

Outline

- Digital CMOS design
 - Boolean algebra
 - Basic digital CMOS gates
 - Combinational and sequential circuits
 - **Coding - Representation of numbers**



Representing Numbers

How values can be coded ?

In a digital circuit each signal can take 2 values (0, 1) (Boolean world)

A vector of n bits can represent up to 2^n values



Representing Numbers

How values can be coded ?

What is the meaning of 0100 0110 ?

The character 'F'

The character 'Φ'

The number 46

The number 70

The number 123

Any symbol in a set where the *Card* = 256



Representing Numbers

How values can be coded ?

by itself a code has no signification



Representing Numbers

How values can be coded ?

arithmetic : dealing with numbers

How can I represent a number ?

Natural numbers

Relative numbers

Rational numbers

Real numbers



Representing Numbers

How can I represent a Natural number ?

I need at least n bits for a Natural ranging from 0 to $2^n - 1$

Standards



Representing Numbers

How can I represent a Natural number ?

Natural Binary Code :

The bits represent the successive powers of 2

$$\begin{array}{c} 0100\ 0110 = 2^1 + 2^2 + 2^6 = 70 \\ \swarrow \quad \nwarrow \\ 2^7 \quad \quad 2^0 \end{array}$$



Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal :

The bits represent the successive powers of 2

The quartets represent the successive powers of 10

$$\begin{array}{c}
 \text{0100 0110} = (2^1 + 2^2) \times 10^0 + 2^2 \times 10^1 = 46 \\
 \begin{array}{cccc}
 \swarrow & \searrow & \swarrow & \searrow \\
 2^3 & & 10^1 & & 10^0 & & 2^0
 \end{array}
 \end{array}$$

packed



Representing Numbers

How can I represent a Natural number ?

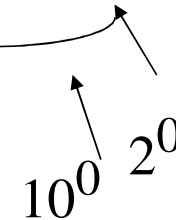
Binary Coded Decimal - Unpacked :

The bits represent the successive powers of 2

The bytes represent the successive powers of 10

In each byte the 4 Msb are 0

0100 0110 = Illegal 0000 0110 = 6



Representing Numbers

How can I represent a Relative number ?

Sign + Value

The bits represent the successive powers of 2

The Msb represents the sign (1 means negative)



Representing Numbers

How can I represent a Relative number ?

Sign+Value :

$$\begin{array}{c}
 + \\
 \nearrow \quad \nearrow \quad \nwarrow \\
 \mathbf{0100\ 0110} = 2^1 + 2^2 + 2^6 = 70 \\
 \quad \quad \quad \nearrow \quad \quad \quad \nwarrow \\
 \quad \quad \quad 2^6 \quad \quad \quad 2^0
 \end{array}$$

$$\begin{array}{c}
 - \\
 \nearrow \quad \nearrow \quad \nwarrow \\
 \mathbf{1100\ 0110} = -1 \times (2^1 + 2^2 + 2^6) = -70 \\
 \quad \quad \quad \nearrow \quad \quad \quad \nwarrow \\
 \quad \quad \quad 2^6 \quad \quad \quad 2^0
 \end{array}$$



Representing Numbers

How can I represent a Relative number ?

2's complemented

The bits represent the successive powers of 2

The Msb represents -2^{n-1}



Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$\begin{array}{c}
 \text{0100 0110} = 2^1 + 2^2 + 2^6 = 70 \\
 \begin{array}{ccc}
 \nearrow & \nearrow & \nwarrow \\
 -2^7 & 2^6 & 2^0
 \end{array}
 \end{array}$$

$$\text{1100 0110} = 2^1 + 2^2 + 2^6 - 2^7 = -58$$



Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$$

$$\begin{array}{r}
 70 \\
 \swarrow \quad \downarrow \quad \searrow \\
 2^7 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 1 \\
 2^7 = 2^0 \qquad \qquad \qquad + 2^3 + 2^4 + 2^5 \qquad \qquad + 1 + 70 \\
 1011\ 1010 \quad -70 = 2^0 \qquad \qquad \qquad + 2^3 + 2^4 + 2^5 \qquad \qquad + 1 - 2^7 \\
 \qquad \qquad \qquad -70 = 2^1 \qquad \qquad \qquad + 2^3 + 2^4 + 2^5 \qquad \qquad - 2^7
 \end{array}$$



Representing Numbers

How can I represent a Real number ?

Range
Precision



Representing Numbers

How can I represent a Real number ?

2's complement Fixed Point :

The bits represents the successive powers of 2

$$0100.0110 = 2^{-3} + 2^{-2} + 2^2 = 4.325$$

The diagram illustrates the bit weights for the binary number 0100.0110. The bits are arranged as follows: 0 (bit 3), 1 (bit 2), 0 (bit 1), 0 (bit 0), . (decimal point), 0 (bit -1), 1 (bit -2), 1 (bit -3), 0 (bit -4). The bit weights are: -2^3 (pointing to the first '1'), 2^0 (pointing to the second '1'), 2^{-1} (pointing to the first '1' after the decimal), and 2^{-4} (pointing to the second '1' after the decimal).



Representing Numbers

How can I represent a Real number ?

Wide range
High precision

Floating Point



Representing Numbers

How can I represent a Real number ?

Normalized scientific representation

$$R = (-1)^S \times M \times 10^E$$

S : Sign (1 if negative)

M : Mantissa ($\in [1, 10[$) in radix 10

E : Exponent (Relative number)



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

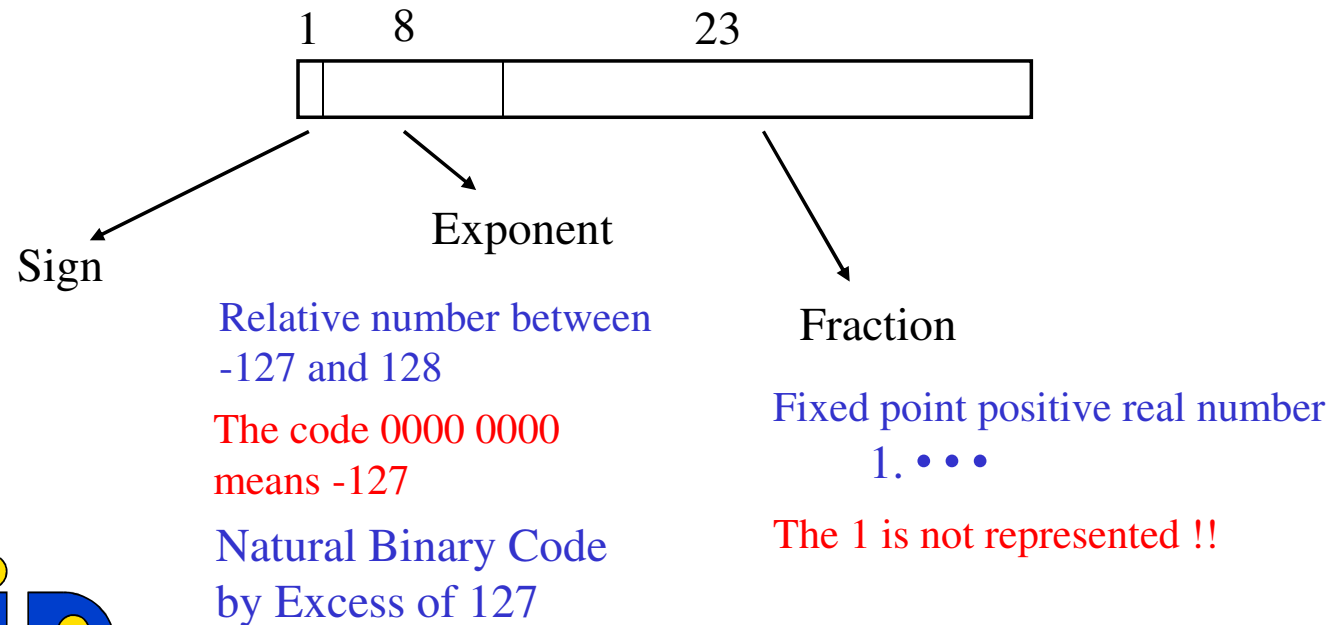
	Single Precision 32 bits	Double Precision 64 bits
S : Sign (1 if negative)	1 bit	1 bit
M : Mantissa ($\in [1, 2[$)	23 bits	52 bits
E : Exponent	8 bits	11 bits



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

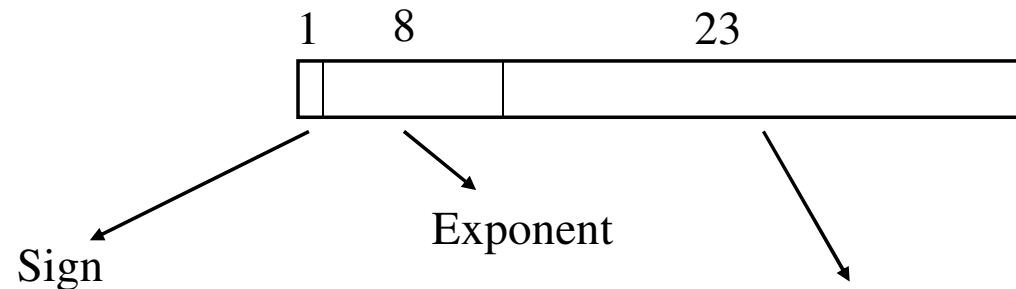
Single precision :



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 1111 1111
(128) means $\pm\infty$ or
an error

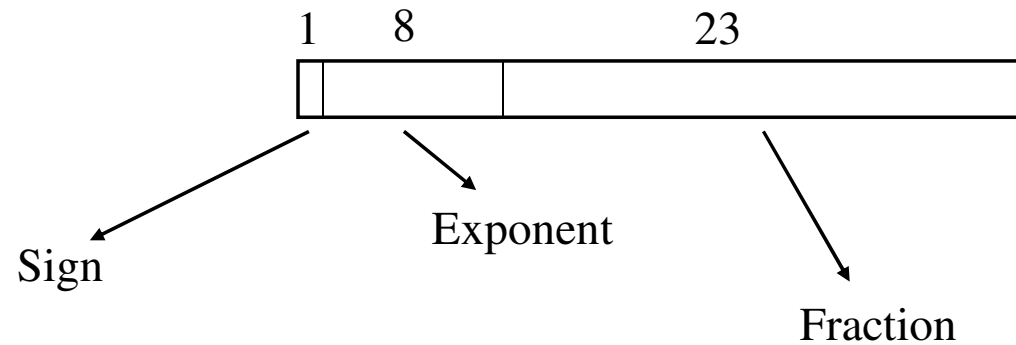
Fraction
.000...000 means $\pm\infty$
other values mean error (NaN)



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Range and precision



$$R \in]-2^{128}, 2^{128}[$$

$$\text{Precision} = 2^{-23+E}$$



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases

