Technische Universiteit Delft

Nonlinear optomechanics

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- Non-linearity and radiation pressure
- Non-linear resonator: Optomechanically induced transparency
- Non-linear cavity: dc
- Non-linear cavity: ac?

Quantum state transfer

We can prepare a cavity in pretty much any state (e.g. coupling to a qubit)

If the interaction is linear we can transfer this state to the mechanical resonator (state swap)

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g(\hat{a}^{\dagger} + \hat{a})(b^{\dagger} + b)$$

But it is difficult. Can we use non-linearity and start from a simple state?

What is non-linear?

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} - \hbar g_0 \hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

- Radiation pressure
- Mechanical resonator?
- Cavity?

Non-linear radiation pressure

A. Nunnenkamp, K. Børkje, and S. M. Girvin
Phys. Rev. Lett. **107**, 063602 (2011)

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_m\hat{b}^{\dagger}\hat{b} - \hbar g_0\hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

$$\Delta = -ng_0^2 / \omega_m$$

multiphoton resonances

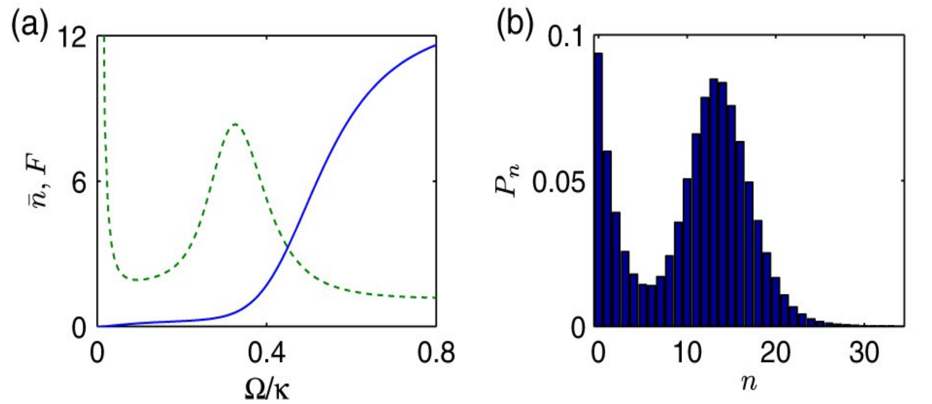
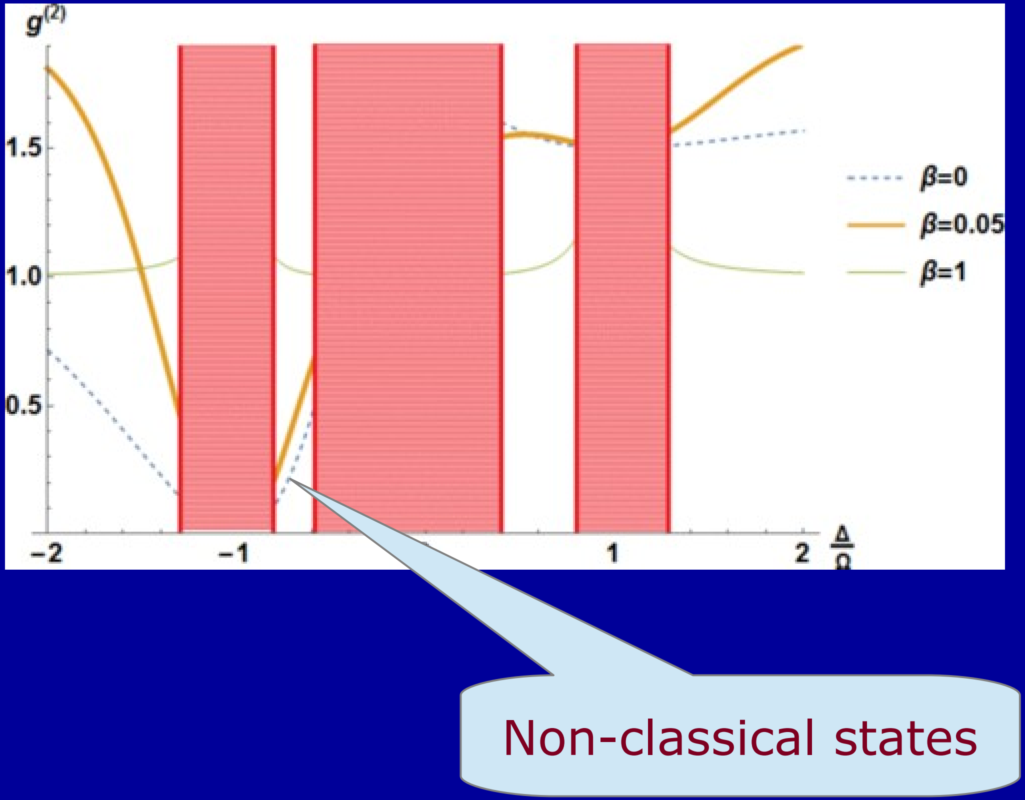


FIG. 4 (color online). Non-Gaussian steady states via multiphoton transitions. (a) Steady-state mean phonon number $\langle \hat{b}^{\dagger}\hat{b}\rangle$ (blue solid line) and the second-order coherence of the mechanical oscillator $F = \langle \hat{b}^{\dagger}\hat{b}^{\dagger}\hat{b}\,\hat{b}\rangle/(\langle \hat{b}^{\dagger}\hat{b}\rangle)^2$ (green dashed line) as a function of drive strength Ω. (b) Phonon number distribution P_n at $\Omega/\kappa = 0.6$. Parameters are $\Delta = -3g^2/\omega_M$, $\omega_M/\kappa = 2$, $\omega_M/\gamma = 1000$, and $g/\kappa = 1$.

Strongly driven optomechanical cavity

J.D.P. Machado and YMB, Phys. Rev. A **94**, 063835

Phonon second-order correlation function for weak coupling and the initial state $\left|1_{phot}, \beta_{phon}\right\rangle$



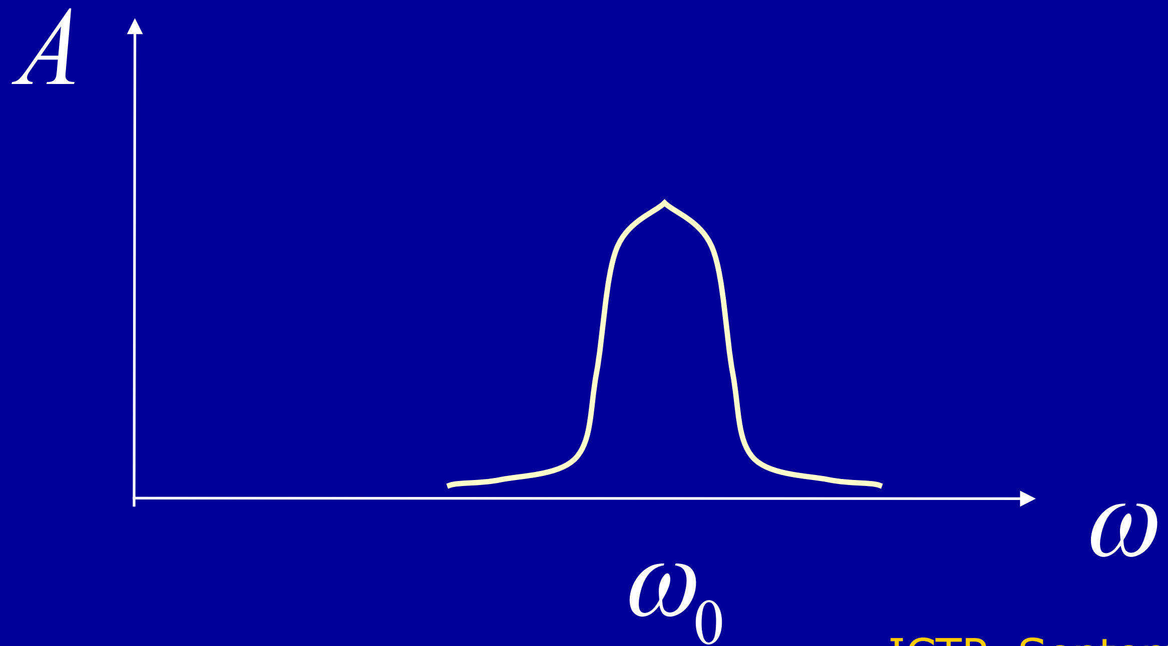
$$g^{(2)}(\tau) = \frac{\left\langle b^\dagger(t) b^\dagger(t+\tau) b(t) b(t+\tau) \right\rangle}{\left\langle b^\dagger(t) b(t) \right\rangle^2}$$

Duffing oscillator

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x + \alpha x^3 = \frac{F}{M}\cos\omega t$$

Driven harmonic oscillator: Resonance $x = A\cos\left(\omega t + \theta\right)$

A peak of the amplitude and a jump of the phase



How to solve the Duffing oscillator

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x + \alpha x^3 = \frac{F}{M}\cos\omega t$$

Going to a rotating frame:

$$x = u\cos\omega t - v\sin\omega t$$

$$\dot{x} = -\omega u\sin\omega t - v\omega\cos\omega t$$

$$\begin{bmatrix}\dot{u}\\ \dot{v}\end{bmatrix} = -\frac{1}{\omega}\begin{bmatrix}\sin\omega t\\ \cos\omega t\end{bmatrix}\left\{\begin{matrix}\left(\omega^2-\omega_0^2\right)\left(u\cos\omega t - v\sin\omega t\right) + \frac{\omega\omega_0}{Q}\left(u\sin\omega t + v\cos\omega t\right)\\ -\alpha\left(u\cos\omega t - v\sin\omega t\right)^3 + \frac{F}{M}\cos\omega t\end{matrix}\right\}$$

How to solve the Duffing oscillator

Rotating wave approximation: average over the time

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = -\frac{1}{\omega}\begin{bmatrix} \sin\omega t \\ \cos\omega t \end{bmatrix}\left\{\begin{array}{l} \left(\omega^2-\omega_0^2\right)\left(u\cos\omega t - v\sin\omega t\right) + \frac{\omega\omega_0}{Q}\left(u\sin\omega t + v\cos\omega t\right) \\ -\alpha\left(u\cos\omega t - v\sin\omega t\right)^3 + \frac{F}{M}\cos\omega t \end{array}\right\}$$

$$\left\langle \cos^2\Omega t\right\rangle = 1/2; \quad \left\langle \cos^4\Omega t\right\rangle = 3/8$$

Result for the "stationary" state $r=\sqrt{u^2+v^2}$

$$\left(r\omega\gamma\right)^2 + r^2\left(\omega^2-\omega_0^2-\frac{3\alpha}{4}r^2\right) = \left(\frac{F}{M}\right)^2$$

Driven Duffing oscillator

r

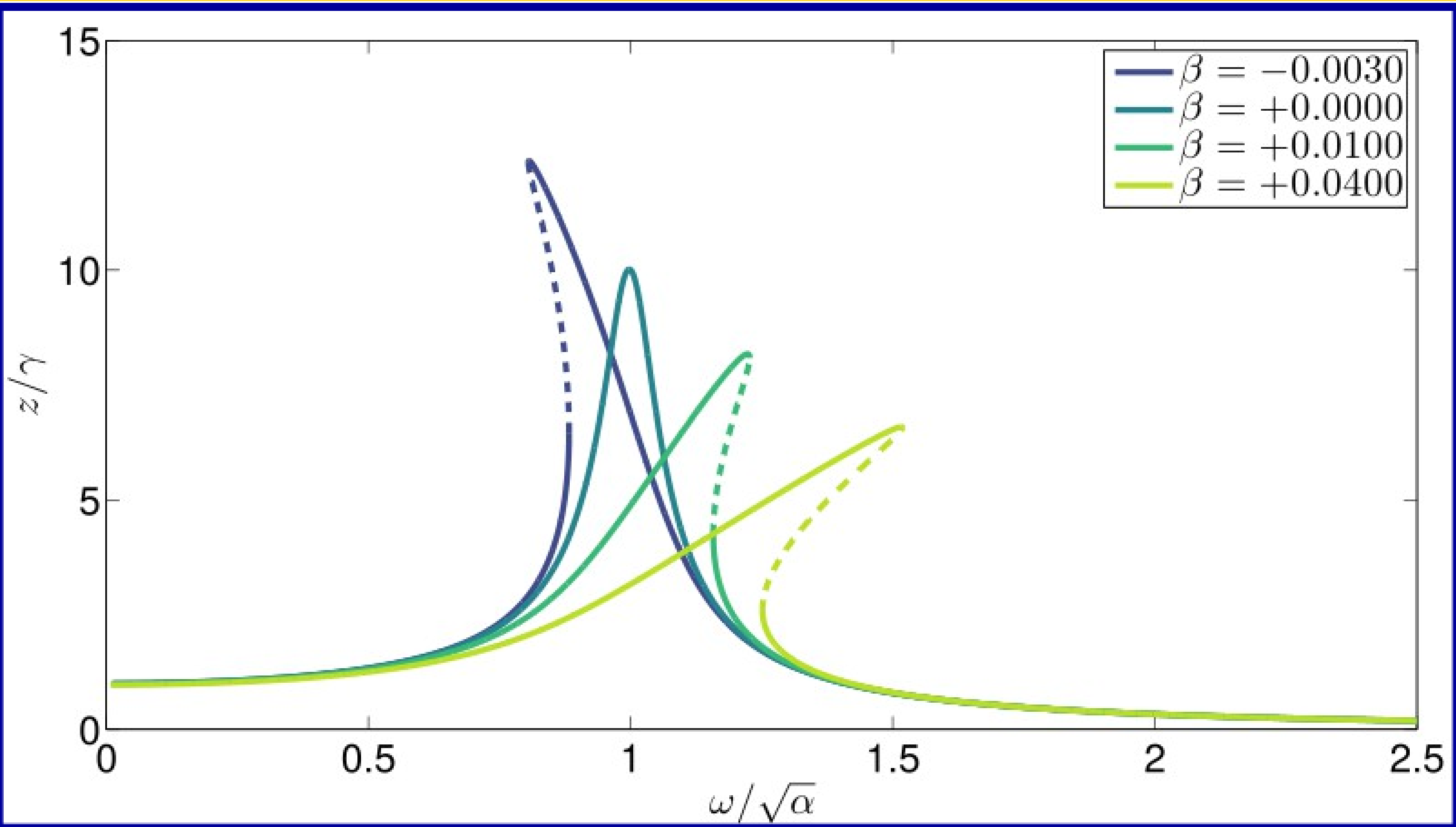


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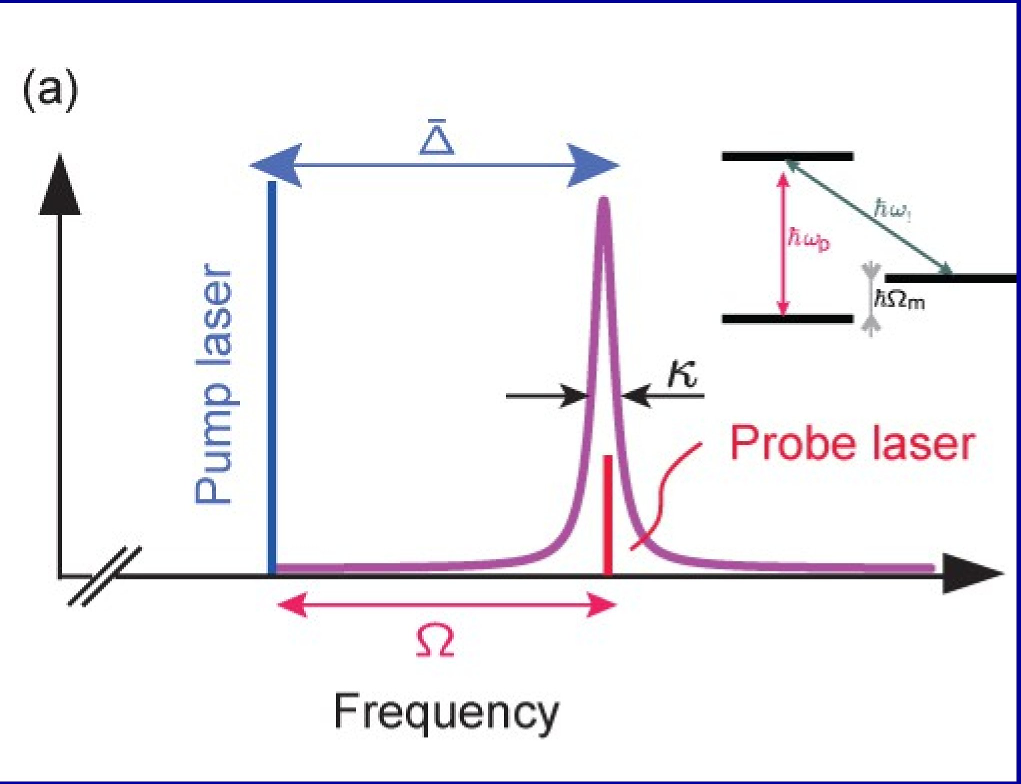
Image by User:Kraaiennest, Wikimedia Commons

Optomechanically induced transparency



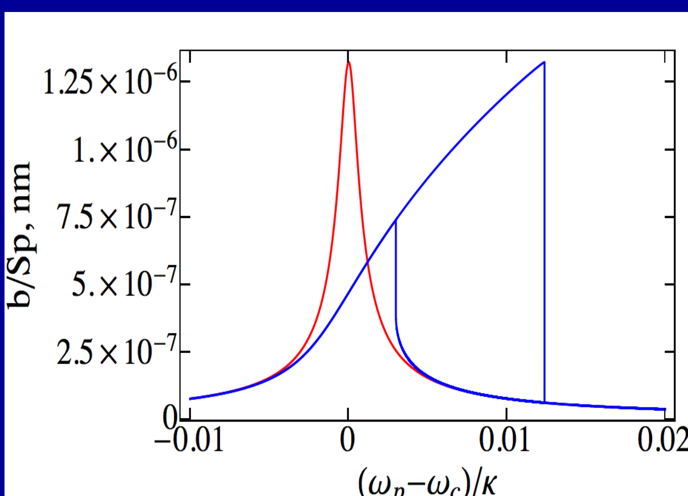
From: Aspelmeyer, Kippenberg, and Marquardt Rev. Mod. Phys. **86**, 1391 (2014)

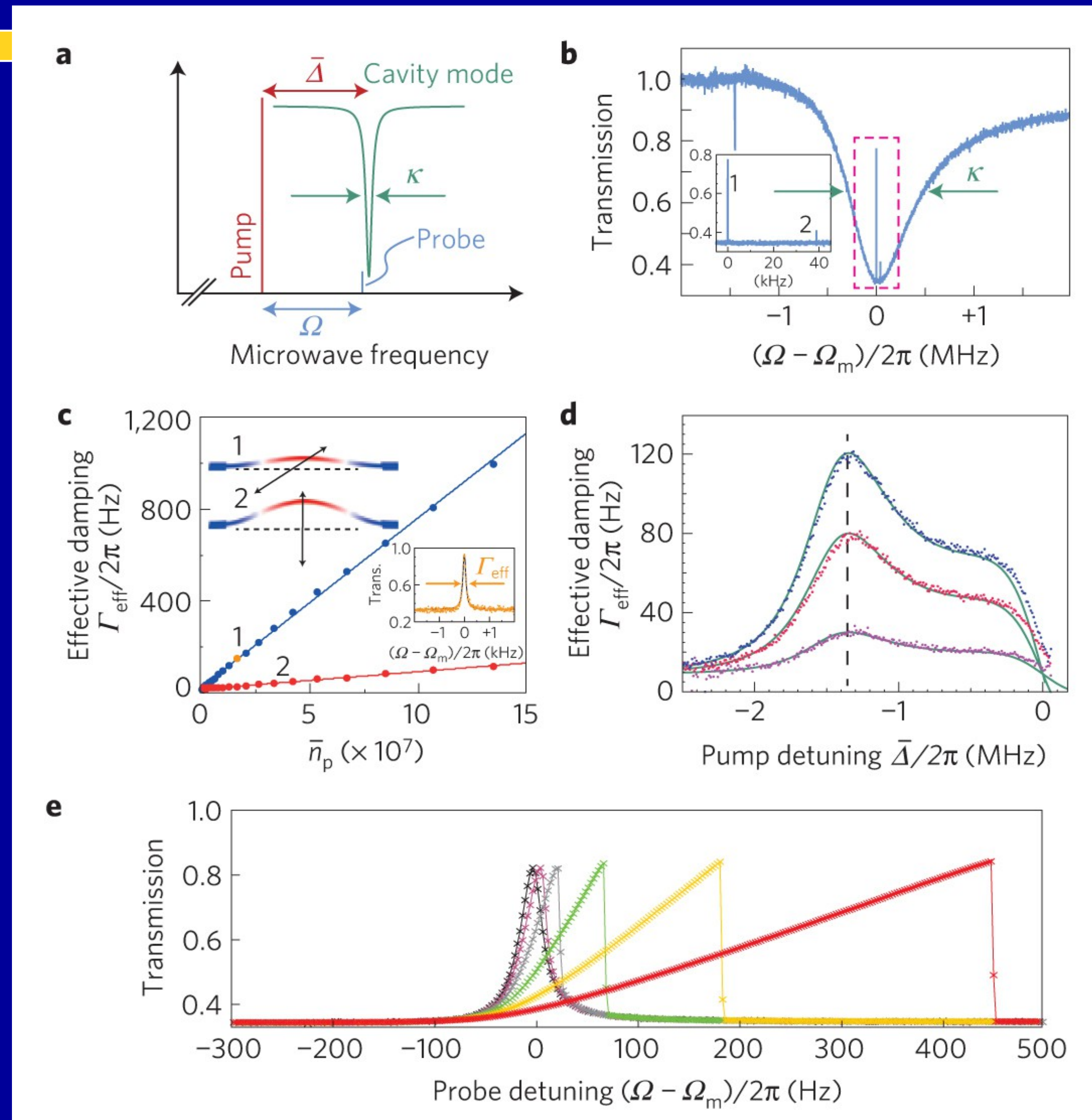
Cavity is strongly red-driven at $\omega_{cav} - \omega_m$ (red-detuned)

Probe laser measures the transmission around the cavity resonance

Non-linear OMIT

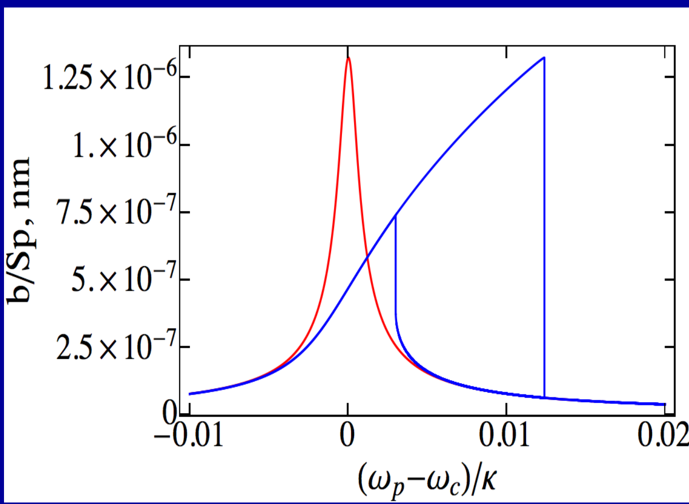
X. Zhou , F. Hocke, A. Schliesser, A. Marx, H. Huebl, R. Gross, T. J. Kippenberg
Nature Physics **9**, 179 (2013)

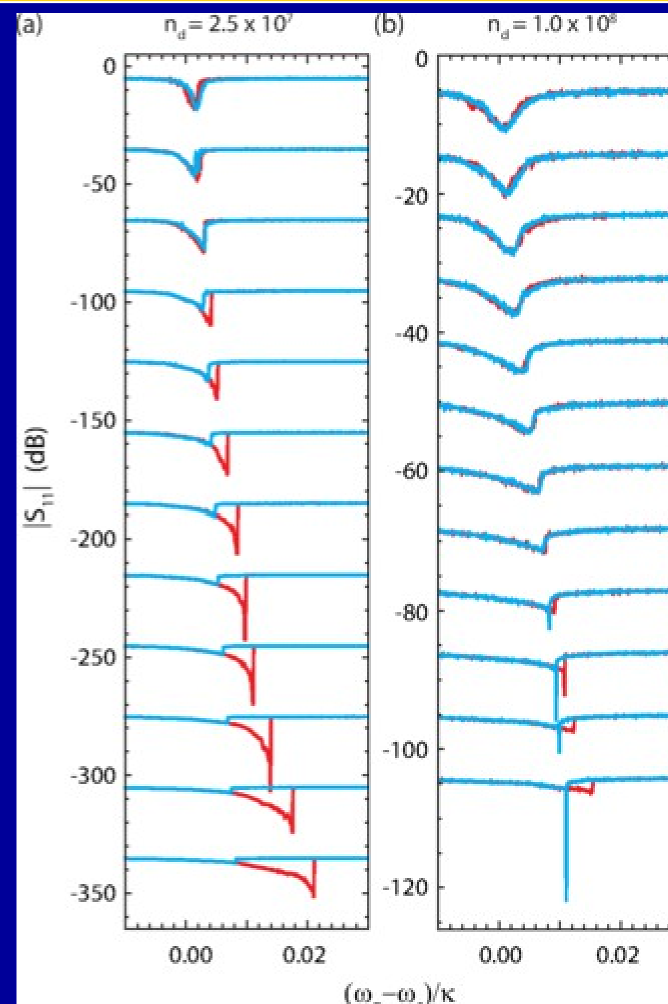




Non-linear OMIT

Duffing oscillator:



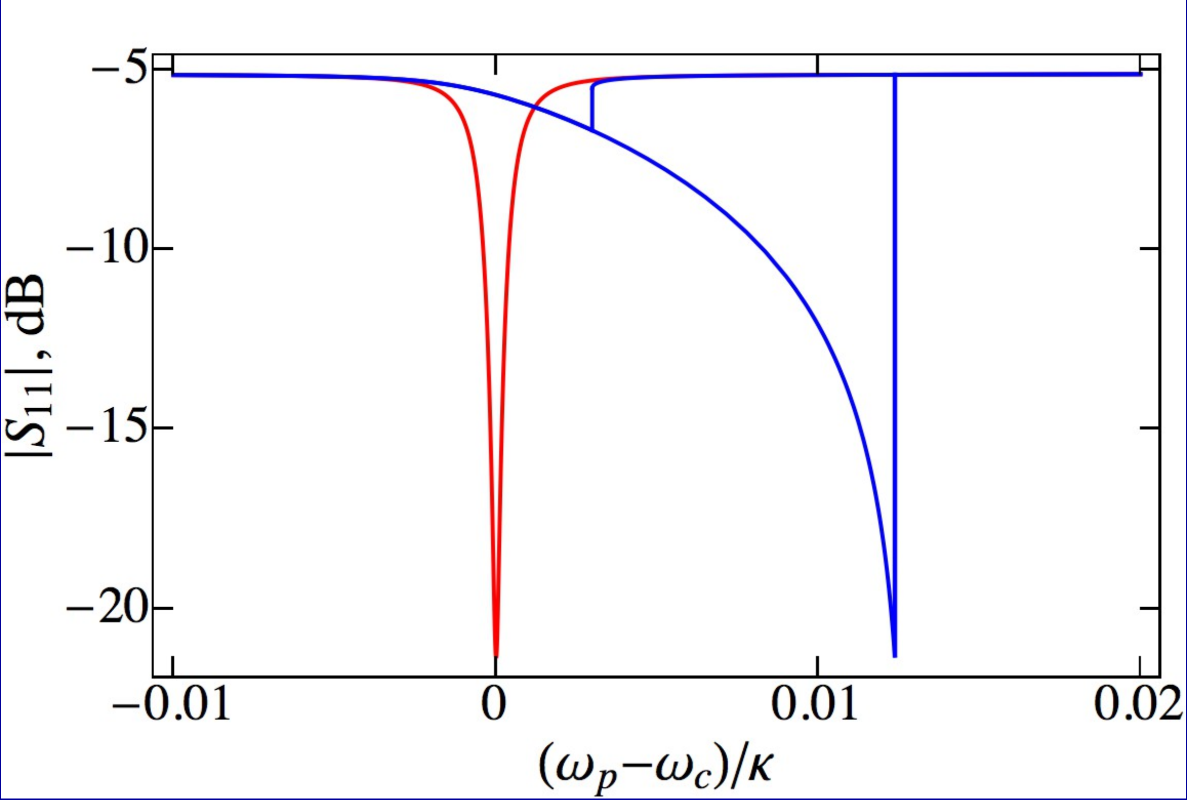


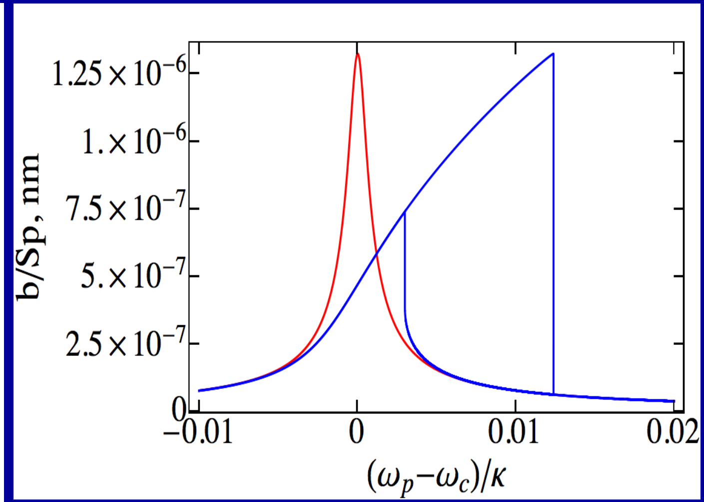
Does the shape of the transmission maximum repeat the response of the driven Duffing ocsillator?

Not always, the phase dynamics is important.

V. Singh, O. Shevchuk, YMB, G. A. Steele, Phys. Rev. B 93, 245407 (2016)

Non-linear OMIA





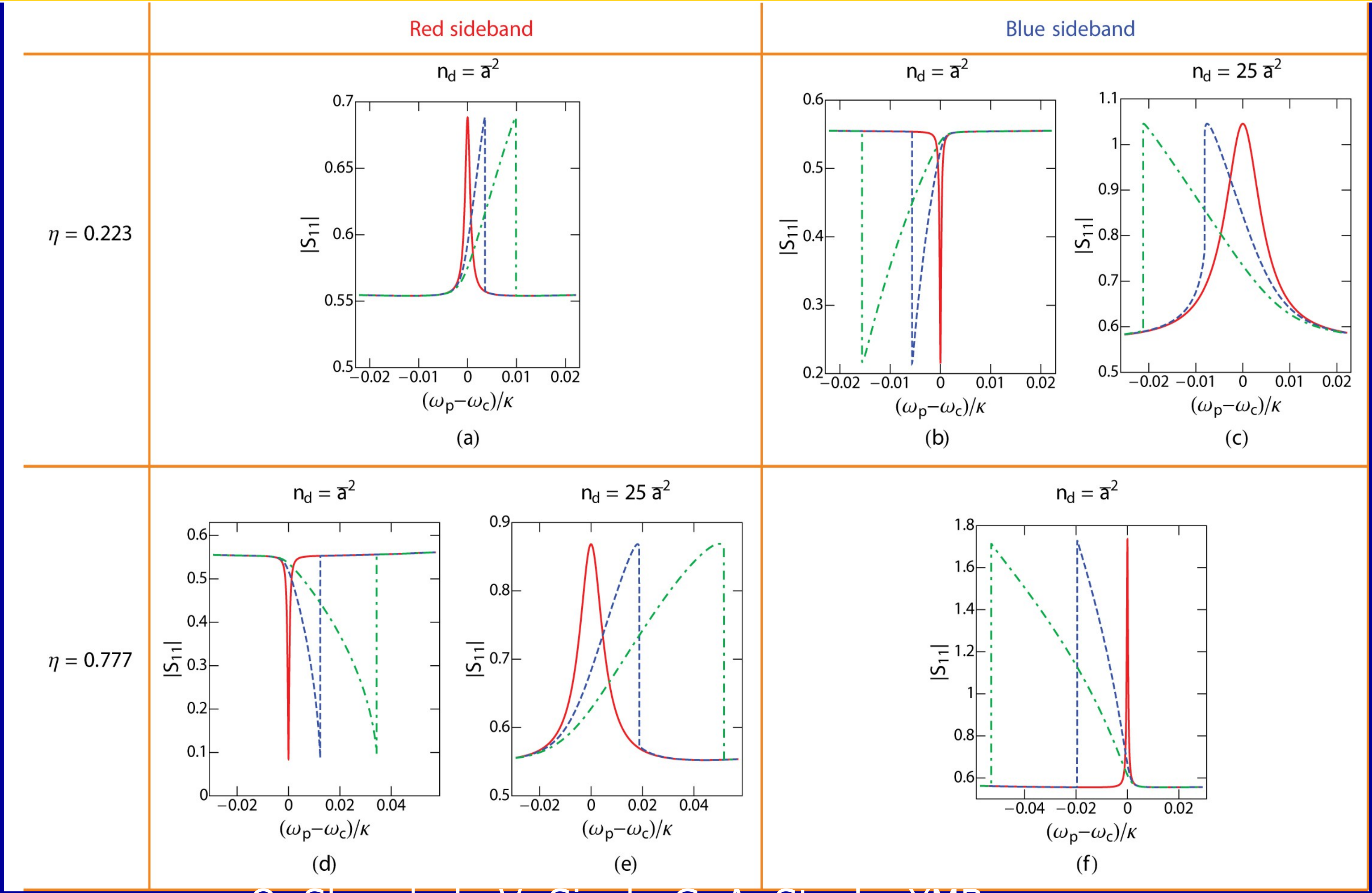
Red-detuned drive

Overcoupled cavity

$$\eta = \frac{\kappa}{\kappa + \kappa_{ext}} > \frac{1}{2}$$

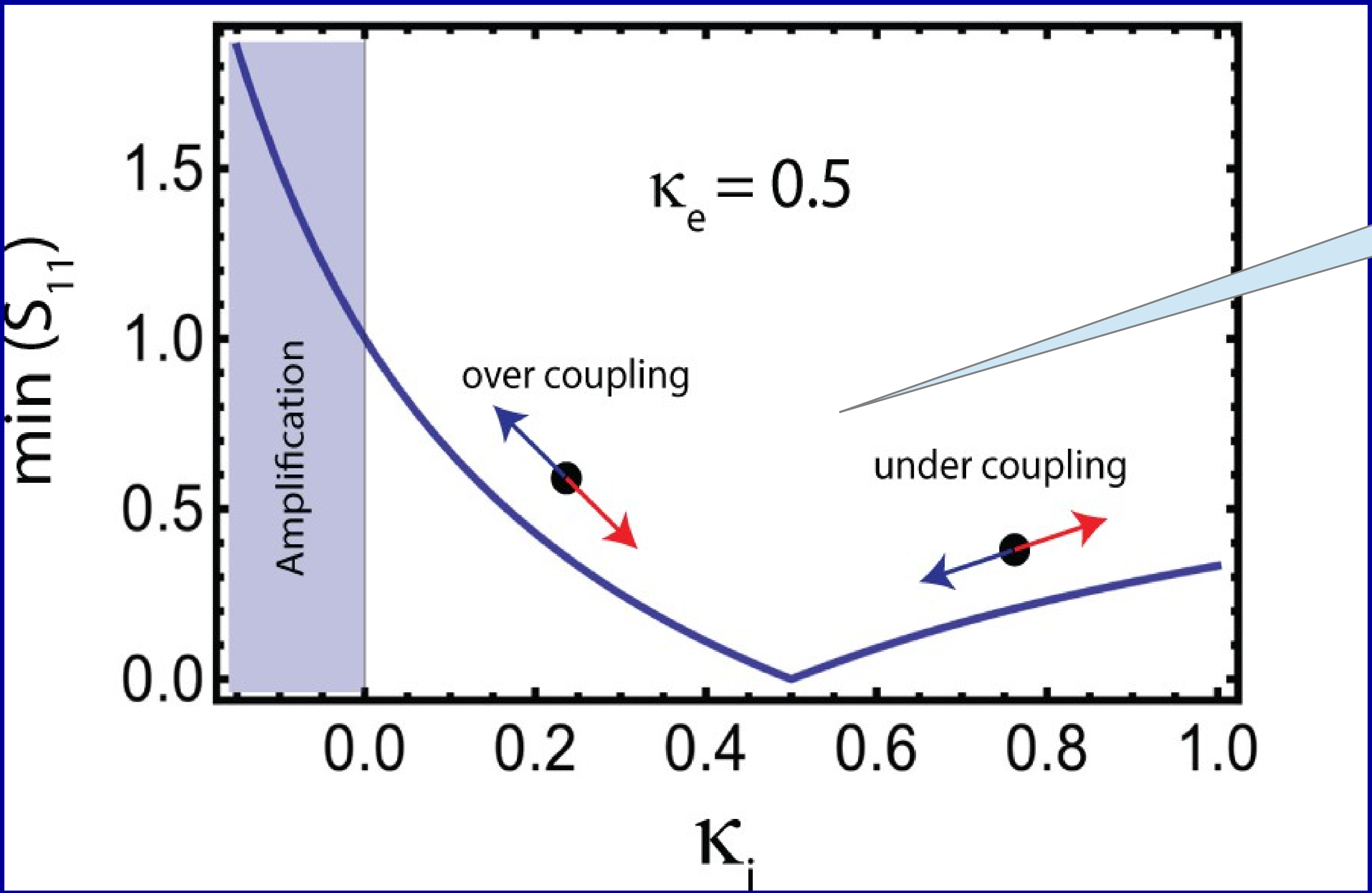
O. Shevchuk, V. Singh, G. A. Steele, YMB
Phys. Rev. B 92, 195415 (2015)

Non-linear OMIA



O. Shevchuk, V. Singh, G. A. Steele, YMB
Phys. Rev. B 92, 195415 (2015)

Non-linear OMIA



Impedance matching

$$\kappa \to \kappa \pm 4g^2 / \Gamma_m$$

V. Singh, S. J. Bosman, B. H. Schneider, YMB, A. Castellanos-Gomez, G. A. Steele, Nature Nanotech. **9**, 820 (2014)

Superconductivity

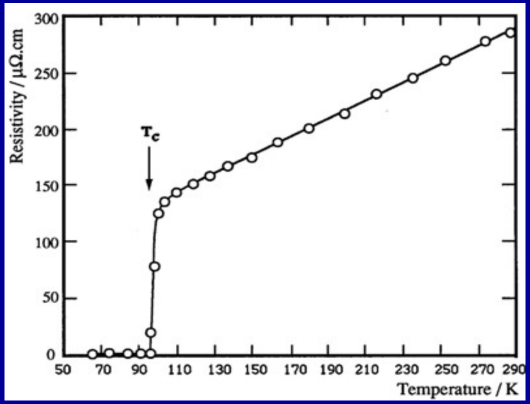
Superconductivity – a state of matter realized at low temperatures

Properties of superconductors:

Absence of electrical resistance

Magnetic field does not penetrate (Meissner effect)

Specific heat exponential with temperature



Mechanism of superconductivity:

Phonon-mediated attraction between electrons

Electrons bound in Cooper pairs

Cooper pairs form condensate characterized by a complex number

Excitations have a gap

$$\Delta e^{i\varphi}$$

Josephson effect

What happens if we bring in contact two superconductors with different phases?

$$\Delta e^{-i\varphi/2} \qquad \Delta e^{i\varphi/2}$$

Energy: must be proportional to the product

$$E = -E_J \cos\varphi$$

"Penetration of Cooper pairs"

Electrostatic potential: only enters in the gauge invariant combination

Gauge invariance: the wave function in the presence of scalar potential can only enter in the combination

$$\frac{\partial \Psi}{\partial t} + \frac{2ie\phi}{\hbar}\Psi$$

$$\Psi = |\Psi| e^{i\theta}$$

$$\Rightarrow \quad \frac{\partial \theta}{\partial t} + \frac{2e}{\hbar}\phi$$

Josephson relation:

$$\frac{\partial \varphi}{\partial t} = \frac{2eV}{\hbar}$$

Constant voltage *V* across the barrier rotates the phase difference

Josephson effect

Phase-dependent energy means current in the ground state!

Let us calculate the work needed to increase the phase difference of the junction from 0 to φ:

$$\frac{\partial \varphi}{\partial t} = \frac{2eV}{\hbar}$$

$$W = \int IV dt = \frac{\hbar}{2e} \int I d\varphi \quad \Rightarrow \quad I = \frac{2e}{\hbar} \frac{dW}{d\varphi} = I_c \sin\varphi$$

We can consider phase as "coordinate" of a particle and quantize it . Kinetic energy from the capacitance:

$$\frac{CV^2}{2} \rightarrow \frac{C\hbar^2}{4e^2} \dot{\varphi}^2$$

Josephson junction as inductor

$$\frac{\partial\varphi}{\partial t} = \frac{2eV}{\hbar} \qquad I = I_c \sin\varphi$$

Linear regime: $I = I_c\varphi$ → $\dot{I} = \frac{2e}{\hbar} I_c V$

Inductance: $L = \frac{2e}{\hbar} I_c$

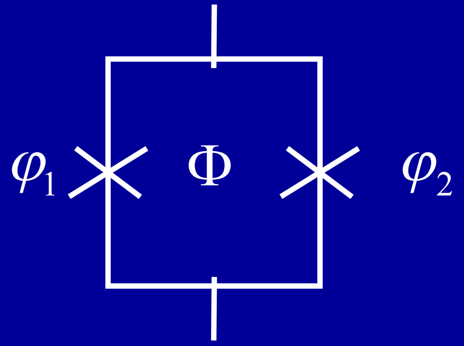
SQUID

SQUID – Superconducting Quantum Interference Device

$$I = I_0 \left(\sin\varphi_1 + \sin\varphi_2 \right)$$

$$\varphi_1 - \varphi_2 = 2\pi n \Phi / \Phi_0; \Phi_0 = \pi\hbar c / e$$

Flux quantum



↓

$$I = I_c \sin\varphi$$

$$I_c = 2 I_0 \cos\left(\pi\Phi / \Phi_0 \right)$$

High sensitivity to magnetic flux

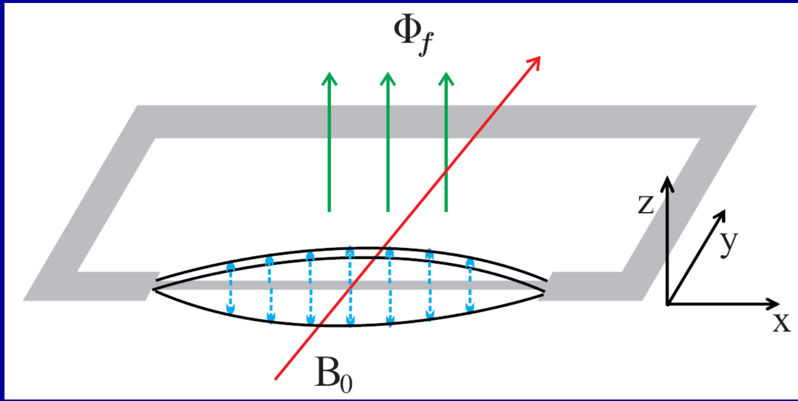
Inductive coupling

SQUID – Superconducting Quantum Interference Device

$I_c = 2I_0 \cos(\pi\Phi/\Phi_0)$ – depends on the position

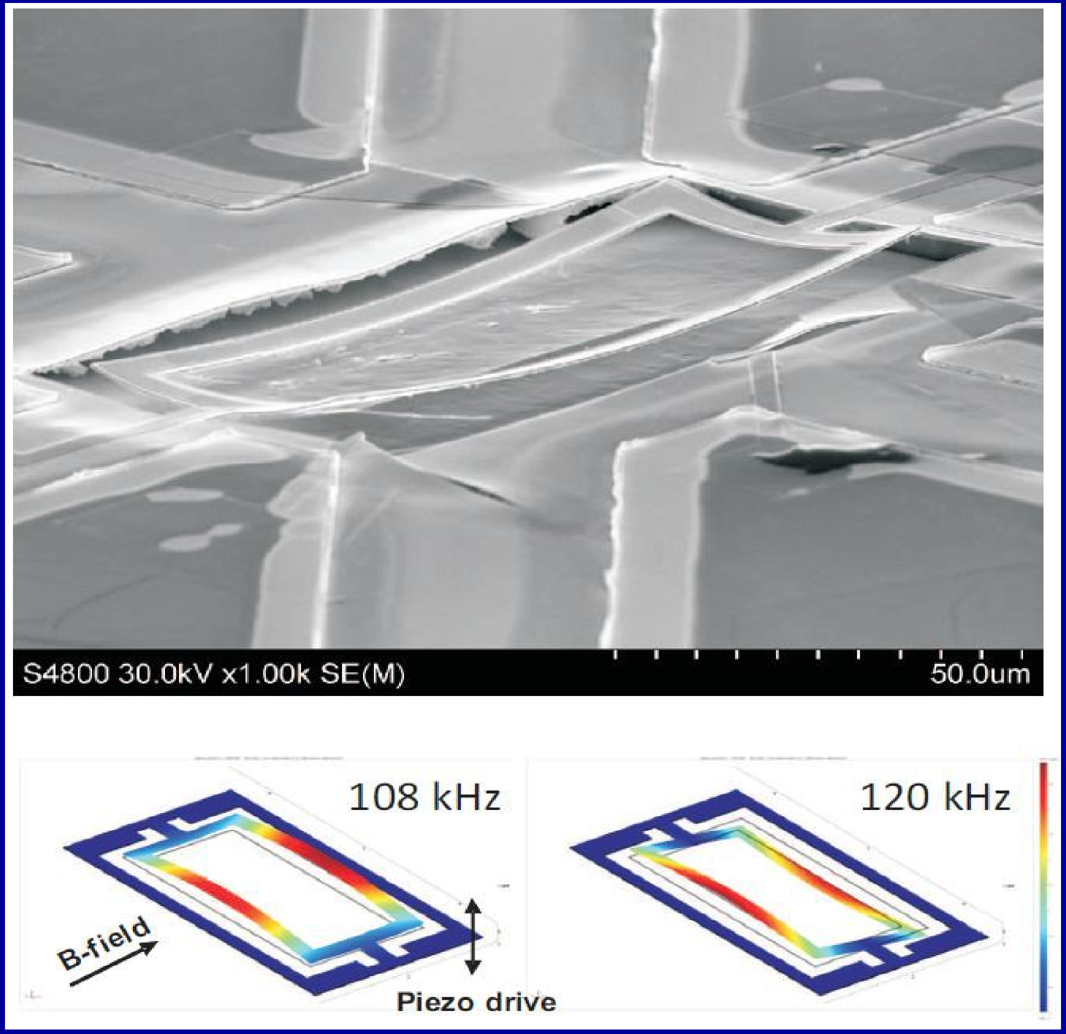
$$L = \frac{4e}{\hbar} I_0 \cos\frac{\pi\Phi(x)}{\Phi_0}$$

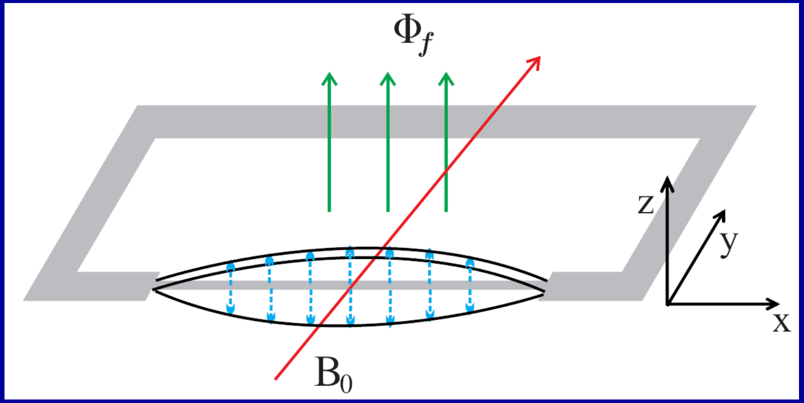
X. Zhou, A. Mizel, Phys. Rev. Lett. 97, 267201 (2006)
E. Buks, M. P. Blencowe, Phys. Rev. B 74, 174504 (2006)



Non-linear cavity

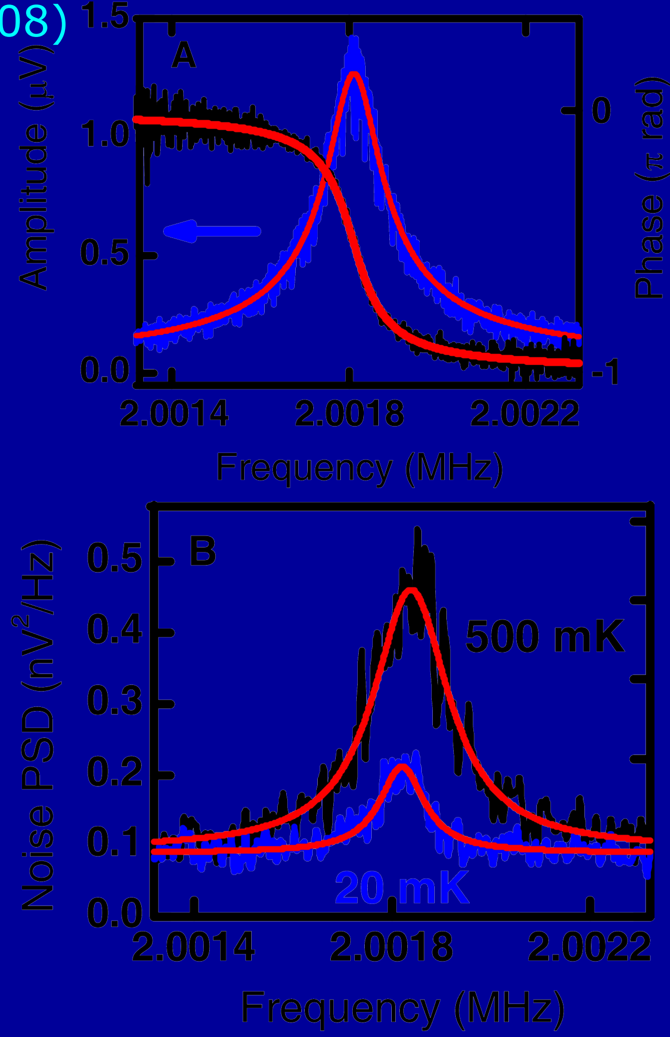
S. Etaki, F. Konschelle, H. Yamaguchi, YMB, H. S. J. van der Zant, Nature Comm. **4**, 1803 (2013)





Classical SQUID: experiment

S. Etaki, M. Poot, I. Mahboob, K. Onomitsu, H. Yamaguchi, H. S. J. van der Zant, Nature Physics **4**, 785 (2008)

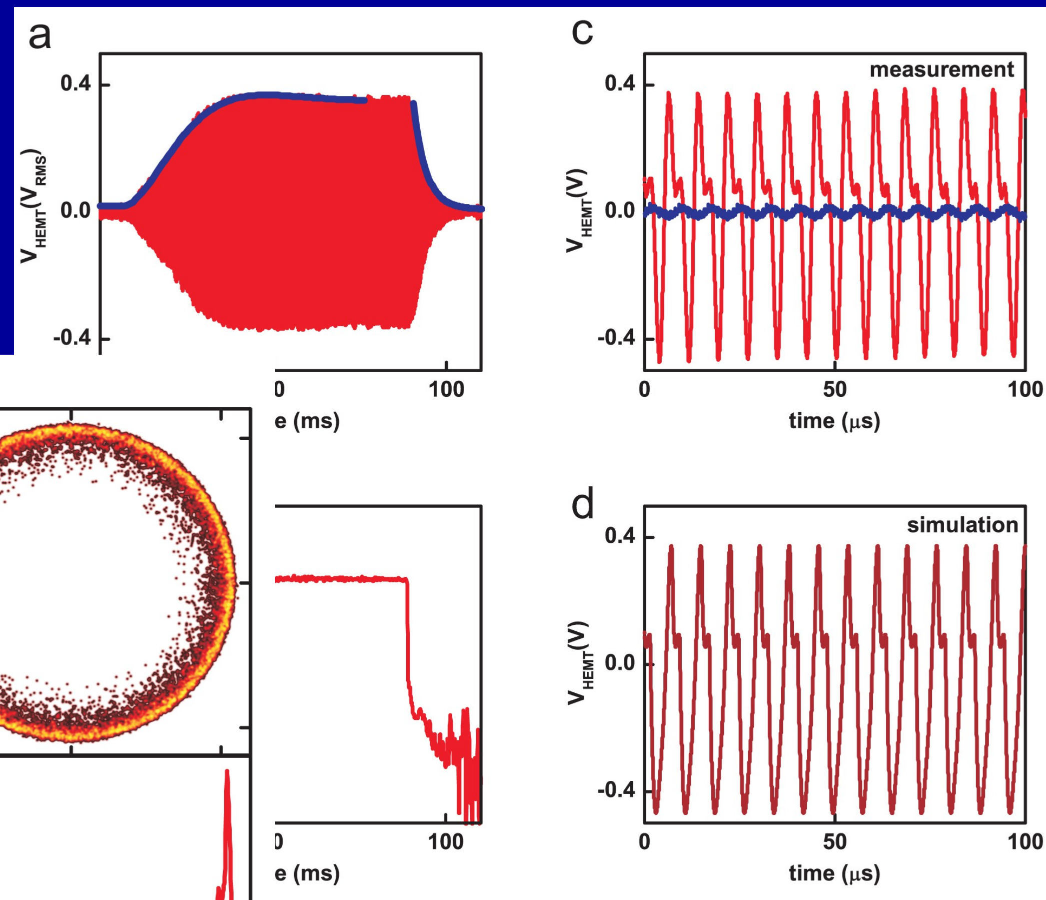


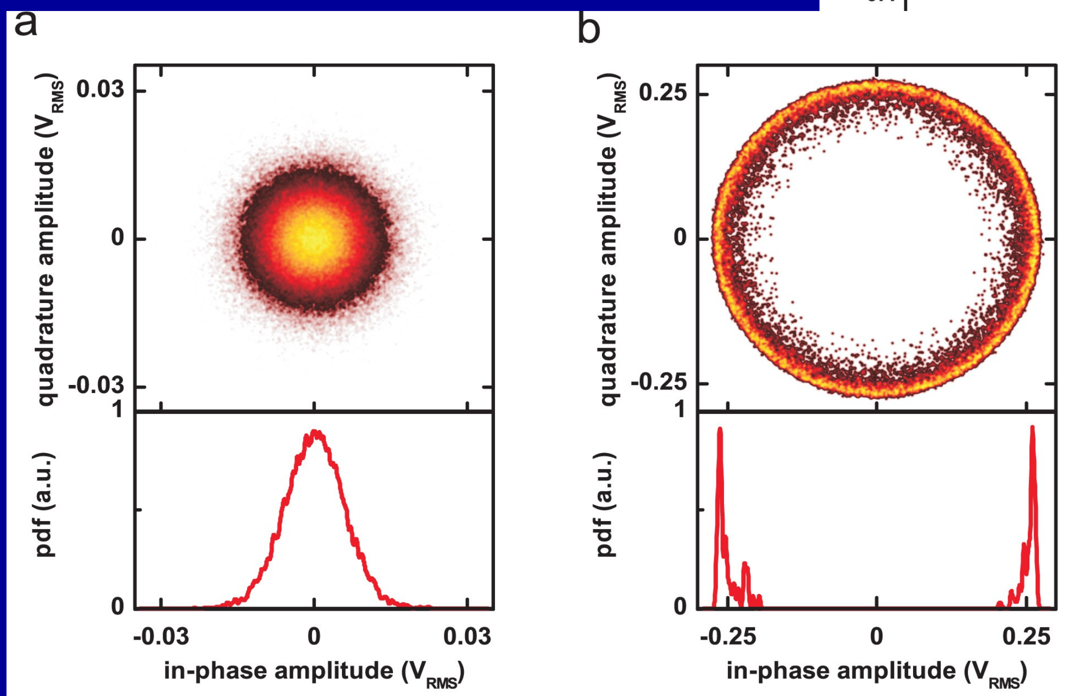
Harmonic oscillator response at

f = 2 MHz and Q = 18 000

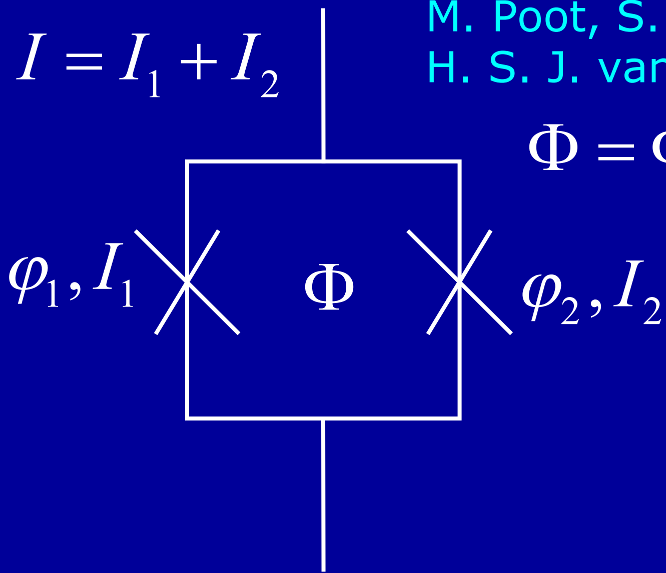
Self-sustained oscillations

S. Etaki, F. Konschelle, H. Yamaguchi, YMB, H. S. J. van der Zant, Nature Comm. **4**, 1803 (2013)





Lorentz force backaction



M. Poot, S. Etaki, I. Mahboob, K. Onomitsu, H. Yamaguchi, YMB, H. S. J. van der Zant, Phys. Rev. Lett. **105**, 207203 (2010)

$$\Phi = \Phi_a + Blax + L(I_1 - I_2)/2 = \Phi_0(\varphi_2 - \varphi_1)/(2\pi)$$

Motion

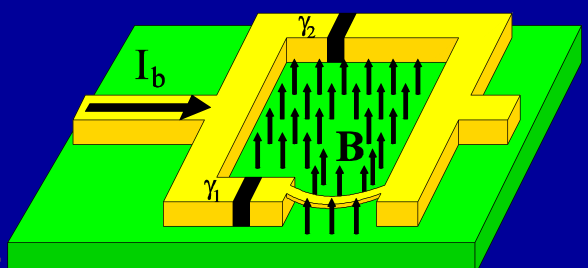
Inductive coupling

Josephson junctions:

$$I_{1,2} = I_0 \sin\varphi_{1,2} + \frac{V_{1,2}}{R} + C\dot{V}_{1,2}, \quad V_{1,2} = \frac{\Phi_0}{2\pi}\dot{\varphi}_{1,2}$$

Oscillator:

$$M\ddot{x} + \frac{M\omega}{Q}\dot{x} + M\omega^2 x = F\cos\omega t + aBlI_1$$



Lorentz force

Back-action and self-sustained oscillations

$$M\ddot{x} + \frac{M\omega}{Q}\dot{x} + M\omega^2 x = F\cos\omega t + aBlI_1$$

For self-sustained oscillations we need Q < 0

Overdamped:

$$I_1 = V / R \propto \sqrt{\left(\frac{I}{I_c}\right)^2 - 1},\ I_c(x) = 2I_c \cos\frac{\pi\Phi(x)}{\Phi_0}$$

– renormalization of the frequency

Finite capacitance: correction

$$\delta I_1 = C\dot{V} \propto \dot{x}\sin\frac{\pi\Phi}{\Phi_0}\left(\sqrt{\left(\frac{I}{2I_c\cos\frac{\pi\Phi(x)}{\Phi_0}}\right)^2 - 1}\right)^{-1}$$

Renormalizes the quality factor and may yield self-oscillations

Quantization of a non-linear cavity

O. Shevchuk, G. A. Steele, YMB Phys. Rev. B 96, 014508 (2017)

SQUID becomes a Kerr cavity: $\hat{H}_{cav} = \hbar\omega\hat{a}^{\dagger}\hat{a} + K\hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}$

Can not quantuize the interaction generally

Cavity operated at dc or the frequency of the cavity is comparable to the mechanical frequency: Beam-splitter + cross-Kerr

$$\hat{H}_{\text{int}} = \hbar g_{bs}\left(\hat{a}^{\dagger}\hat{b} + \hat{a}\hat{b}^{\dagger}\right) + \hbar g_{CK}\hat{a}^{\dagger}\hat{a}\hat{b}^{\dagger}\hat{b}$$

Disappears for a symmetric SQUID

Quantization of a non-linear cavity

O. Shevchuk, G. A. Steele, YMB Phys. Rev. B 96, 014508 (2017)

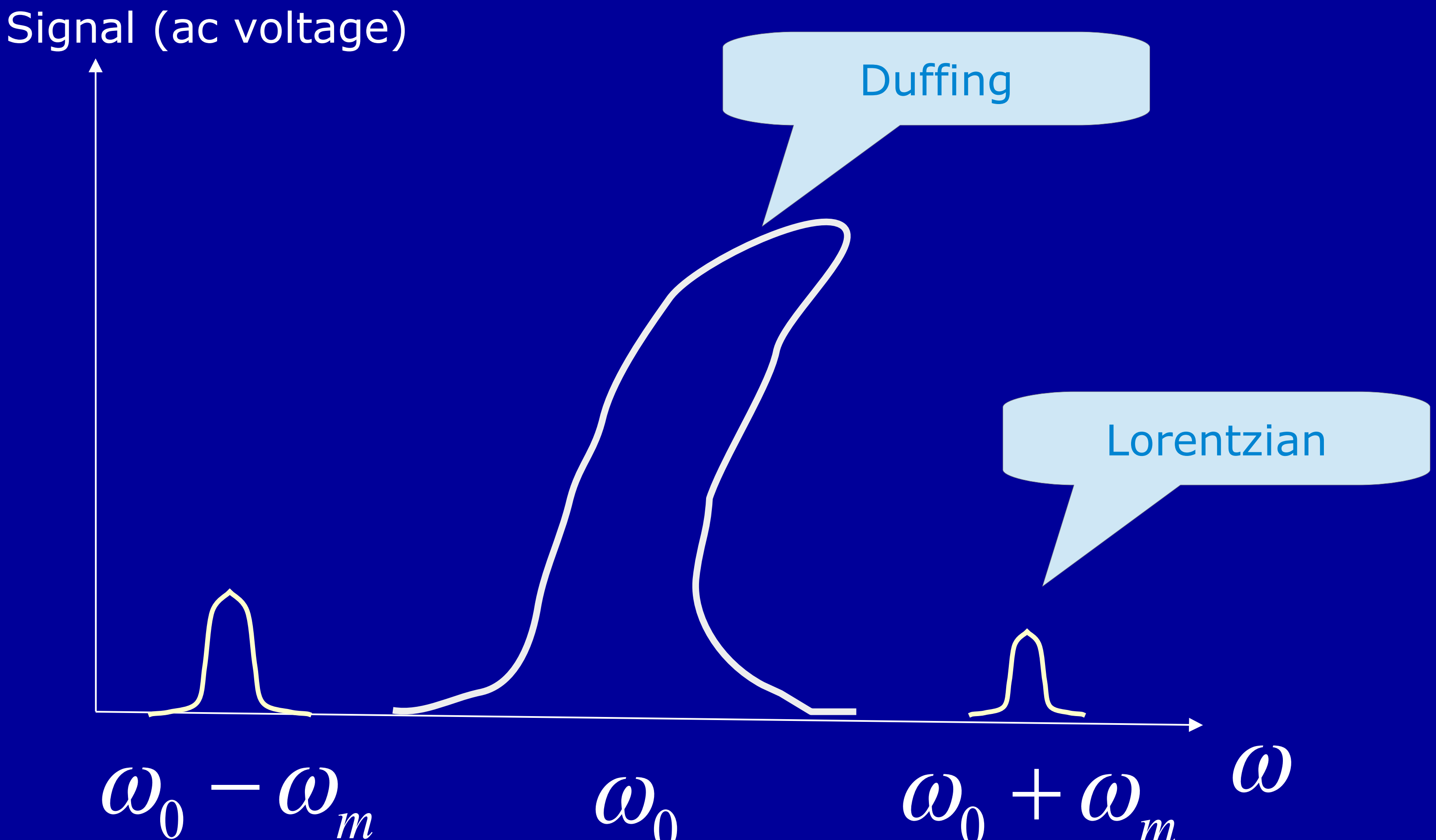
Cavity operated close to the resonance: radiation pressure + cross-Kerr

$$\hat{H}_{\text{int}} = \hbar g_{rp} \hat{a}^{\dagger} \hat{a} \left(\hat{b} + \hat{b}^{\dagger} \right) + \hbar g_{CK} \hat{a}^{\dagger} \hat{a} \hat{b}^{\dagger} \hat{b}$$

What should we expect for such non-linear cavity?

Mechanical subbands

We drive a cavity at resonance



Quantum state transfer

B. Yurke and D. Stoler, Phys.Rev.Lett. **57**, 13 (1986)

What is an evolution of a quantum state in a non-linear cavity?

$$\hat{H} = \hbar\omega \hat{a}^{\dagger}\hat{a} + K\left(\hat{a}^{\dagger}\hat{a}\right)^{p}$$

Initially: coherent state

$$\psi(0) = \left|\alpha\right\rangle = \exp\left(-\frac{\left|\alpha\right|^{2}}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^{n}}{\sqrt{n!}}\left|n\right\rangle$$

Evolution of the state:

$$\psi(t) = \exp\left(-\frac{\left|\alpha\right|^{2}}{2}\right)\sum_{n=0}^{\infty}\exp\left(-iKn^{p}t\right)\frac{\alpha^{n}}{\sqrt{n!}}\left|n\right\rangle$$

Quantum state transfer

B. Yurke and D. Stoler, Phys.Rev.Lett. **57**, 13 (1986)

$$\psi(t) = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty} \exp\left(-iKn^p t\right)\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Periodic: the same for *t* and $t + 2\pi / K$

After a quarter of a period: A cat state

$$\psi\left(\frac{\pi}{2K}\right) = \frac{1}{\sqrt{2}}\left(e^{-i\pi/4}|\alpha\rangle + e^{i\pi/4}|-\alpha\rangle\right)$$