Photonics **8**, 821 (2014)

…and ours coming later this year…

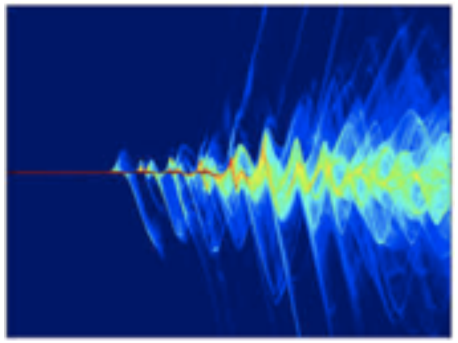
Photonics

Carusotto & Ciuti, RMP **85**, 299 (2013)

Always bosons



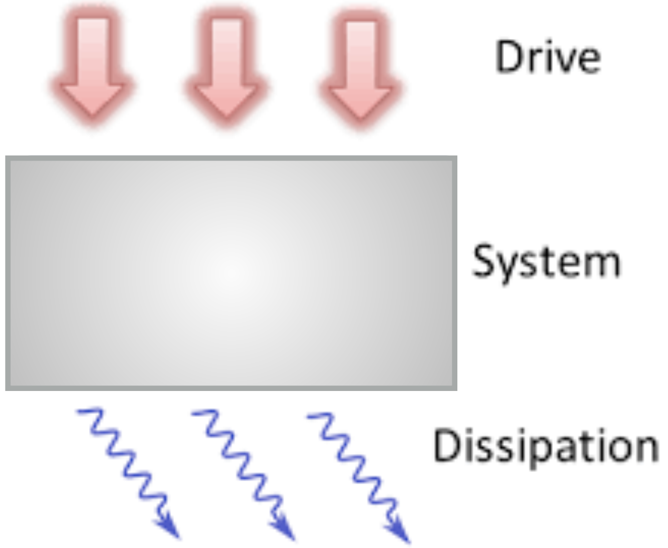
Tuneable effective interactions (mediated by nonlinearities of the medium)



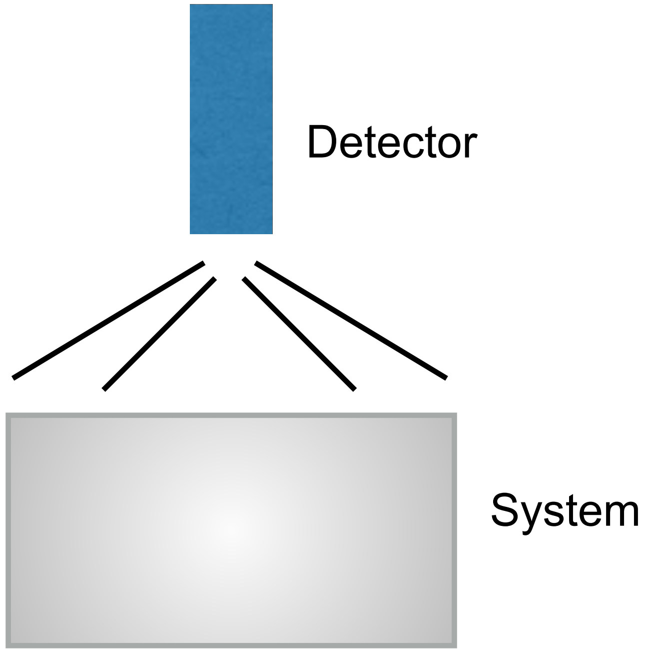
Designer structures

(Fabrication imperfections)

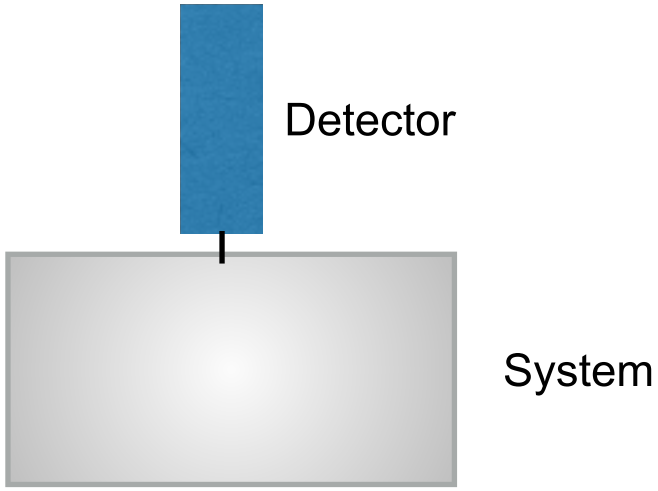
Photon pumping and losses



Access field in momentum space (far-field imaging)



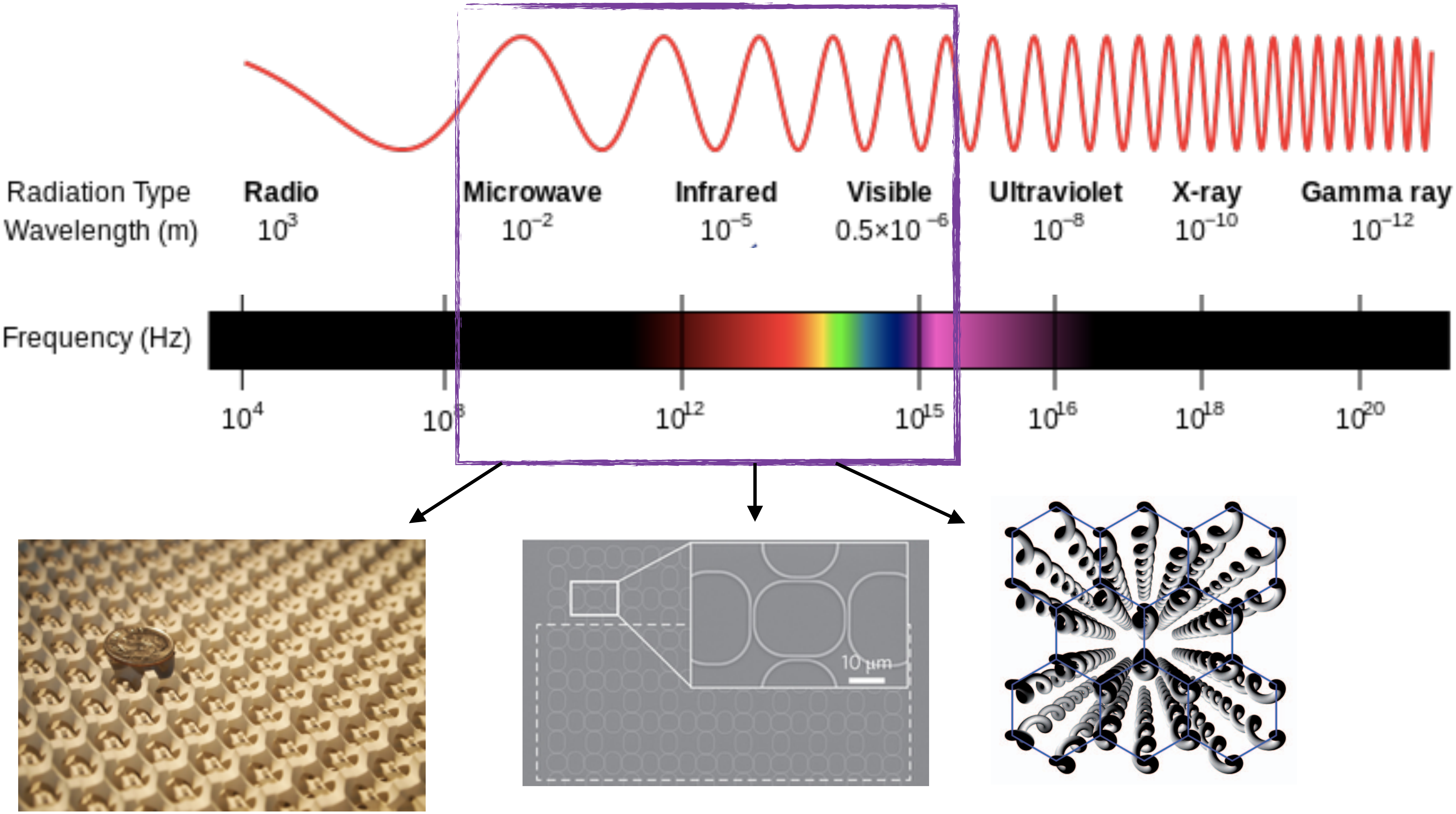
Access field in real space (near-field imaging)



Topological Photonics

Remember that we will be talking about:

1. Different parts of the EM spectrum —> very different physical systems



Topological Photonics

Remember that we will be talking about:

2. Mostly about classical effects

Properties of waves, not of quantum mechanics

Lectures 1 & 2: Quadratic Hamiltonians $\hat{H} = \Psi^\dagger \underset{\uparrow}{H} \Psi$

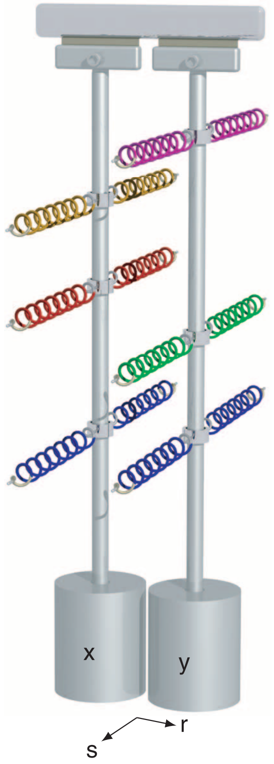
classification of topological properties of this matrix

But more generally, we can talk about topology if a system obeys a linear equation: $H\vec{\psi} = \omega\vec{\psi}$

(where ω is the normal mode frequency),

Topology in classical photonics, phononics, mechanics…

e.g. Introduction to topological classical mechanics:
Huber et al., Nature Physics 12, 621–623 (2016)



Lecture 4

- How can we engineer topology for photons?
  - Quantum Hall systems
  - Quantum spin Hall systems
  - SSH Model & Topological Pumps
  - Topological superconductors?

- How can we probe topology with photons?

- Future perspectives

Lecture 4

- How can we engineer topology for photons?
  - **Quantum Hall systems**
  - Quantum spin Hall systems
  - SSH Model & Topological Pumps
  - Topological superconductors?

- How can we probe topology with photons?

- Future perspectives

Photonic quantum Hall system

Topological photonics started with seminal theoretical works of Haldane and Raghu:

Haldane and Raghu, PRL 100, 013904 (2008)
Raghu and Haldane, PRA 78, 033834 (2008)

Maxwell's equations without source:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

Assuming linear, isotropic, loss-free medium,

$$\mathbf{D} = \boldsymbol{\varepsilon}\mathbf{E} \qquad \mathbf{B} = \boldsymbol{\mu}\mathbf{H}$$

permittivity permeability

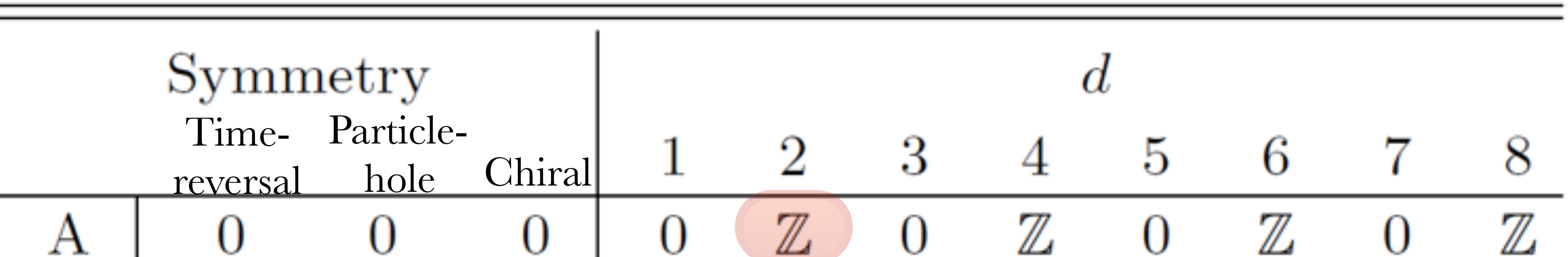
Find normal modes

How to make this topological?

$$i\begin{pmatrix} 0 & \nabla\times \\ -\nabla\times & 0 \end{pmatrix}\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \omega \begin{pmatrix} \boldsymbol{\varepsilon}(\mathbf{r}) & 0 \\ 0 & \boldsymbol{\mu}(\mathbf{r}) \end{pmatrix}\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \quad \Longrightarrow \quad i\begin{pmatrix} \boldsymbol{\varepsilon}(\mathbf{r})^{-1} & 0 \\ 0 & \boldsymbol{\mu}(\mathbf{r})^{-1} \end{pmatrix}\begin{pmatrix} 0 & \nabla\times \\ -\nabla\times & 0 \end{pmatrix}\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \omega \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}$$

Concept: Try to engineer photonic energy bands with non-trivial Chern number

- Electrons in a lattice —> photons in a periodic structure
- Magnetic field —> time-reversal symmetry breaking (*magneto-optical materials*)



	Symmetry			d							
	Time-reversal	Particle-hole	Chiral	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

Magneto-optic material

magneto-optic material in presence of magnetic field

Faraday effect

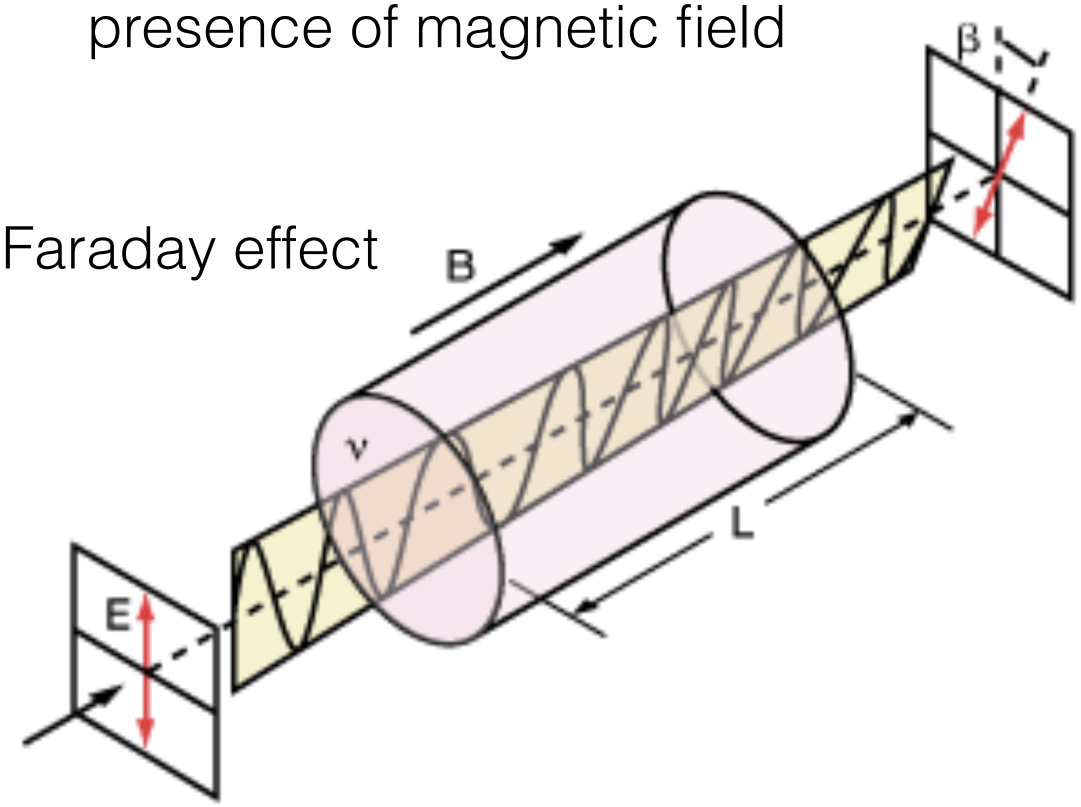


Figure from: https://en.wikipedia.org/wiki/Faraday_effect#/media/File:Faraday-effect.svg

Applications in optical isolators

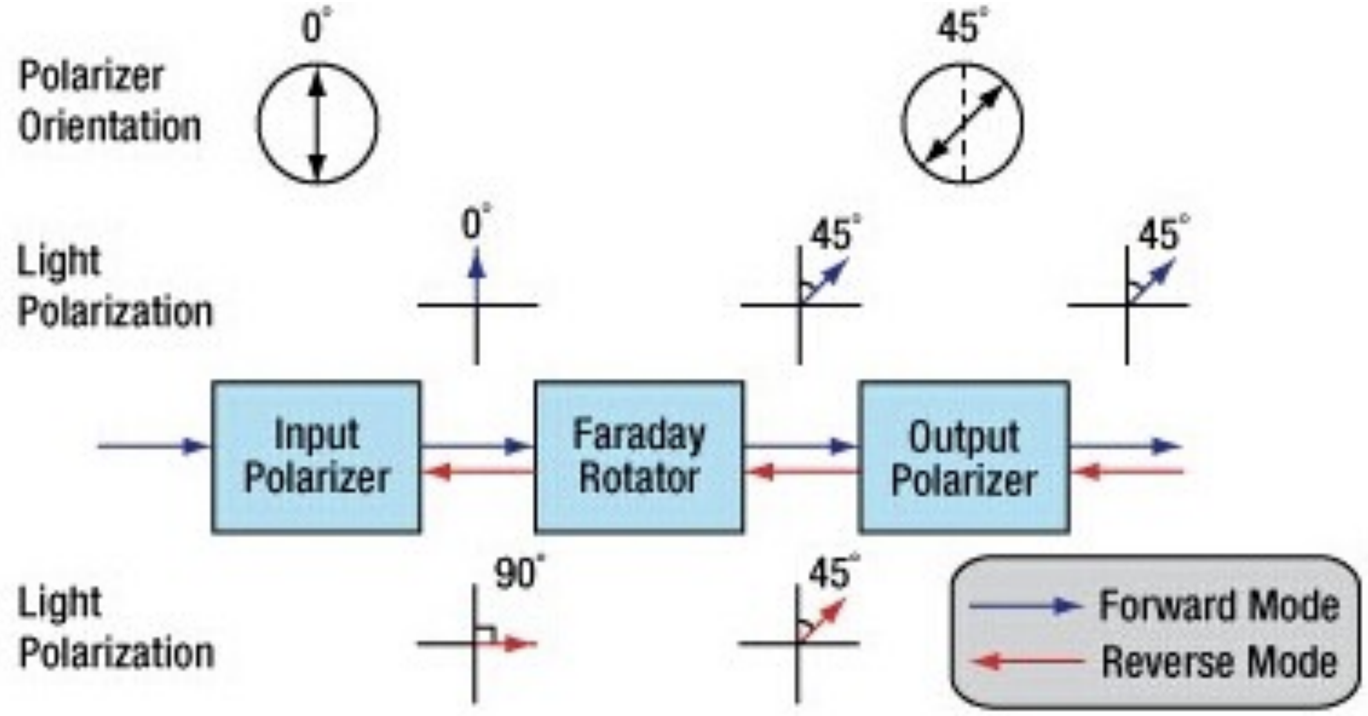
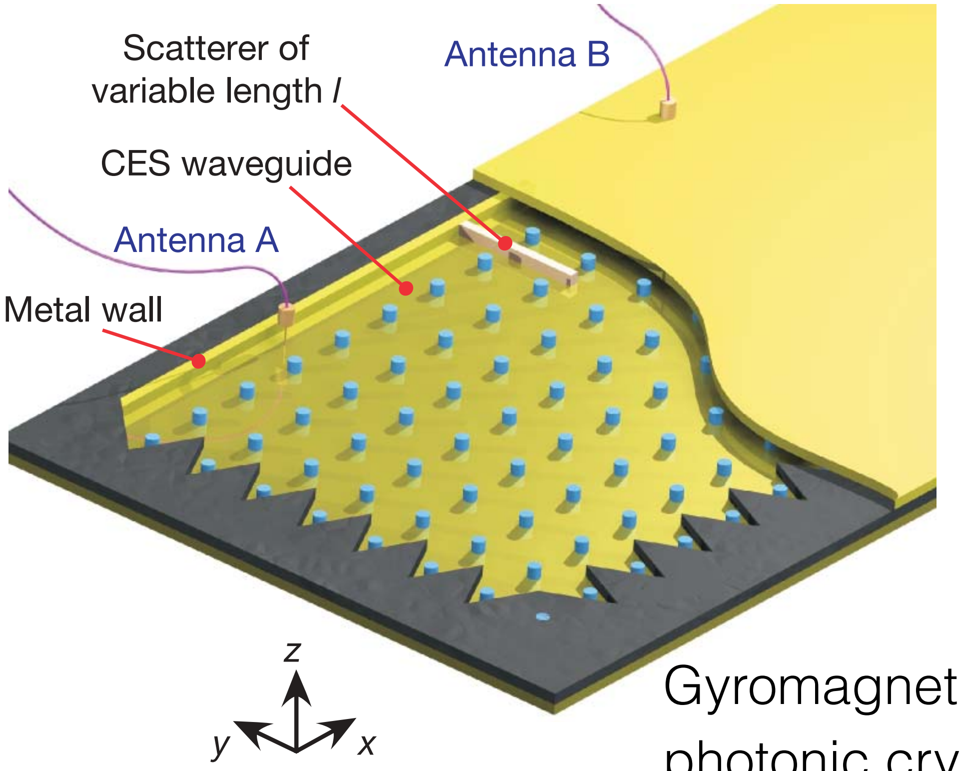


Figure from: http://www.fiber-optic-components.com/tag/optical-isolator

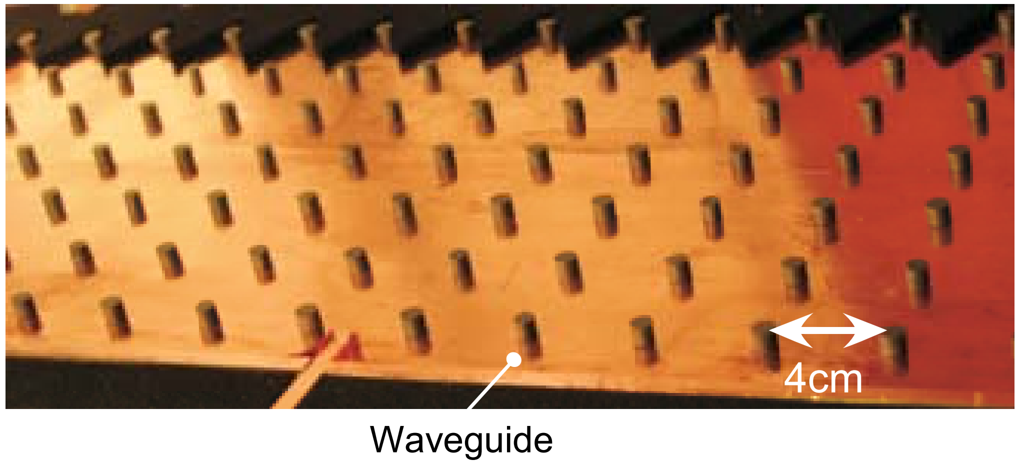
Magnetic Photonic Crystals

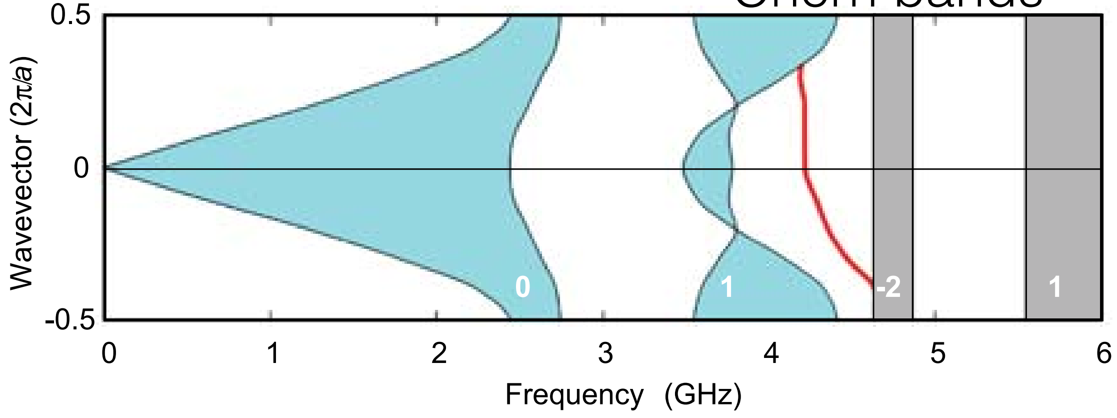
Experimental realisation of Haldane-Raghu idea by MIT group: Wang et al., Nature 461, 772–775 (2009).

- Microwaves propagate through a lattice of ferrite rods [lattice optimised numerically]
- TRS broken by strong magnetic field, coupling to ferrite rods —> gyromagnetic

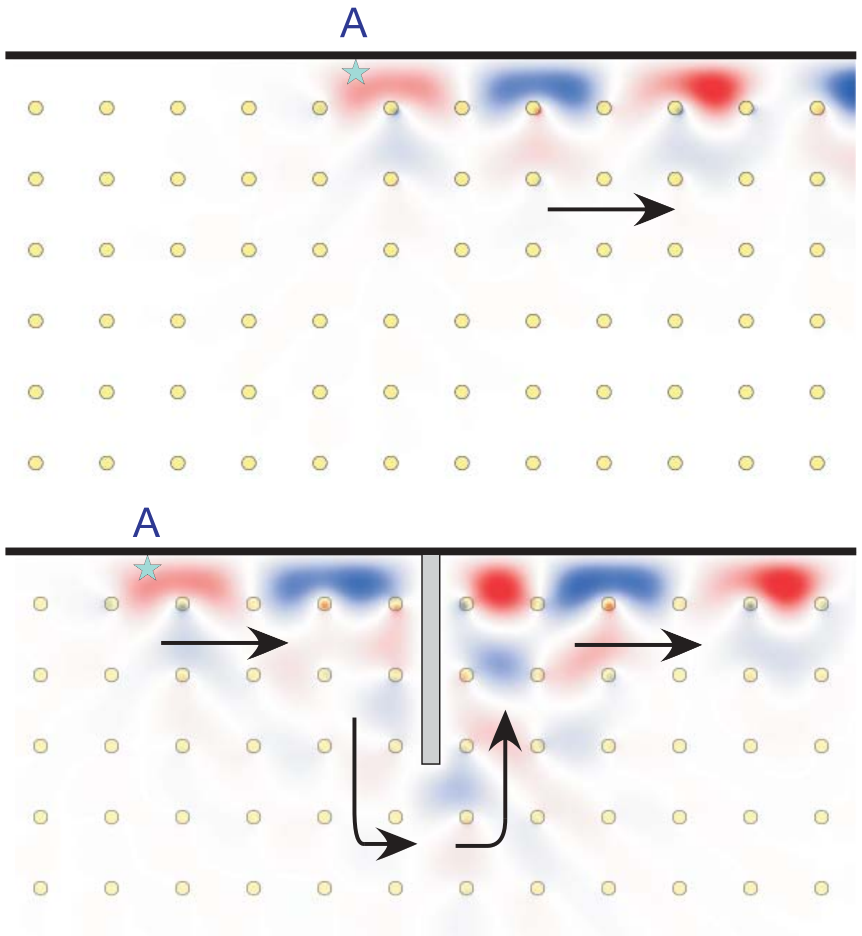


Gyromagnetic 2D photonic crystal





Chern bands



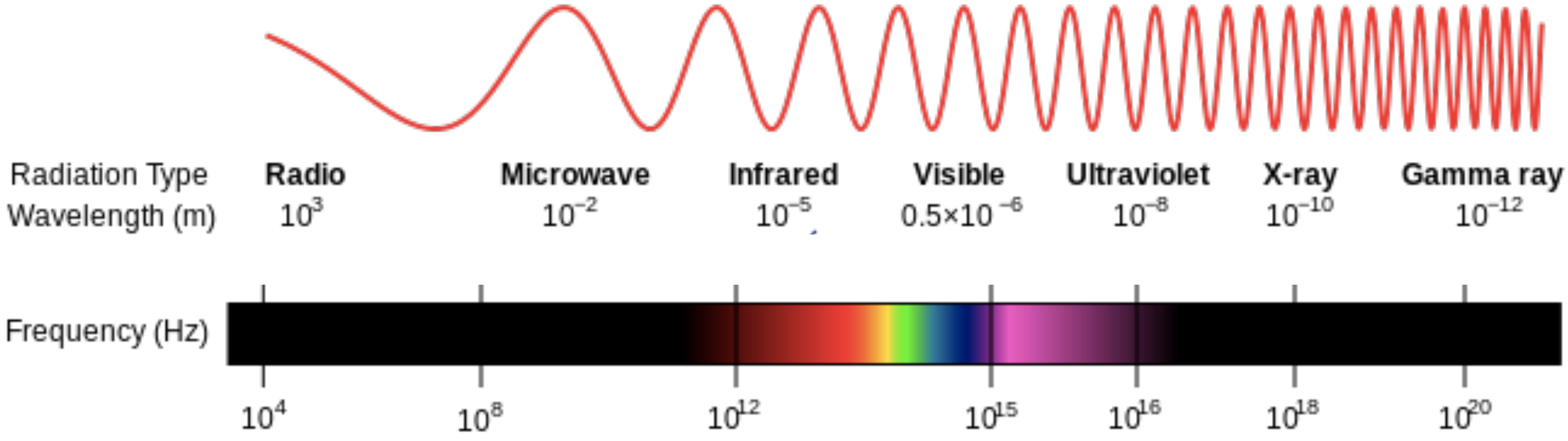
Many related experiments since… see e.g. Lu et al, Nature Phys., 12, 626 (2016)

[NB photonic crystals also used for first observation of Weyl points!}

Lu, et al., Science **349**, 622 (2015)

The end of the story?

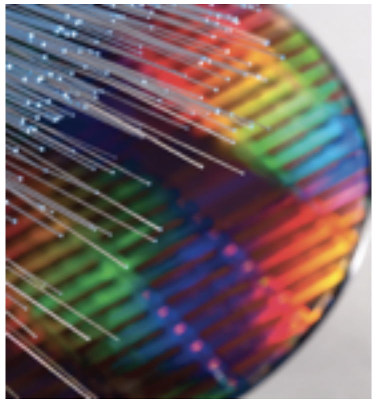
Magneto-optical effect works well for breaking TRS at microwave frequencies



but (i) this effect is weak at optical frequencies and (ii) having real magnetic fields is not good for many applications (e.g. on-chip devices)…. so we need other tricks!

- Floquet engineering
- Synthetic dimensions
- ….

c.f. Lecture 3!



Floquet engineering

Very(!) brief intro to Floquet theory:

System modulated periodically in time

static periodic driving

$$H = H_0 + V(t)$$

$$V(t+T) = V(t)$$

$$T = 2\pi/\omega$$

$$U(T) = \mathcal{T}\exp\left(-i\int_0^T dt H(t)\right)$$

Stroboscopic evolution captured by time-independent effective Hamiltonian:

$$U(T) = \exp\left(-iTH_{\text{eff}}\right)$$

H_0 and H_{eff} can be in **different** topological classes

Typically assume high-frequency driving ($\omega \gg$ all other frequencies) and then calculate effective Hamiltonian perturbatively, e.g. at lowest order:

$$H^{\text{eff}} = \frac{1}{T}\int_0^T H(t)\mathrm{d}t$$

Concept: Design driving to engineer an artificial magnetic field in the effective Hamiltonian

For lots more about Floquet theory, see e.g:
M. Bukov et al. Advances in Physics, 64, 139, (2015)
N. Goldman et al., arXiv:1507.07805

Shaking: propagating waveguides

Propagation of light in source-free non-magnetic material $\nabla \times \nabla \times \mathbf{E} = \varepsilon \left(\frac{\omega}{c}\right)^2 \mathbf{E},$

In the “paraxial” approximation, for light propagating along z $k_0 \gg k_{x,y}$ $\mathbf{E}(x,y,z) = \psi(x,y,z) \exp\left[ik_0 z\right] \mathbf{x}$

slowly varying envelope

$$i\partial_z \psi = -\frac{1}{2k_0}\nabla_\perp^2 \psi - \frac{k_0 \Delta n}{n_1}\psi.$$

refractive index deviation

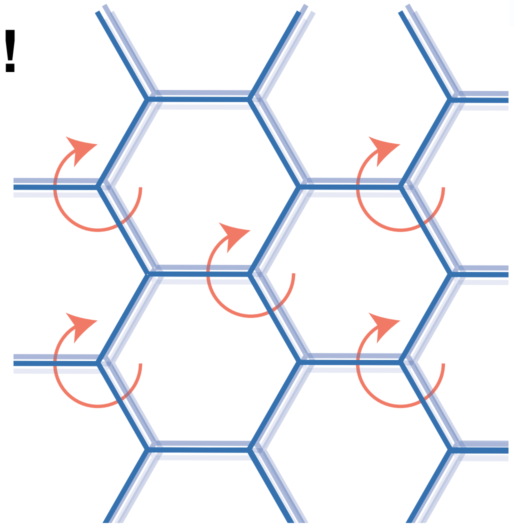
background refractive index

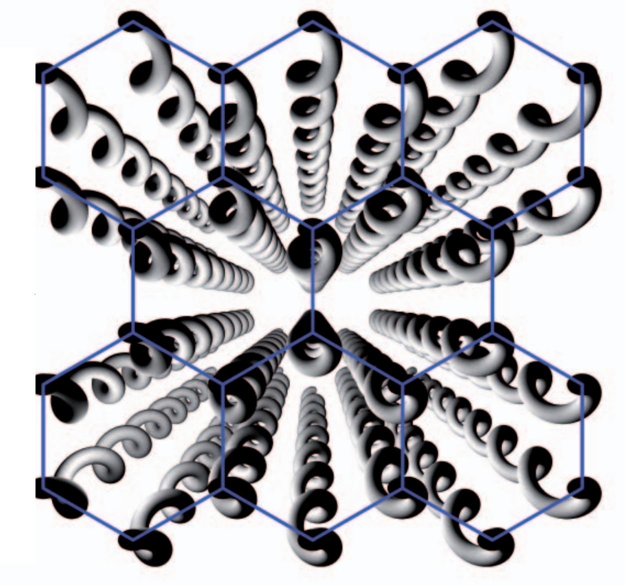
$$\varepsilon = \varepsilon_1 + \Delta\varepsilon(x,y,z)$$
$$n = \sqrt{\varepsilon}$$

Like Schrodinger but with roles of t and z reversed!

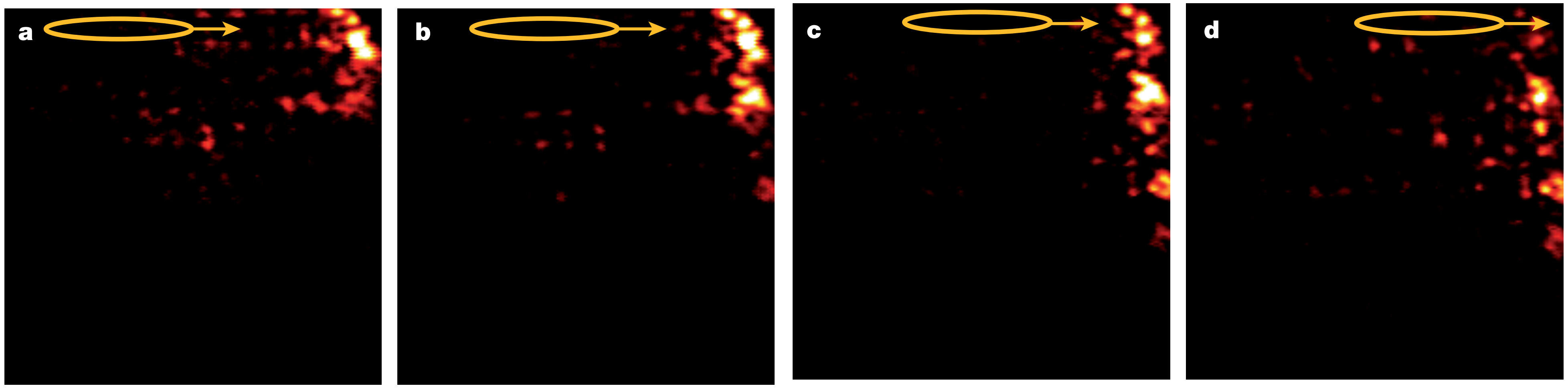
Shaking in time ↔ spatially-varying refractive index along direction of propagation, z

Rechtsman, et al., Nature 496, 196 (2013)





Pumping on edge : chiral edge state



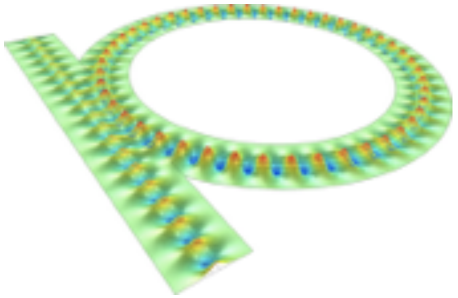
Propagating waveguides also recently used to realise *anomalous* Floquet topological states

Maczewsky et al., Nat. Comm. 8 (2017).
Mukherjee et al., Nat. Comm. 8 (2017).

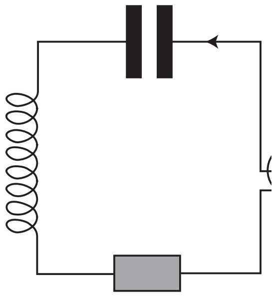
Dynamical modulation: resonator lattices

Resonator Lattice

Lattice sites are resonators, e.g.



optical/near-IR ring resonators



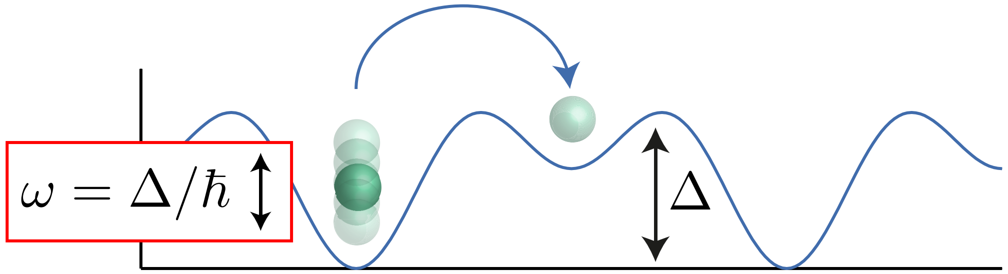
microwave RLC resonators

Coupled together e.g. by waveguides or auxiliary resonators

Often can be well-described by **tight-binding Hamiltonians**

Dynamical modulation for resonator lattices

General concept same as:
superlattice + resonant modulation (cold atoms)



Proposed model: Fang, et al., Nat. Photon. 6, 782 (2012)

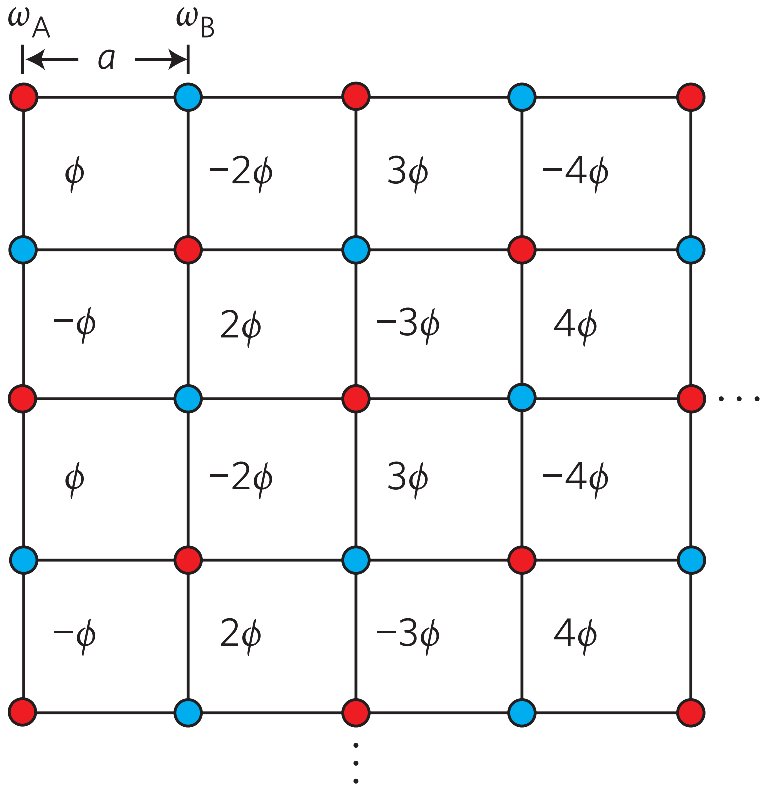
$$H = \omega_{\mathrm{A}} \sum_i a_i^\dagger a_i + \omega_{\mathrm{B}} \sum_j b_j^\dagger b_j$$

$$+ \sum_{\langle ij \rangle} V \cos(\Omega t + \phi_{ij})(a_i^\dagger b_j + b_j^\dagger a_i)$$

modulated hoppings

In rotating wave approx, in rotating frame:

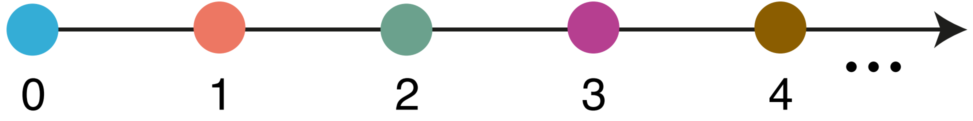
$$H = \sum_{\langle ij \rangle} \frac{V}{2} (\mathrm{e}^{-i\phi_{ij}} c_i^\dagger c_j + \mathrm{e}^{i\phi_{ij}} c_j^\dagger c_i)$$



Synthetic Dimensions in Photonics

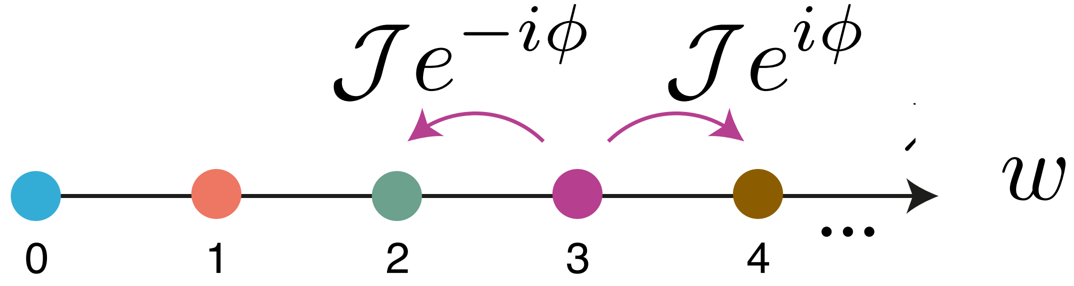
Concept:

1. Identify a set of states and reinterpret as sites in a synthetic dimension



First proposed by:
Boada et al., PRL, 108, 133001 (2012),
Celi et al., PRL, 112, 043001 (2014)

2. Couple these modes to simulate a tight-binding "hopping"

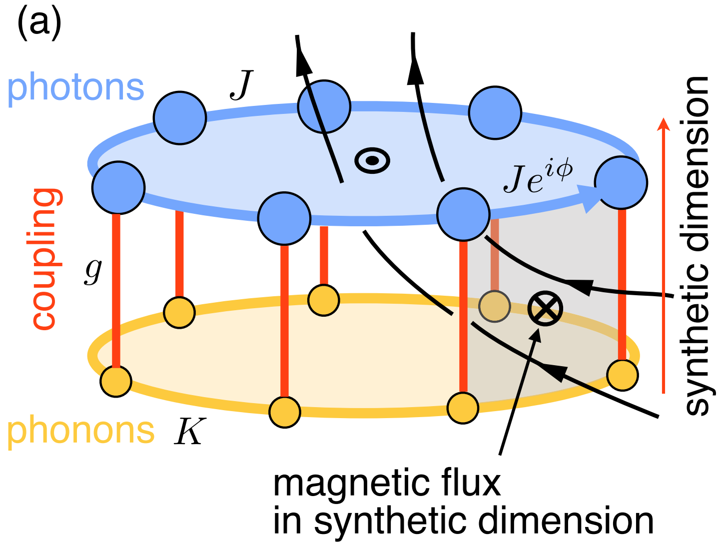


3. Combine with real spatial dimensions or more synthetic dimensions as desired

As yet no experimental realisation for photons, but interesting proposals…

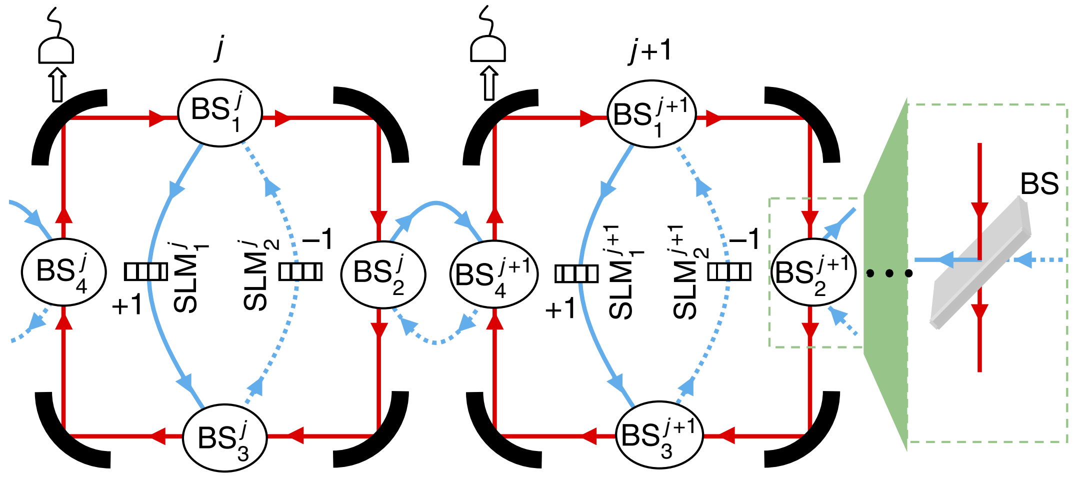
Optomechanics — photons & phonons

Schmidt et al., Optica **2**, 635 (2015)



Optical cavities — orbital angular momentum

Luo et al., Nature Comm. 6, 7704 (2015)



Synthetic Dimensions in Photonics

Ring resonator lattice — different modes of ring resonators

Ozawa et al, PRA **93**, 043827 (2016)
Yuan et al. Opt. Lett. **41**, 741 (2016)
Ozawa et al., PRL **118**, 013601 (2017)

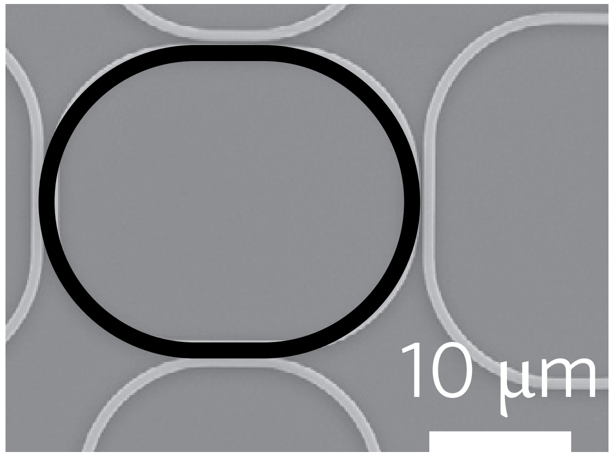
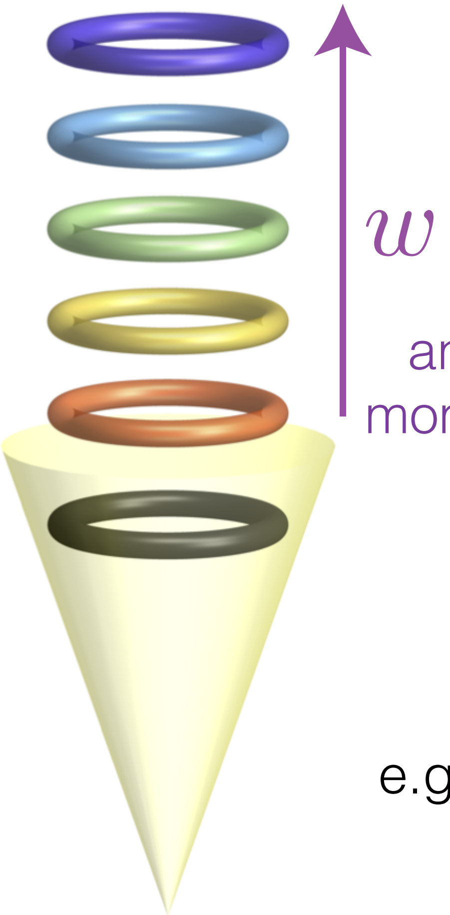


Figure from: Hafezi et al, Nat. Photon. 7, 1001, (2013)



angular momentum

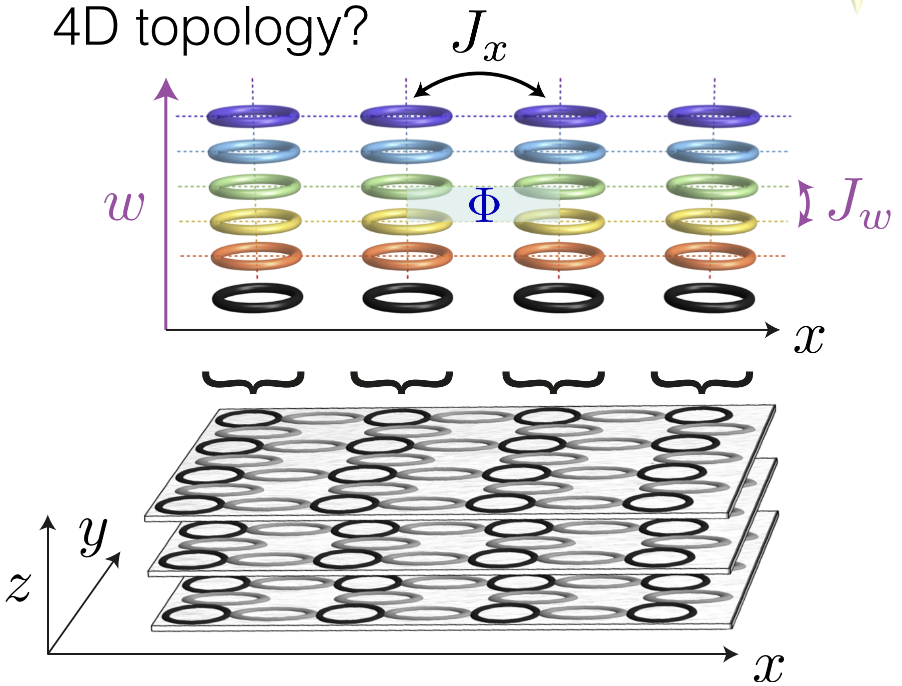
frequency comb $\omega_w = \omega_{w_0} + \Delta\omega\,(|w| - w_0) + \ldots$

free spectral range (FSR) $\Delta\omega = 2\pi c / n_{\text{eff}} R$

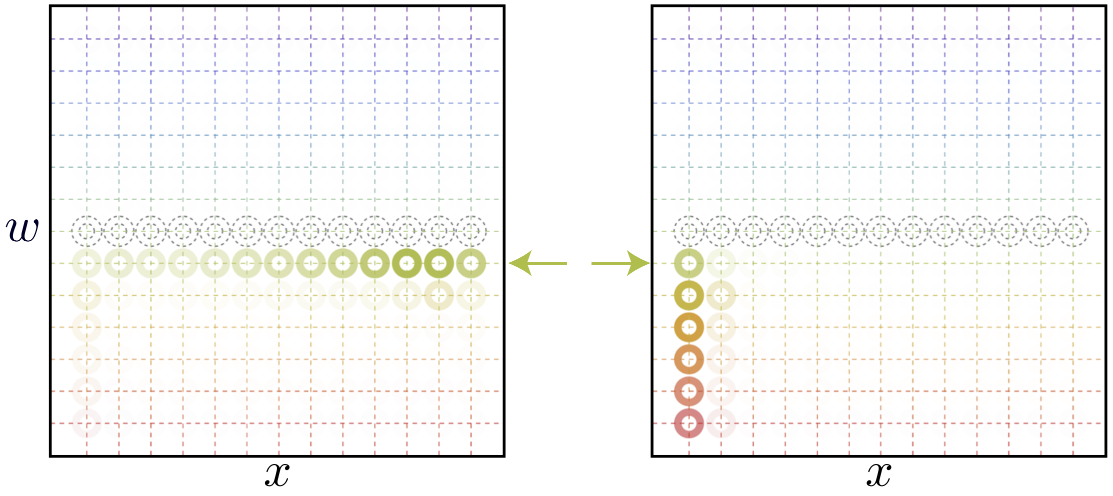
Couple modes by modulating the refractive index with frequency $\Delta\omega$

e.g. through **material nonlinearities** $\chi^{(3)}$ $\chi^{(2)}$ or **electro-optic phase modulators**

4D topology?



Applications in on-chip optical isolators?



Lecture 4

- How can we engineer topology for photons?
  - Quantum Hall systems
  - **Quantum spin Hall systems**
  - SSH Model & Topological Pumps
  - Topological superconductors?

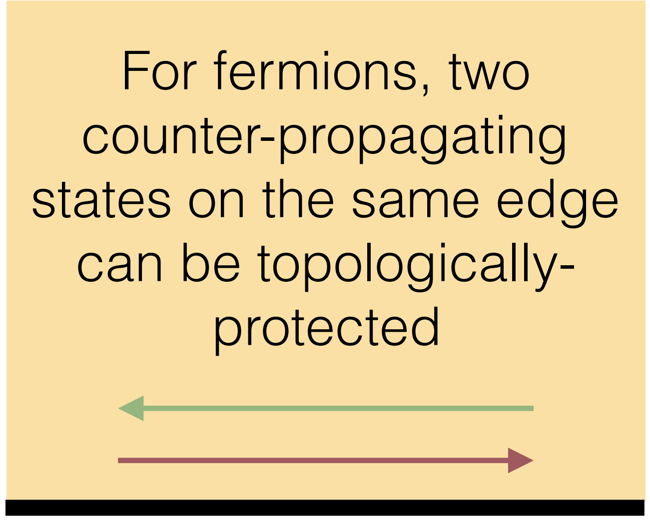
- How can we probe topology with photons?

- Future perspectives

Time-reversal symmetry?

Remember from Lecture 2, for bosons there is no Kramer's theorem as $\mathcal{T}^2 = +1$

Problem: Two counter-propagating bosonic edge states (e.g. like in a topological insulator) will generally couple and backscatter —> not topologically-robust!



For fermions, two counter-propagating states on the same edge can be topologically-protected

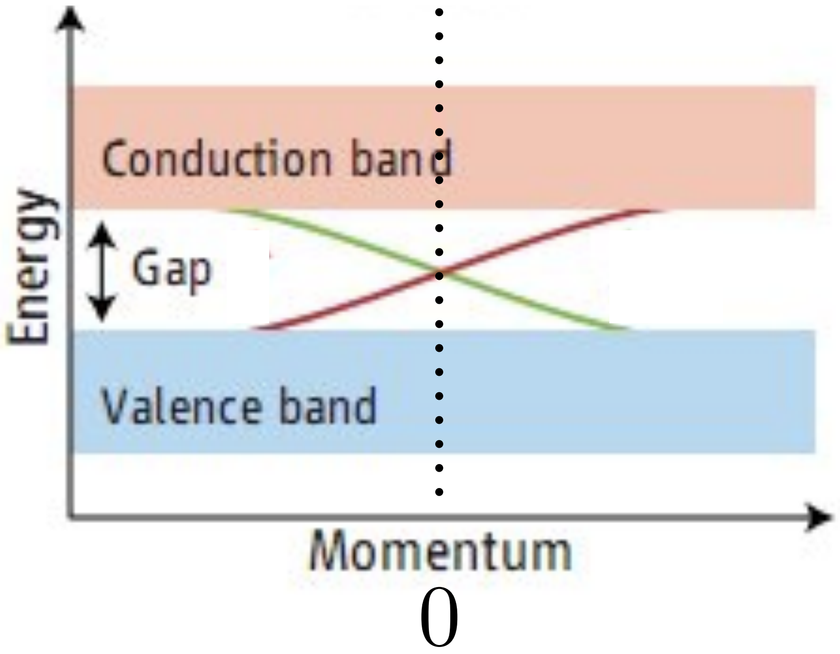


Figure adapted from
C. L. Kane & E. J. Mele, Science 314, 5806, 1692 (2006)

Work-around solution: *Design* the system to suppress the inter-mode coupling

Caveat: The following photonics set-ups are inspired by Class AII systems but they are *not truly topological*

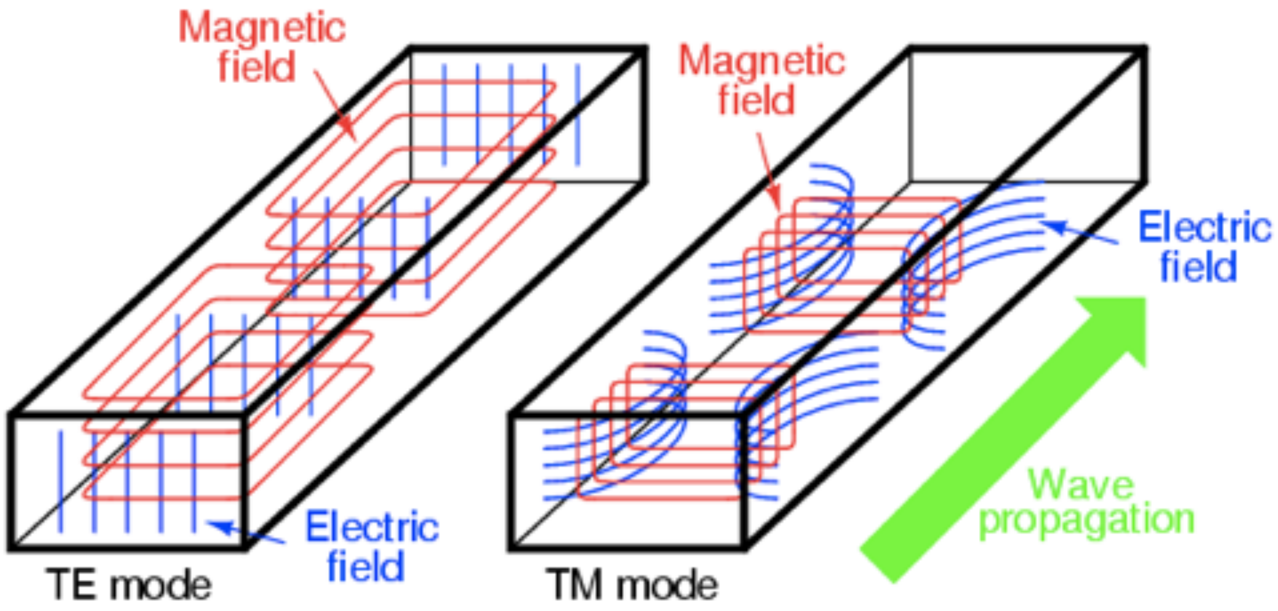
	Symmetry			d							
	Time-reversal	Particle-hole	Chiral	1	2	3	4	5	6	7	8
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$

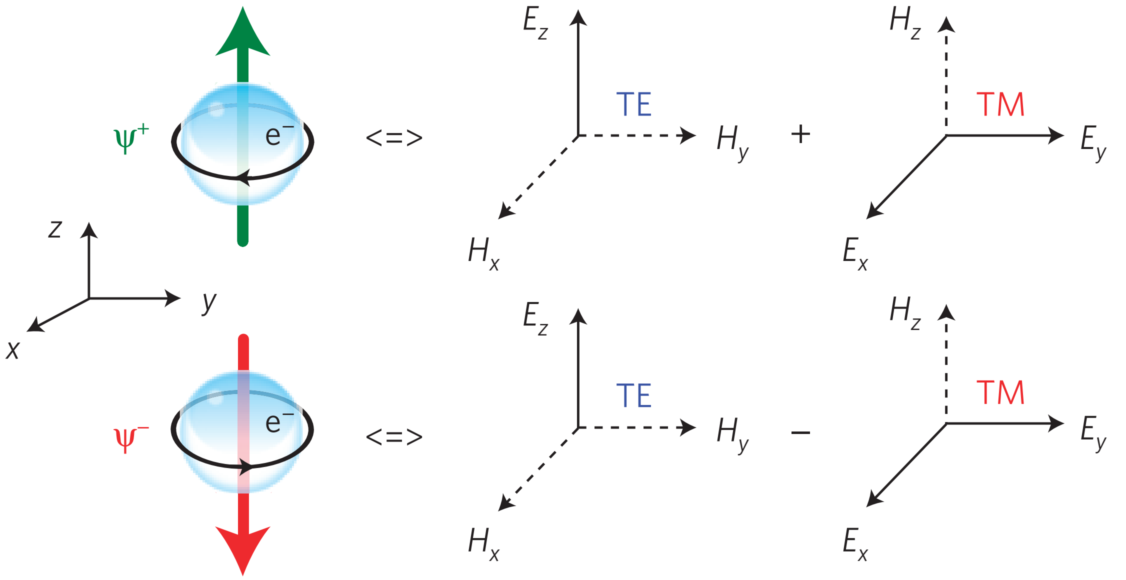
Bianisotropy: metamaterials

Ingredients for a photonic “topological insulator”: Proposal: Khanikaev, et al. , Nature Materials **12**, 233 (2013)

1) A “pseudo” spin-1/2?

Usually, TE and TM modes have different wave-vectors, but in special metamaterials (where ε = μ), the wave-vectors are identical. Then:

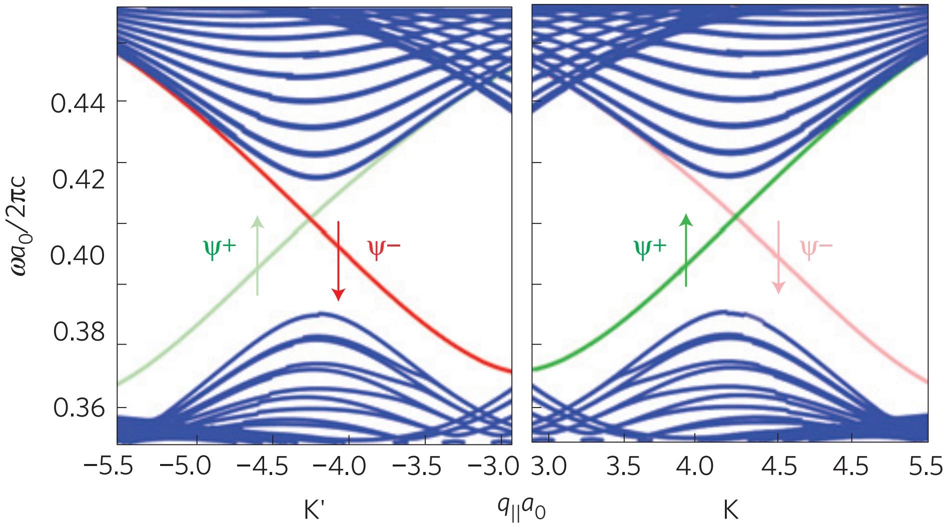


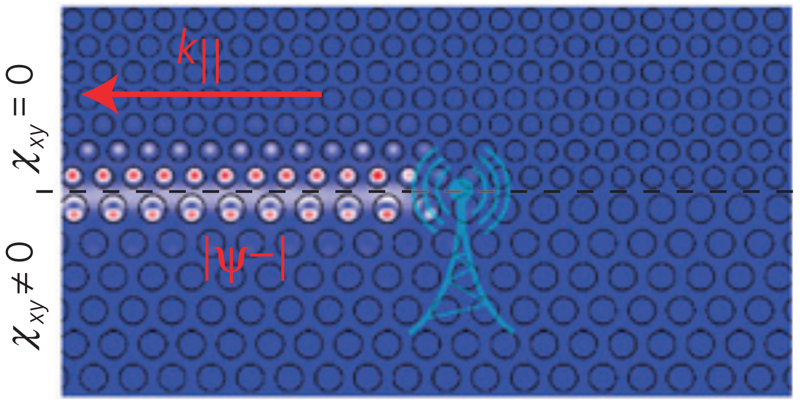


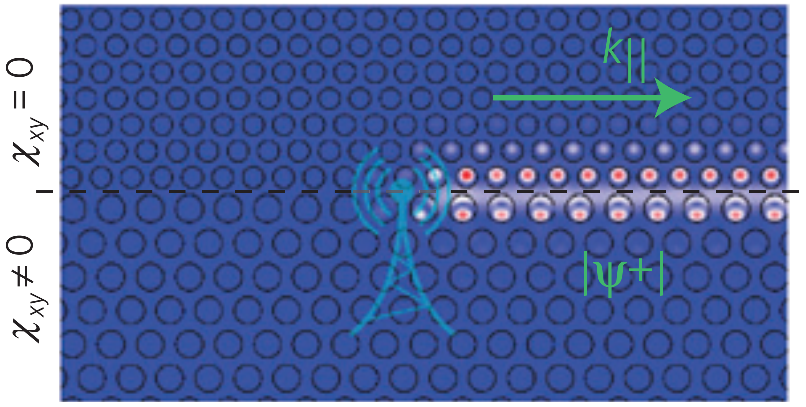
2) A “spin-orbit” coupling? Bianisotropic material

$$i\begin{pmatrix}0 & \nabla\times\\ -\nabla\times & 0\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix} = \omega\begin{pmatrix}\boldsymbol{\varepsilon}(\mathbf{r}) & \boldsymbol{\chi}(\mathbf{r})\\ \boldsymbol{\chi}(\mathbf{r})^{\dagger} & \boldsymbol{\mu}(\mathbf{r})\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}$$

Bianisotropy

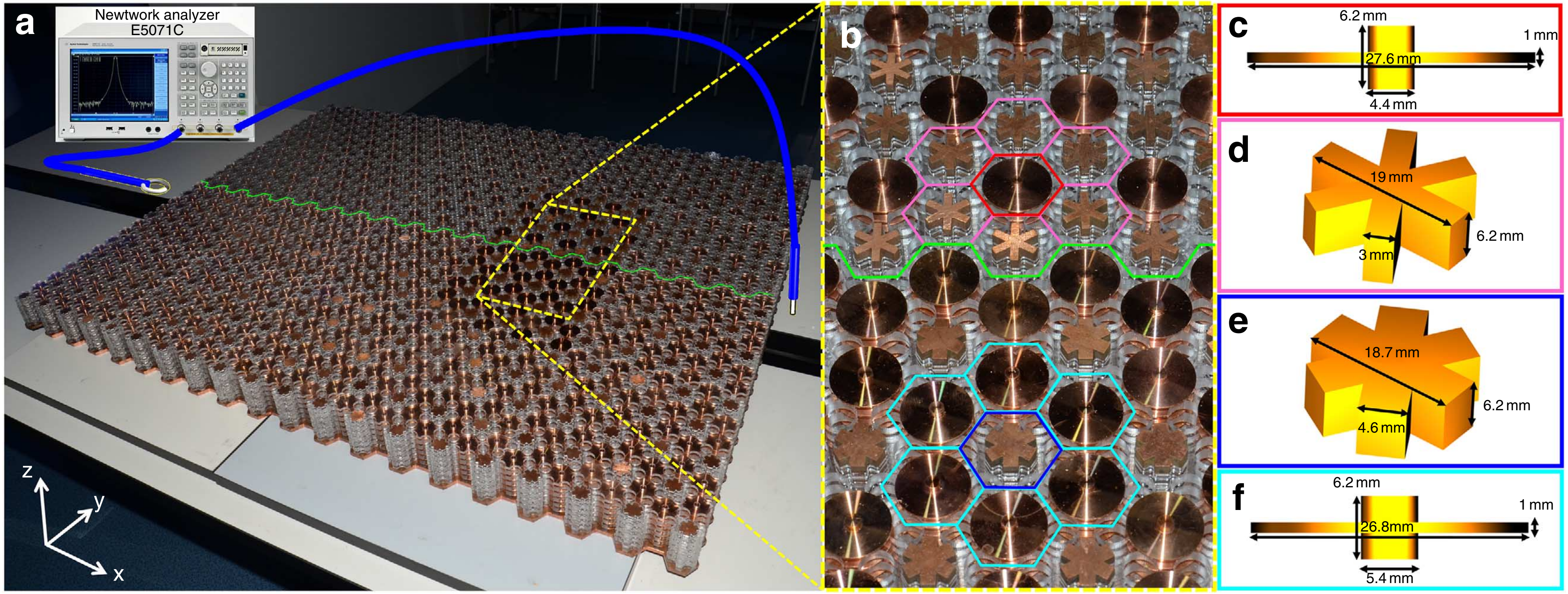






Bianisotropy: metamaterials

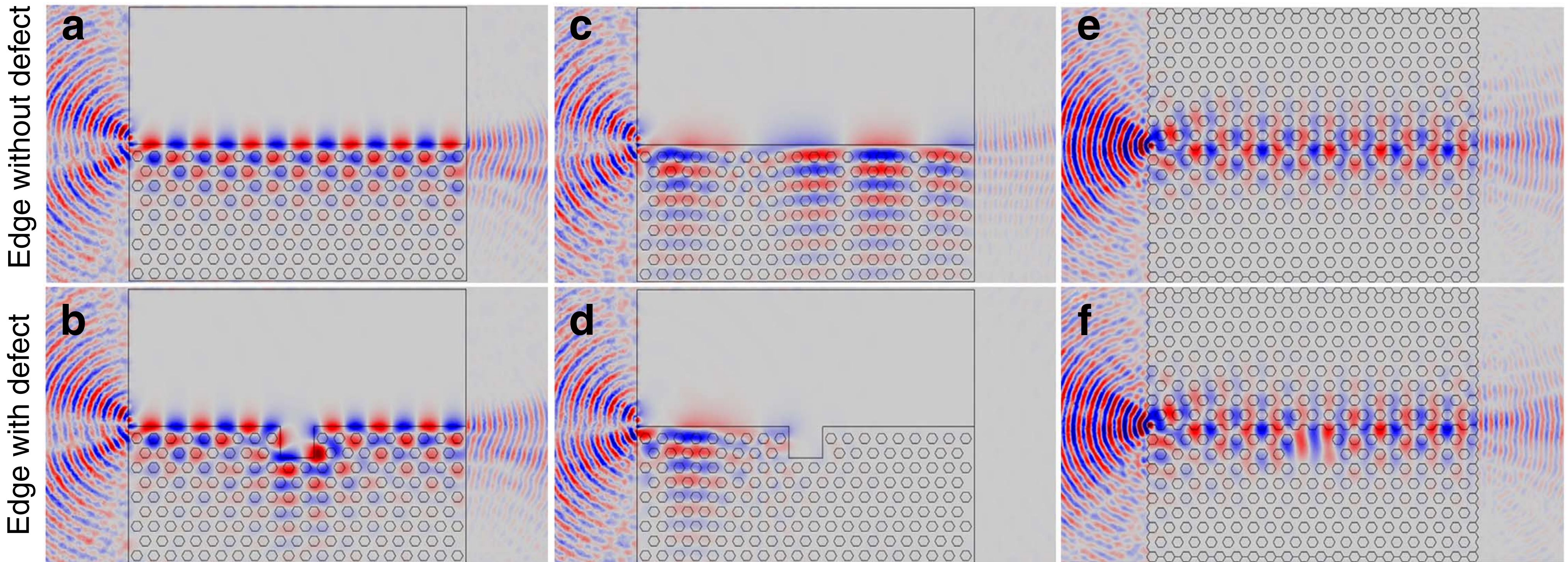
Experimental realisation (Hong Kong): Chen, et al., Nature Comm. 5, 5782 (2014)


Experimental setup

Edge between PTI and POI

Geometries of meta-atoms

Microwave regime

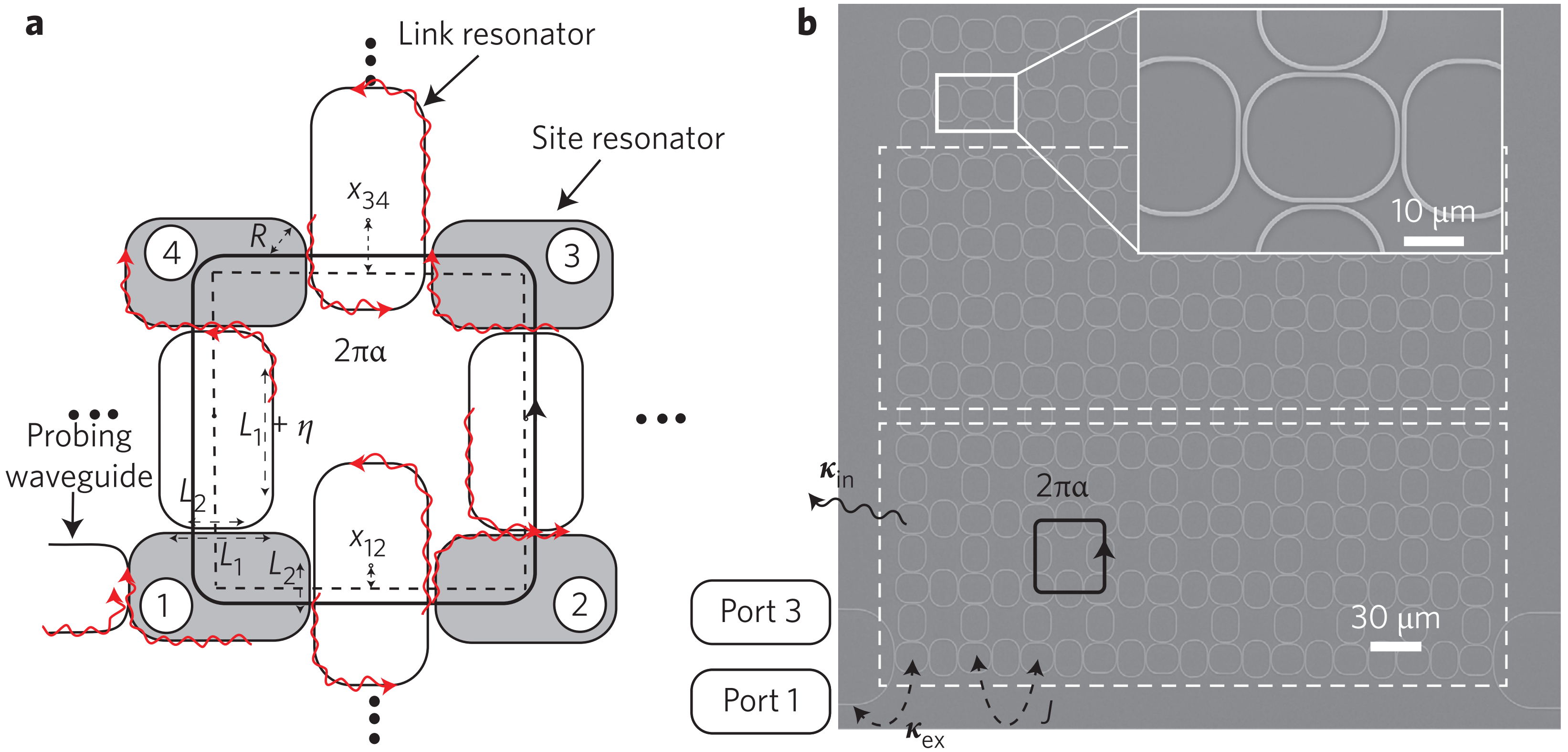

PTI and low index waveguide

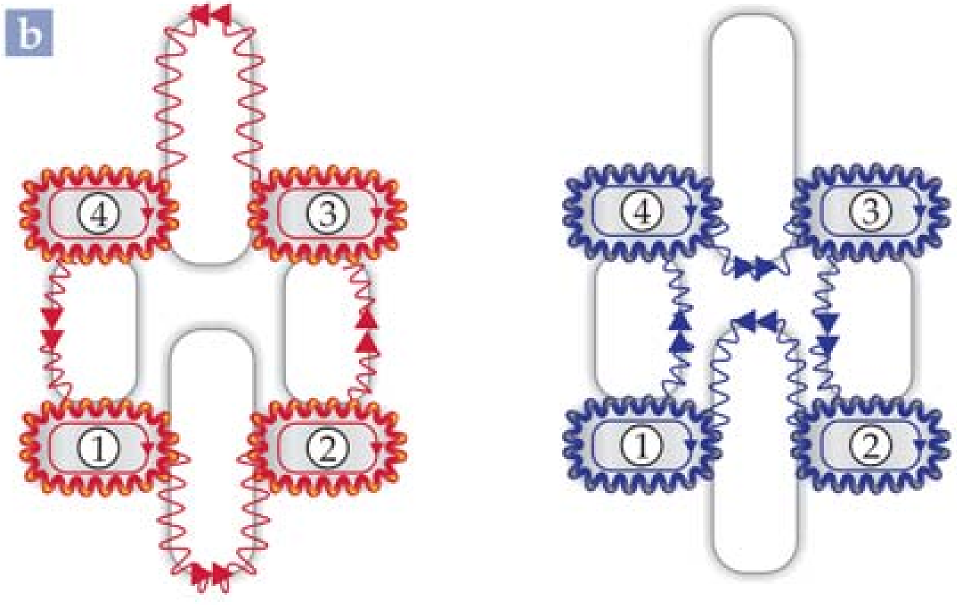
POI and low index waveguide

Edge between PTI and POI

Differential Optical Paths: Resonator Lattice

Maryland group: Hafezi, et al., Nat. Photonics 7, 1001 (2013).



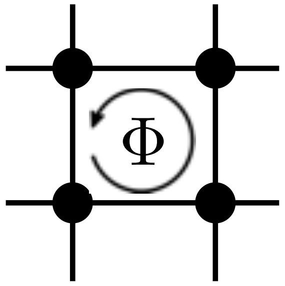


M. Hafezi, and J. M. Taylor , Physics Today, 67, 68–69 (2014)

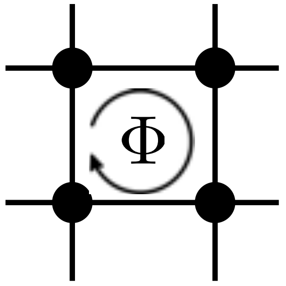
Due to displacement of link resonators, photons travel a different distance and hence acquire a different phase from (1) to (2) than from (2) to (1) —> Analogous to the Peierls phase

By making this displacement vary over the lattice, can engineer an artificial magnetic field

Quantum spin Hall system:
2 copies of Harper-Hofstadter model



For modes circulating clockwise in the site resonators (pseudo-spin-up)



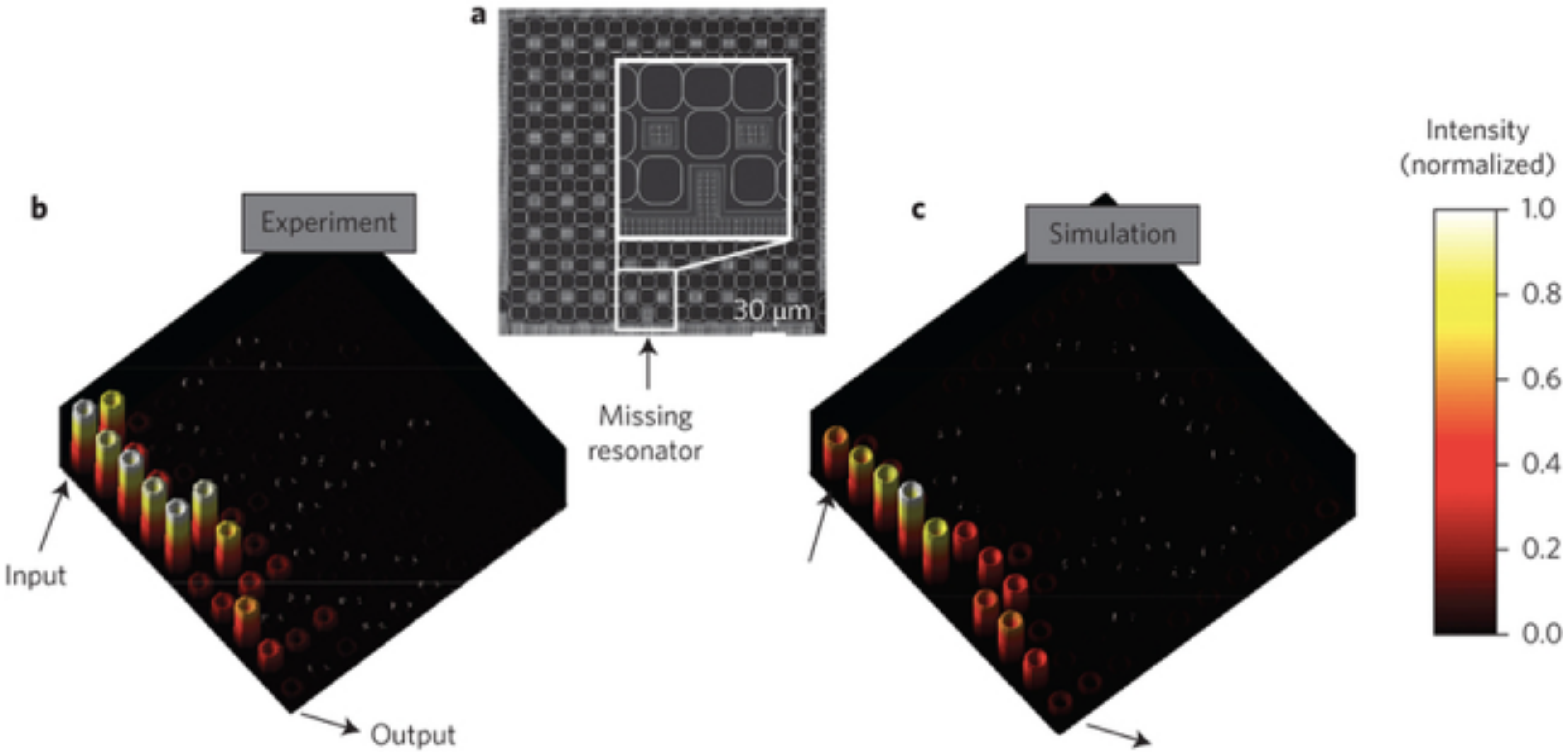
For modes circulating anti-clockwise in the site resonators (pseudo-spin-down)

Differential Optical Paths: Resonator Lattice

Maryland group: Hafezi, et al., Nat. Photonics 7, 1001 (2013).

Injecting light in pseudo-spin-up channel, see chiral edge states

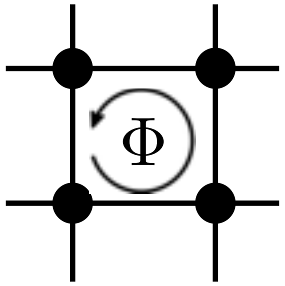
but not protected — backscattering is just weak



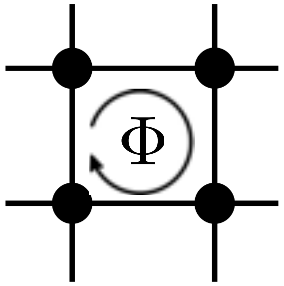
Due to displacement of link resonators, photons travel a different distance and hence acquire a different phase from (1) to (2) than from (2) to (1) —> Analogous to the Peierls phase

By making this displacement vary over the lattice, can engineer an artificial magnetic field

Quantum spin Hall system: 2 copies of Harper-Hofstadter model



For modes circulating clockwise in the site resonators (pseudo-spin-up)

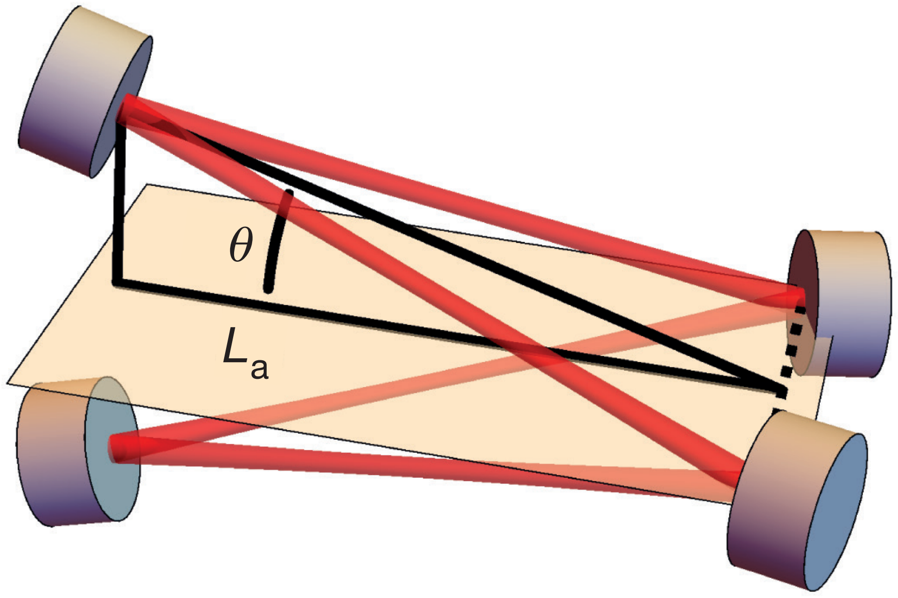


For modes circulating anti-clockwise in the site resonators (pseudo-spin-down)

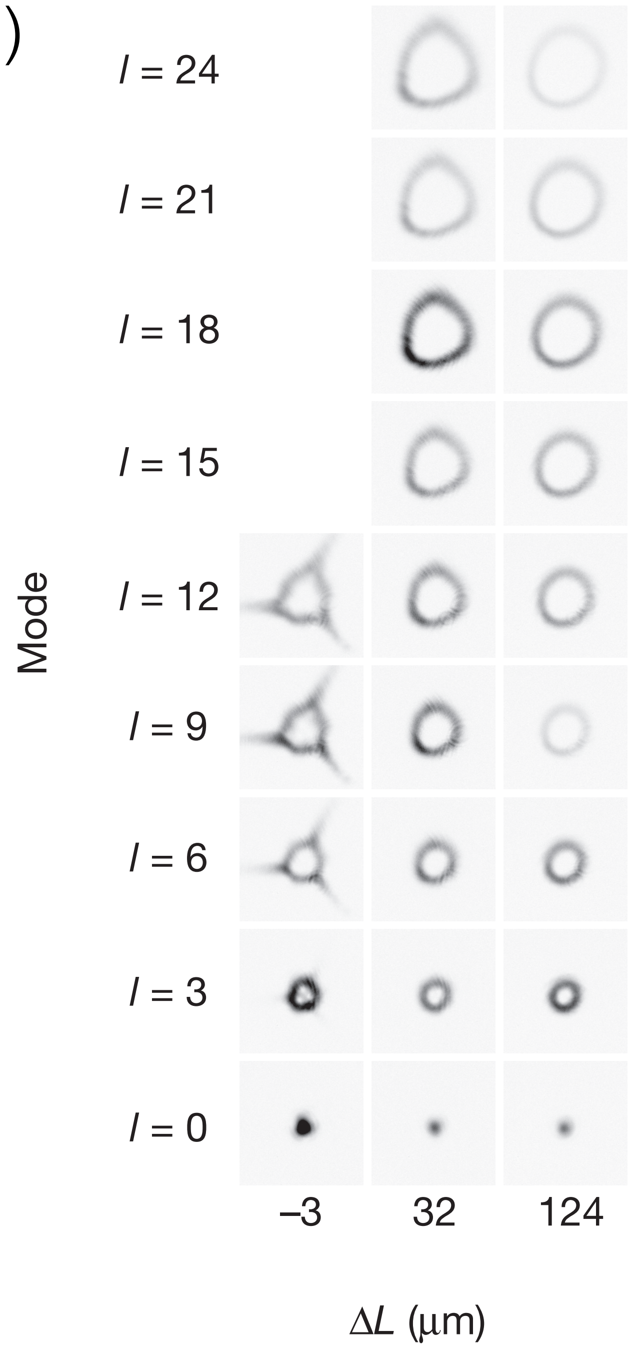
Twisted Optical Resonators

Chicago group: Schine, et al., Nature 534, 671 (2016).

Non-planar multimode cavity such that light experiences an image-rotation after a roundtrip —> Analogous to rotation of a gas (see Lecture 3)



Landau levels of photons



(Residual harmonic trapping)

Coriolis force in rotating frame —> Lorentz force

Photons traversing the cavity in one direction experience opposite “magnetic field” to the opposite direction: quantum spin Hall

Particularly promising set-up for observing strongly-correlated states (when combined with atoms & Rydberg-EIT)

Lecture 4

- How can we engineer topology for photons?
    - Quantum Hall systems
    - Quantum spin Hall systems
    - **SSH Model & Topological Pumps**
    - Gapless topology: Dirac & Weyl points
    - Topological superconductors?

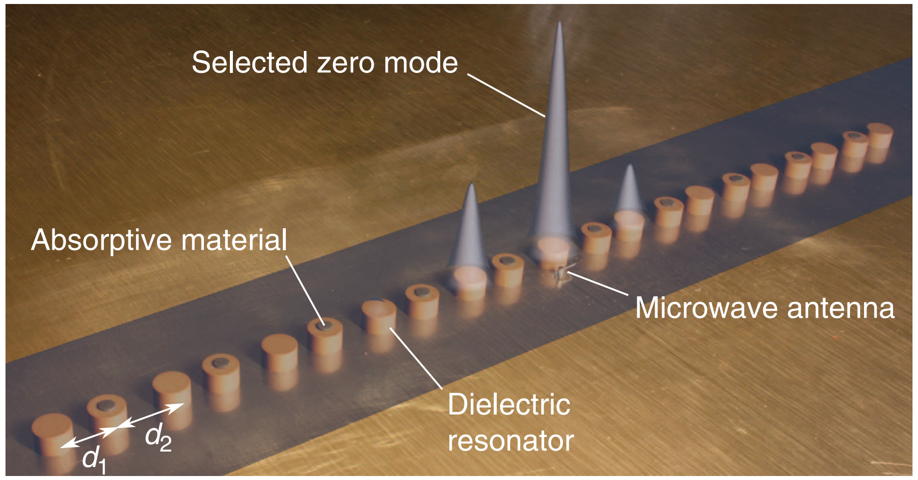
- How can we probe topology with photons?

- Future perspectives

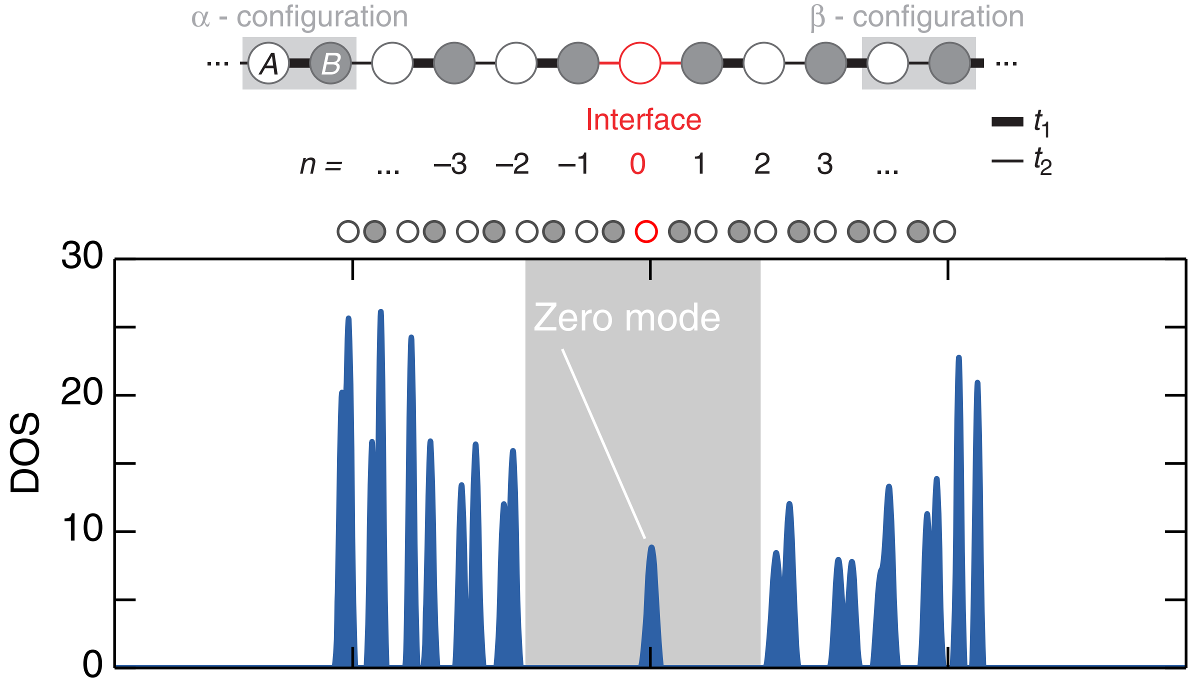
SSH Model

In photonic crystals, metamaterials, exciton-polaritons, microwave resonator arrays, etc…

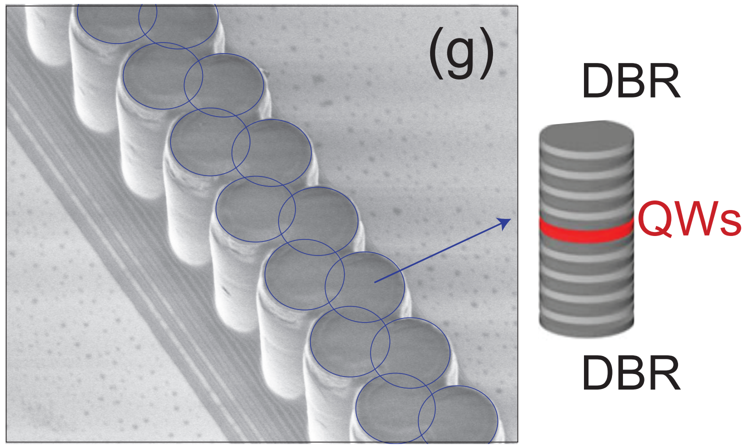
e.g. Nice: Poli, et al., Nature Comm. 6, 6710 (2015)



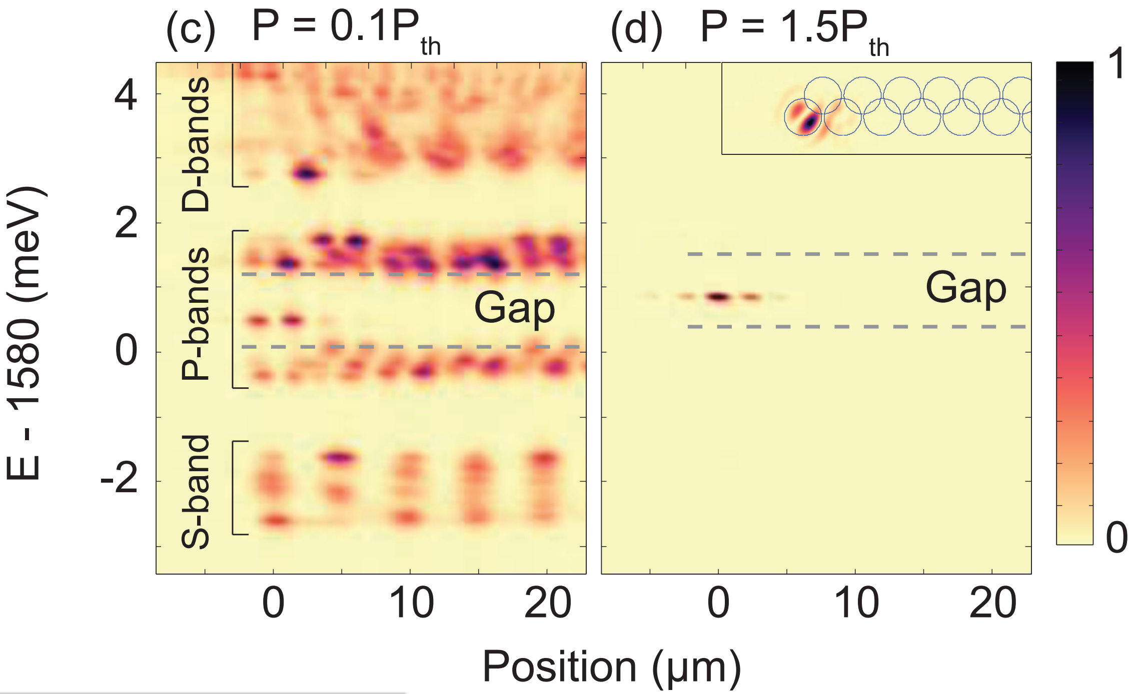
Spacing between resonators controls hopping



e.g. Marcoussis: St-Jean, et al., arXiv:1704.07310



exciton-polariton micropillars



	Symmetry			*d*							
	Time-reversal	Particle-hole	Chiral	1	2	3	4	5	6	7	8
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$

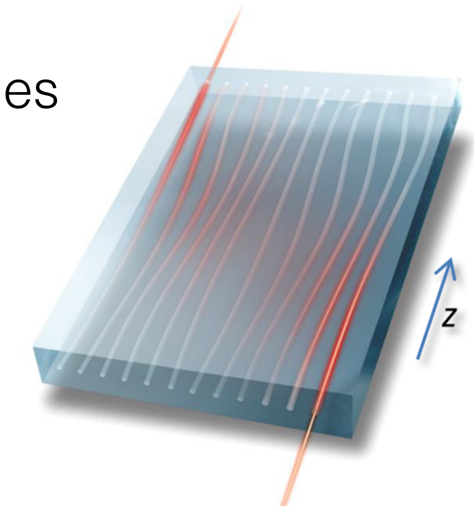
Lasing in a topological edge state!

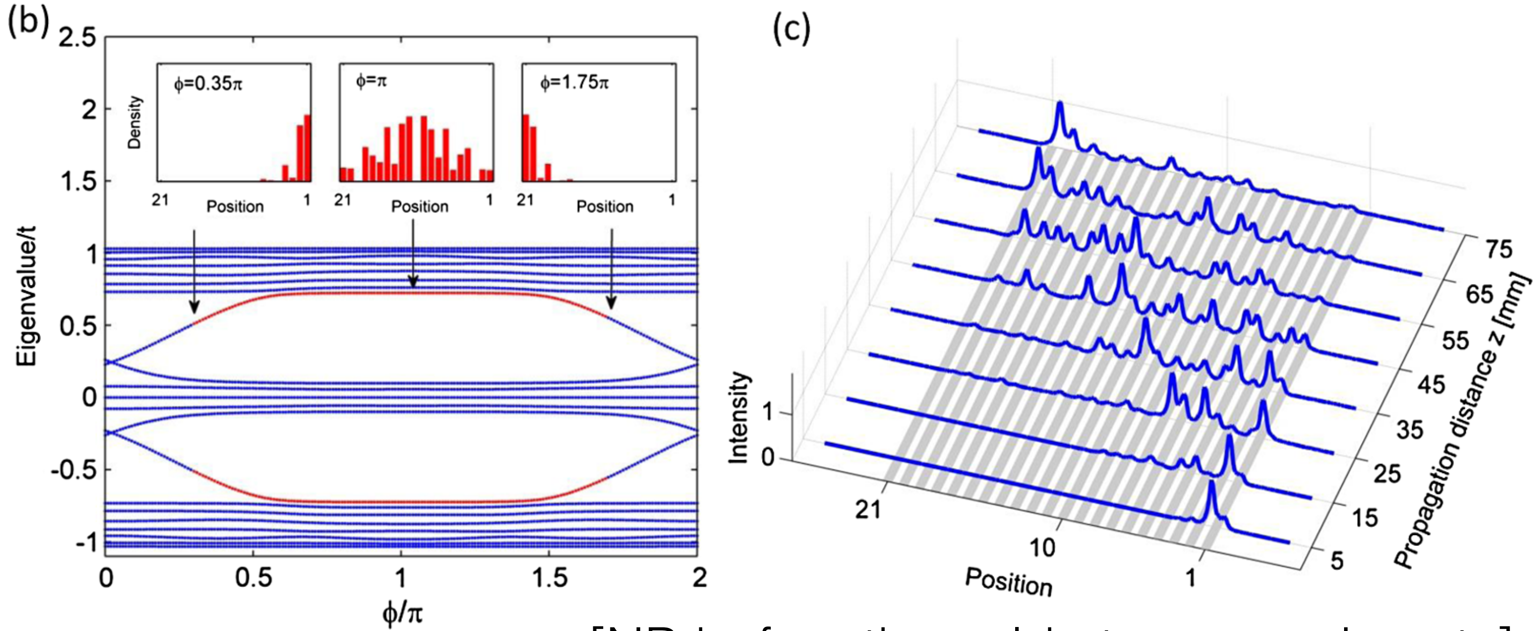
Topological Pumps: Propagating Waveguides

1D Pump —> Dynamical 2D QH Effect (*First Chern Number*)

Spacing between waveguides controls hopping

Remember that we exchange z and t



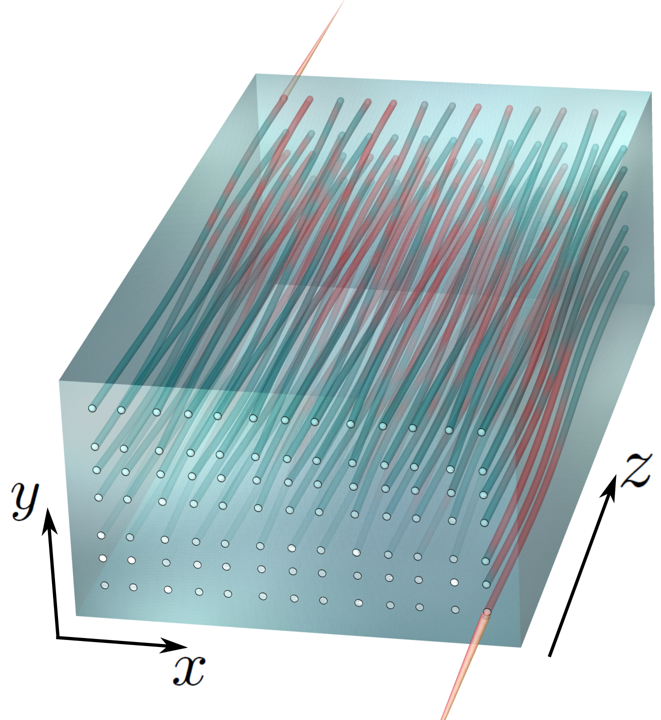


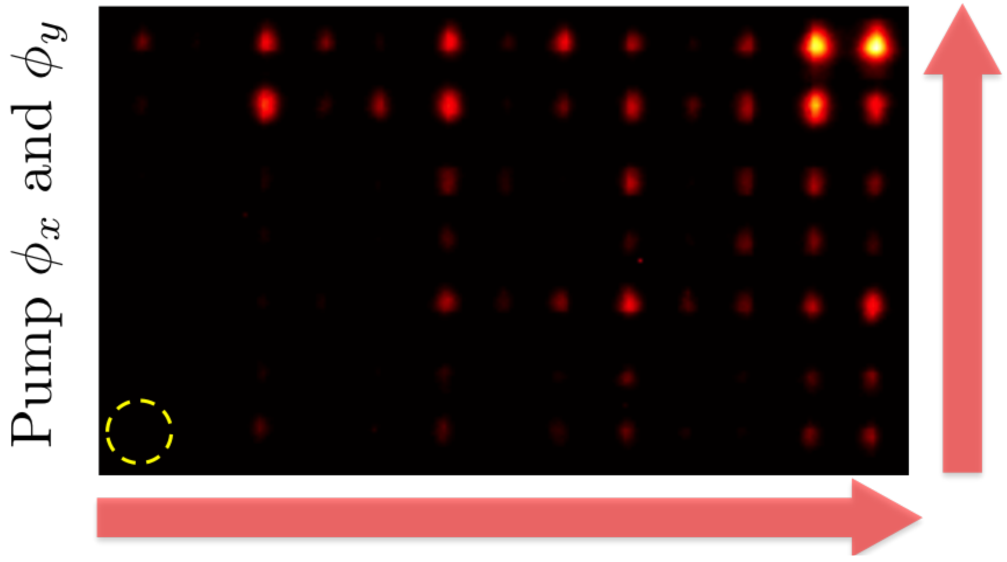
So pumping in time is now pumping in space

Kraus et al, PRL, 109, 106402 (2012) [NB before the cold atom experiments]

2D Pump —> Dynamical 4D QH Effect (*Second Chern Number*)

Zilberberg et al, arXiv:1705.08361





In cold atom experiments, probed bulk response, here probed edge states —> very complementary!

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  - SSH Model & Topological Pumps
  - Gapless topology: Dirac & Weyl points
  - **Topological superconductors?**

- How can we probe topology with photons?

- Future perspectives

Topological Superconductors?

Peano, et al., Nature Comm. 7, 10779 (2016) & PRX, 6, 041026, (2016)

A nonlinear cavity under parametric driving can have a Hamiltonian of the form:

$$H_{\text{cavity}} = i\hbar\chi^{(2)}\left(\beta^* \hat{a}^2 - \beta \hat{a}^{\dagger 2}\right)$$

i.e. can inject two photons at a time

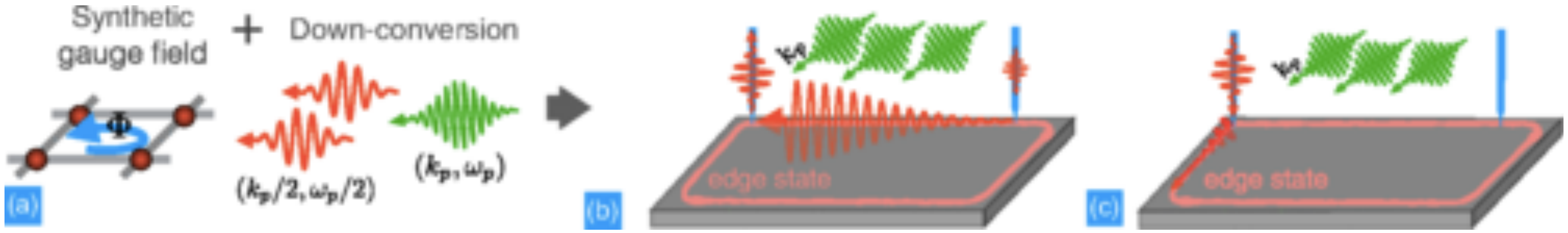
Aligning cavities to form a lattice, the momentum-space Hamiltonian takes

$$H_{\text{lattice}} = \frac{1}{2}\sum_{\mathbf{k}} \begin{pmatrix} \hat{\Psi}_{\mathbf{k}}^{\dagger} & \hat{\Psi}_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} A(\mathbf{k}) & B(\mathbf{k}) \\ B(-\mathbf{k})^* & A(-\mathbf{k})^t \end{pmatrix} \begin{pmatrix} \hat{\Psi}_{\mathbf{k}} \\ \hat{\Psi}^{\dagger}_{-\mathbf{k}} \end{pmatrix},$$

This reminds us of BdG in topological superconductors….

… but actually very different physics, e.g. no limit to occupancy of a state means there can be instabilities. **Need new topological classification for bosons!**

Can exploit instabilities to make *non-reciprocal* travelling-wave parametric amplifiers?



Also see Bardyn, et al., PRL 109, 253606 (2012) for proposal linked to Kitaev chain with parametric driving & strong interactions

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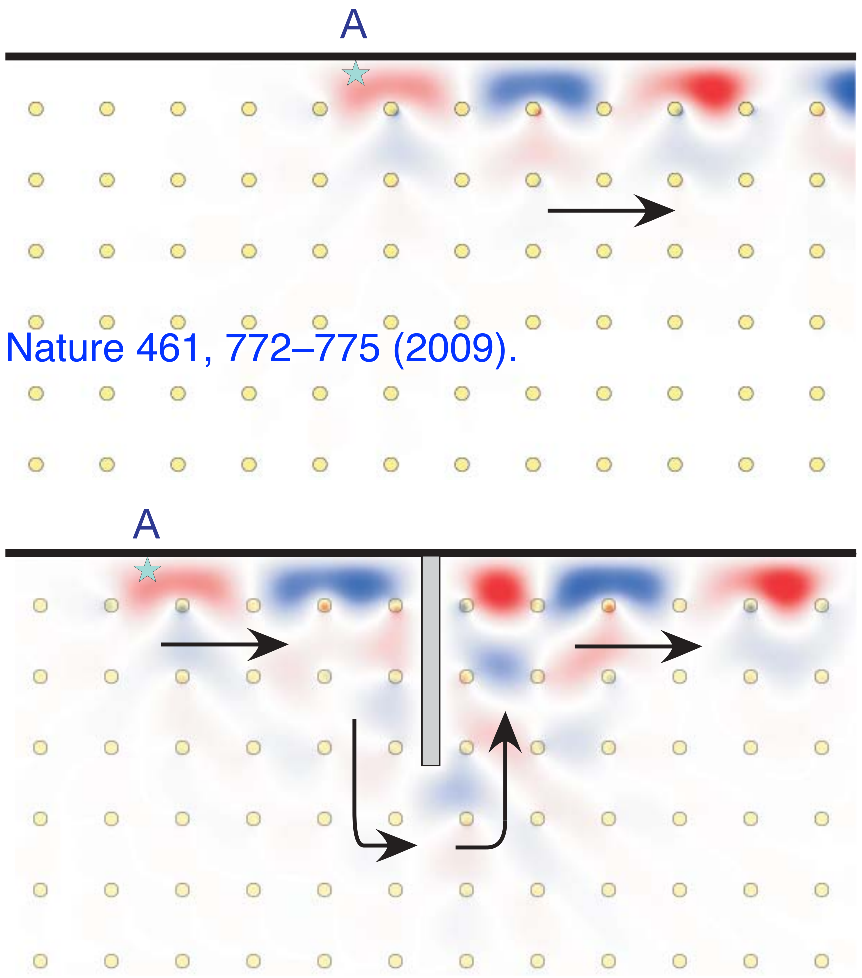
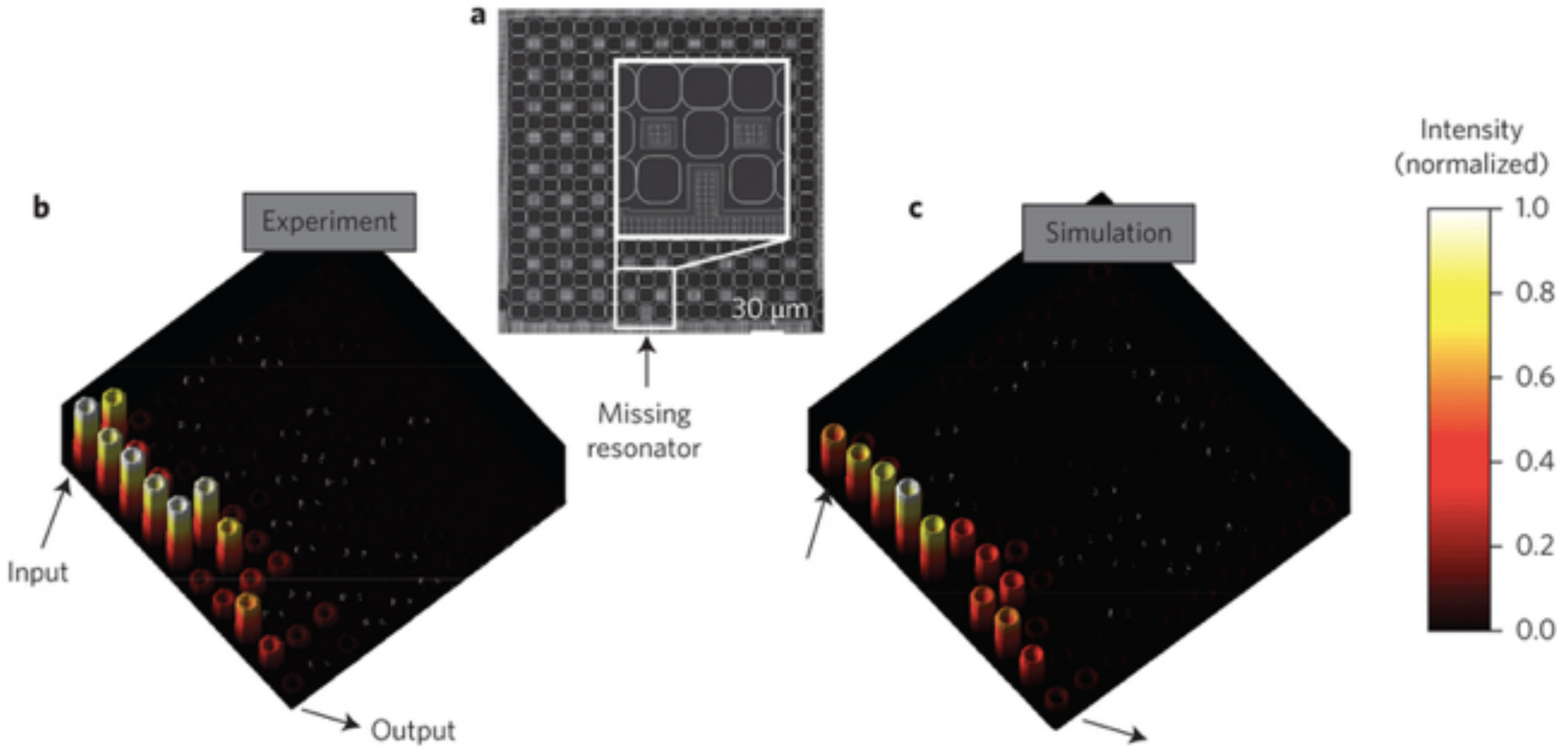
Probing topology with photons

Most straightforward: pump at the edge of the system to see topological edge states

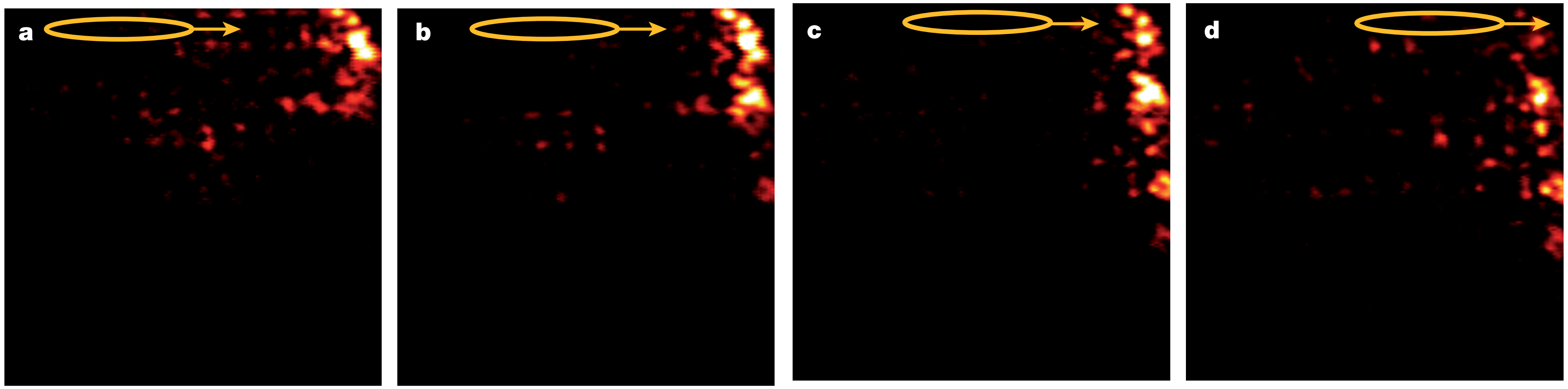


Wang et al., Nature 461, 772–775 (2009).

Hafezi, et al., Nat. Photonics 7, 1001 (2013).



Rechtsman, et al., Nature 496, 196 (2013)

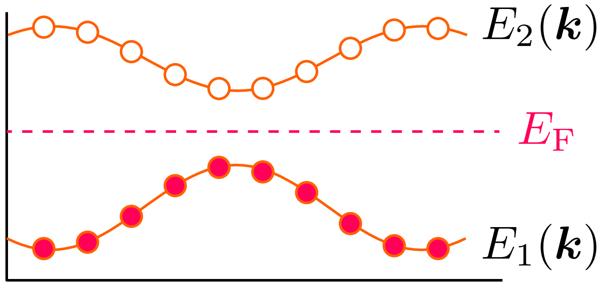


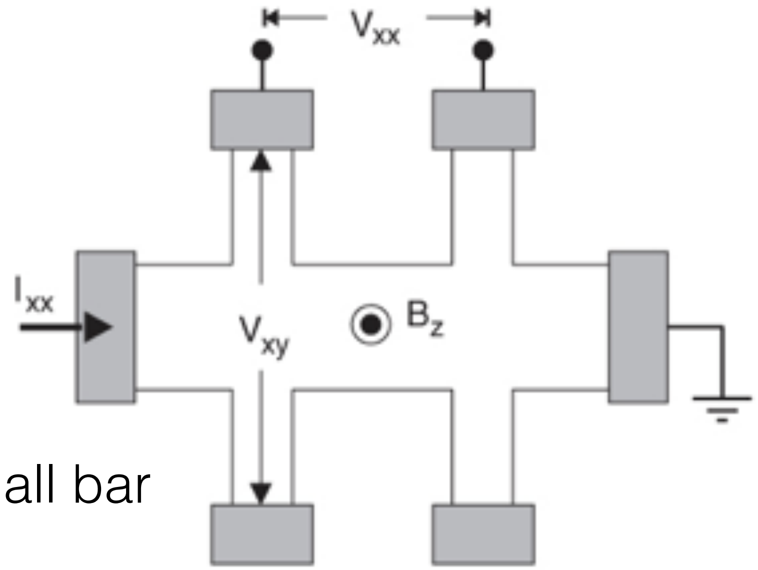
See e.g. topological robustness to disorder

Probing topology with photons

in solid state:

electrons fill bands up to Fermi level





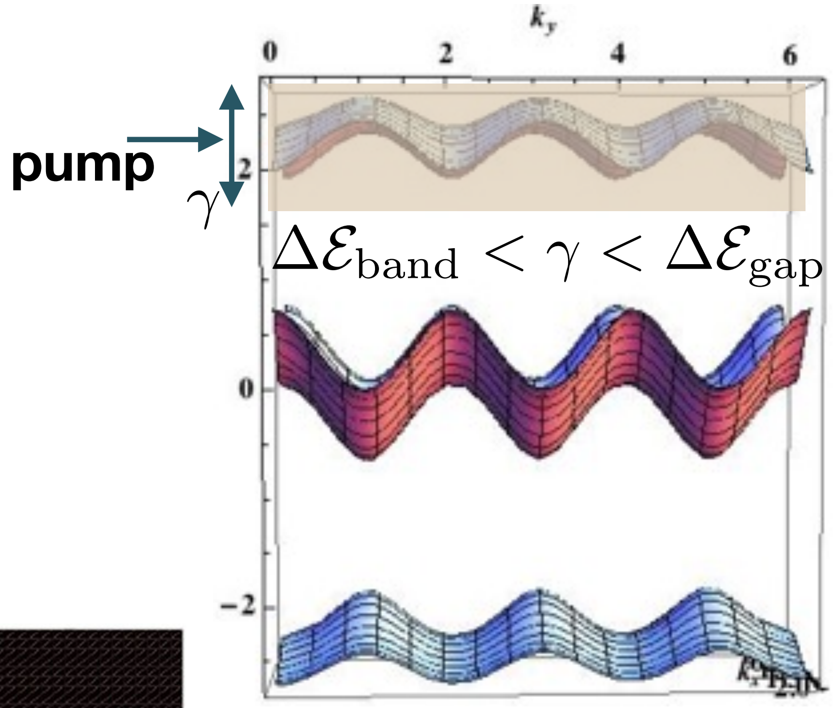
Hall bar

measurements of
currents and *voltages*

$$\sigma_{xy} = -\frac{e^2}{h} \sum_{n \in \text{occupied}} \nu_n$$

With photons:

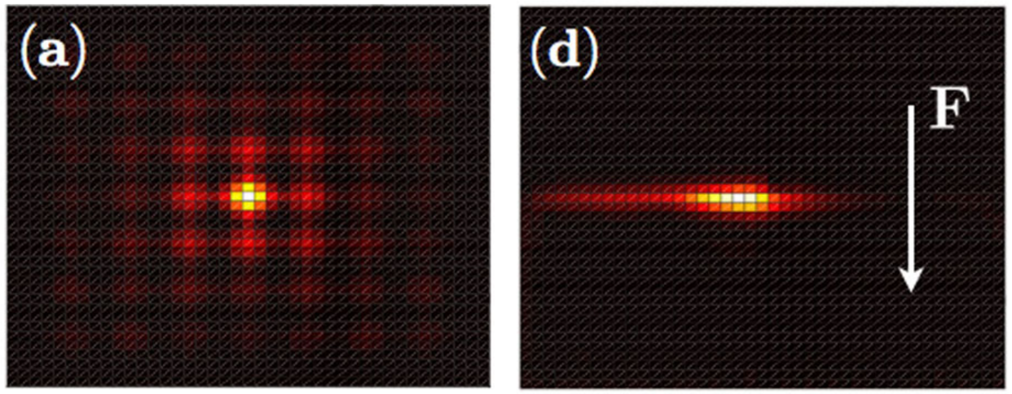
Proposal:
Ozawa & Carusotto, PRL, 112, 133902, (2014)



$\Delta\mathcal{E}_{\text{band}} < \gamma < \Delta\mathcal{E}_{\text{gap}}$

- Excite the system with a frequency of a bulk band
- Measure center-of-mass shift *of the photonic steady-state*

$$\langle x \rangle \approx \frac{2\pi \overbrace{\nu_1}^{\text{1st Chern number}} \overbrace{F}^{\text{external force}}}{\underbrace{A_{\mathrm{BZ}}}_{\text{BZ volume}} \underbrace{\gamma}_{\text{Loss}}}$$



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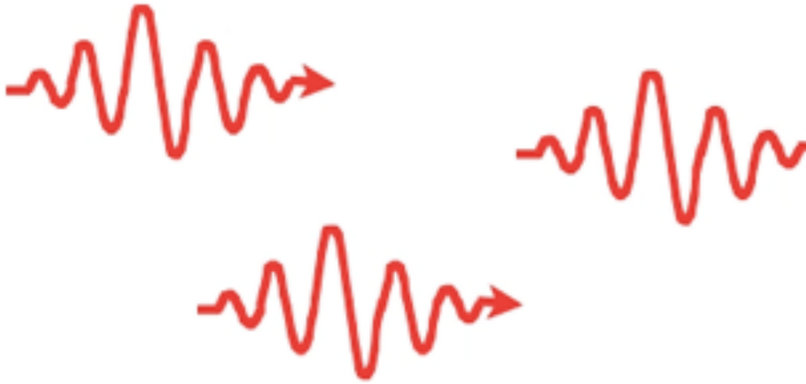
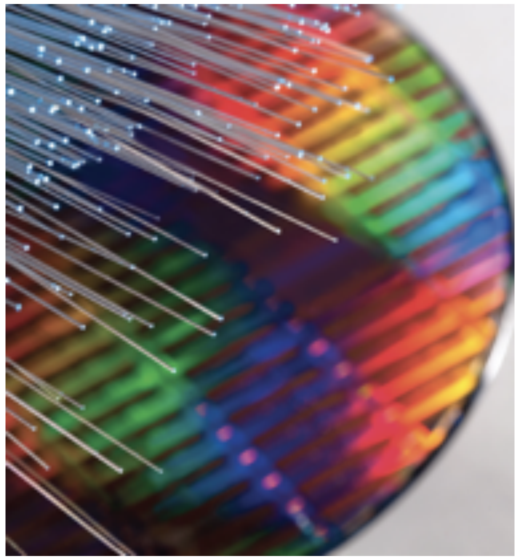
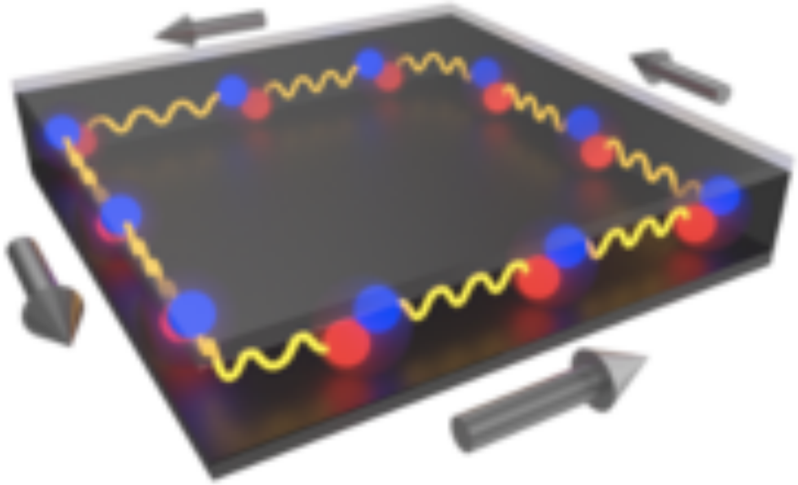
Future Perspectives

- Topological phases of fermions have been classified: what about bosons?



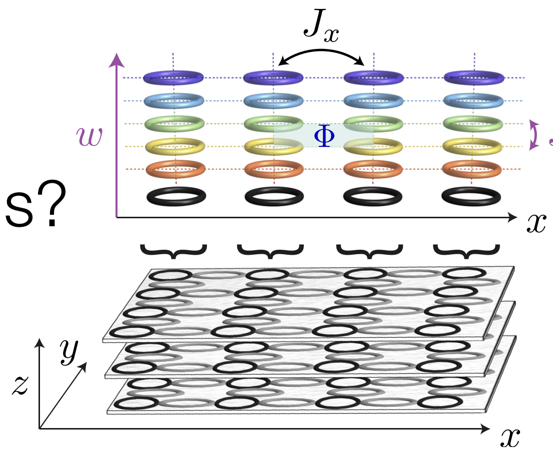
- Topological protection useful for quantum information?
- Practical photonics devices with topological protection? e.g. optical isolator?





Future Perspectives

- Can we reach new topological phases of matter, e.g. in higher dimensions?

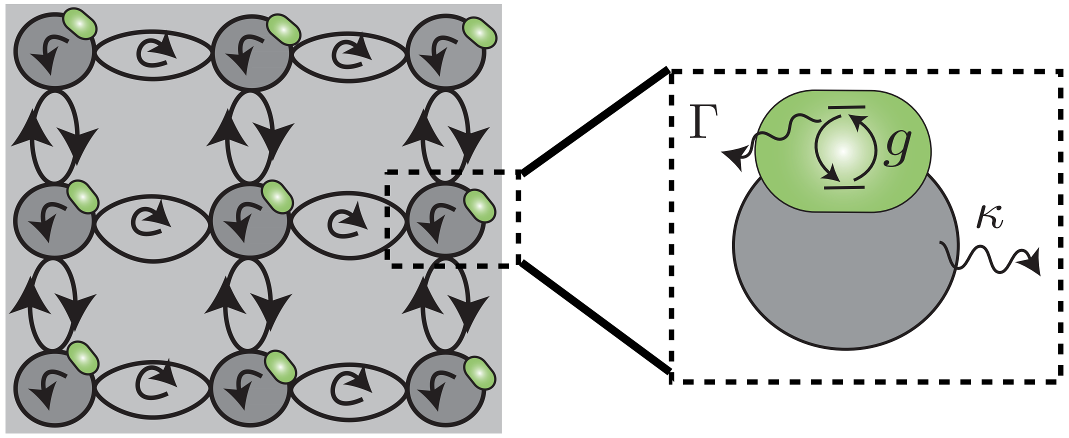


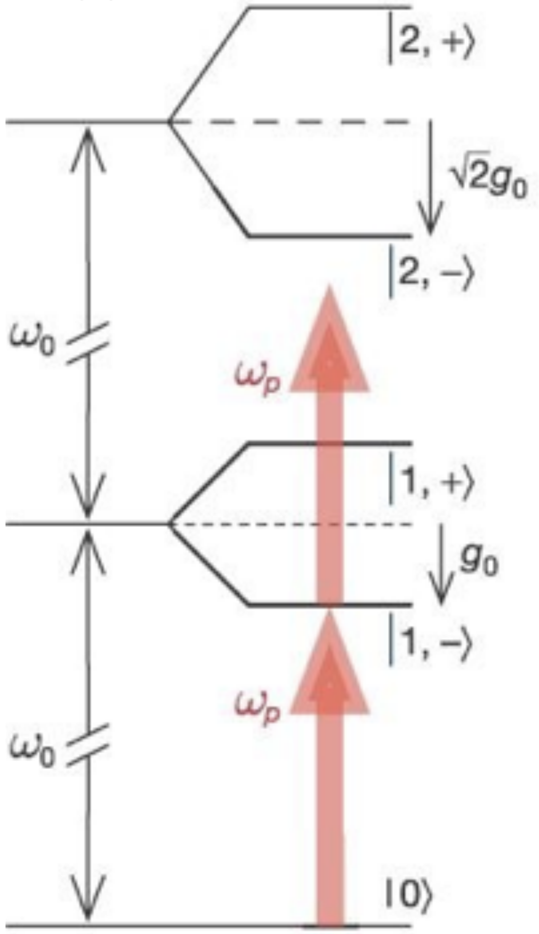
- What happens when the photonic material is weakly or strongly nonlinear?

  See e.g. review of Carusotto et al, RMP (2013)

- How to prepare a fractional quantum Hall state of light?

  See e.g. Kapit, et al PRX (2014)





e.g. combine synthetic gauge fields
& photon blockade?

Overview

Lectures 1 & 2

Introduction to Topological Phases of Matter

Lecture 3

Topological Phases of Matter with Ultracold Atoms

Lecture 4

Topological Phases of Matter with Photons

Thanks very much for your attention!