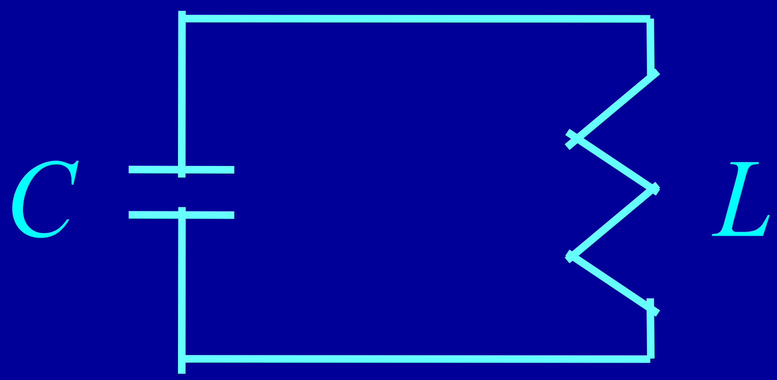
Microwave optomechanics

Yaroslav M. Blanter

Kavli Institute of Nanoscience, Delft University of Technology

- Microwave cavities
- Optomechanical coupling
- Optomechanically induced transparency
- Quantum states

Microwave cavities



V – voltage
I – current
Q – charge
Φ – flux

$$Q = CV \Rightarrow \dot{Q} = I = C\dot{V}$$

$$\Phi = LI \Rightarrow -\dot{\Phi} = V = -L\dot{I}$$

$$I + CL\ddot{I} = 0$$

Harmonic oscillator with the frequency $\omega = \frac{1}{\sqrt{LC}}$

Frequency in the microwave range: 1 GHz to 10 GHz
(actually, even more narrow)

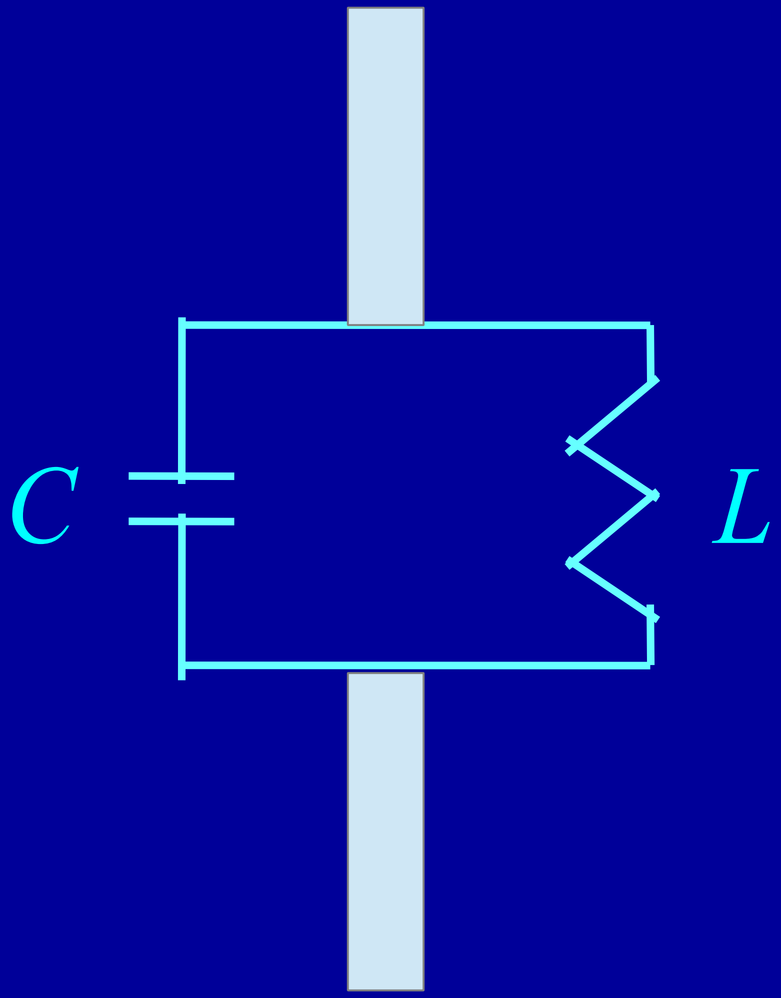
Microwave cavities

Harmonic oscillator with the frequency $\omega = \frac{1}{\sqrt{LC}}$

$$f = \frac{\omega}{2\pi} = 5\ \mathrm{GHz}$$

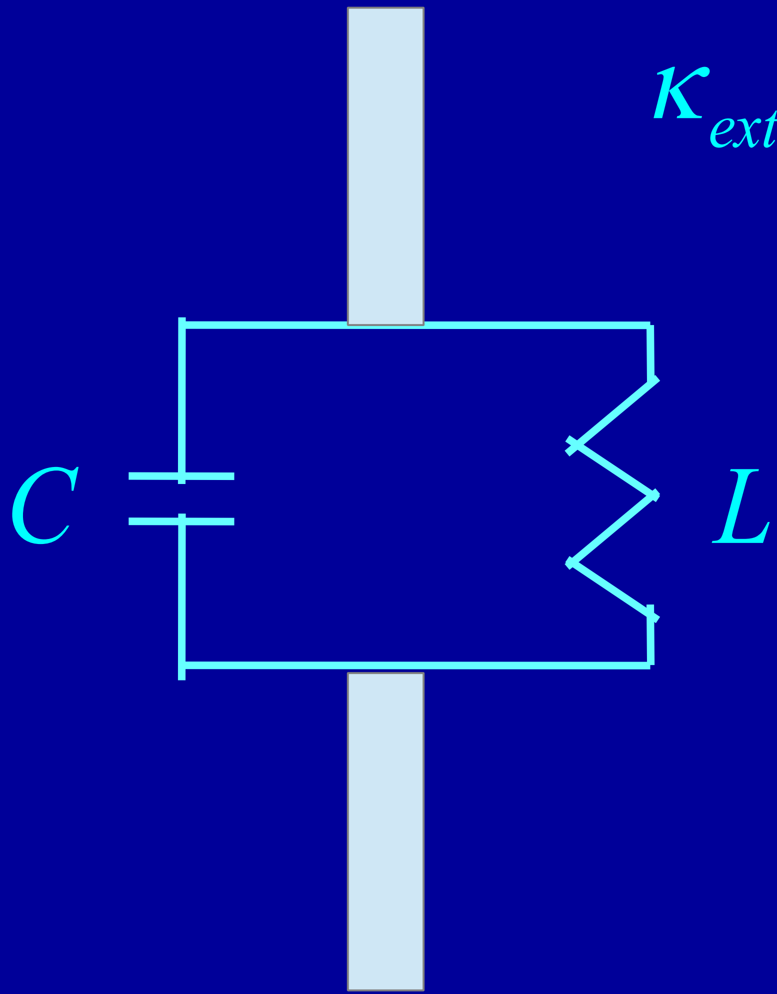
$\hbar\omega = k_B T$ gives $T = 30\ \mathrm{mK}$

Reflection/Transmission



Adding a waveguide

Losses

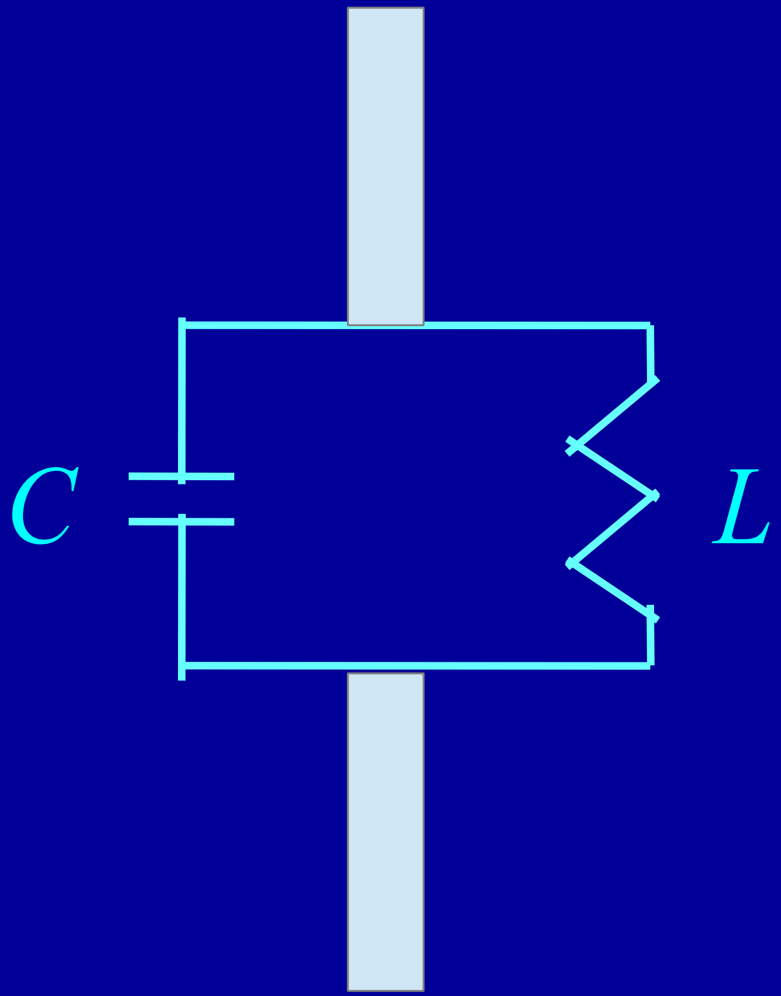


κ_{ext} External

κ_{in} Internal (resistance)

$$\kappa = \kappa_{in} + \kappa_{ext}$$

Quantization



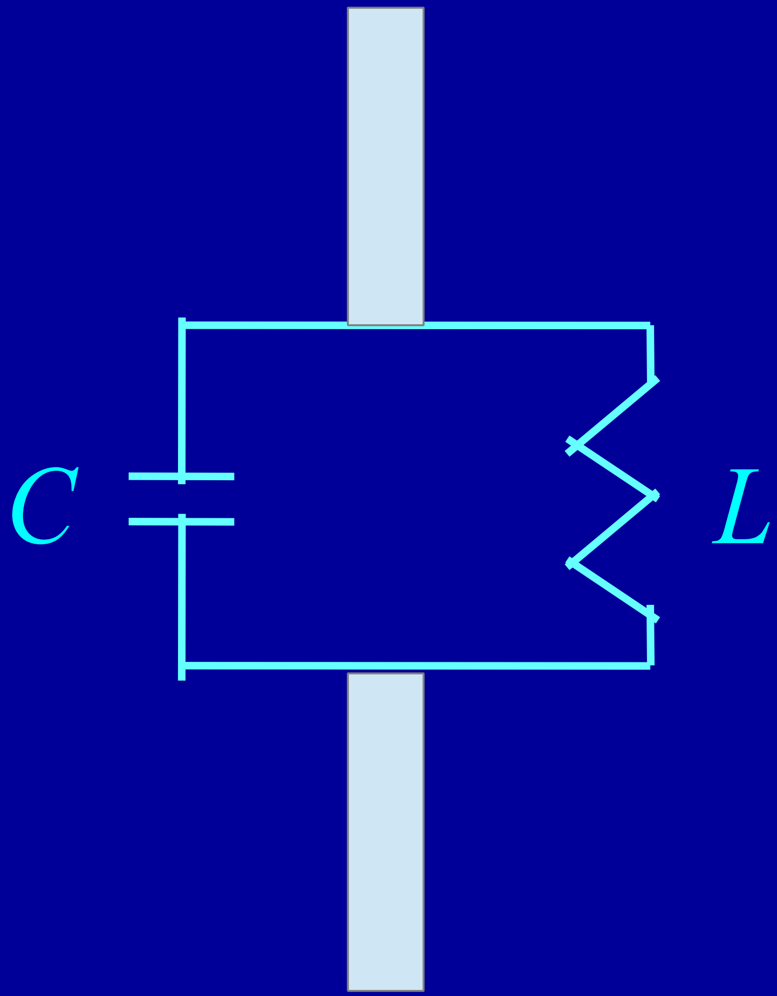
Energy: $$W = \frac{CV^2}{2} + \frac{LI^2}{2}$$

$$V = \sqrt{\frac{\hbar\omega}{2C}}\left(\hat{a} + \hat{a}^\dagger\right)$$

$$I = i\sqrt{\frac{\hbar\omega}{2L}}\left(\hat{a} - \hat{a}^\dagger\right)$$

$$\left[\hat{a}, \hat{a}^\dagger\right] = 1$$

Quantization



Energy: $$W = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)$$

Also reproduces the equations of motion

Capacitive coupling

Energy: $W = \frac{C(x)V^2}{2}$ $\quad C(x) = C_0 + \frac{dC}{dx}x$

Coupling: $H_{\text{int}} = \frac{dC}{dx}\frac{xV^2}{2}$

$$H_{\text{int}} = g\left(\hat{a} + \hat{a}^\dagger\right)^2\left(\hat{b} + \hat{b}^\dagger\right)$$

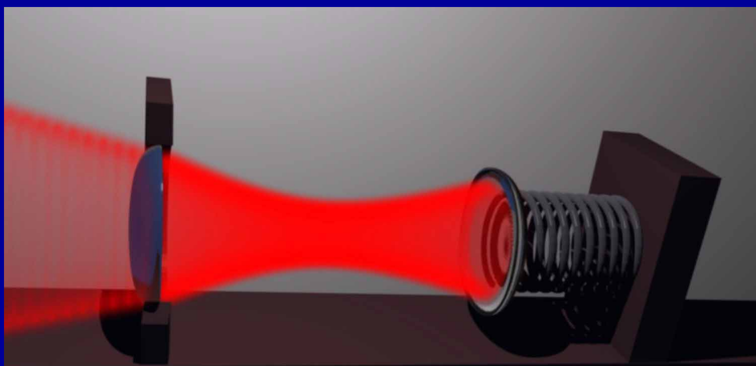
Resonant terms: Radiation pressure

$$H_{\text{int}} = g\hat{a}^\dagger\hat{a}\left(\hat{b} + \hat{b}^\dagger\right)$$

Radiation pressure in optomechanics

Movable mirror

Static mirror



Kippenberg's group website

Radiation pressure coupling

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g_{0}\hat{a}^{\dagger}\hat{a}(b^{\dagger} + b) \quad \omega_{cav}(x)$$

Cavity

Mechanical resonator

$$\hat{x} = x_{ZPM}\left(\hat{b} + \hat{b}^{\dagger}\right)$$

Coupling

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} - \hbar g_0 \hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

Dissipation rate in the cavity

Sideband-resolved regime

$$\Gamma, \kappa \ll \omega_m \ll \omega_{cav}$$

Where is g_0?

Weak coupling ← → Strong coupling

Driving and linearization: $g = g_0\sqrt{n_{cav}}$

Coupling

$$H_{\text{int}} = -\hbar g_0 \hat{a}^\dagger \hat{a}(b^\dagger + b) \to -\hbar g(\hat{a}^\dagger + \hat{a})(b^\dagger + b)$$

Non-resonant? Depends how we drive. $g = g_0\sqrt{n_{cav}}$

In the rotating frame: $\sqrt{n_{cav}} \propto e^{i\omega_d t}; a \propto e^{i\omega_{cav} t}; b \propto e^{i\omega_m t}$

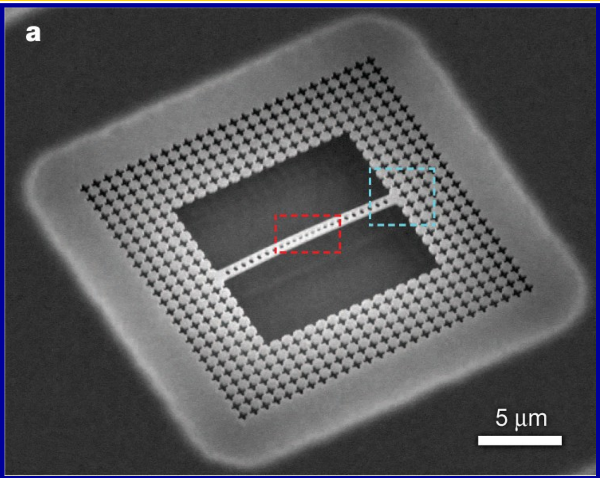
Red-detuned drive: $\omega_d = \omega_{cav} - \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b + \hat{a} b^\dagger)$$

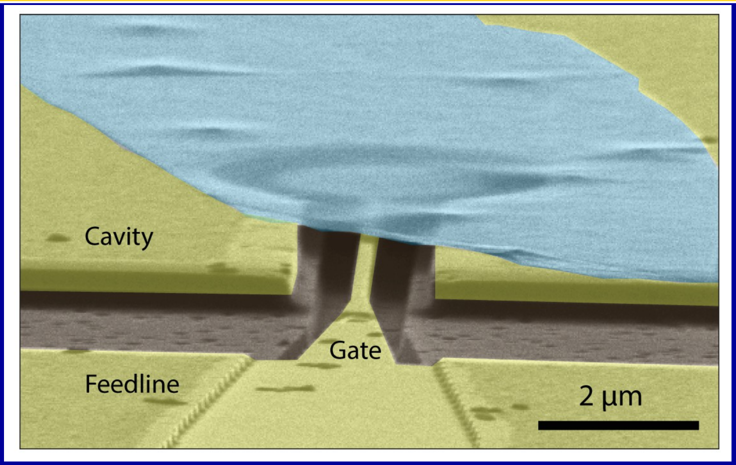
Blue-detuned drive: $\omega_d = \omega_{cav} + \omega_m$

$$H_{\text{int}} = -\hbar g(\hat{a}^\dagger b^\dagger + \hat{a} b)$$

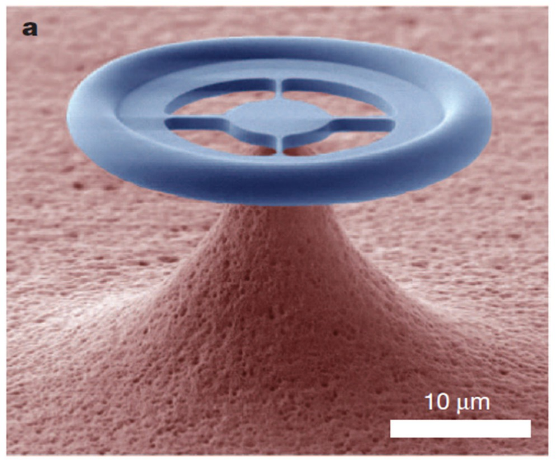
Cavity/circuit optomechanics



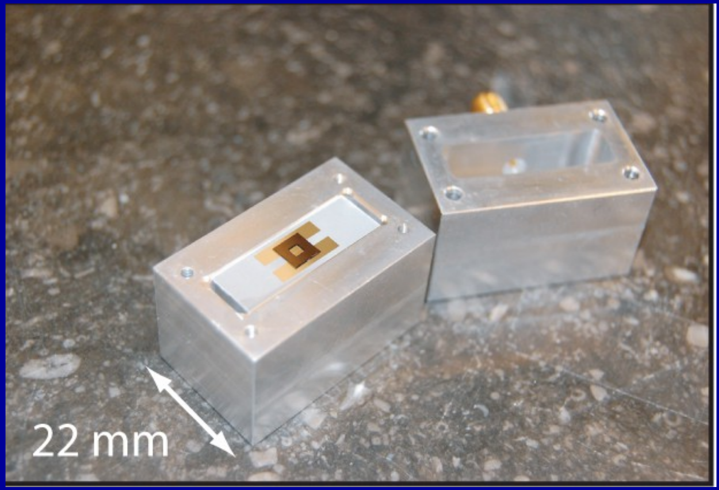
Chan et al, Nature **478**, 89 (2011)



Singh et al, Nature Nanotech. **9**, 820 (2014)



Verhagen et al, Nature **482**, 63 (2012)



Yuan et al, Nature Comms. 6, 8491 (2015)

Microwave optomechanics

What can we do with microwaves?

- The same things as with visible light
- The cavity frequency can be comparable to the mechanical frequency
- Non-linearity (via Josephson effect)

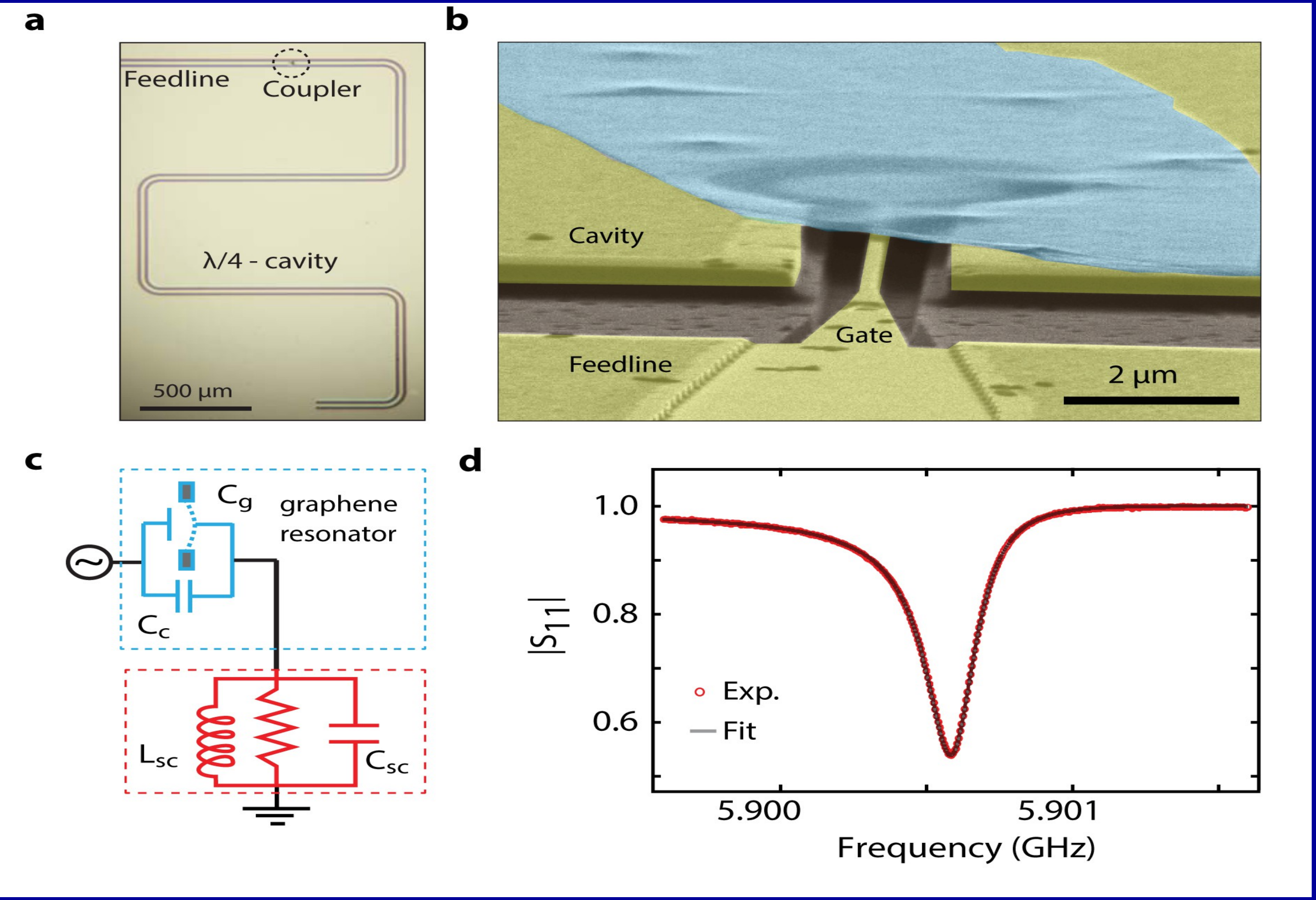
Static effects

If we only look at the mechanical resonator:
- Equilibrium position is shifted
- Frequency is renormalized
- Damping coefficient is renormalized
- Non-linearity appears and can lead to instabilities

Same with the cavity: frequency shift and renormalization of the damping

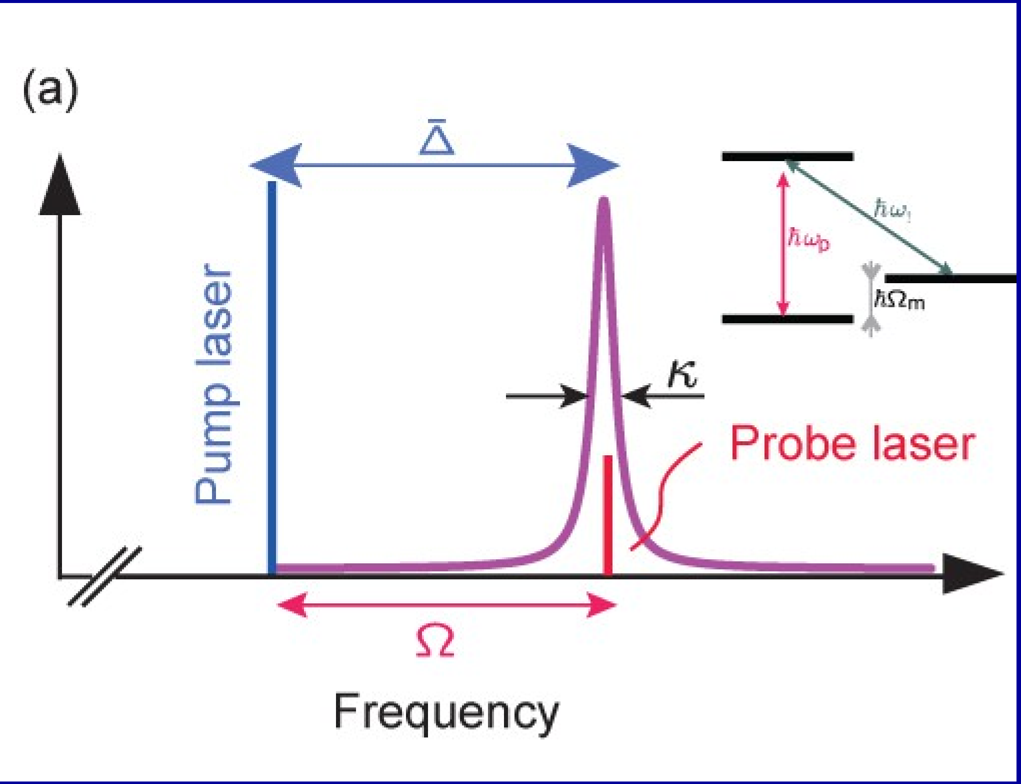
$$\kappa \to \kappa \pm 4g^2 / \Gamma$$

Superconducting microwave cavity



V. Singh, S. J. Bosman, B. H. Schneider, YMB, A. Castellanos-Gomez, and G. A. Steele, Nature Nanotech. **9**, 820 (2014)

Optomechanically induced transparency



From: Aspelmeyer, Kippenberg, and Marquardt Rev. Mod. Phys. **86** 1391 (2014)

Cavity is strongly red-driven at $\omega_{cav} - \omega_m$ (red-detuned)

Probe laser measures the transmission around the cavity resonance

Optomechanically induced transparency

Langevin equations for the creation/annihilation operators in the frame rotating with the drive:

$$\frac{d\hat{a}}{dt} = \left(i\Delta - \frac{\kappa}{2} \right) \hat{a} - iG\hat{x}\hat{a} + \sqrt{\kappa_{ext}} s_{in} + \sqrt{(1-\eta_c)\kappa} \delta \hat{s}_{vac}(t)$$

Detuning and dissipation in the cavity

Input signal

Quantum noise

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}$$

$$\eta_c = \frac{\kappa_{ext}}{\kappa}$$

$$\frac{d\hat{p}}{dt} = -m\omega_m{}^2 \hat{p} - \hbar G \hat{a}^\dagger \hat{a} - \Gamma_m \hat{p} + \delta F_{th}(t)$$

Coupling

Mechanical dissipation

Thermal noise

Optomechanically induced transparency

Transmission:

Drive

Probe

$$s_{in} = \bar{s}_{in} + s_P \exp\left(i\left(\omega_d - \omega_p\right)t\right) \equiv \bar{s}_{in} + s_P \exp\left(-i\Omega t\right)$$

$$\hat{a} = \bar{a} + A^{-} \exp\left(-i\Omega t\right) + A^{+} \exp\left(i\Omega t\right)$$

$$s_{out} = s_{in} - \sqrt{\eta_c \kappa}\,\hat{a}$$

$$t_p = \left(s_p - \sqrt{\eta_c \kappa} A^{-}\right) / s_p$$

S. Weis, R. Rivière, S. Deléglise, E. Gavartin, O. Arcizet, A. Schliesser, T. J. Kippenberg, Science **330**, 1520 (2010)

Optomechanically induced transparency

Result: Additional peak at the cavity resonance
Width: mechanical linewidth

$$\Gamma_{OMIT} = \Gamma_m + \frac{4g^2}{\kappa} = \Gamma_m (1 + C)$$

Height:

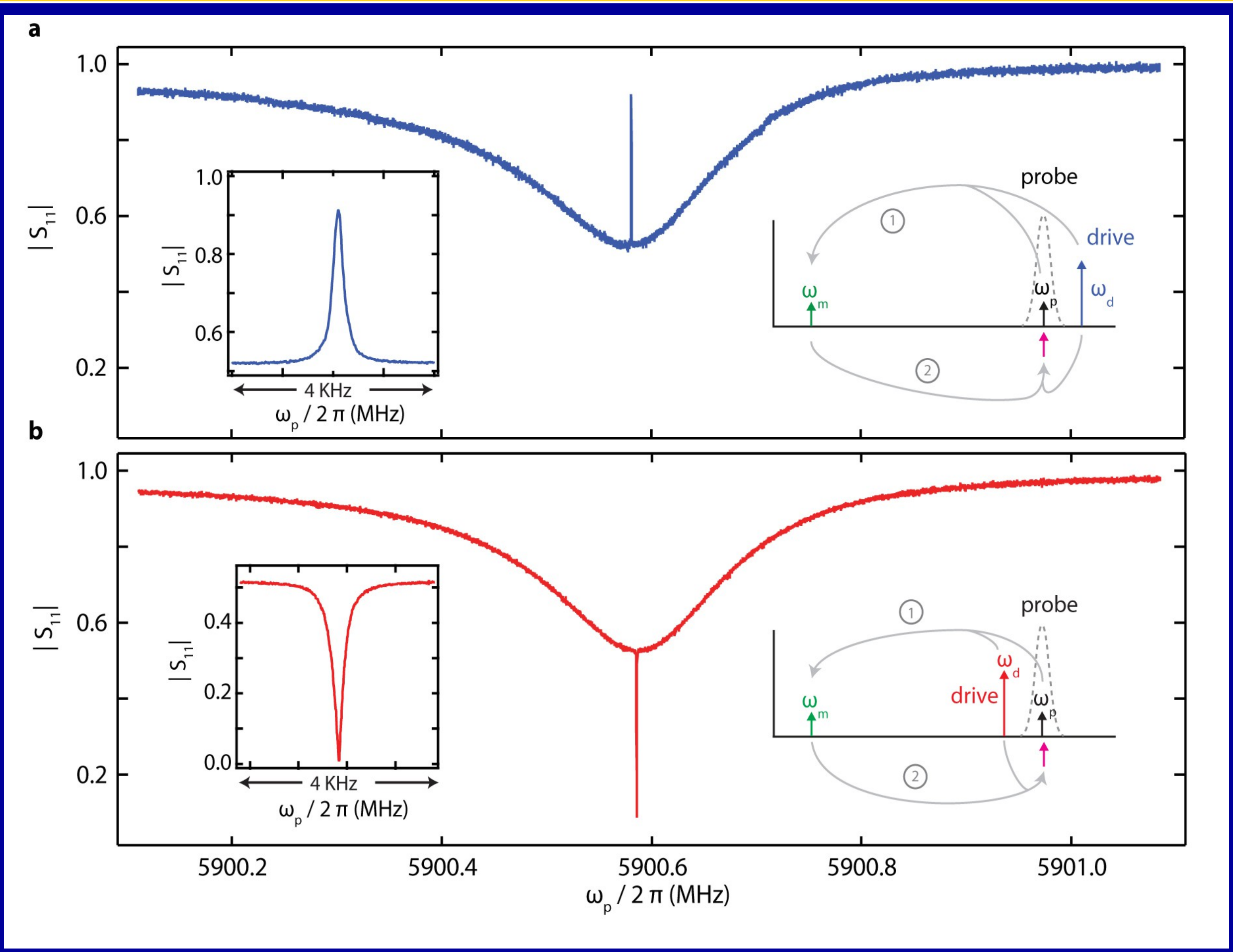
$$T = \left(1 - 2\eta_c \frac{1}{1 + C} \right)^2$$

Cooperativity:

$$C = \frac{4g^2}{\kappa \Gamma_m}, \quad g = G\bar{a}x_{ZPF}$$

S. Weis, R. Rivière, S. Deléglise, E. Gavartin, O. Arcizet, A. Schliesser, T. J. Kippenberg, Science **330**, 1520 (2010)

Optomechanically induced (transparency) reflection

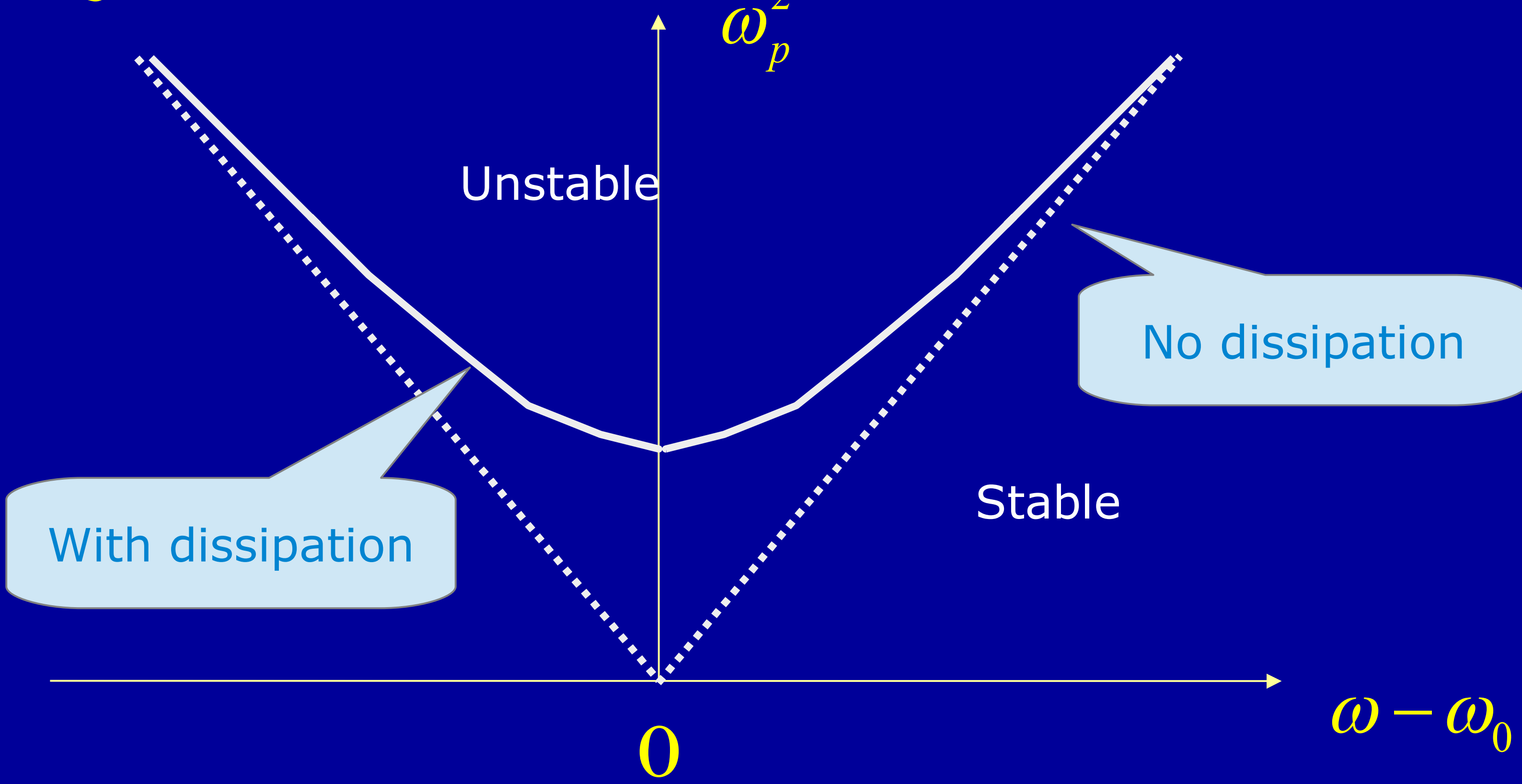


Constructive interference between the two probes results in OMIT

V. Singh, S. J. Bosman, B. H. Schneider, YMB, A. Castellanos-Gomez, and G. A. Steele, Nature Nanotech. **9**, 820 (2014)

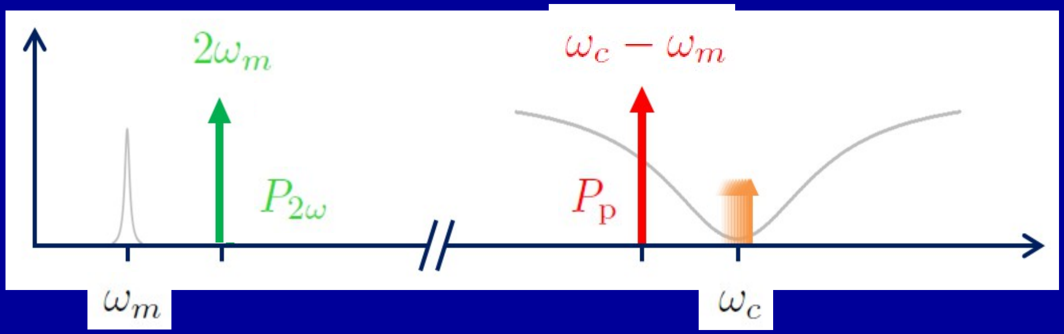
Parametric driving

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \left[\omega_0^2 + \omega_p^2 \cos 2\omega t\right] x = 0$$

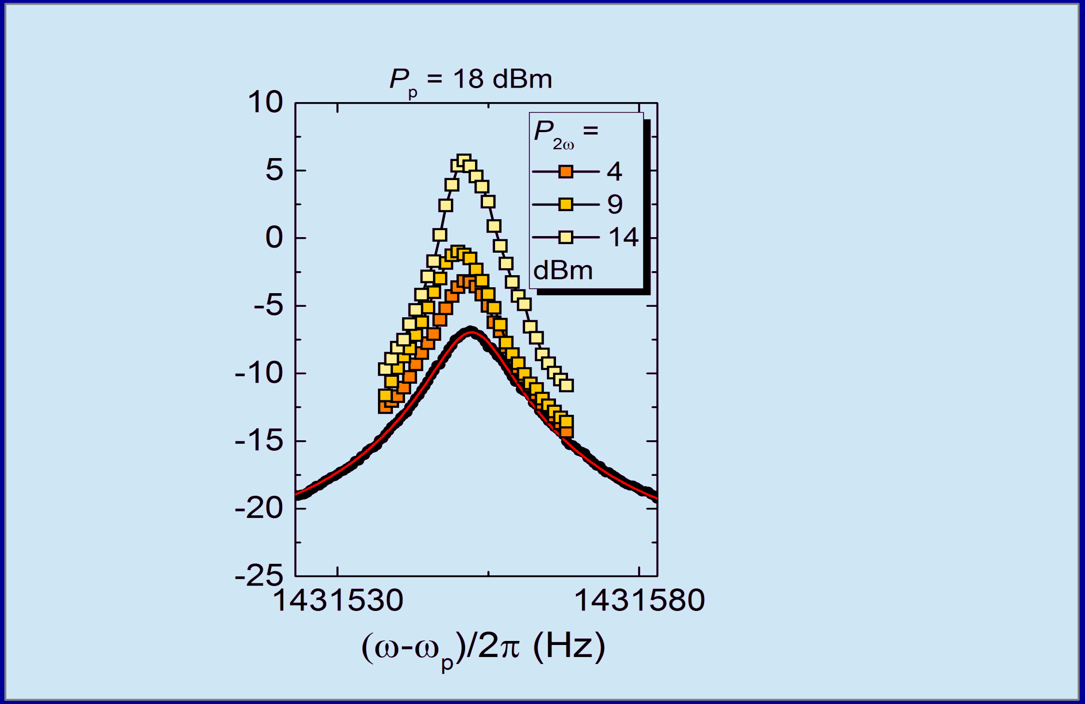


OMIT with parametrically driven resonator

D. Bottner, S. Hanai, M. Yuan, YMB, G. A. Steele, in preparation



Parametric excitation of a mechanical resonator leads to the transmission of light (microwaves) above 1 – amplification of light



Quantum detection of mechanical oscillations

Can we see quantum effects in mechanical motion?

Issues:

1. Need low temperatures $k_B T \ll \hbar\omega$

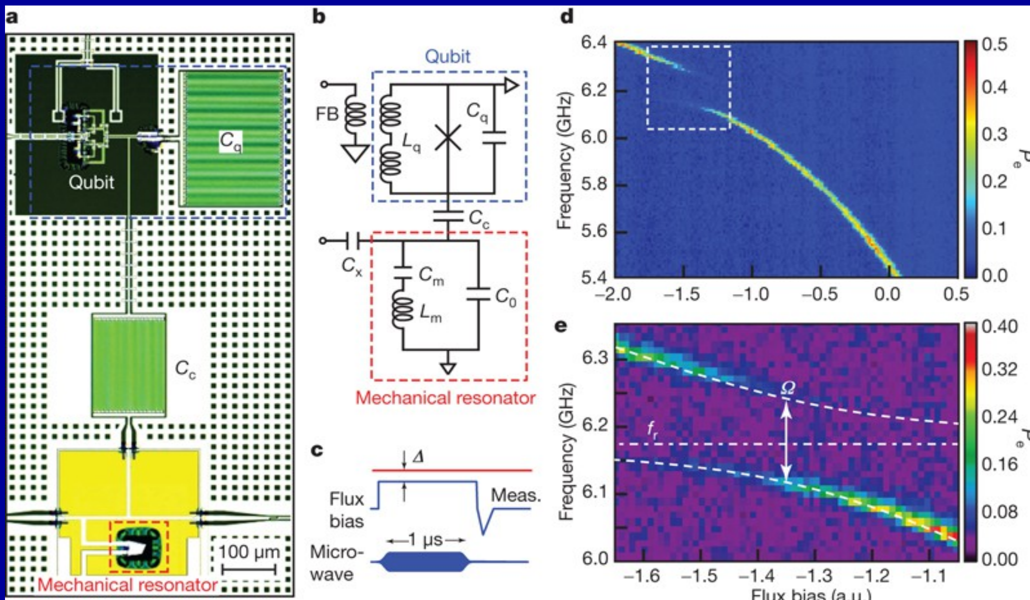
$$T = 1K \quad \Rightarrow \quad \omega \gg 100\,\text{GHz}$$

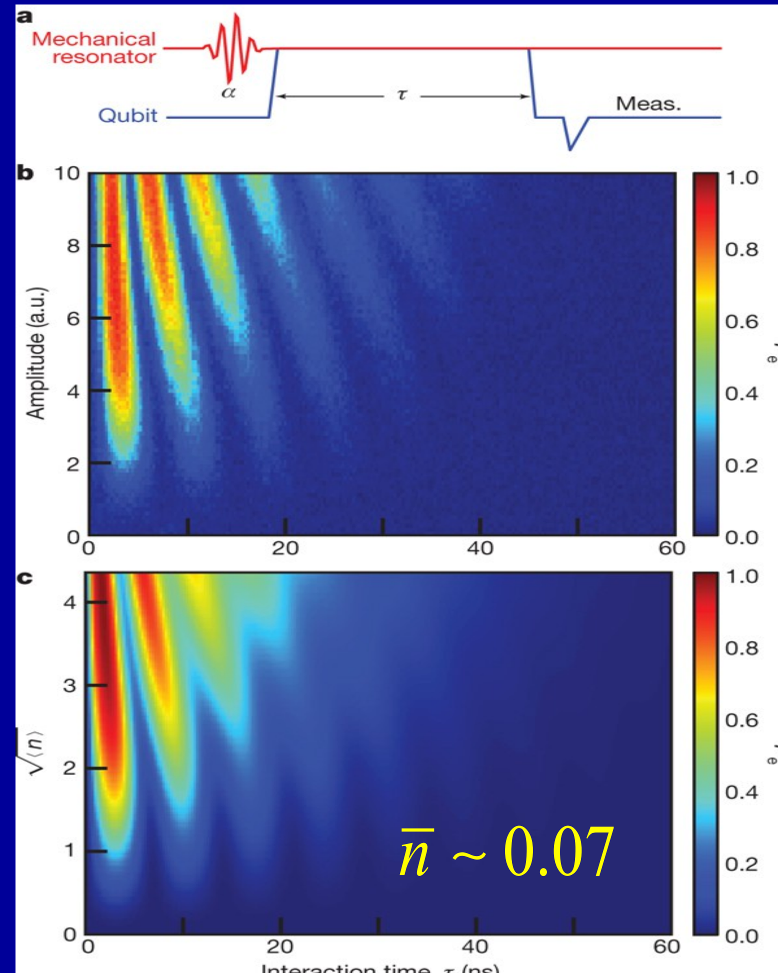
Either need to cool the mechanical resonator down or need to work with very high frequerncies

2. Need to decide what are the signatures of the quantum behavior and need a quantum detector to measure them

TU Delft
Technische Universiteit Delft

Quantum detection of mechanical oscillations





$\bar{n} \sim 0.07$

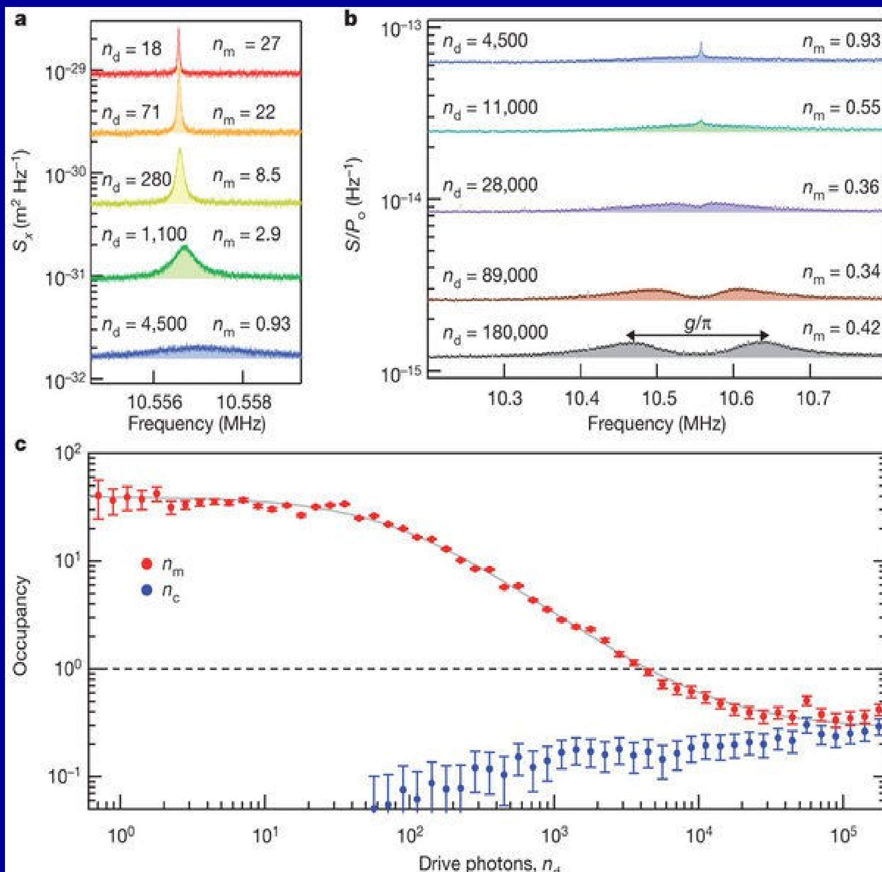
A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, J. M. Martinis, A. N. Cleland
Nature **464**, 697 (2010)

A mechanical resonator capacitively coupled to a superconducting qubit

$f \sim 6\,\mathrm{GHz}$

TU Delft
Technische Universiteit Delft

Quantum detection of mechanical oscillations



J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, R. W. Simmonds
Nature **475**, 359 (2011)

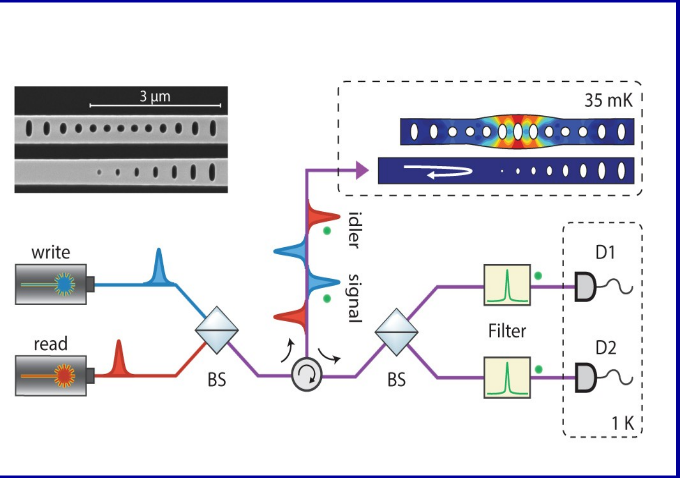
Cavity: $f_c \sim 7.5\ \text{GHz}$

Mechanical resonator: $f \sim 10\ \text{MHz}$

Sideband cooling

Quantum behavior of mechanical resonator

S. Hong, R. Riedinger, I. Marinkovic, A. Wallucks, S. G. Hofer, R. A. Norte, M. Aspelmeyer, S. Gröblacher, arXiv:1706.03777



Two-point correlation function:

$$g^{(2)}(\tau) = \frac{\left\langle b^\dagger(t) b^\dagger(t+\tau) b(t) b(t+\tau) \right\rangle}{\left\langle b^\dagger(t) b(t) \right\rangle^2}$$

Signature of non-classical states: $g^{(2)}(0) < 1$

Generally: $0 < g^{(2)}(0) < 2$

Quantum behavior of mechanical resonator

S. Hong, R. Riedinger, I. Marinkovic, A. Wallucks, S. G. Hofer, R. A. Norte, M. Aspelmeyer, S. Gröblacher, arXiv:1706.03777

Signature of non-classical states: $g^{(2)}(0) < 1$