



INVITED SCIENTIFIC REPORT 2

RELAXATION ASPECTS OF THE GLASS TRANSITION

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Buryat State University named after Dorjee Banzarov





Lake Baikal is the largest freshwater lake by volume in the world, containing 22–23% of the world's fresh surface water. With 23,615.39 km³ of fresh water, it contains more water than the North American Great Lakes combined. With a maximum depth of 1,642 m, Baikal is the world's deepest lake. It is considered among the world's clearest lakes and is considered the world's oldest lake — at 25 million years, making it the most ancient lake in geological history.









Republic of Buryatia, Ulan-Ude, Buddhist temple



Ulan-Ude, Monument of Lenin







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Russia, Republic of Buryatia, Lake «Baikal»





Fig. 1.1. Volume change during the liquid–glass transition in the process of cooling. ΔT_g – glass transition region, T_g – glass transition temperature. T_{g1} – corresponds to cooling rate q_1 , and T_{g2} – to cooling rate q_2 , $q_2 < q_1$. Bartenev proposed in 1951 the equation that is often called the main equation of glass transition:

$$q\tau_g = C,$$
 $\tau_g \approx (1 \div 2) \cdot 10^2 \,\mathrm{s},$

G.M. Bartenev, Dokl. Akad. Nauk SSSR 76 (1951) 227-230.

This equation is successfully applied in relaxation spectrometry of polymers and glasses as a condition of structural relaxation transition at $T = T_g$.

The glass transition temperature T_g depends on the cooling rate of the glass-forming melt q = dT/dt. The lower the cooling rate, the lower the glass transition temperature.





Fig. 1.4. Linear correlation between Bartenev equation parameters a_1 and a_2 . 1 – inorganic silicate glasses; 2 – amorphous organic polymers.

Weak dependence: e.g. a changes of q by a order of magnitude (10 times) typically result in about 3% change of T_g ($\Delta T_g/T_g \sim 0.03$)

Bartenev	equation	parameters	(1.8) and	fraction	of fluctuation	volume f_g f	or anoxic and a	ļ
number o	of oxide g	lasses [49].						

Glass	<i>Тg</i> , К	$a_1 \cdot 10^{-3}, K^{-1}$	$\begin{array}{l} a_2 \cdot 10^5, \\ K^{-1} \end{array}$	a_2/a_1	f_g
Se-Bi	309	3.12	10.45	0.034	0.031
Se-Ga	315	3.09	9.29	0.030	0.027
In-Se	316.7	3.09	5.54	0.018	0.017
As-Sb-Se	444.8	2.15	6.74	0.031	0.028
GeO ₂	762	1.23	3.45	0.028	0.026
P ₂ O ₅ .TeO ₂	578	1.67	4.99	0.030	0.027
SiO ₂ -Al ₂ O ₃ -B ₂ O ₃ -P ₂ O ₅ -	1064	0.89	3.20	0.036	0.032
MgO-Na ₂ O-K ₂ O					11

Theories

Mandelstamm-Leontovich theory

 $V = V(P, T, \xi)$

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 $|q| \cdot \tau_g = \Delta T_0$

Fig. 2.1. To the explanation of the interrelation between periodic temperature change and its approximation in the form of linear function on certain intervals. 1 – sinusoid, Eq. (2.5); 2 – linear parts of the curve as corresponding to approximation $dT/dt = \Delta T_0 \cdot \omega \cdot \cos(\omega t)$ (see text).

Relaxation theory of Volkenshtein-Ptitsyn

$$\frac{dn_2}{dT} = -\frac{1}{q\tau}(n_2 - n_{20}),$$

Condition for glass transition

$$\left(\frac{d\tau}{dT}\right)_{T=T_g} = -\frac{1}{|q|}$$



Fig. 3.1. The theoretical scheme of Volkenshtein-Ptitsyn [6]. 1 – ground state of the particle, 2 – its excited state. U_1 and U_2 – kinetic barriers for the 1 \rightarrow 2 and 2 \rightarrow 1, respectively, $\Delta E = (U_1 - U_2)$, the energy of the excited state is greater than the energy of the ground state by an amount ΔE .

$$|q|\tau_g = \frac{1}{\Psi(T_g)}$$

Configuron percolation theory

A configuron is an elementary configurational excitation in an amorphous material, formed by breaking of a chemical bond and the associated strain-releasing local adjustment of centres of atomic vibration. Formation of a broken bond can be represented as a reaction when a lattice phonon is absorbed by a bond resulting in a un-relaxed broken bond which after relaxation of the surrounding environment results in formation of a quasiparticle termed a configuron (o):





J. Phys.: Condens. Matter 18 (2006) 11507-11520

Fig. 2. CBL of A_2B_3 with several broken bonds (non-relaxed configurons). The system is shown before structure relaxation which will involve rearrangements near the broken bond (dashed circle).

$$T_g = \frac{H_d}{S_d + R \ln[(1 - f_c)/f_c]}$$

$$\int_{(T_q \to \infty)}^{(T_q)} \frac{1}{T_g = \frac{S_d}{H_d} + \frac{[\ln(1/\vartheta_c) + \ln((V_c - V_d)/V_d)]R}{H_d} - \frac{mR}{H_d} \ln\left(\frac{q}{q_0}\right)}{\frac{1}{T_g} = a_1 - a_2 \ln q}$$



Fig. 4. Degree of brokenness of bond networks in GeO_2 and SiO_2 as a function of temperature.



Williams-Landel-Ferry equation for relaxation and viscosity

$$\ln a_T = -C_1 \frac{T - T_g}{T - T_g + C_2}, \quad a_T = \frac{\tau(T)}{\tau(T_g)} \cong \frac{\eta(T)}{\eta(T_g)}, \qquad \left(\frac{d\tau}{dT}\right)_{T = T_g} = -\frac{1}{|q|}$$
$$q\tau_g = \frac{C_2}{C_1},$$

from which follows the formula for calculating δT_g using parameters of the WLF equation C_1 and C_2

$$\delta T_g = \frac{C_2}{C_1}.$$

$$\delta T_g = (8 \div 12) \text{ K}$$



Fig. 7.2. Linear correlation between δT_g and T_g for amorphous organic polymers. 1 polyhexene-1, 2 – polyurethane, 3 – polyvinylchloroacetate, 4 – polymethylacrylate, ξ polyvinyl acetate, 6 – natural rubber, 7 – methacrylate ethyl. Data from [36].

Fig. 7.3. Correlation between δT_g and T_g for metallic glasses. Data from [77,78]. 1 – Fe₈₃B₁₇, 2 – Fe₈₀P₁₃C₇, 3 – Fe_{41.5}Ni_{41.5}B₁₇, 4 – Pd₈₂Si₁₈, 5 – Pd_{77.5}Cu₆Si_{16.5}, 6 – Pd₄₀Ni₄₀P₂₀.

Nemilov estimations

$$\delta T_g = -\left(\frac{\partial T}{\partial \lg \eta}\right)_{T=T_g} = -\left(\frac{\Delta T}{\Delta \lg \eta}\right) = \frac{T_{12} - T_{13}}{\lg \eta_{12} - \lg \eta_{13}}$$

Overestimated data

$$\delta T_g = (T_{12} - T_{13}) \cong 20 \text{ K}$$

Volkeistein-Ptitsyn estimations

$$\delta T_g = \frac{1}{2.3}(T_{12} - T_{13}) = (6 - 13) \text{ K}$$

Glass (content, mol%)	T ₁₂ , K	T ₁₃ , K	$\boldsymbol{\delta T_g, K}$ $\boldsymbol{\delta T_g = T_{12} - T_{13}}$	$\delta T_{a}, K$ $\delta T_{g} = \frac{1}{2.3}(T_{12} - T_{13})$	$\delta T_g = \frac{C_2}{C_1}, \boldsymbol{K} \ (6.8)$	<i>C</i> ₁	C2, K	$ au_g, \ \mathbf{c}$
Sodium-silicate glasses Na ₂ O – S	SiO ₂ [49]							
15 Na ₂ O-85 SiO ₂	819	790	29	13	12	36	430	239
20 Na ₂ O-80 SiO ₂	792	766	26	11	11	36	390	217
25 Na ₂ O-75 SiO ₂	769	745	24	10	10	35	355	202
30 Na ₂ O-70 SiO ₂	749	727	22	10	9	35	322	184
33 Na ₂ O-67 SiO ₂	738	717	21	9	9	35	304	174
35 Na ₂ O-65 SiO ₂	726	705	21	9	8	35	291	166
Window glass [19]	846	825	21	9	8	36	305	160
Poly-alkali silicate glass [76]								
69.04SiO ₂ ·30.96Na ₂ O	736	718	18	8	7	46	340	147
79.29SiO2·12.97Na2O·7.75Li2O	700	683	17	7	7	45	315	140
43.22SiO ₂ .9.55Na ₂ O.47.23CsO	721	704	17	7	6	31	200	129
71.59SiO ₂ ·24.4Na ₂ O·4.01Li ₂ O	695	681	14	6	6	36	231	128
Amorphous polymers [58]								
Polyisobutylene	-	202	6	-	2.7	38	104	54
Polyvinyl acetate	-	305	9	-	1.3	36	47	26
Polyvinylchloroacetate	-	296	9	-	1.0	40	40	20
Polymethyl acrylate	-	276	8	-	1.1	42	45	22
Polyurethane	-	238	7	-	0.9	36	33	18
Natural rubber	-	300	9	-	1.4	38	54	57
Methacrylate polymers								
ethyl	-	335	10	-	1.6	40	65	32
n-butyl	-	300	9	-	2.5	39	97	50
n-octyl	-	253	8	-	2.9	37	107	58

Calculation of temperature range δT_g for silicate glasses and amorphous polymers.

Note: Composition of window glass (mass%) [40]: SiO₂ – 72.7; CaO – 8.6; MgO – 3.4; Al₂O₃ – 1.3; Na₂O – 13.6; K₂O – 0.4; $\tau_g = C_2 / qC_1$, $C_1 \mu C_2$ – parameters of the Williams-Landel-Ferry equation, q = 0.05 K/s.

inorganics

 $q\tau_g\approx (5\,-\,10)\,{\rm K}$

organics

$$\delta T_g \approx (1-3) \text{ K}$$

Model of delocalized atoms

D.S. Sanditov, J. Exp. Theor. Phys. 115 (2012) 112-124.

Fluctuation volume Fraction of fluctuation volume $\Delta V_e = N_e \Delta v_e$ $f = \frac{\Delta V_e}{V}$ $f = \frac{\Delta V_e}{V}$ $f = \frac{\Delta V_e}{V}$

WLF constants and delocalized atoms model:

 $C_1 = \frac{1}{f_g}$, $C_2 = \frac{f_g}{\beta_f}$, $f_g = (\Delta V_e/V)_{T=T_g}$ is the fraction of fluctuation volume frozen

The fraction of fluctuation volume frozen at glass transition is practically a constant

$$f_g = \left(\frac{\Delta V_e}{V}\right)_{T=T_g} = \frac{1}{C_1} \approx const \approx 0.024 - 0.028$$

A double-exponent equation for viscosity

$$\eta = \eta_o \exp\left\{\frac{\Delta F_{\infty}}{kT} + \left[\exp\left(\frac{\Delta \varepsilon_e}{kT}\right) - 1\right]\right\}$$

Amorphous	T_g ,	Cl	С ₂ ,	$\delta T_g = \frac{C_2}{C_1},$	f_g	$\frac{\delta T_g}{T} \cdot 10^3$	$\delta T_{g}, \boldsymbol{K}$	$\tau_g,$	C ₀	$\left(\frac{N_{e}}{N_{e}}\right)$
substance	K		K	K		Ig	$\frac{\delta T_g}{T_g} \cong \frac{f_g}{\ln\left(1/f_g\right)}$	c		$\binom{N}{T=T_g}$ %
Sodium silicate glass l	Na ₂ O–SiO ₂	[49]								
Na ₂ O, mol%										
15	782	36	430	12	0.028	7.8	6.1	240	0.5	3.5
20	759	36	390	11	0.028	7.8	5.9	220	0.5	3.4
25	739	35	355	10	0.028	7.8	5.8	200	0.5	3.5
30	721	35	322	9	0.028	7.8	5.6	180	0.5	3.5
33	712	35	304	9	0.028	7.8	5.6	180	0.6	3.5
35	705	35	291	8	0.028	7.8	5.5	160	0.6	3.5
Metallic glass (amorp)	hous alloys	s) [77]								
Pd40Ni40P20	602	39	93	2.4	0.026	7.1	4.3	48	0.8	3.6
Pt ₆₀ Ni ₁₅ P ₂₅	500	37	95	2.6	0.027	7.5	3.7	52	0.8	3.6
Pd _{77.5} Cu ₆ Si _{16.5}	653	38	100	2.6	0.026	7.1	4.6	52	0.8	3.7
Fe ₈₀ P ₁₃ C ₇	736	38	120	3.2	0.026	7.1	5.2	64	0.8	3.6
Amorphous organic p	olymers [5	8] and seleni	ium [5]							
Polyvinyl acetate	305	36	47	1.3	0.028	7.8	2.4	61	0.8	3.5
Natural rubber	300	38	54	1.4	0.026	7.1	2.1	57	0.8	3.6
Methacrylate ethyl	335	40	65	1.6	0.025	6.8	2.3	50	0.8	3.6
Selenium	303	32	58	1.8	0.031	8.9	2.7	44	0.8	3.6
Low molecular weight	organic gl	ass [61]								
Propanol	98	41	25	0.6	0.024	6.4	0.6	12	0.7	3.6
Prothylene Glycol	160	44	40	0.9	0.023	6.1	1.0	18	0.7	3.7
Glycerol	185	42	53	1.3	0.024	6.4	1.2	26	0.7	3.9

WLF equation parameters C_1 , C_2 and the glass transition characteristics of amorphous substances.

Note: $f_g = 1/C_1$, $\tau_g = C_2/C_1q$.

Sanditov equation of viscosity

D.S. Sanditov, JETP 110 (2010) 675-688.



Fig. 12.2. Temperature dependence of the viscosity of sodium silicate (*a*), lead silicate (*b*), lithium silicate (*c*) and sodium germanate (*d*) glasses. Points – experimental data, curves – calculation by the Eq. (12.11).

Characteristics of viscous flow	of inorganic	glasses	R ₂ O-SiO ₂	(R =	Li, Na,	K),	PbO-SiO ₂ ,
Na2O-GeO2 and Na2O-B2O3.							

3.77

4.45

0.026

0.025





T, °C M.I. Ojovan, Phys. Chem. Glass. 53 (2012) 143–150.

Kinetic criterion of glass transition

Gutzov suggested the criterion:

$$\left(\frac{1}{T}\left|\frac{dT}{dt}\right|\tau\right)\Big|_{T=T_g} = C_3, \text{ with } C_3 \approx 1.$$

It gives unrealistic $q\tau_g \approx 800$ K (Nemilov).

It is suggested:

$$\frac{q\iota_g}{T_g} = C_g,$$

where value of C_g is actually a universal constant

$$C_g = \frac{\delta T_g}{T_g} = \frac{f_g}{\ln\left(1/f_g\right)} \approx const \approx 0.007.$$

In terms of Gutzov equation e.g.:

$$\left(\frac{1}{T} \left| \frac{dT}{dt} \right| \right) \tau \bigg|_{T=T_g} = C_g, \quad C_g \approx 7 \cdot 10^{-3}$$

Generalized Bartenev equation

$$\frac{1}{T_g} = a_1 + b_1 \ln \left(1 - \frac{\ln q}{b_2}\right)$$

 $b_1 = R/\Delta \varepsilon_e$; $b_2 = \ln(C/\tau_0)$; $a_1 = b_1 \ln b_2 \ \tau_0 \approx 10^{-12}$ s and $C \approx 10$ K $b_2 = \ln(C/\tau_0) \approx 30$



Fig. 11.1. Dependence of T_g on cooling rate for glasses As-Sb-Se and Te-Ge in the coordinates of Eq. (11.3): $1/T_g - \ln[1 - (\ln q/30)]$. Data from [49]: (Mahadevan S., 1986 and Lasocka M., 1976). Parameters of Eq. (11.3) for As-Sb-Se: $a_1 = 2, 1 \cdot 10^{-3} \text{ K}^{-1}$, $b_1 = 4, 5 \cdot 10^{-3} \text{ K}^{-1}$; and for Te-Ge: $a_1 = 2, 4 \cdot 10^{-3} \text{ K}^{-1}$, $b_1 = 5, 6 \cdot 10^{-3} \text{ K}^{-1}$.



Fig. 11.2. Dependence 1/Tg on $\ln(1 - \ln q/30)$ for Se-Ga glass. Data from [49]: (El-Oyoun M.A., 2003). Parameters of Eq. (11.3): $a_1 = 3 \cdot 10^{-3} \text{ K}^{-1}$, $b_1 = 5,7 \cdot 10^{-3} \text{ K}^{-1}$.



$$\frac{1}{T_g} = a_1 + b_1 \ln\left(1 - \frac{\ln q}{b_2}\right)$$

When Inq<<30 this reduces to original Bartenev equation



$$\frac{1}{T_g} = a_1 - a_2 \ln q$$

$$a_2 = b_1/b_2$$

Fig. 11.3. Dependence of the glass transition temperature on the heating rate for lead silicate glasses Nº1 and Nº2 in coordinates $1/T_g - \ln(1 - \ln q/30)$. Data from [45]. Parameters of Eq. (11.3) for glass Nº 1: $a_1 = 1,3\cdot10^{-3}$ K⁻¹, $b_1 = 7,4\cdot10^{-3}$ K⁻¹; and glass Nº 2: $a_1 = 1,2\cdot10^{-3}$ K⁻¹, $b_1 = 6,2\cdot10^{-3}$ K⁻¹.

Conclusions

- Glass transition of liquid has a pronounced relaxation character and obeys kinetic laws.
- The relation between structural relaxation time τ and cooling rate q = dT/dt is expressed by glass transition equation: $q\tau_q = \delta T_q$.
- Calculation of temperature interval δT_g using the data on parameters of Williams-Landel-Ferry equation and the model of delocalized atoms for inorganic oxide glasses is in an agreement with the product $q\tau_q \approx (5 - 10) K$.
- An analogous statement is true for organic glasses, whose δT_g have lower values $\delta T_g \approx (1-3) K$.
- Bartenev equation is true at relatively low cooling rates lnq<<30.

- Fraction of the fluctuation volume f_g , calculated from the data on the dependence of T_g on cooling rate of the melt coincides with calculation of f_g from the data on viscosity near the glass transition region.
- A refined version of kinetic criterion of glass transition is proposed.
- Excited delocalized atoms are responsible for viscous flow of glass-forming materials, their concentration during cooling decreases and in the glass transition region reaches negligibly small values (around 3%), which is equivalent to freezing.

