An empirical dynamical approach to modelling teleconnections using the DREAM model

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DREAM: Dynamical Research Empirical Atmospheric Model

A Simple GCM with Empirical Forcing

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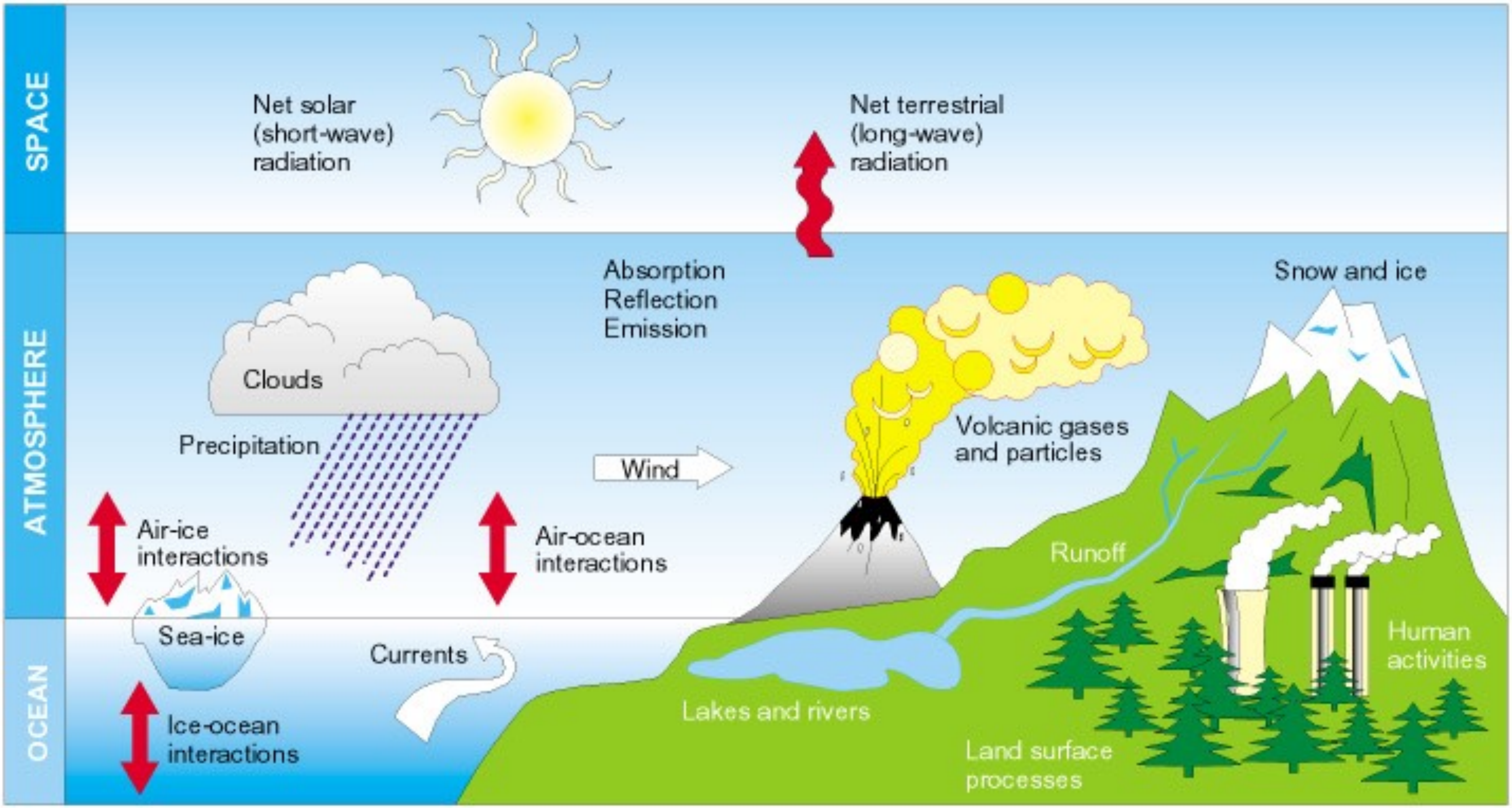
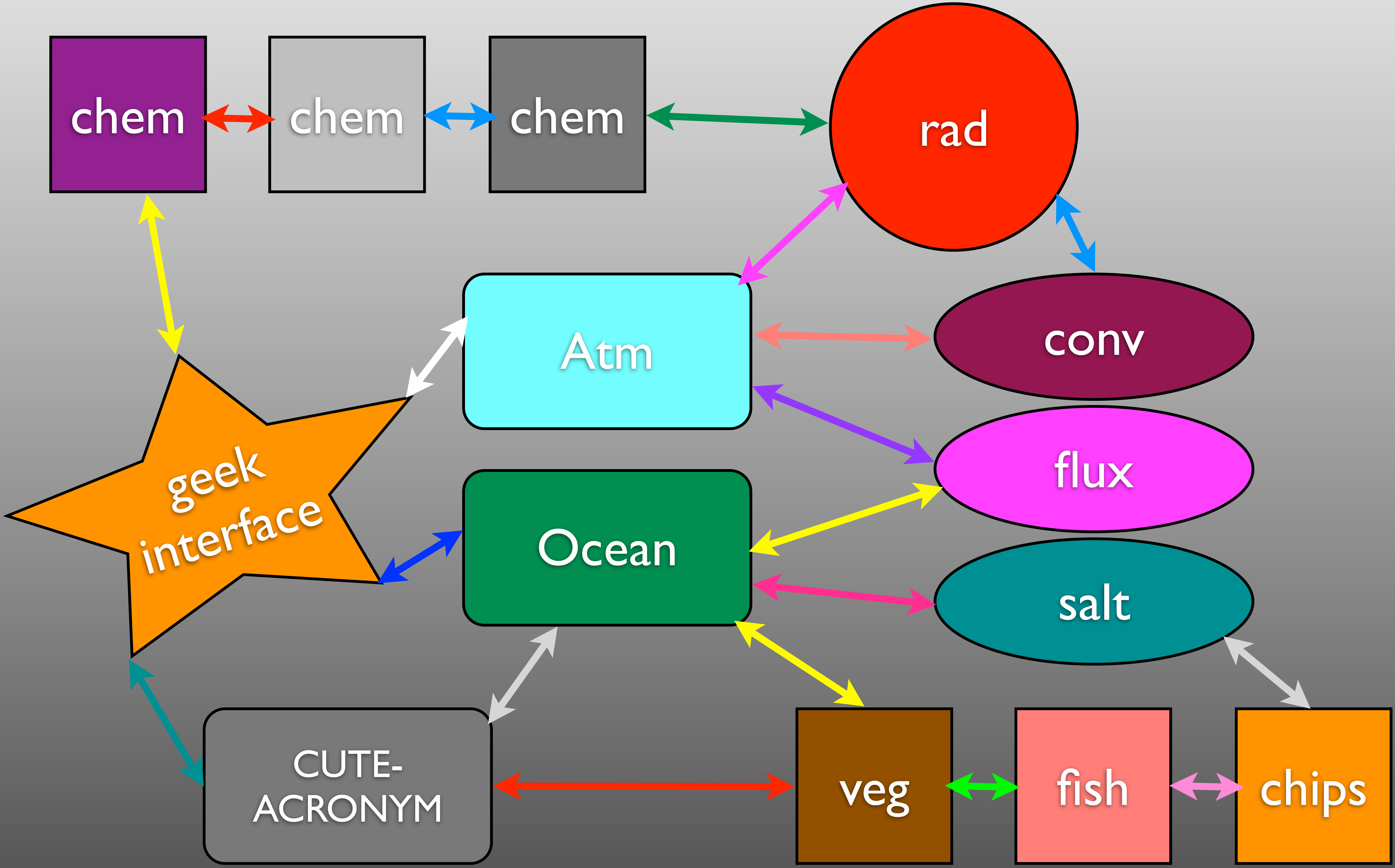
the model is a dream...
... the code is a nightmare !

The Climate System



courtesy N. Noreiks, L. Bengtsson, MPI

AV/Global/0101


chem
chem
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Atm
conv
flux
geek interface
Ocean
salt
CUTE-ACRONYM
veg
fish
chips

Introducing... DREAM

Dynamical Research Empirical Atmospheric Model

http://www.legos.obs-mip.fr/members/hall/DREAM_training_handout.pdf?lang=en

1) Model overview
Equations
Datasets
Forcing specification

2) Examination of the forcing terms
Application to the annual cycle

3) Perturbation runs
Response to tropospheric heating
Remote influences on rainfall over South America

4) Condensation heating
Impact on propagating tropical signals

Basic equations

$$\frac{\partial u}{\partial t} = F_u, \quad \frac{\partial v}{\partial t} = F_v \quad \rightarrow \quad \frac{\partial \xi}{\partial t} = \frac{\partial F_v}{\partial x} - \frac{\partial F_u}{\partial y}$$

Vorticity equation

$$\frac{\partial u}{\partial t} - fv = -uu_x - vu_y - wu_z - p_x/\rho \quad (1)$$

$$\frac{\partial v}{\partial t} + fu = -uv_x - vv_y - wv_z - p_y/\rho \quad (2)$$

$$\frac{\partial}{\partial x}(2) - \frac{\partial}{\partial y}(1) \quad \rightarrow$$

$$\frac{\partial \xi}{\partial t} + fD + \beta v = -u\xi_x - v\xi_y - \xi D - \frac{\partial}{\partial x}(wv_z + p_y/\rho) + \frac{\partial}{\partial y}(wu_z + p_x/\rho)$$

which leads to

$$\frac{\partial \zeta}{\partial t} = \frac{\partial}{\partial x}\underbrace{\{-u\zeta - wv_z - p_y/\rho\}}_{F_v} - \frac{\partial}{\partial y}\underbrace{\{-v\zeta - wu_z - p_x/\rho\}}_{F_u}$$

Divergence equation $\frac{\partial}{\partial x}(1) + \frac{\partial}{\partial y}(2) \quad \rightarrow$

$$\frac{\partial D}{\partial t} - f\xi + \beta u = -u_x^2 - uu_{xx} - v_x u_y - vu_{xy} - (wu_z + p_x/\rho)_x - u_y v_x - uv_{xy} - v_y^2 - vv_{yy} - (wv_z + p_y/\rho)_y$$

which eventually leads to

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial x}F_u + \frac{\partial}{\partial y}F_v - \frac{1}{2}\nabla^2(u^2 + v^2)$$

In sigma coordinates

From Hoskins and Simmons (1975): λ = longitude, μ = sin(latitude)

vorticity **ZT**
$$\frac{\partial \zeta}{\partial t} = \frac{1}{1-\mu^2}\frac{\partial}{\partial \lambda}\mathscr{F}_V - \frac{\partial}{\partial \mu}\mathscr{F}_U, \qquad (1)$$

divergence **DT**
$$\frac{\partial D}{\partial t} = \frac{1}{1-\mu^2}\frac{\partial}{\partial \lambda}\mathscr{F}_U + \frac{\partial}{\partial \mu}\mathscr{F}_V - \nabla^2\left(\underbrace{\frac{U^2+V^2}{2(1-\mu^2)}}_{\text{EG}} + \phi + \bar{T}\ln p_*\right), \qquad (2)$$

thermodynamic **TT**
$$\frac{\partial T'}{\partial t} = -\frac{1}{1-\mu^2}\underbrace{\frac{\partial}{\partial \lambda}(UT')}_{\text{UTG}} - \underbrace{\frac{\partial}{\partial \mu}(VT')}_{\text{VTG}} \underbrace{+ DT' - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \kappa\frac{T\omega}{p}}_{\text{TNLG}}, \qquad (3)$$

continuity **VP**
$$\frac{\partial \ln p_*}{\partial t} = \underbrace{-\mathbf{V}\cdot\nabla\ln p_*}_{\text{VGPG}} - D - \frac{\partial \dot{\sigma}}{\partial \sigma}, \qquad (4)$$

hydrostatic
$$\frac{\partial \phi}{\partial \ln \sigma} = -T. \qquad (5)$$

$$\underbrace{\mathscr{F}_U = V\zeta - \dot{\sigma}\frac{\partial U}{\partial \sigma} - T'\frac{\partial \ln p_*}{\partial \lambda}}_{\text{FUG}}, \quad \underbrace{\mathscr{F}_V = -U\zeta - \dot{\sigma}\frac{\partial V}{\partial \sigma} - T'(1-\mu^2)\frac{\partial \ln p_*}{\partial \mu}}_{\text{FVG}}.$$

Semi-implicit time scheme

2-5 can be summarized as

$$\partial \mathbf{D}/\partial t = \mathscr{D} - \nabla^2(\phi + \mathbf{T}\ln p_*),$$
$$\partial \mathbf{T}'/\partial t = \mathscr{T} - \tau \mathbf{D},$$
$$\partial \ln p_*/\partial t = \mathscr{P} - \pi \mathbf{D},$$
$$\phi - \phi_* = \text{TMPB}$$

Flux

Source

Eliminating for divergence gives

$$\left(\frac{\partial^2}{\partial t^2} - \mathbf{B}\nabla^2\right)\mathbf{D} = \frac{\partial \mathscr{D}}{\partial t} - \nabla^2(\mathbf{g}\mathscr{T} + \mathbf{T}\mathscr{P}).$$

gravity wave source

which is discretized as

$$(\mathbf{I} - \mathbf{B}\Delta t^2\nabla^2)\overline{\mathbf{D}}^t = \text{RCN *DMI}^{\Delta t} + \Delta t[\text{RCN *DT}\ \nabla^2(\text{TMPB +OROG} + \mathbf{T}\ \text{T0*SPMI})] - \Delta t^2\nabla^2\ \text{TMPA} + \text{T0*VP}.$$

where bar denotes average across a centred time difference - effectively filtering the generation of gravity waves and allowing a longer time step.

Note that the Laplacian operator becomes an algebraic multiplier in spectral space.

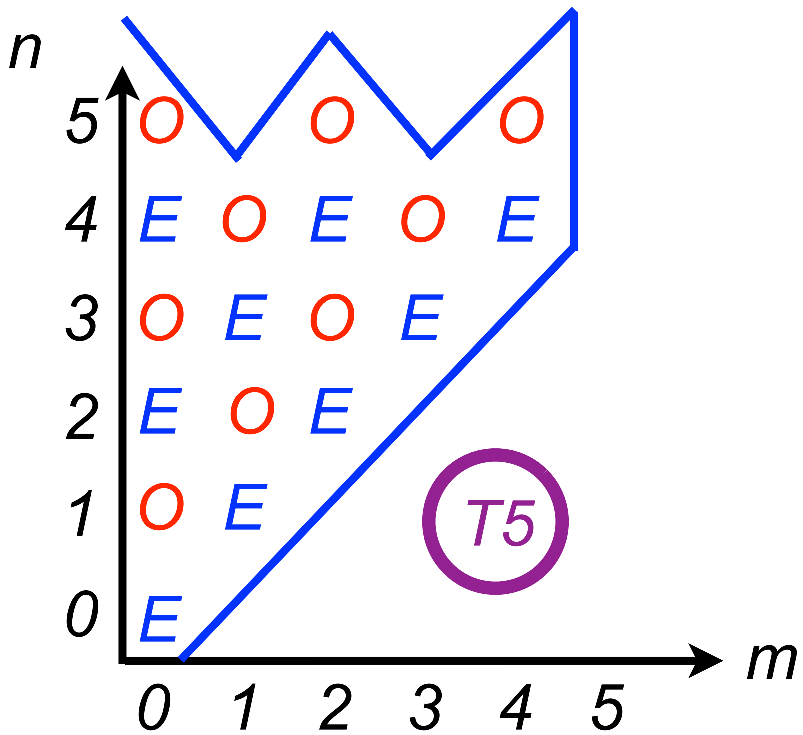
Spectral transforms

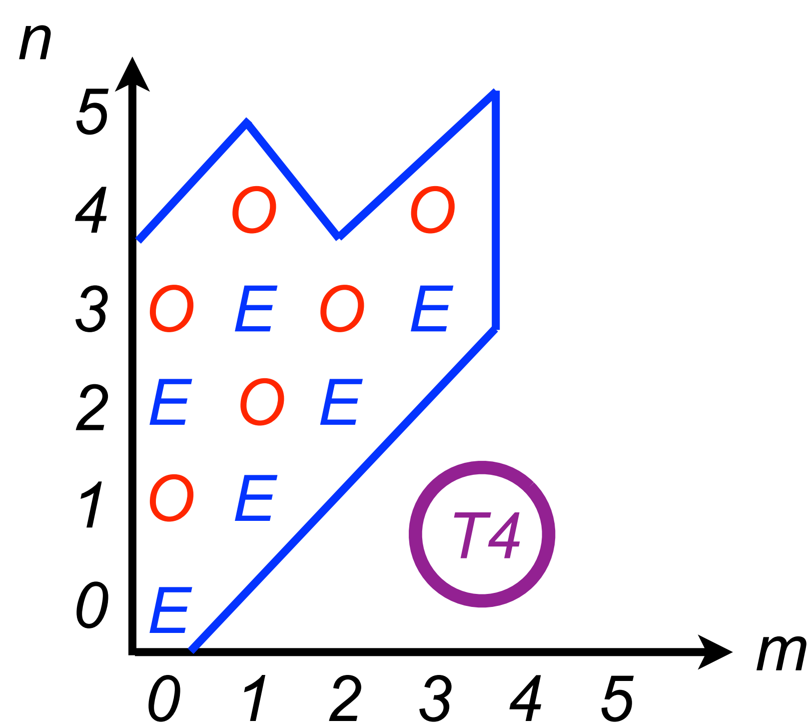
Model variables are projected onto Fourier transforms in the zonal direction and Legendre polynomials in the meridional direction.

$$X = \Sigma X_n^m P_n^m(\mu) e^{im\lambda}$$

Where n=meridional wave number, m=zonal wave number

"Jagged triangular truncation" - gives equal numbers of even and odd symmetries about the equator for each zonal wavenumber





Data for each level (L=1,15 top to bottom) is stored as complex numbers increasing n,m,parity

for example T5: EEE,EE,EE,E,E,OOO,OO,OO,O,O for divergence, temperature and pressure
but: OOO,OO,OO,O,O,EEE,EE,EE,E,E for vorticity

Horizontal and vertical diffusion

Horizontal diffusion: 12h ∇^6

on vorticity, divergence, temperature and specific humidity

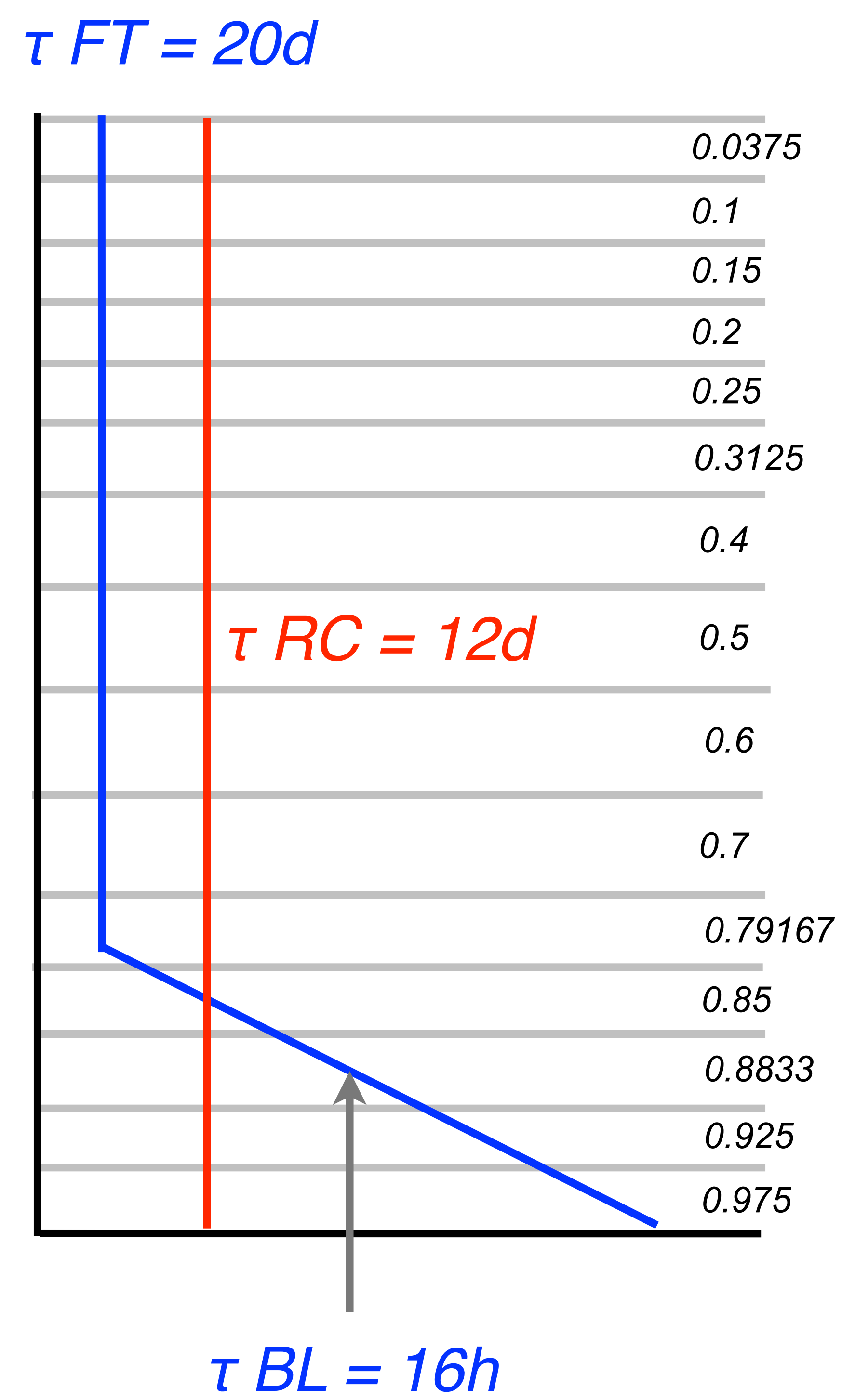
Vertical diffusion:
on vorticity, divergence, temperature and specific humidity.
Linear finite differences with a profile of timescales set at layer boundaries.
Top and bottom boundary conditions:
=> relaxation to observed mean at top and bottom levels.

Extra linear surface drag over land:
on momentum at the same timescale as for vertical diffusion in lowest layer only.

Radiative convective restoration:
on temperature only, independent of height.

These timescales are tunable parameters.

Remember: every time you change a parameter, you have to recalculate the forcing **g**.



Data structure

A binary history record is written in the form

WRITE(9)RKOUNT,RNTAPE,DAY,Z,D,T,SP,Q,RNTAPE

where RKOUNT is the current time step, RNTAPE=100., DAY=RKOUNT/TSPD = real day number

Z,D,T,SP,Q are complex arrays for absolute vorticity, divergence, temperature, ln(p(bar)),specific humidity

these are non-dimensionalized using

*time: Ω=WW=2π/23.93*3600 (s^{-1})*
speed: CV=aΩ (m/s)*
temperature: CT=CV^2/GASCON (GASCON=R=287.) T(non-d) = (T(K) - 250.) / CT
humidity: already dimensionless (kg/kg).

Data is regularly transformed to grid space where it is stored on a MG x JGG grid in latitude pairs, closing in from the poles to the equator i.e. for a given level:

J=1(north):(MG+2), J=JGG(south):(MG+2), J=2, J=JGG-1,... (note JGG=2JG).

Gridpoint operations are carried out one latitude-pair at a time for all levels.

Note that in some routines the data is in grid space in the meridional direction but in Fourier coefficients in the zonal direction. At these points the data is still complex.

Code structure

Blue - operations in spectral space
Red - operations in grid space

Initialize constants

----------- START TRAINING LOOP

Initialize variables (read initial conditions)

----------- START TIME LOOP

Read forcing functions and reference state
Calculate associated grid point fields

Write history, restart, diagnostics (for first time step this is just the initial condition)

Check counters - if finished go to end of time loop

Preset spectral tendencies to zero
Calculate dynamical advective tendencies: MGRMLT
Perform adiabatic semi-implicit time step: TSTEP

Reset spectral tendencies to zero
Calculate physical diabatic tendencies: DGRMLT
    Deep convection, Vertical diffusion, Surface fluxes,
    Large scale condensation, Radiative-Convective relaxation.
Calculate horizontal diffusion and add empirical forcing to spectral tendency: DIFUSE
Perform diabatic time step: DSTEP

----------- END TIME LOOP

Write final restart record
Check training index and repeat

----------- END TRAINING LOOP

https://github.com/stephanieleroux/igcm_T42L15

README.md

Welcome to your DREAM's github page!

Dynamical **R**esearch **E**mpirical **A**tmospheric **M**odel

Credits:

- Nick Hall, LEGOS, Université de Toulouse, Toulouse, France.
- Stephanie Leroux, IGE, Grenoble, France.

To download the latest version of the model:

- Click on the green "Clone or download" button.
- More info on this wiki page: "How to download, install the model, and how to run your first test simulation?".

Any QUESTIONS?

- Check out our wiki pages here.
- If you don't find your answer in the wiki pages, post an "issue" on github (click on "issues" in grey in the menu of the igcm_T42L15 page).

Disclaimer:

- This github page for the model is very recent and still under construction. Don't expect to find here a complete documentation for the model yet. We are working on it and we will add more and more information on these pages with time. Any suggestion is welcome.

The dataset

Reanalysis data is used to provide initial conditions, reference states and to calculate the empirical forcing for the model.

We use gridded ERAi fields of u,v,T,q,φ to calculate spectral coefficients of model variables:

u,v -> vorticity (Z) and divergence (D)
T,q -> temperature and specific humidity (T,Q)
φ,T at 1000mb -> ln(msl pressure)

$$ln\left(p_{msl}/1000\right) = \left(\frac{gz}{RT}\right)_{1000}$$

Mean sea level pressure is used instead of surface pressure since orography is not included in the model.

The mean effect of orography is represented indirectly by the empirical forcing (see later).

Timing

4x daily data for 38 years: 1979-2016

This is 13880 days, 55520 records, 901864 bytes per record, 50 GB of data.
There are 1461 records every 365.25 days

record	year	month	date	hour	RKOUNT	DAY	DAY modulo 365.25	record modulo 1461	cycle number
1	1979	Jan	1	0	0	0.00	0.00	1	1
2	1979	Jan	1	6	16	0.25	0.25	2	
3	1979	Jan	1	12	32	0.50	0.50	3	
4	1979	Jan	1	18	48	0.75	0.75	4	
5	1979	Jan	2	0	64	1.00	1.00	5	
1 459	1979	Dec	31	12	23 328	364.50	364.50	1 459	1
1 460	1979	Dec	31	18	23 344	364.75	364.75	1 460	
1 461	1980	Jan	1	0	23 360	365.00	365.00	1 461	
1 462	1980	Jan	1	6	23 376	365.25	0.00	0	2
1 463	1980	Jan	1	12	23 392	365.75	0.25	1	
55 517	2016	Dec	31	0	888 256	13 879.00	364.75	1 460	38
55 518	2016	Dec	31	6	888 272	13 879.25	365.00	1 461	
55 519	2016	Dec	31	12	888 288	13 879.50	0.00	1	39
55 520	2016	Dec	31	18	888 304	13 879.75	0.25	2	

Flow separation: Time-mean and transients

Consider the development of the observed atmosphere

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = f - \mathcal{D}(q)$$

Separate into time-mean and transients

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}', \quad q = \overline{q} + q'$$

Time-mean of development equation

$$\overline{\mathbf{v}}.\nabla\overline{q} + \mathcal{D}(\overline{q}) = \overline{f} - \overline{\mathbf{v}'.\nabla q'}$$

Transient fluxes can be viewed as a forcing.

Lets represent the entire system as a state vector

$$\mathbf{\Phi} = (u, v, q, ...)$$

which thus develops according to

$$\frac{d\mathbf{\Phi}}{dt} + (\mathcal{A} + \mathcal{D})\mathbf{\Phi} = \mathbf{f}$$

Separate into time-mean and transients

$$\frac{d\mathbf{\Phi}'}{dt} + (\mathcal{A} + \mathcal{D})(\mathbf{\Phi}_c + \mathbf{\Phi}') = \overline{\mathbf{f}} + \mathbf{f}'$$

or

$$\frac{d\mathbf{\Phi}'}{dt} + (\mathcal{A} + \mathcal{D})\mathbf{\Phi}_c + \mathcal{L}_c(\mathbf{\Phi}') + O(\mathbf{\Phi}'^2) = \overline{\mathbf{f}} + \mathbf{f}'$$

The time-mean budget is

$$(\mathcal{A} + \mathcal{D})\mathbf{\Phi}_c + \overline{O(\mathbf{\Phi}'^2)} = \overline{\mathbf{f}}$$

and the transient eddy budget is

$$\frac{d\mathbf{\Phi}'}{dt} + \mathcal{L}_c(\mathbf{\Phi}') + \left[O(\mathbf{\Phi}'^2) - \overline{O(\mathbf{\Phi}'^2)}\right] = \mathbf{f}'$$

- Time-mean advection is balanced by transient eddy fluxes and forcing.
- Each term in the eddy budget has a zero time-mean - there is no large cancellation.
- The time-mean state is a realistic basis.
- So perturbations may be compared with observed transient systems.
- The structure of small perturbations may be relevant to observed transient systems.

Forcing a simple GCM

Back to our development equation

$$\frac{d\mathbf{\Phi}}{dt} + (\mathcal{A}+\mathcal{D})\mathbf{\Phi} = \mathbf{f}(t)$$

Introduce a model

$$\frac{d\mathbf{\Psi}}{dt} + (\mathcal{A}+\mathcal{D})\mathbf{\Psi} = \mathbf{g}$$

Key assumption: ***g*** *is time-independent.*
Set

$$\mathbf{g} = \overline{\mathbf{f}}$$

How do we find ***g*** *?*

Run the model without forcing, for one timestep.

$$\frac{d\mathbf{\Psi}}{dt} + (\mathcal{A}+\mathcal{D})\mathbf{\Psi} = 0 \quad \Rightarrow \quad (\mathcal{A}+\mathcal{D})\mathbf{\Psi} = -\frac{d\mathbf{\Psi}}{dt}$$

Do this many times, initialising with a series of data realisations $\mathbf{\Phi}_i$

$$\mathbf{g} = \frac{1}{n}\sum_{i=1}^{n}(\mathcal{A}+\mathcal{D})\mathbf{\Phi}_i$$

If we use this forcing to perform a long integration of the model we can compare our simulation with the dataset we used to generate the empirical forcing.

This method guarantees that the total generalised flux from the model simulation will be the same as in the observations, i.e.

$$\overline{(\mathcal{A}+\mathcal{D})\mathbf{\Psi}} = \overline{(\mathcal{A}+\mathcal{D})\mathbf{\Phi}}$$

But it does not guarantee that the simulated time-mean flow will be realistic

$$\overline{\mathbf{\Psi}} \neq \mathbf{\Phi}_c$$

Neither does it guarantee that the transient fluxes will be realistic. This is because mean flow and transient fluxes can balance differently in

$$(\mathcal{A}+\mathcal{D})\overline{\mathbf{\Psi}} + \overline{O(\mathbf{\Psi}'^2)} = (\mathcal{A}+\mathcal{D})\mathbf{\Phi}_c + \overline{O(\mathbf{\Phi}'^2)}$$

Forcing a perturbation model

Back to our development equation

$$\frac{d\mathbf{\Phi}}{dt} + (\mathcal{A} + \mathcal{D})\mathbf{\Phi} = \mathbf{f}(t)$$

What happens if we define a forcing

$$\mathbf{h} = (\mathcal{A} + \mathcal{D})\mathbf{\Phi}_c$$

And then intialise the model with $\mathbf{\Psi} = \mathbf{\Phi}_c$?

$$\frac{d\mathbf{\Psi}}{dt} + (\mathcal{A} + \mathcal{D})\mathbf{\Psi} = \mathbf{h}$$

... Nothing of course !

The model with run and stick on its initial condition without developing. But if we add a perturbation $\mathbf{\Psi}_1$ to the initial condition, the development equation becomes

$$\frac{d\mathbf{\Psi}_1}{dt} + \mathcal{L}_c(\mathbf{\Psi}_1) + O(\mathbf{\Psi}_1^2) = 0$$

If we make sure $\mathbf{\Psi}_1$ is small (and remains small) we have a linear perturbation model

$$\frac{d\mathbf{\Psi}_1}{dt} + \mathcal{L}_c(\mathbf{\Psi}_1) = 0$$

The solution to this equation gives the normal mode structure associated with the climatology $\mathbf{\Phi}_c$ (or any other basic state, with appropriate ***h***)

If $\mathcal{L}_c \mathbf{e}_n(x, y, z) = \lambda_n \mathbf{e}_n(x, y, z)$

and $\lambda_n = \sigma + i\omega$

Then $\mathbf{\Psi}_1 = \mathbf{e}_n(x, y, z) e^{(\sigma + i\omega)t}$ or

$$\mathbf{\Psi}_1 = [A(x, y, z)\cos\omega t + B(x, y, z)\sin\omega t]\, e^{\sigma t}$$

If instead of adding a perturbation to the initial condition we add a perturbation to the forcing, we can obtain the linear response to a forcing anomaly

$$\frac{d\mathbf{\Psi}_1}{dt} + \mathcal{L}_c(\mathbf{\Psi}_1) = \mathbf{h}_1$$

with solution $\Psi_{1j}(t) = \dfrac{h_{1j}}{\lambda_j}\left(e^{\lambda_j t} - 1\right)$

(for j^{th} projection of Ψ_1 and h_1 onto L^T and j^{th} eigenvalue of L)

or steady (asymptotic) solution $\mathbf{\Psi}_1 = \mathcal{L}_c^{-1}\mathbf{h}_1$

(which we can find by timestepping provided all eigenmodes have negative σ)

The difference between g and h

Recall our tracer advection formulation

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = f - \mathcal{D}(q)$$

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$$

$$q = \overline{q} + q'$$

time mean of this

$$\overline{\mathbf{v}}.\nabla\overline{q} + \mathcal{D}(\overline{q}) = \overline{f} - \overline{\mathbf{v}'.\nabla q'}$$

time-mean advection/dissipation

diabatic forcing

transient-eddy forcing

which translates to

$$(\mathcal{A} + \mathcal{D})\mathbf{\Phi}_c = \overline{\mathrm{f}} - \overline{O({\mathbf{\Phi}'}^2)} \quad \mathbf{h}$$

$$\mathbf{g}$$

$$\mathbf{h} = \mathbf{g} + \mathbf{TE}$$

fli = frc + fed

A word on damping and restoration

So far we have not discussed the nature of *A* and *D* except to say they represent advection and dissipation. A popular way to force simple advective models is through “restoration” to a radiative-convective equilibrium state or a reference climatology.

So it’s worth outlining the connection between our empirical forcing approach and the more intuitive restoration approach, which we imagine as a spring, which returns the atmospheric state to what it would be if there were no dynamical fluxes, or at least prevents a model from straying too far from a realistic state.

If *D* is linear and diagonal, the two approaches are mathematically identical.

$$\frac{d\mathbf{\Psi}}{dt} + \mathcal{A}(\mathbf{\Psi}) = \mathbf{g} - \mathcal{D}(\mathbf{\Psi})$$

$$\mathbf{g} = R\mathbf{\Phi}^*, \quad \mathcal{D} = R$$

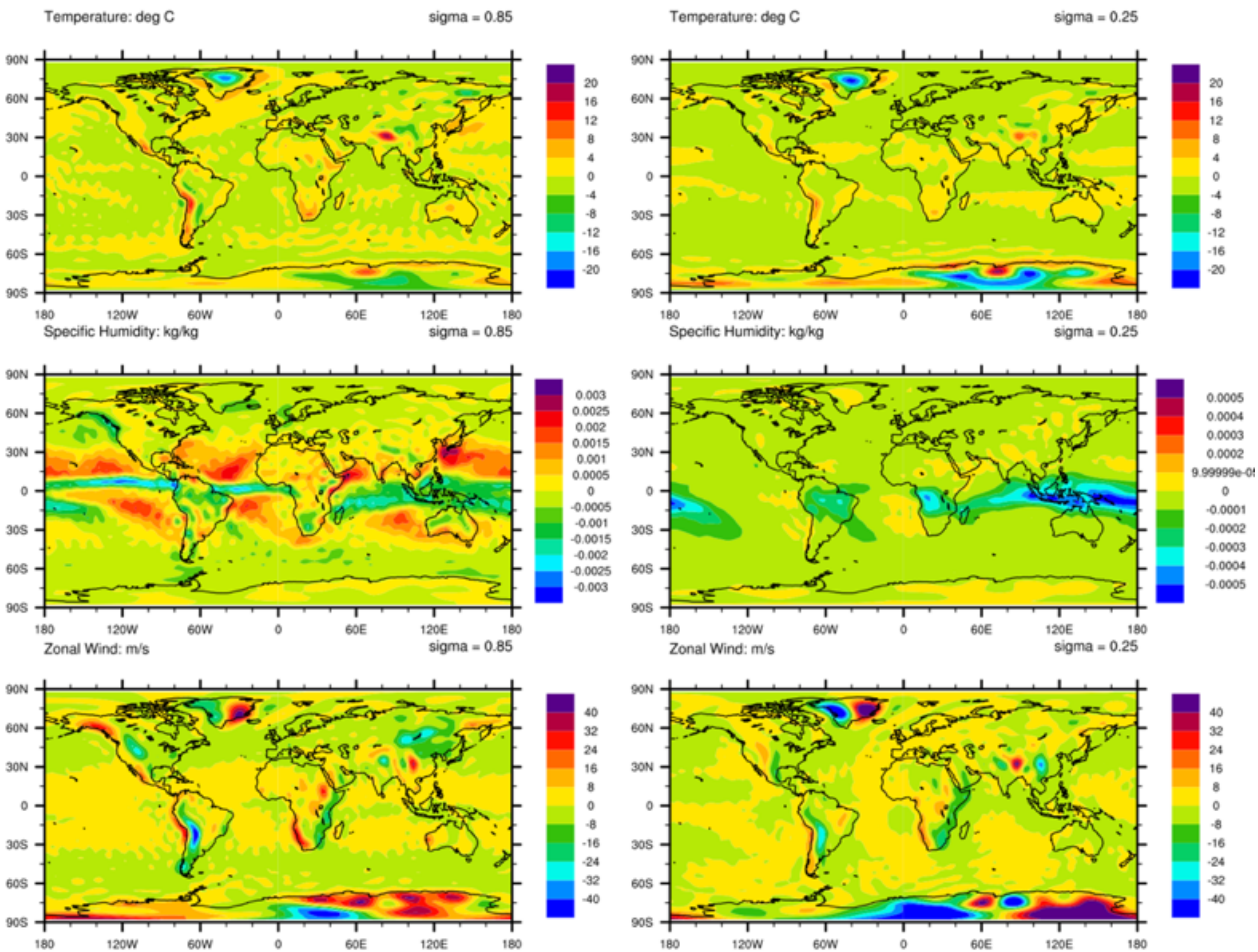
$$\frac{d\mathbf{\Psi}}{dt} + \mathcal{A}(\mathbf{\Psi}) = R(\mathbf{\Phi}^* - \mathbf{\Psi}) = R\mathbf{\Phi}^* - R\mathbf{\Psi}$$

So instead of specifying a damping rate and an ad-hoc restoration state,
we specify a damping rate and an objective empirical forcing.

(we could of course calculate the associated restoration state if interested, although our specification of D has off-diagonal elements)

Distribution of forcing

*Changes in one day associated with simple GCM forcing **g** for DJF*

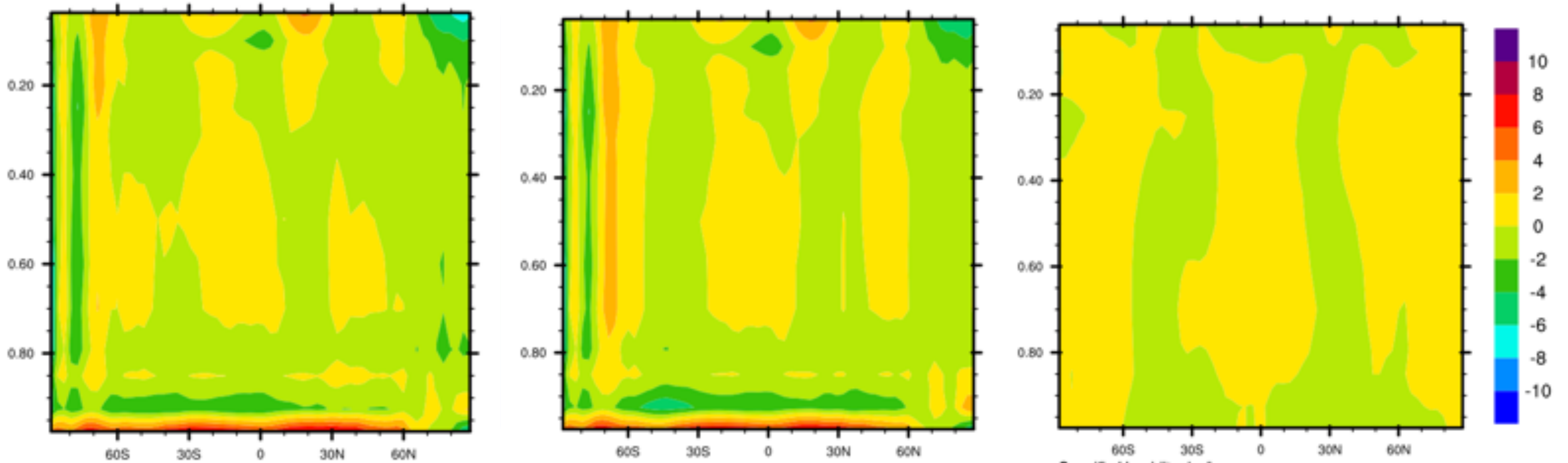


Distribution of forcing

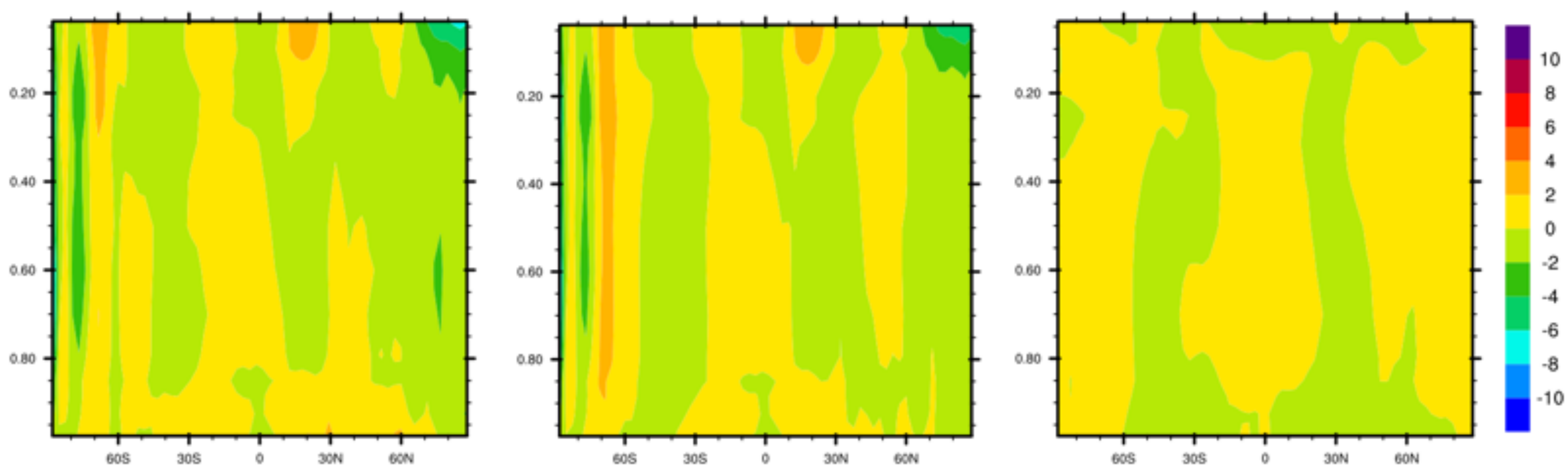
Changes in one day associated with different forcing components for DJF: TEMPERATURE

g *(.frc)* ***h*** *(.fli)* ***h – g*** *(.fed)*

Simple GCM (dry)



Purely dynamical model (no vertical diffusion, boundary fluxes or temperature restoration)

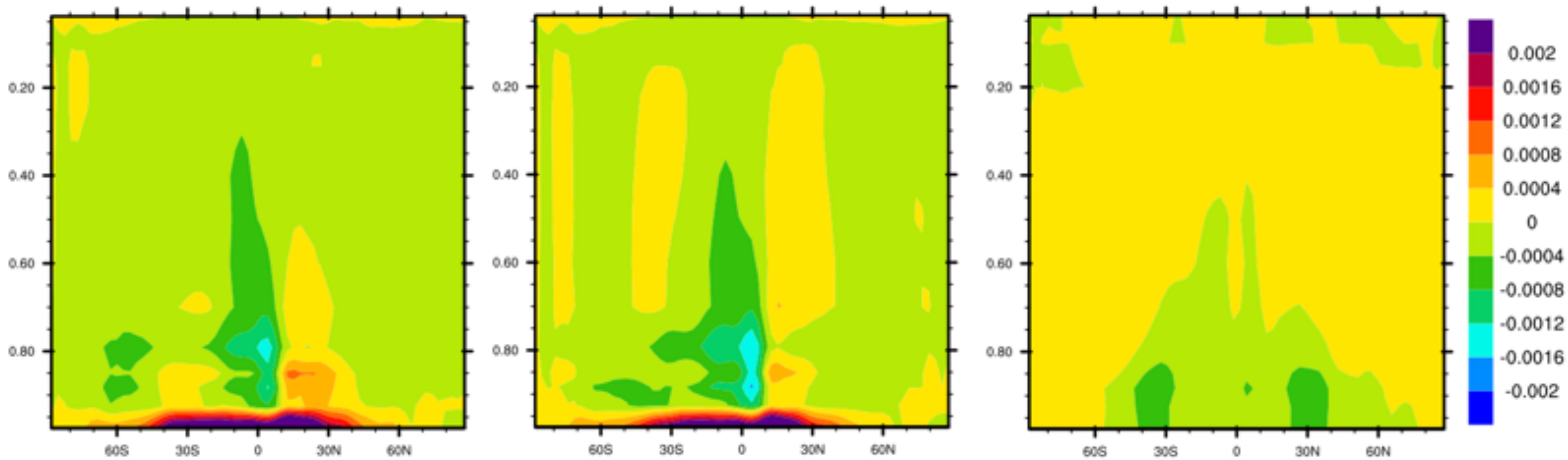


Distribution of forcing

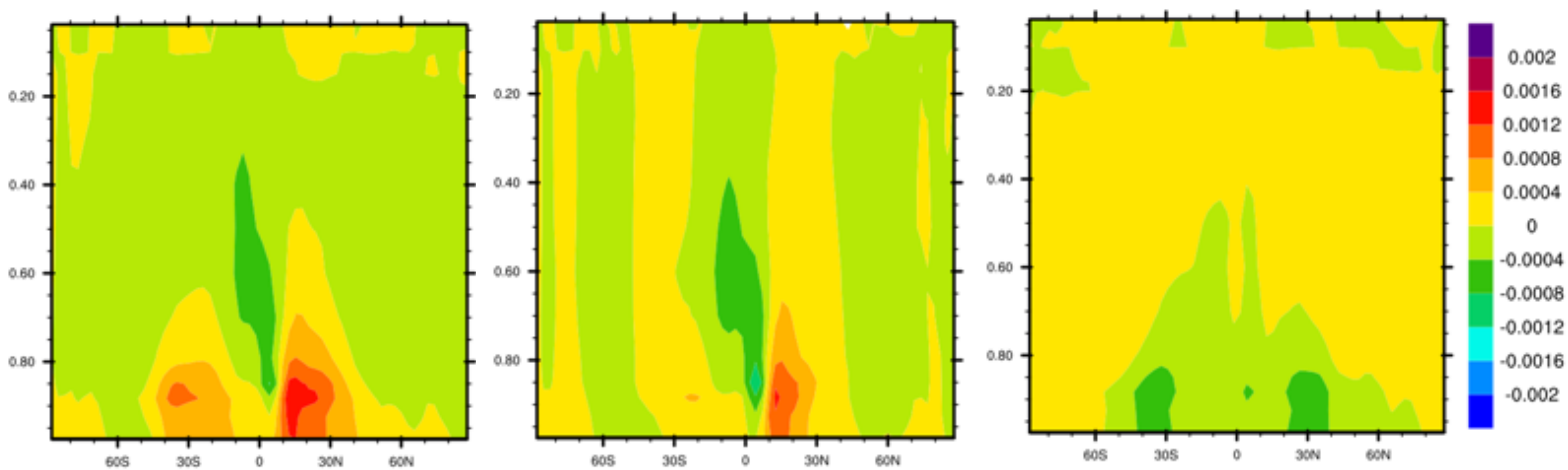
Changes in one day associated with different forcing components for DJF: SPECIFIC HUMIDITY

g *(.frc)* ***h*** *(.fli)* ***h – g*** *(.fed)*

Simple GCM (dry)



Purely dynamical model (no vertical diffusion, boundary fluxes or temperature restoration)



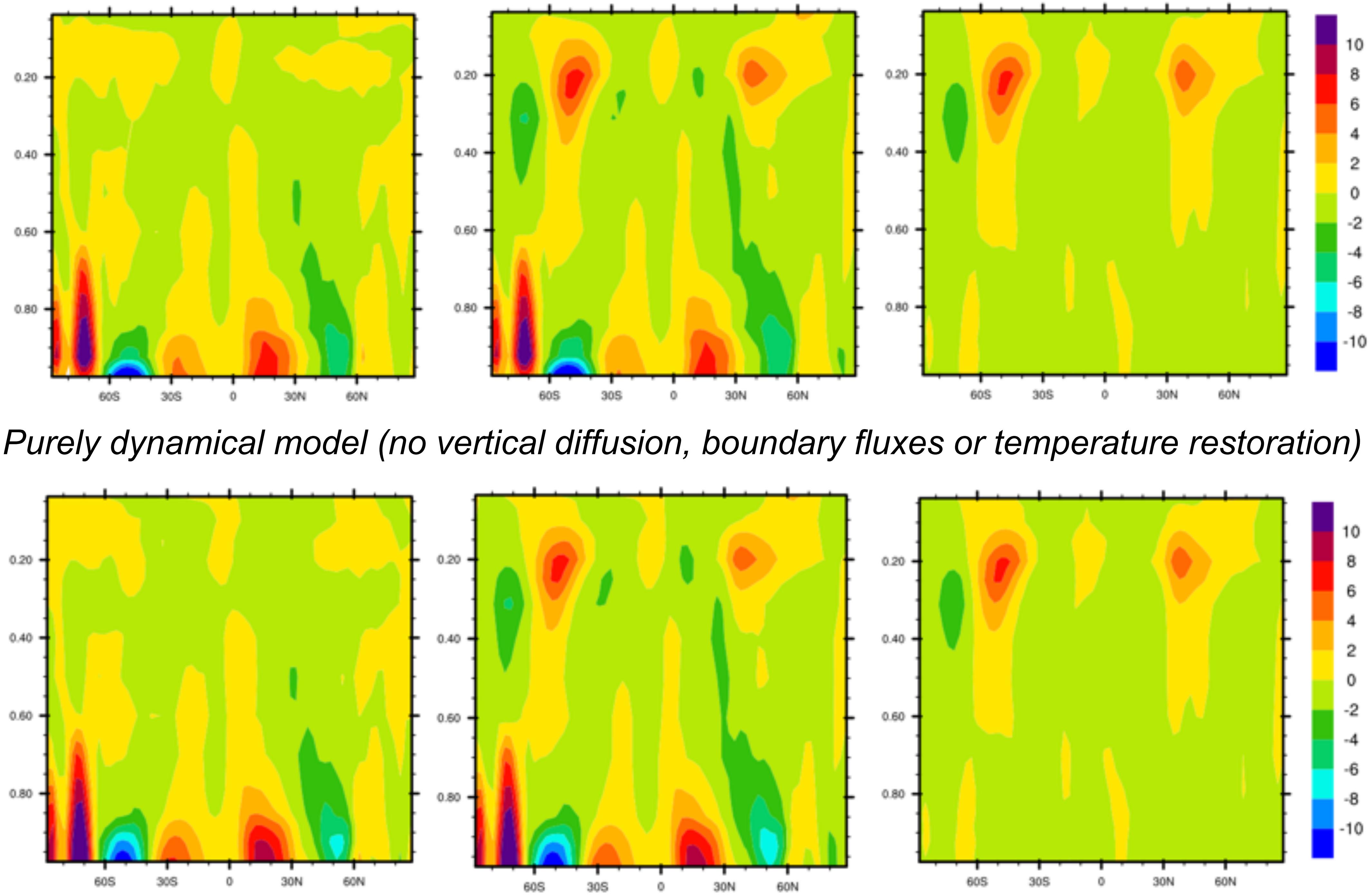
Distribution of forcing

Changes in one day associated with different forcing components for DJF: ZONAL WIND

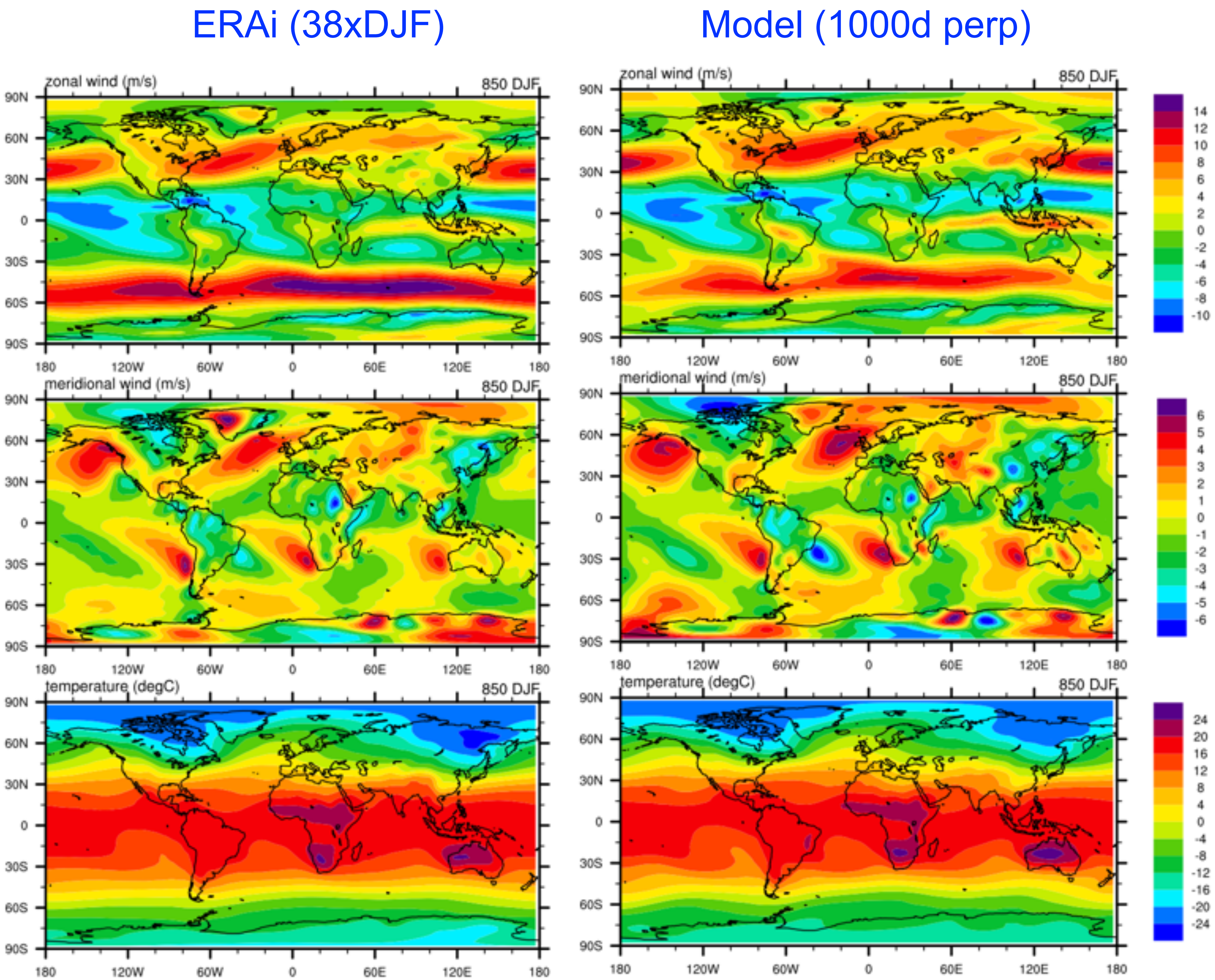
g *(.frc)* ***h*** *(.fli)* ***h – g*** *(.fed)*

Simple GCM (dry)

Purely dynamical model (no vertical diffusion, boundary fluxes or temperature restoration)



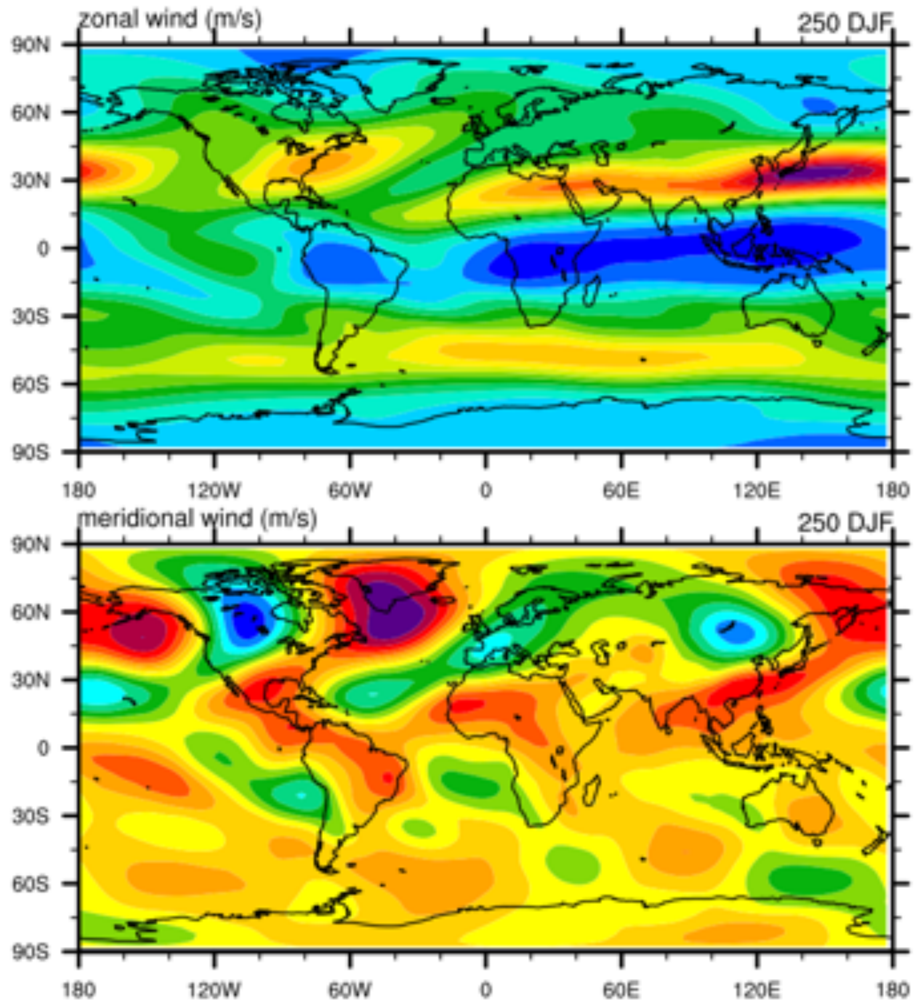
So... does it work ?



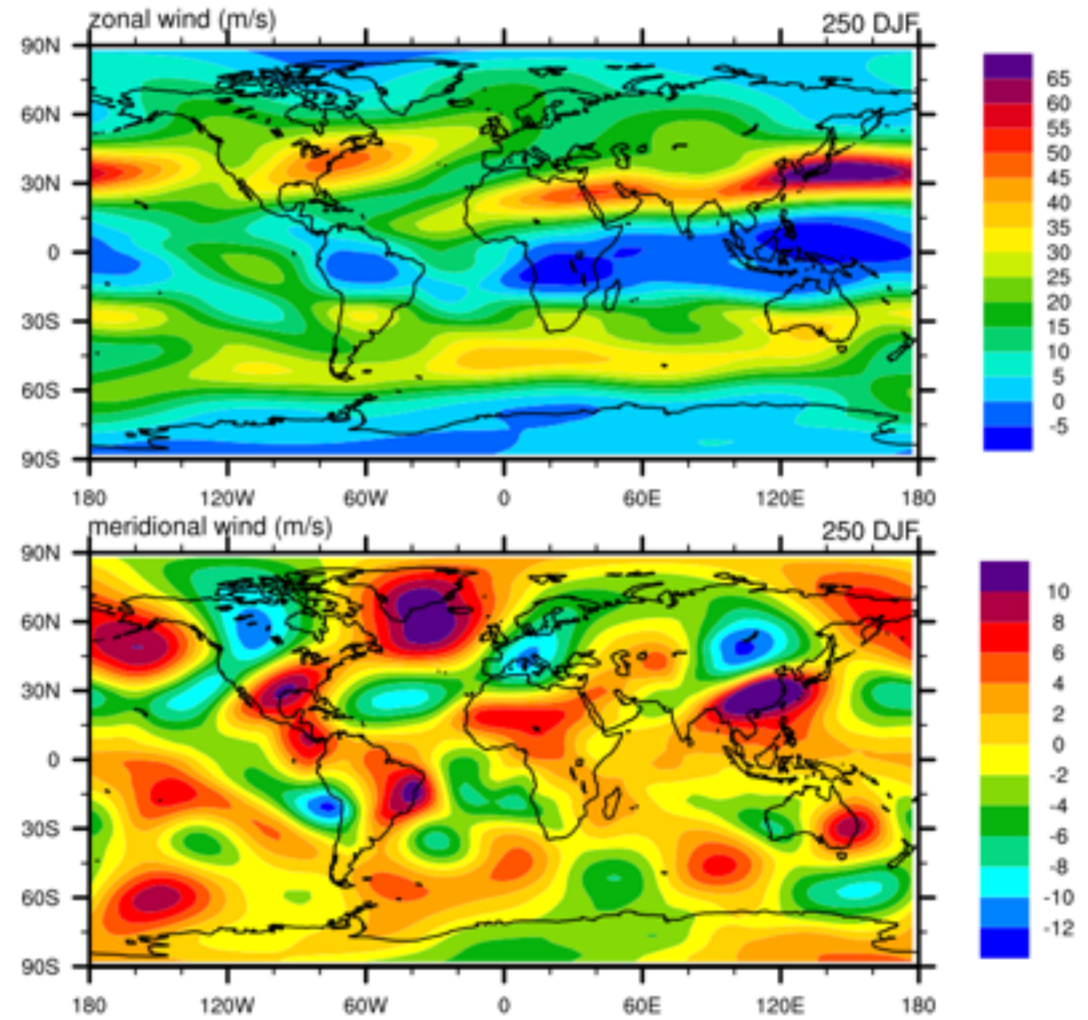
σ = 0.85

Mean fields

ERAi (38xDJF)



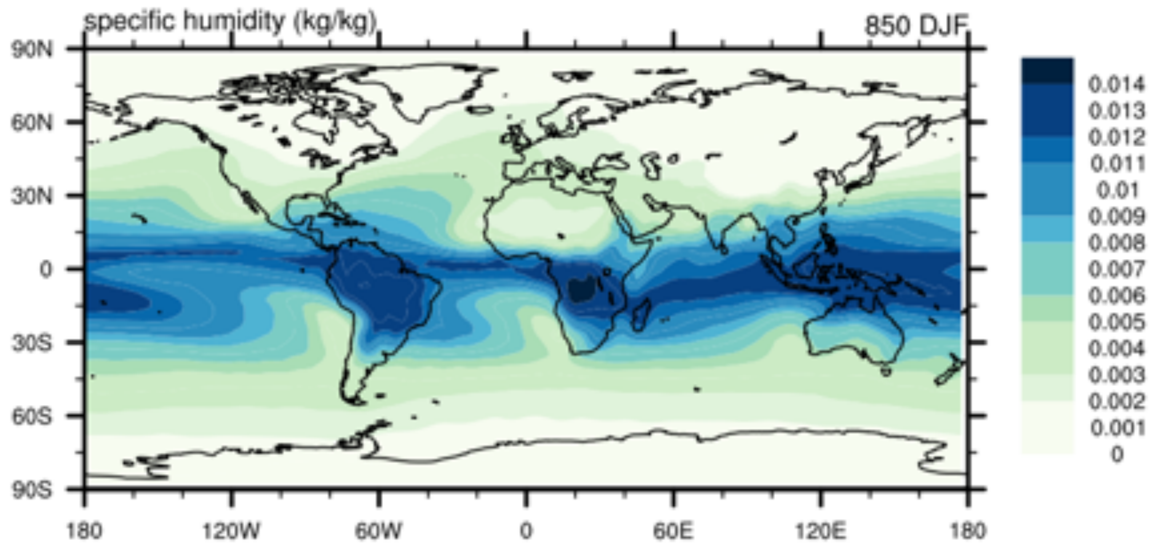
Model (1000d perp)

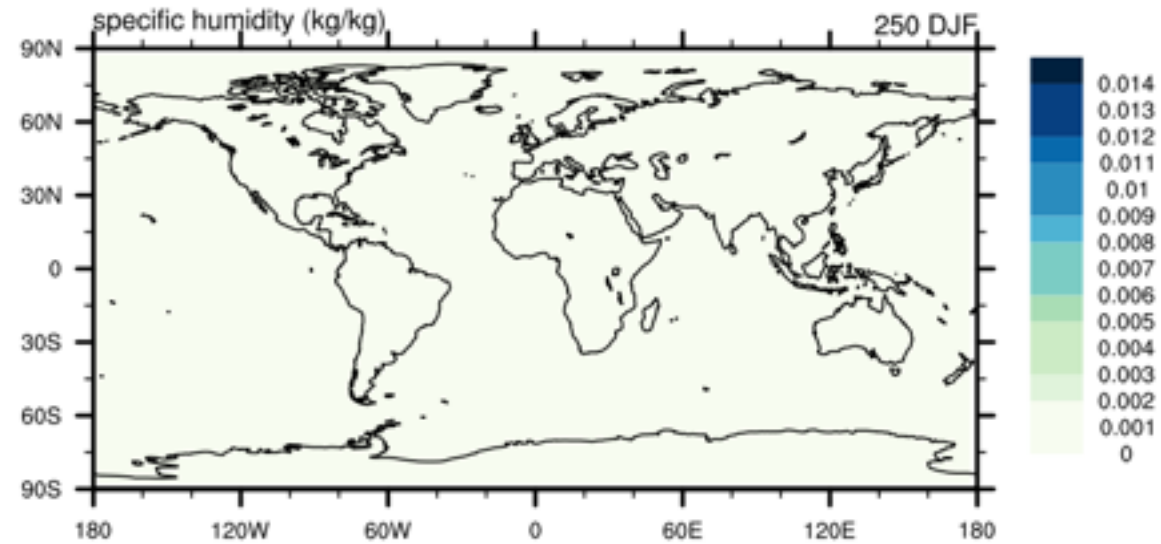


σ = 0.25

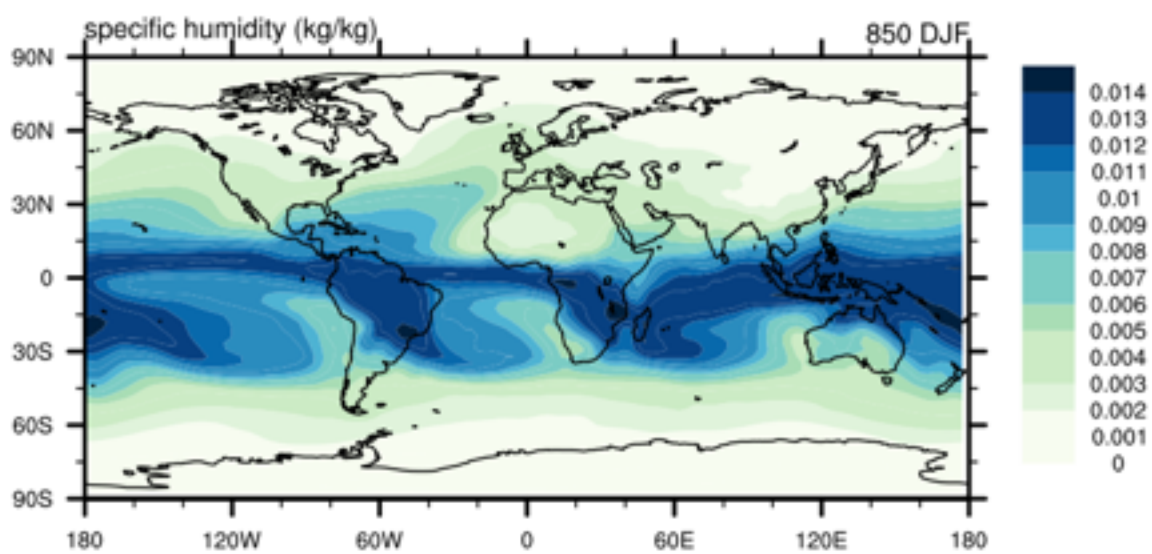
Mean fields

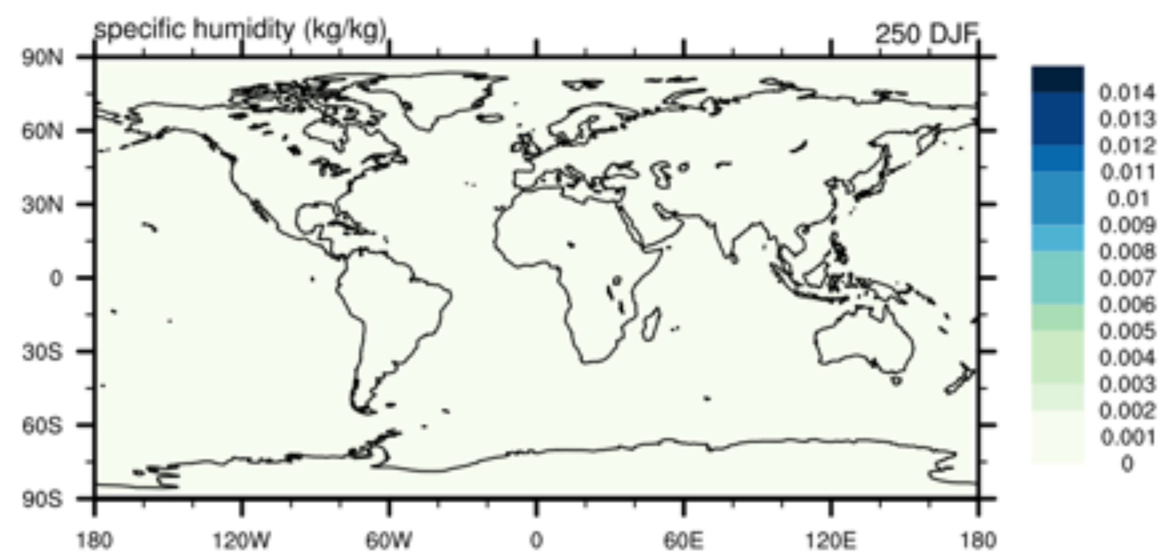
ERAi (38xDJF)





Model (1000d perp)





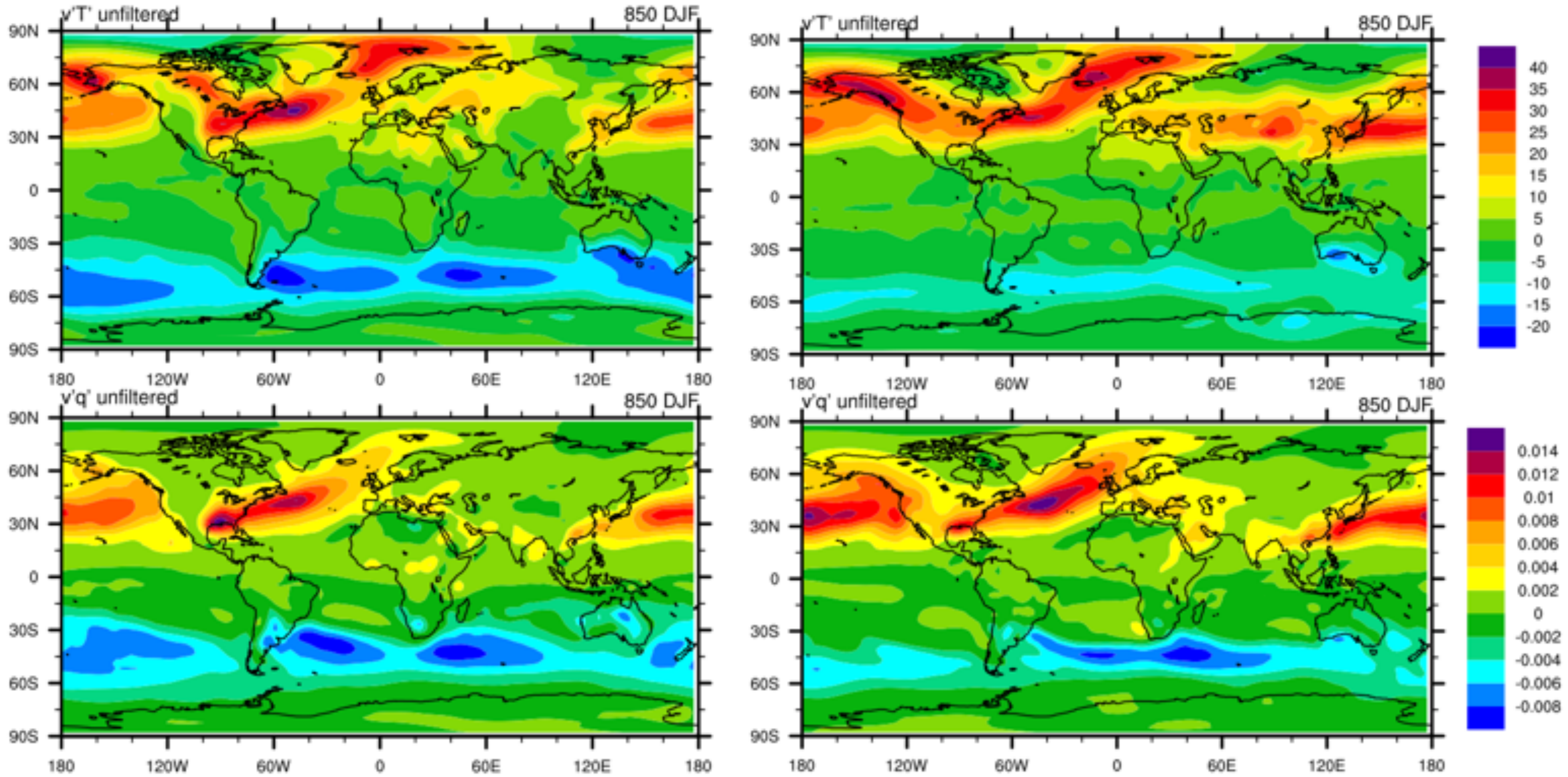
$\sigma = 0.85$

$\sigma = 0.25$

Unfiltered transients

ERAi (38xDJF)

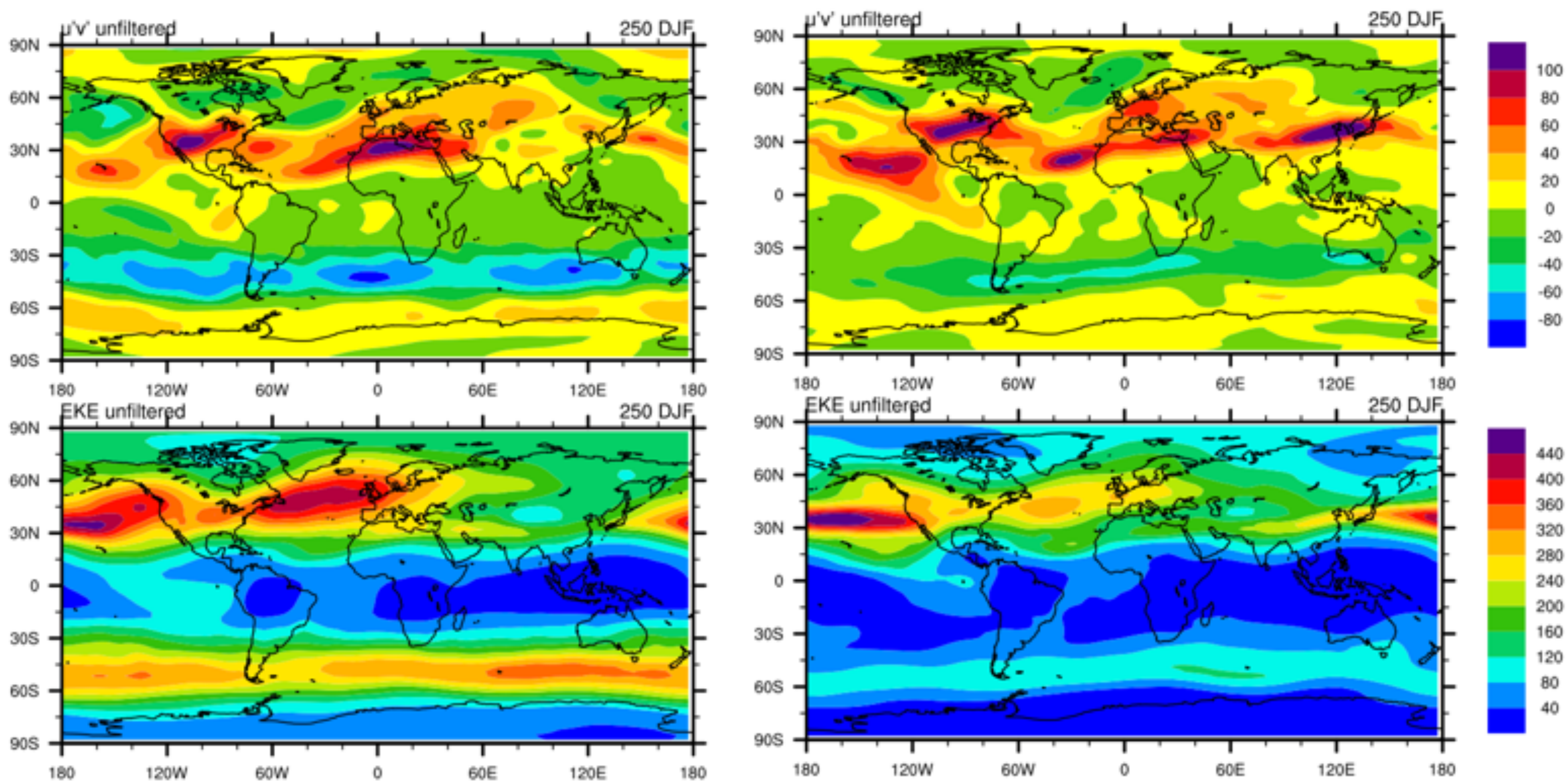
Model (1000d perp)



σ = 0.85

Unfiltered transients

ERAi (38xDJF) | Model (1000d perp)

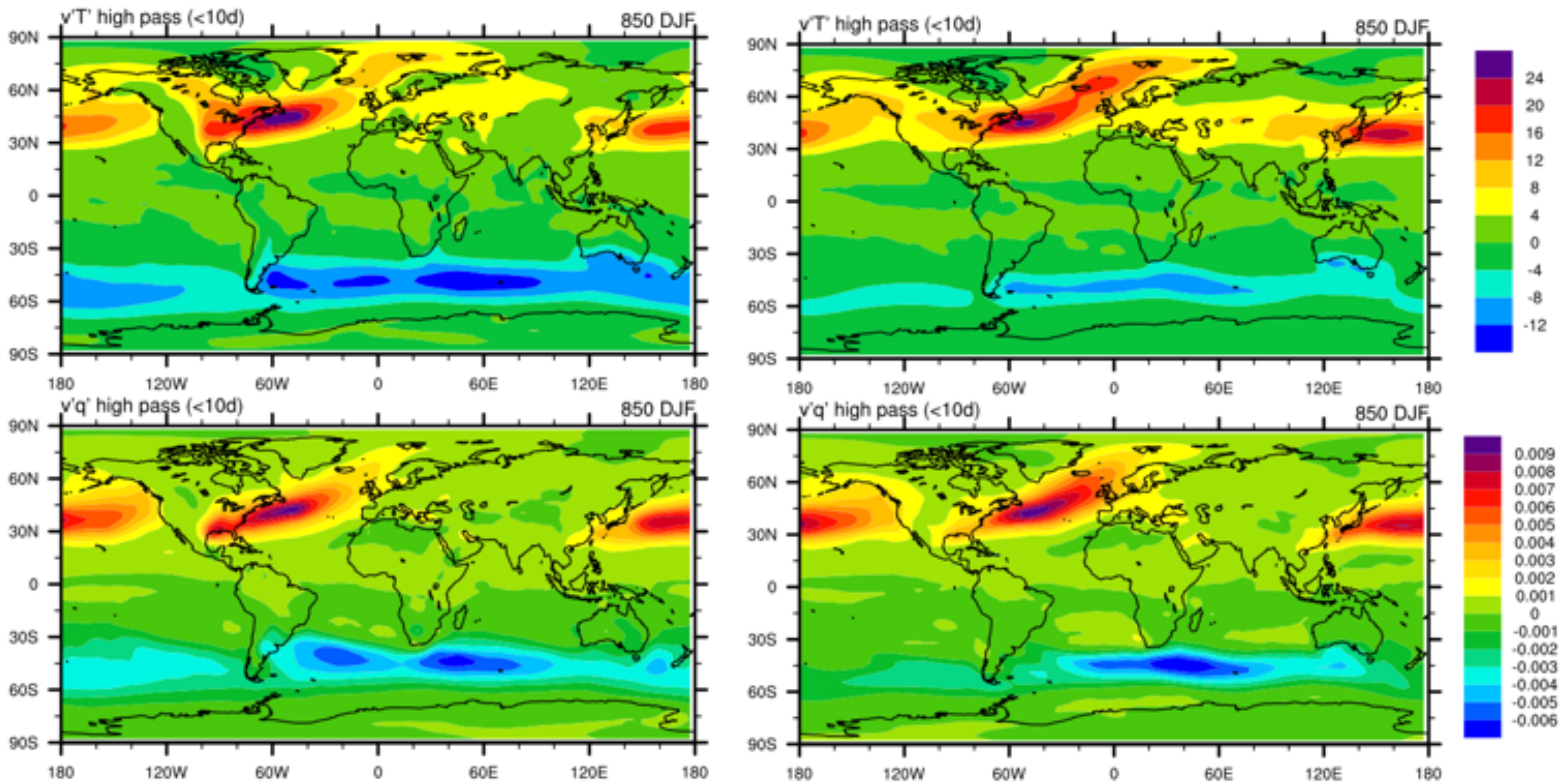


σ = 0.25

< 10-day filtered transients

ERAi (38xDJF)

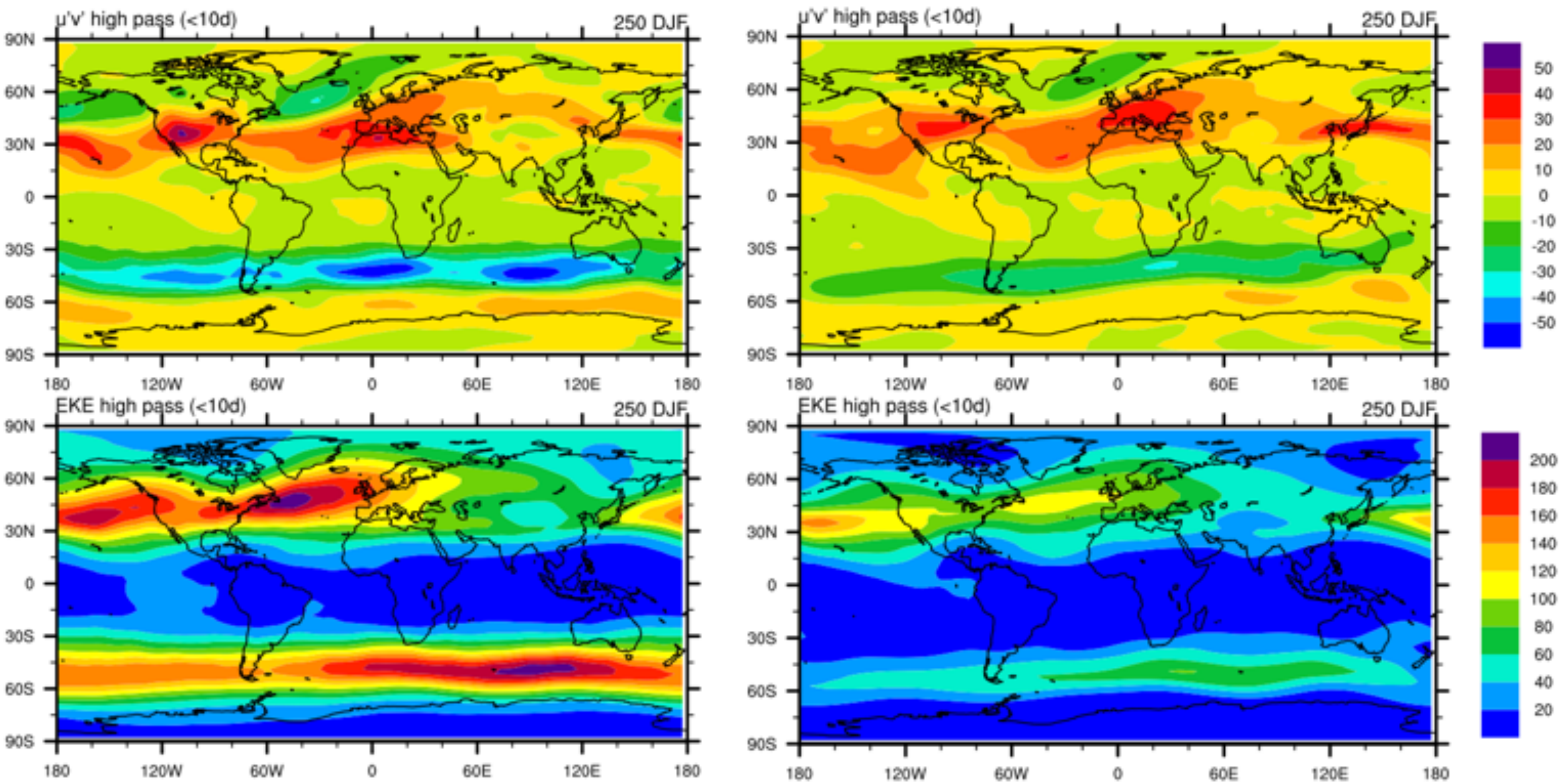
Model (1000d perp)



σ = 0.85

< 10-day filtered transients

ERAi (38xDJF)　　　　Model (1000d perp)



$\sigma = 0.25$


Everything you never wanted to know about...
The Annual Cycle
... but you were too polite to leave
Nick Hall: LEGOS, Univ. Toulouse.
Stephanie Leroux, IGE, Grenoble.
Tercio Ambrizzi, IAG, Univ. Sao Paulo.

Energy balance

Consider a linear dissipative system with a cyclic forcing. D essentially represents radiative cooling.

$$\frac{dT}{dt} + DT = f_o \sin \omega t$$

The solution is $$T = e^{-Dt} + \frac{D}{(\omega^2 + D^2)} f_0 \sin \omega t + \frac{\omega}{(\omega^2 + D^2)} f_0 \cos \omega t$$

homogeneous free damped response

equilibrium solution for strong dissipation (atmosphere)

out of phase forced response for weak dissipation (ocean)

If we assume that the effect of the last term is to modify the phase of the forcing, the atmospheric response is approximately

$$T = e^{-Dt} + \frac{f_0}{D} \sin \omega t$$

by the way, if we call this term T* we can write $$\frac{dT}{dt} = D(T^* - T)$$

Let's try to add some atmospheric dynamics: what is the effective forcing associated with departures of the atmospheric state from the fixed annual cycle solution ?

$$T = \widetilde{T} + T'$$

Add a quadratic term: $$\frac{dT}{dt} + DT + \alpha T^2 = f_o \sin \omega t$$

The solution for the annual cycle will involve a response to "forcing" associated with timescale interactions

$$\frac{d\widetilde{T}}{dt} + D\widetilde{T} = f_0 \sin \omega t - \alpha \left[\widetilde{T}^2 + 2\widetilde{\widetilde{T}T'} + \widetilde{T'^2} \right]$$

$$T = e^{-Dt} + \frac{f_0}{D} \sin \omega t - \frac{f_\alpha}{D}$$

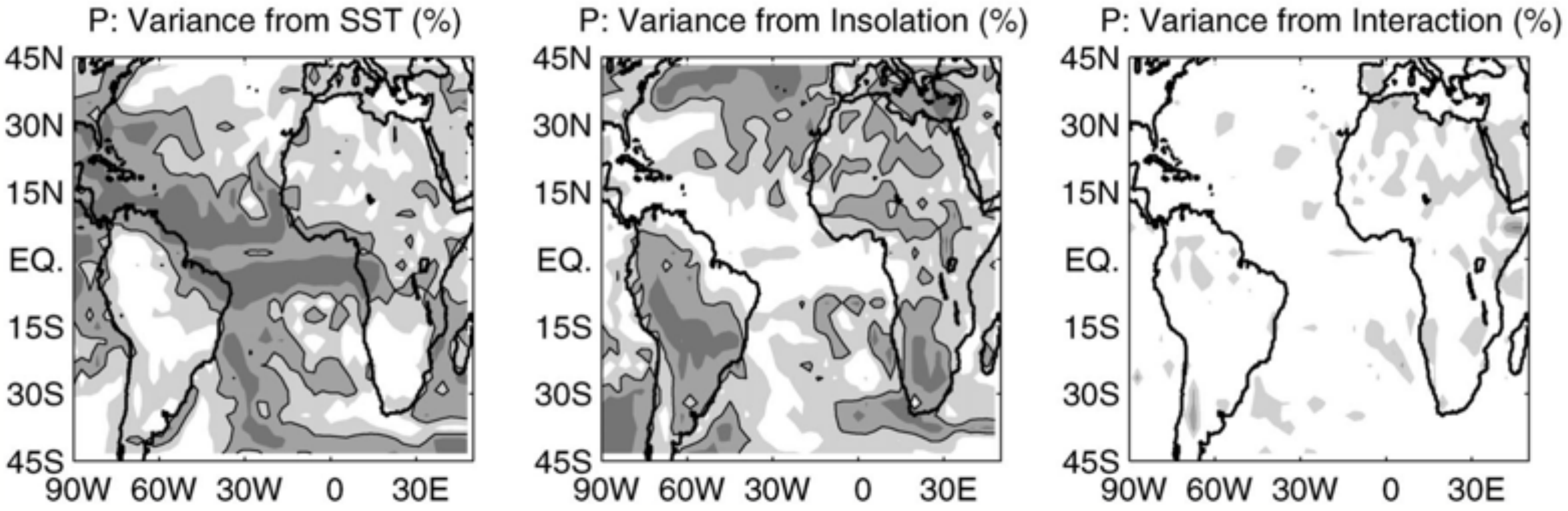
how important is this term ?

Contributions from TOA and SST

Both are important but in different regions as this analysis of variance from GCM experiments shows.

Three experiments: Control; fixed SST; fixed TOA insolation.

The annual cycle of SST determines a large part of the precipitation variance. But insolation is crucial to monsoon dynamics, generation of heat lows and monsoon fluxes.



Biasutti, Battisti and Sarachik (2003)

Dynamical balance: Annual cycle

Return to our development equation $$\frac{d\mathbf{\Phi}}{dt} + (\mathcal{A}+\mathcal{D})\mathbf{\Phi} = \mathbf{f}(t)$$

Separate state vector into time-man, annual cycle and transients, with cyclic forcing

$$\mathbf{\Phi} = \overline{\mathbf{\Phi}} + \widetilde{\mathbf{\Phi}} + \mathbf{\Phi}', \quad \mathbf{g} = \overline{\mathbf{f}} + \widetilde{\mathbf{f}}$$

So for the annual cycle we can write an equation for the forcing

$$\overline{\mathbf{f}} + \widetilde{\mathbf{f}} = \frac{d\widetilde{\mathbf{\Phi}}}{dt} + \widetilde{\overline{\mathcal{A}}}(\overline{\mathbf{\Phi}} + \widetilde{\mathbf{\Phi}} + \mathbf{\Phi}') + \mathcal{D}(\overline{\mathbf{\Phi}} + \widetilde{\mathbf{\Phi}})$$

Which expands into

$$\overline{\mathbf{f}} + \widetilde{\mathbf{f}} = \mathrm{TEND} + \overbrace{\underbrace{\mathrm{MM}}_{\text{MEAN ADV / DISS}} + \underbrace{\mathrm{MC} + \mathrm{CC}}_{\text{CYCLIC ADV / DISS}} + \underbrace{\mathrm{CT} + \mathrm{TT}}_{\text{TRANSIENT ADVECTION}}}^{\text{TOTAL ADVECTION / DISSIPATION}}$$

(NEGATIVE TRANSIENT EDDY FORCING)

The tendency term must be calculated directly from data.

The other terms can be found by carefully designed one-timestep experiments with the unforced model.

$$\mathrm{TEND} = \frac{d\widetilde{\mathbf{\Phi}}}{dt}$$

$$\mathrm{MM} = \mathcal{A}(\overline{\mathbf{\Phi}}, \overline{\mathbf{\Phi}}) + \mathcal{D}(\overline{\mathbf{\Phi}})$$

$$\mathrm{MC} = \mathcal{A}(\overline{\mathbf{\Phi}}, \widetilde{\mathbf{\Phi}}) + \mathcal{D}(\widetilde{\mathbf{\Phi}})$$

$$\mathrm{CC} = \mathcal{A}(\widetilde{\mathbf{\Phi}}, \widetilde{\mathbf{\Phi}})$$

$$\mathrm{CT} = \mathcal{A}(\widetilde{\mathbf{\Phi}}, \mathbf{\Phi}')$$

$$\mathrm{TT} = \mathcal{A}(\mathbf{\Phi}', \mathbf{\Phi}')$$

Finding the terms in the budget

Our definition of the annual cycle is the average for a given point in a 365.25 day cycle with 6-hourly ERAi data from 1/1/1979 to 31/12/2016 (38 years). This is then smoothed with a 10-day (41 pt) running mean.

The tendency term is evaluated as $(\mathbf{\Phi}^{+} - \mathbf{\Phi}^{-})$ */ 12 hours*

For MM: initialise unforced model with $\overline{\mathbf{\Phi}} \rightarrow (\mathcal{A}+\mathcal{D})(\overline{\mathbf{\Phi}})$

1 RUN

Separate the linear dissipative part by comparing with a model that has no dissipation.

For MC and CC: initialise with $\overline{\mathbf{\Phi}} + \widetilde{\mathbf{\Phi}}$

This gives MM+MC+CC so we now know MC+CC.

Now initialise with $\overline{\mathbf{\Phi}} + \alpha\widetilde{\mathbf{\Phi}}$ *(set α = 1.1)*

1461 RUNS

If A is quadratic this gives MM + 2αMC + α²CC and we can deduce MC and CC with simple algebra.

For CT and TT initialise with $\overline{\mathbf{\Phi}} + \widetilde{\mathbf{\Phi}} + \mathbf{\Phi}'$

This gives MM+MC+CC+CT+TT, thence CT+TT

Now initialise with $\overline{\mathbf{\Phi}} + \widetilde{\mathbf{\Phi}} + \alpha\mathbf{\Phi}'$

55518 RUNS

This gives MM+MC+CC + 2αCT + α²TT

and thus CT and TT, so now we've collected them all !

$$\mathrm{TEND} = \frac{d\widetilde{\mathbf{\Phi}}}{dt}$$

$$\mathrm{MM} = \mathcal{A}(\overline{\mathbf{\Phi}}, \overline{\mathbf{\Phi}}) + \mathcal{D}(\overline{\mathbf{\Phi}})$$

$$\mathrm{MC} = \mathcal{A}(\overline{\mathbf{\Phi}}, \widetilde{\mathbf{\Phi}}) + \mathcal{D}(\widetilde{\mathbf{\Phi}})$$

$$\mathrm{CC} = \mathcal{A}(\widetilde{\mathbf{\Phi}}, \widetilde{\mathbf{\Phi}})$$

$$\mathrm{CT} = \mathcal{A}(\widetilde{\mathbf{\Phi}}, \mathbf{\Phi}')$$

$$\mathrm{TT} = \mathcal{A}(\mathbf{\Phi}', \mathbf{\Phi}')$$

Summary table for forcing terms

TOTAL ADVECTION / DISSIPATION

(NEGATIVE TRANSIENT EDDY FORCING)

$$\overline{\mathbf{f}} + \widetilde{\mathbf{f}} = \mathrm{TEND} + \mathrm{MM} + \mathrm{MC} + \mathrm{CC} + \mathrm{CT} + \mathrm{TT}$$

MEAN ADV / DISS

CYCLIC ADV / DISS

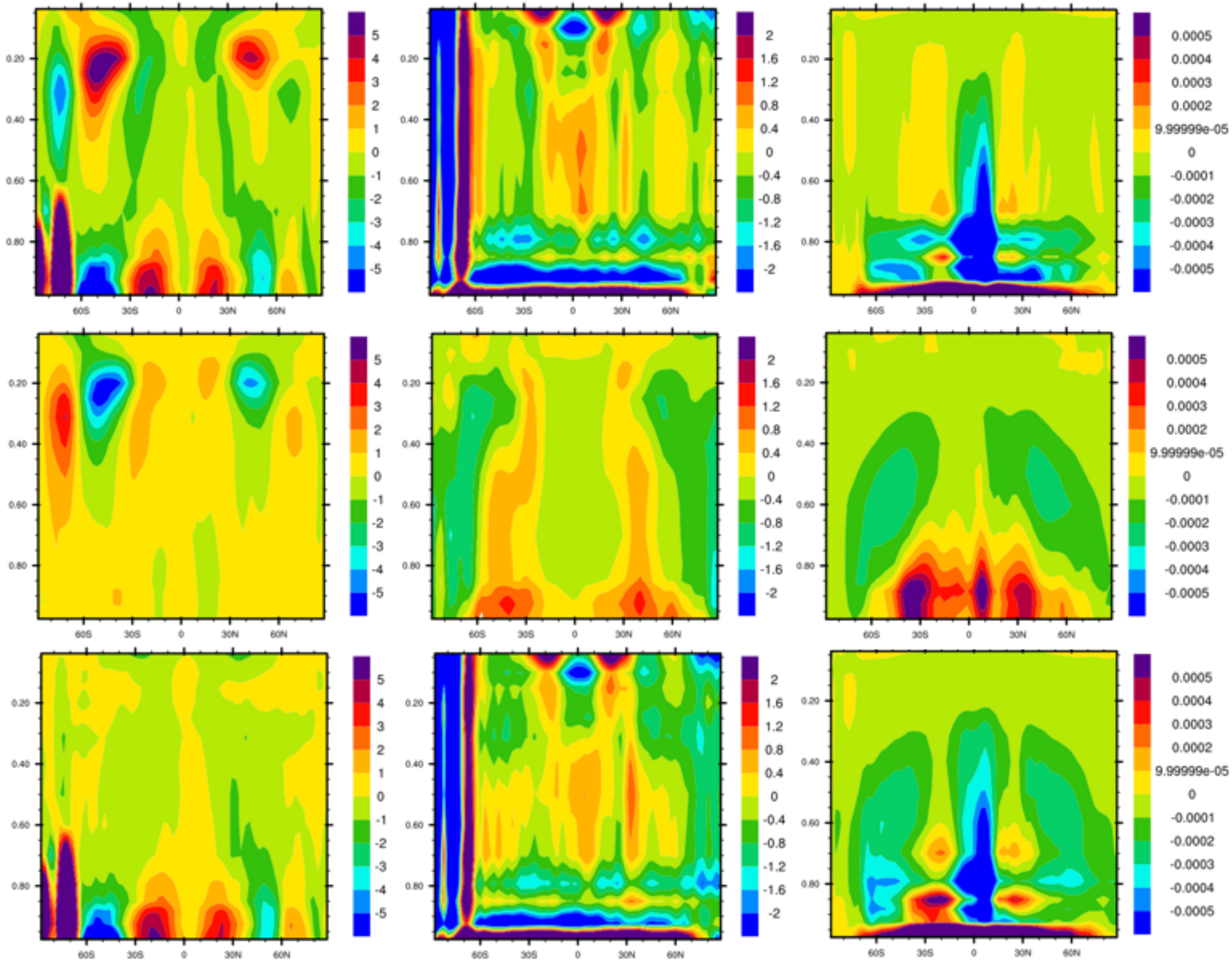
TRANSIENT ADVECTION

Term	*Contributes to*		*Linear*	*Nonlinear*
	Mean	*Cycle*		
$\mathrm{TEND} = \frac{d\widetilde{\mathbf{\Phi}}}{dt}$				
$\mathrm{MM} = \mathcal{A}(\overline{\mathbf{\Phi}}, \overline{\mathbf{\Phi}}) + \mathcal{D}(\overline{\mathbf{\Phi}})$				
$\mathrm{MC} = \mathcal{A}(\overline{\mathbf{\Phi}}, \widetilde{\mathbf{\Phi}}) + \mathcal{D}(\widetilde{\mathbf{\Phi}})$				
$\mathrm{CC} = \mathcal{A}(\widetilde{\mathbf{\Phi}}, \widetilde{\mathbf{\Phi}})$				
$\mathrm{CT} = \mathcal{A}(\widetilde{\mathbf{\Phi}}, \mathbf{\Phi}')$				
$\mathrm{TT} = \mathcal{A}(\mathbf{\Phi}', \mathbf{\Phi}')$				

Annual mean advection

u (ms^{-1}/day) T (deg/day) q (day^{-1})

Forcing required to balance:



advection by mean: MM

transient advection: MC+CC+CT+TT

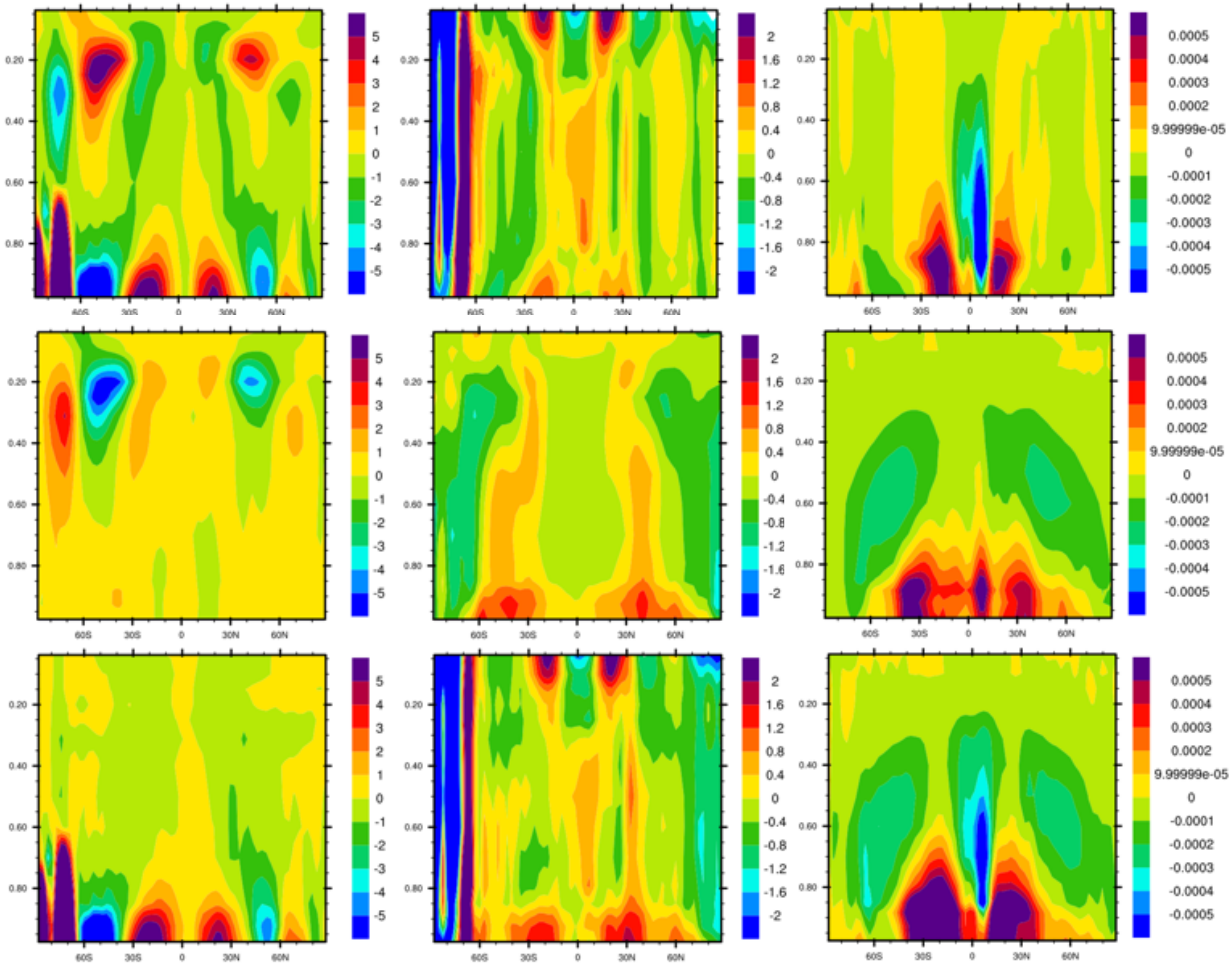
total forcing: TEND+MM+MC +CC+CT+TT

(TEND and MC have zero annual mean)

Annual mean advection: No dissipation

Following plots will be without dissipation

u (ms^{-1}/day) | T (deg/day) | q (day^{-1})



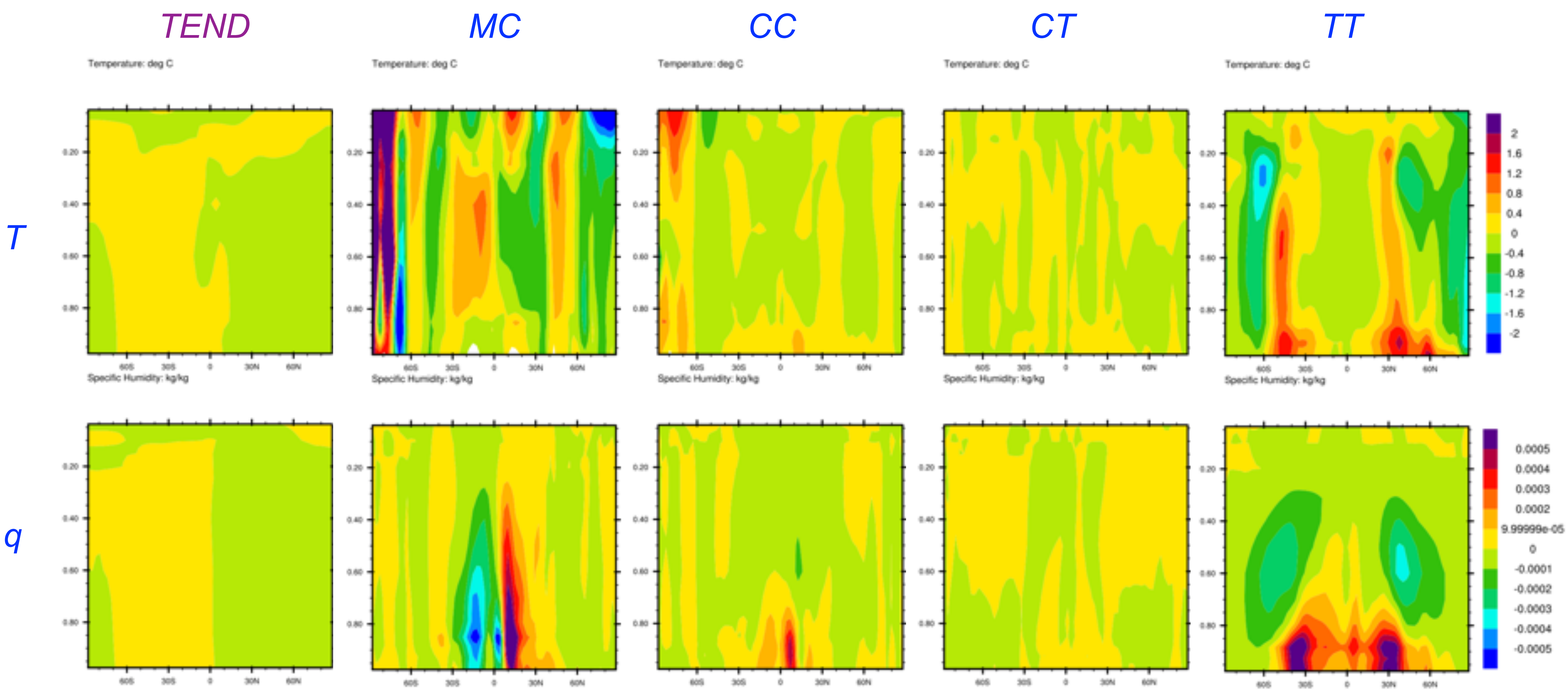
advection by mean: MM

transient advection: MC+CC+CT+TT

total forcing: TEND+MM+MC +CC+CT+TT

(TEND and MC have zero annual mean)

DJF



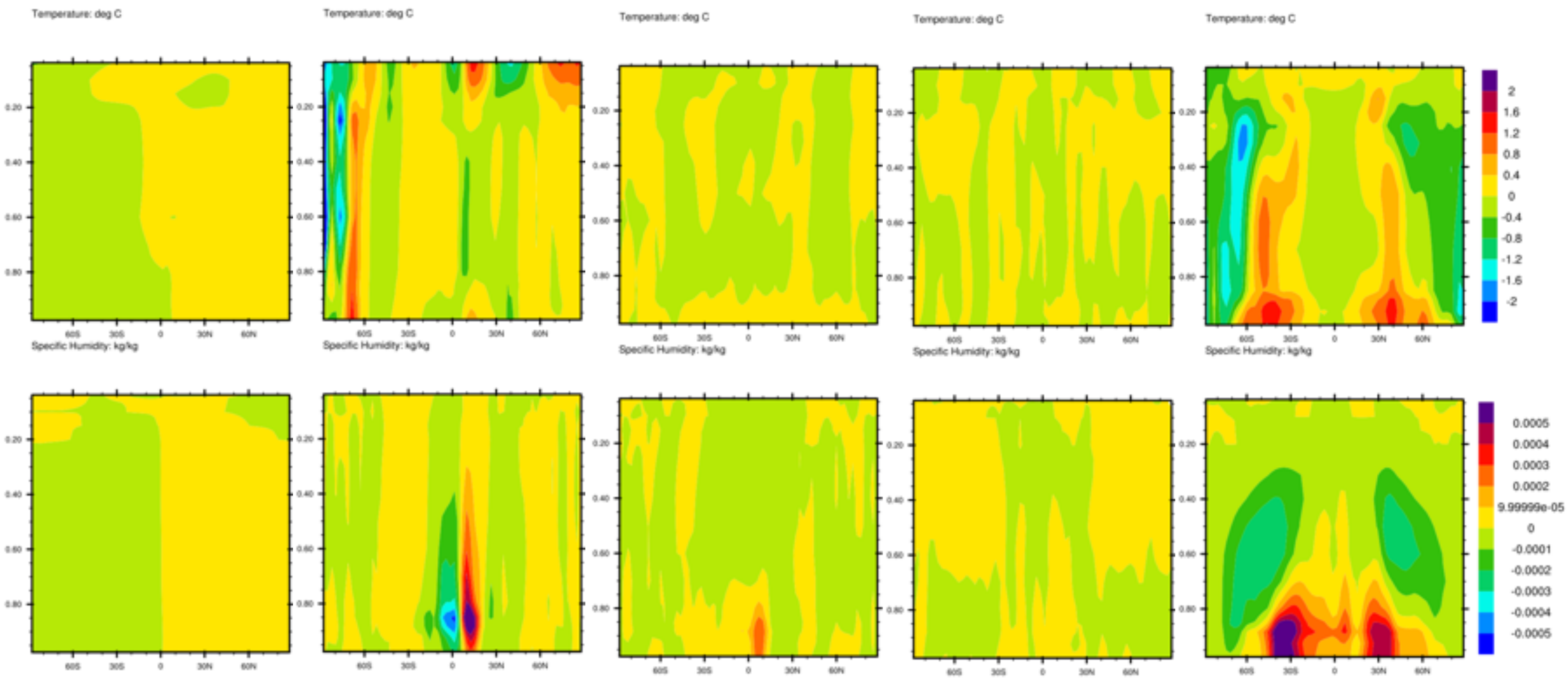
the tendency term - absent from a perpetual run - is small

MAM

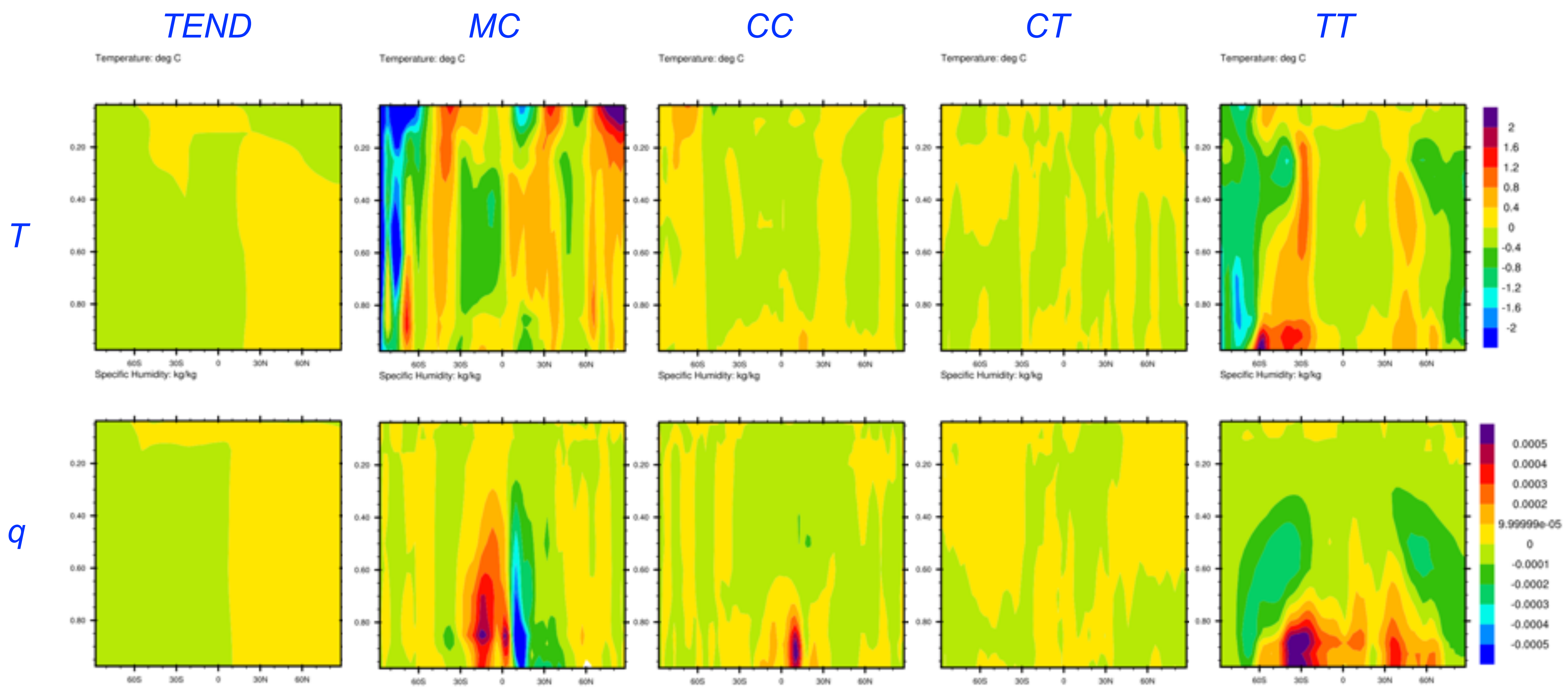
TEND MC CC CT TT

T

q

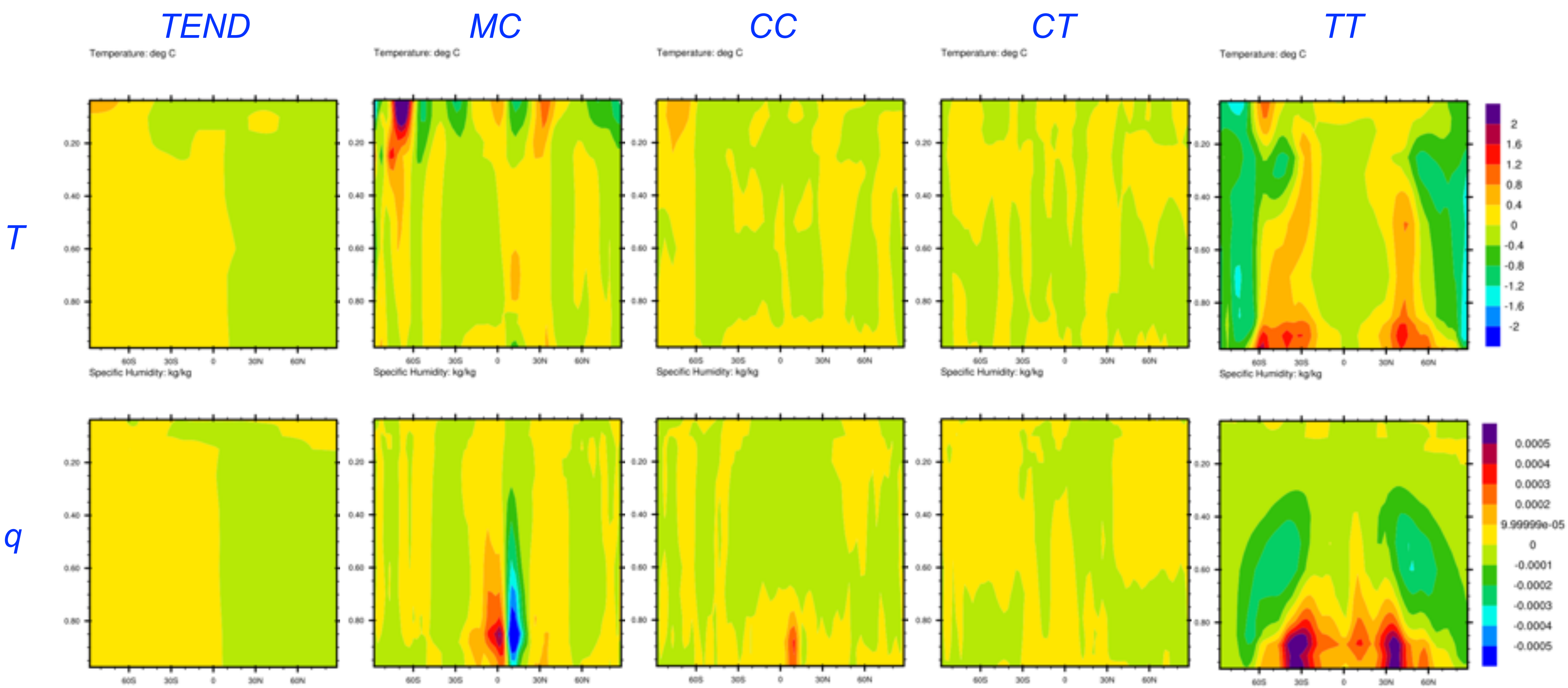


JJA



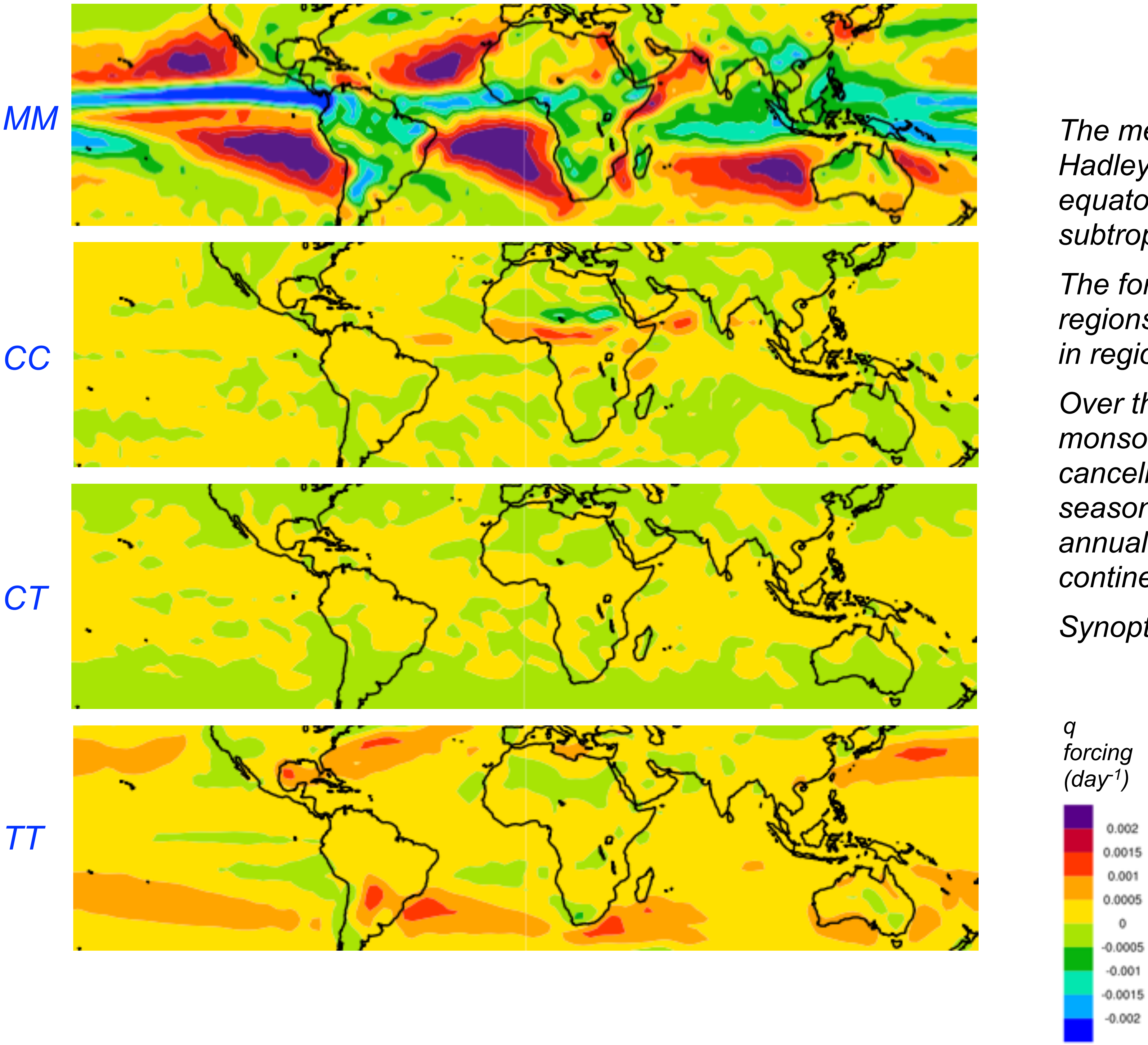
TEND
MC
CC
CT
TT
T
q
Temperature: deg C
Specific Humidity: kg/kg
0.20
0.40
0.60
0.80
60S
30S
0
30N
60N
2
1.6
1.2
0.8
0.4
0
-0.4
-0.8
-1.2
-1.6
-2
0.0005
0.0004
0.0003
0.0002
9.99999e-05
0
-0.0001
-0.0002
-0.0003
-0.0004
-0.0005

SON



TEND
MC
CC
CT
TT
T
q
Temperature: deg C
Specific Humidity: kg/kg
0.20
0.40
0.60
0.80
60S
30S
0
30N
60N
2
1.6
1.2
0.8
0.4
0
-0.4
-0.8
-1.2
-1.6
-2
0.0005
0.0004
0.0003
0.0002
9.99999e-05
0
-0.0001
-0.0002
-0.0003
-0.0004
-0.0005

Maintenance of tropical humidity: Annual Mean



TEND and MC have zero annual mean

The mean flow is a double-branched Hadley cell ascending just north of the equator and descending in the subtropical oceans.

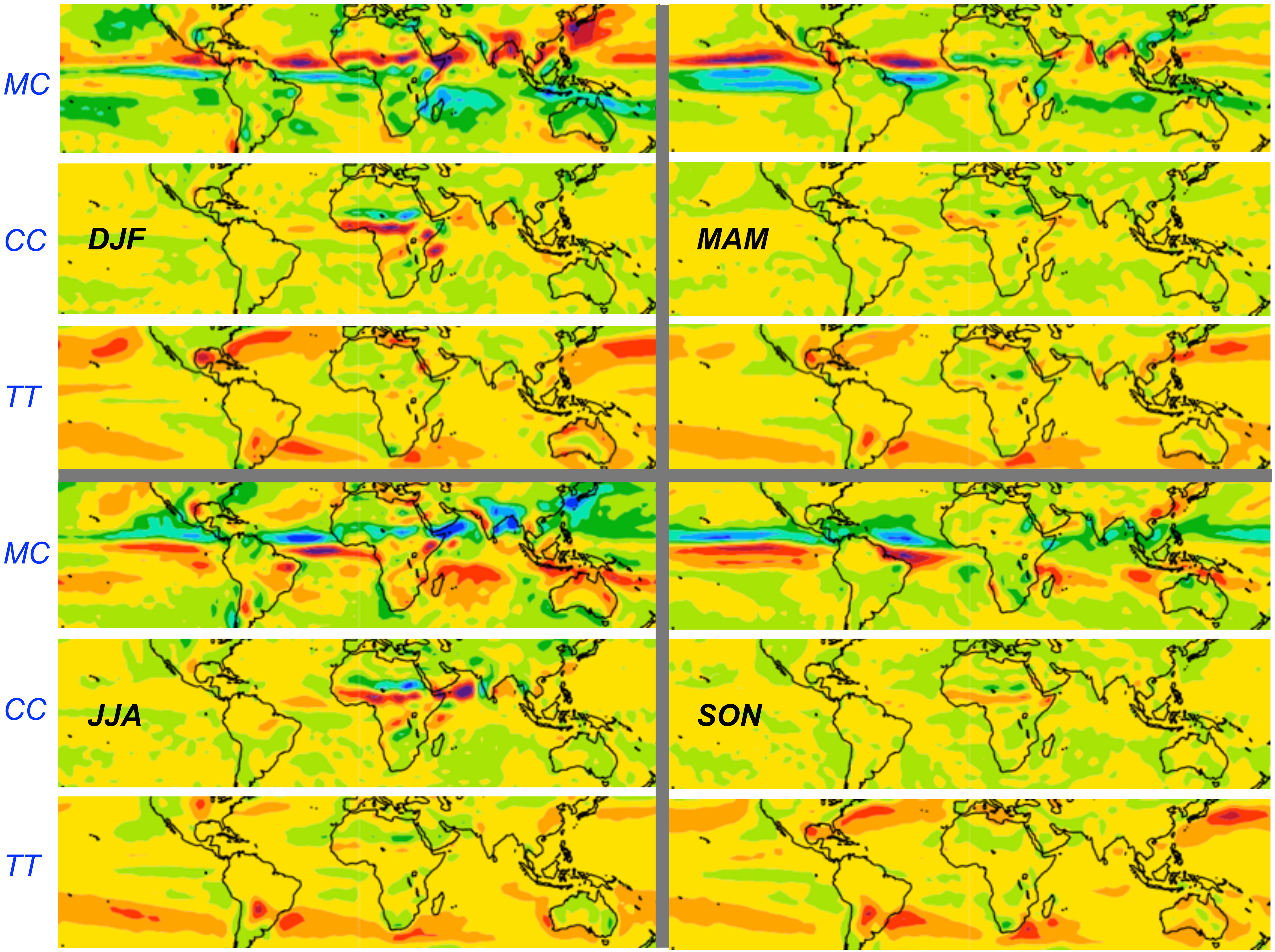
The forcing must supply moisture in regions of evaporation and remove it in regions of precipitation.

Over the terrestrial West African monsoon region there is an interesting cancellation between mean flow and seasonal covariance. The resultant annual mean forcing over the continent is weak.

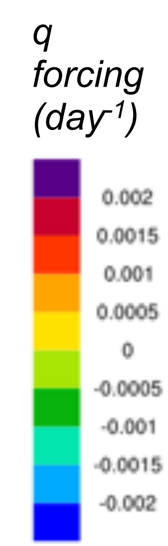
Synoptic transients contribute little.

$\sigma = 0.85$

Maintenance of tropical humidity: Annual Cycle



MM has no cycle. TEND and CT are very small



σ = 0.85

Onset of the African Monsoon

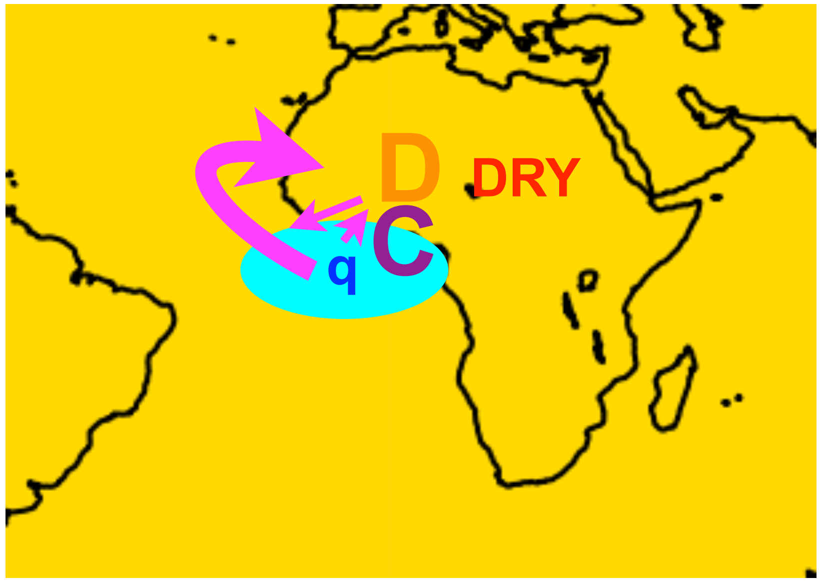
The CC term over West Africa remains the same sign in opposite seasons, leading to partial cancellation of MC in DJF and reinforcement in JJA.

We can explain this in terms of seasonal anomaly covariance. The flow reverses, but crucially, so do the seasonal anomaly humidity gradients.

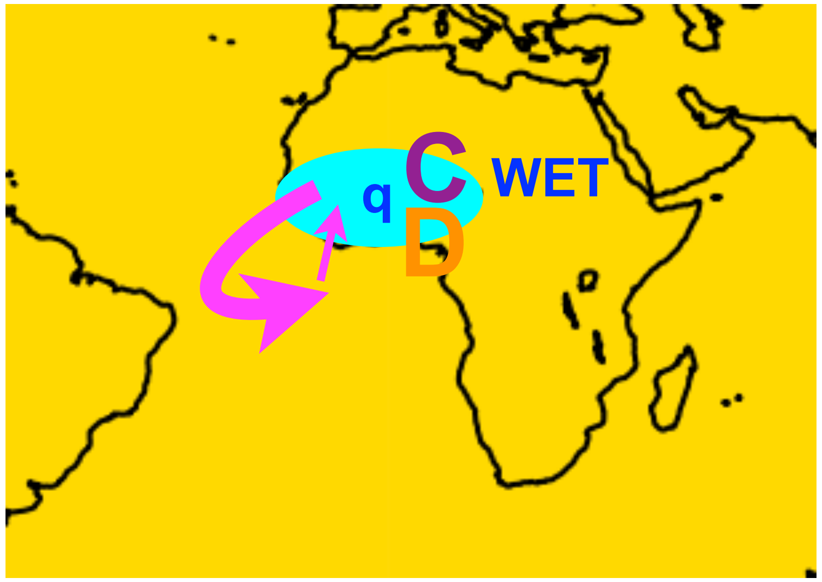
Covariance between divergence anomaly and humidity anomaly retains the same sign, leading to drying in the Guinean zone and moistening of the Sahel in both summer and winter, and, indeed, all year round.

In the winter this partially cancels the linear component, and in the summer it reinforces it.

DJF



JJA



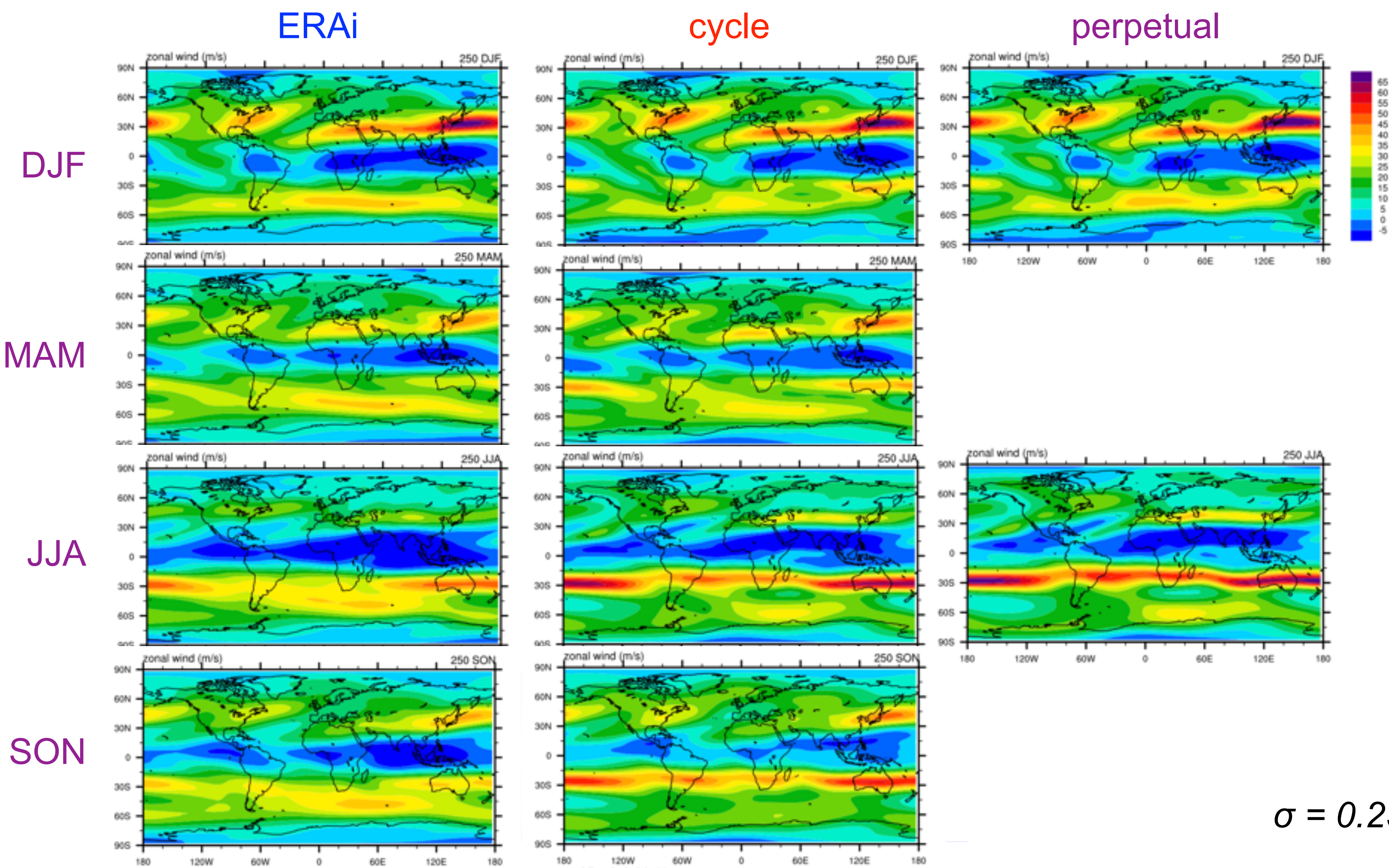
Cyclic changes in wind direction shift the humidity distribution, which then interacts with the seasonal anomaly flow. It is this covariant interaction that characterizes the African monsoon.

Upper level zonal wind: GCM simulations

$$\frac{d\boldsymbol{\Psi}}{dt} + (\mathcal{A} + \mathcal{D})\boldsymbol{\Psi} = \mathbf{g}$$

$$\mathbf{g} = \overline{(\mathcal{A} + \mathcal{D})\boldsymbol{\Phi}} + (\widetilde{\mathcal{A} + \mathcal{D}})\boldsymbol{\Phi} + \widetilde{\frac{d\boldsymbol{\Phi}}{dt}}$$

$$\mathbf{g} = \overline{(\mathcal{A} + \mathcal{D})\boldsymbol{\Phi}_s}$$



σ = 0.25

Summary

1) The moisture budget over West Africa depends on seasonal anomaly advection of seasonal anomaly specific humidity

2) This budget separation highlights the two phases of the monsoon onset

   - reversed winds transport humidity

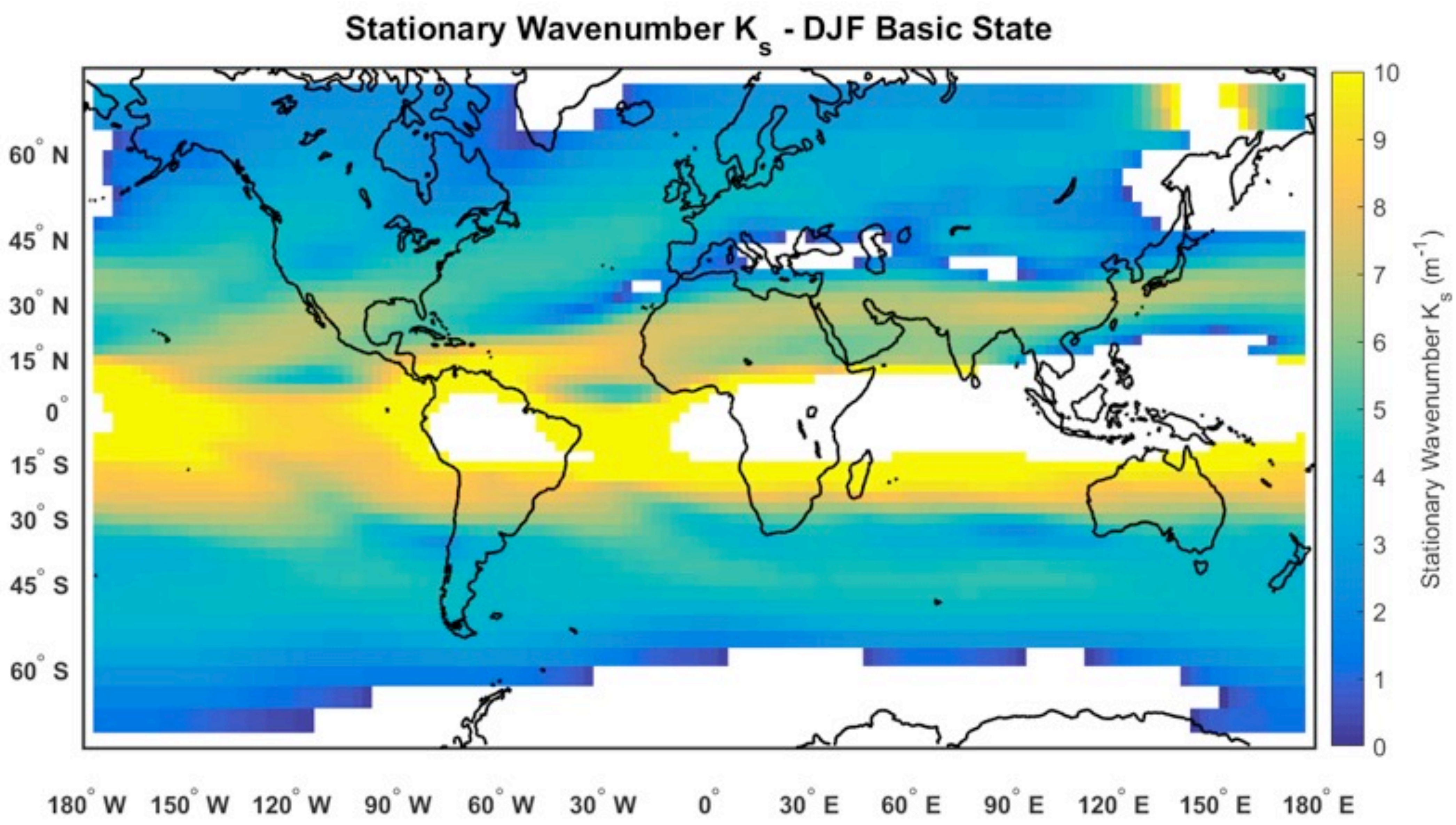
   - modified humidity and divergence determine moisture supply

3) Perpetual runs give results consistent with the small tendency term

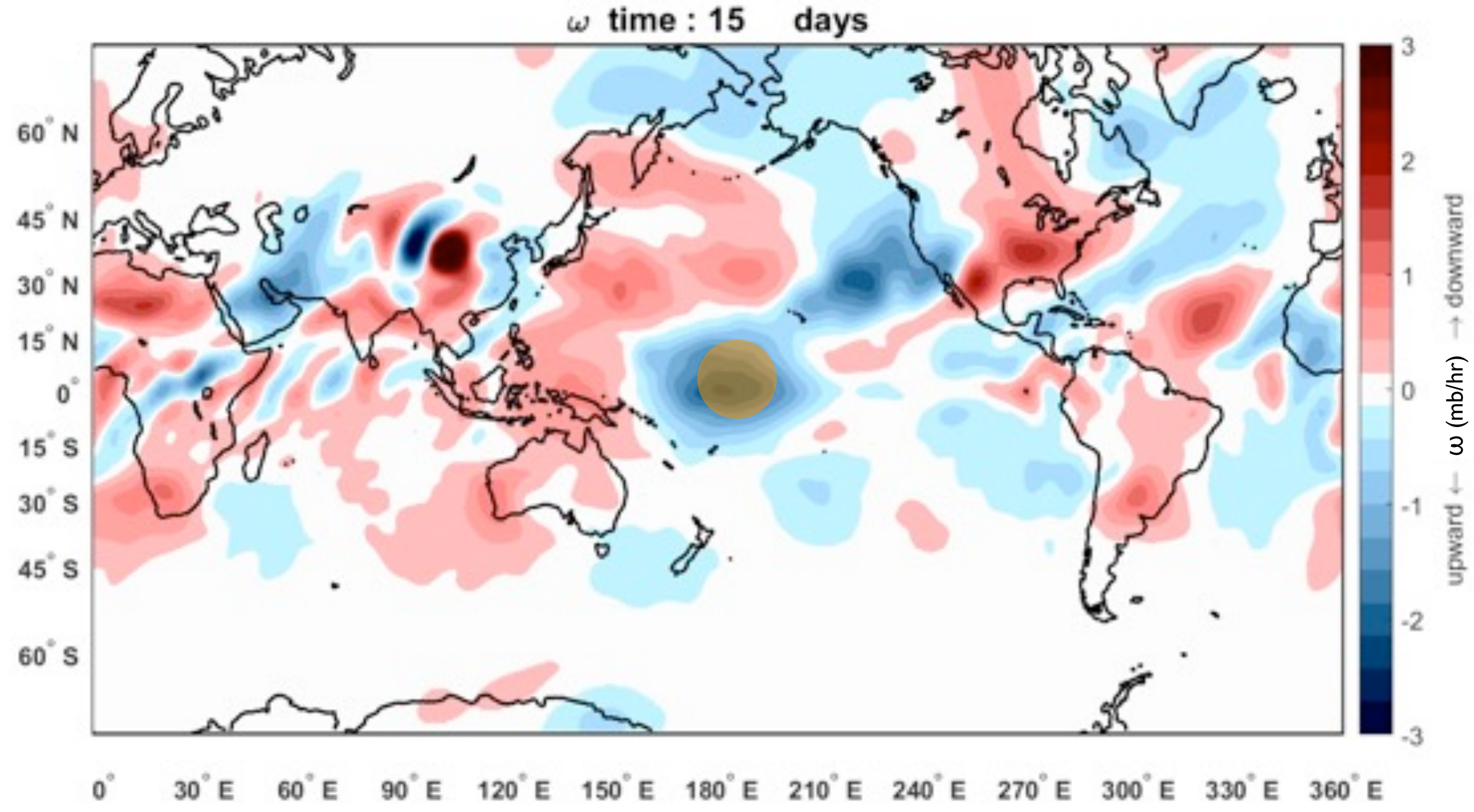
4) This technique can be used for other timescale separations

Teleconnections to South American rainfall

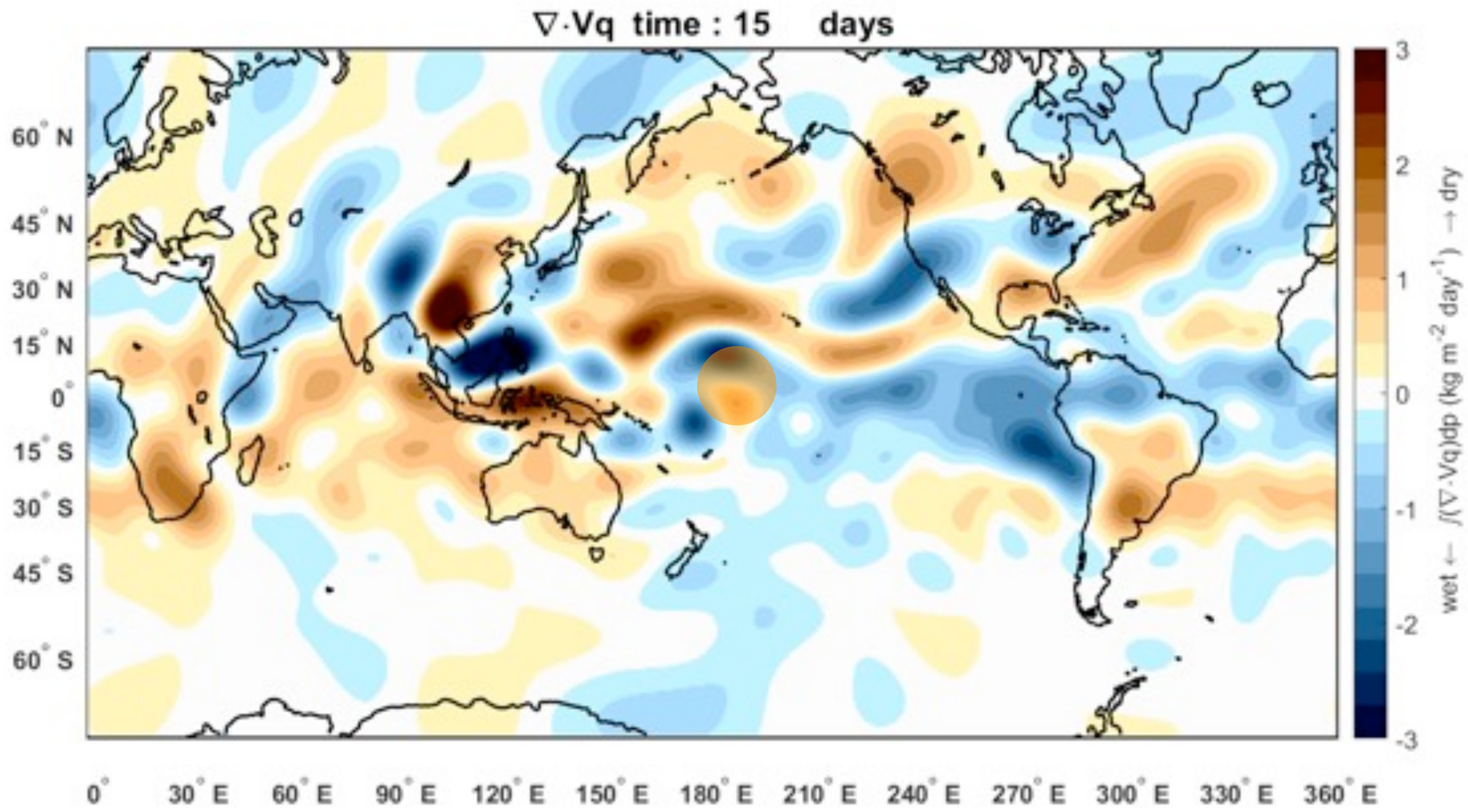
$$K_s = \sqrt{\frac{\beta_*}{\overline{U}}}$$



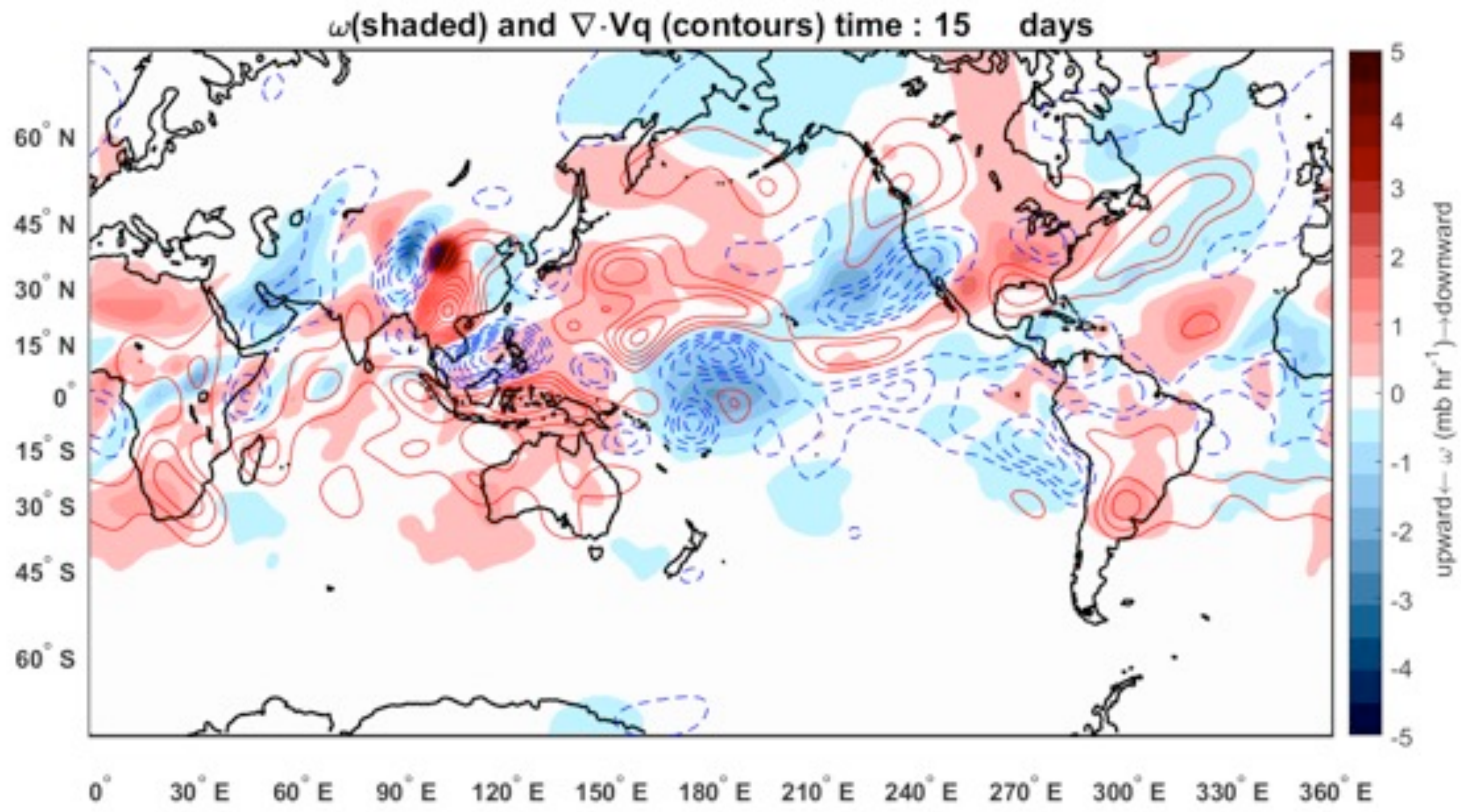
Vertical velocity at σ=0.5 (mb/h)



Vertically integrated humidity flux divergence (mm/day)

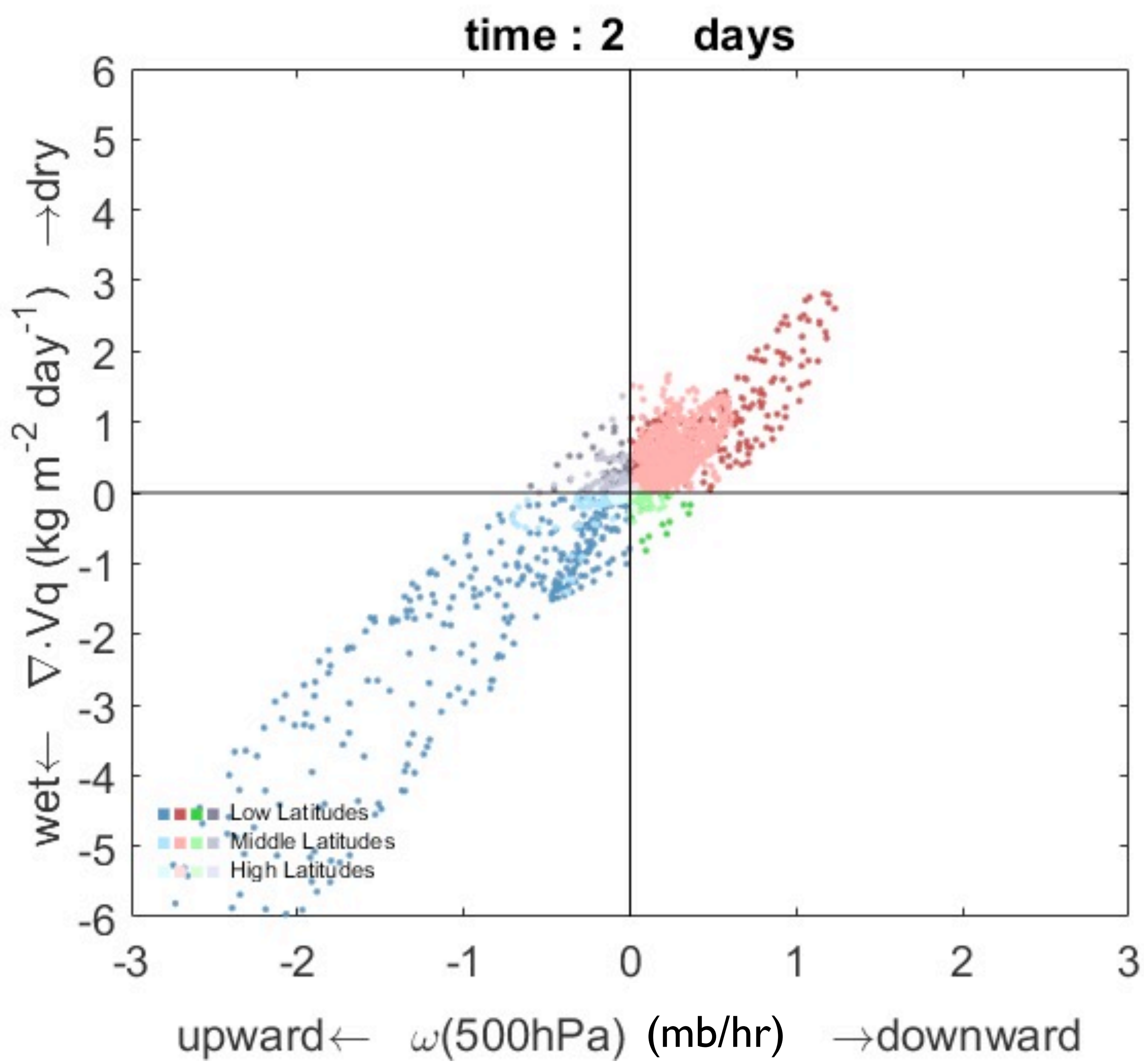


Both at once

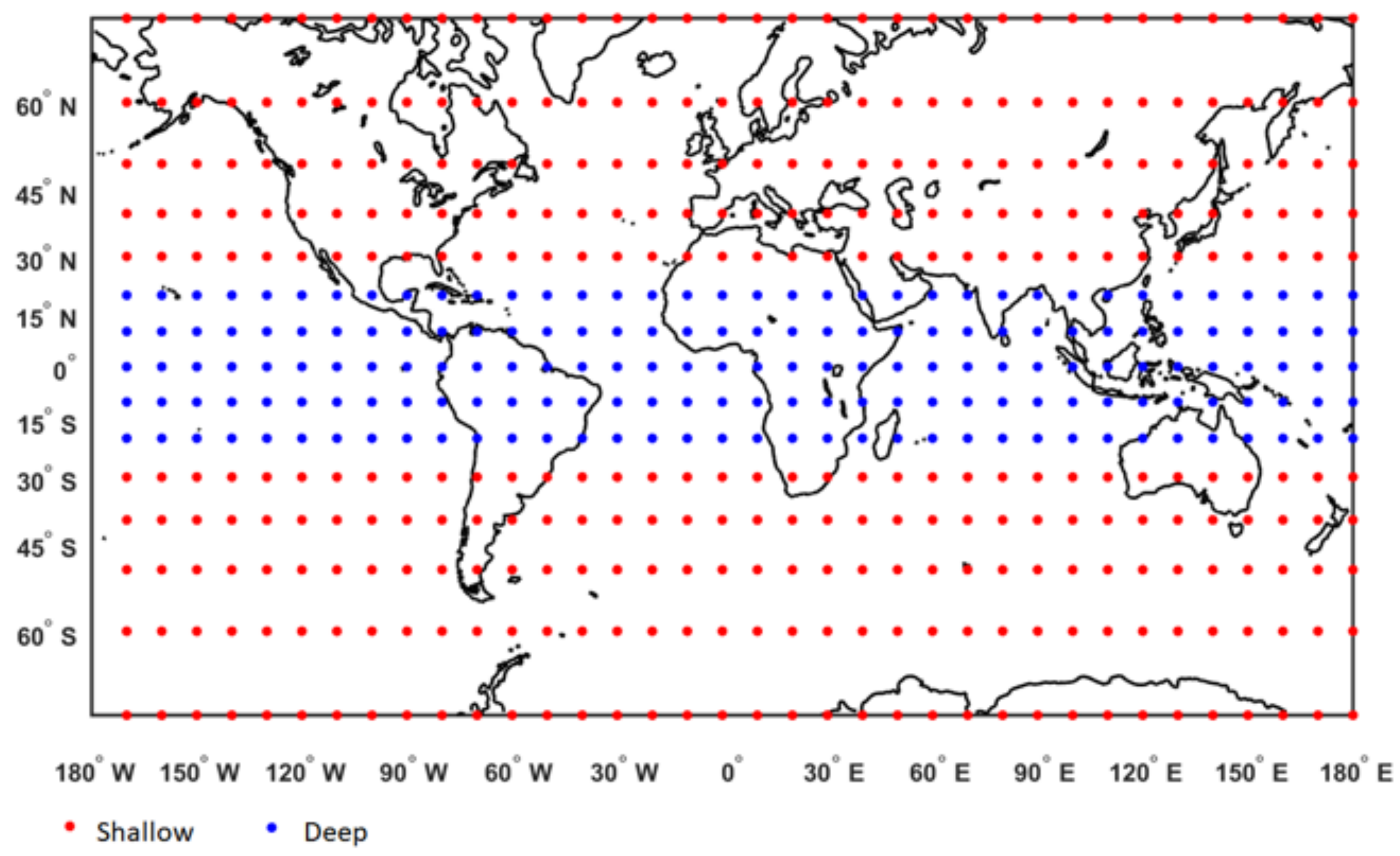


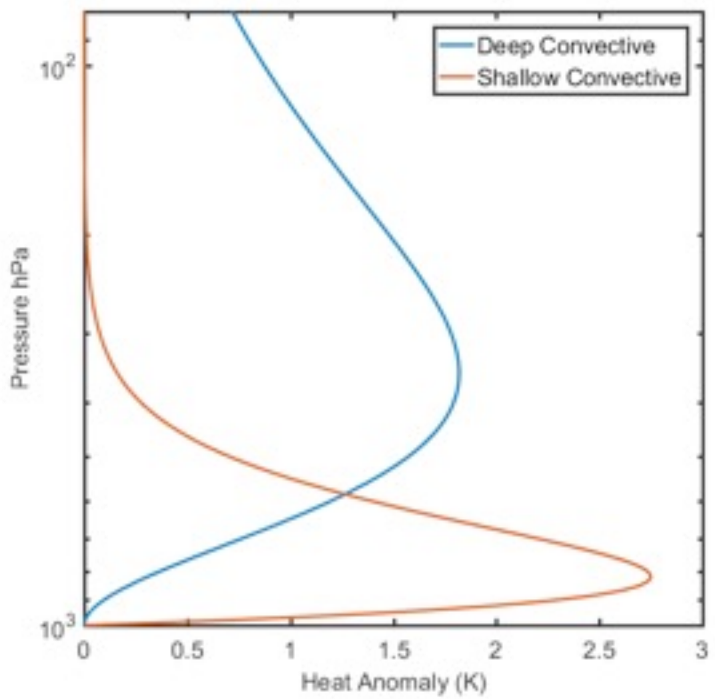
ω(shaded) and ∇·Vq (contours) time : 15 days
60° N
45° N
30° N
15° N
0°
15° S
30° S
45° S
60° S
0°
30° E
60° E
90° E
120° E
150° E
180° E
210° E
240° E
270° E
300° E
330° E
360° E
5
4
3
2
1
0
-1
-2
-3
-4
-5
upward← ω (mb hr⁻¹)→downward

Scatterplot

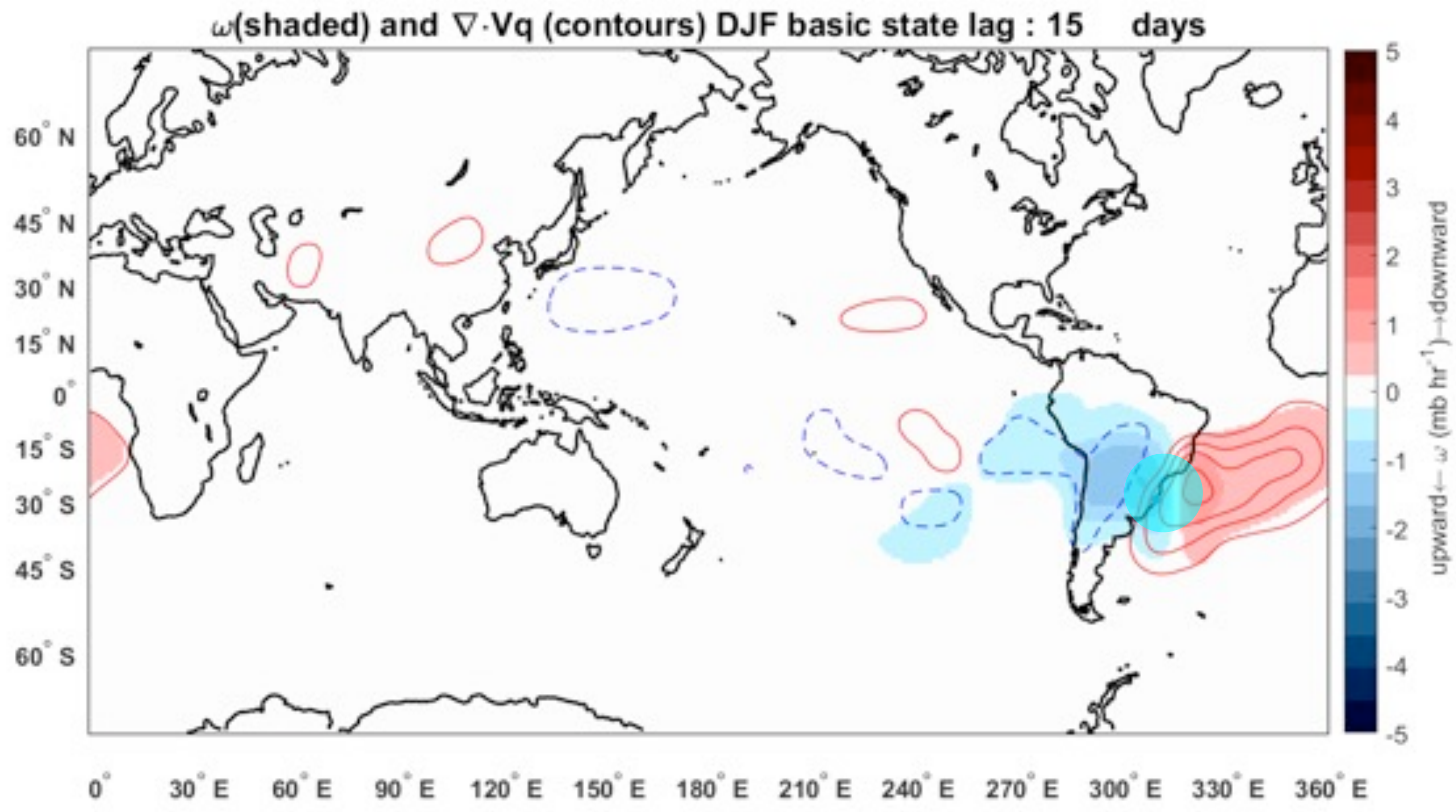


Multiple experiments

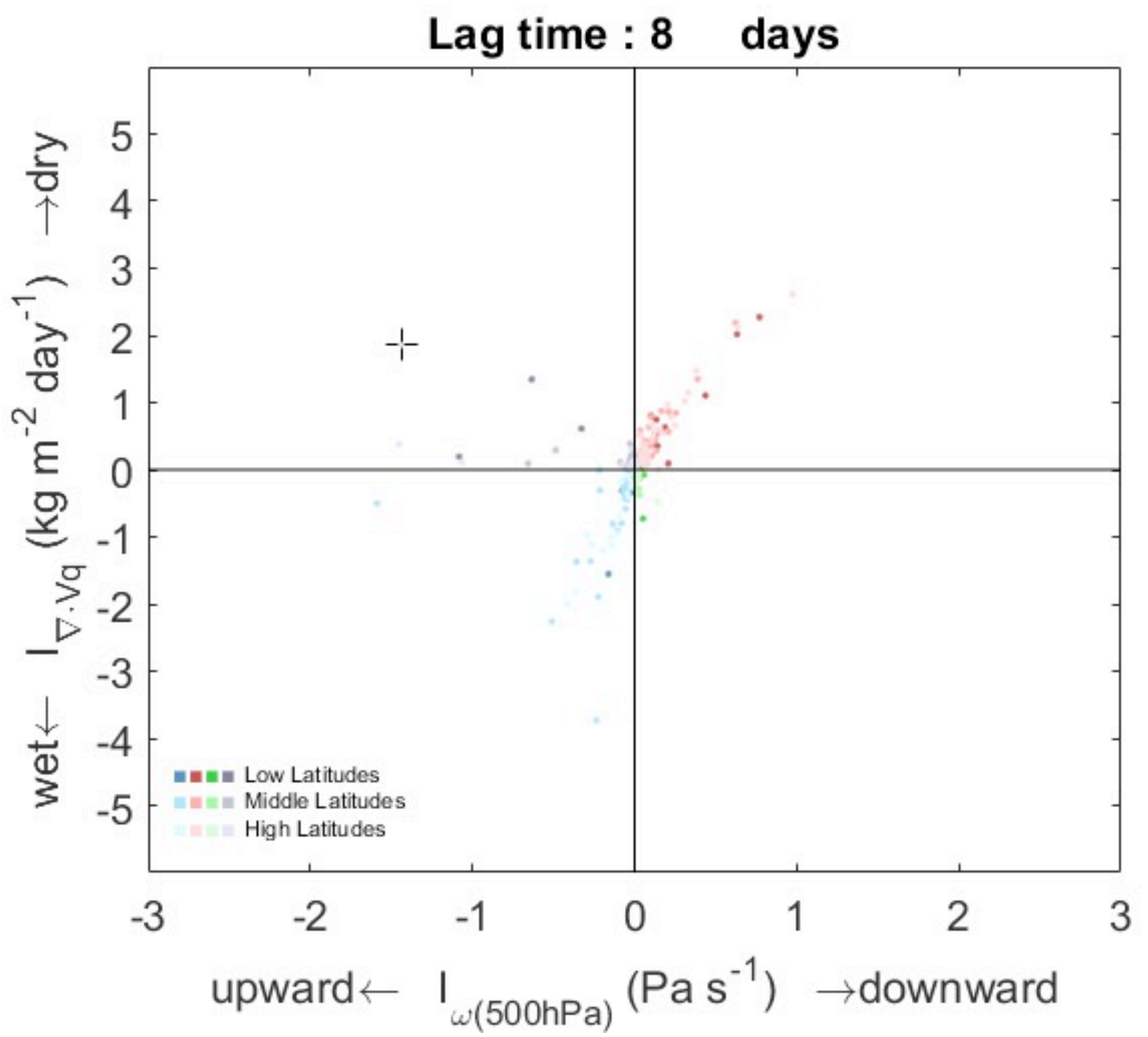




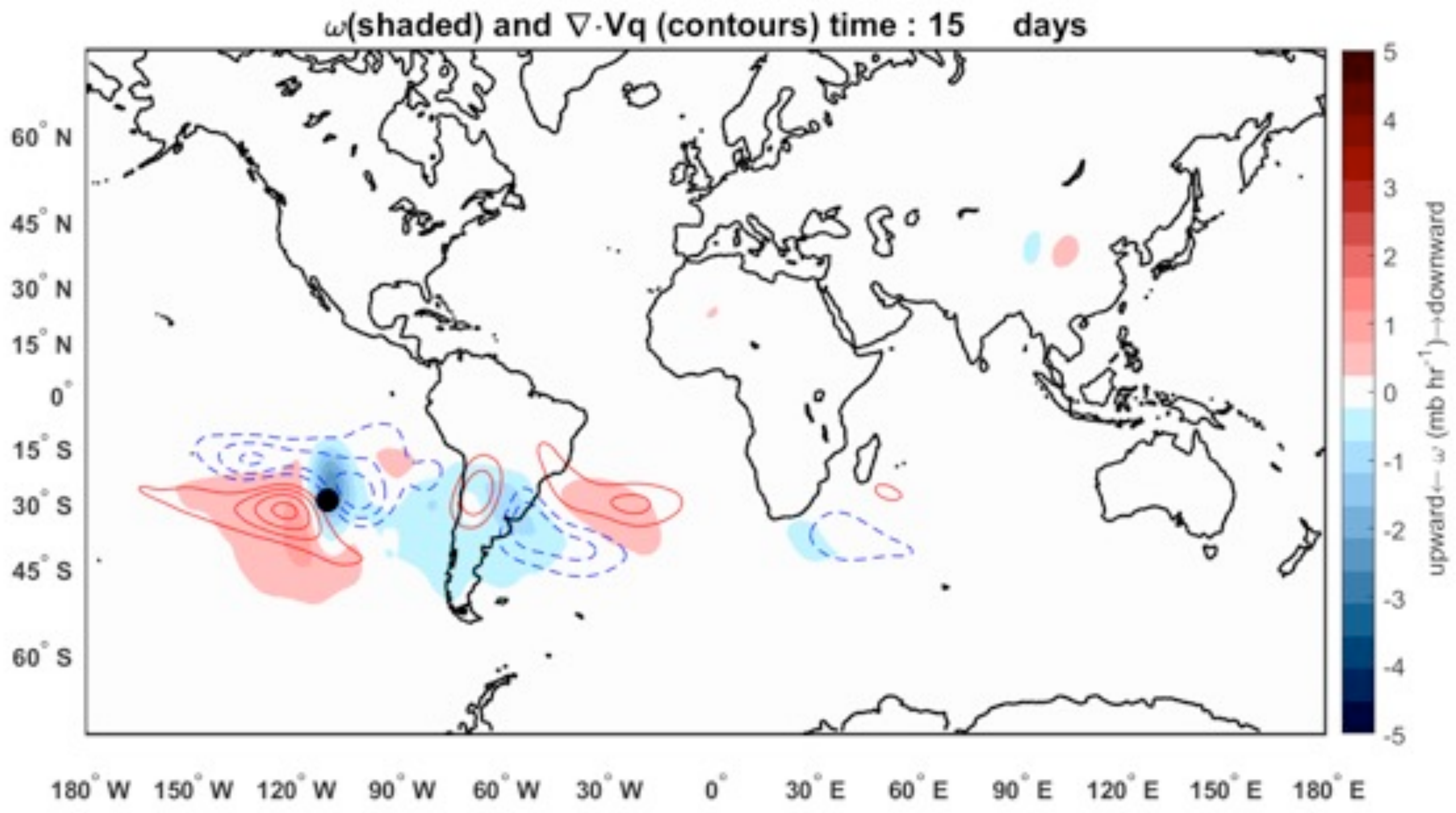
Influence functions



Influence scatterplot

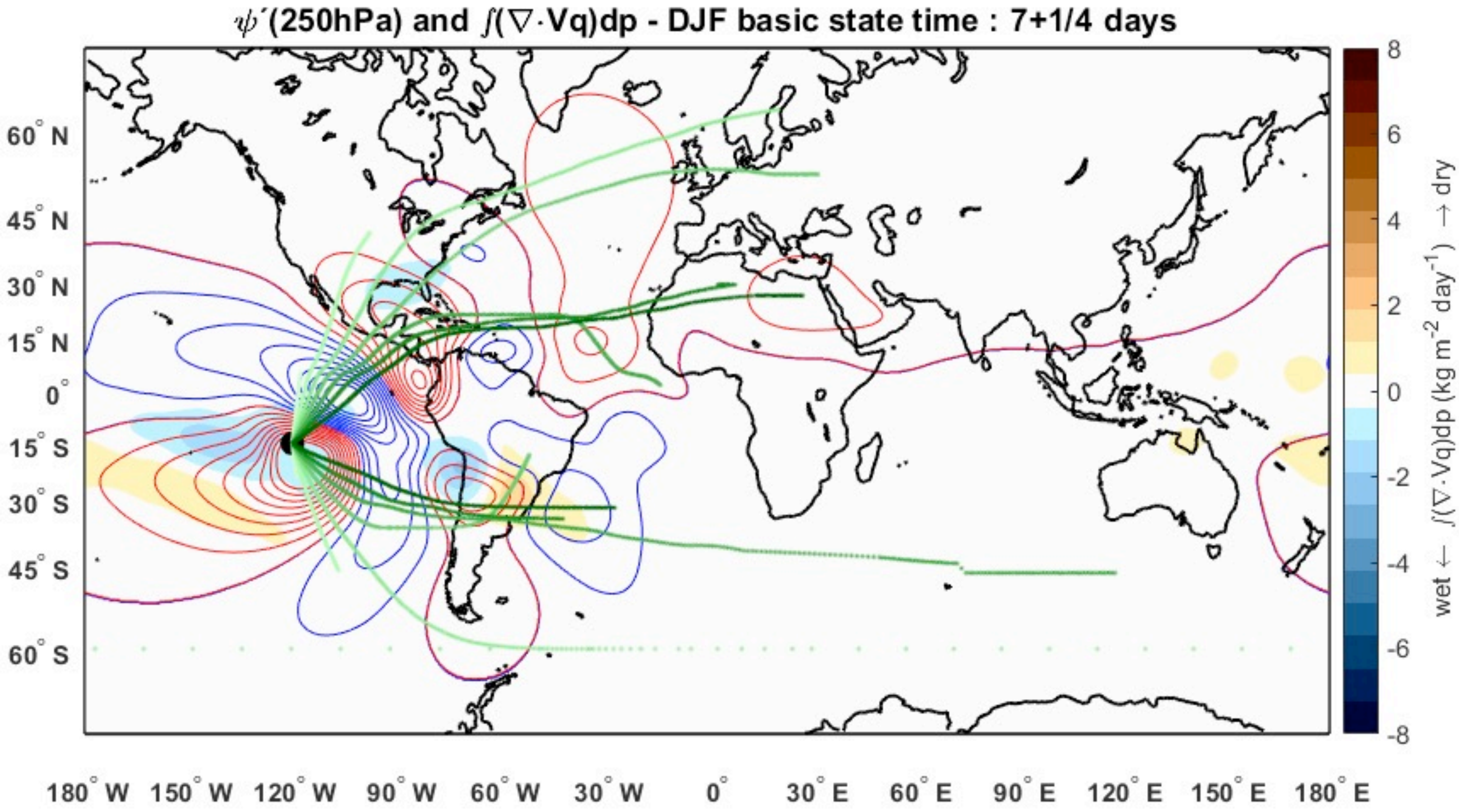


Hot spot



ω(shaded) and $\nabla \cdot$Vq (contours) time : 15 days
60° N
45° N
30° N
15° N
0°
15° S
30° S
45° S
60° S
180° W 150° W 120° W 90° W 60° W 30° W 0° 30° E 60° E 90° E 120° E 150° E 180° E
5
4
3
2
1
0
-1
-2
-3
-4
-5
upward← ω (mb hr^{-1})→downward

Example with Ray Tracing



Summary

- Remote Rossby-wave influence on Southern Brazil region can originate from tropics or extratropics and can even cross the equator.

- Strongest remote influence appears in the South-East Pacific

- Vertical velocity and moisture flux convergence are affected differently and this varies throughout the run - so even without moist processes, the dynamics of the rainfall response is likely to be complex.

Condensation heating: Work in progress

The model currently has a deep convection scheme (LDEEP) and a large scale condensation scheme (LLSR).

Deep convection is triggered if the smoothed local value of column total water vapour convergence exceeds the reference value by a certain amount, AND the smoothed local boundary layer static stability is less than the reference value.

In this case, the total amount of water converging into the column is rained out over a given timescale (τ_{cond}) provided this amount does not exceed the current column total water. Specific humidity is decreased on a pro-rata basis from the current profile. The associated heating is added to the temperature tendency over a deep convective ($\sin \pi\sigma$) profile. The remaining humidity is subject to enhanced vertical diffusion throughout the troposphere.

Large scale rain takes place after deep convection. Any local super-saturation of specific humidity is rained out and the associated heating is applied locally over the same timescale τ_{cond}.

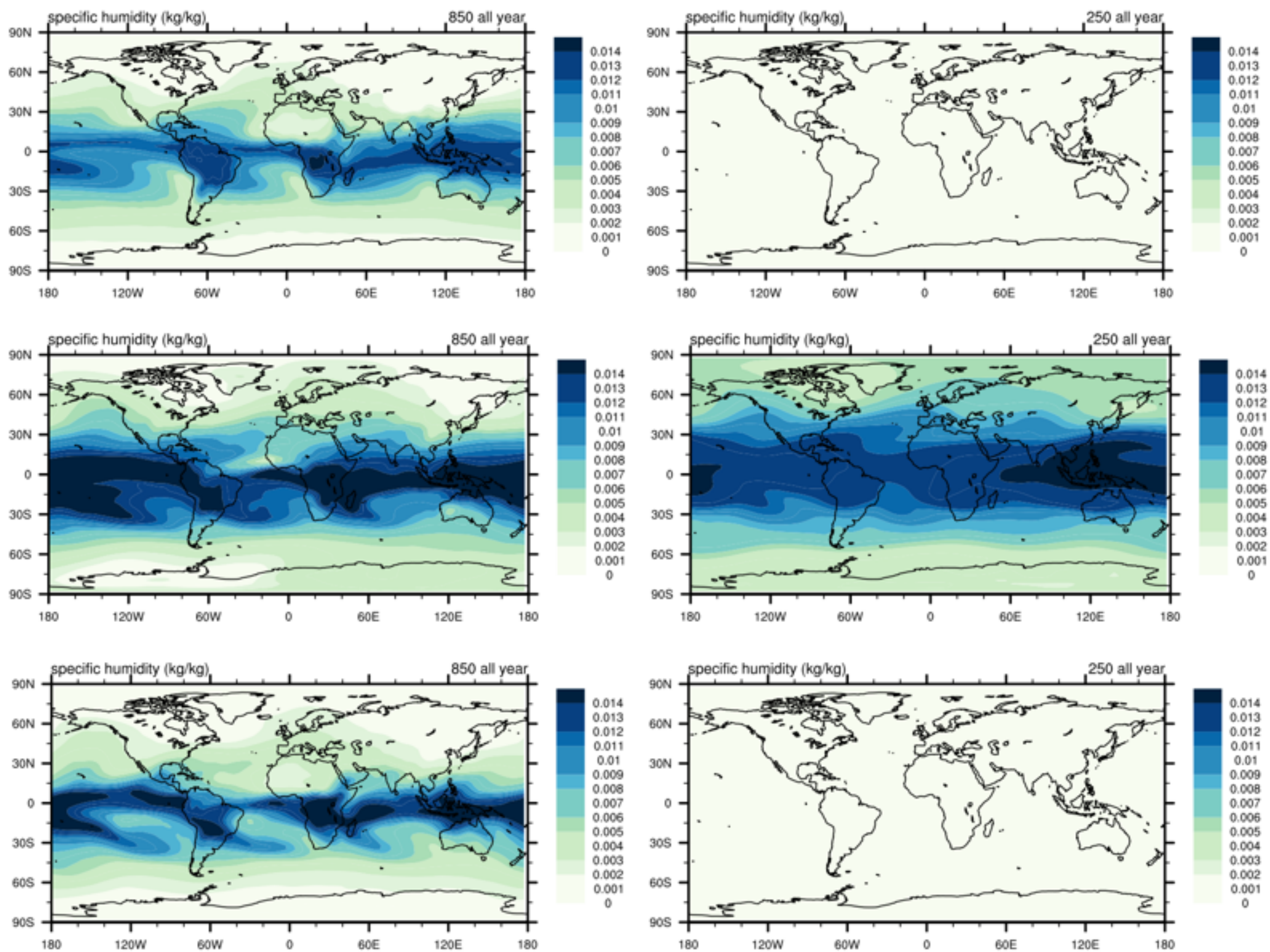
We are still exploring the interaction between these schemes, and with the forcing applied to q.

Specific humidity

ERAi

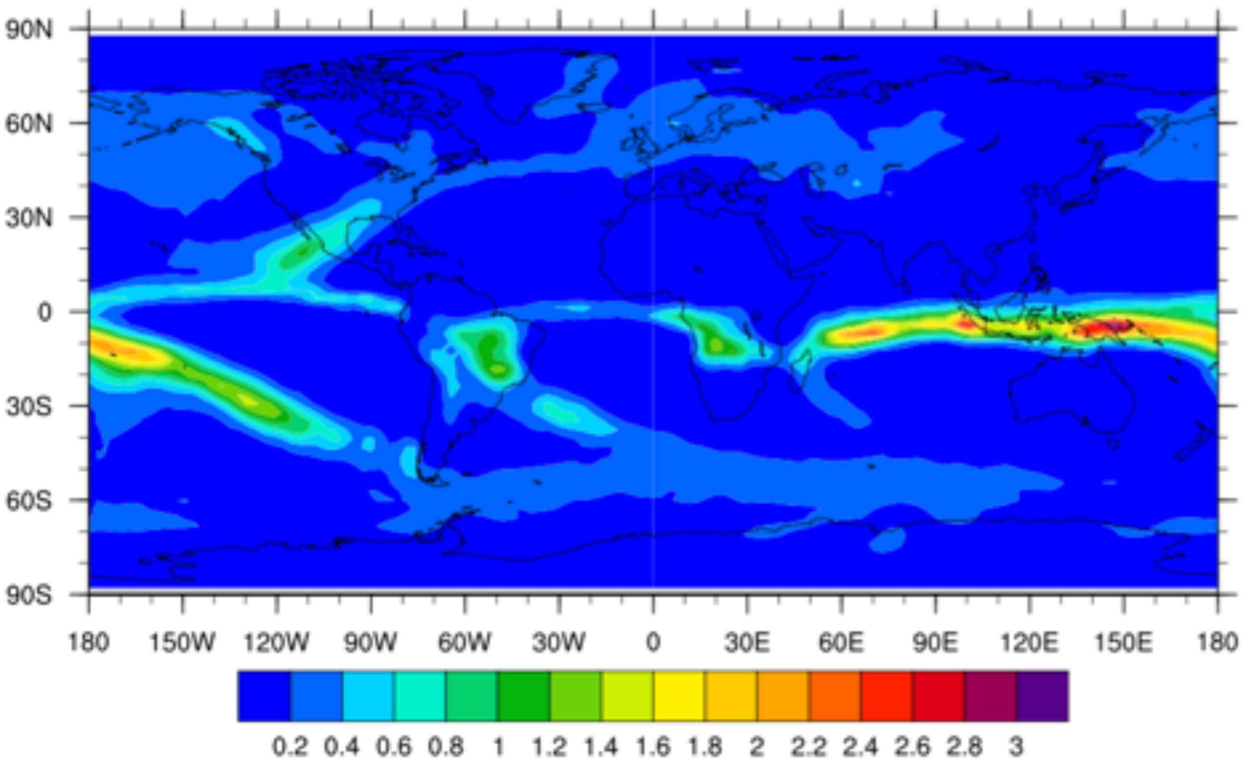
dry model

moist model

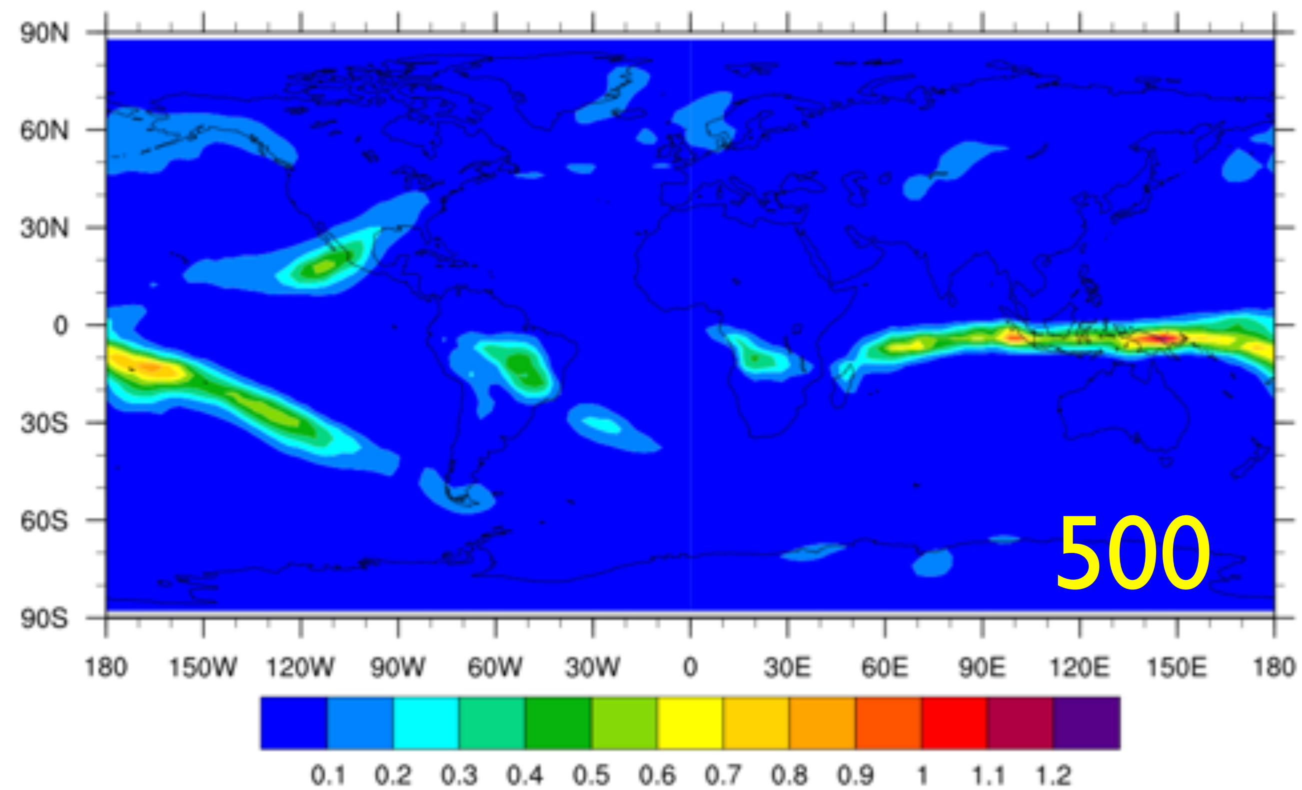


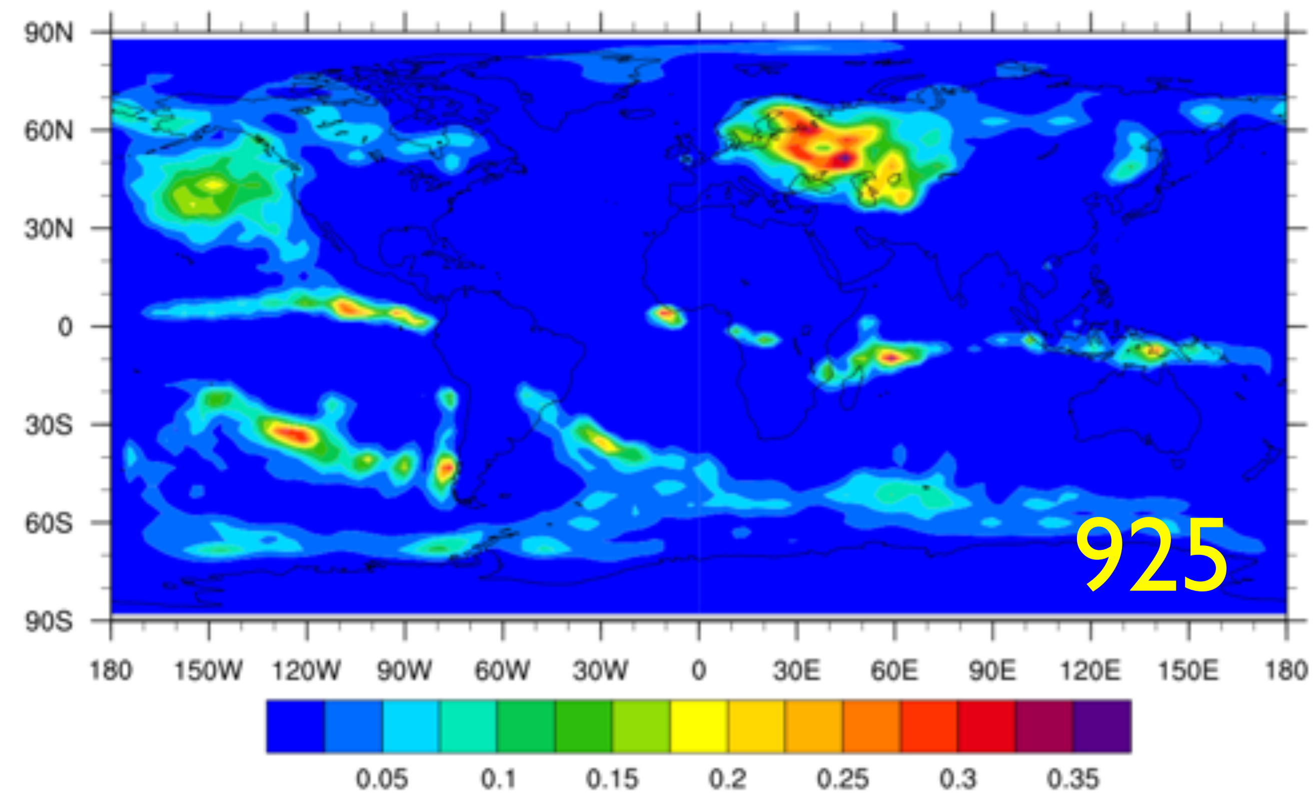
Precipitation

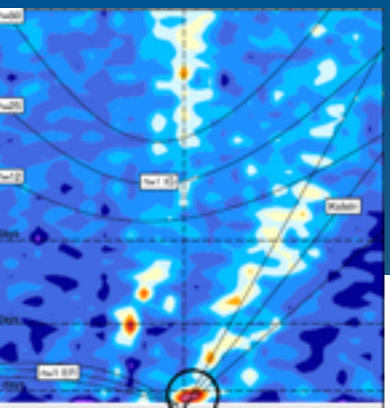
rain rate mm/day



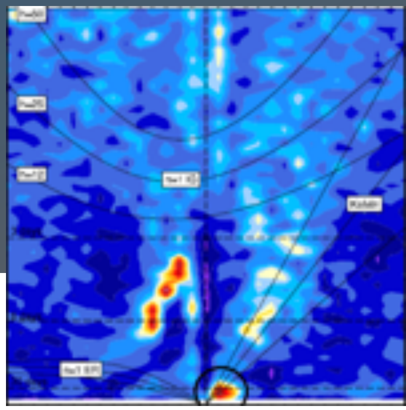
diabatic heating degs/day





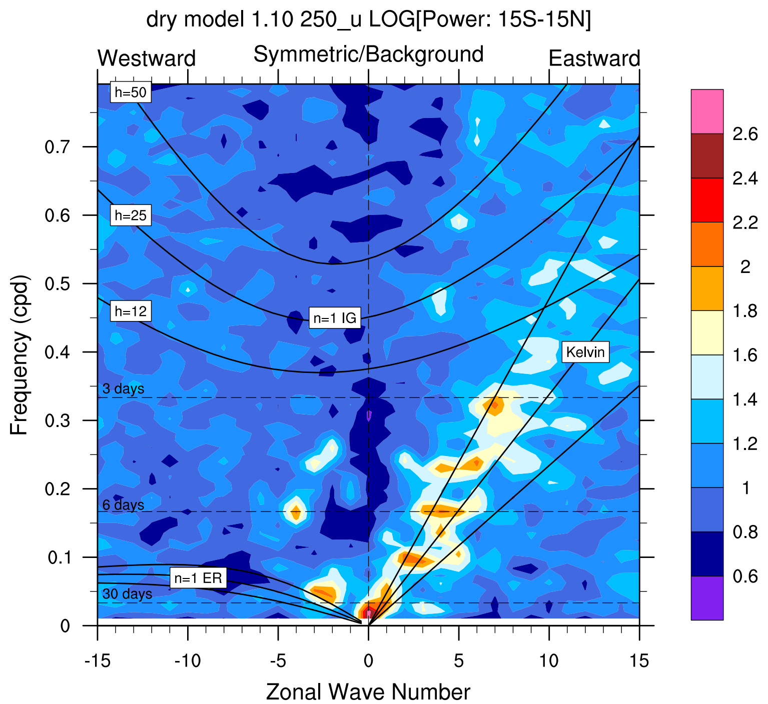


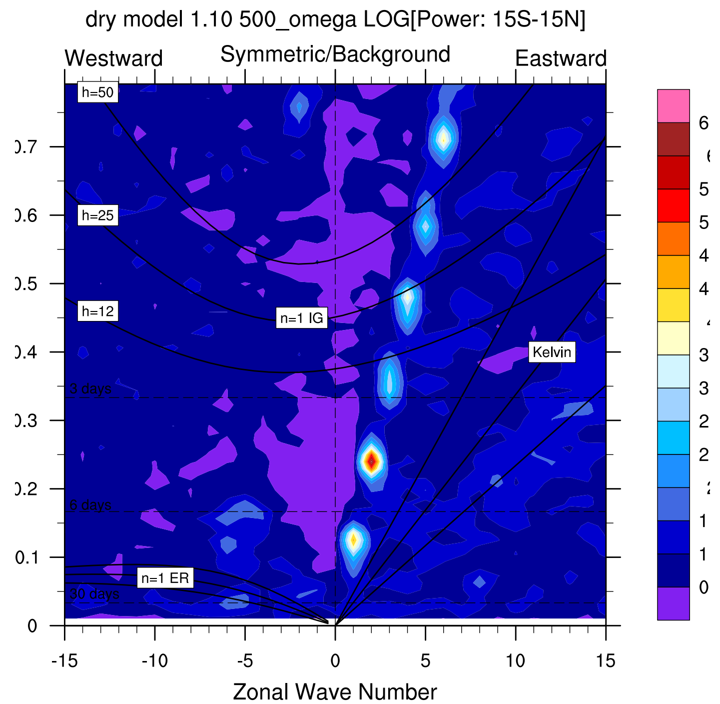
WK spectra: symmetric

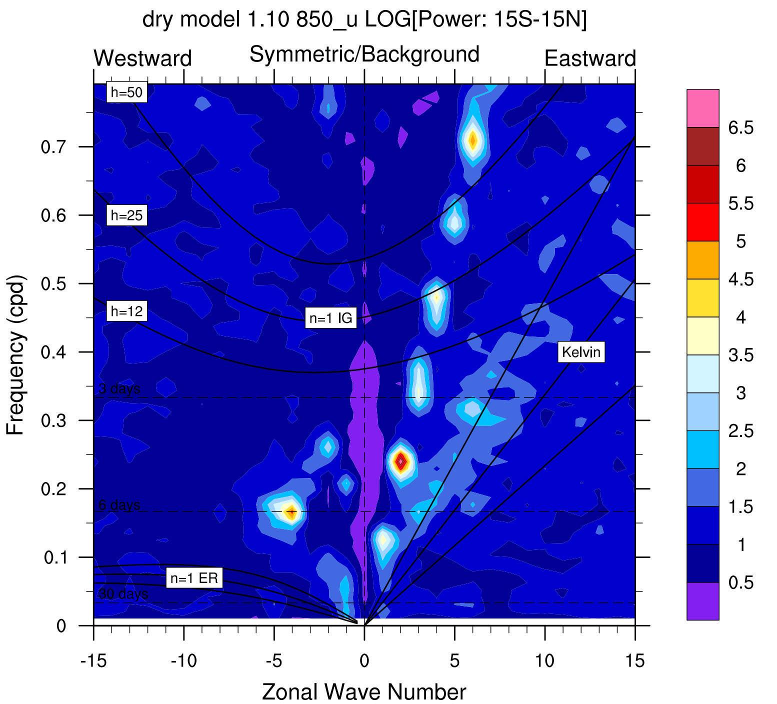


u 250 *omega 500* *u850*

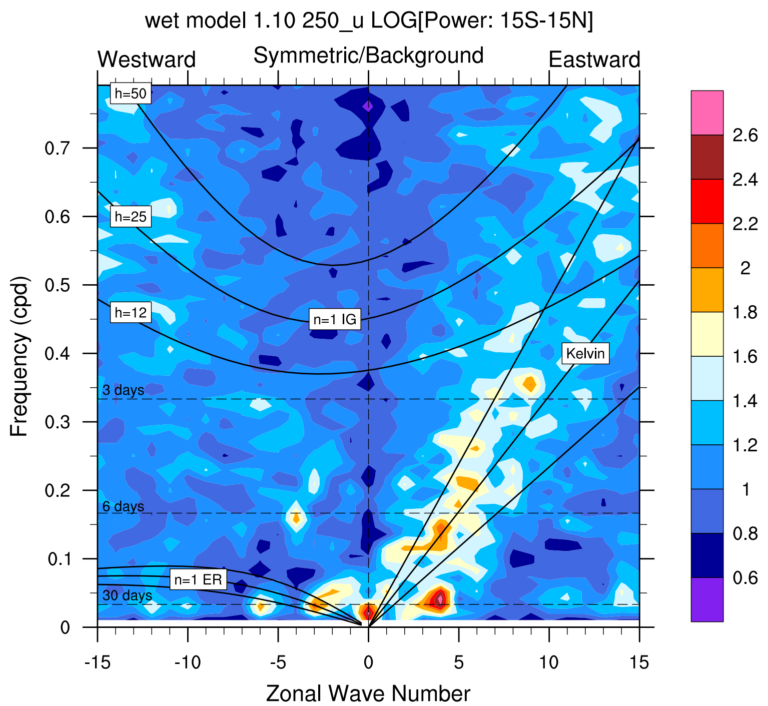
dry model

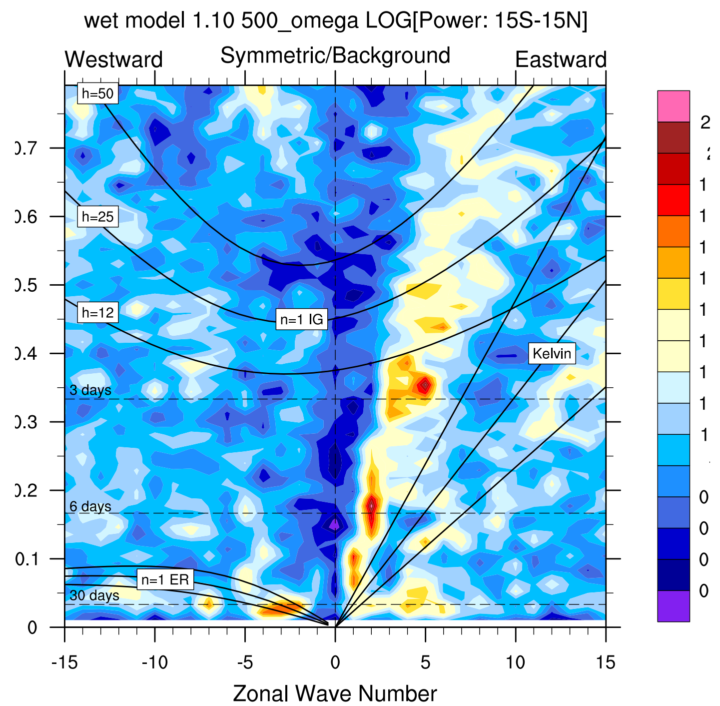


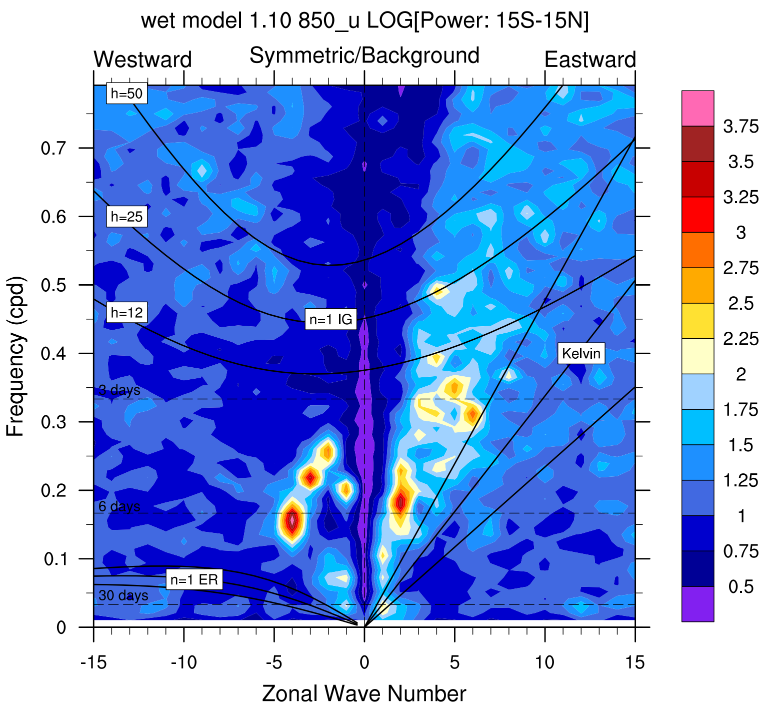




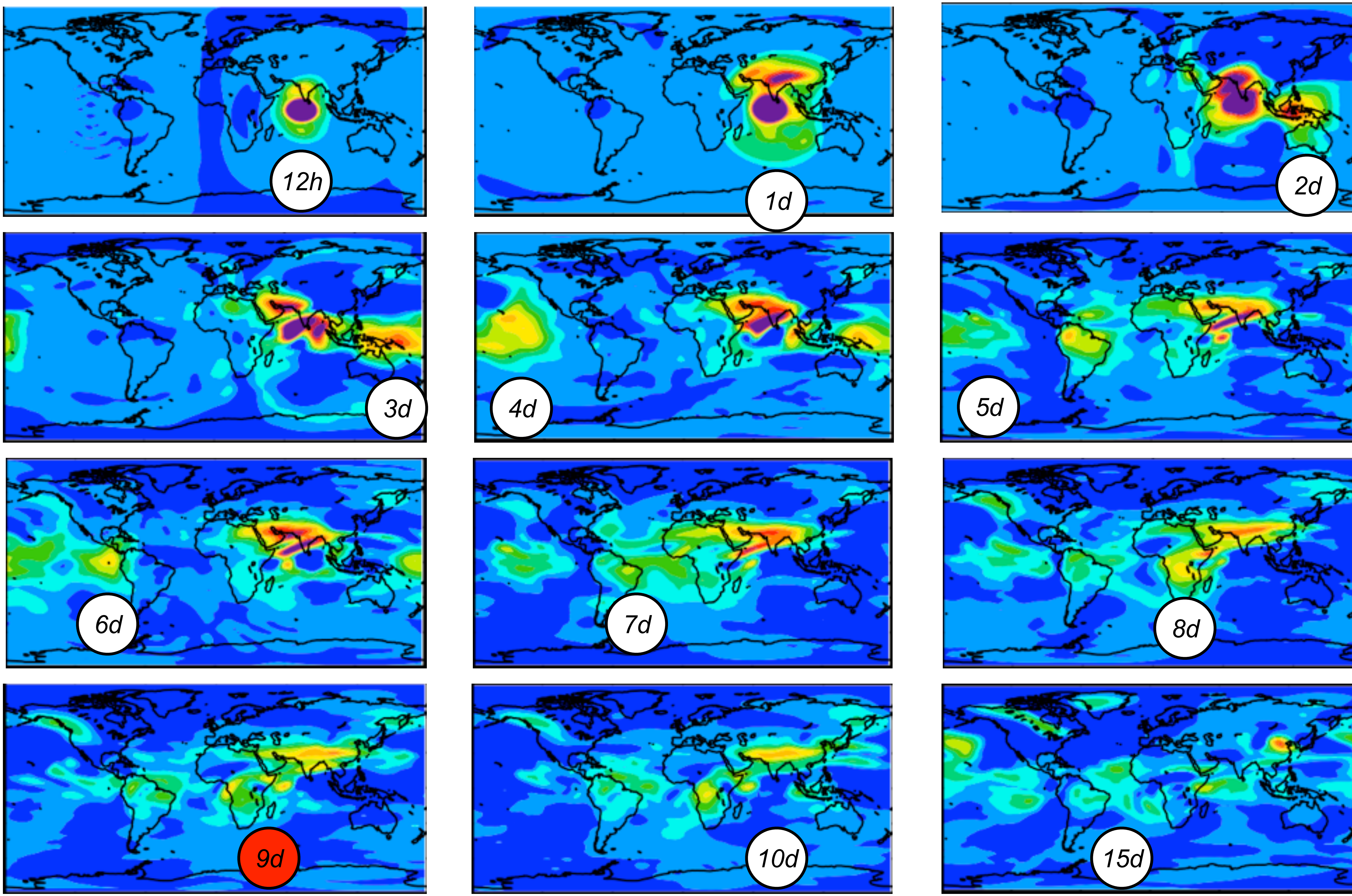
moist model



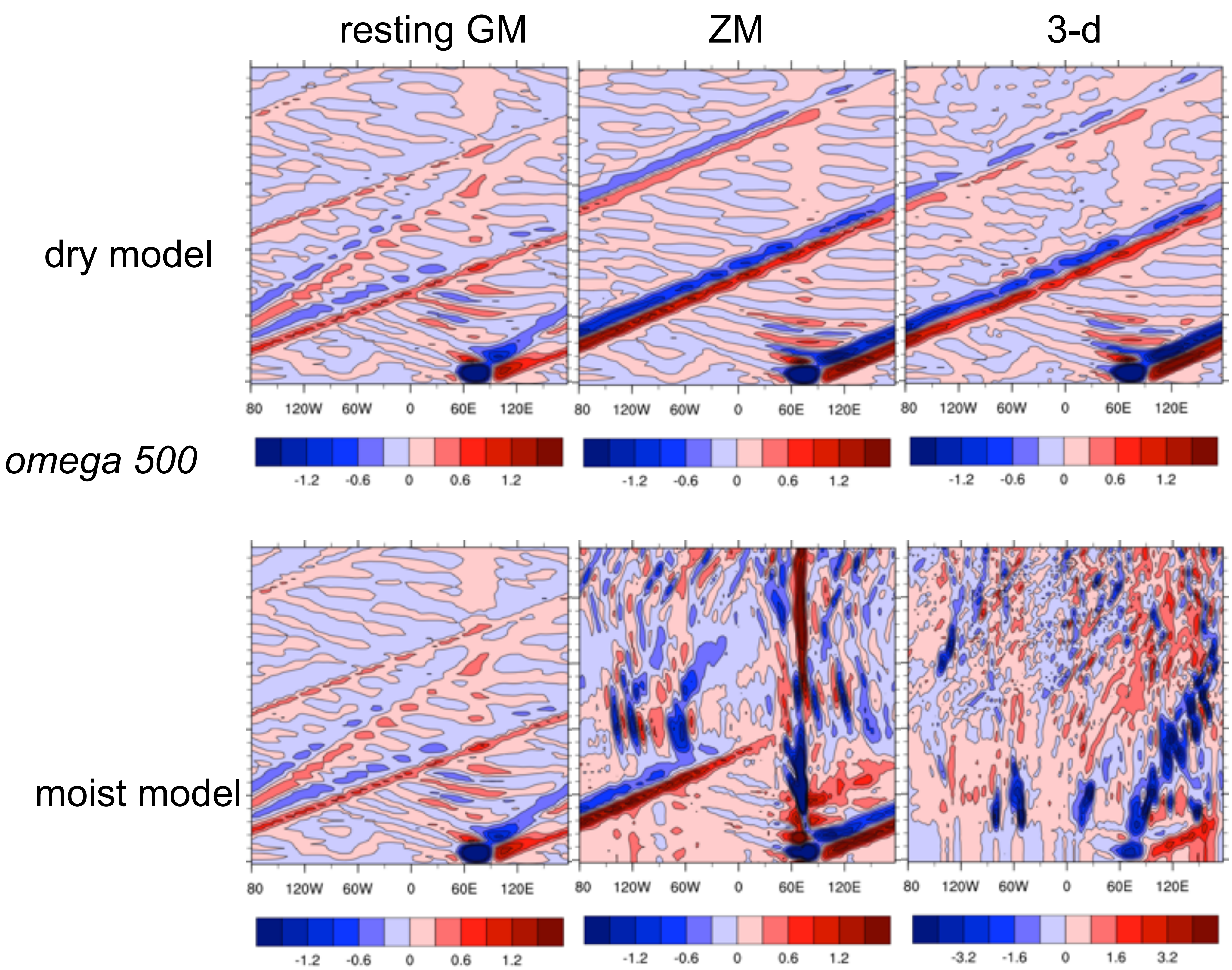




3d basic state: 1-day deep profile heating, T_{250}



Basic state dependence - 20d runs

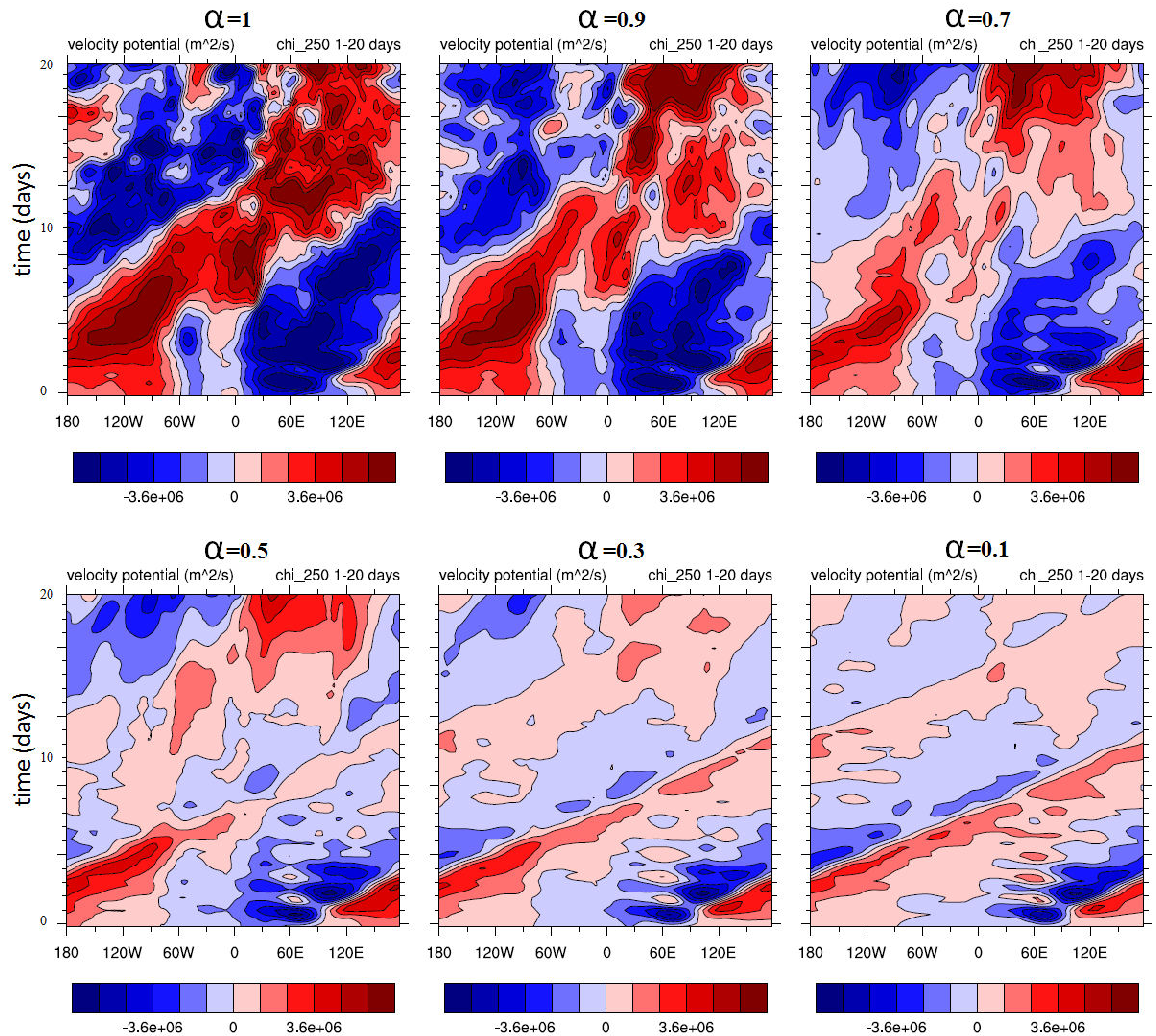


Effect of coupling to condensation

An experiment with the large scale rain scheme and a very simple zonally uniform basic state, looking at the effect of condensation heating on Kelvin waves in the tropical band.

The specific latent heat L is varied from its full value to just 10% of its full value.

This appears to influence the phase speed of propagating tropical disturbances.



Conclusions

- The dry GCM exhibits fast and slow Kelvin type propagation in the tropics.

- Condensation heating appears to select the slower mode.

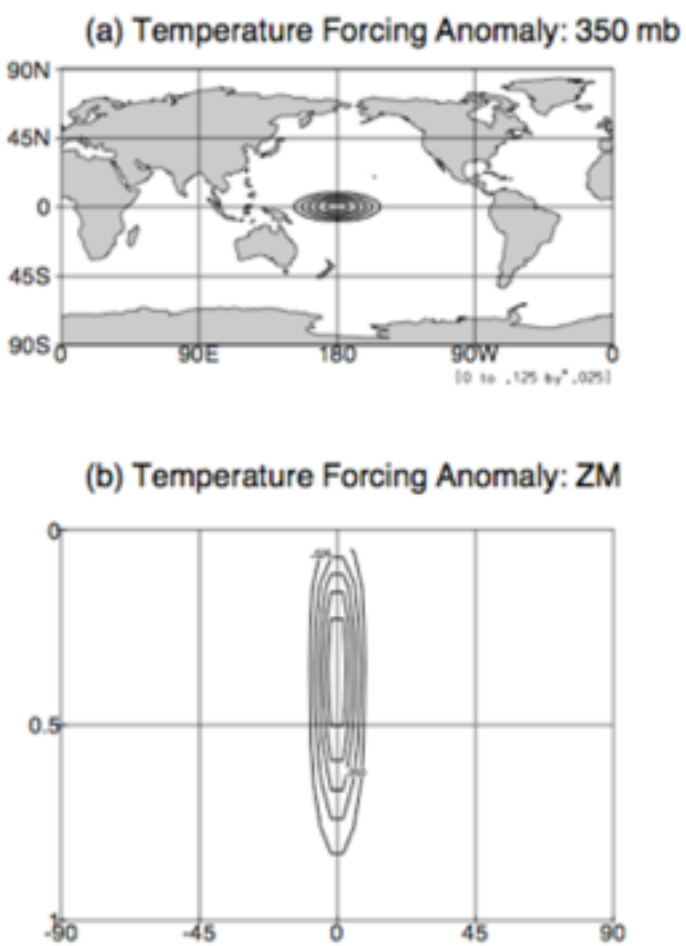
- Idealised perturbation experiments show fast and slow modes even on a resting basic state with a realistic temperature profile.

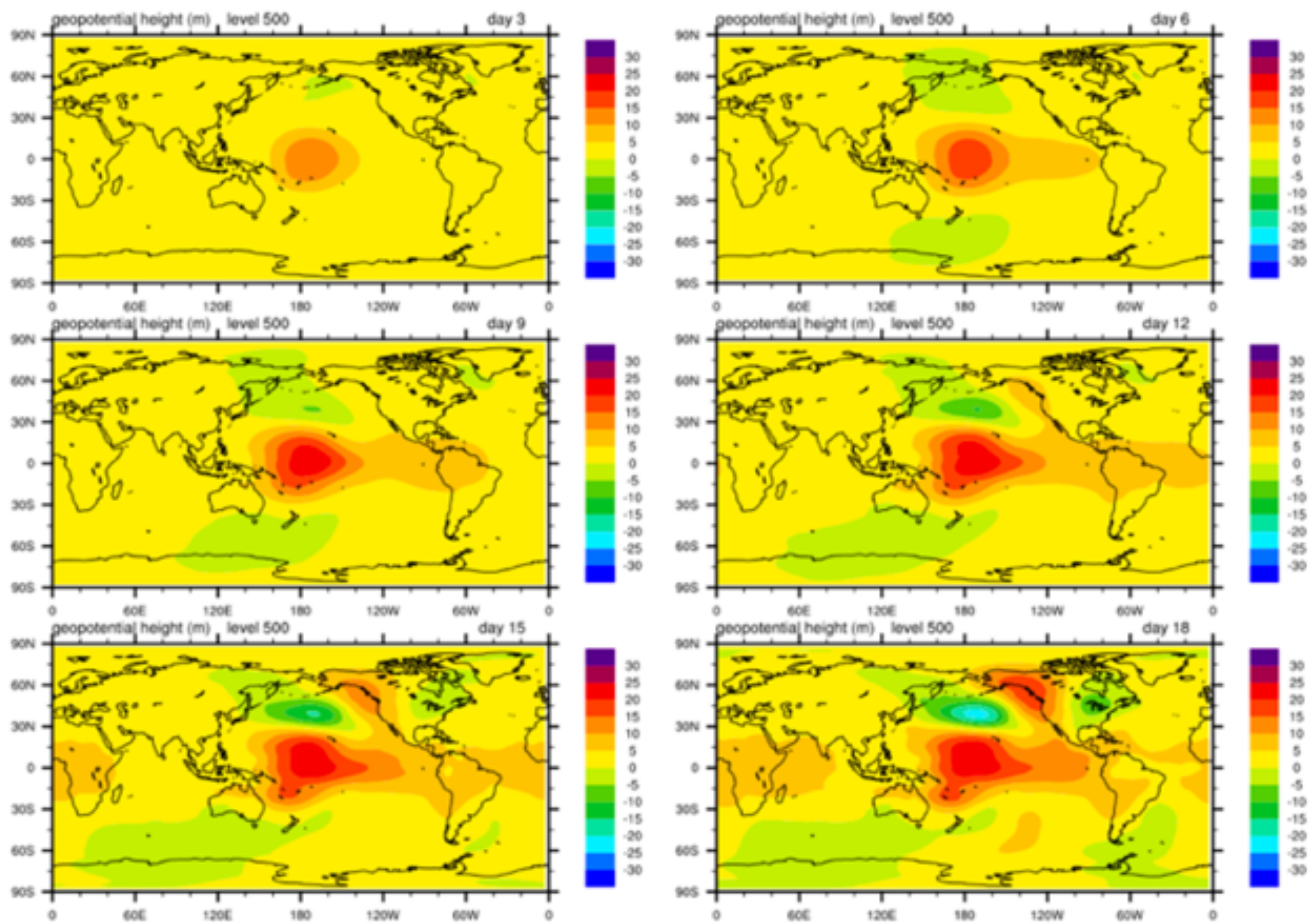
- Condensation heating leads to more complex behaviour, selects the slower mode and disrupts the fast mode.

- This behaviour is also longitude dependent, and considerably more complex on a 3-d basic state even in a dry simulation.

Response to tropical heating revisited

Fixed deep equatorial heat source.
Transient Rossby and Kelvin wave response
(500 mb height)

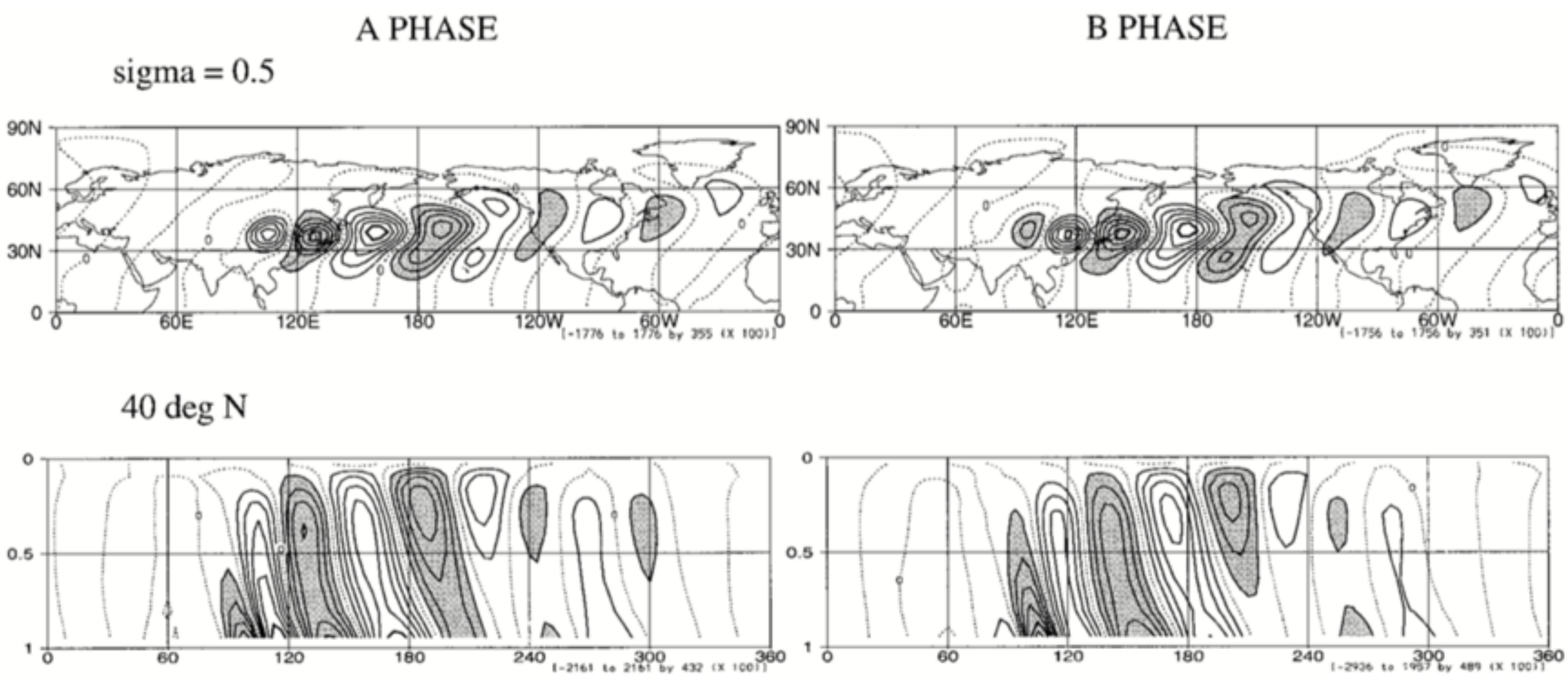




Unstable modes

Normal modes of the midlatitude wintertime flow (streamfunction)

recall $$\mathbf{\Psi}_1 = [A(x,y,z)\cos\omega t + B(x,y,z)\sin\omega t]\, e^{\sigma t}$$



Tangent linear / nonlinear response

Mid-pacific heating anomaly and now we allow the basic state to evolve

single realisation (climatological IC) | *ensemble mean* | *ensemble mean (nonlinear)*

Time-independent response

Time-independent solution to the linear problem

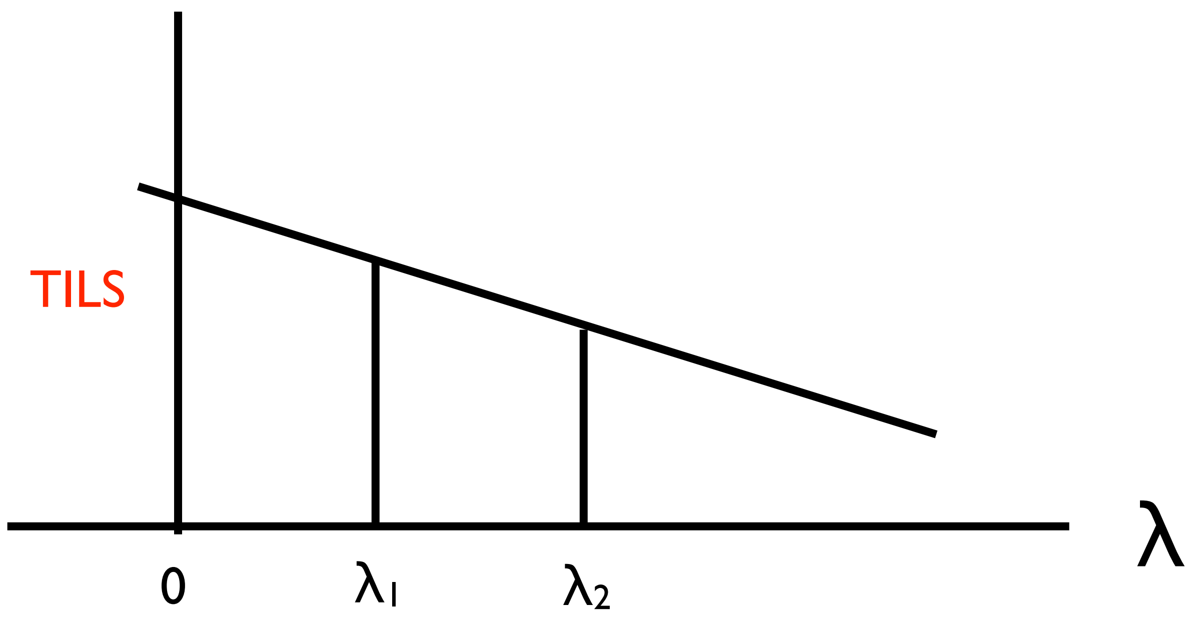
$$\mathcal{L}_c \mathbf{\Psi}_1 = \mathbf{f}_1$$

Not easy if L_c is unstable (i.e. has positive values of σ).
If this is the case any integration of the model will end up with a growing mode, unrelated to the forcing f'.

We can stabilize L_c by subtracting a multiple of the identity matrix I.
This does not affect the modal structure of L.
We can then find the time independent solution by integration of

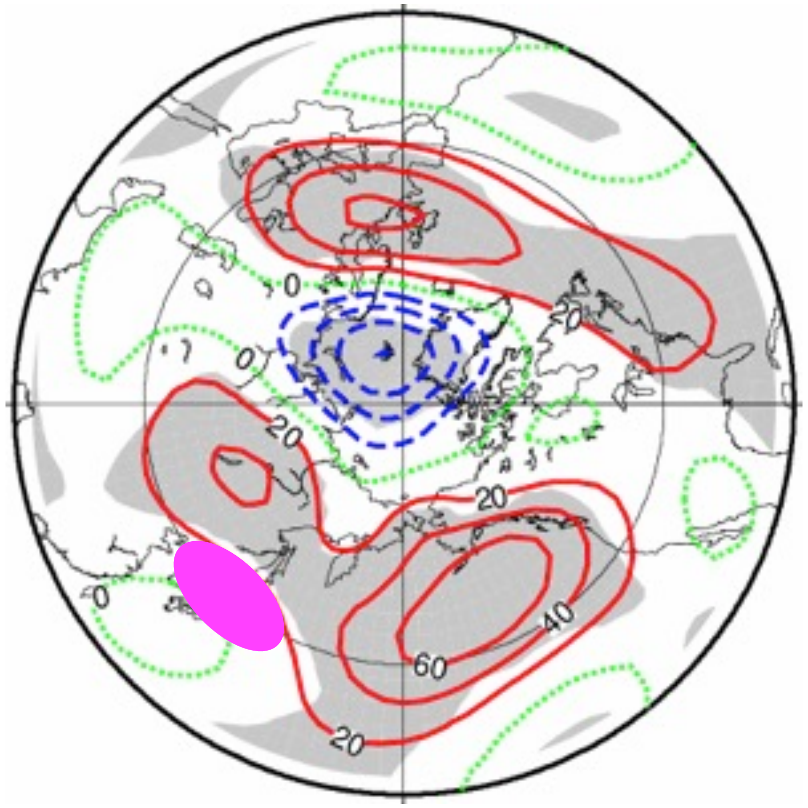
$$\frac{d\mathbf{\Psi}_1}{dt} + (\mathcal{L}_c - \lambda I)\mathbf{\Psi}_1 = \mathbf{f}_1$$

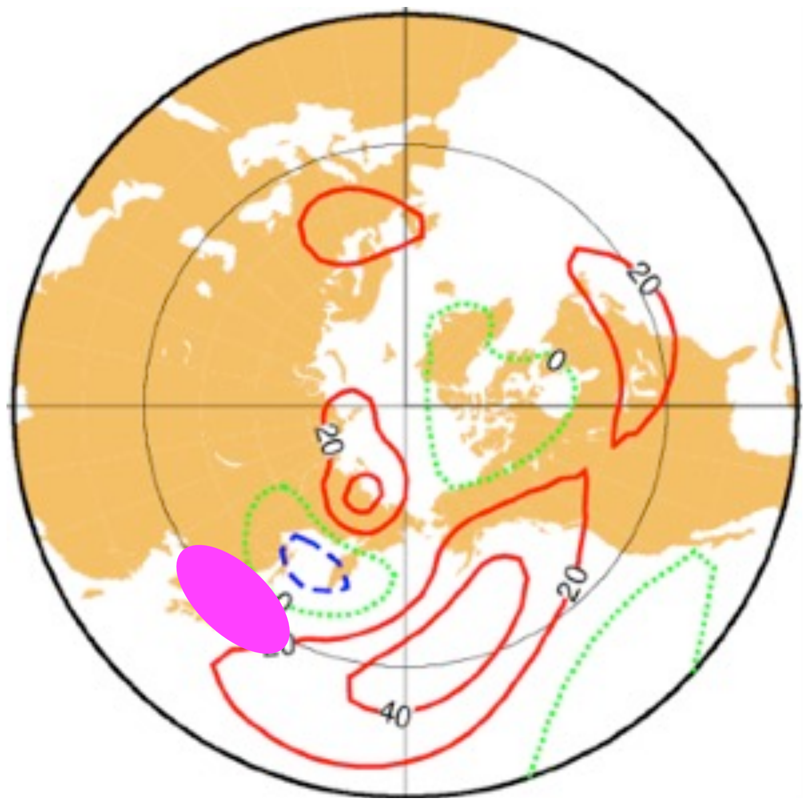
and then extrapolate back to λ = 0 to get our time independent linear solution.



Equilibrium experiments

This is the equilibrium response to a mid-latitude heating anomaly:
i.e. the difference between two long runs - one with and the other without.

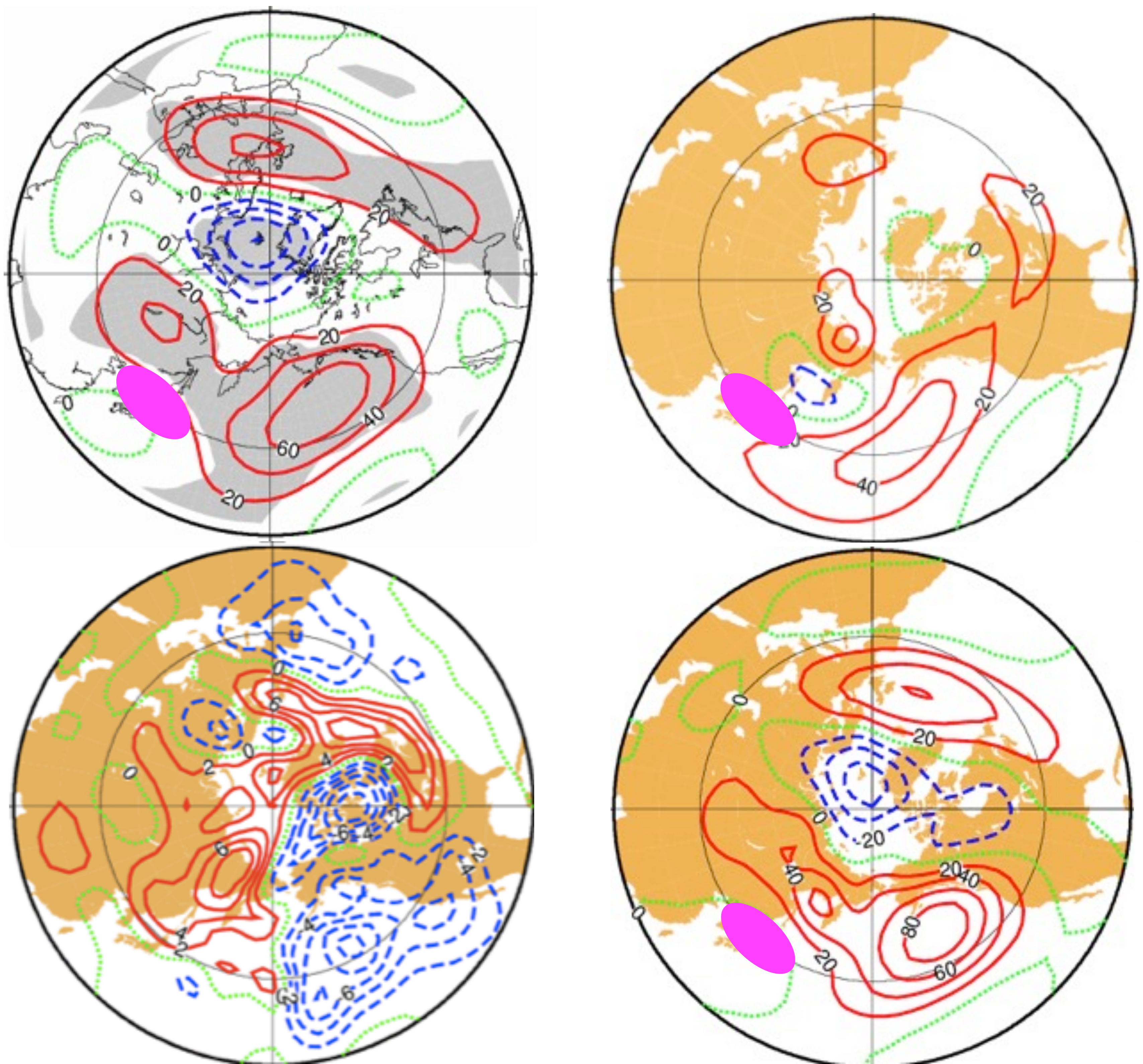




This is the time independent linear response to the same forcing

Response to transient-eddy forcing

We can diagnose the transient eddy component of the difference between the equilibrium runs and treat it as a forcing to find the associated TILS



This is the time independent linear response with added transient eddy forcing

nudge nudge

Another way of forcing a model is to push it towards a desired climatology in a restricted region, and look at the effect on the solution outside that region. This is called nudging.

$$\frac{d\mathbf{\Psi}}{dt} + (\mathcal{A} + \mathcal{D})\mathbf{\Psi} = \mathbf{g} + \left(\frac{\mathbf{\Phi}_n - \mathbf{\Psi}}{\tau}\right)$$

Nudging involves an additional constant forcing term and a damping term.

In a linear experiment, the appropriate model is:

$$\frac{d\mathbf{\Psi}_1}{dt} + \mathcal{L}_c\mathbf{\Psi}_1 = \epsilon\left(\frac{\mathbf{\Phi}_n - \mathbf{\Phi}_c}{\tau}\right) - \frac{\mathbf{\Psi}_1}{\tau}$$

This can be a useful technique for diagnosing climate anomalies or simulating other people's GCMs with a simple model.

A tropical source for the 2003 European heat wave ?

Selecting different "monsoon" regions on a summer (JJAS) basic state we can look at the effect of nudging the model towards the 2003 observations in these regions.

Here the equilibrium solution is compared to the TILS, showing in which cases transient eddy forcing is important.