Quantum Information Processing & Quantum Optics with Superconducting Circuits

Trieste Summer School, Italy

September 2017

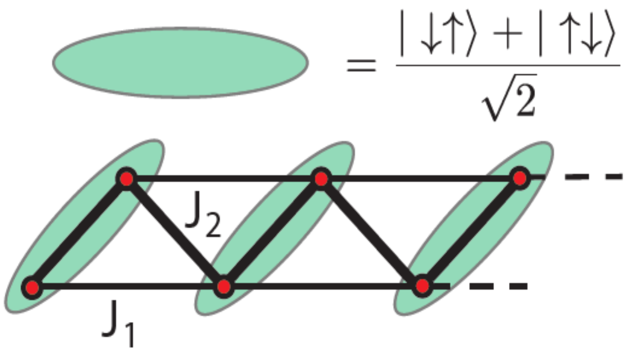
Gerhard Kirchmair

Institute for Quantum Optics and Quantum Information, Innsbruck
Institute for Experimental Physics, University of Innsbruck

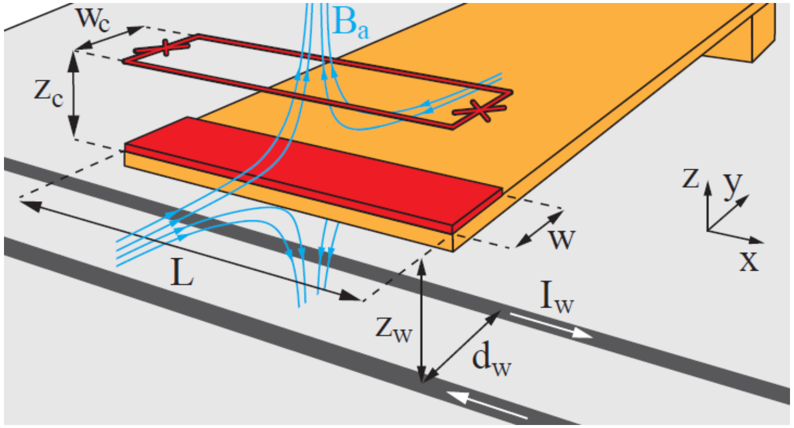
Experiments in Innsbruck on circuit QED

Quantum circuits group at IQOQI Innsbruck: https://iqoqi.at/en/group-page-kirchmair

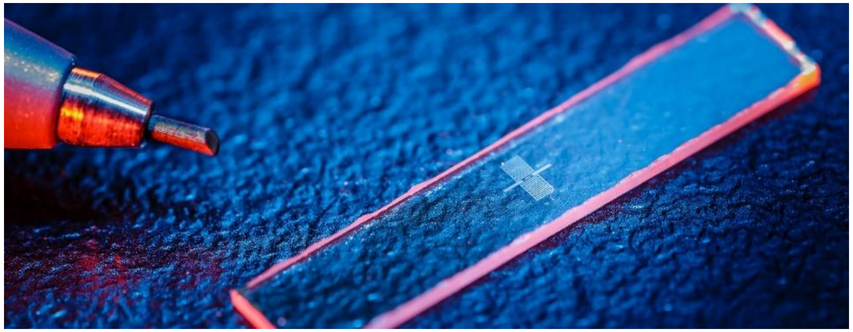
Quantum Simulation using cQED



Quantum Magnetomechanic



Josephson Junction array resonators



More information

- **Low Temperature Physics & Superconductivity**
  - http://www.wmi.badw.de/teaching/Lecturenotes/
  - Groß & Marx – Festkörper Physik, De Gryter

- **SC circuits, Circuit Quantization, SC Qubits**
  - Introduction to Quantum Electromagnetic Circuits
    U. Vool, M. Devoret, https://arxiv.org/abs/1610.03438
  - S.M. Girvin, SC Qubits and Circuits, Les Houches Lectures, 2011

- **SC Qubits and Circuit QED**
  - Superconducting quantum bits
    J. Clarke & F.K. Wilhelm, Nature 453, 1031 (2008)
  - Cavity quantum electrodynamics for SC electrical circuits
    Blais et.al., PRA 69, 062320 (2004)
  - SC circuits for Quantum Information: An Outlook
    M.H. Devoret & R.J. Schoelkopf, Science 339, 1169 (2013)

- **IBM quantum experience:** https://www.research.ibm.com/ibm-q/

Superconductivity & Josephson Effect

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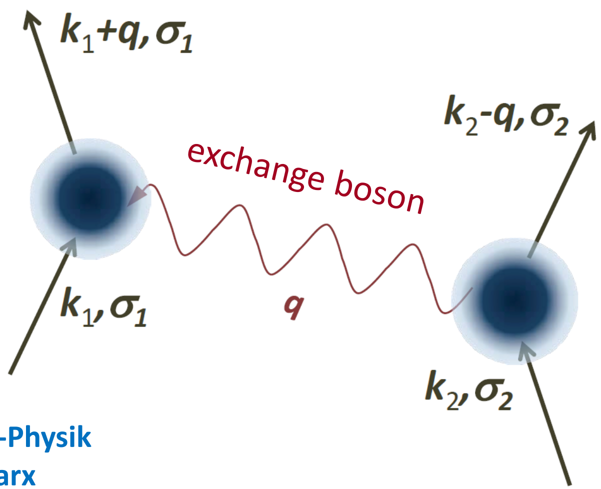
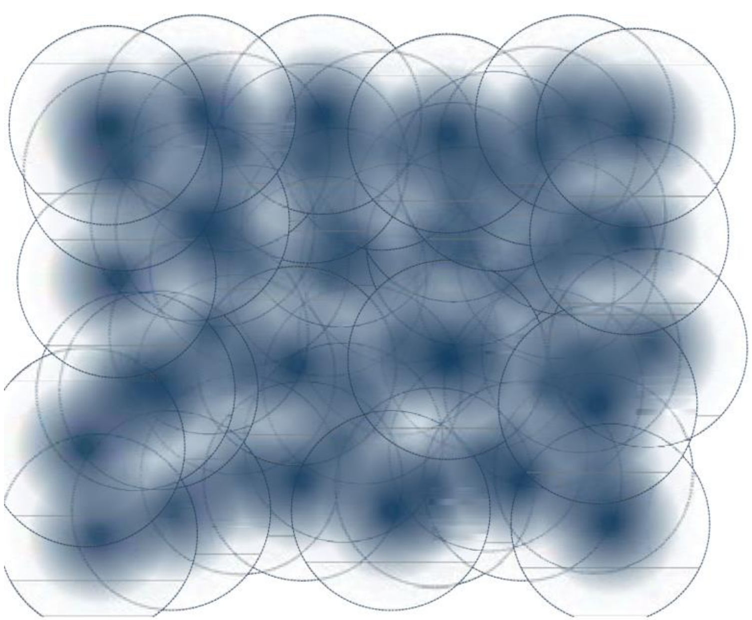
Superconductivity



Festkörper-Physik
Gross & Marx
De Gruyter



- phonon mediated interaction between two e⁻
  range ≈ 100nm
- formation of cooper pairs
  anti correlated in moment q,-q
  $V_{cp} \approx (100\text{nm})^3$
- cooper pairs condense into coherent quantum state
  describe with Macroscopic wave function

$$\Psi(\boldsymbol{r}, t) = \Psi_0 e^{-i\,\theta(\boldsymbol{r},t)}$$

$$|\Psi(\boldsymbol{r}, t)|^2 = n_s(\boldsymbol{r}, t)$$

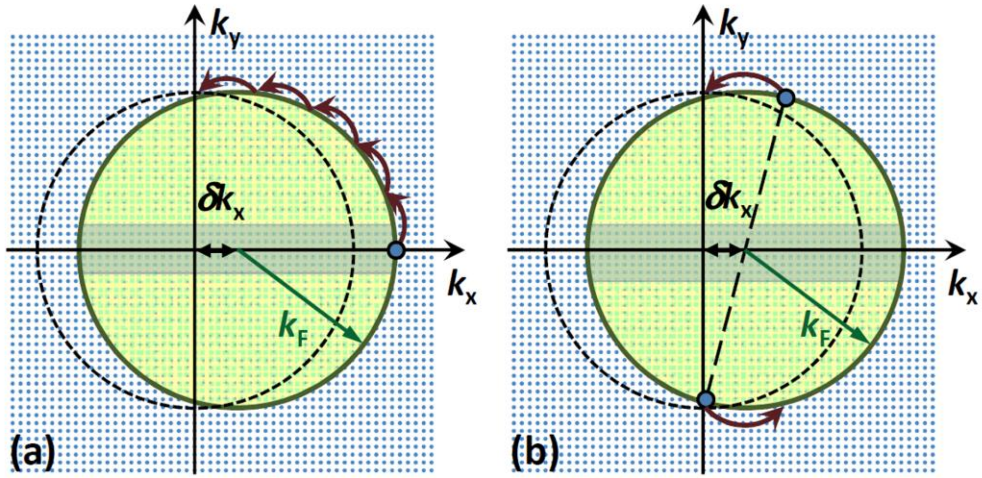
$$n_e \approx 10^6 / V_{cp}$$

Superconductivity

Why does a super-current not vanish?

Electrons

Cooper pairs

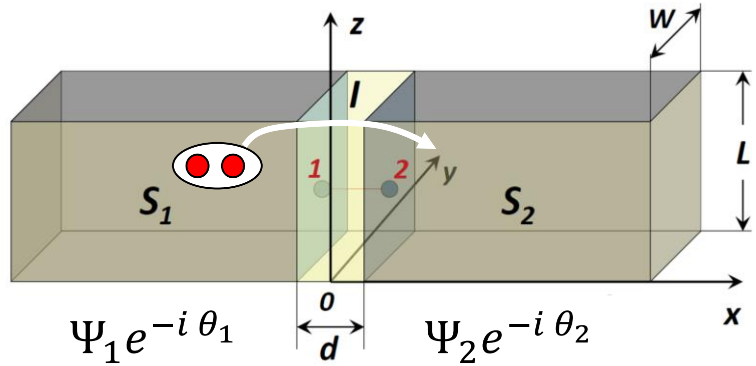


Festkörper-Physik
Gross & Marx
De Gruyter

Momentum correlation ensures super current does not vanish!
Quantum effect!!

Josephson Junction

Festkörper-Physik
Gross & Marx
De Gruyter



Brian Josephson

Tunnel current
1. Josephson equation $I(\varphi) = I_c \sin\varphi$ $\varphi = \theta_2 - \theta_1$ Quantum interference!

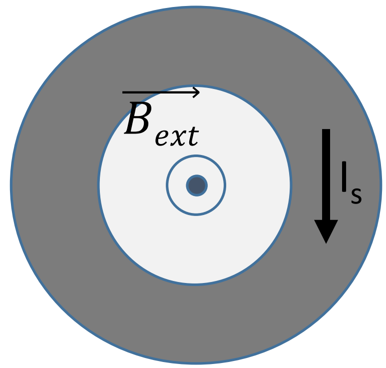
2. Josephson equation $\dot{\varphi} = \frac{2e}{\hbar} V(t)$ → $V_{jj} = \underbrace{\frac{\hbar}{2e}\frac{1}{I_c\cos\varphi}}\dot{I}$

L_j…Josephson Inductance

Energy: $E = \int_0^t I\,V\,dt = I_c\,\frac{\hbar}{2e}\int_0^t \sin(\varphi)\frac{d\varphi}{dt}dt = \underbrace{I_c\,\frac{\hbar}{2e}}\cos(\varphi)$

E_j…Josephson Energy

Fluxoid quantization



From London equations:

$$\oint_C \Lambda \mathbf{J_s} \cdot d\ell + \int_F (\nabla \times \mathbf{A}) \cdot \hat{n}\, dF = \frac{\hbar}{q_s} \underbrace{\oint_C \nabla\theta \cdot d\ell}_{2\pi n} = \frac{h}{2e} n = n\, \phi_0$$

(2π n: $\oint_C \nabla\theta \cdot d\ell$)

Super-current in the loop ($\oint_C \Lambda \mathbf{J_s} \cdot d\ell$)

Magnetic field ($\int_F (\nabla \times \mathbf{A}) \cdot \hat{n}\, dF$)

Gradient of the phase θ of Ψ ($\oint_C \nabla\theta \cdot d\ell$)

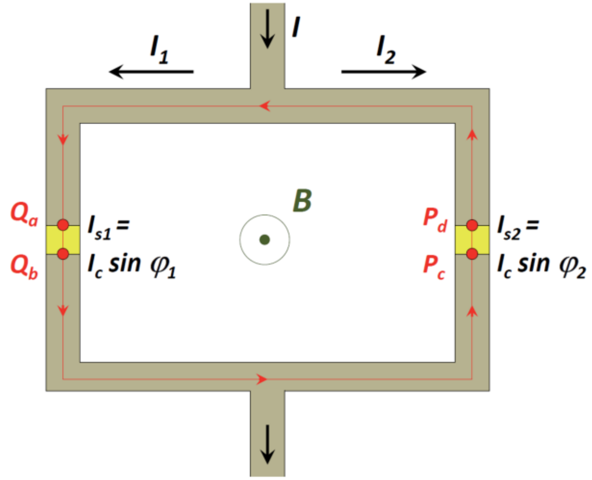
Flux Quant (ϕ_0)

Applying a B-field to a ring creates a super-current such that the total flux through the loop is

$$n\, \phi_0$$

Phase and flux are linked!!

SQUID



Festkörper-Physik
Gross & Marx
De Gruyter

$$I(\varphi) = I_c \sin\varphi_1 + I_c \sin\varphi_2 = I_c\,(\sin\varphi_1 + \sin\varphi_2) = 2I_c\,(\cos(\frac{\varphi_1 - \varphi_2}{2})\,\sin(\frac{\varphi_1 + \varphi_2}{2})$$

Fluxoid quantization: $$\varphi_1 - \varphi_2 + \frac{\Phi_{ext}}{\Phi_0} = n\,2\pi$$

direction of $\int d\vec{l}$

$$I(\varphi) \sim I_c \cos\Phi_{ext}$$

External field can tune the effective critical current of the SQUID and thus E_j!

Introduction to SC circuits

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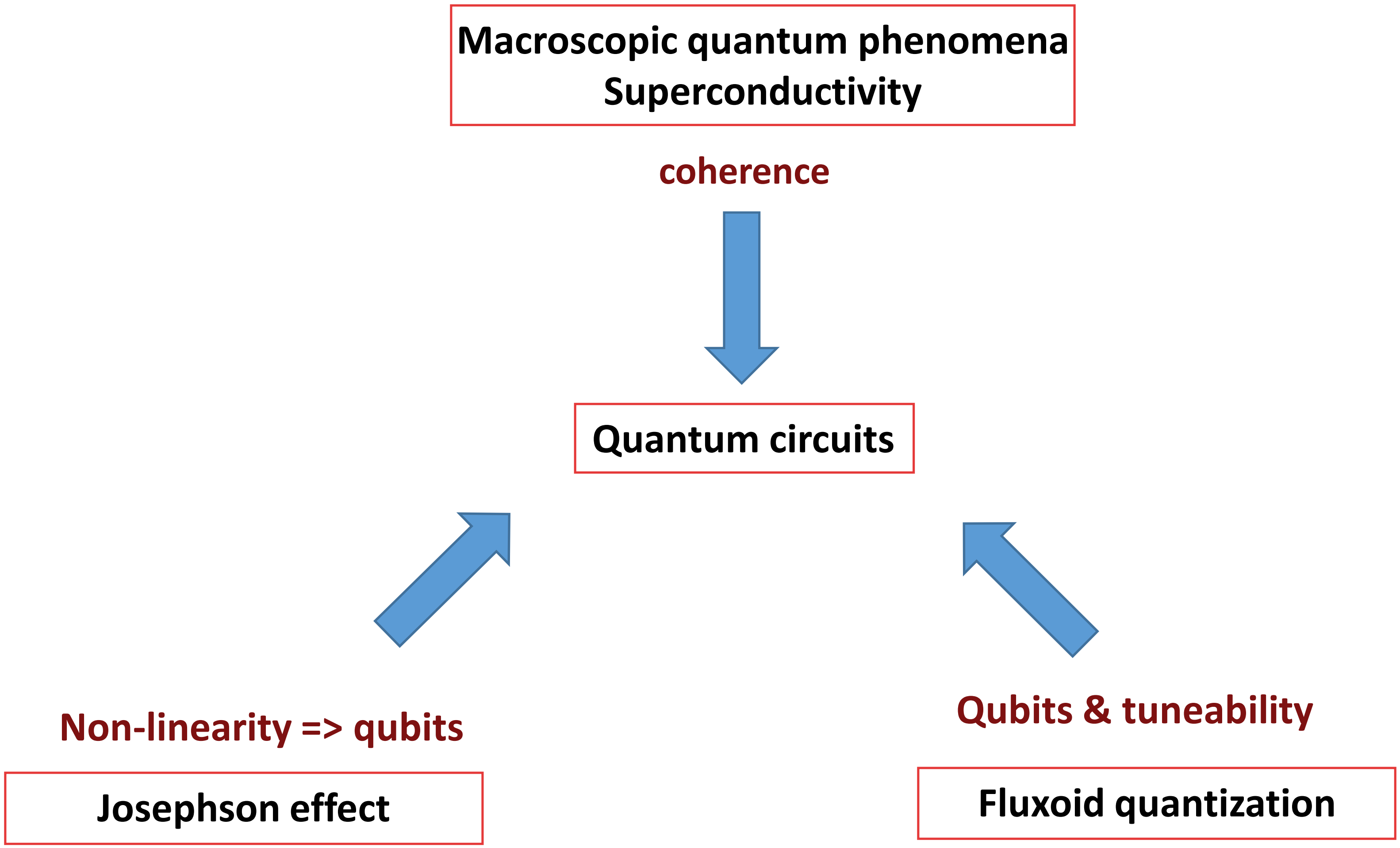
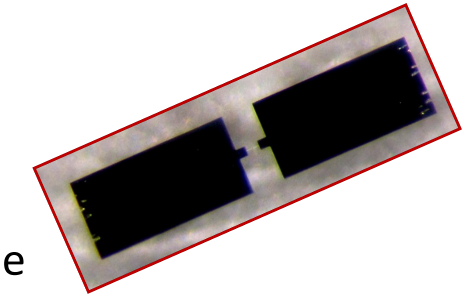
Superconducting Quantum circuits



Why superconducting qubits & circuits?

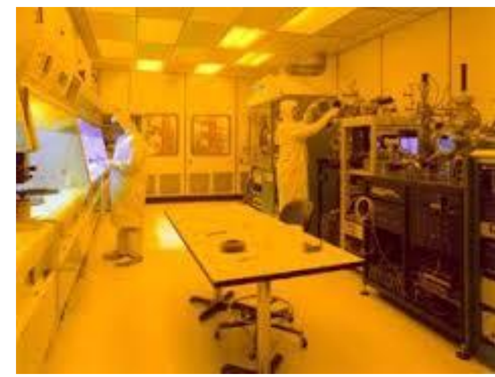
- **Advantages:**



Manmade objects → energy spectrum can be engineered

Manmade objects → micro fabrication => in principle scalable

Manmade objects → design flexibility, tunable parameters



Engineered dipoles → Strong interactions

Microwave control → commercially available components

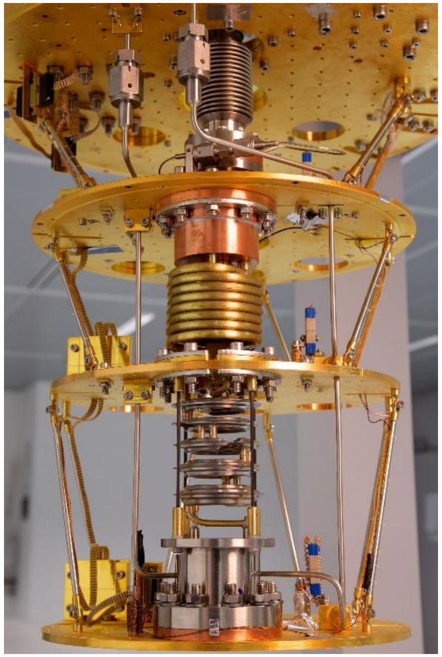
Why superconducting qubits & circuits?

- **Drawbacks:**

Manmade objects → spread in parameters

Strong coupling to environment → need protect against thermal microwave fields
→ avoid two level fluctuators
→ good qubit design

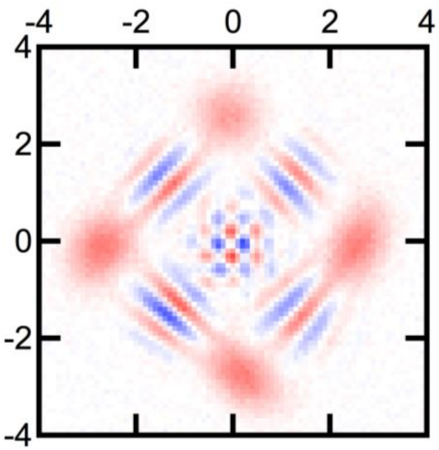
Low energy scales → Cryogenic environment



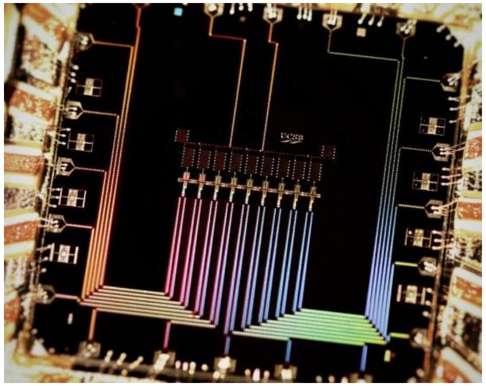
QIP and QO with SC circuits

Many groups around the world:

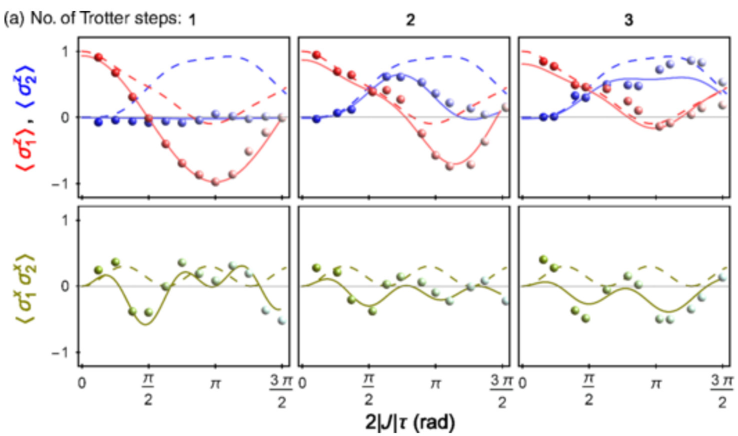
Yale University, UC Santa Barbara, ETH Zurich, TU Delft, Princeton, University of Chicago, Chalmers, Saclay Paris, KIT Karlsruhe, IBM …



Cat states & QEC
Devoret & Schoelkopf
Yale University



Chip for Error correction
Martinis
UCSB, Google



Quantum Simulation
Wallraff
ETH Zurich

QIP, Quantum Optics, Quantum Measurement, Quantum Simulation…

Requirements for QIP (DiVincenzo criteria)

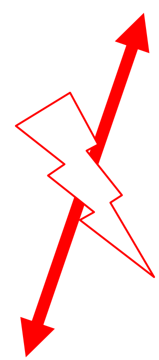
- Scalable Qubit

  Quantized energy levels in a elect. Circuit, micro-fabrication

- State initialization

  Cooling such that $\hbar\omega > k_b\, T \quad (20\ \mathrm{GHz}\ \approx 1\ \mathrm{K})$

- Single and two qubit operations
  - Coherent control using microwave signals, strong coupling to outside world



- State measurement

  Coupling of qubits to other circuit elements e.g. cavity

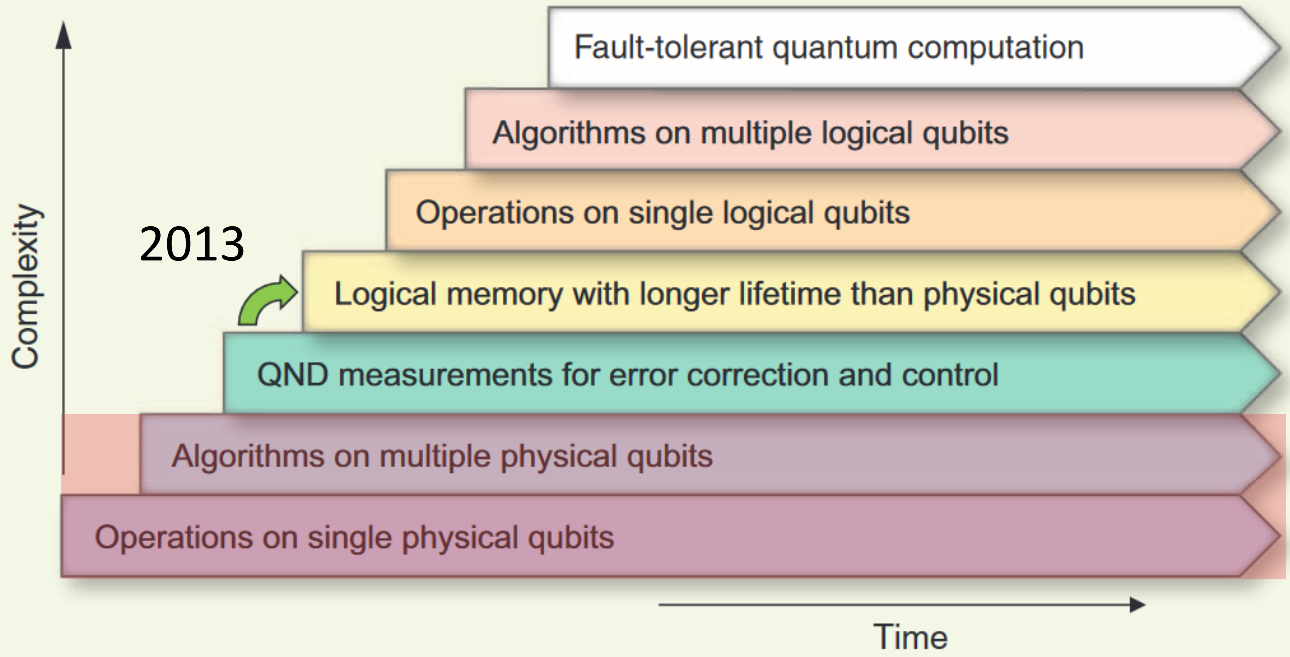
- Long coherence times

  Superconducting materials, decoupling from environment

QIP with Superconducting Qubits

Where are we and what are the steps to a Quantum Computer?

M.H. Devoret & R.J. Schoelkopf, Science 339, 1169 (2013)



This lecture:

A little there

and

How to do circuits, create qubits and get the right Hamiltonian

mostly here

How to quantize a simple circuit – LC Oscillator

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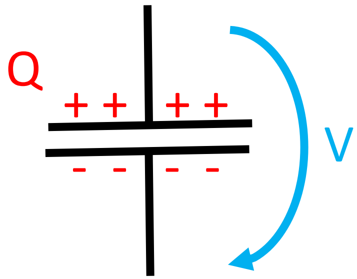
Building Blocks for SC circuits

Lumped element: size << λ neglect retardation effects $\lambda_{\mu w}$ = cm

Capacitance

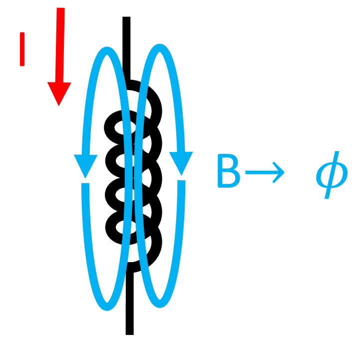


$$V = \frac{Q}{C}$$

$$E_c = \int_0^Q V(q)dq = \int_0^Q \frac{q}{c} dq =$$

$$E_c = \frac{Q^2}{2\,C}$$

Inductance

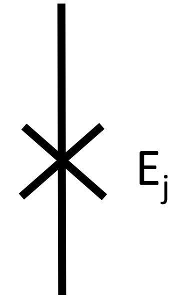


$$\left.\begin{aligned} V &= L\,\dot{I} \\ V &= -\,\dot{\phi} \end{aligned}\right\} \quad \phi = -L\,I$$

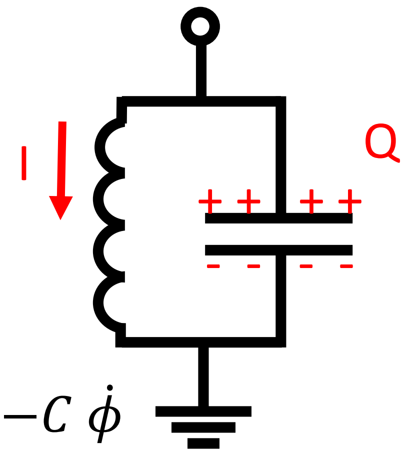
$$E_L = \int_0^t VI\,dt = \int_0^t L\,I\,\dot{I}dt = \int_0^I L\,I\,dI =$$

$$E_L = L\frac{I^2}{2} = \frac{\phi^2}{2\,L}$$

Josephson Junction



Quantum LC Oscillator



"Potential" Energy

$$E_L = \frac{\phi^2}{2\,L}$$

coordinate

"Kinetic" Energy

momentum

$$E_c = \frac{Q^2}{2\,C}$$

$$Q = C\,V = -C\,\dot{\phi}$$

Lagrangian

$$\mathcal{L} = T - U = \frac{Q^2}{2\,C} - \frac{\phi^2}{2\,L} = \frac{C\,\dot{\phi}^2}{2} - \frac{\phi^2}{2\,L}$$

Euler Lagrange

$$\frac{d}{dt}\frac{d\mathcal{L}}{d\dot{\phi}} = \frac{d\mathcal{L}}{d\,\phi} \Rightarrow \ddot{\phi} = -\frac{1}{LC}\phi$$

$$\omega_0^2 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

Conjugate Variables

$$\frac{d\mathcal{L}}{d\dot{\phi}} = C\dot{\phi} = C\;U = Q$$

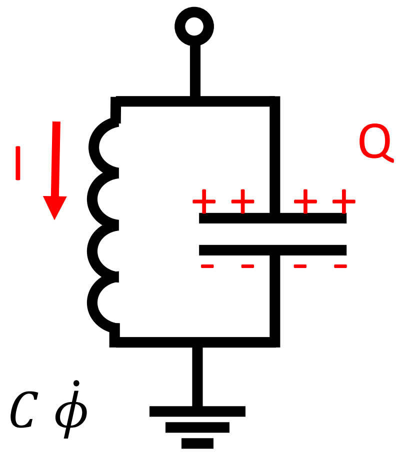
charge and flux are conjugate variables!

Quantum LC Oscillator

"Potential" Energy

$$E_L = \frac{\phi^2}{2\,L}$$

coordinate



"Kinetic" Energy

momentum

$$E_c = \frac{Q^2}{2\,C}$$

$$C\,V = C\,\dot{\phi}$$

Lagrangian

$$\mathcal{L} = T - U = \frac{Q^2}{2\,C} - \frac{\phi^2}{2\,L} = \frac{\dot{\phi}^2}{2\,C} - \frac{\phi^2}{2\,L}$$

Hamilton

$$H = Q\,\dot{\phi} - \mathcal{L} = C\,\dot{\phi}^2 - \frac{C}{2}\dot{\phi}^2 + \frac{\phi^2}{2\,L} = \frac{C}{2}\dot{\phi}^2 + \frac{\phi^2}{2\,L}$$

$$H = \frac{Q^2}{2\,C} + \frac{\phi^2}{2\,L}$$

$Q \rightarrow \hat{Q} \quad \phi \rightarrow \hat{\phi}$

$$[\hat{Q}, \hat{\phi}] = -\mathrm{i}\,\hbar$$

$$H = \frac{\hat{Q}^2}{2\,C} + \frac{\hat{\phi}^2}{2\,L}$$

Quantum LC Oscillator

Introduce raising and lowering operators such that:

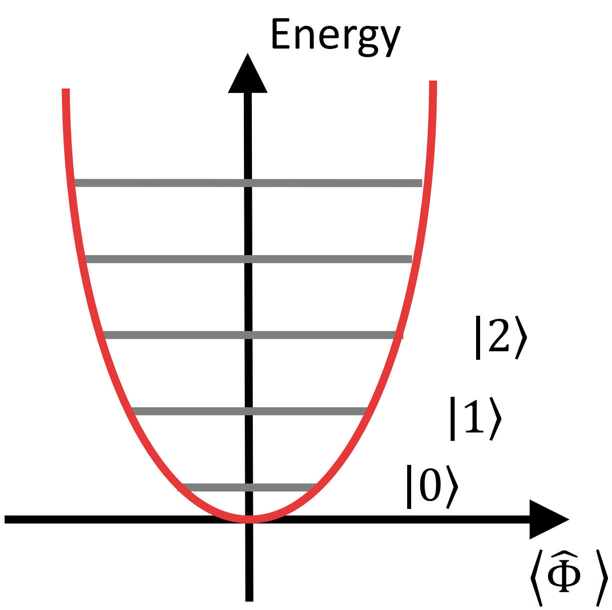
$$\hat{\phi} = \sqrt{\frac{\hbar\, Z}{2}}(a + a^{\dagger})$$ ← ϕ_{zpf}

$$\hat{Q} = -i\sqrt{\frac{\hbar}{2\, Z}}(a - a^{\dagger})$$ ← Q_{zpf}

Just like a QM harmonic oscillator with the characteristic impedance $Z = \sqrt{\frac{L}{C}}$

Hamilton of an LC Oscillator

$$H = \hbar\omega\left(a^{\dagger}a + \frac{1}{2}\right)$$

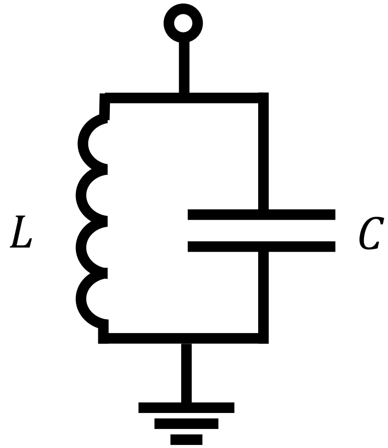


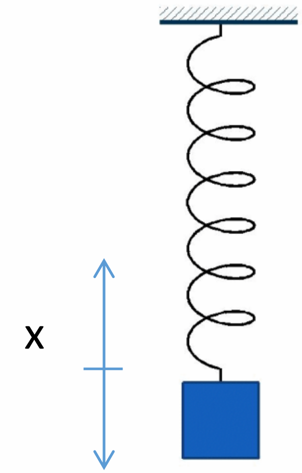
Ground state size

$$\langle 0|\hat{Q}^2|0\rangle = \frac{\hbar}{2Z} = Q_{zpf}^2$$

$$\langle 0|\hat{\phi}^2|0\rangle = \frac{\hbar\, Z}{2} = \phi_{zpf}^2$$

LC Resonator - Harmonic Oscillator





$$H = \frac{\hat{Q}^2}{2\,C} + \frac{\hat{\phi}^2}{2\,L} \qquad [\hat{\phi}, \hat{Q}] = \mathrm{i}\,\hbar$$

$$a = \sqrt{\frac{1}{2\hbar Z}}(\hat{\phi} + iZ\hat{Q}) \qquad \omega = \sqrt{\frac{1}{L\,C}}$$

$$a^\dagger = \sqrt{\frac{1}{2\hbar Z}}(\hat{\phi} - iZ\hat{Q}) \qquad Z = \sqrt{\frac{L}{C}}$$

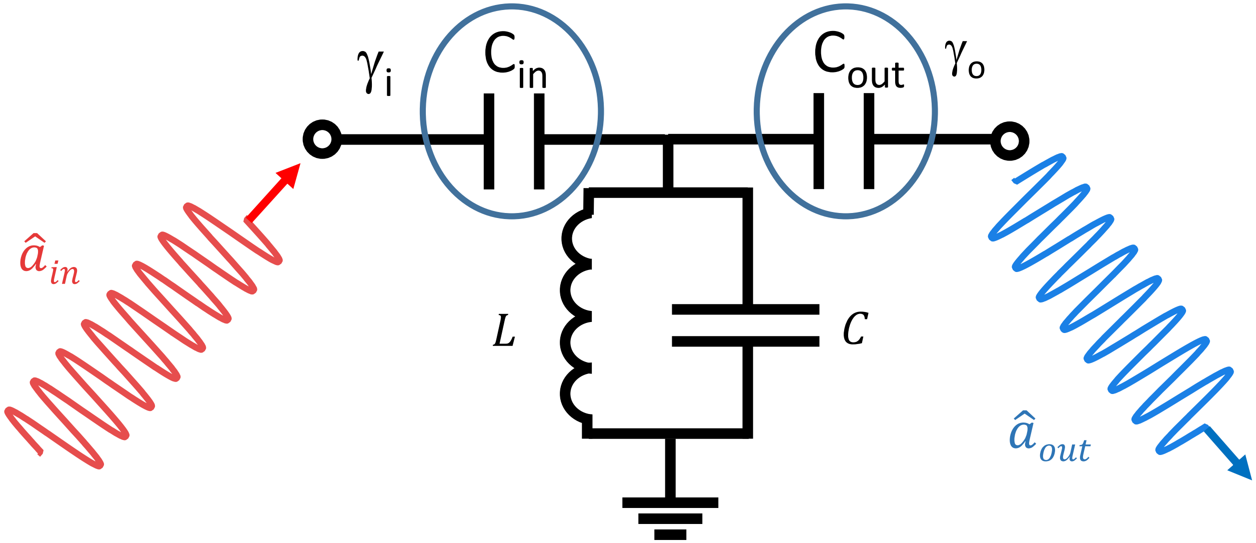
$$H = \frac{\hat{p}^2}{2\,m} + \frac{m\,\omega^2\hat{x}^2}{2} \qquad [\hat{x}, \hat{p}] = \mathrm{i}\,\hbar$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}(\hat{x} + i\,\frac{\hat{p}}{m\omega}) \qquad \omega = \sqrt{\frac{k}{m}}$$

$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}(\hat{x} - i\,\frac{\hat{p}}{m\omega})$$

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

How to measure the spectrum of a resonator



- Spectrum tells you H_{LC}
- Losses given by coupling to the outside world
  - γ_i and γ_o typically combined to γ_c
- We want to know internal losses γ_{int}
- $\gamma_{tot} = \gamma_c + \gamma_{int}$

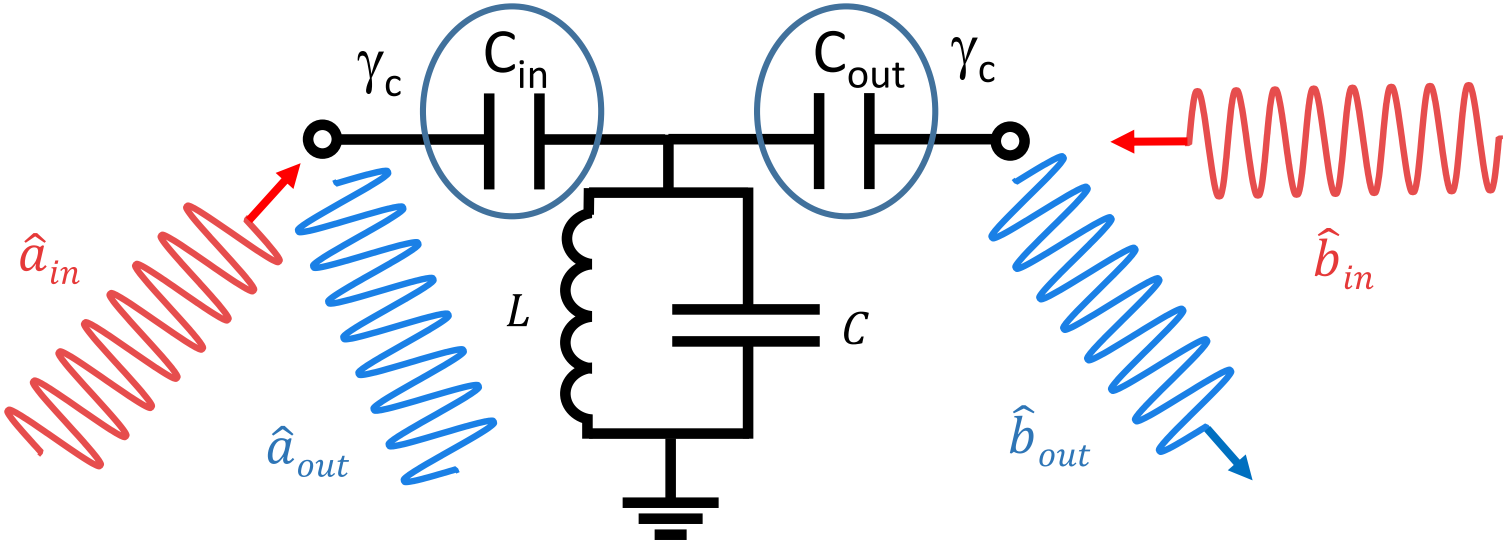
measurement requires at least two components

- **know input probe field** $\hat{a}_{in}$
- **detected output field** $\hat{a}_{out}$

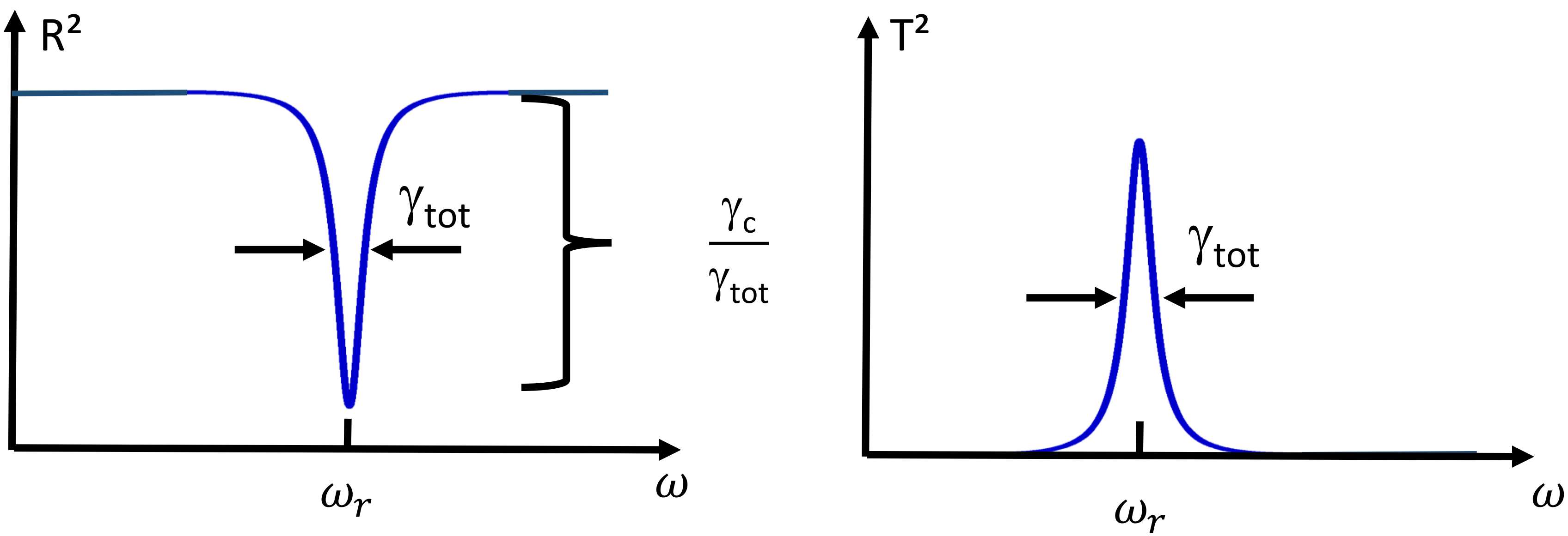
→ input output formalism from quantum optics!!

D. F. Walls & G. Milburn, *Quantum Optics* (Springer, Berlin-Heidelberg, 2008)

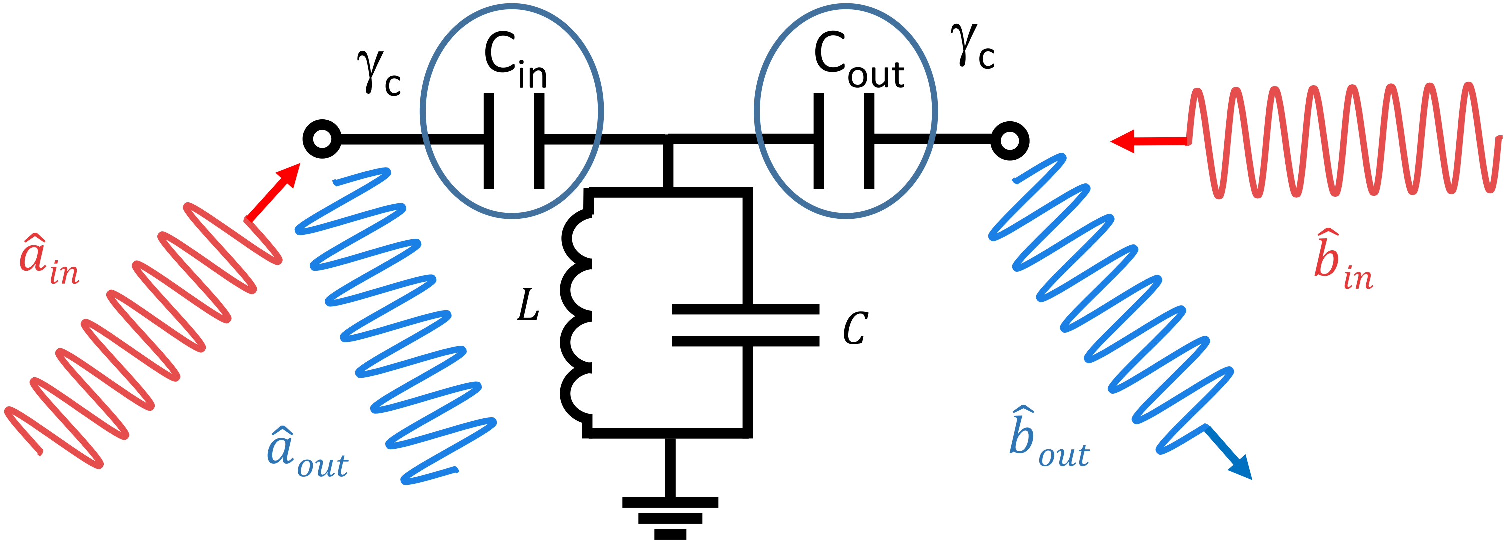
How to measure the spectrum of a resonator



- measurement transmission T and reflection R coefficients
- Transmitted power is a Lorentzian!



How to measure the spectrum of a resonator



- Quality factor is used to describe Resonator storage time

$$Q = \frac{\omega_r}{\gamma}$$

- γ describes the dephasing rate of the resonator!!

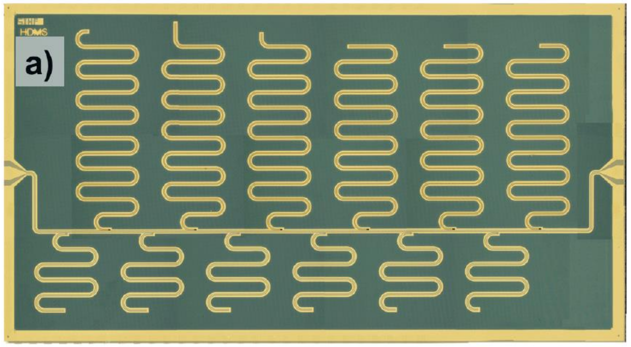
$$T_2 = \frac{1}{\gamma}$$

- coupling losses Q_c – desired losses e.g. qubit readout
- internal losses Q_{Int} – all undesired losses, many reasons

$$\frac{1}{Q_{tot}} = \frac{1}{Q_c} + \frac{1}{Q_{int}}$$

Resonators and Cavities

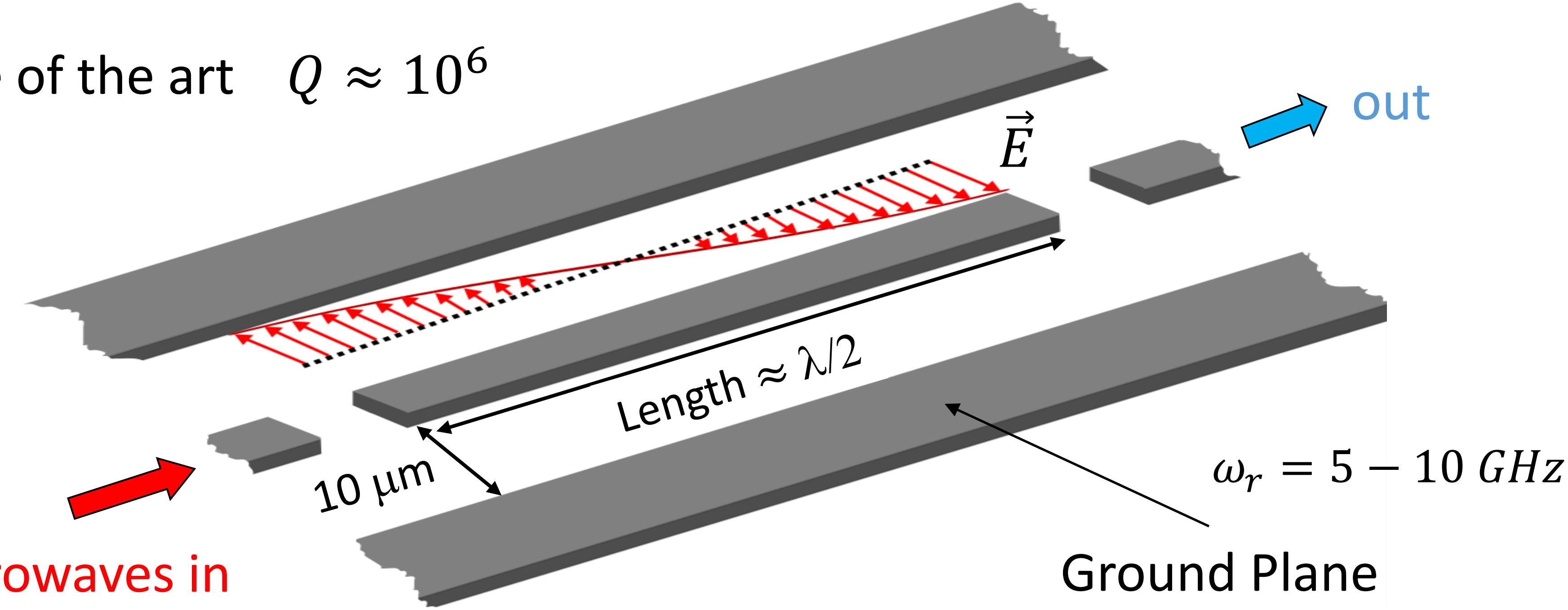
Coplanar Waveguide Resonators



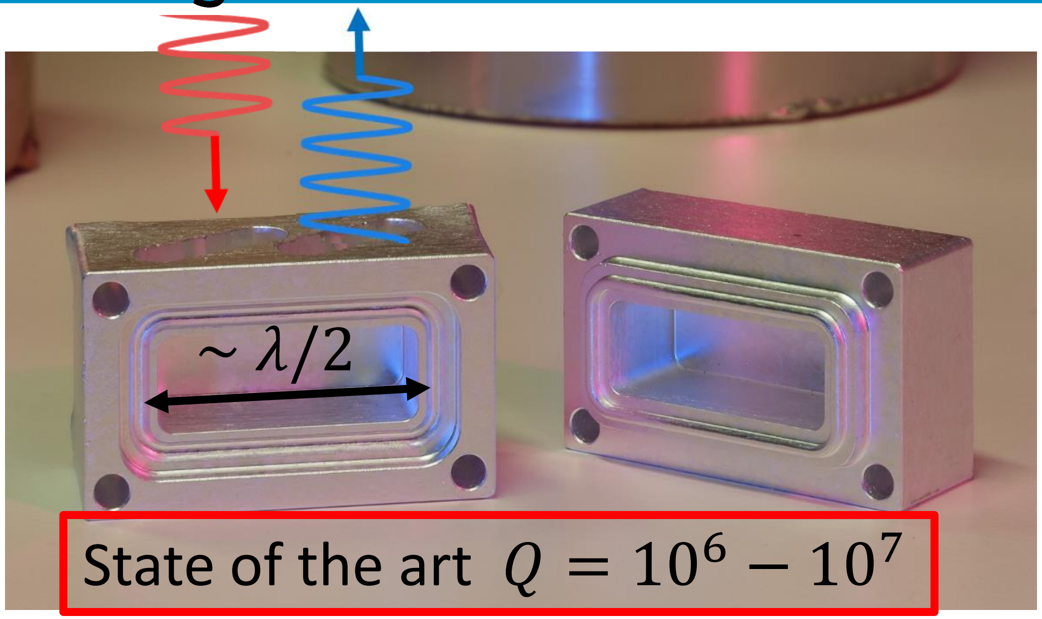
Bruno et.al. APL 106, 182601 (2015)

also: Megrant et.al. APL 100, 113510 (2012)

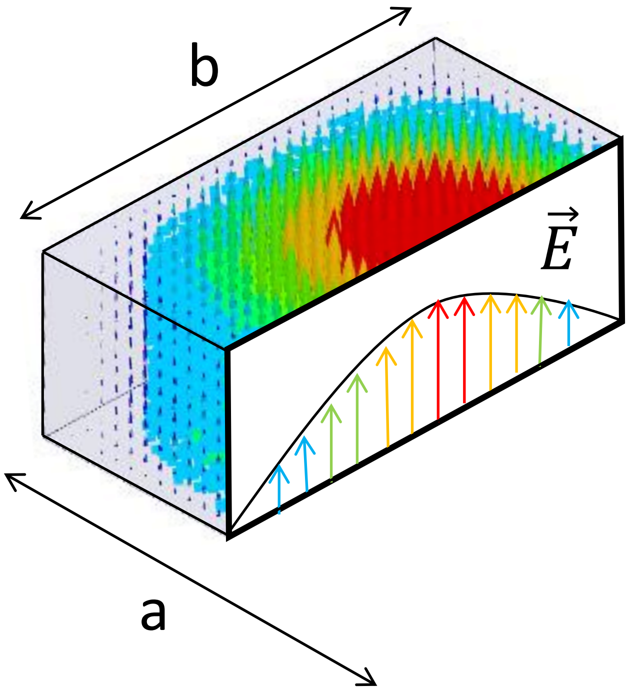
State of the art $Q \approx 10^6$



Waveguide microwave resonator



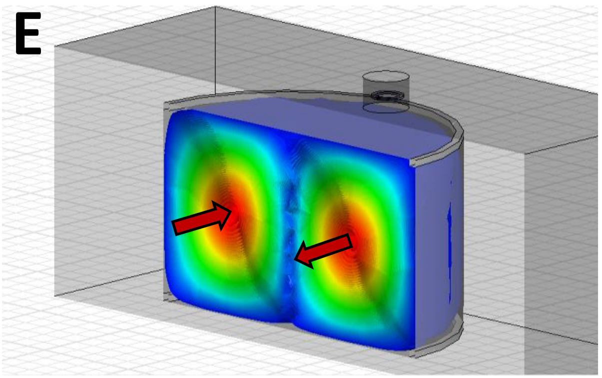
$$\upsilon_{m,n} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

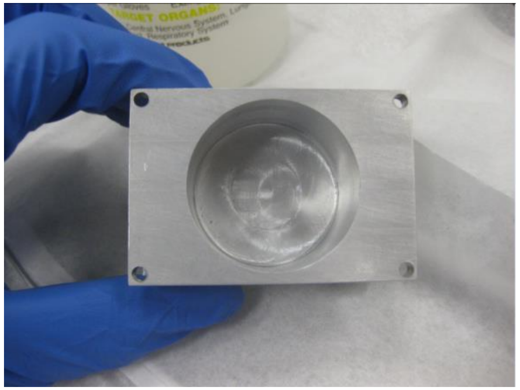


Paik et.al. PRL 107, 240501, (2011)

Other resonators

"Coke Can" resonator

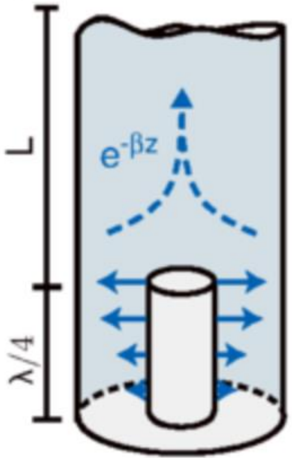




$Q \geq 10^8$

Reagor et.al. APL 102, 192604 (2013)

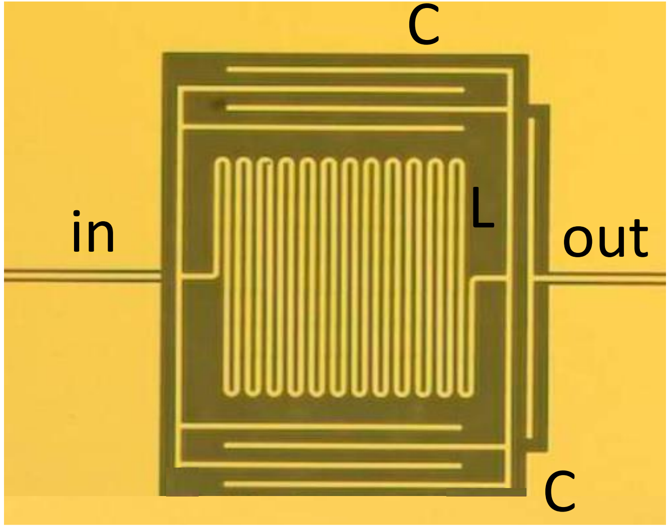
Coaxial λ/4 resonator



$Q \geq 10^7$

Reagor et.al. PRB 94, 014506, (2016)

Lumped element resonator



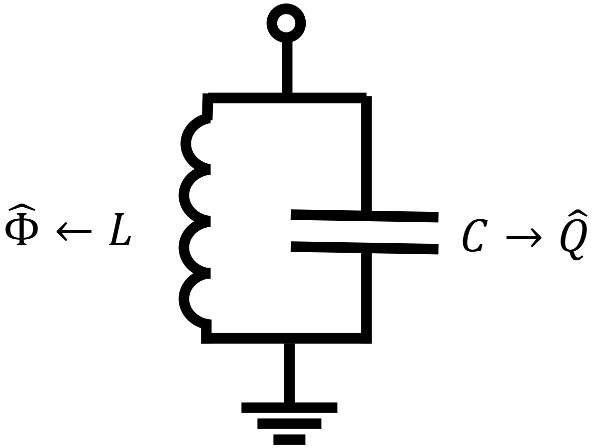
$$\omega = \frac{1}{\sqrt{L\,C}}$$

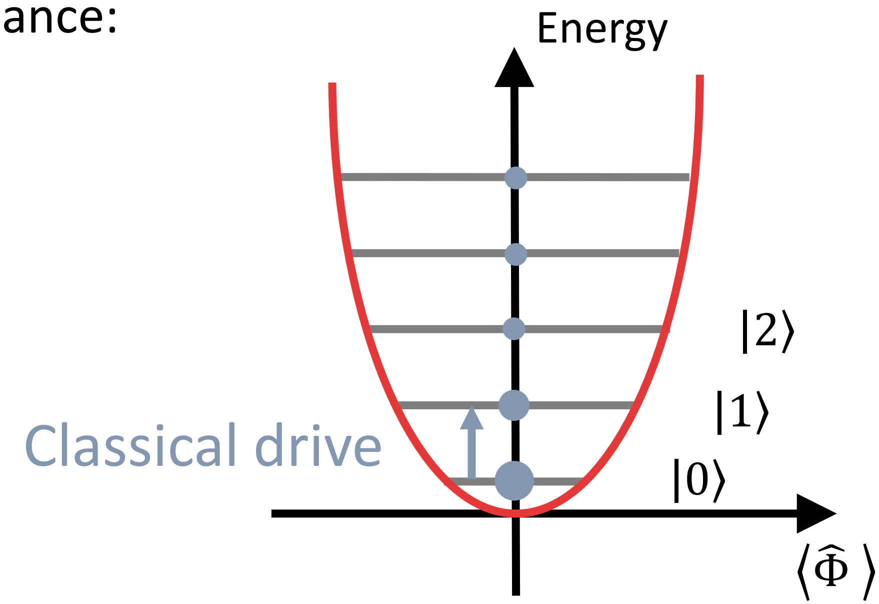
length ≪ λ

$Q \geq 10^5$

Geerlings et.al. APL 100, 192601 (2012)

For **all** resonators around a single resonance:



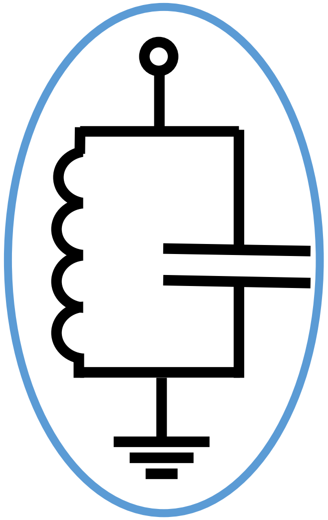


$$H = \hbar\omega_c\left(a^\dagger a + \frac{1}{2}\right)$$

Quantum Harmonic Oscillator

Applications of Superconducting resonators

Readout of superconducting qubits
Slichter et.al. PRL 106, 110502, (2011)
Hatrdige et.al. SCIENCE 339 (181 2013)



Mediate interactions
DiCarlo, et al. Nature 460, 240 (2009)
Steffen et.al. Nature 500, 319-322 (2013)

Quantum memory
Ofek et.al. Nature 536, 441, (2016)

Quantum simulation
Fitzpatrick et.al. PRX 7, 011016 (2017)
Sundaresan et.a. PRX 5, 021035 (2015)

Quantum Optics
Hofheinz et.al. Nature 459, 546 (2009)
Kirchmair et.al. Nature 495, 205 (2013)

How to build a qubit (Transmon)

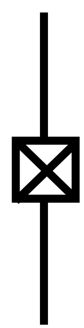
Institute for Quantum Optics and Quantum Information, Innsbruck
Institute for Experimental Physics, University of Innsbruck

Josephson Junction

Regular inductance		Josephson Junction
$V_L = L\dot{I}$	$\Leftrightarrow$	$V_{jj} = \frac{\hbar}{2e}\frac{1}{I_c \cos\varphi}\dot{I}$
$E = \frac{\Phi^2}{2L}$	$\Leftrightarrow$	$E = -E_j \cos(\varphi)$

$$\varphi = \frac{2e}{\hbar}\Phi = 2\pi\frac{\Phi}{\Phi_0}$$

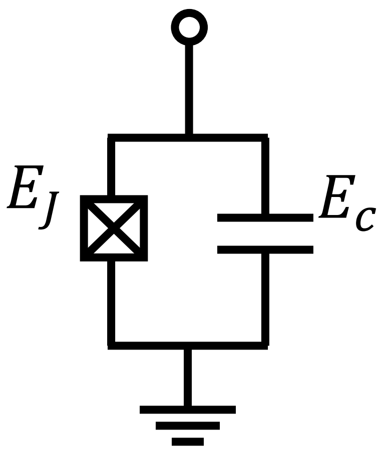
Circuit element including charging energy:

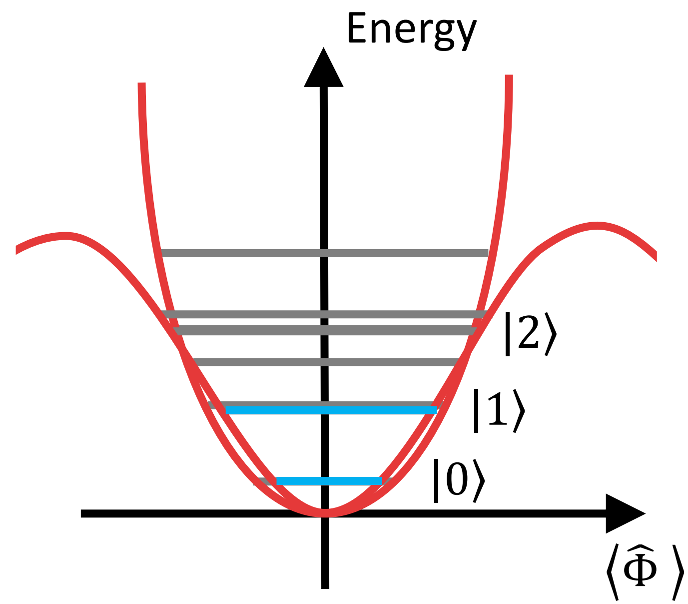


$$\hat{H} = -E_j \cos\left(2\pi\frac{\hat{\Phi}}{\Phi_0}\right) + \frac{\hat{Q}^2}{2C}$$

Superconducting Qubit - Transmon

Transmon: charge insensitive qubit





$$\hat{H} = -E_j \cos\left(2\pi \frac{\hat{\Phi}}{\Phi_0}\right) + \frac{\hat{Q}^{\,2}}{2\,C_\Sigma}$$

Replace charge and flux with raising and lowering operators as before.

$$H = \hbar\omega_q\, b^\dagger b - \frac{E_c}{2}\left(b^\dagger b\right)^2$$

$$\omega_q = \sqrt{8\,E_j E_c}$$

$$H = \hbar\frac{\omega_q}{2}\sigma_z$$

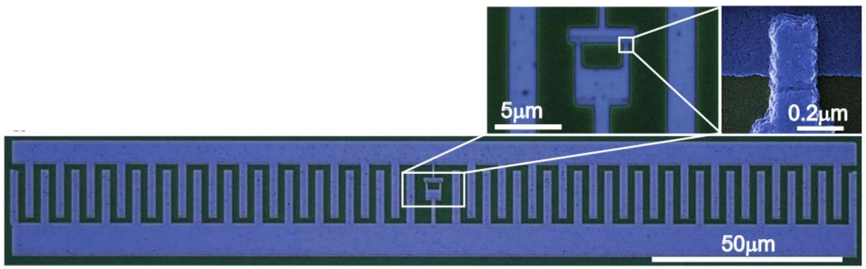
$$\omega_q = 5 - 10\ GHz$$

$$E_c = 300\ MHz = \alpha$$

Koch et.al., PRA 76 042319 (2007)

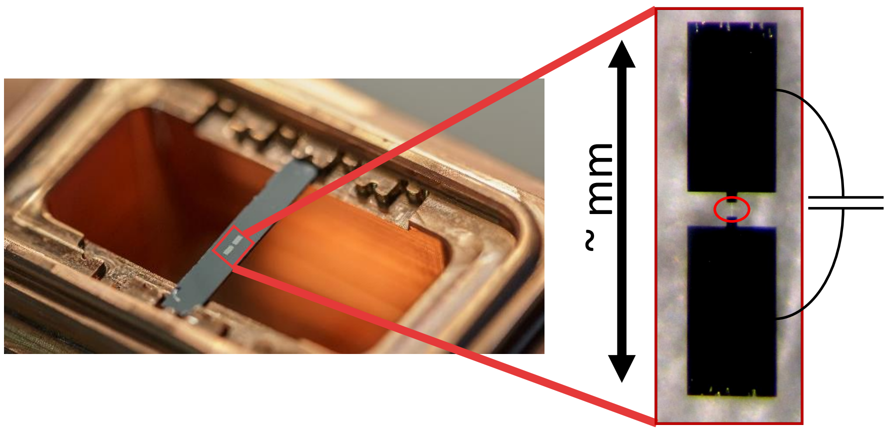
Some Transmon designs

Transmon



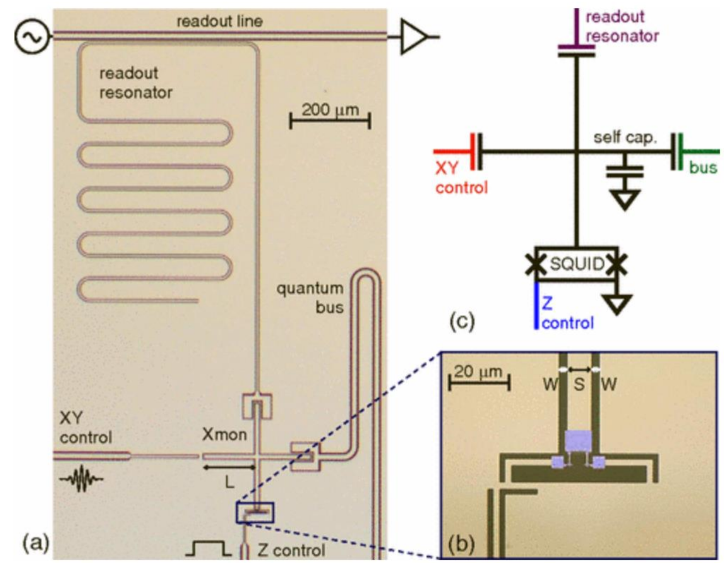
Yale, Delft, ETH Zürich….
Fink PhD Thesis, ETH Zürich

3D Transmon



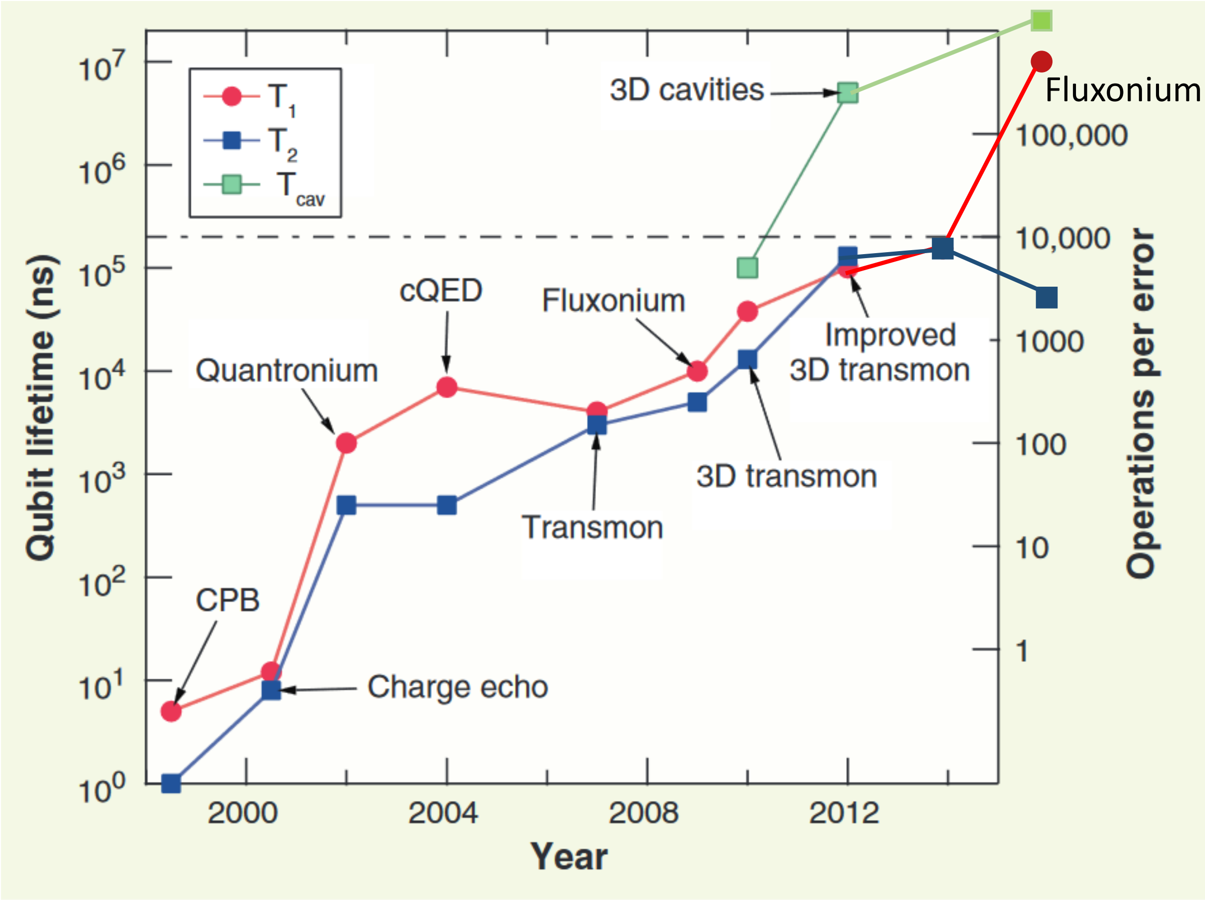
Yale, IBM, Delft, Innsbruck,….

X-mon



Martinis group, Google
Phys. Rev. Lett. 111, 080502 (2013)

Coherence properties of Charge Qubits

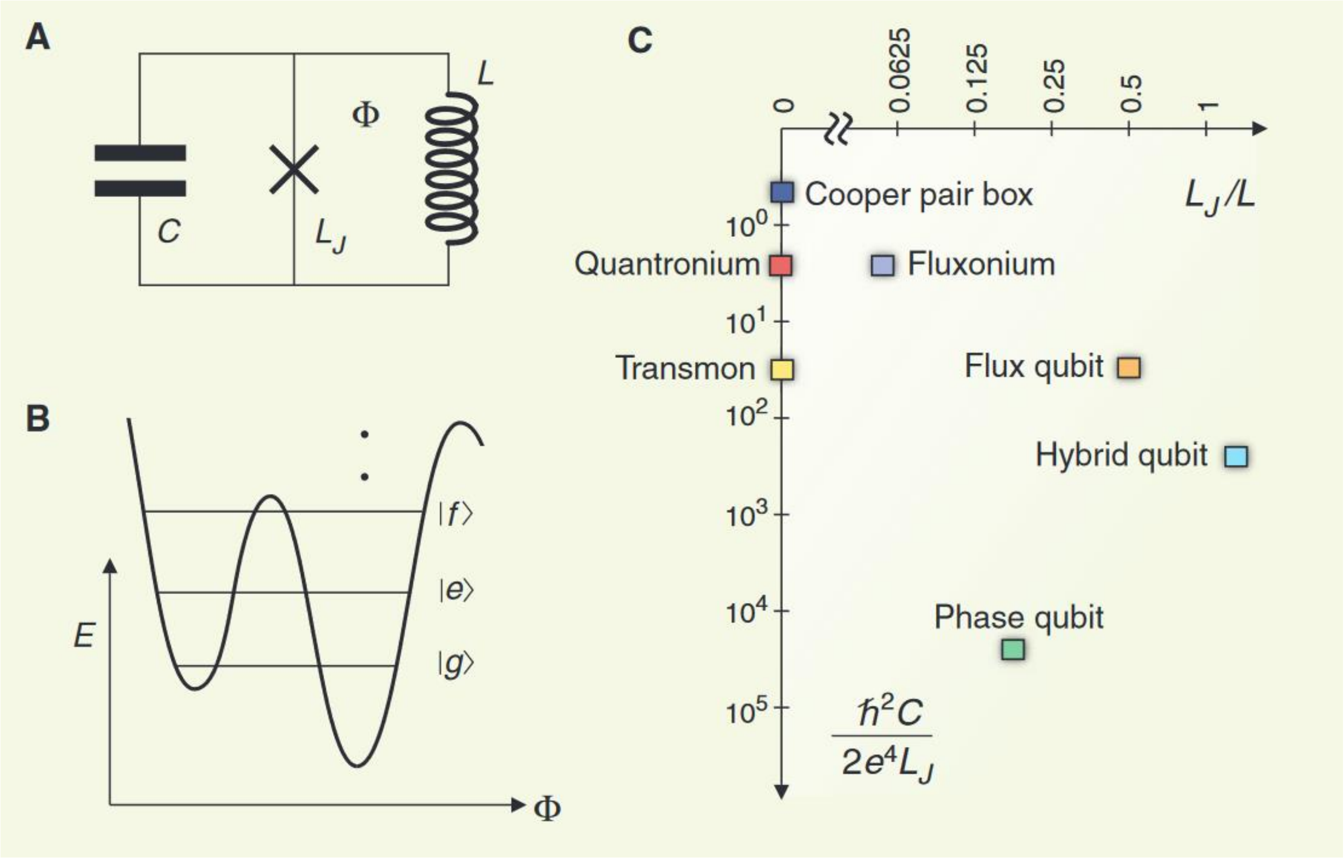


M.H. Devoret & R.J. Schoelkopf, Science 339, 1169 (2013)

Transmon $T_1 \leq 100\ \mu s, T_2 \leq 110\ \mu s$ Fluxonium $T_1 \leq 3\ \text{ms}, T_2 \leq 20\ \mu s$

Qubit Zoo

Combination of L, C & Josephson Junction leads to a number of different Qubits



M.H. Devoret & R.J. Schoelkopf, Science 339, 1169 (2013)

How to couple a qubit and a resonator?

Circuit QED

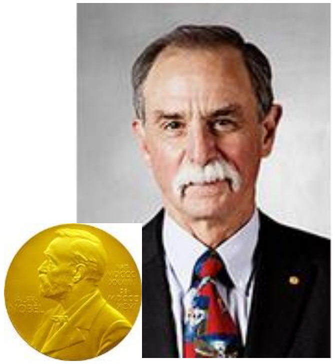
Institute for Quantum Optics and Quantum Information, Innsbruck
Institute for Experimental Physics, University of Innsbruck

ÖAW ÖSTERREICHISCHE AKADEMIE DER WISSENSCHAFTEN

IQI

universität innsbruck

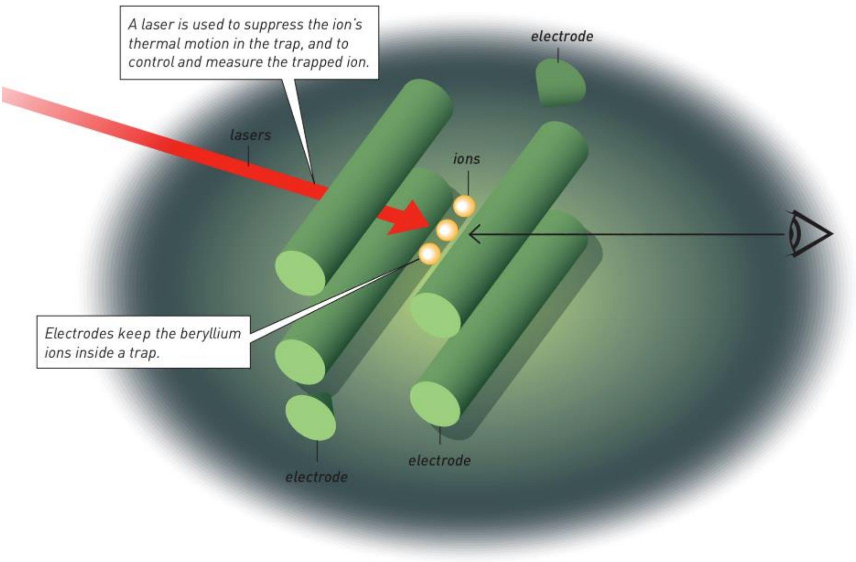
Nobelpreis 2012

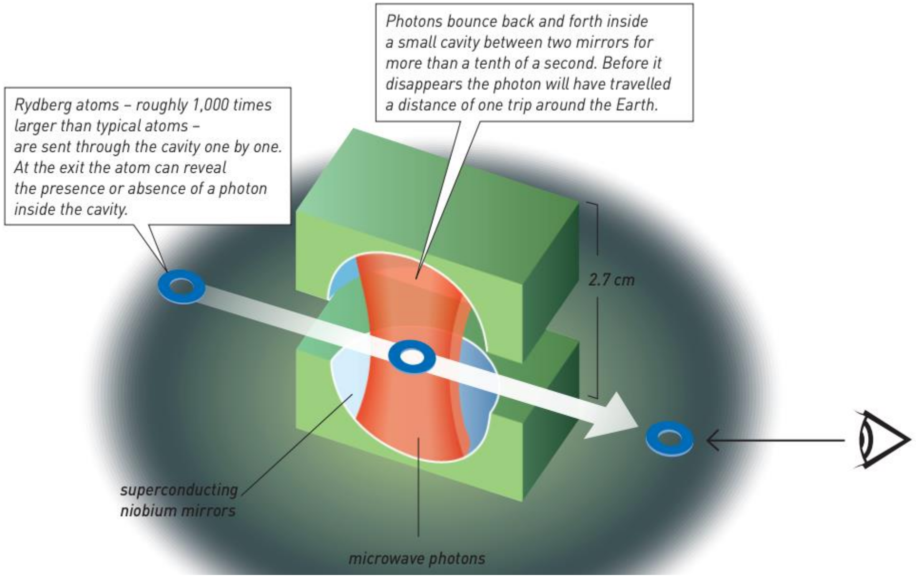


Dave Wineland

…for ground-breaking experimental methods
that enable measuring and manipulation
of individual quantum systems

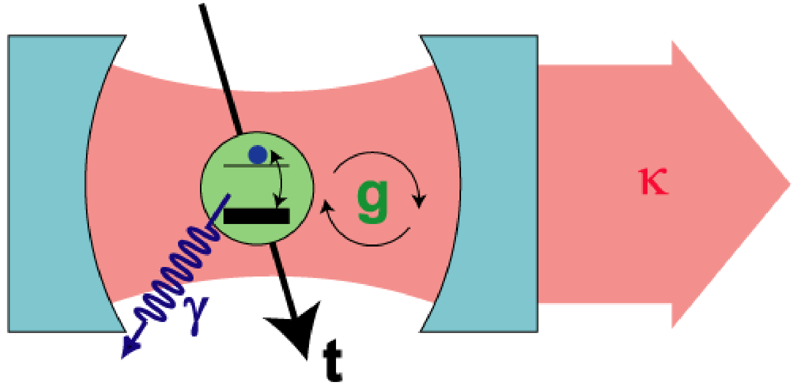
Serge Haroche





http://www.nobelprize.org/nobel_prizes/physics/laureates/2012/

cavity QED → circuit QED



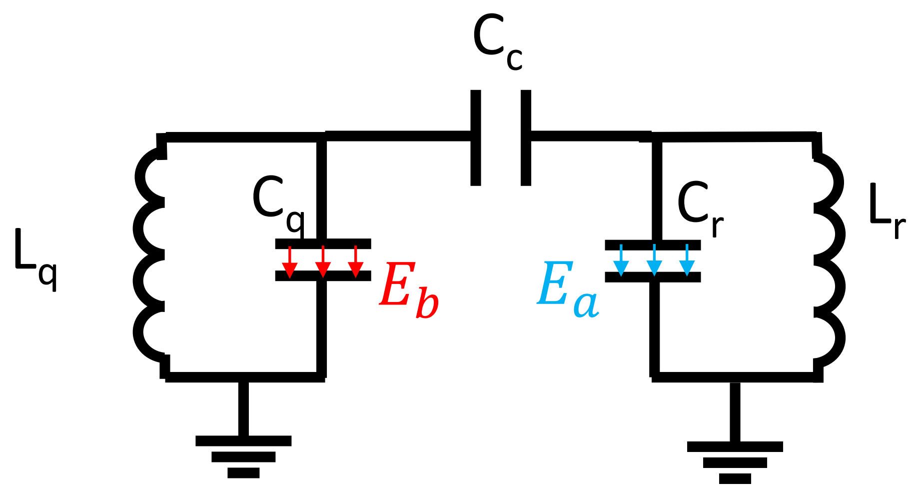
optical photons
⇓
microwave photons

optical resonators
⇓
microwave resonators

atoms as two level systems
⇓
nonlinear quantum circuits

QIP, quantum optics, quantum measurement…

Transmon coupled to a Resonators



$$E_a = \beta\ E_b$$

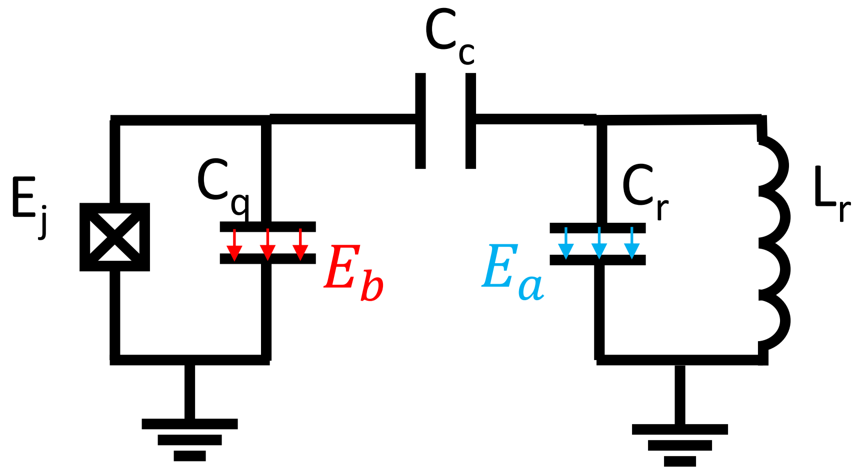
β given by capacitive divider

E-field in qubit creates field in resonator and vice versa!

Proper derivation:

coupling $\sim\beta\ (b + b^\dagger)(a + a^\dagger)$

Transmon coupled to a Resonators



$$H = \hbar\frac{\omega_q}{2}\sigma_z + \hbar\omega_r\, a^\dagger a$$

$$H_{int} \sim \beta\,(b+b^\dagger)(a+a^\dagger) \xrightarrow{\text{Qubit}} \beta\,(\sigma+\sigma^\dagger)(a+a^\dagger)$$

RWA

$$H_{int} = \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

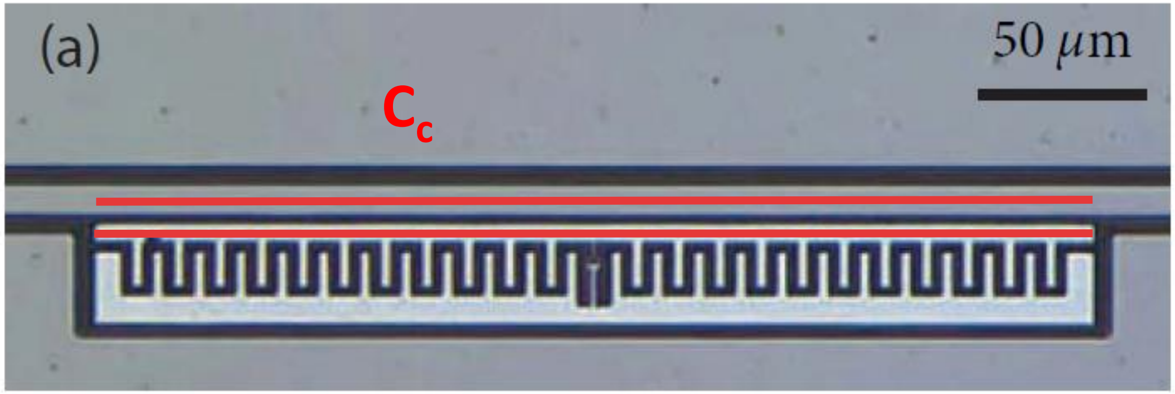
Jaynes Cummings Hamiltonian

⇩

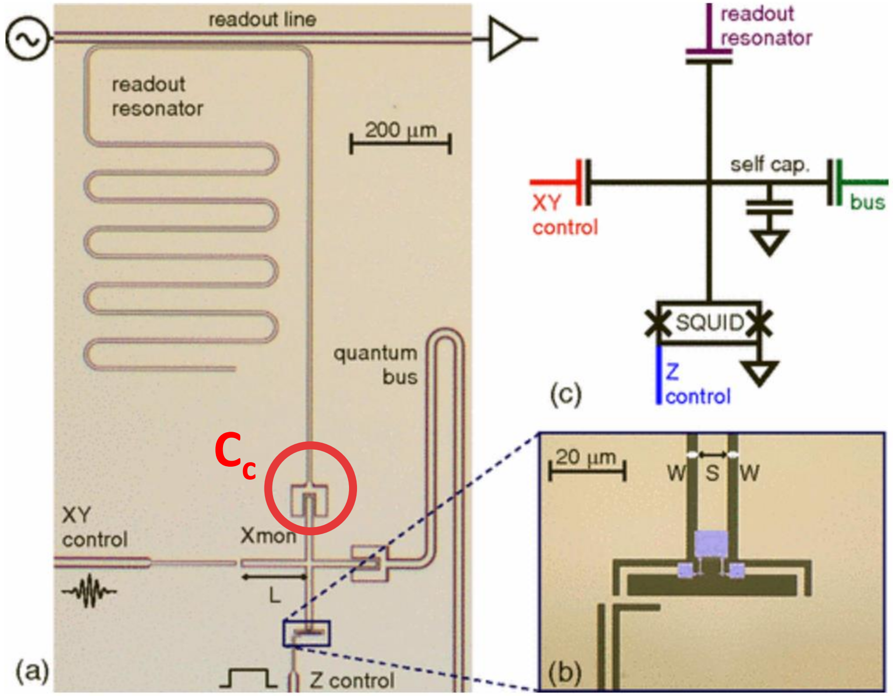
on resonance exchange of excitations

Planar Qubit coupled to Resonator

Small mode volume => large E field => large coupling => g ≈ $50 - 200\ MHz$



J. Chow PhD Thesis Yale
Schoelkopf group

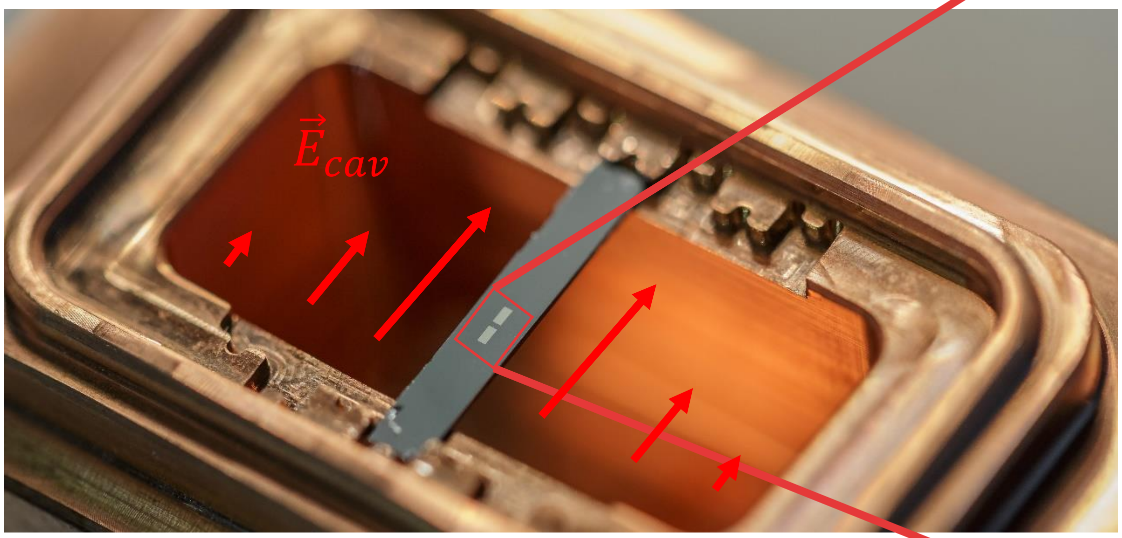


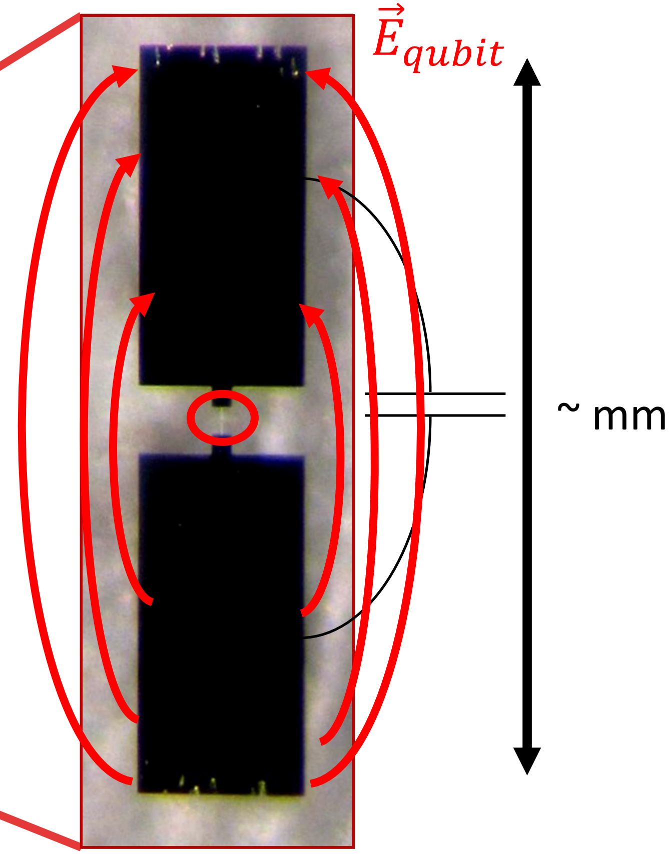
$g \approx 50 - 200\ MHz$

Martinis group, Google
Phys. Rev. Lett. 111, 080502 (2013)

3D Transmon coupled to a Resonator

Large mode volume compensated by large "Dipolemoment" of the qubit





$$\left|\vec{d}\right| = 2e \cdot 1mm \approx 10^7 \text{Debye}$$

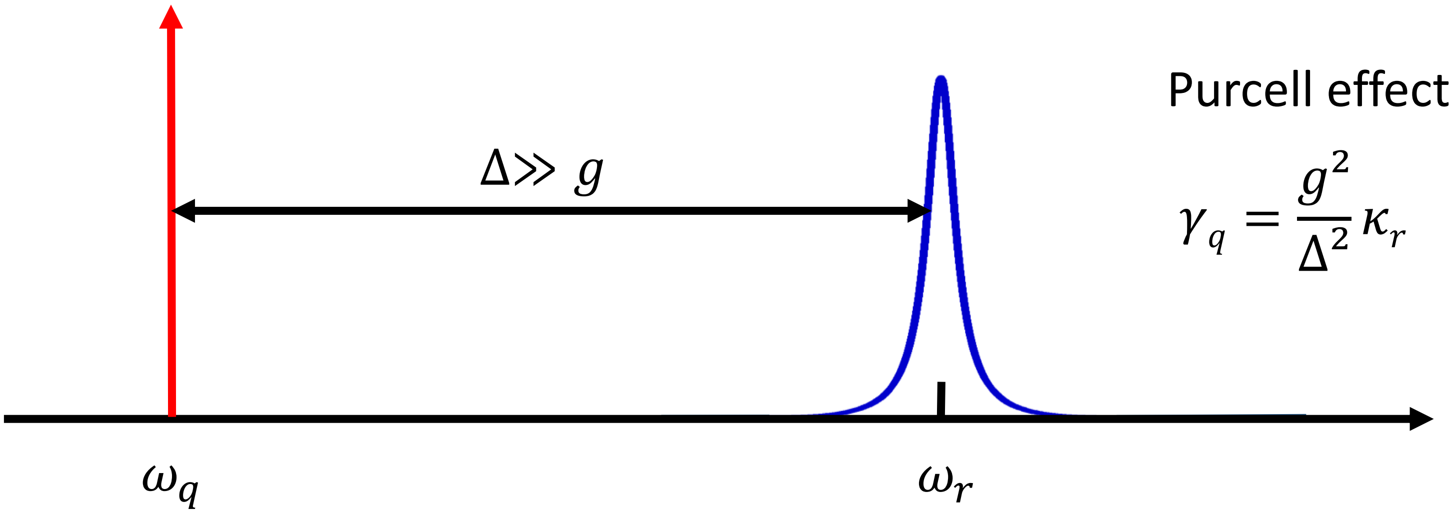
$$g \approx 50 - 200\ MHz$$

Qubit in cavity: Purcell filtering

- Qubits couple very strongly to EM fields!

  e.g. 3D Transmon in free space life time < 1ps

- Cavity protects qubit from vacuum fluctuations -> reduces DOS

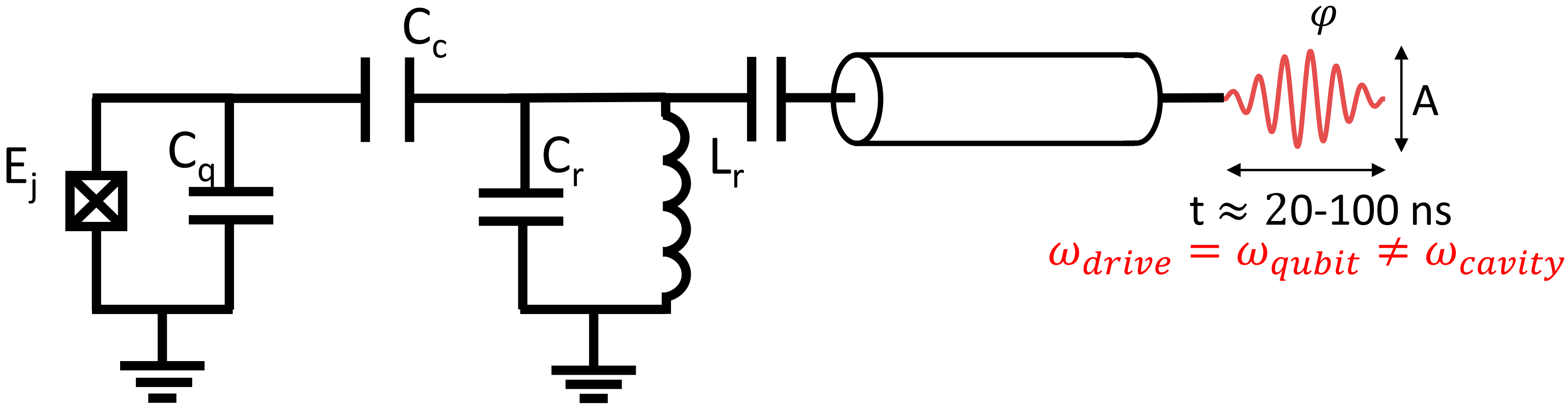


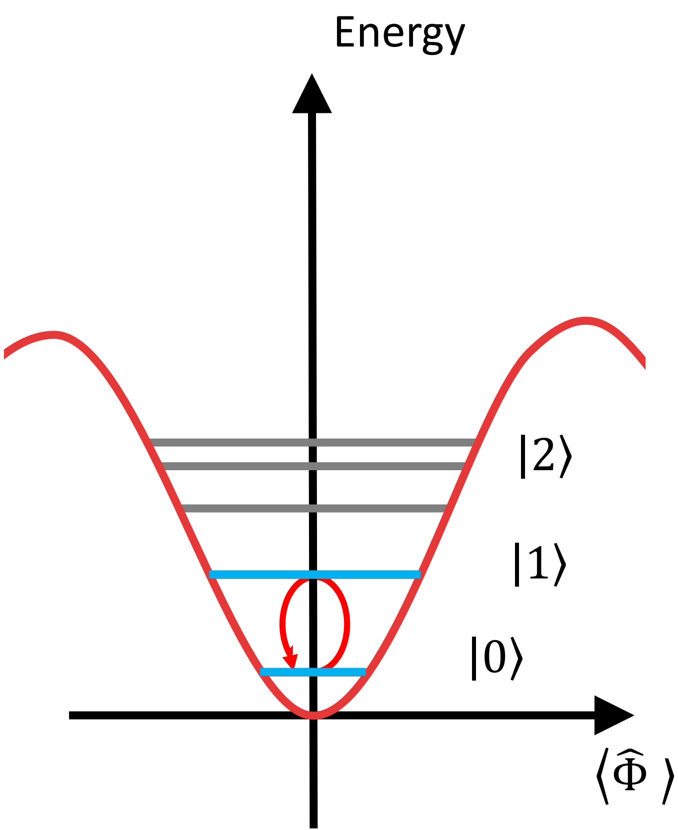
Purcell effect

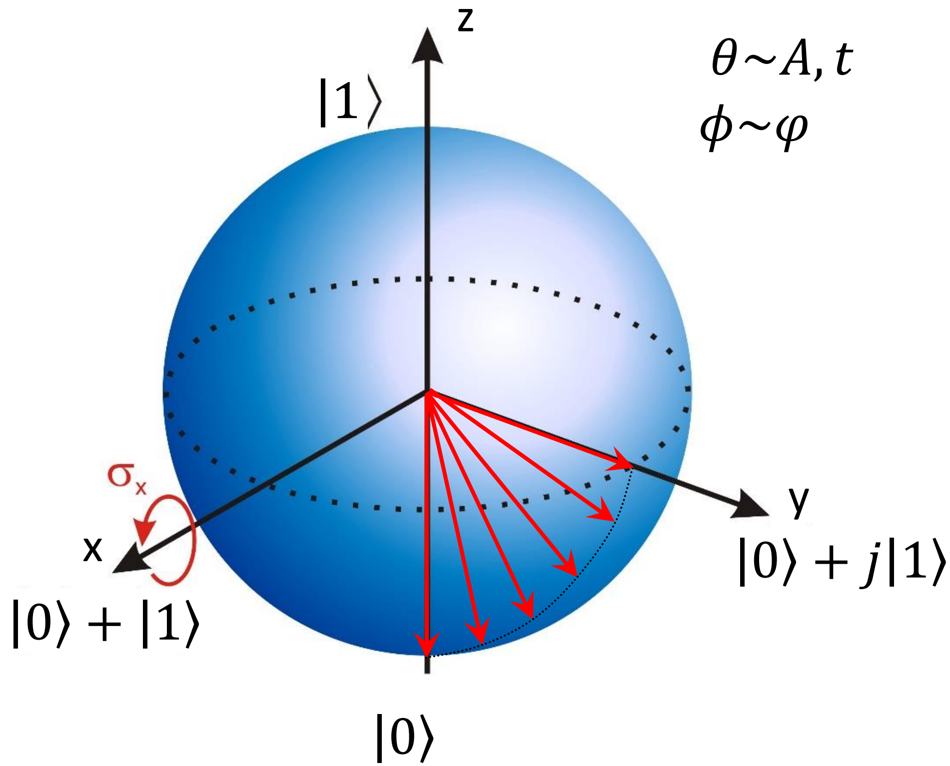
$$\gamma_q = \frac{g^2}{\Delta^2}\kappa_r$$

- High Q cavity required!
- Does that limit operation speed to κ_r?
- NO! - off resonant cavity rings up ~ $1/\Delta$

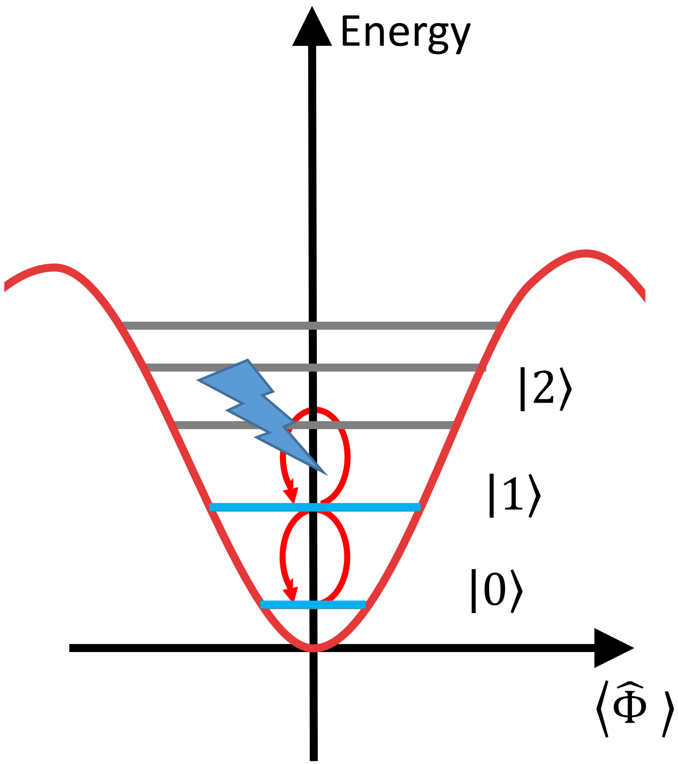
Single qubit gates

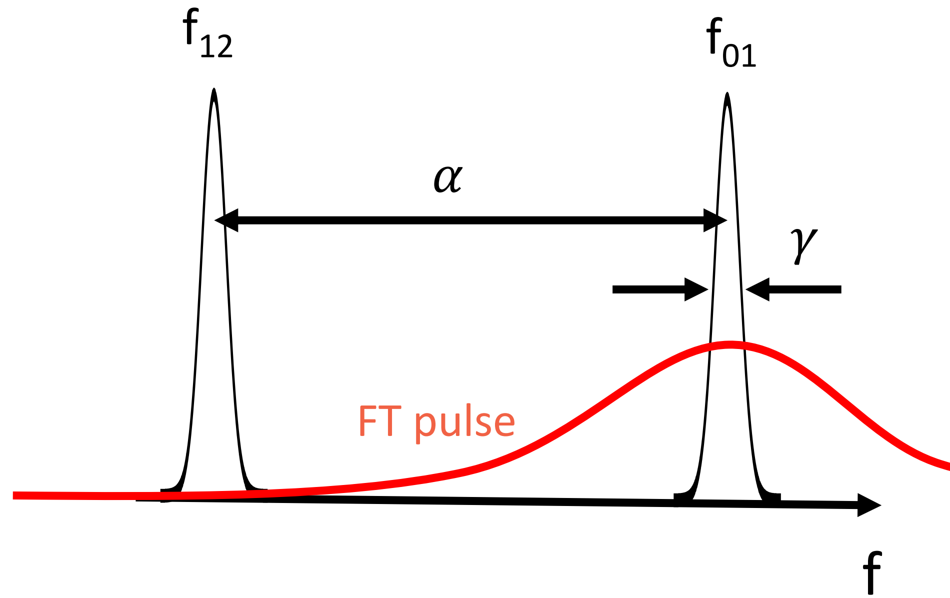






Single qubit gates - limits





Coherence time limit: $\frac{1}{\gamma} \gg t_{op}$

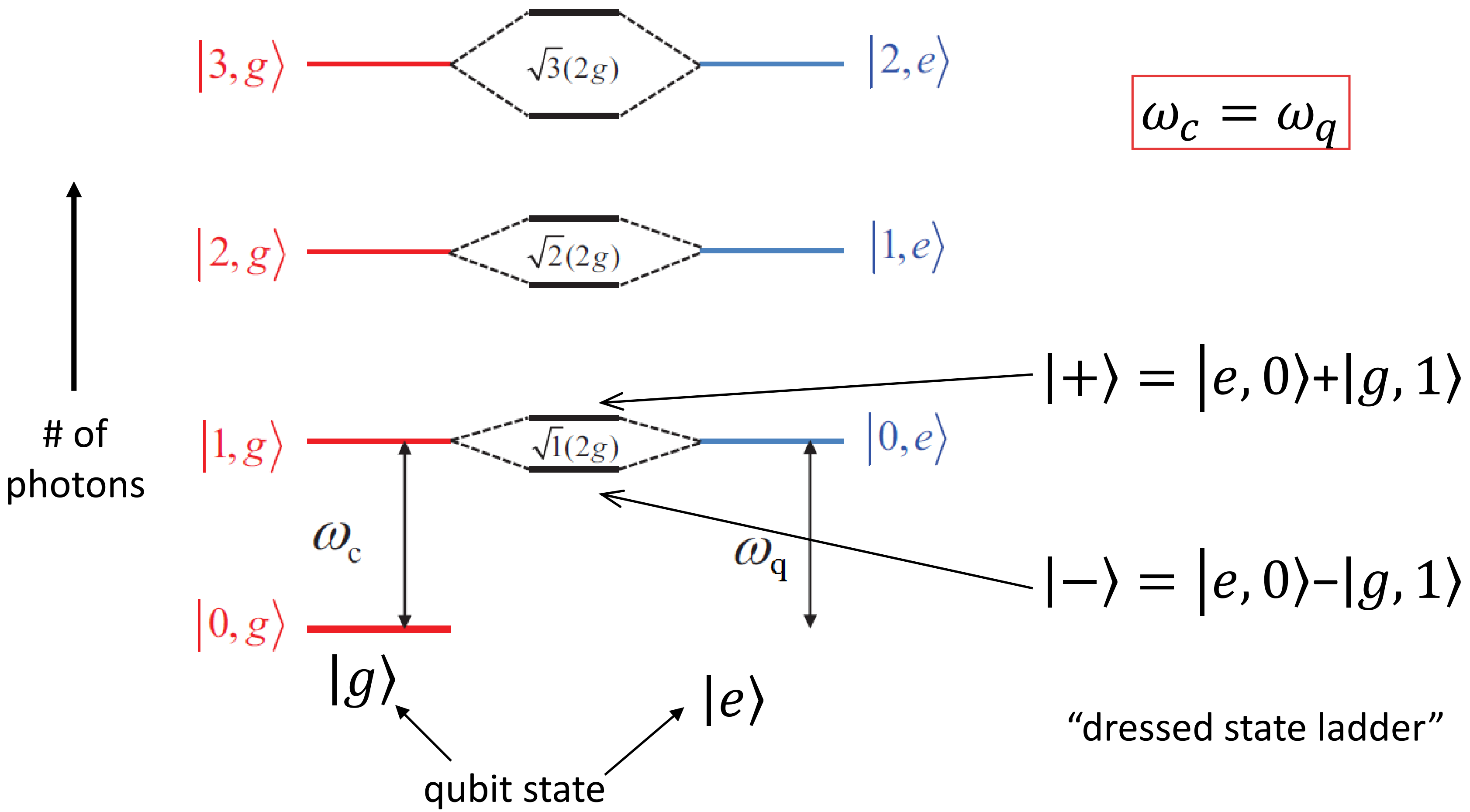
Anharmonicity limit: $\frac{1}{\alpha} < t_{op}$

operations $< \frac{\alpha}{\gamma} \approx \frac{250\text{ MHz}}{10\text{ kHz}} = 25 \cdot 10^3$

$< \; \rightarrow \; \approx$ with optimal control & pulse shaping

e.g. Motzoi et.al. PRL 103, 110501, (2009)

cQED: Resonant Case



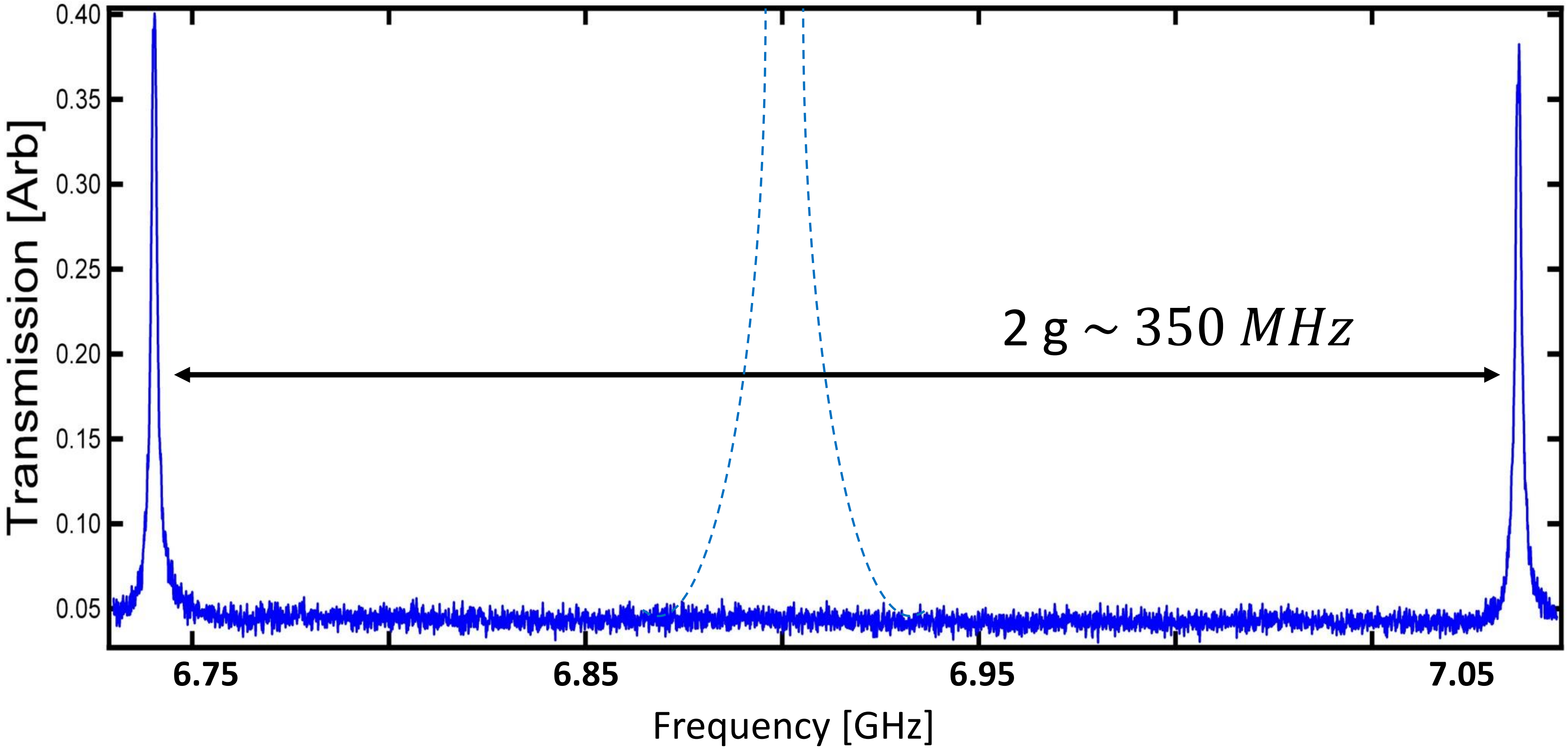
$$H = \hbar \frac{\omega_q}{2} \sigma_z + \hbar \omega_r \, a^\dagger a + \hbar g (a^\dagger \sigma^- + a \sigma^+)$$

(see e.g. "Exploring the Quantum…," S. Haroche & J.-M. Raimond)

cQED: Resonant Case

$$\frac{g}{\omega} \sim \frac{200\ Mhz}{5\ Ghz} \sim 0.04$$

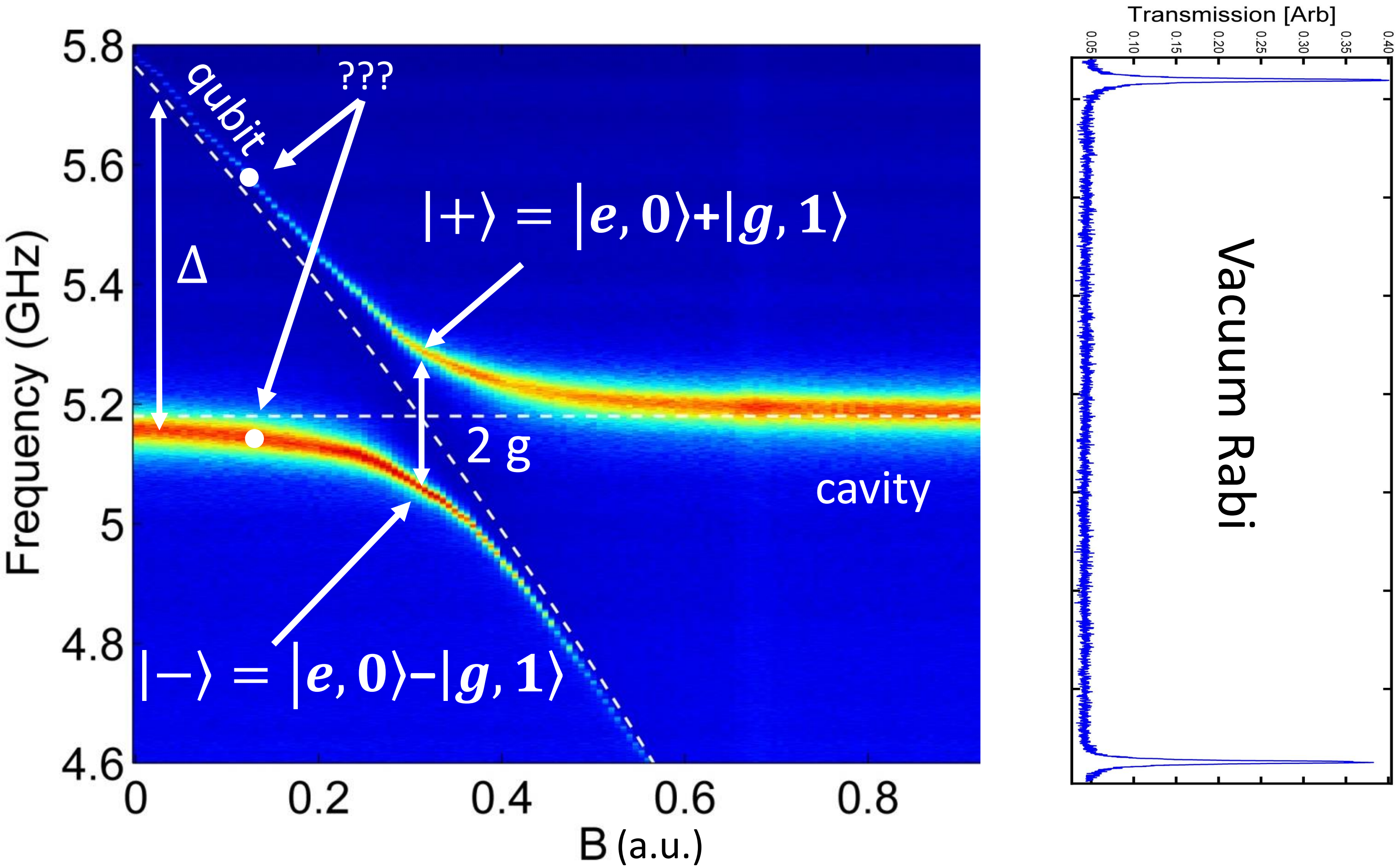
g >> [κ, γ]



*Review: RS and S.M. Girvin, Nature **451**, 664 (2008).*

Nonlinear behavior: Bishop et al., Nature Physics (2009).

cQED: Tuning the qubit



Small hybridization for Δ >: "qubit is mostly a qubit" & "cavity is mostly cavity"

Only a small part of the qubit energy is stored in the cavity!

Dispersive cQED

$$H = \hbar\frac{\omega_q}{2}\sigma_z + \hbar\omega_c\, a^\dagger a + \hbar g(a^\dagger\sigma^- + a\sigma^+)$$

Now strong dispersive regime: $\Delta = \omega_c - \omega_q \gg g$ & $g \gg [\kappa, \gamma]$

Do perturbation theory for small $\frac{g}{\Delta}$

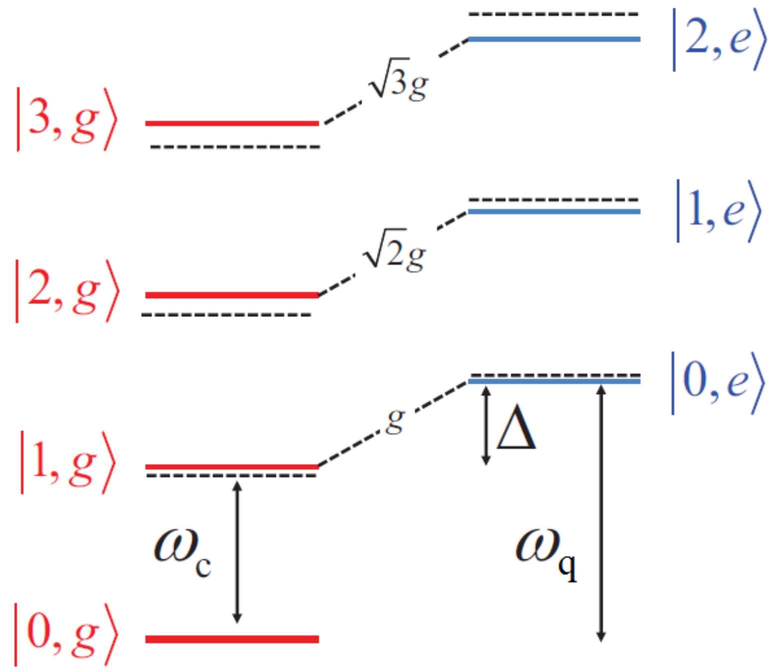
$$H = \hbar\frac{\omega_q}{2}\sigma_z + \hbar\omega_c\, a^\dagger a - \hbar\frac{\chi}{2}a^\dagger a\sigma_z \qquad \chi = \frac{g^2}{\Delta}$$

Note: Transmon is <u>not</u> a two level system!!!

$$\rightarrow \chi = 2\frac{g^2}{\Delta^2}\alpha$$

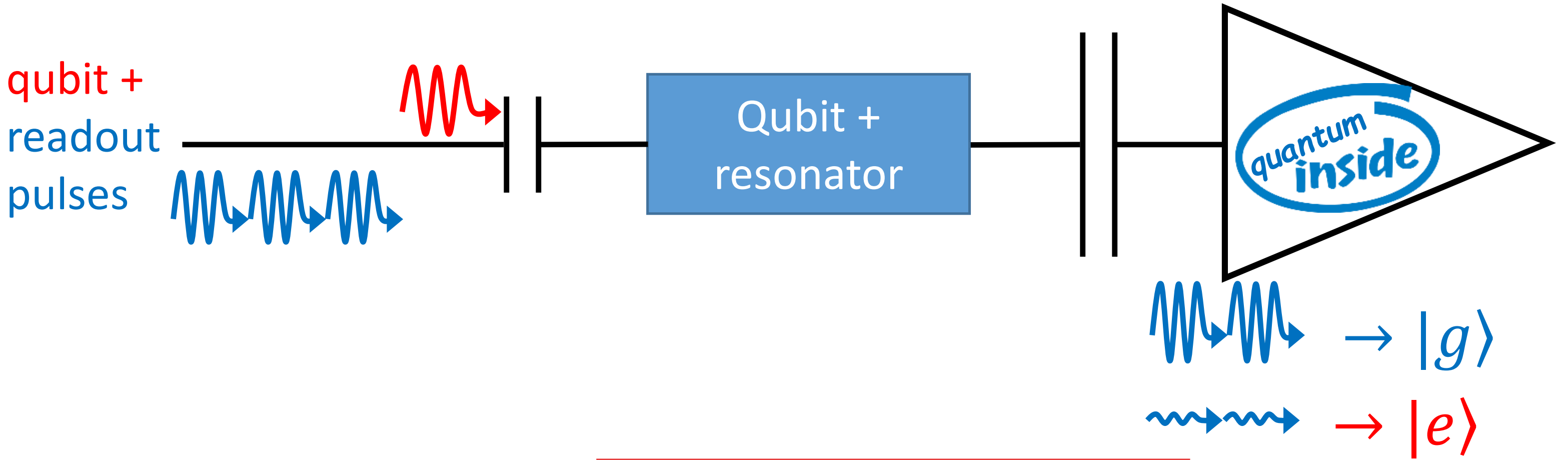
Dispersive Energy levels

$$\Delta = \omega_q - \omega_c \gg g$$



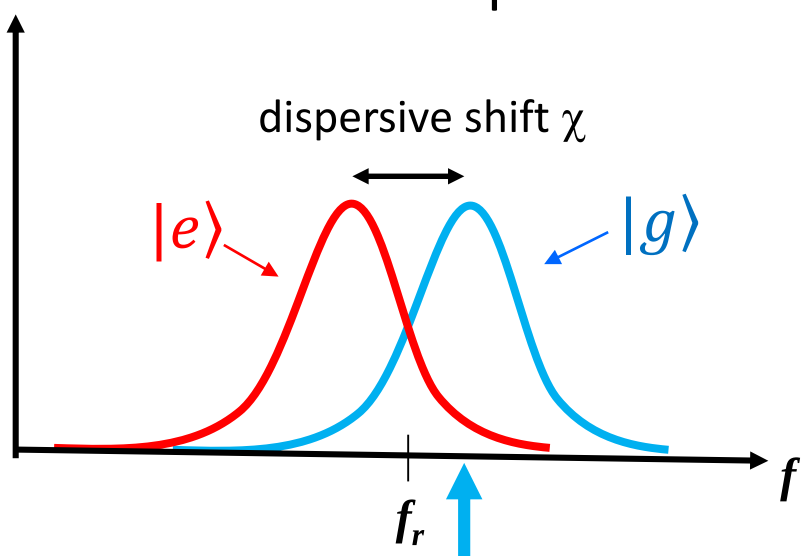
S. M. Girvin (2011). **Circuit QED: Superconducting Qubits Coupled to Microwave Photons.** (Lecture Notes of the Les Houches Summer School)

Dispersive cQED - readout



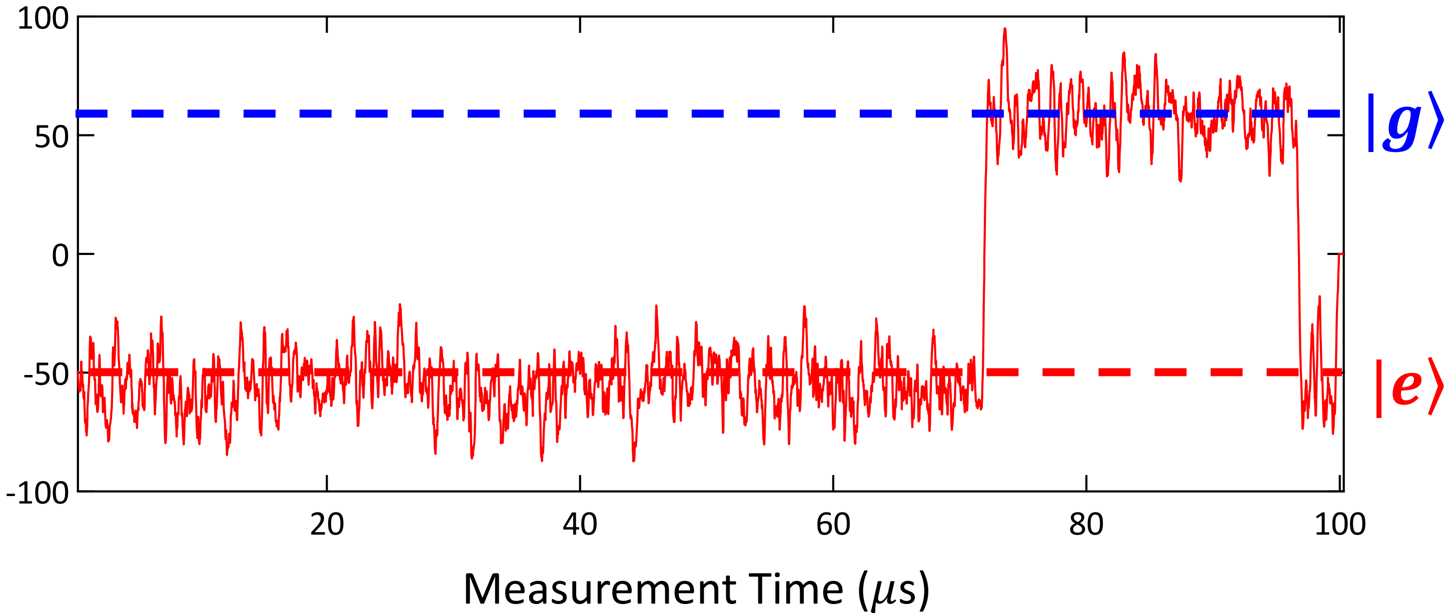
$$H = \hbar\frac{\omega_q}{2}\sigma_z + \left(\hbar\omega_c - \hbar\frac{\chi}{2}\sigma_z\right)a^\dagger a$$

Readout amplitude



Dispersive cQED - readout

Single measurement trace with quantum limited amplifier:

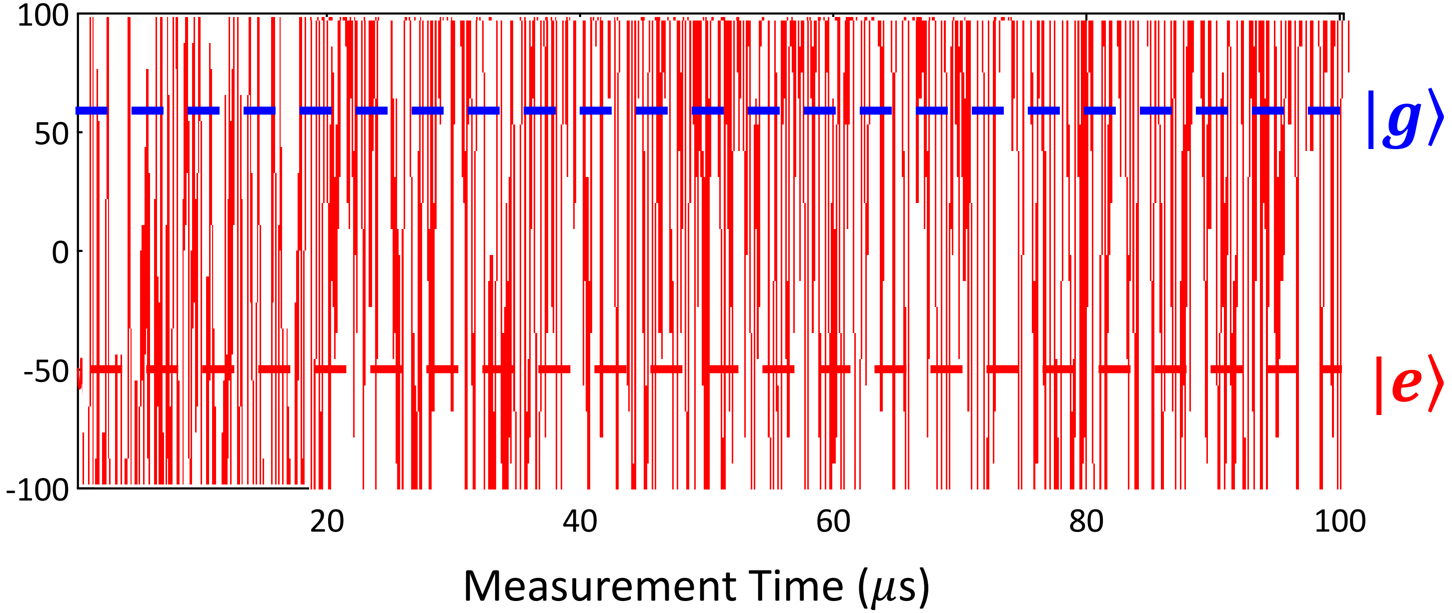


For T_m=300ns F_{det} > 99%

Data courtesy of Yale's Qlab Group
See also: Science 339, 178 (2013)

Dispersive cQED - readout

Single measurement trace <u>with</u> <u>regular</u> amplifier:



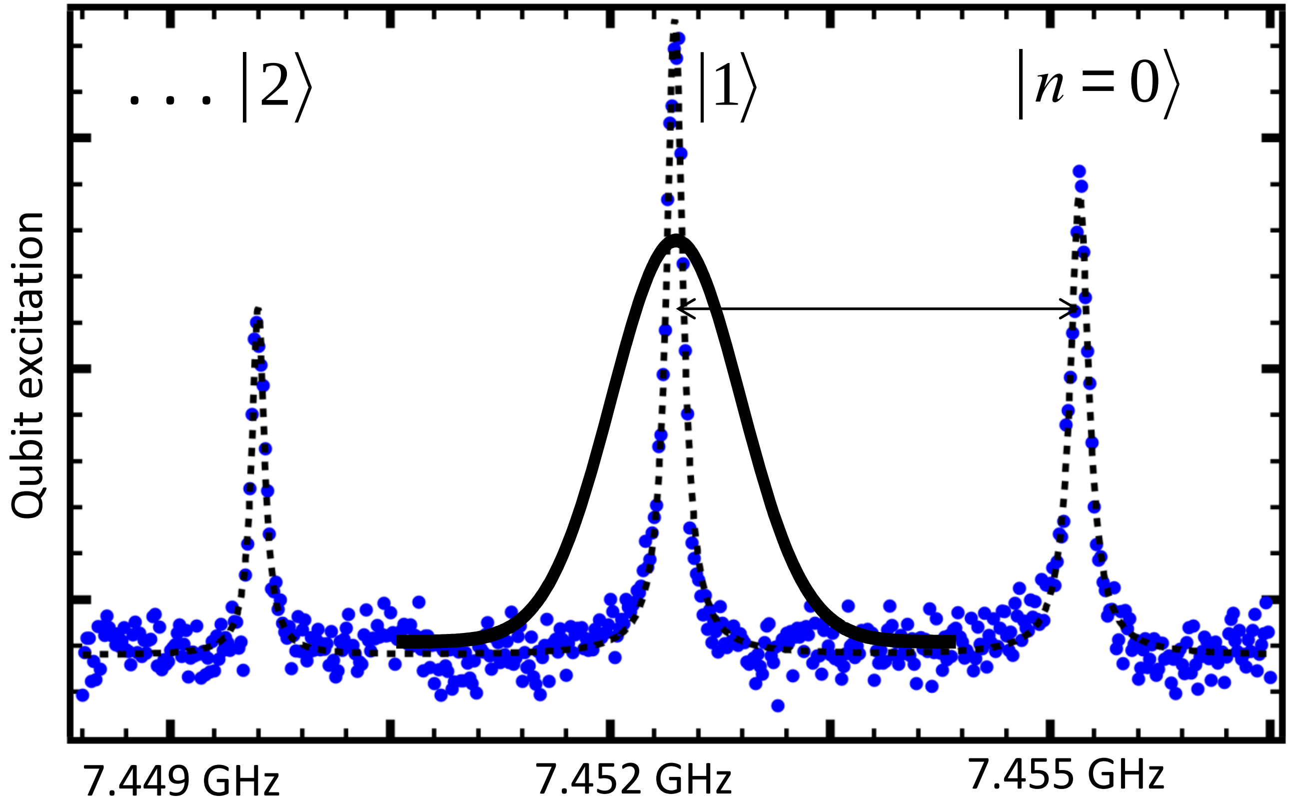
=> only expectation values can be measured

Data courtesy of Yale's Qlab Group
See also: Science 339, 178 (2013)

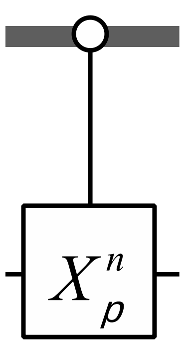
Dispersive cQED– Photon Number splitting

Strong Dispersive Hamiltonian: "doubly-QND" interaction

$$H = \left(\hbar\frac{\omega_q}{2} - \hbar\frac{\chi}{2}a^\dagger a\right)\sigma_z + \hbar\omega_c\, a^\dagger a \qquad \chi = \frac{g^2}{\Delta^2}\alpha \gg \gamma, \kappa$$



π pulse on the n photon qubit peak



Experimental techniques

Institute for Quantum Optics and Quantum Information, Innsbruck
Institute for Experimental Physics, University of Innsbruck

Materials for superconducting circutis

Typical superconductors

- **Al**
  - type-I superconductor, $Tc \approx 1.2$ K
  - shadow evaporation possible
  - Perfect oxide for barrier – self limiting and very clean
  - Used for Junctions, Qubits
- **Nb**
  - type-II superconductor, $Tc \approx 9$K
  - shadow evaporation for nanoscale junction is hard to do
  - Used for resonators
- **Nb-Ti-Ni**
  - type-II superconductor, $Tc \approx 15$K
  - better interface quality
  - used for resonators and qubit capacitors

Dielectric substrates

- **silicon, sapphire**
  - contribute to dielectric losses ($T1$)

Fabrication of superconducting circuits

Lithography – to define the pattern

- optical lithography (UV)
- electron beam lithography (EBL)

Thin-film deposition

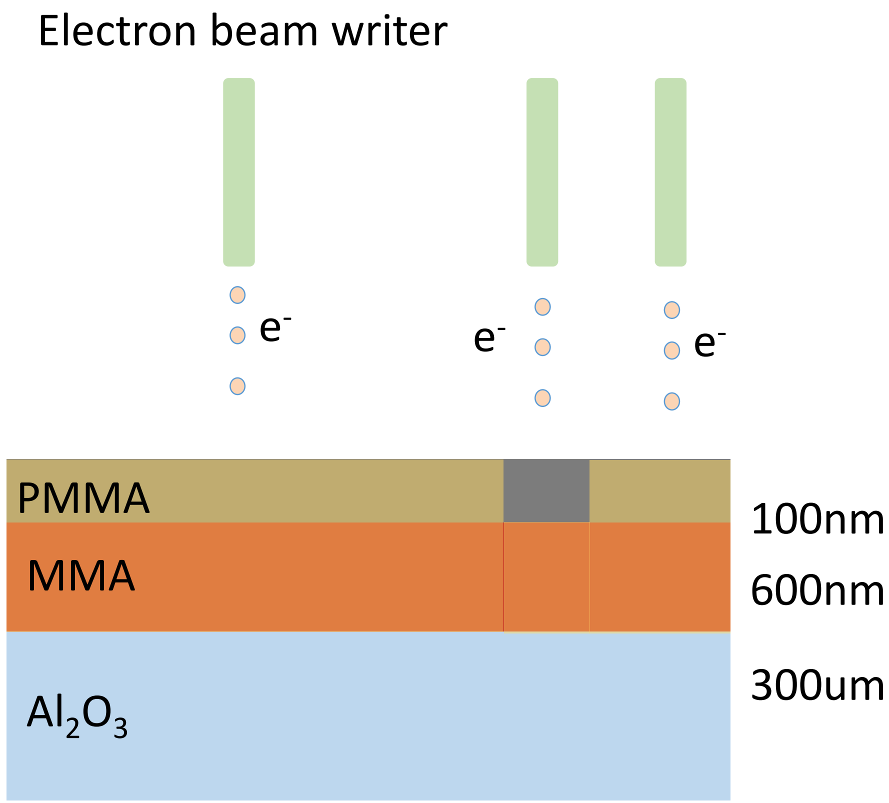
- deposit materials
- DC sputtering (metals, e.g. Nb)
- RF sputtering (insulators)
- electron beam evaporation (metals, e.g. Al)

Processing

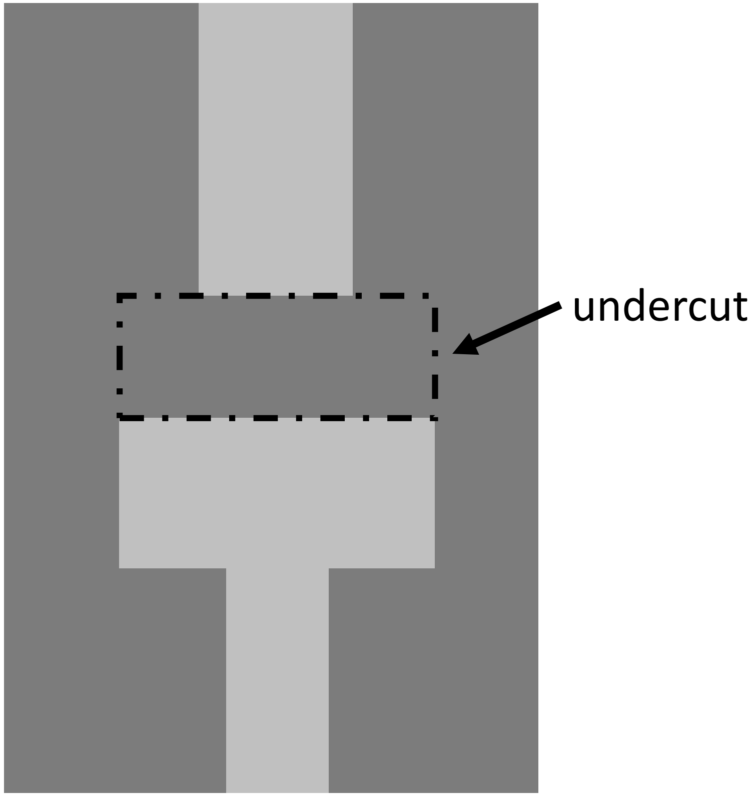
- positive pattern => lift-off
  - deposit material only where you want it
- negative pattern => etching
  - deposit material everywhere
  - remove what you don‘t want

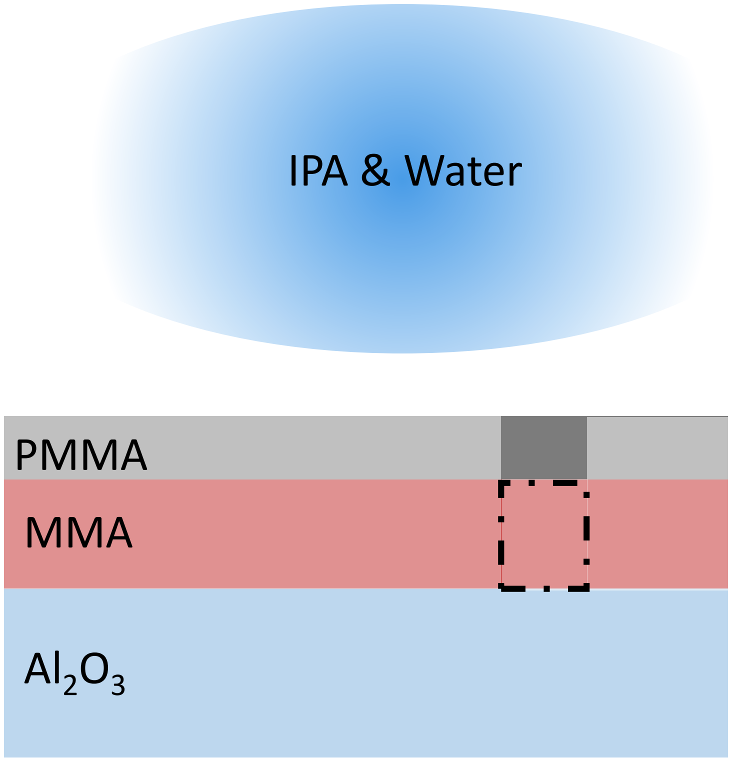
How to build a Josephson Junction

Dolan Bridge technique – Shadow evaporation

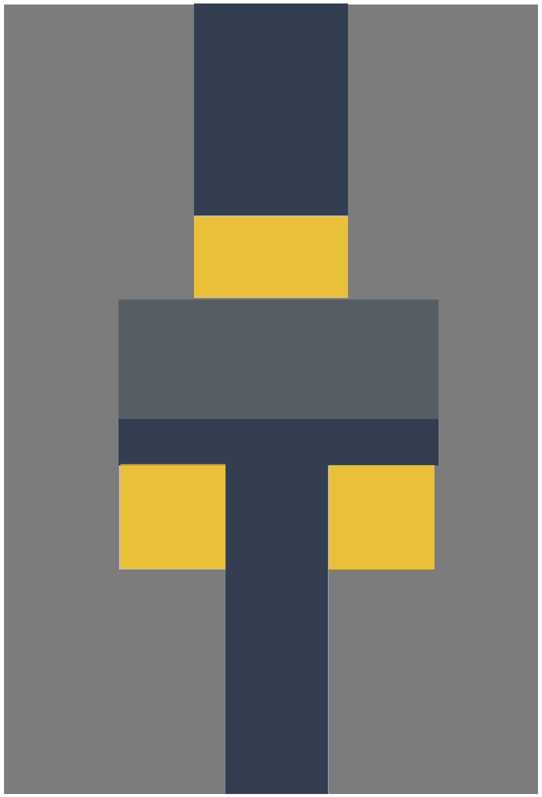


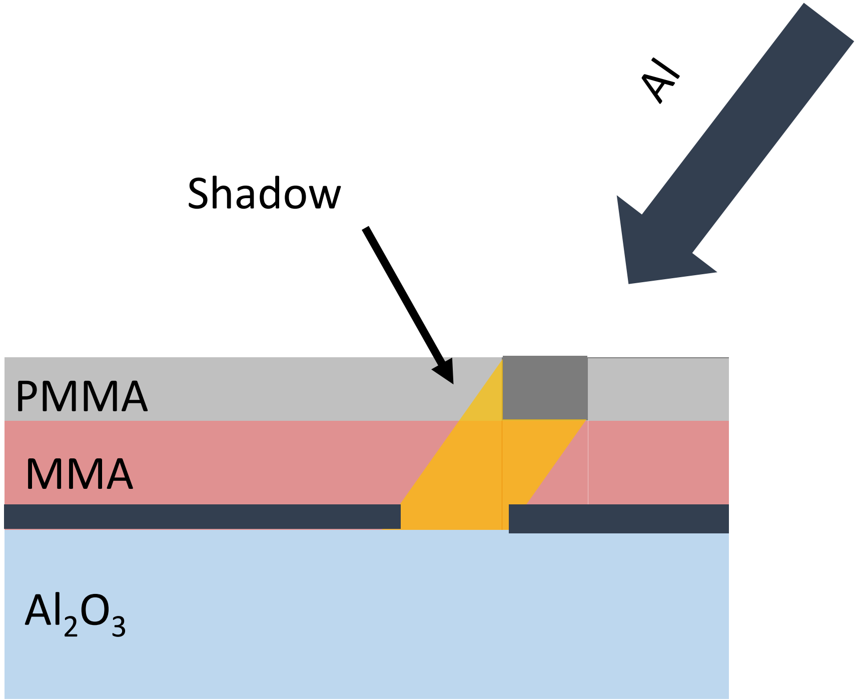
How to build a Josephson Junction



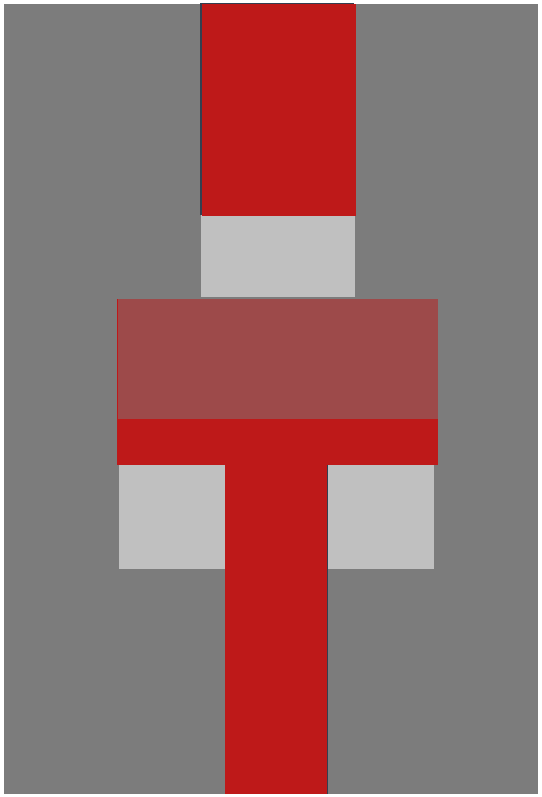


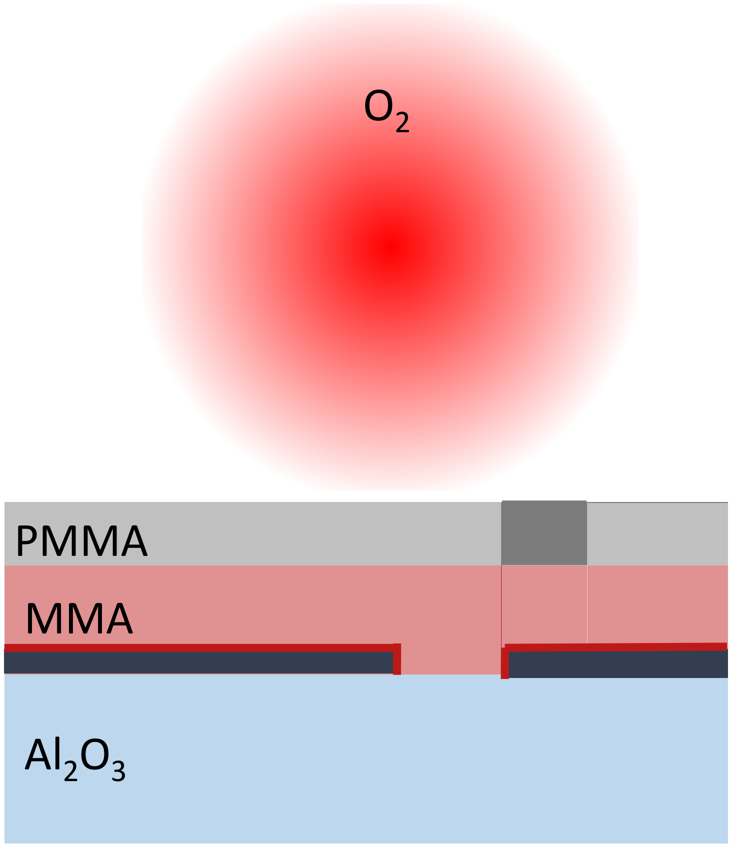
How to build a Josephson Junction



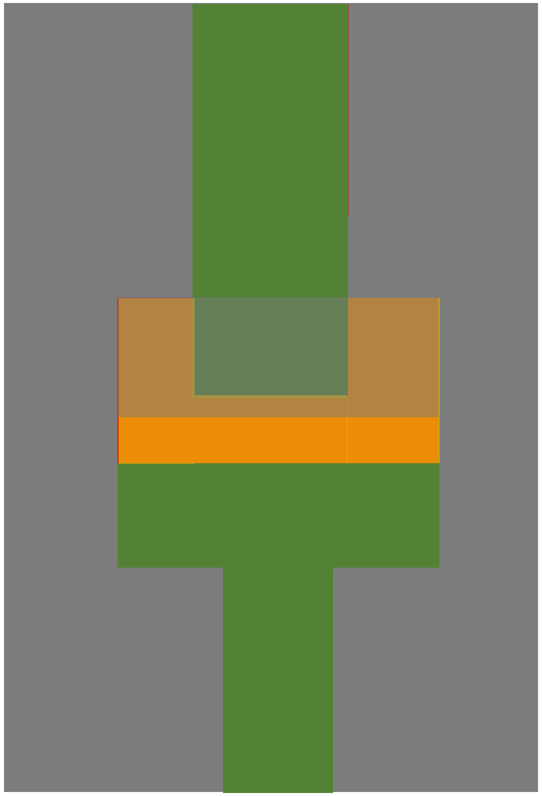


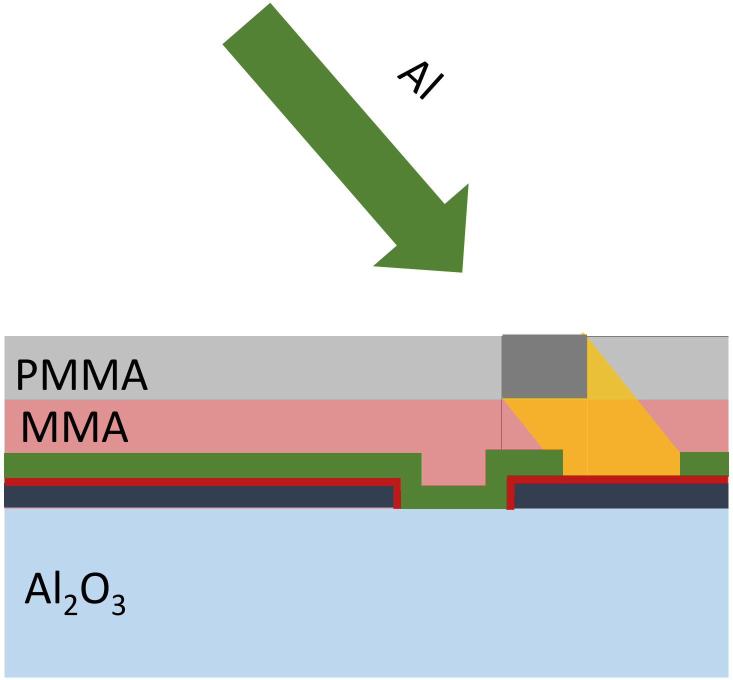
How to build a Josephson Junction



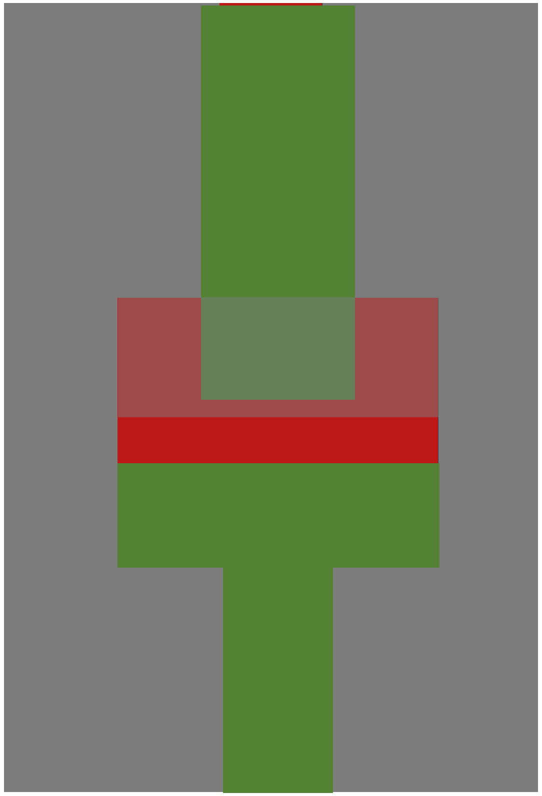


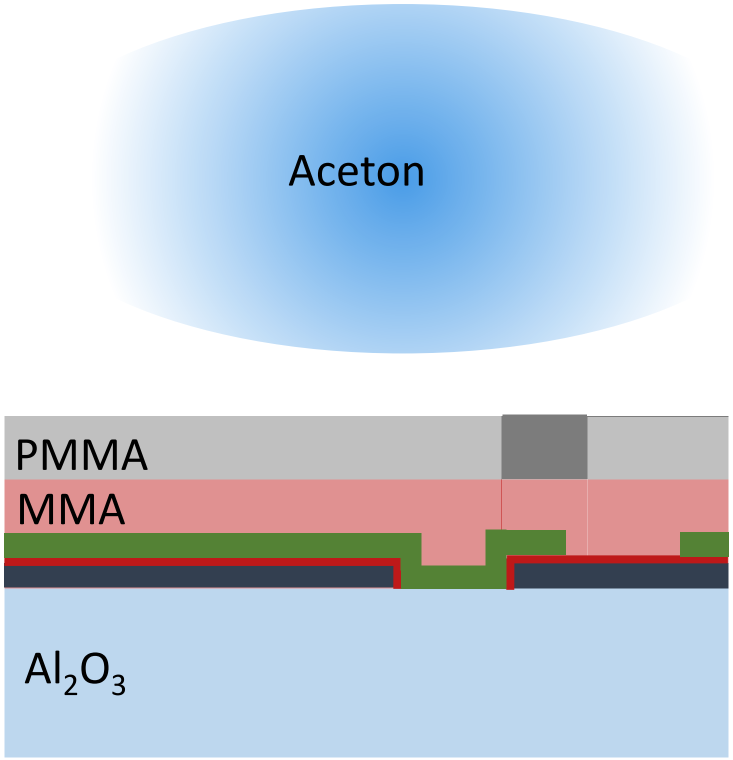
How to build a Josephson Junction



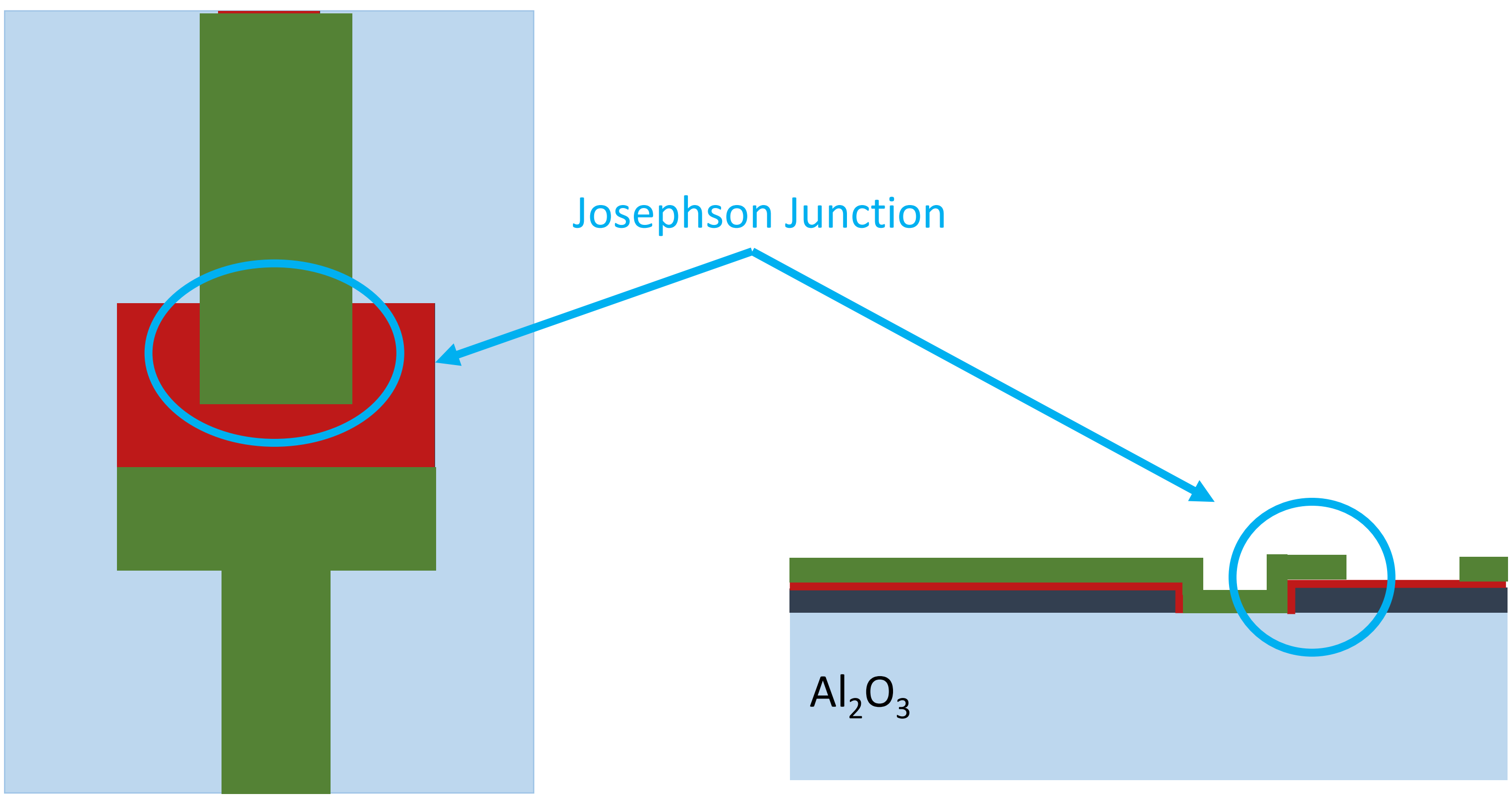


How to build a Josephson Junction



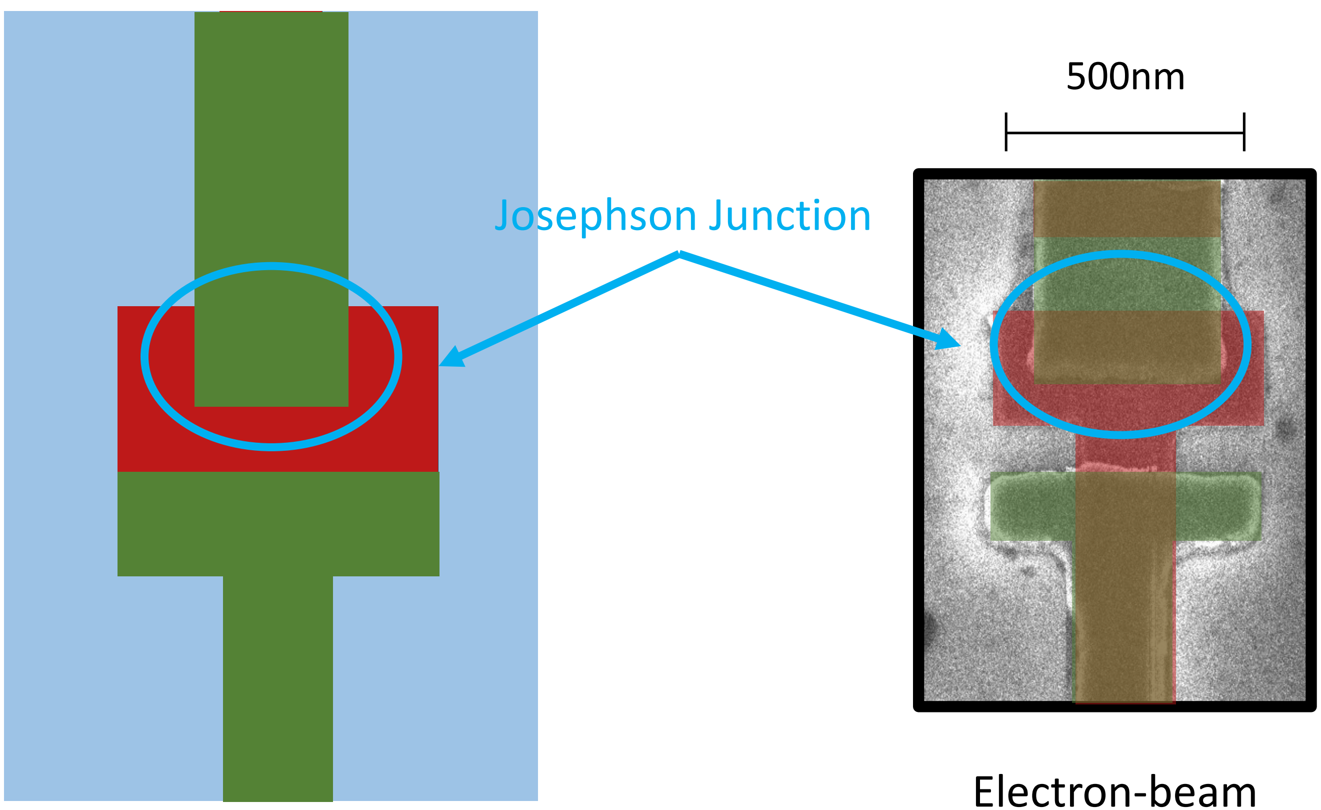


How to build a Josephson Junction



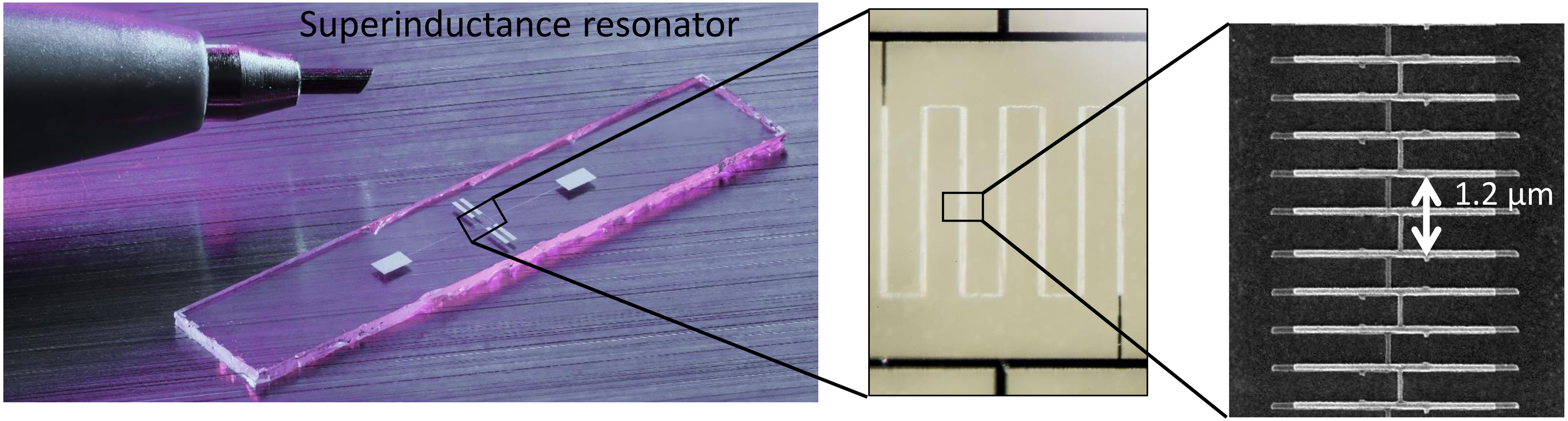
How to build a Josephson Junction

Dolan Bridge shadow technique

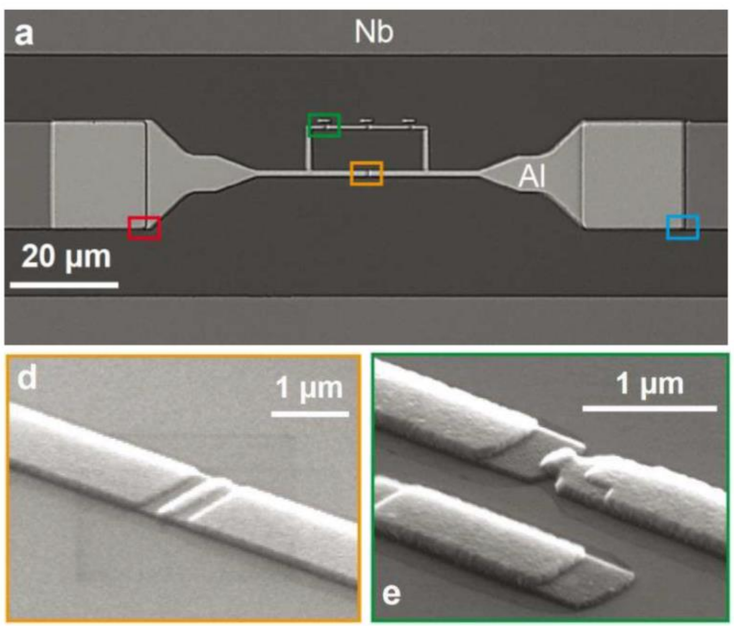


Electron-beam microscope image

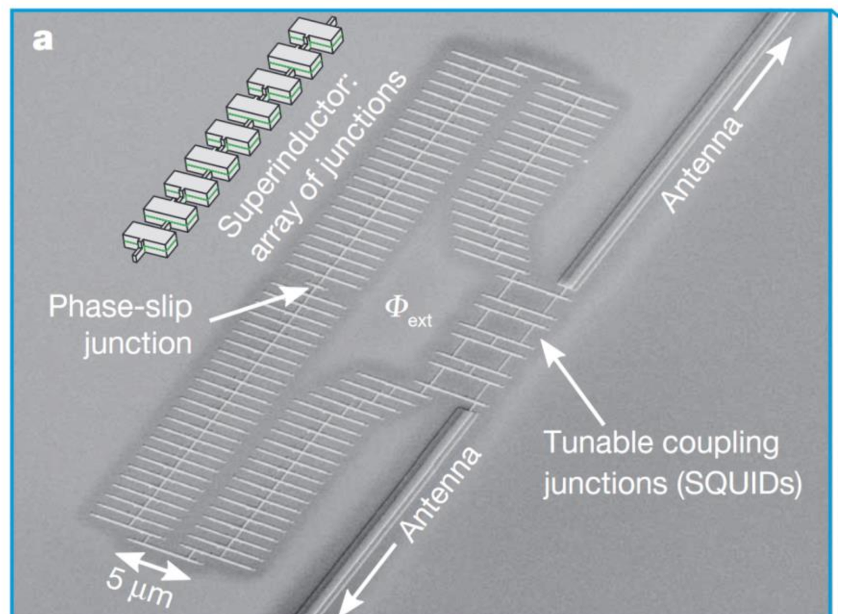
Microfabricated structures



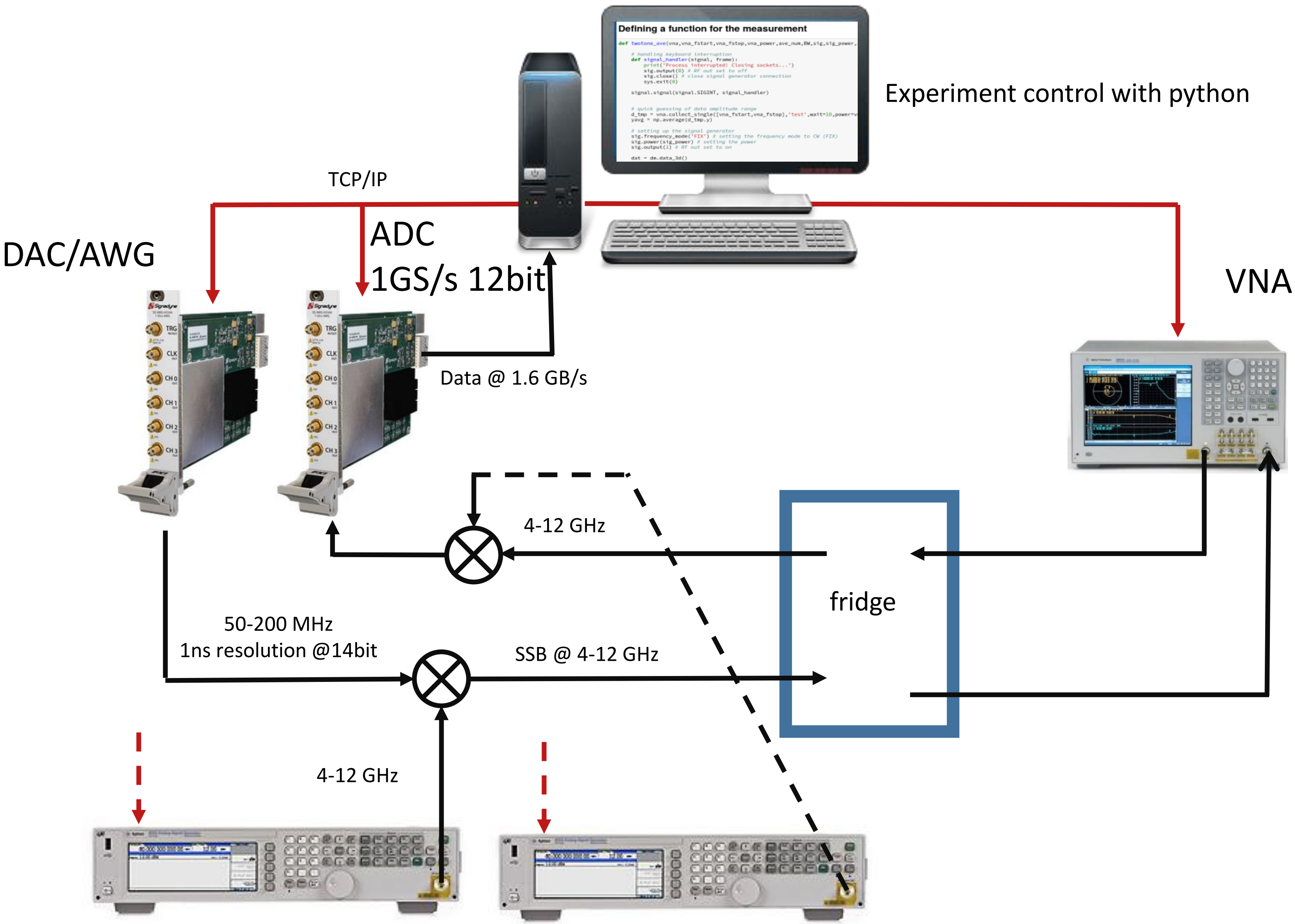
Flux Qubit WMI – ultrastrong coupling



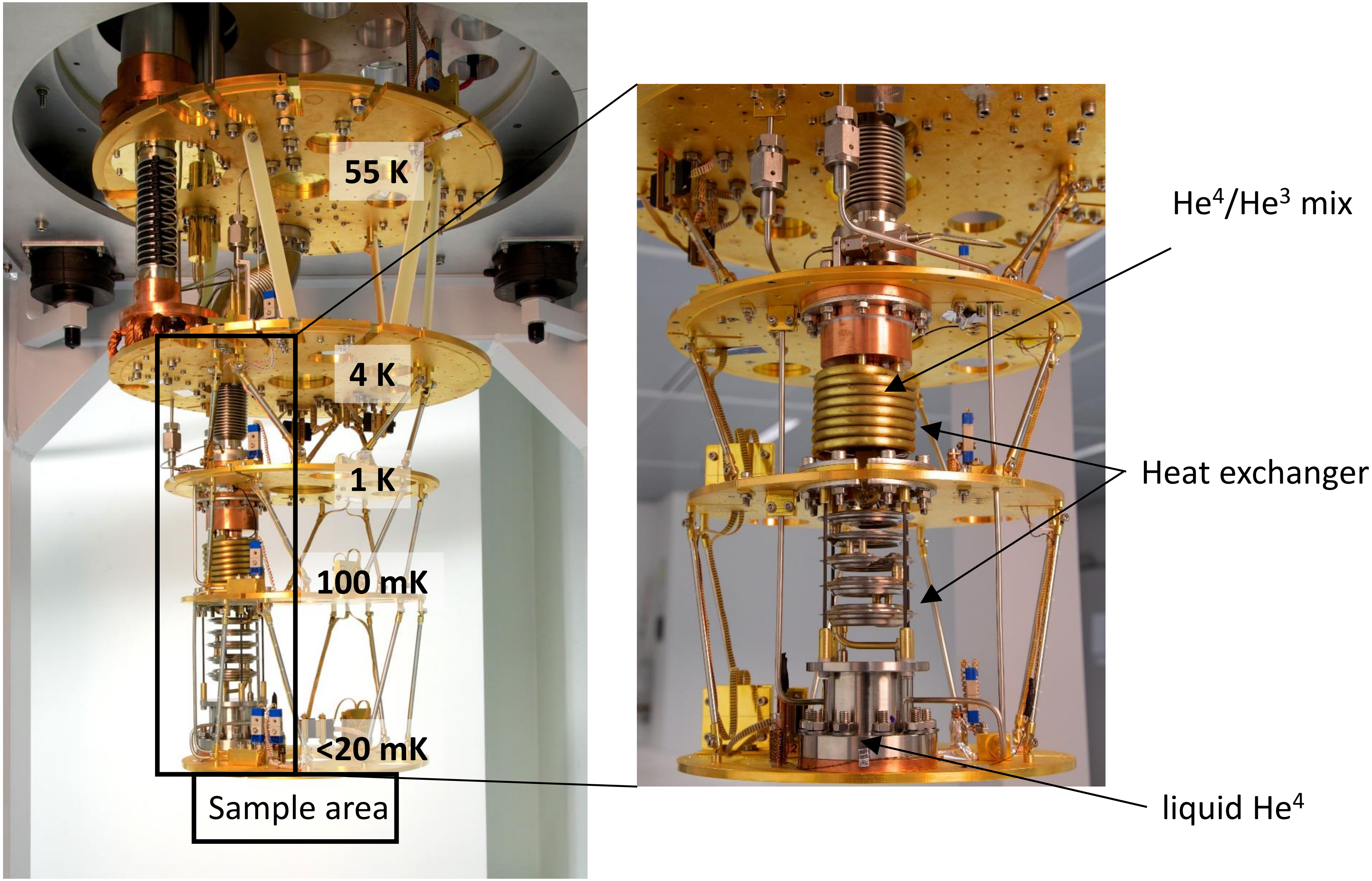
Fluxonium Qubit Yale



Microwave setup room temperature

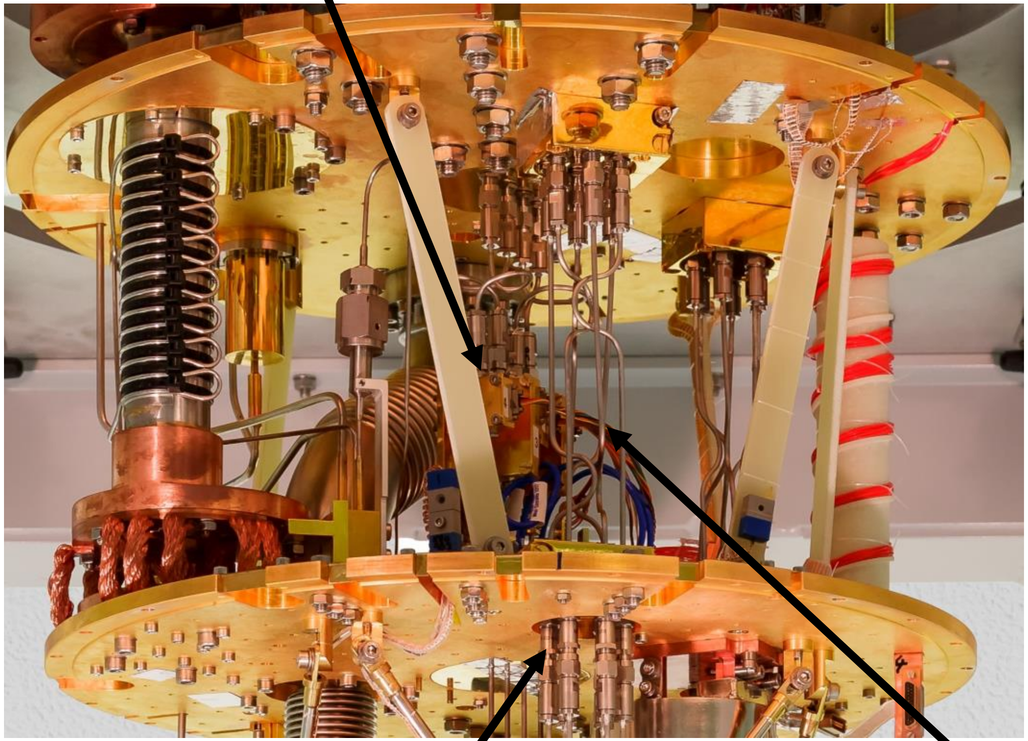


The Cryostat

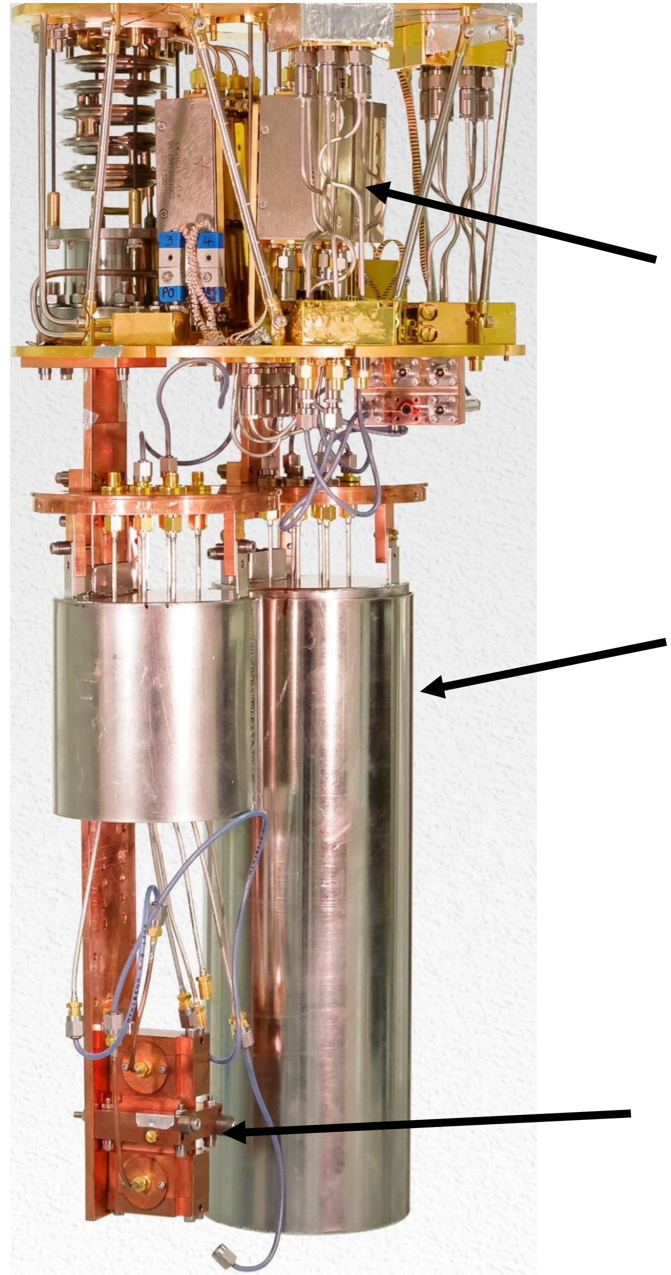


[Explanation of a Cryostat by Andrea Morello: https://www.youtube.com/watch?v=7jT5rbE69ho&t=185s](https://www.youtube.com/watch?v=7jT5rbE69ho&t=185s)

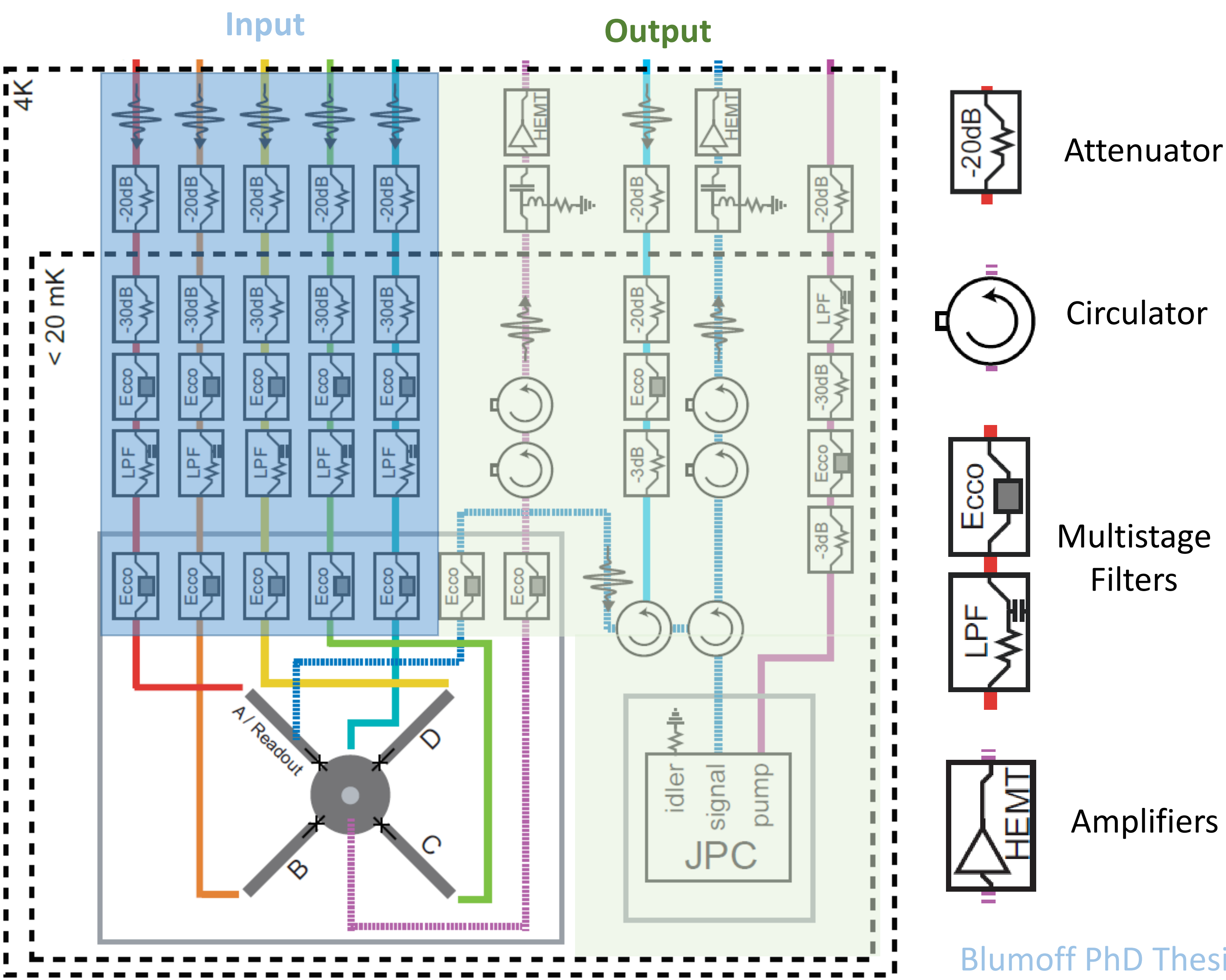
The Cryostat



The Cryostat



Microwave setup cryogenic



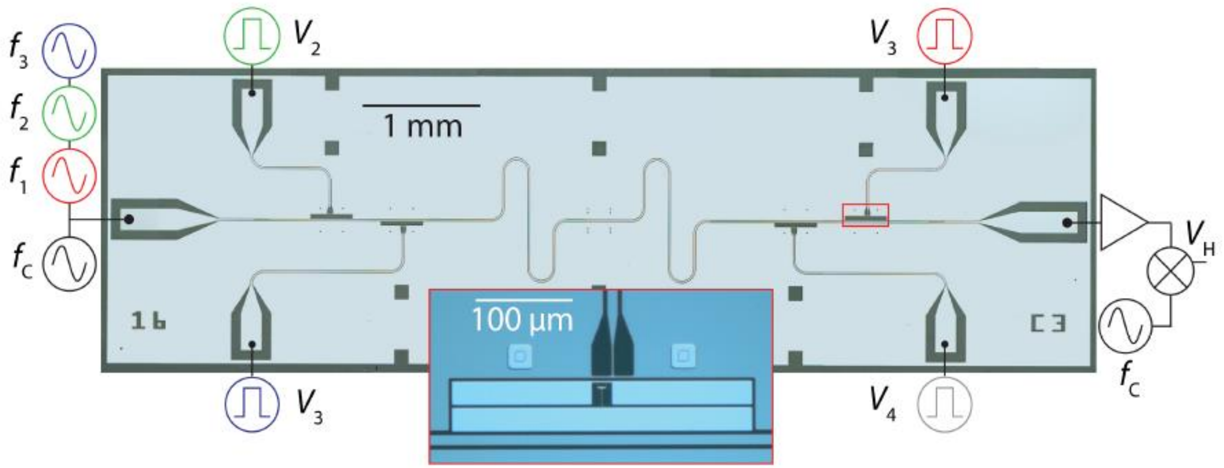
Blumoff PhD Thesis Yale

Quantum information processing with cQED

Institute for Quantum Optics and Quantum Information, Innsbruck
Institute for Experimental Physics, University of Innsbruck

Four-qubit cQED device

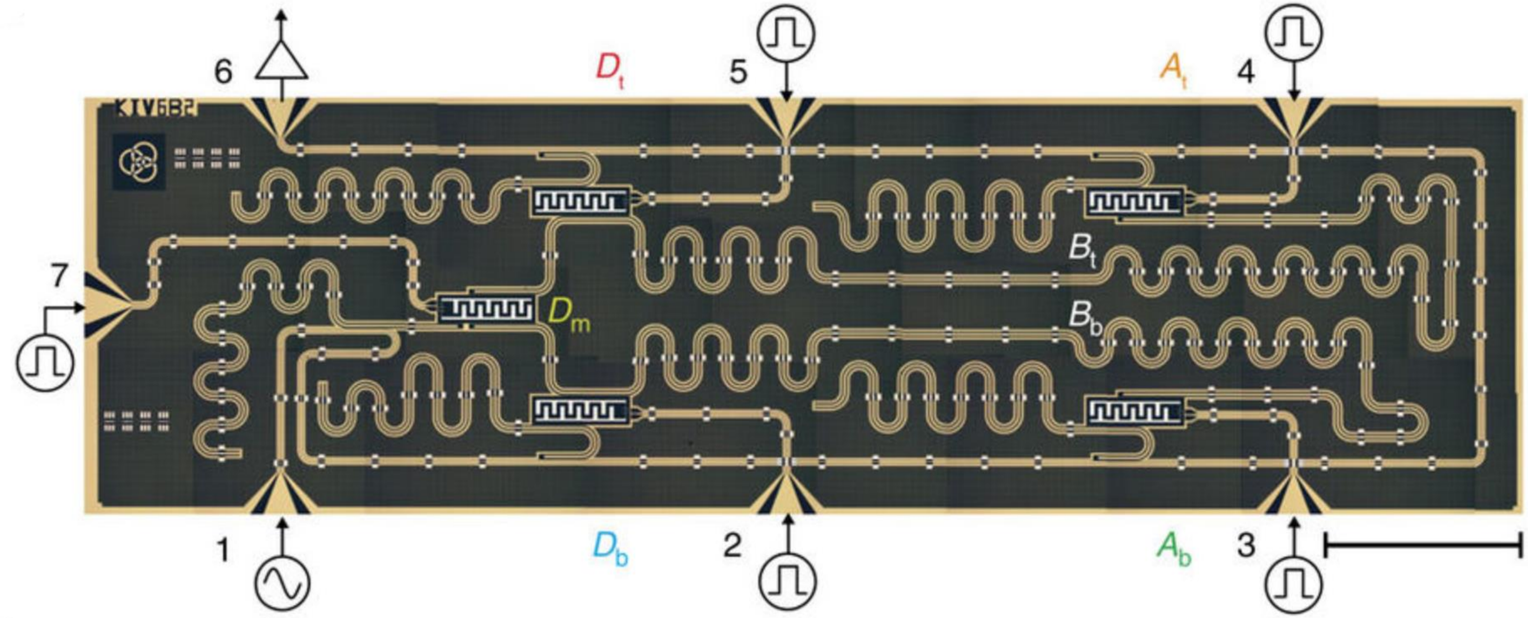
- **Four** transmon qubits coupled to single 2D microwave resonator



- Three qubits biased at **6**, **7**, and **~8** GHz
  - Fourth qubit above cavity and unused
  - T_1 ~ 1 μs, T_2 ~ 0.5 μs
- **Flux bias lines** to control frequency

DiCarlo, *et al. Nature* **467,** 574 (2010), Reed et.al. **482**, 382 Nature (2012)

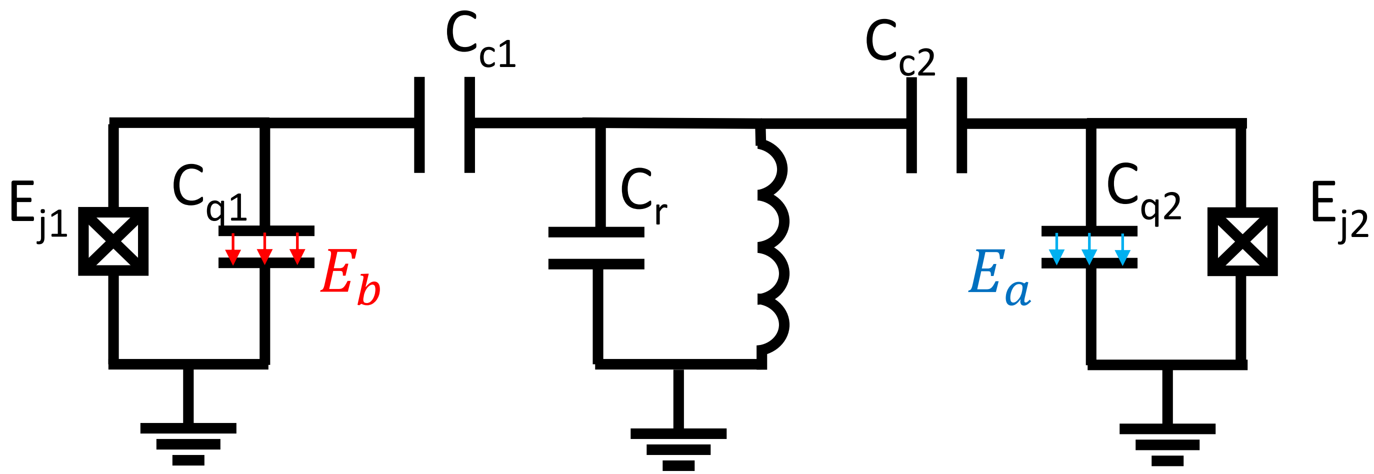
Five-qubit cQED device



Riste et.al. Nature communications 6, 6983 (2015)

- 5 qubits
- 5 readout resonators coupled to one readout CPW
- 2 resonators for mediating coupling
- All flux tunable

Transmon Transmon coupling

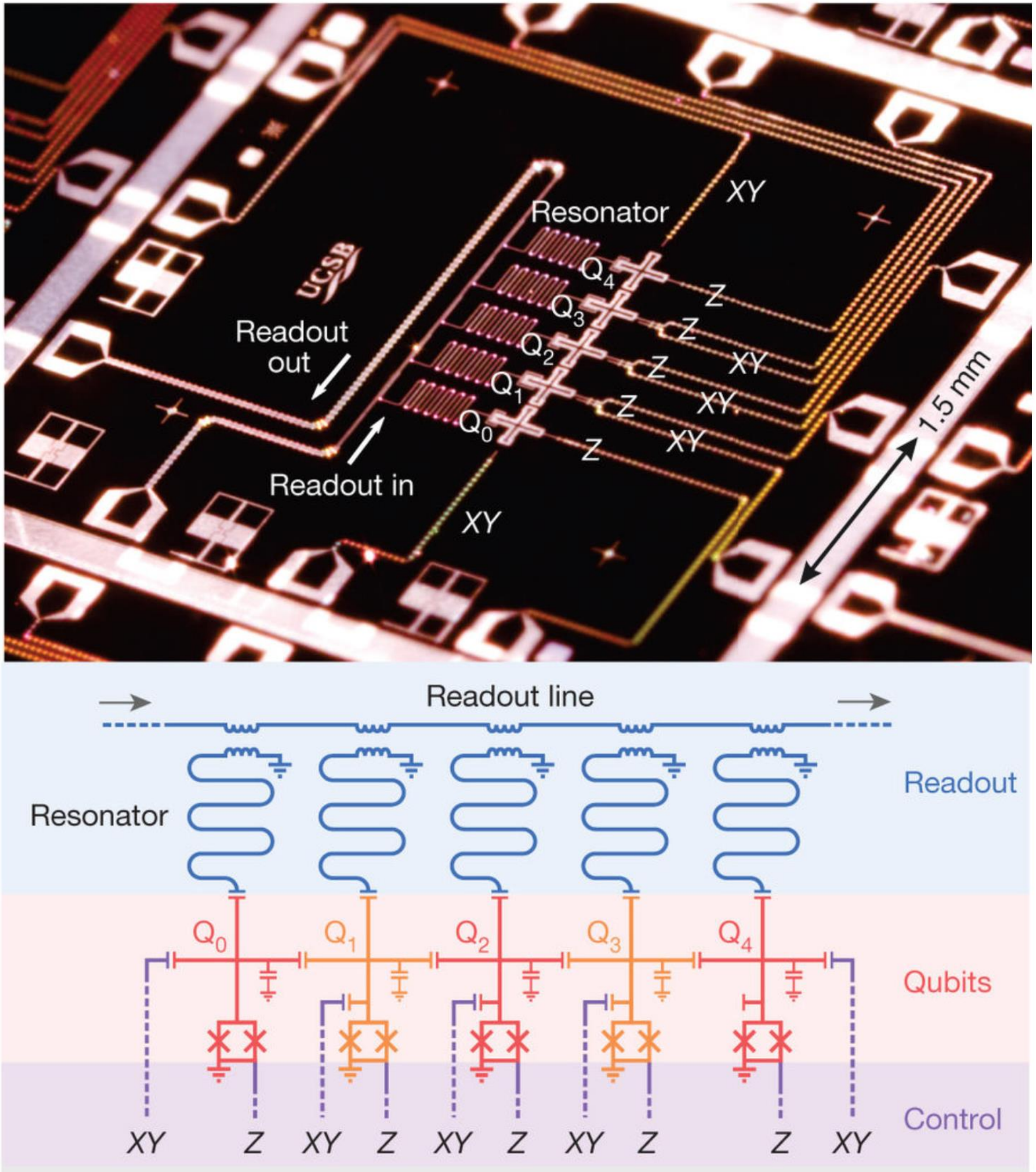


Qubits interact via Cavity => serves as a bus

$$H_{int} = \hbar J(\sigma^+\sigma^- + \sigma^-\sigma^+)$$

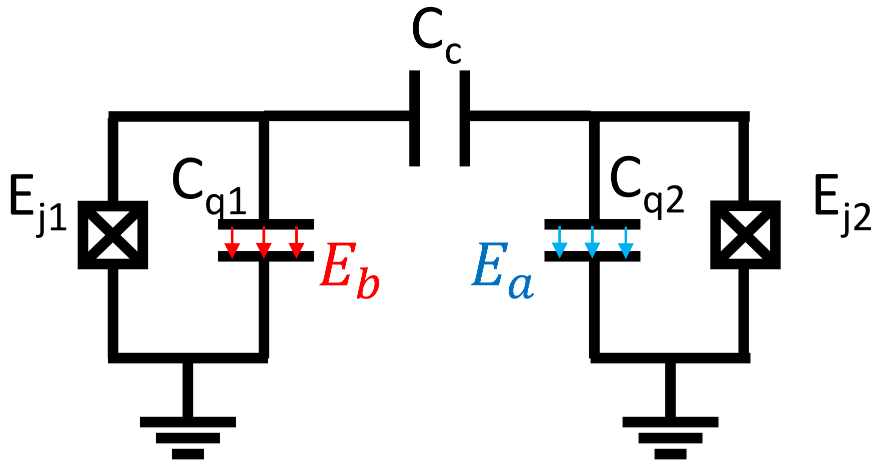
On resonance exchange of excitations!!

5 Transmon qubits – direct coupling



- 5 qubits directly coupled to Neighboring qubit
- 5 readout resonators with common readout CPW
- Individual flux control and single qubit pulses
- ZZ gates through direct qubit qubit interaction

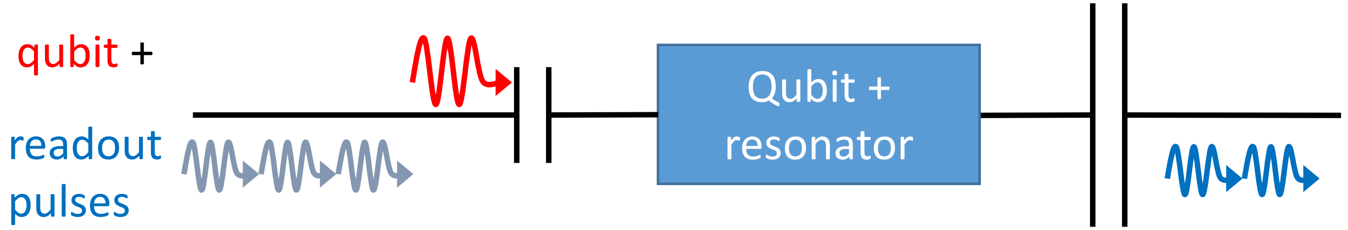
R Barends *et al.* *Nature* **508**, 500-503 (2014) doi:10.1038/nature13171

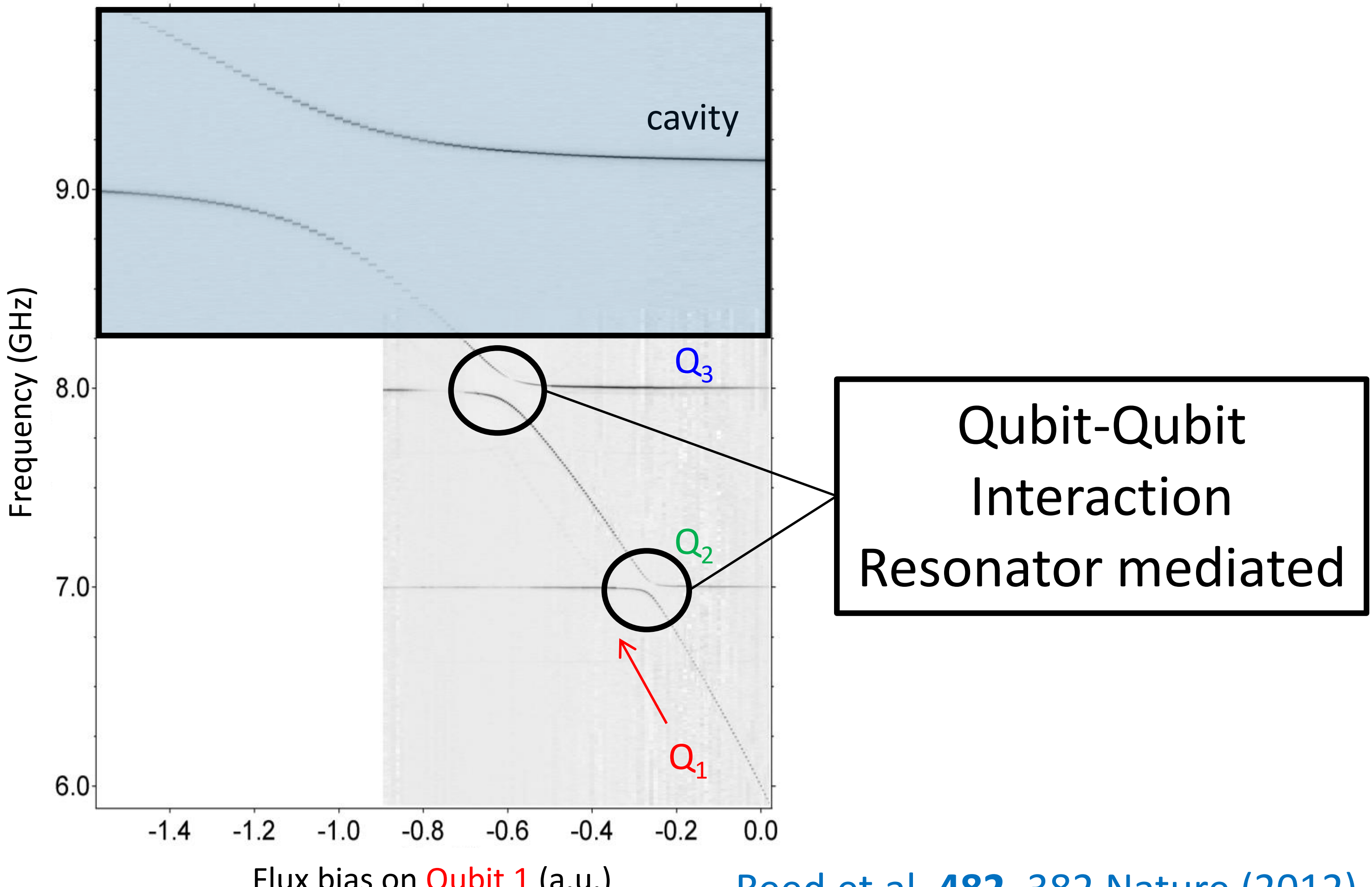


Direct capacitive qubit-qubit interaction

$$H_{int} = \hbar J(\sigma^{+}\sigma^{-} + \sigma^{-}\sigma^{+})$$

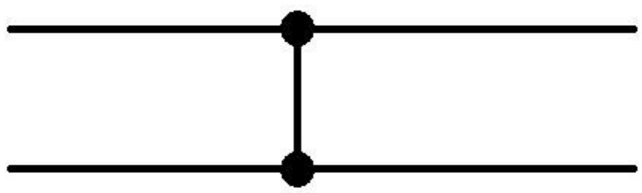
Spectroscopy





Reed et.al. **482**, 382 Nature (2012)

Phase Gate



$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Single qubit phases

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i(\phi_{01}+\phi_{10}+\phi_{11})} \end{bmatrix}$$

Time evolution of a state

$$|\Psi_i(t)\rangle = e^{i\frac{E_i}{\hbar}t}|\Psi_i\rangle$$

$|00\rangle \longrightarrow E_{00} = 0$ $\quad |01\rangle \longrightarrow E_{01}$ $\quad |10\rangle \longrightarrow E_{10}$ $\quad |11\rangle \longrightarrow E_{01} + E_{10}$

$$U(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\frac{E_{01}}{\hbar}t} & 0 & 0 \\ 0 & 0 & e^{i\frac{E_{10}}{\hbar}t} & 0 \\ 0 & 0 & 0 & e^{i\frac{E_{01}+E_{10}}{\hbar}t} \end{pmatrix}$$

for time τ

We actually want:

$$U(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\,\phi_{01}} & 0 & 0 \\ 0 & 0 & e^{i\,\phi_{10}} & 0 \\ 0 & 0 & 0 & e^{i(\phi_{01}+\phi_{10}+\phi_{11})} \end{pmatrix}$$

How do we get ϕ_{11}?

$$E_{11} \neq E_{10} + E_{01}$$

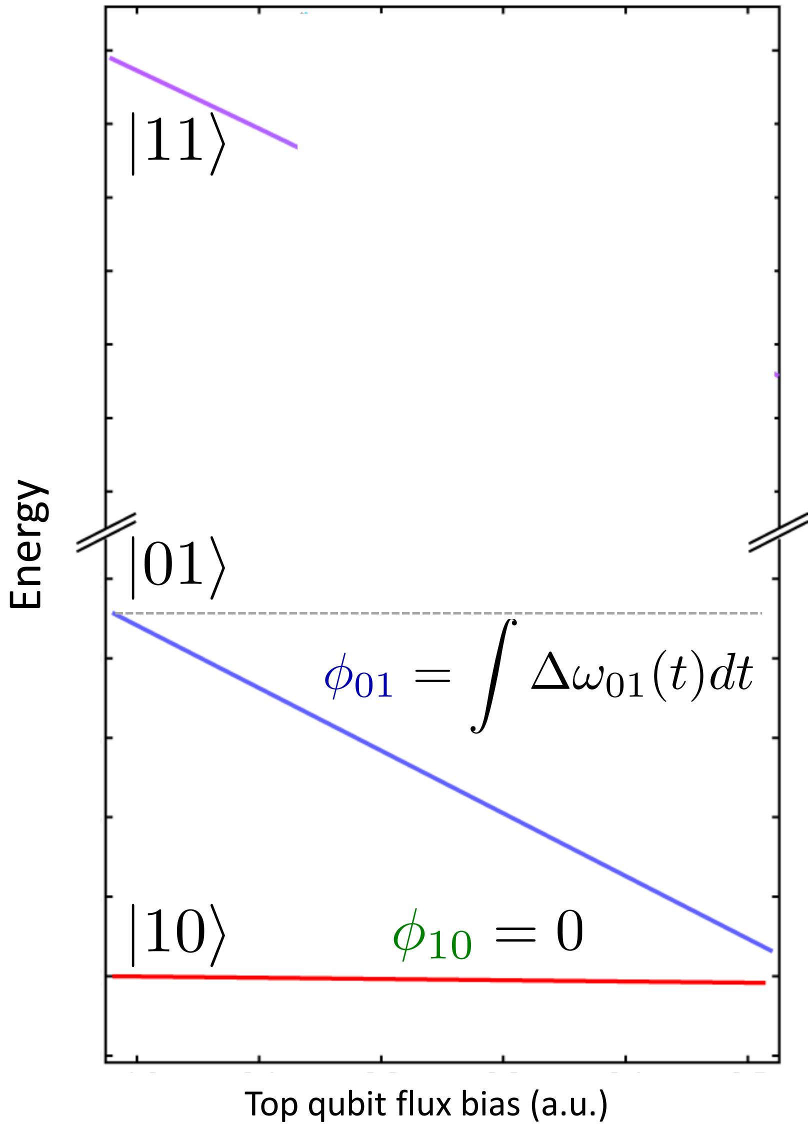
→ Avoided crossing

Adiabatic multiqubit phase gates

Two qubit phase:

$$|00\rangle \longrightarrow |00\rangle$$
$$|01\rangle \rightarrow e^{i\phi_{01}}|01\rangle$$

Interactions on **two excitation manifold** give entangling two-qubit conditional phases



DiCarlo, *et al. Nature* **460**, 240 (2009)

Adiabatic multiqubit phase gates

A **two qubit** phase gate can be written:

$$
\begin{aligned}
|00\rangle &\longrightarrow |00\rangle \\
|01\rangle &\rightarrow e^{i\phi_{01}}|01\rangle \\
|10\rangle &\rightarrow e^{i\phi_{10}}|10\rangle \\
|11\rangle &\rightarrow e^{i(\phi_{01}+\phi_{10}+\phi_{11})}|11\rangle
\end{aligned}
$$

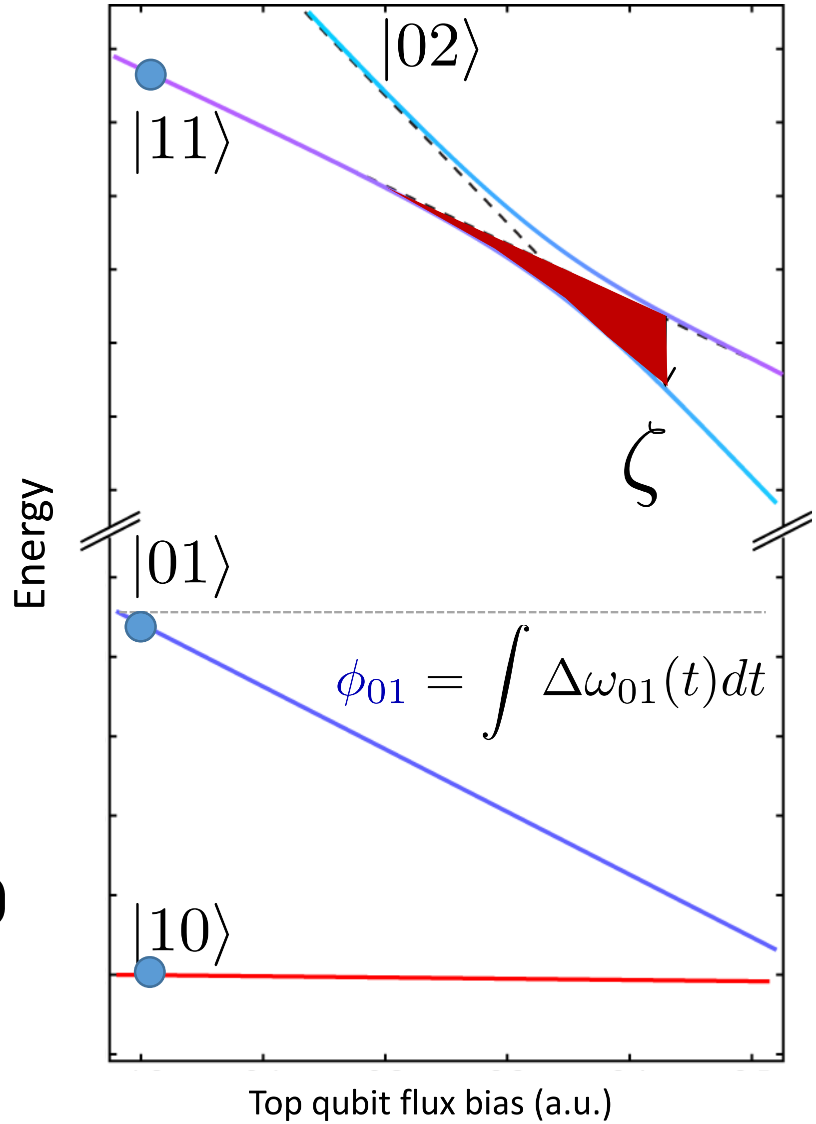
Interactions on **two excitation manifold** give entangling two-qubit conditional phases

$$\phi_{11} = -2\pi \int \zeta(t)dt$$

Can give a **universal** "Conditional Phase Gate"

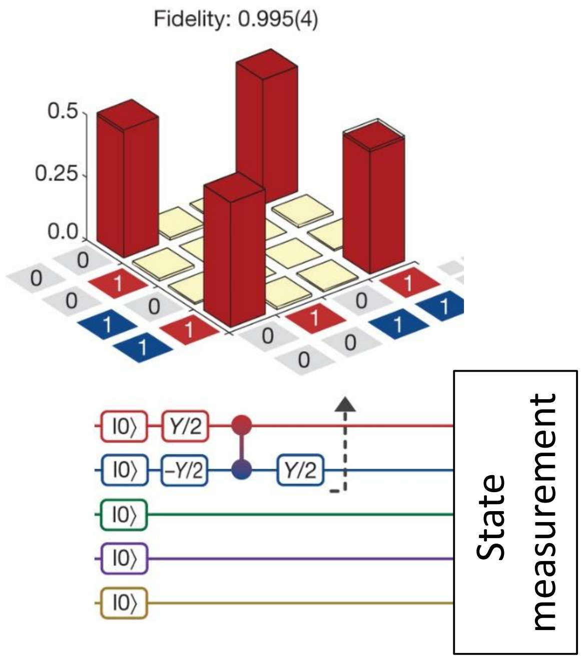
$$
\begin{aligned}
|00\rangle &\rightarrow |00\rangle \\
|01\rangle &\rightarrow |01\rangle \\
|10\rangle &\rightarrow |10\rangle \\
|11\rangle &\rightarrow -|11\rangle
\end{aligned}
$$

$$\phi_{01} = \phi_{10} = 0$$
$$\phi_{11} = \pi$$



DiCarlo, *et al. Nature* **460**, 240 (2009)

Generation of GHZ states.



$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Sequence:

$$|00\rangle$$

↓ $\pi/2$ pulses

$$(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle) =$$

$$|00\rangle + |10\rangle - |01\rangle - |11\rangle =$$

↓ zz gate

$$|00\rangle + |10\rangle - |01\rangle + |11\rangle =$$

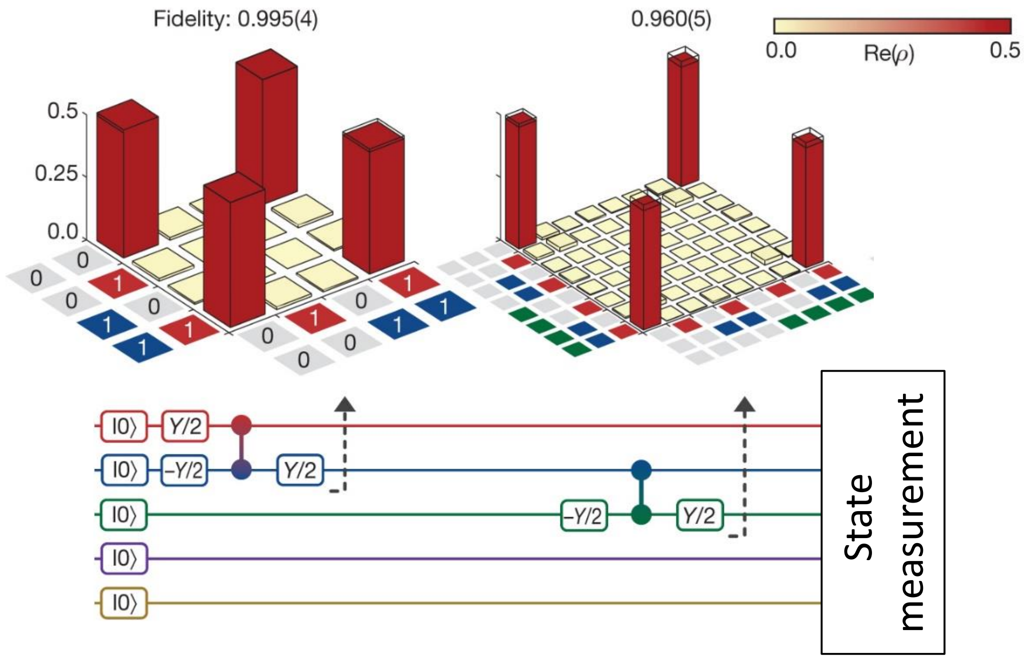
$$|0\rangle \otimes (|0\rangle - |1\rangle) + |1\rangle \otimes (|0\rangle + |1\rangle)$$

↓ $\pi/2$ pulse

$$|00\rangle + |11\rangle$$

R Barends *et al.* *Nature* **508**, 500-503 (2014) doi:10.1038/nature13171

Generation of GHZ states.



$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

Sequence:

$$|00\rangle + |11\rangle \otimes |0\rangle$$

↓ $\pi/2$ pulse

$$|00\rangle + |11\rangle \otimes (|0\rangle - |1\rangle)$$

$$|000\rangle - |001\rangle + |110\rangle - |111\rangle$$

↓ zz gate

$$|000\rangle - |001\rangle + |110\rangle + |111\rangle$$

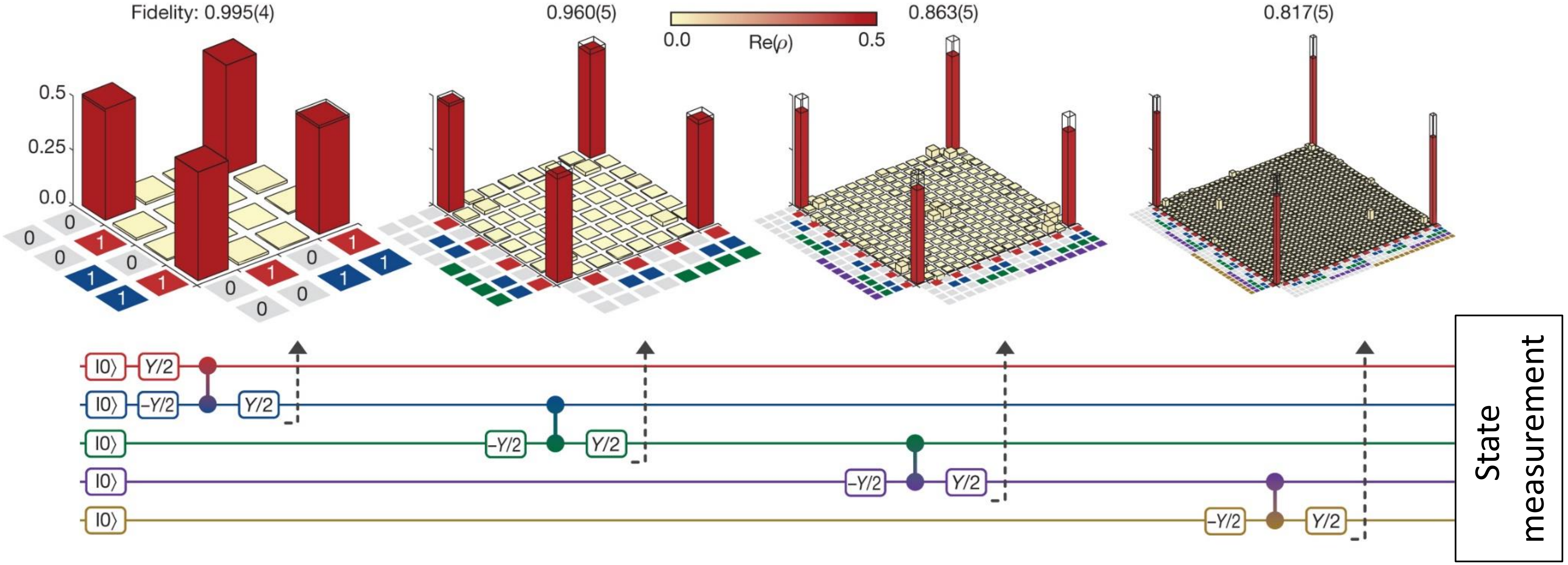
$$|00\rangle \otimes (|0\rangle - |1\rangle) + |11\rangle \otimes (|0\rangle + |1\rangle)$$

↓ $\pi/2$ pulse

$$|000\rangle + |111\rangle$$

R Barends *et al.* *Nature* **508**, 500-503 (2014) doi:10.1038/nature13171

Generation of GHZ states.



$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

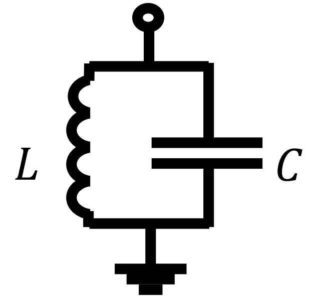
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

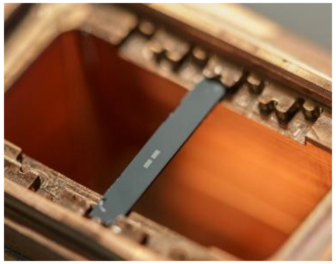
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00000\rangle + |11111\rangle)$$

R Barends *et al.* *Nature* **508**, 500-503 (2014) doi:10.1038/nature13171

Summary

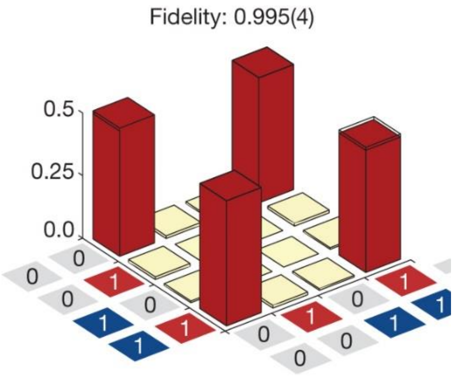


- Circuit Quantization



- Transmon qubit and cQED

- Experimental Techniques



- QIP with cQED

Additional slides

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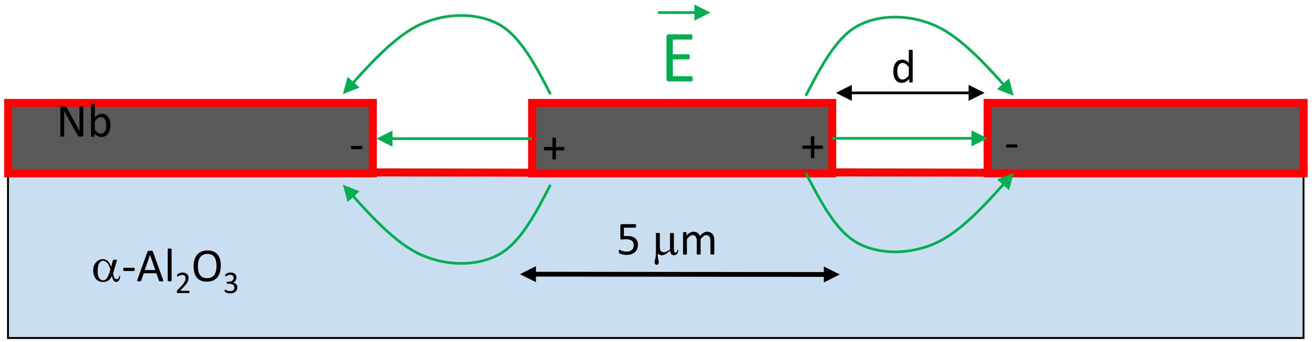
Why surfaces matter… dirt happens



"participation ratio" = fraction of energy stored in material

even a thin (few nanometer) surface layer
will store ~ 1/1000 of the energy

If surface loss tangent is poor ($\tan\delta \sim 10^{-2}$) would limit $Q \sim 10^5$

Increase spacing

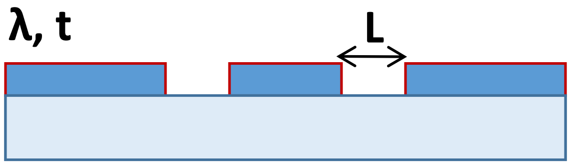
→ decreases energy on surfaces

→ increases Q

But Effects Can Be Reduced Drastically!

$$\frac{1}{Q} = \frac{1}{p_{diel} \tan\delta} + \frac{1}{\alpha Q_s}$$

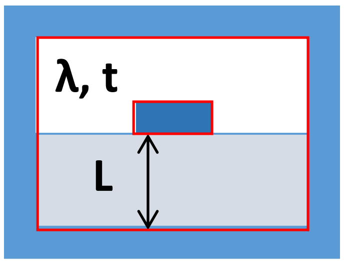
CPW/Compact Resonator



L ~ 10 μm

$$p_{diel} \sim 10^{-2} - 10^{-3}$$
$$p_{cond} \sim 10^{-1} - 10^{-2}$$

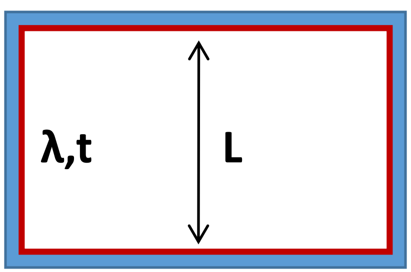
Stripline (vertical transmon)



L ~ 500 μm

$$p_{diel} \sim 10^{-4} - 10^{-5}$$
$$p_{cond} = \alpha^{-1} \sim 10^{-3} - 10^{-4}$$

3D



L ~ 5,000 μm

Assume:
t ~ 3 nm,
λ ~ 50 nm

$$p_{diel} \sim 10^{-6} - 0$$
$$p_{cond} = \alpha^{-1} \sim 10^{-5} - 10^{-6}$$

How to quantize more complicated systems?

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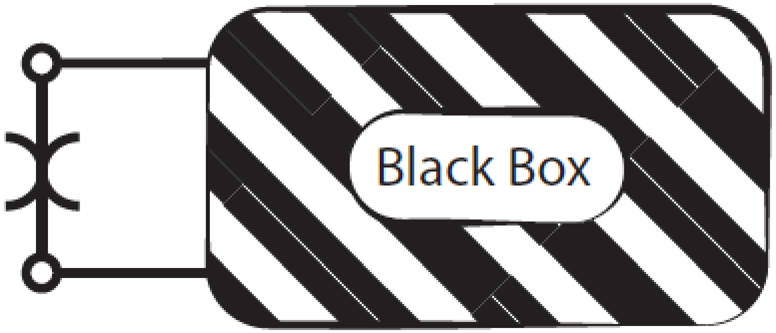
Black Box Quantization - BBQ

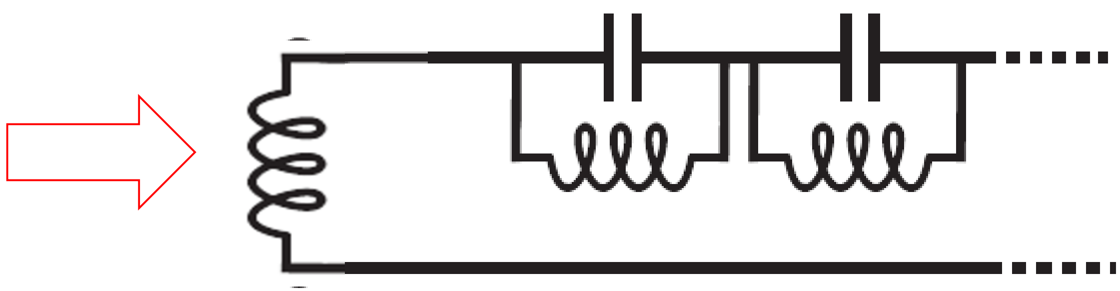
Single qubit coupled to multimode (lossless) circuit = Black Box



Foster's theorem works for any

N coupled modes each with $a, a^\dagger$

Y(ω)



Nigg et.al. PRL 240502 (2012)

Black Box Quantization - BBQ

Diagonalize by finding all zeros of Y(ω)

$$\omega_k = \frac{1}{\sqrt{L_k C_k}}$$

$$H = \sum_k \hbar\omega_k A_k^\dagger A_k$$

All couplings taken into account!!!

New Operators for uncoupled eigenmodes of complete system: A & $A^\dagger$

All we need to do is get the anharmonicity back!

$$H_1 \sim \left(a^\dagger + a\right)^4$$

First order term JJ!
In old coordinates!

rewriting H_1 in the new operators leads to

$$H_1 = \frac{E_J}{24}\left(\frac{2e}{\hbar}\right)^4 \left(\sum_k \Phi_{zpf}^{(k)}(A_k + A_k^\dagger)\right)^4$$

$$\Phi_{zpf}^{(k)} = \sqrt{\frac{\hbar}{\omega_k \mathrm{Im}\{Y'(\omega_k)\}}}$$

We get these from Y(ω)

Black Box Quantization - BBQ

more approximations lead to simple expressions for the anharmonicities, state dependent shifts and resonance frequency shifts

$$\alpha_i = -\frac{1}{2L_j}\frac{e^2}{\hbar^2}\left(\Phi_{zpf}^{(i)}\right)^4 \qquad \chi_{ij} = -\frac{1}{L_j}\frac{e^2}{\hbar^2}\left(\Phi_{zpf}^{(i)}\right)^2\left(\Phi_{zpf}^{(i)}\right)^2$$

all we need to know: ω_k for $Y(\omega) = 0$ & $Y'(\omega_k)$

better way: numerically calculate Eigenenergies of H_0+H_1 to get parameters

The recipe

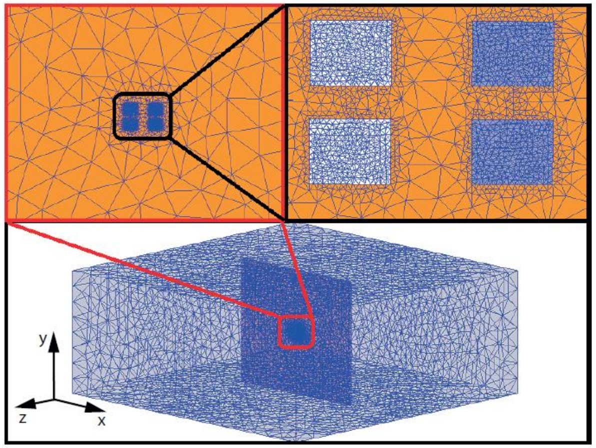
1. Determine $Y(\omega)$: HFSS, Microwave Office, analytical expression,….
2. Determine ω_k where $Y(\omega) = 0$
3. Get the derivative of $Y(\omega)$ @ $\omega=\omega_k$
4. Do a numerical diagonalization or use first order expressions to get α_i, χ_{ij}

For all modes!!

Black Box Quantization - BBQ

- Works also for 3D structures where it is hard to write down an analytical circuit!

  e.g. 2 qubits in a waveguide cavity using finite element solver



- Works for **multiple** qubits coupled to **multiple** resonators (see ***Nigg et.al. PRL 240502 (2012)***)
- Works only well for **transmon** qubits! (more general: **Smith et.al. Phys. Rev. B 94, 144507 (2016)**)

Take away

You can calculate circuit QED Hamiltonians by assuming everything is a harmonic oscillator and add in the anharmonicity later!

Nothing is a harmonic oscillator after it has been coupled to a Josephson Junction

Quantum optics with circuit QED

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Driving a Quantum Harmonic Oscillator

Giving a classical 'drive':

$$D(\alpha) = e^{\left(\alpha a^{\dagger} - \alpha^{*} a\right)}$$

$$D(\alpha)|0\rangle = |\alpha\rangle$$

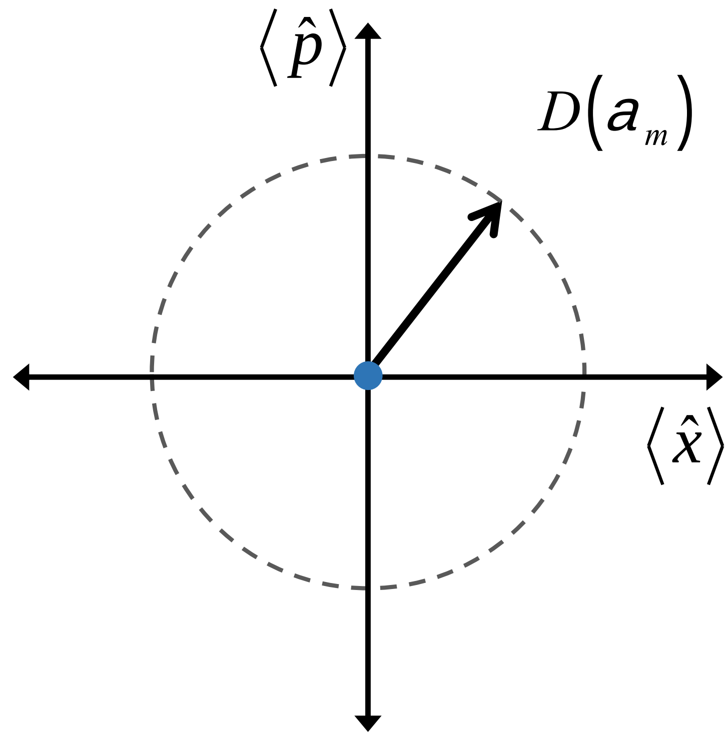
Where:

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

with $\alpha = |\alpha| e^{i\phi}$

Our state is described by **two continuous variables**, an amplitude and phase.
A '**coherent**' state.

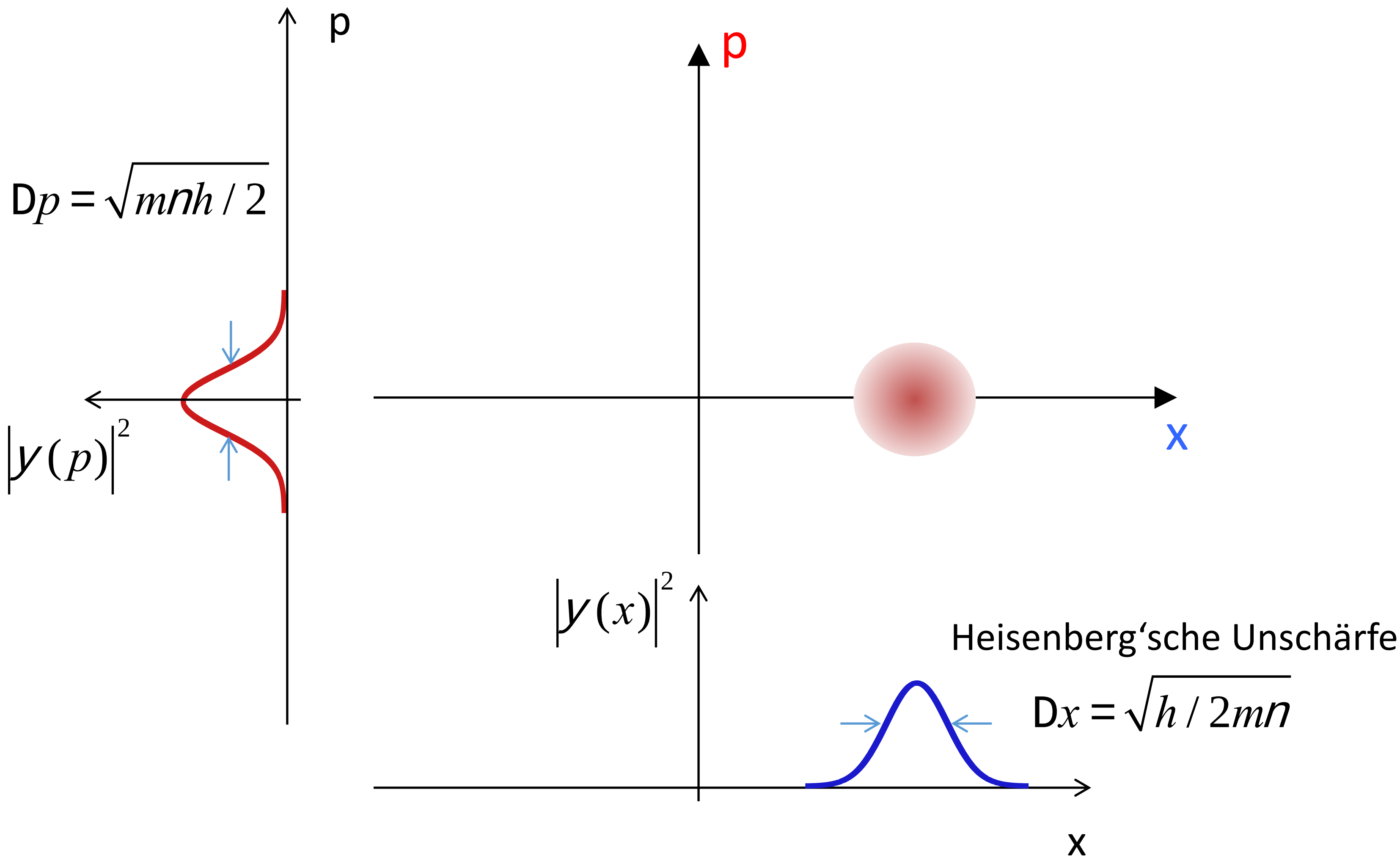
Phase-space picture:



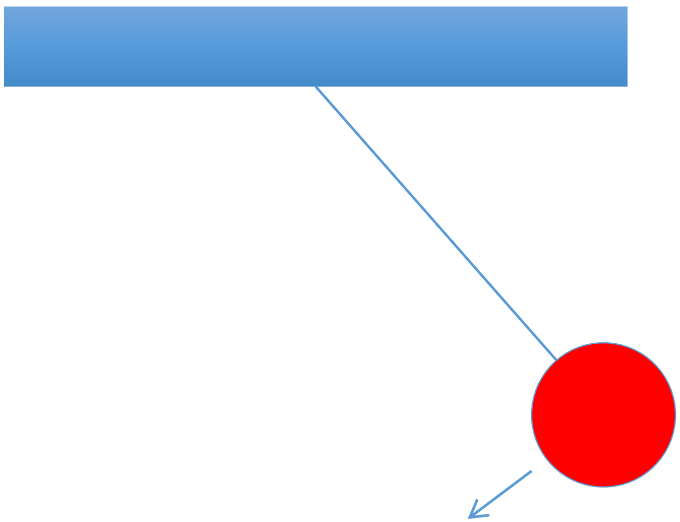
$$\langle \hat{x} \rangle = |\alpha| \cos(\phi)$$

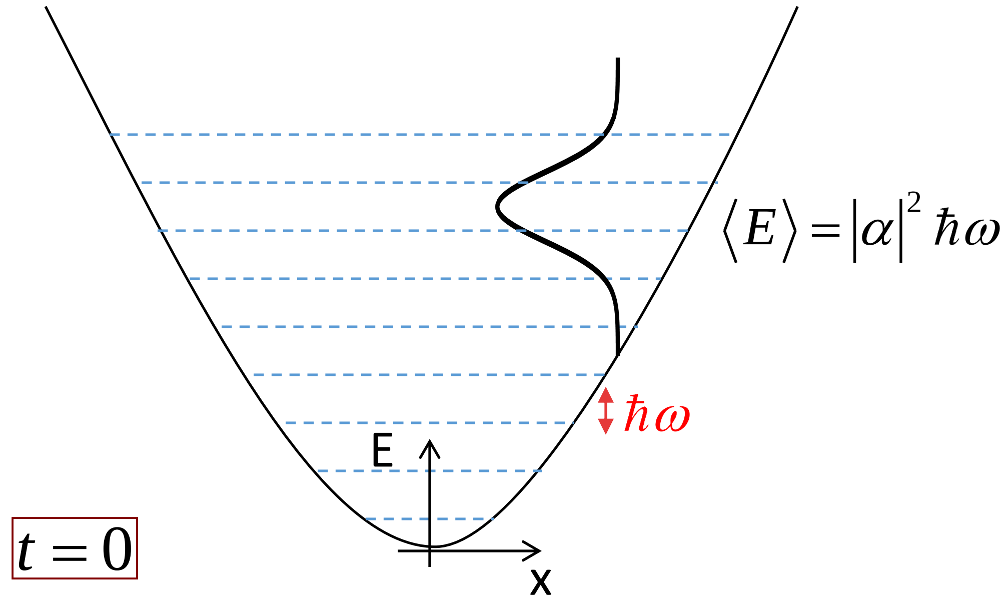
$$\langle \hat{p} \rangle = |\alpha| \sin(\phi)$$

Quantum harmonic Oscillator



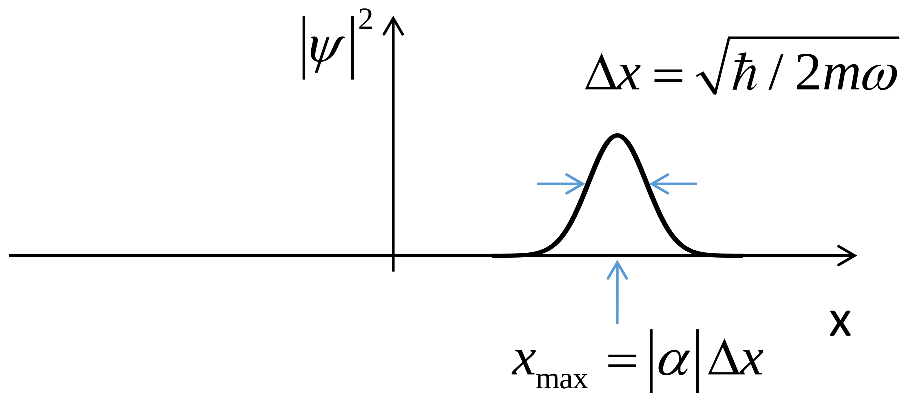
What’s a Coherent State?



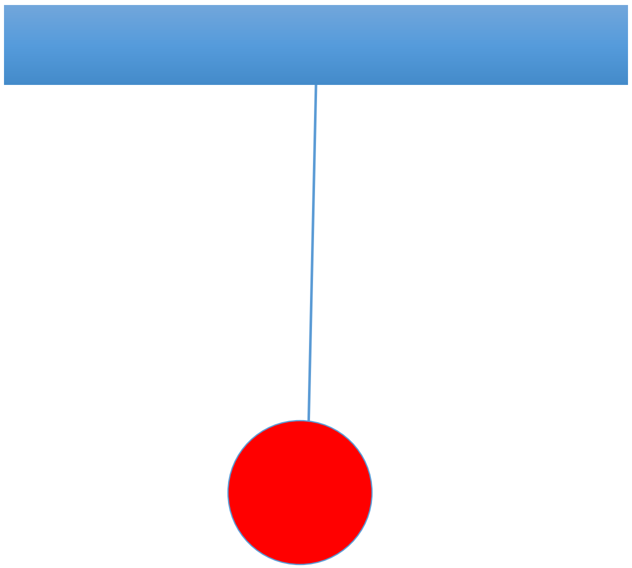


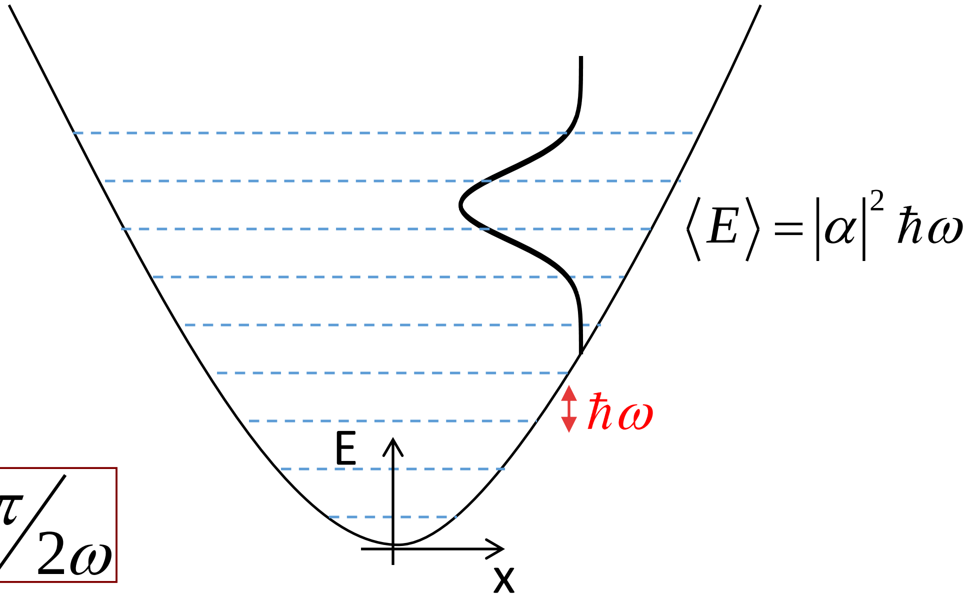
Glauber (coherent) state

$$|a\rangle = e^{-\frac{|a|^2}{2}} \sum_{n=0}^{\infty} \frac{a^n}{\sqrt{n!}} |n\rangle$$



What's a Coherent State?

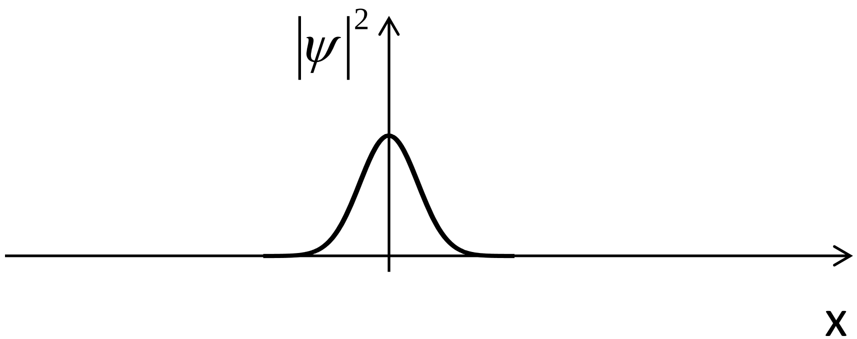




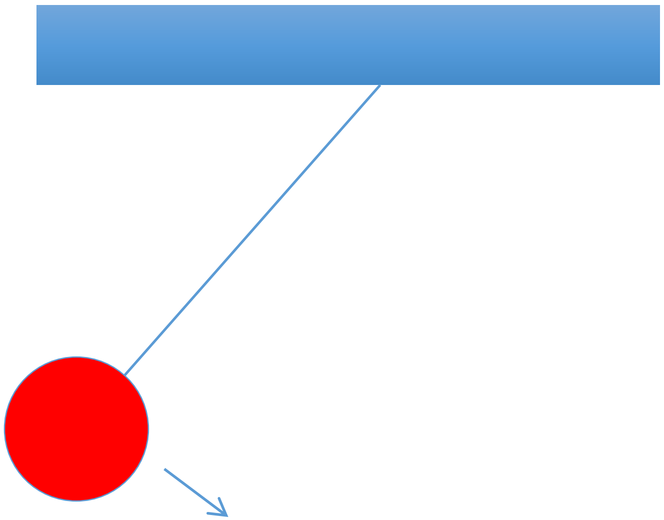
$$t = \pi/2\omega$$

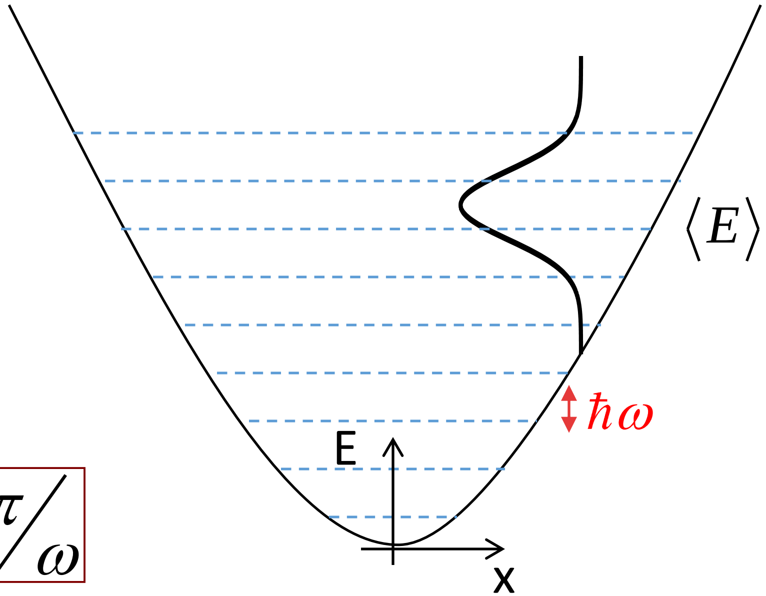
Glauber (coherent) state

$$|a\rangle = e^{-\frac{|a|^2}{2}} \sum_{n=0}^{\infty} \frac{a^n}{\sqrt{n!}} |n\rangle$$



What’s a Coherent State?



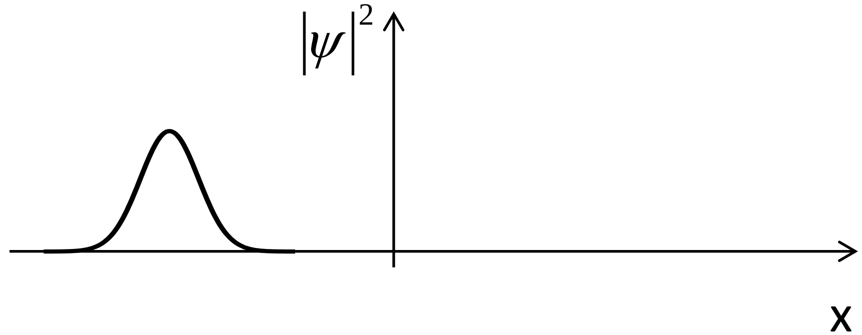


$$\langle E \rangle = |\alpha|^2 \hbar\omega$$

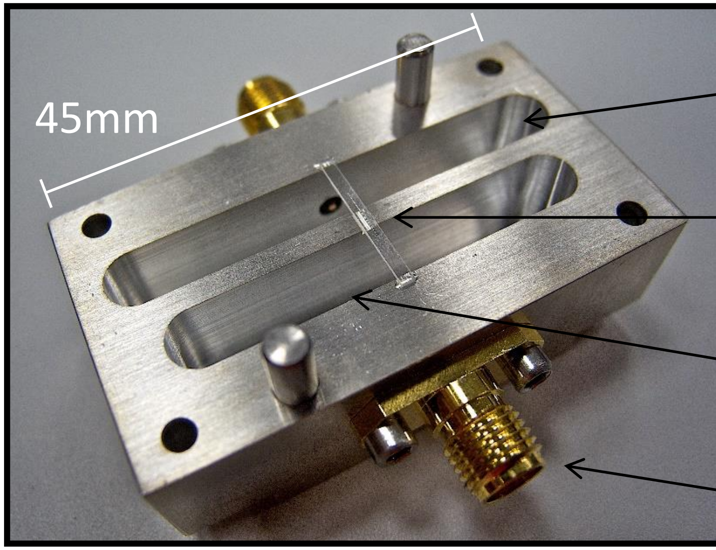
$$t = \pi / \omega$$

Glauber (coherent) state

$$|a\rangle = e^{-\frac{|a|^2}{2}} \sum_{n=0}^{\infty} \frac{a^n}{\sqrt{n!}} |n\rangle$$

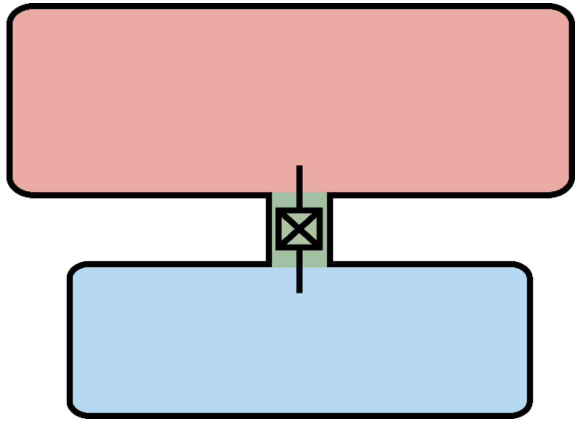


Two-cavity 3D architecture



One **fast probe cavity** for qubit state detection and one **long-lived cavity** for photon manipulation/storage

Quantizing Three Modes



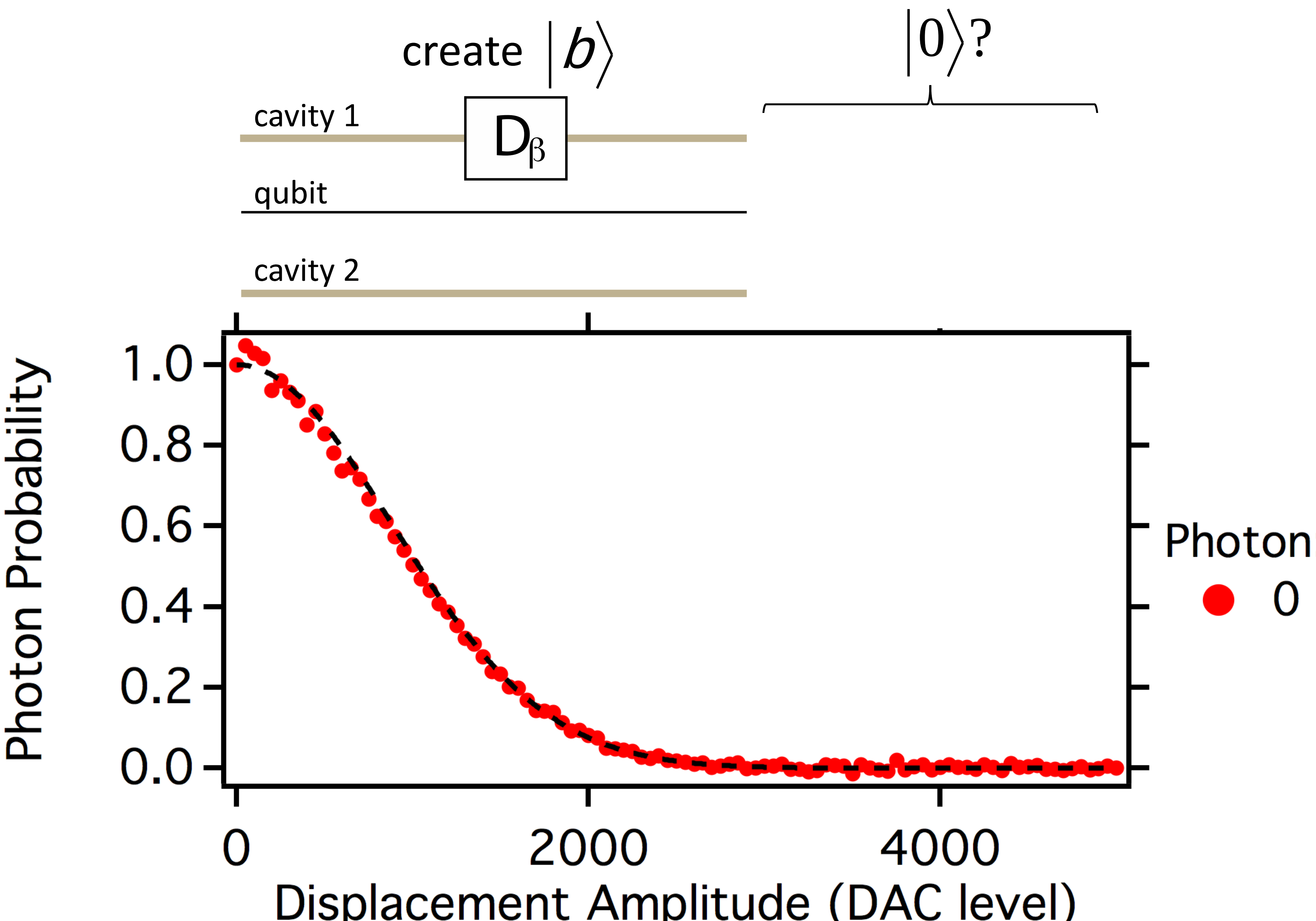
$$H = \hbar \frac{W_q}{2} s_z + \hbar W_1 a_1^\dagger a_1 + \hbar W_2 a_2^\dagger a_2 +$$

$$- \hbar \frac{c_1}{2} a_1^\dagger a_1 s_z - \hbar \frac{c_2}{2} a_2^\dagger a_2 s_z$$

Qubit cavity interactions $\sim \frac{g^2}{\Delta^2} \alpha$

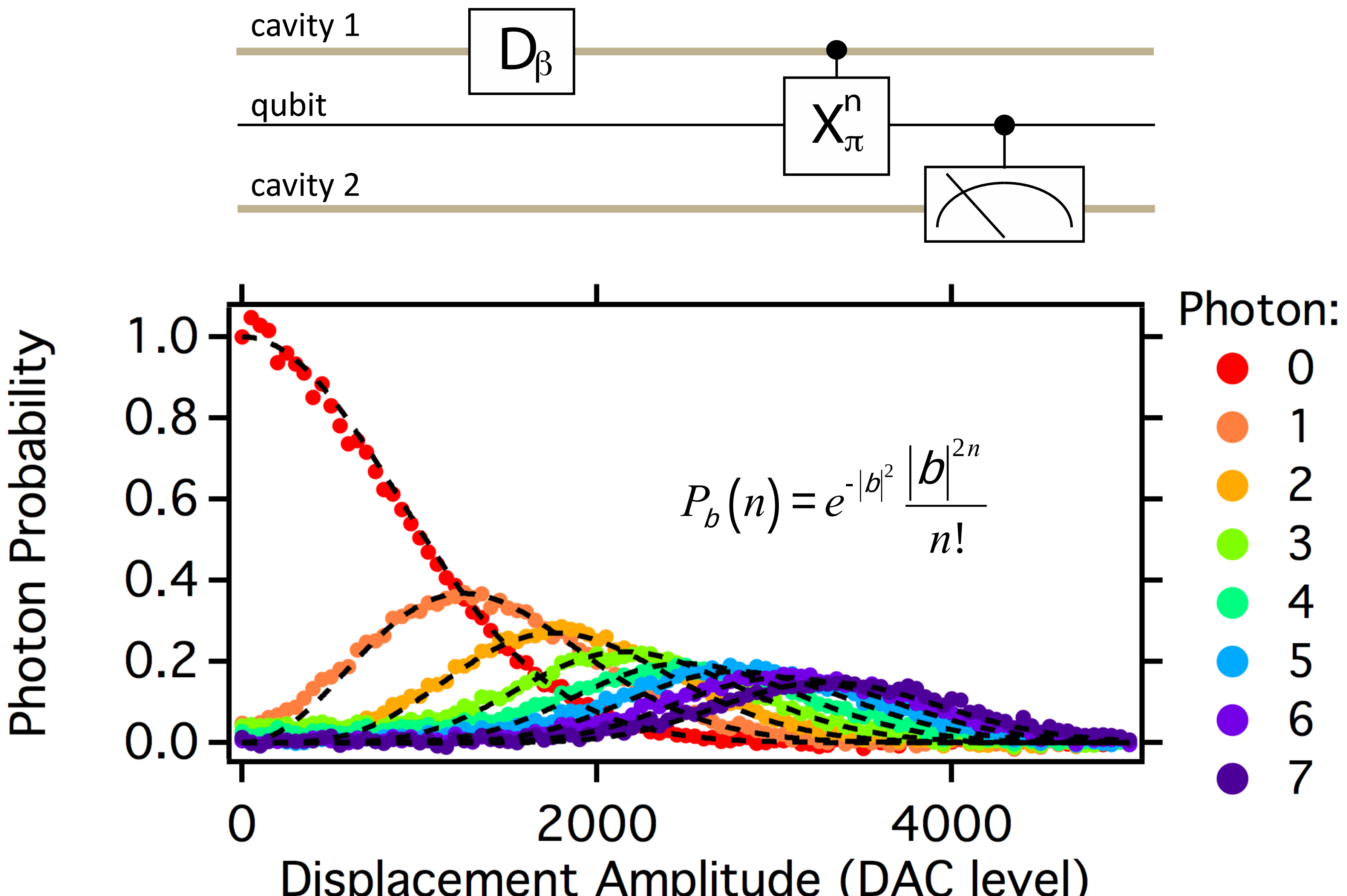
Nigg et.al. PRL 240502 (2012)

Coherent Displacements



Johnson B.R. et al. Nature Phys. 6, 663-667 2010

Coherent Displacements



Measuring the Husimi-Q function

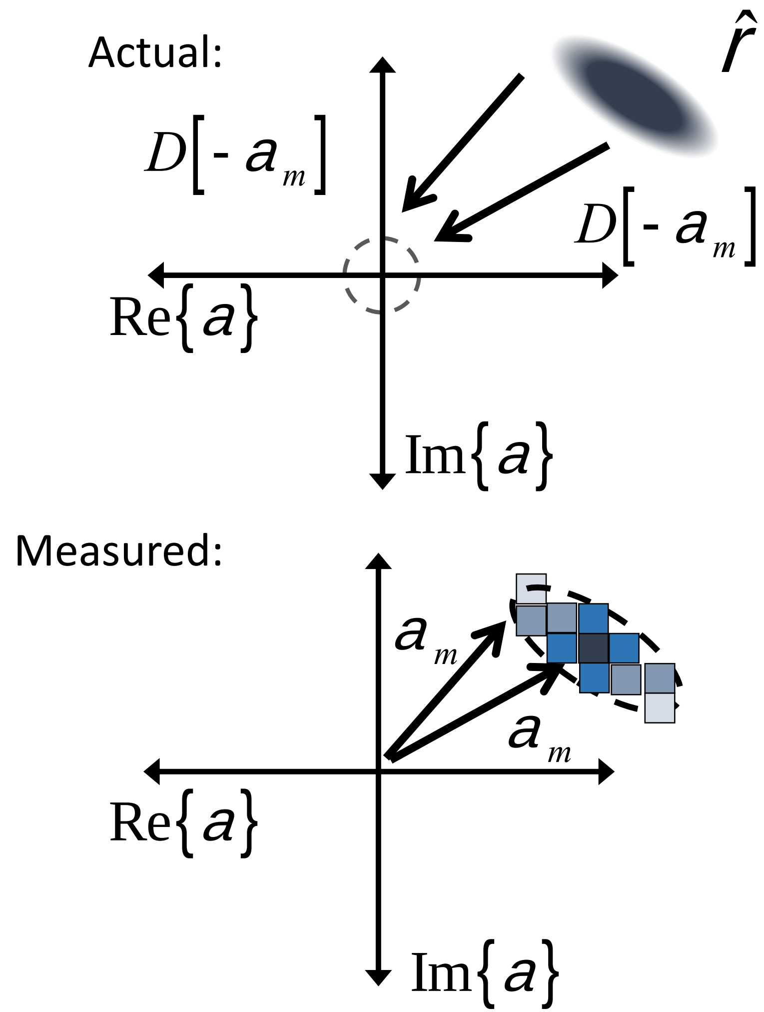
Husimi Q

$$Q(\alpha) = \frac{1}{\pi}\langle \alpha \,|\, \rho \,|\, \alpha \rangle$$

⬇

$$Q(\alpha) = \frac{1}{\pi}\langle 0 \,|\, D(-\alpha)\,\rho\, D(\alpha) \,|\, 0 \rangle$$

- Prepare state
- Displace state by –α
- Detect probability of finding zero photons

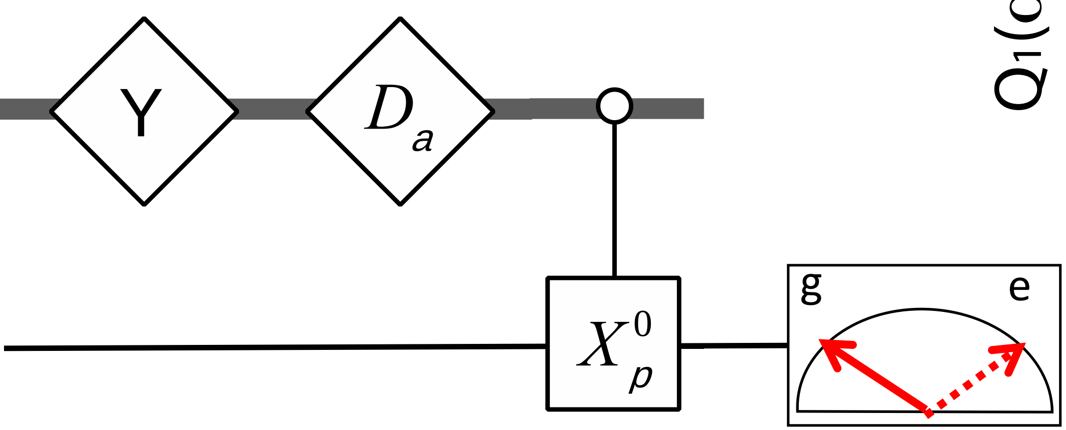


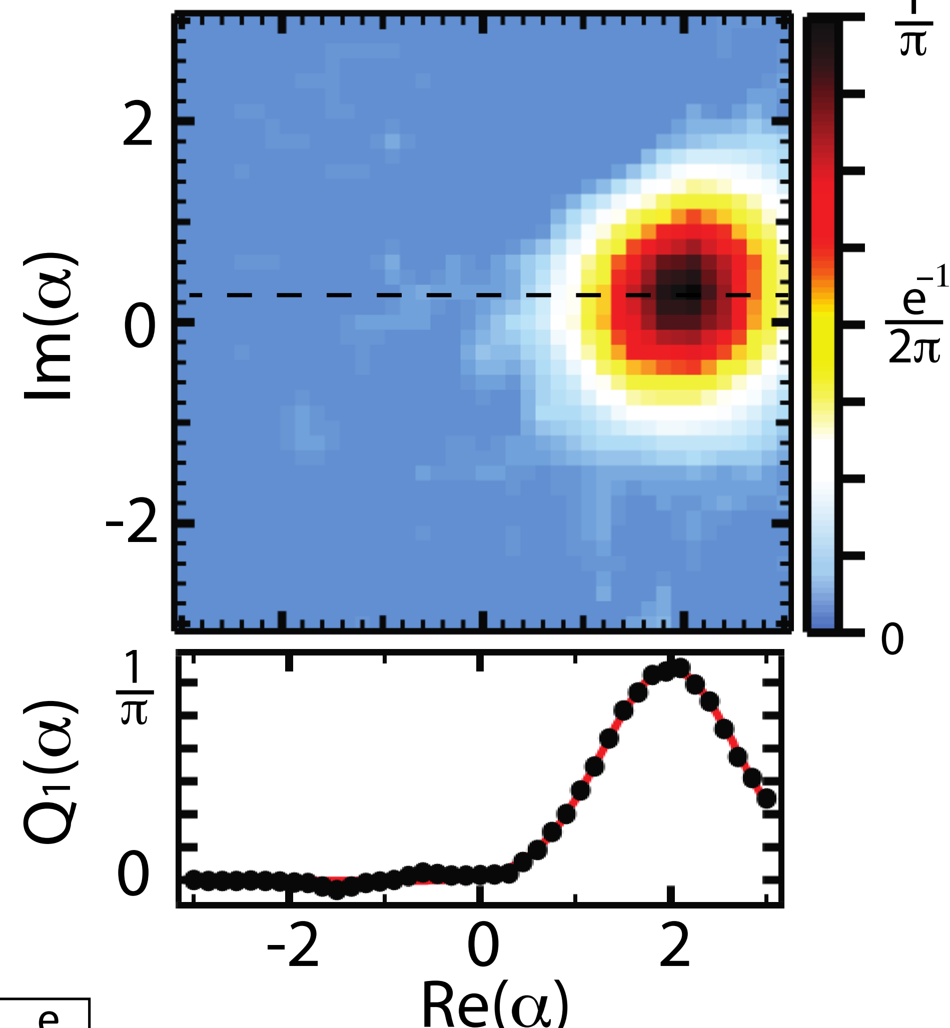
Measure the Quantum State of a resonator

Husimi Q:

$$Q(\alpha)=\frac{1}{\pi}\langle 0|D(-\alpha)\rho D(\alpha)|0\rangle$$

$$Q(\alpha,\beta)=\frac{1}{\pi}e^{-|\alpha-\beta|^2}$$

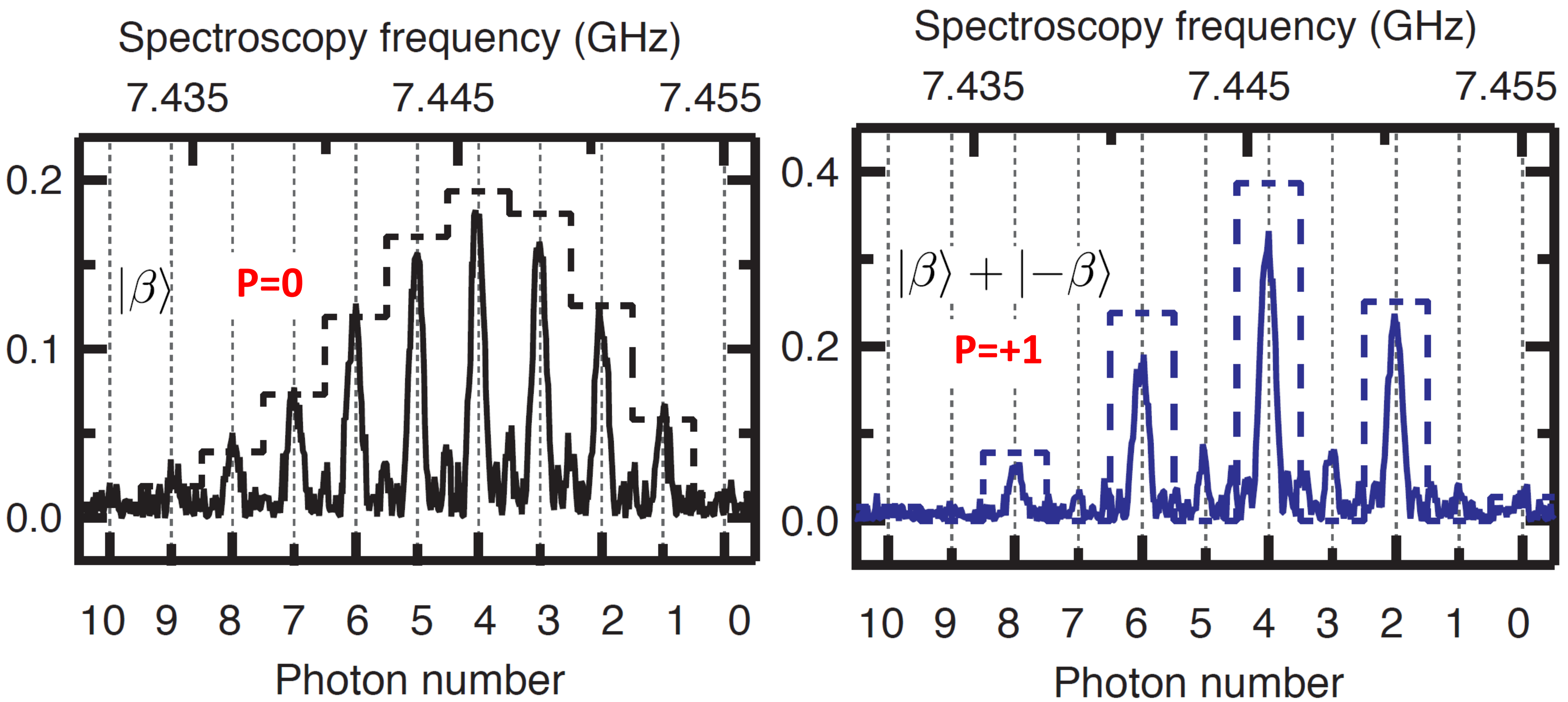




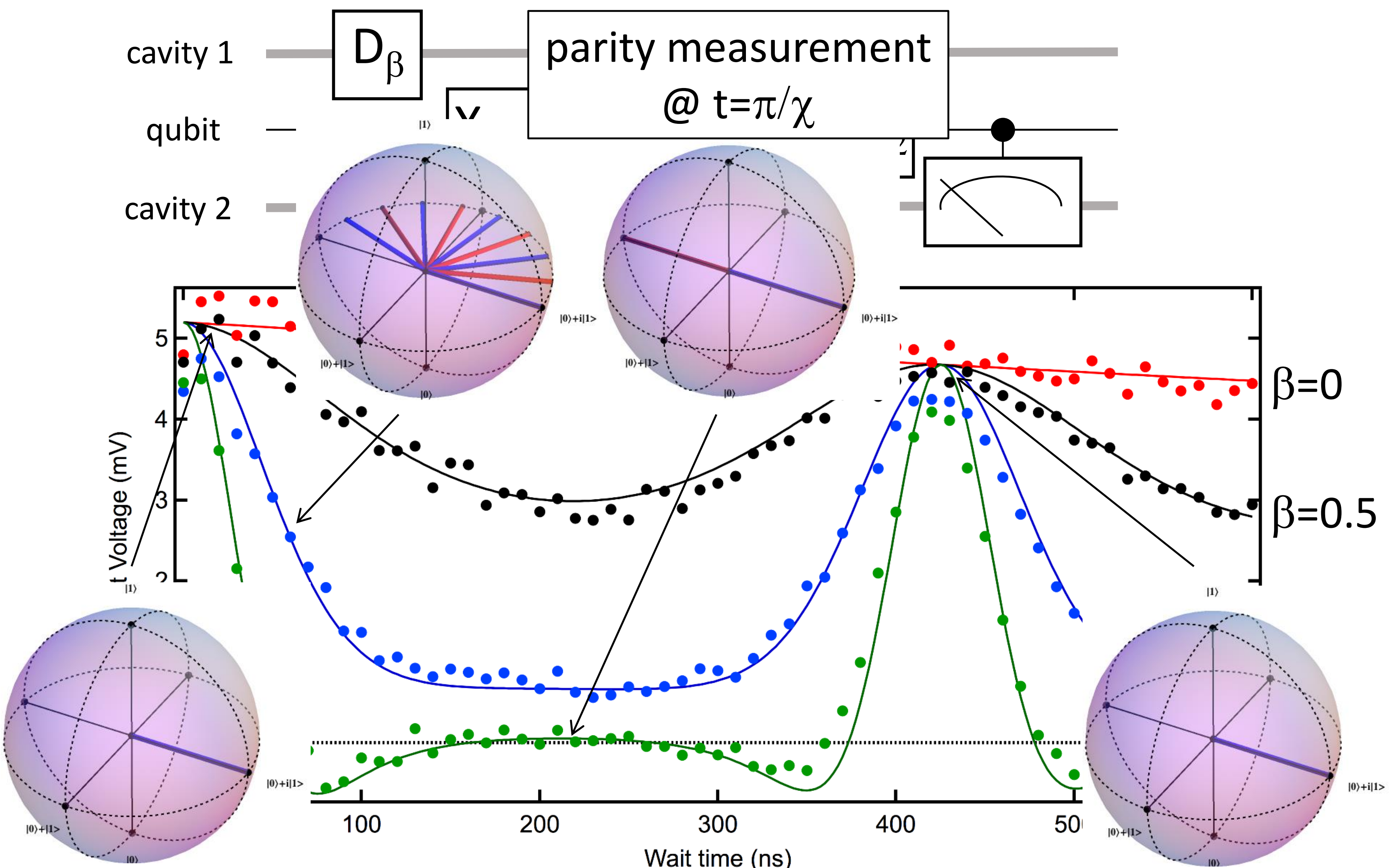
Measure the Quantum State of a resonator

Wigner function: $W(a_m) = \frac{2}{p} Tr\{r' P\}$ $\qquad r' = D(-b_m) r D(b_m)$

$$P = e^{ipa^\dagger a} = (-1)^n \rightarrow P = \sum_n p_n (-1)^n$$



Measuring parity



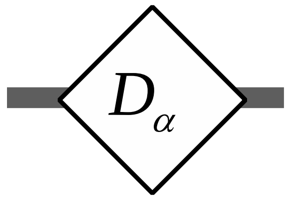
Schrödinger cats on demand - Mapping a qubit onto coherent states

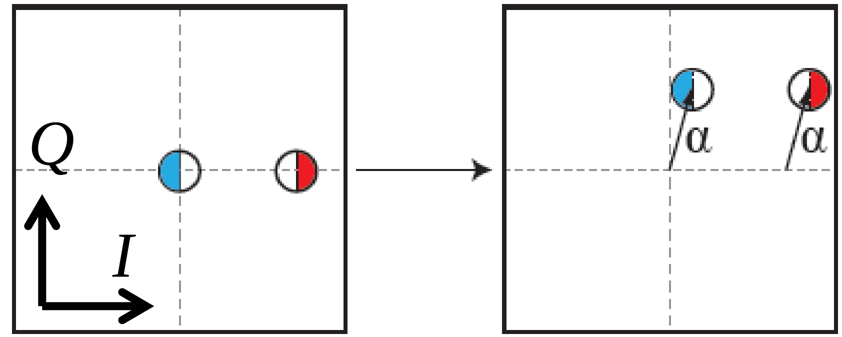
Science 342, 6158 (2013) ,
PRA 87, 042315 (2012)

Institute for Quantum Optics and Quantum Information, Innsbruck
Institute for Experimental Physics, University of Innsbruck

A New Toolbox for Manipulating Light

Cavity displacement

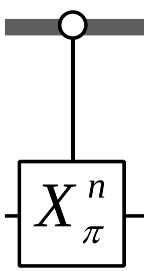




Conditional qubit rotation

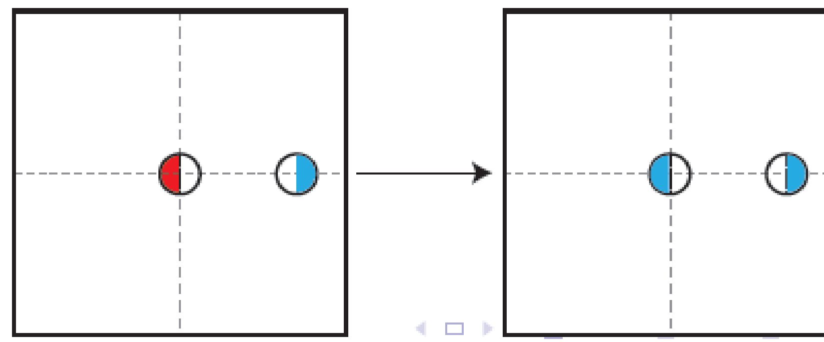
cavity

qubit



$$H = \xi(t)\sigma_+ e^{i(\omega_q + n\chi)t} + h.c.$$

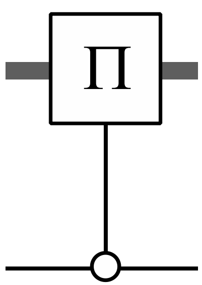
"flip qubit
IFF $|\psi_{\text{cav}}\rangle = |n\rangle$"



Conditional cavity phase

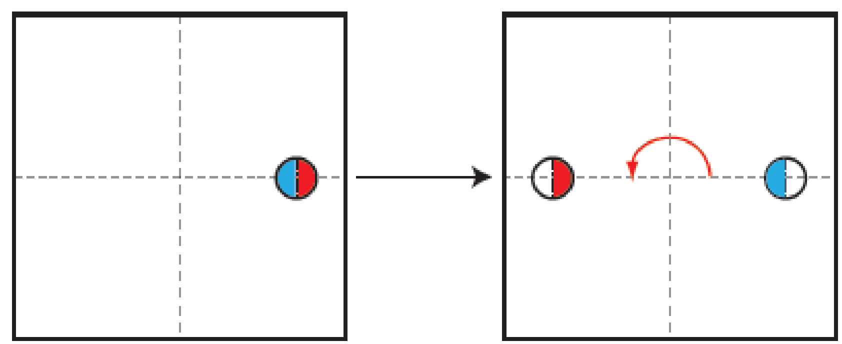
cavity

qubit

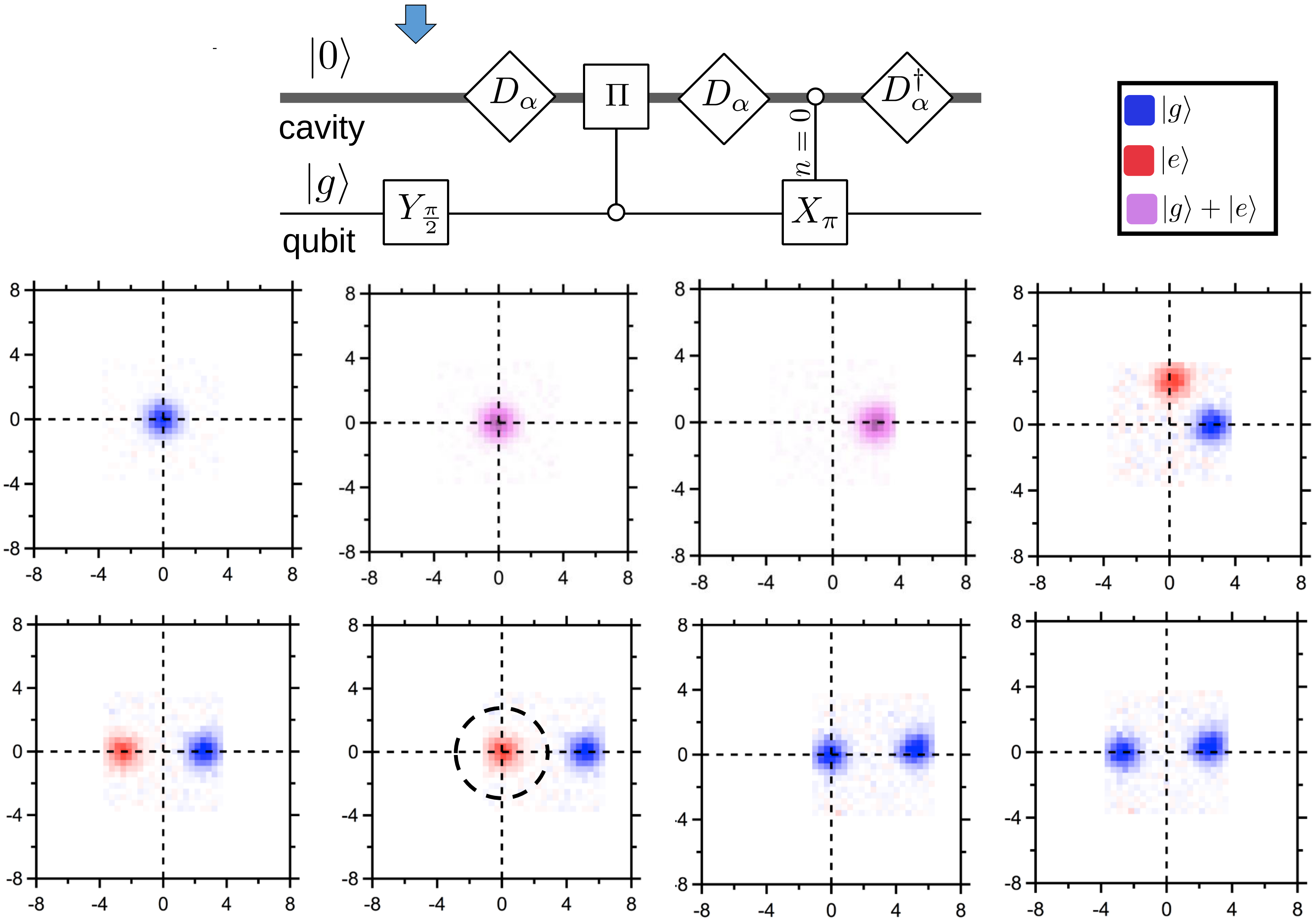


$$H = \chi a^\dagger a \sigma_z$$

"shift cavity
IFF qubit excited"



Deterministic cat creation

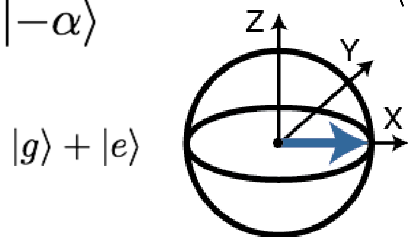


Mapping a qubit to a cavity

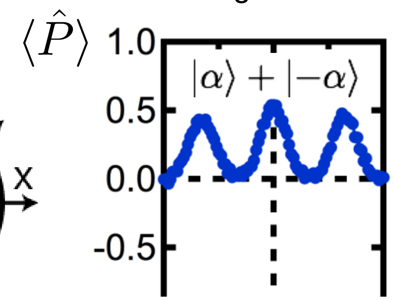
Qubit state: Cavity state:

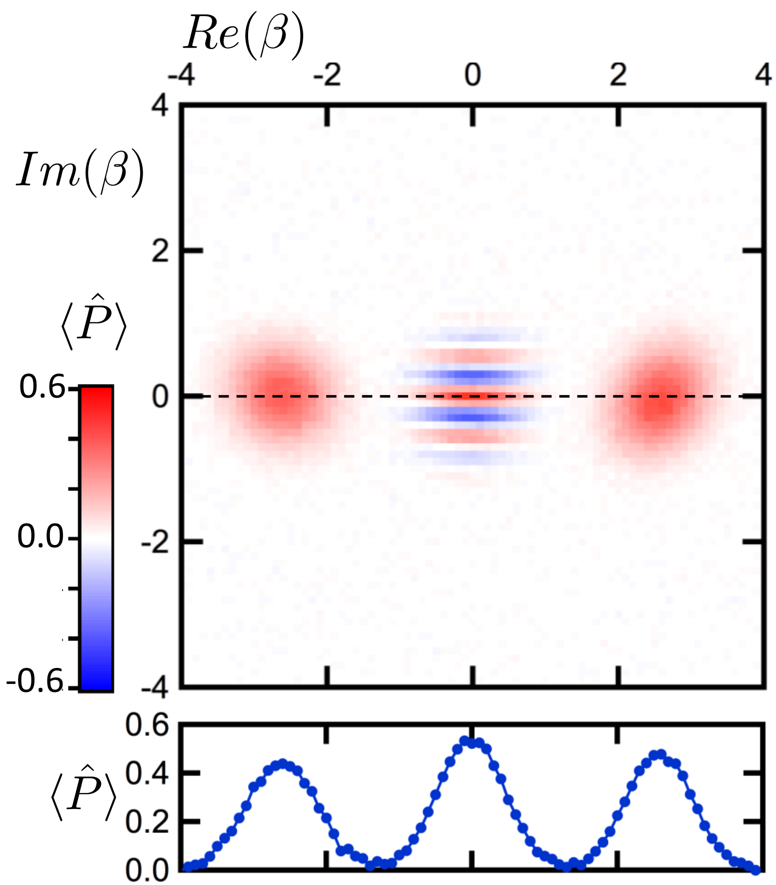
$$\cos(\tfrac{\theta}{2})\,|g\rangle + \sin(\tfrac{\theta}{2})e^{i\phi}\,|e\rangle \Rightarrow \cos(\tfrac{\theta}{2})\,|\alpha\rangle + \sin(\tfrac{\theta}{2})e^{i\phi}\,|-\alpha\rangle$$

Initial qubit state:



Wigner cut:





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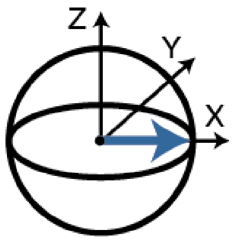
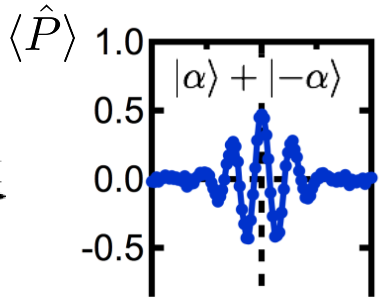
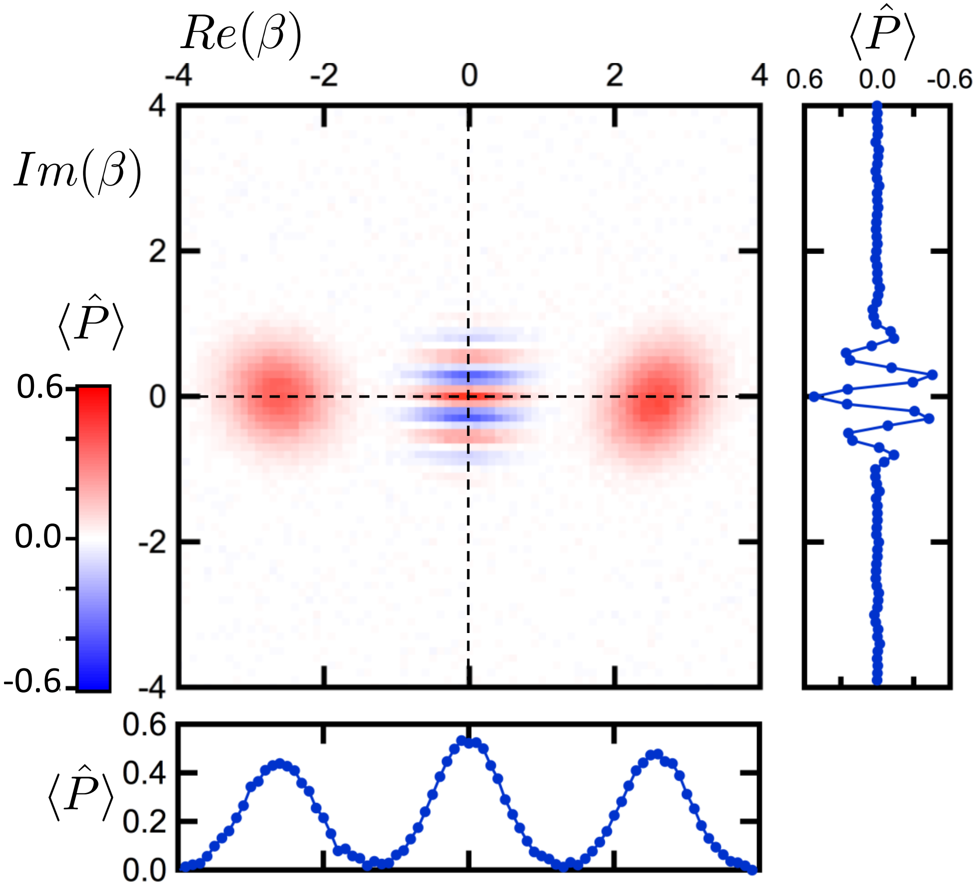
Initial qubit state:

$|g\rangle + |e\rangle$

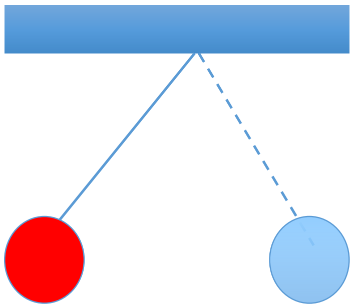


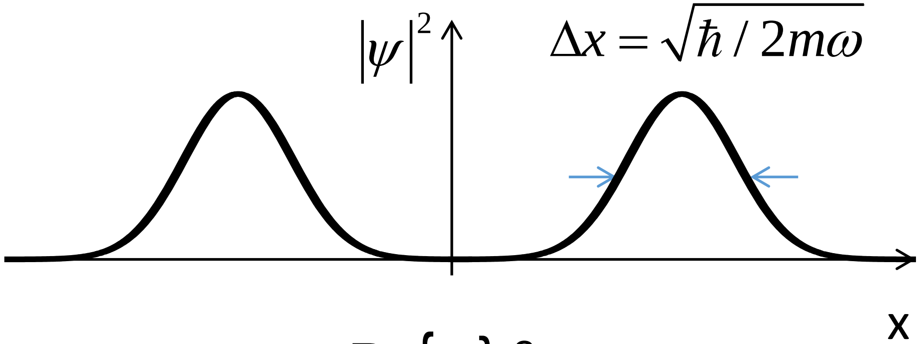
Wigner cut:





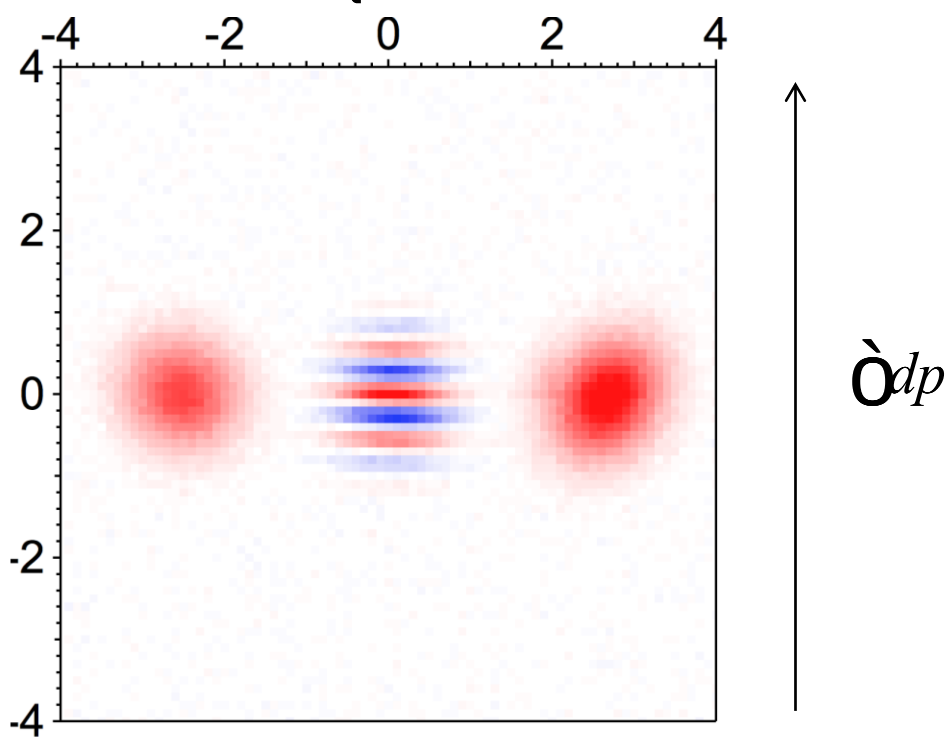
Wigner Function: interpretation





$\mathrm{Re}\{\alpha\} \equiv x$

$\mathrm{Im}\{\alpha\} \equiv p$



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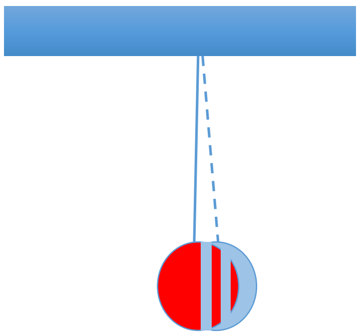
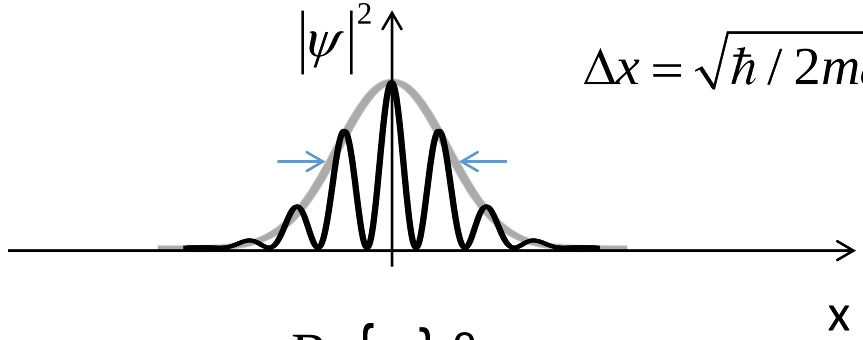


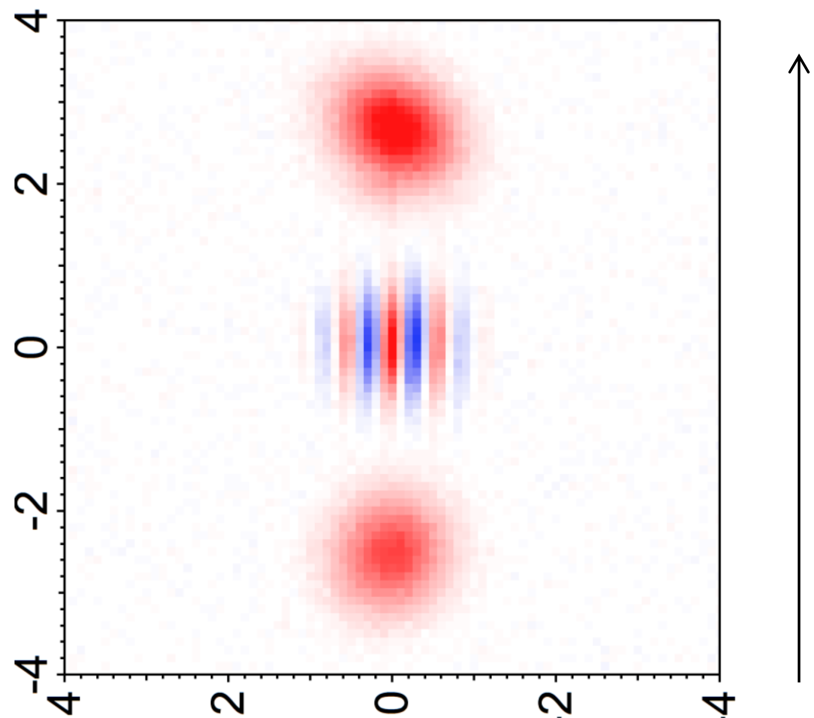


$$\Delta x = \sqrt{\hbar / 2m\omega}$$

$\mathrm{Re}\{\alpha\} \propto x$

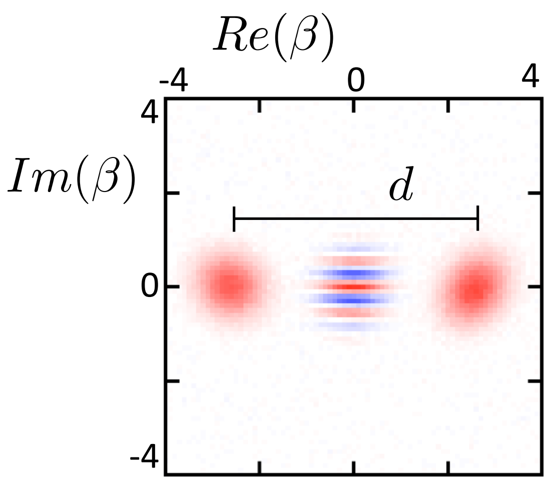
$$t = \pi/2\omega$$

$\mathrm{Im}\{\alpha\} \propto p$



$\int dp$

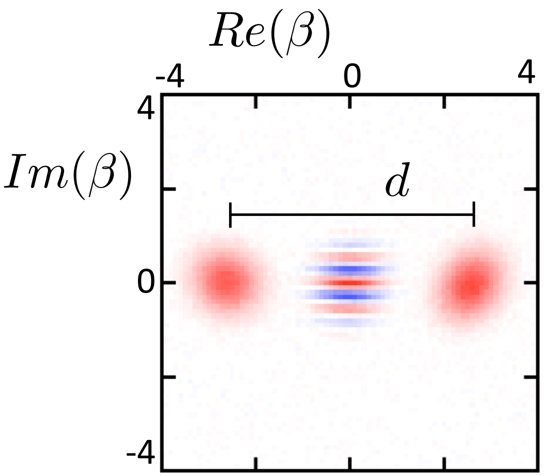
Scaling cat state 'size'



susceptibility to decoherence: $\tau_{\text{eff}} \propto \frac{1}{d^2}\tau_{\text{cav}}$

orthogonality of coherent states: $\langle -\alpha|\alpha\rangle = e^{-d^2}$

Scaling cat state 'size'



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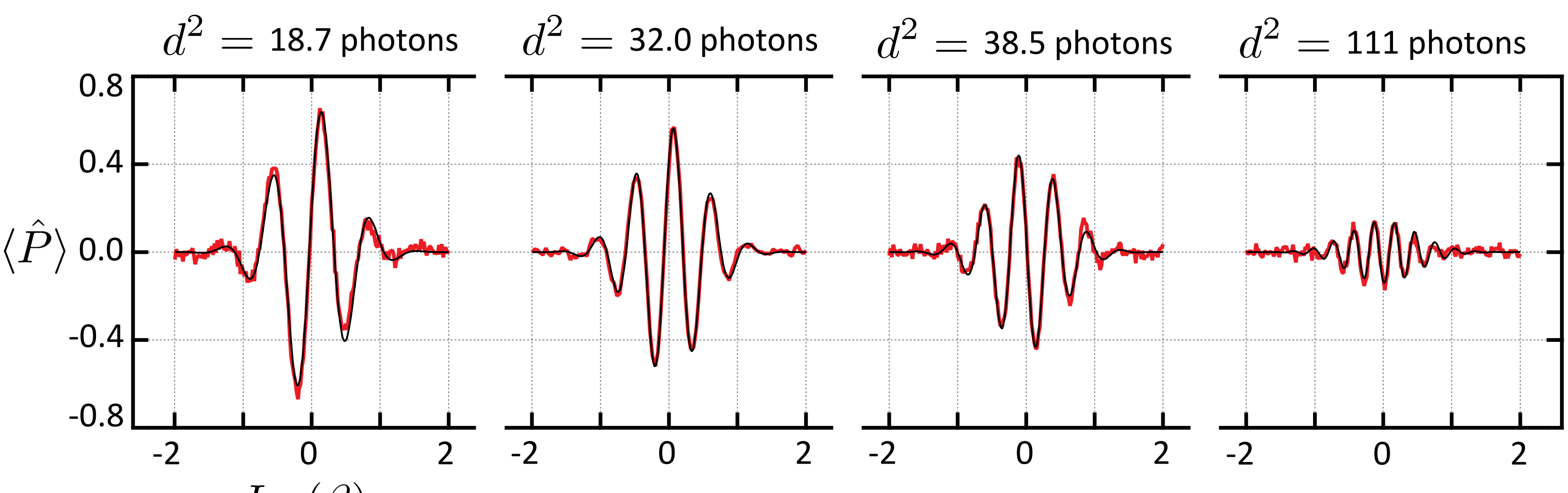
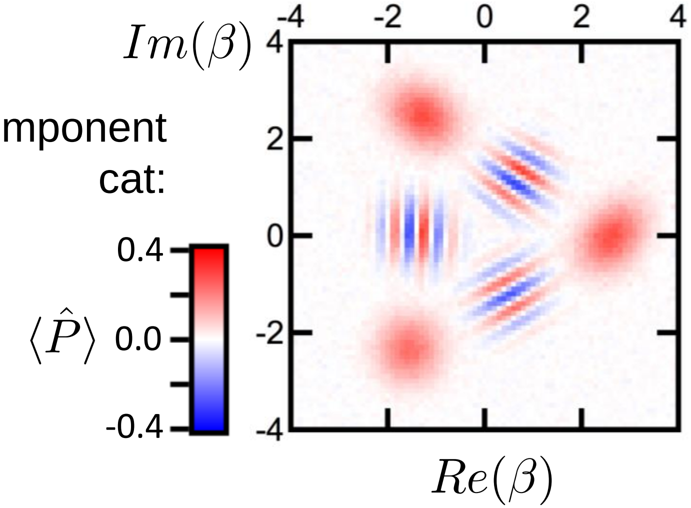


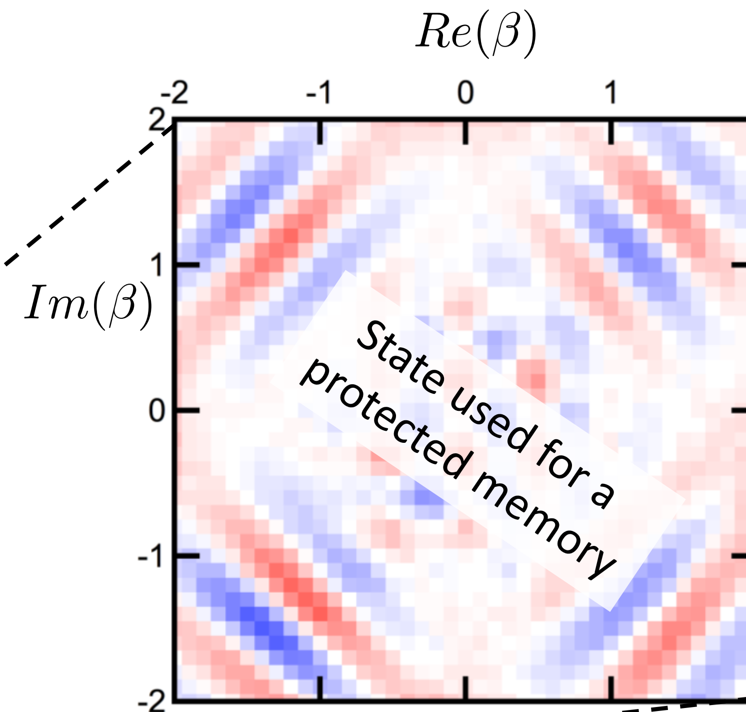
d^2 determined by fringe frequency

Multi-component cat states

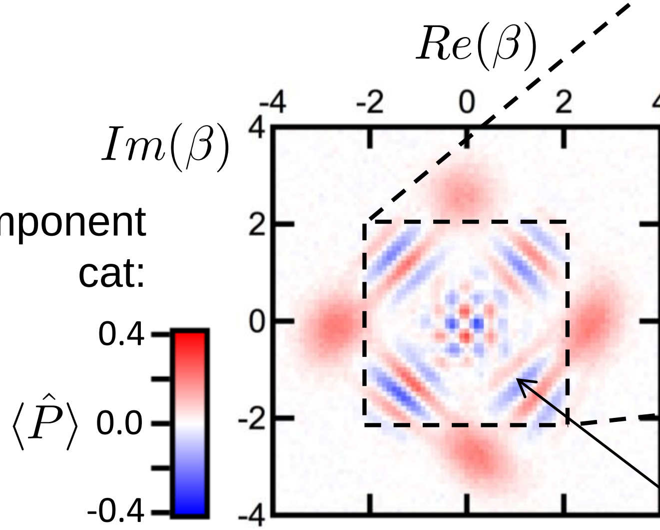
Three-component cat:



$$\Psi_3 = \frac{1}{\sqrt{3}}\left(|\alpha\rangle + |e^{i\pi/3}\alpha\rangle + |e^{-i\pi/3}\alpha\rangle\right)$$



Four-component cat:



$$\Psi_4 = \frac{1}{2}\left(|\alpha\rangle + |-\alpha\rangle + |i\alpha\rangle + |-i\alpha\rangle\right)$$

The four component 'compass state'

Error Correction using a four component Cat state

Institute for Quantum Optics and Quantum Information, Innsbruck
Institute for Experimental Physics, University of Innsbruck

Error Correction using a Cat state

Leghtas, Mirrahimi, et al., Phys. Rev. Lett. 111, 120501 (2013),

- Correction for a single bit / phase flip: at least 5 qubits
- A single cavity mode: infinite dimensional Hilbert Space
- Minimal QEC hardware: a single cavity mode coupled to a qubit

Idea: encode a qubit in a 4 component parity state

also known as a Zurek "compass state"

$$c_g|\downarrow\rangle + c_e|\uparrow\rangle \Rightarrow c_g|0_L\rangle + c_e|1_L\rangle$$

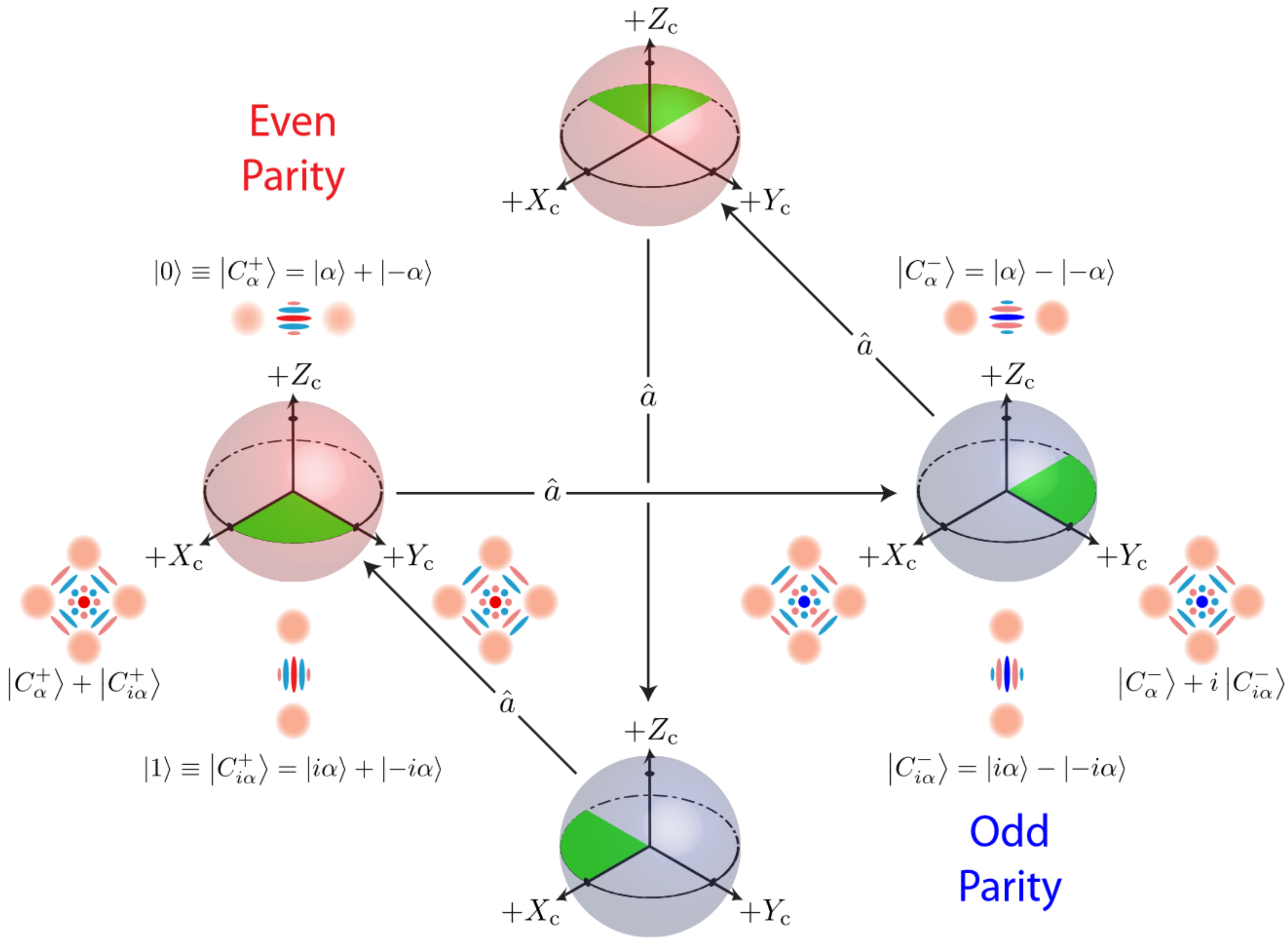
$$|0_L\rangle = |\mathcal{C}_\alpha^+\rangle = \mathcal{N}(|\alpha\rangle + |-\alpha\rangle)$$

$$|1_L\rangle = |\mathcal{C}_{i\alpha}^+\rangle = \mathcal{N}(|i\alpha\rangle + |-i\alpha\rangle)$$

Then photon loss can be monitored/corrected by repeated photon parity measurement using a qubit

Cat code cylce

reminder: $a(|\alpha\rangle + |-\alpha\rangle) = |\alpha\rangle - |-\alpha\rangle)$



Ofek et.al. Nature 536, 441, 2016

Why Cats Are Cool for QEC…

Conventional bit flip repetition code:

Redundantly encode qubit in "fragile" GHZ state

$$c_g\left|\downarrow\right\rangle + c_e\left|\uparrow\right\rangle \Rightarrow c_g\left|\downarrow\downarrow\downarrow\right\rangle + c_e\left|\uparrow\uparrow\uparrow\right\rangle$$

Detect 4 error syndromes with 2 parity checks: $\langle Z_1 Z_2\rangle, \langle Z_2 Z_3\rangle$

Pro: now a single decay doesn't erase all quantum info

Con: but decays happen 3x faster…

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Detect 4 error syndromes with 2 parity checks: $\left\langle Z_1 Z_2 \right\rangle, \left\langle Z_2 Z_3 \right\rangle$

Pro: now a single decay doesn't erase all quantum info
Con: but decays happen 3x faster…

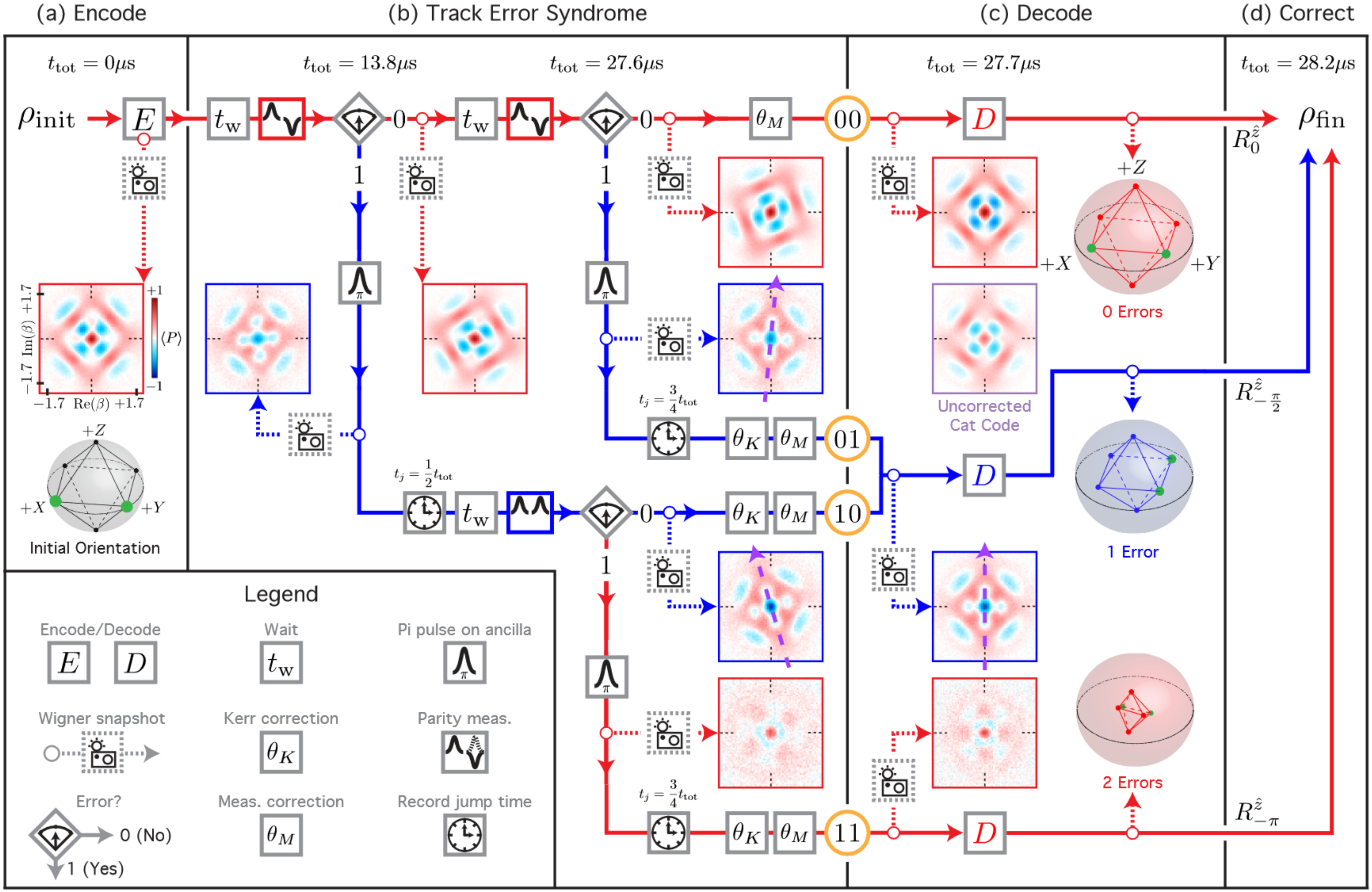
Qubit encoding in parity "cat" of an oscillator?:

Single mode redundantly encodes!

$$\hat{a}^4\left(c_g\left|0_L\right\rangle + c_e\left|1_L\right\rangle\right) = c_g\left|0_L\right\rangle + c_e\left|1_L\right\rangle$$

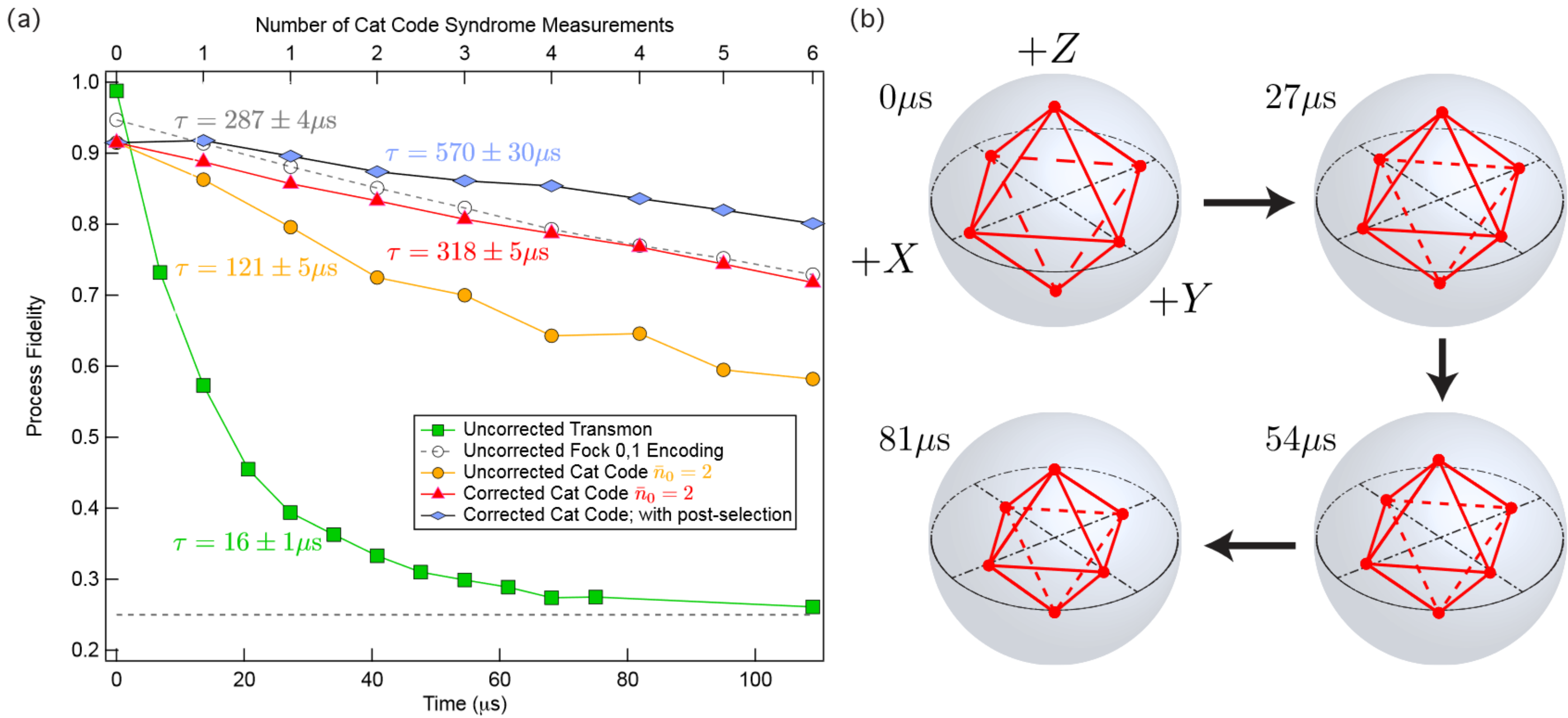
Pro: photon decay doesn't erase all quantum info
Pro: only one error syndrome to measure!
Pro: only one object to build per logical qubit!
Pro: cavity coherence can be much longer
Con: decays also happen <n> times faster

Experimental implementation



Ofek et.al. Nature 536, 441, 2016

Did it work??



Ofek et.al. Nature 536, 441, 2016