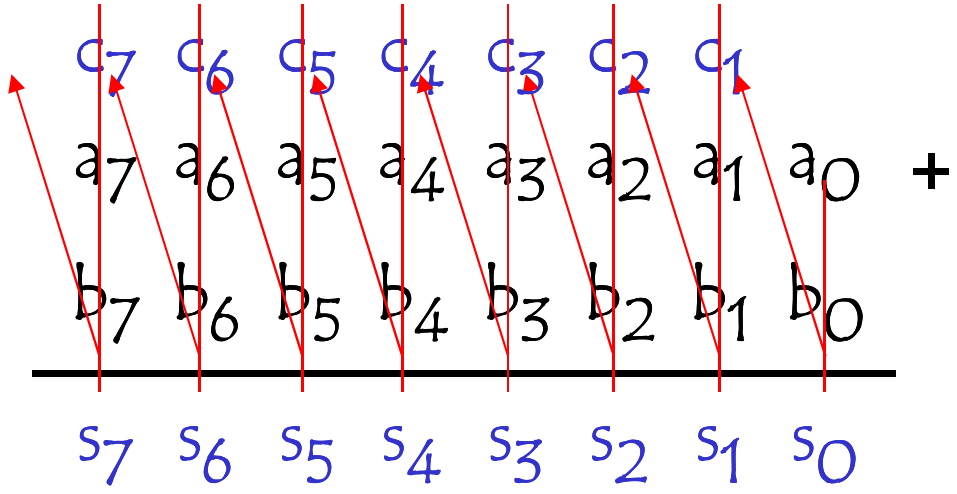
Outline

- Digital CMOS design
- Arithmetic operators
  - Adders
  - Comparators
  - Shifters
  - Multipliers

Adders

Adding two natural numbers

Let consider two natural numbers a and b coded on 8 bits using Natural Binary Code

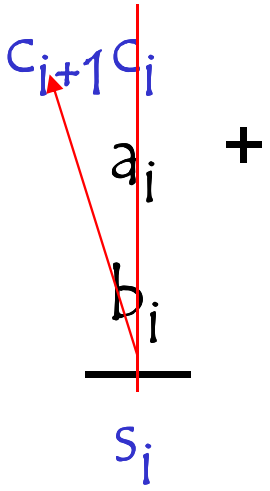


Adders

Adding two natural numbers

At each stage, I need to sum 3 single bit numbers a_i b_i c_i

The carry out of the stage i is the input carry of the next stage



s_i and c_{i+1} are Boolean functions of a_i b_i c_i

Adders

Adding two natural numbers

c_i \ $a_i b_i$	00	01	11	10
0	0	1	0	1
1	1	0	1	0

s_i

$$s_i = a_i \oplus b_i \oplus c_i$$

c_i \ $a_i b_i$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

c_{i+1}

$$c_{i+1} = a_i.b_i + a_i.c_i + b_i.c_i$$

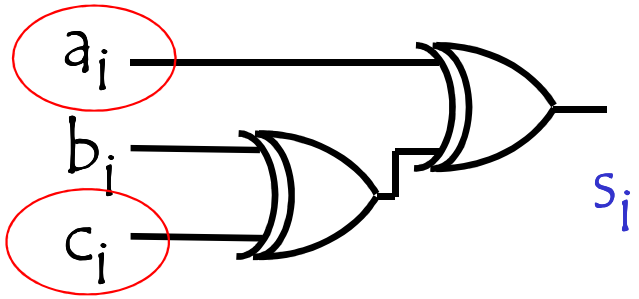
Adders

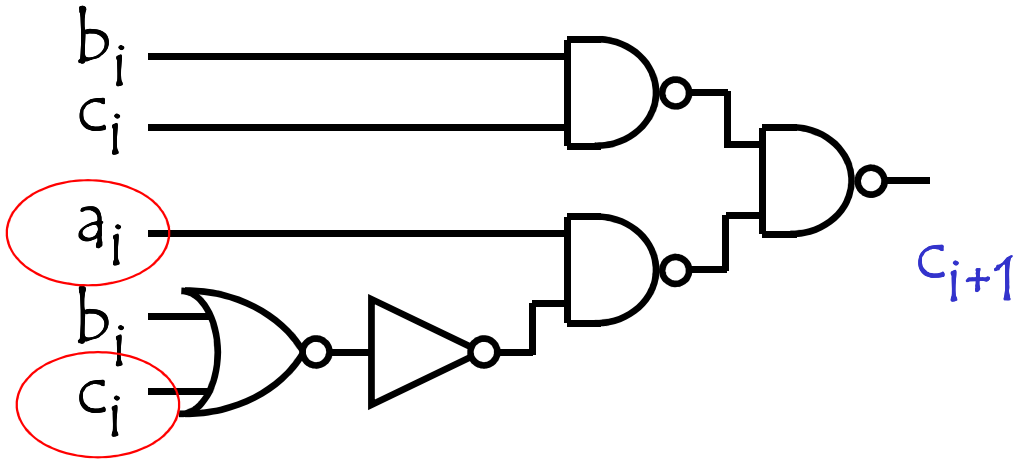
Adding two natural numbers

$s_i = a_i \oplus b_i \oplus c_i$

$c_{i+1} = a_i.b_i + a_i.c_i + b_i.c_i$

$c_{i+1} = a_i \cdot (b_i + c_i) + b_i.c_i$





Addition delay depends on the delay of c_i to c_{i+1}

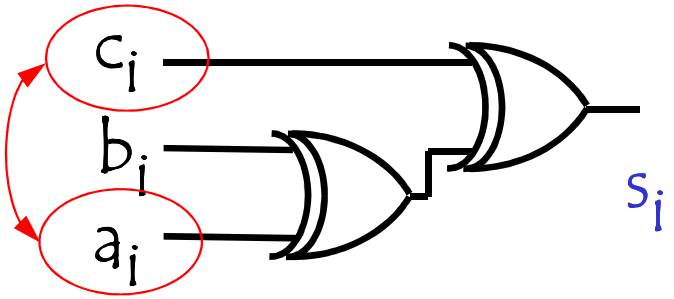
Adders

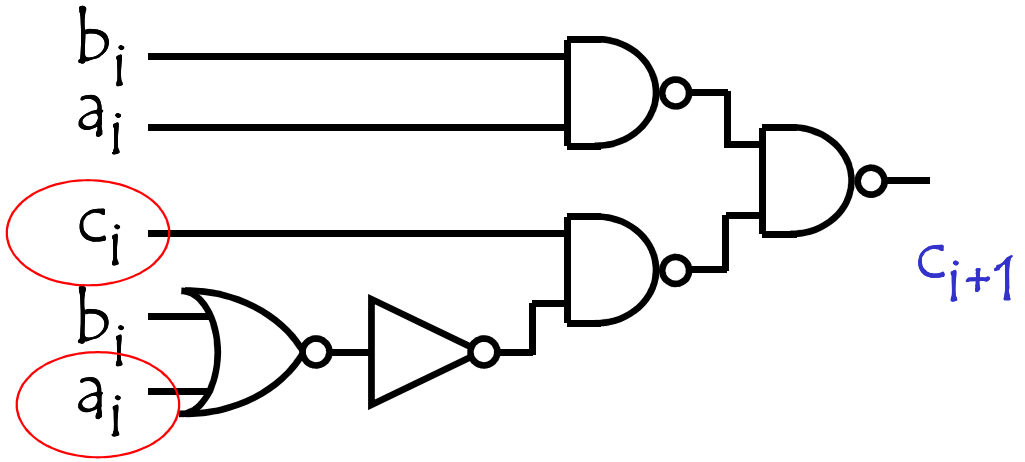
Adding two natural numbers

$s_i = a_i \oplus b_i \oplus c_i$

$c_{i+1} = a_i.b_i + a_i.c_i + b_i.c_i$

$c_{i+1} = a_i.b_i + (a_i+b_i) . c_i$





Adders

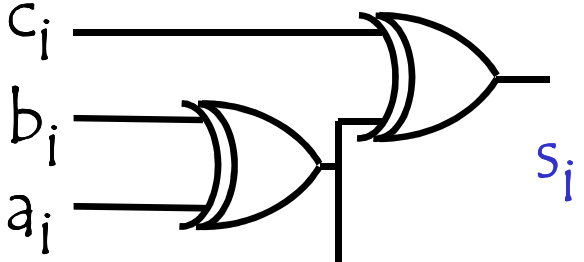
Adding two natural numbers

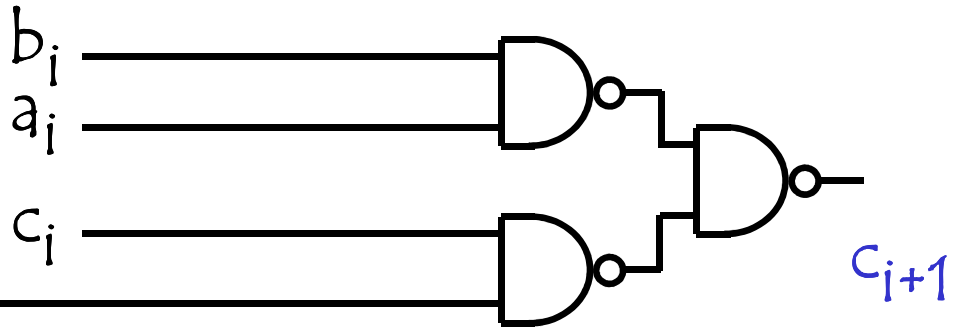
$s_i = a_i \oplus b_i \oplus c_i$

$c_{i+1} = a_i.b_i + a_i.c_i + b_i.c_i$

$c_{i+1} = a_i.b_i + (a_i+b_i) \, . \, c_i$

$c_{i+1} = a_i.b_i + (a_i \oplus b_i) \, . \, c_i$

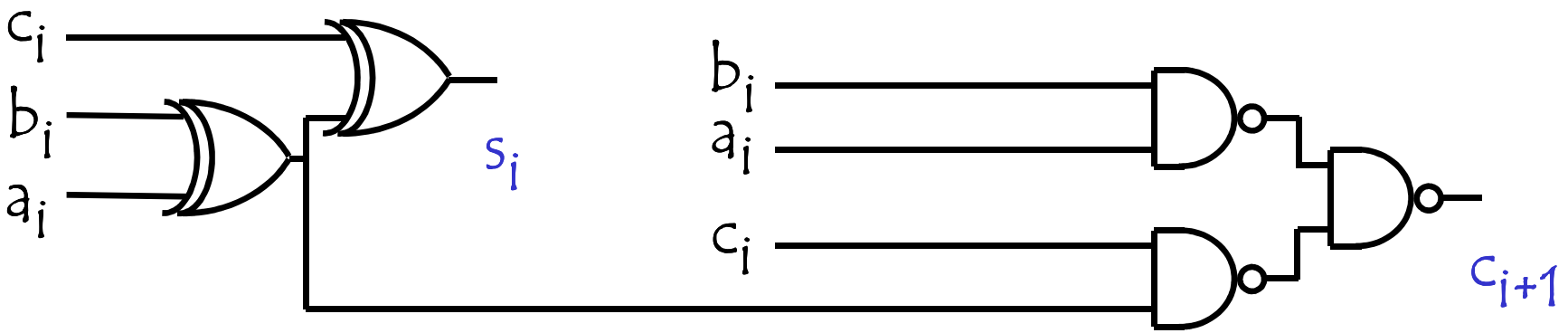




Adders

Adding two natural numbers

The circuit generating s_i and c_{i+1} is called a Full Adder (FA)

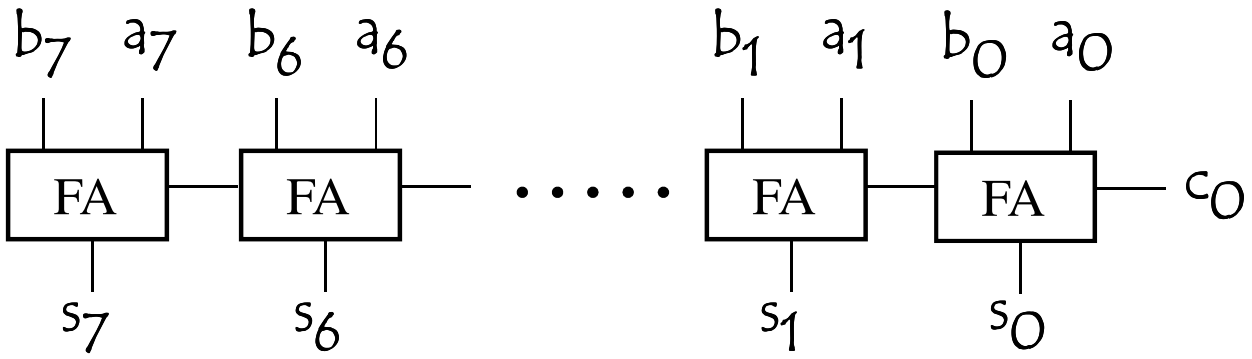


Adders

Adding two natural numbers

At each stage, I need to sum 3 single bit numbers a_i b_i c_i

The carry out of the stage i is the input carry of the next stage



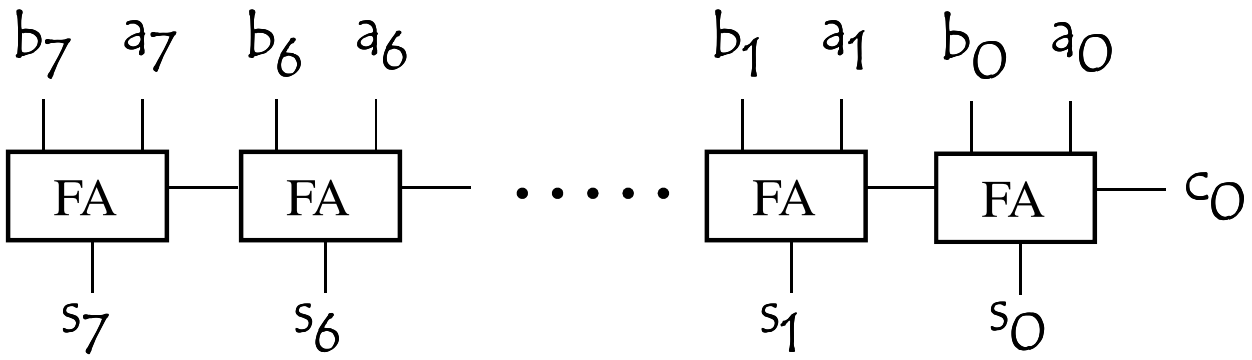
Ripple Carry Adder (RCA)

Adders

Adding two natural numbers

Ripple Carry Adder (RCA)

Area ∝ n Delay ∝ n

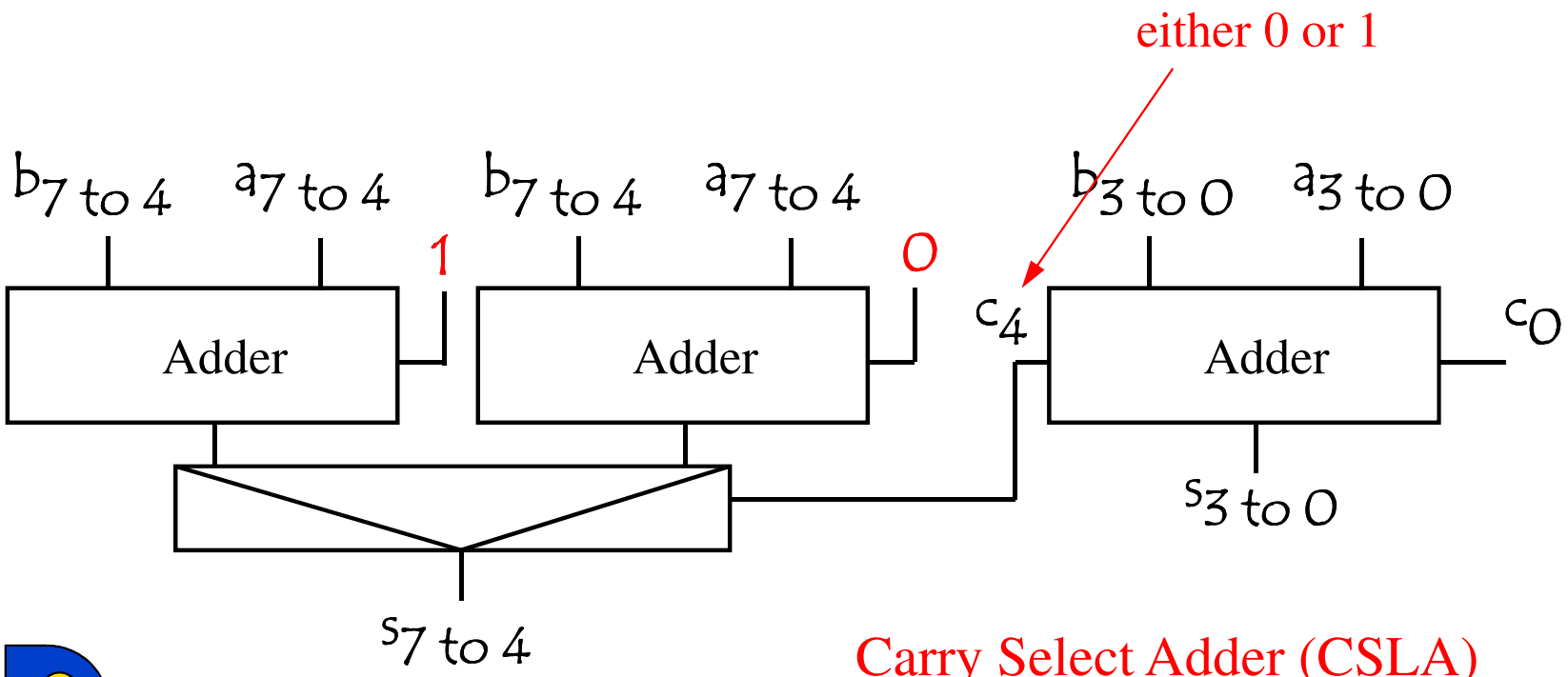


Timing should be improved

Adders

Adding two natural numbers

Acceleration technics



Carry Select Adder (CSLA)

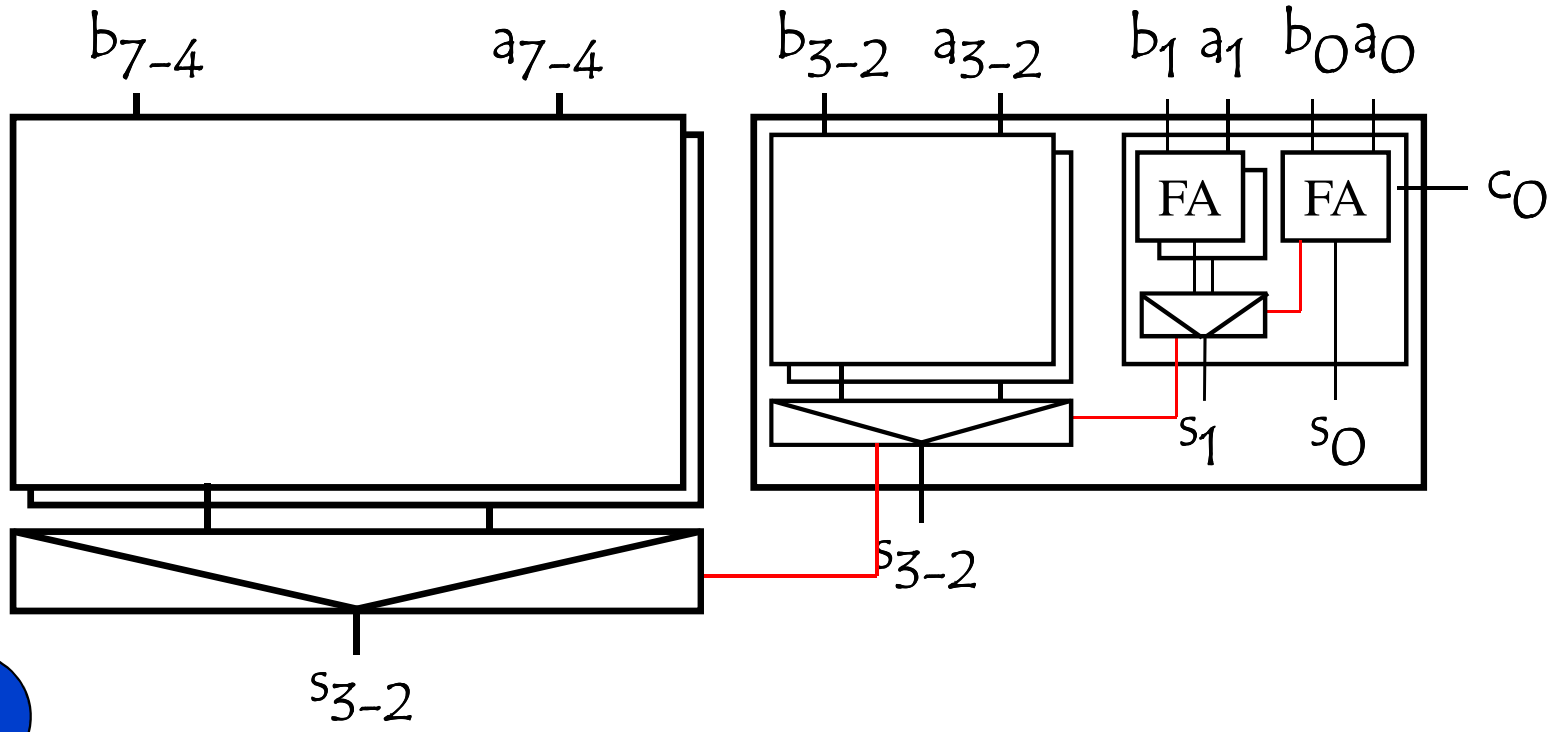
Adders

Adding two natural numbers

Carry Select Adder (CSLA)

Area $\propto n^{log(3)} = n^{1.585}$

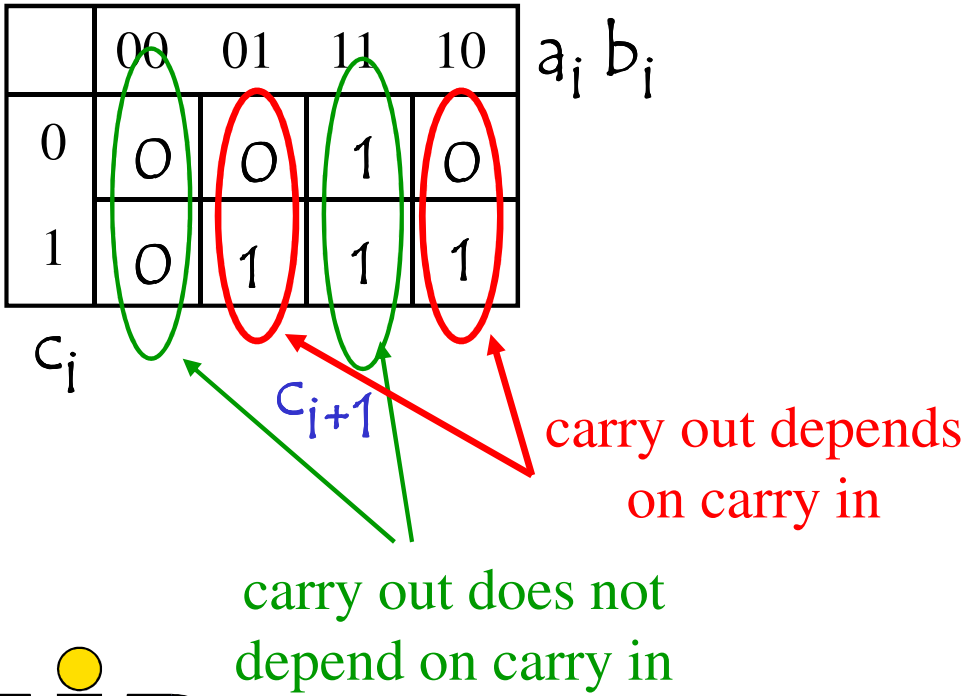
Delay $\propto log\ (n)$



Adders

Adding two natural numbers

Acceleration technics



00	01	11	10	a_i b_i
absorption	propagation	generation	propagation	

Adders

Adding two natural numbers

Acceleration technics

c_i \ $a_i b_i$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

c_{i+1}

00	01	11	10	$a_i b_i$
absorption	propagation	generation	propagation	

$$G_i = a_i b_i$$

$$P_i = a_i \oplus b_i$$

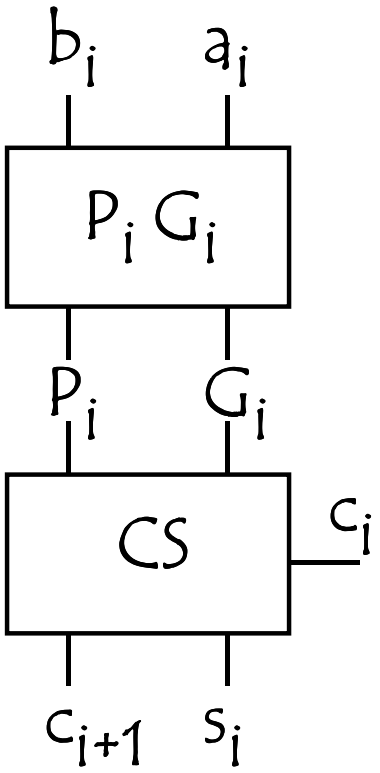
$$c_{i+1} = G_i + P_i c_i$$

$$s_i = P_i \oplus c_i$$

Adders

Adding two natural numbers

Acceleration technics



$$G_i = a_i\, b_i$$

$$P_i = a_i \oplus b_i$$

$$c_{i+1} = G_i + P_i\, c_i$$

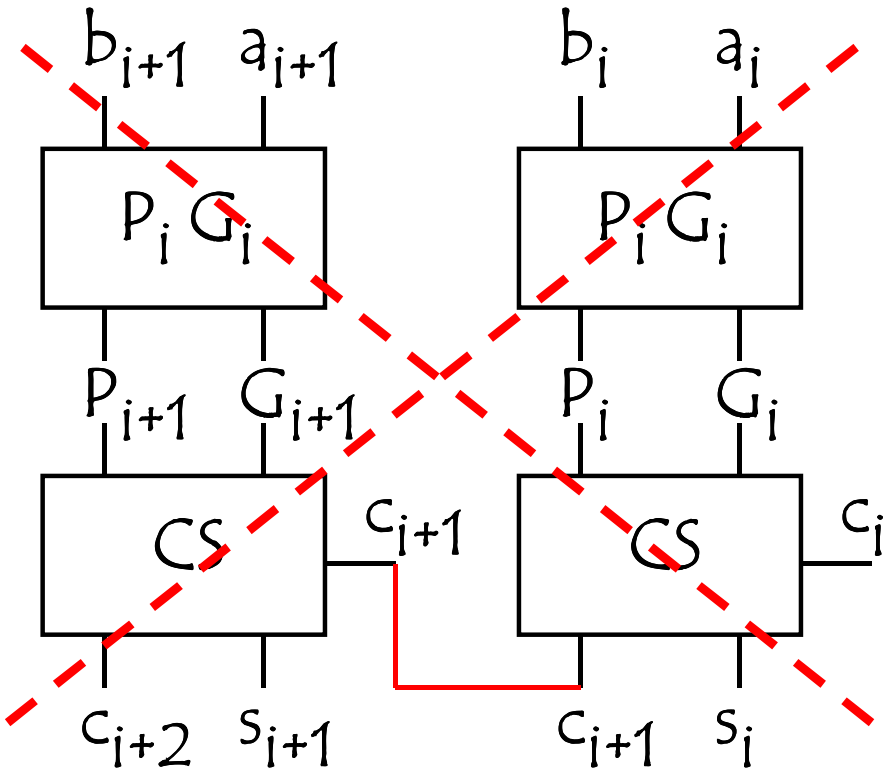
$$s_i = P_i \oplus c_i$$

a_i b_i	00	01	11	10
	absorption	propagation	generation	propagation

Adders

Adding two natural numbers

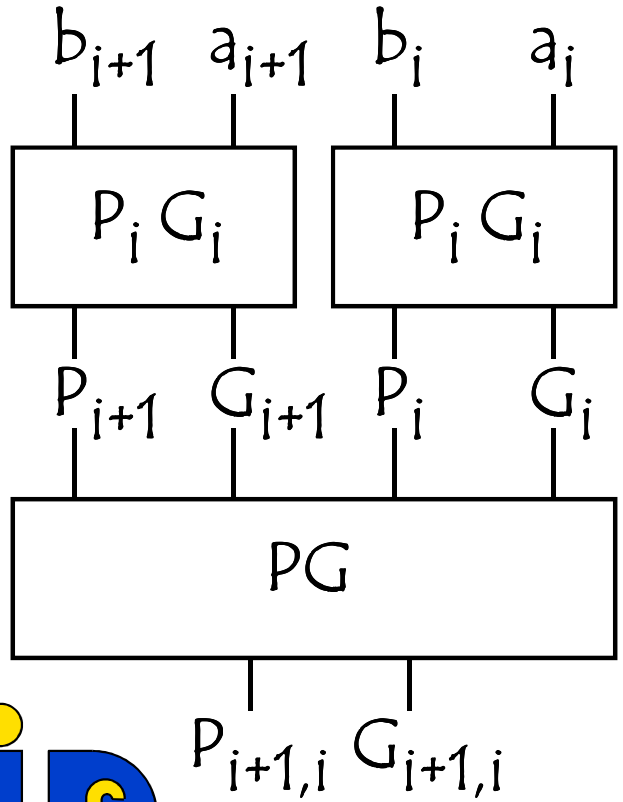
Acceleration technics



Adders

Adding two natural numbers

Acceleration technics



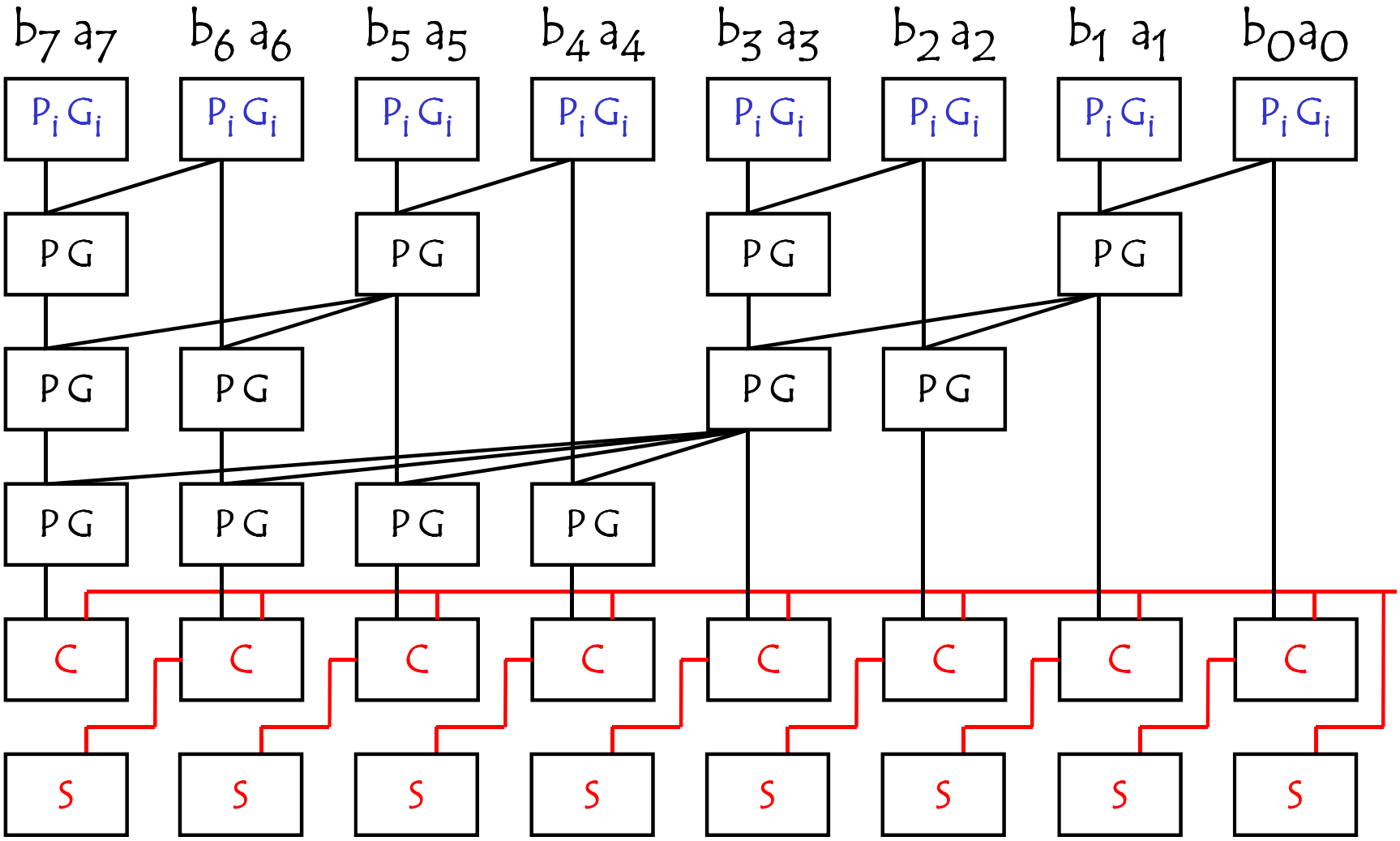
$$G_{i+1} = a_{i+1}\, b_{i+1} \qquad G_i = a_i\, b_i$$

$$P_{i+1} = a_{i+1} \oplus b_{i+1} \qquad P_i = a_i \oplus b_i$$

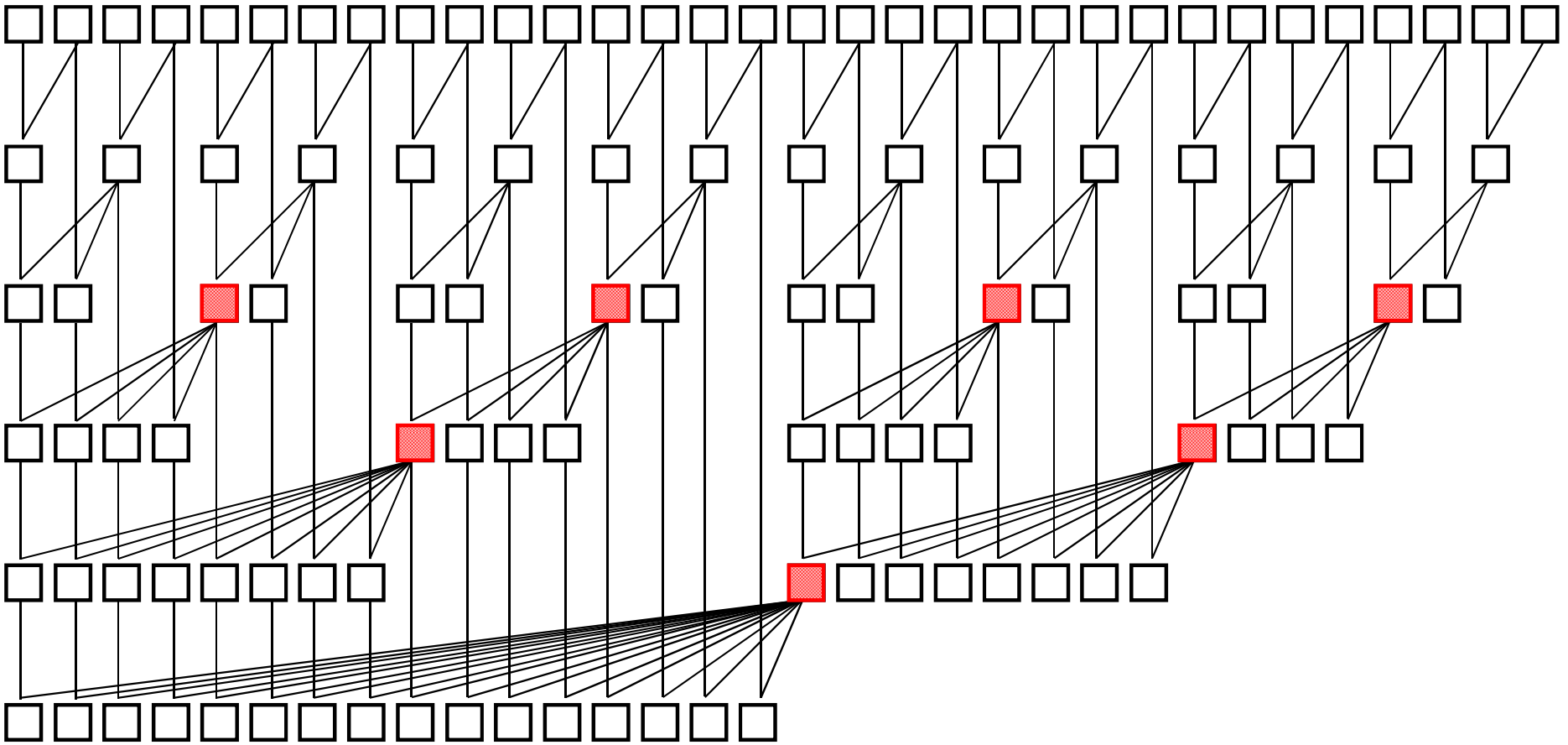
$$G_{i+1,\,i} = G_{i+1} + G_i \cdot P_{i+1}$$

$$P_{i+1,\,i} = P_i \cdot P_{i+1}$$

Adders

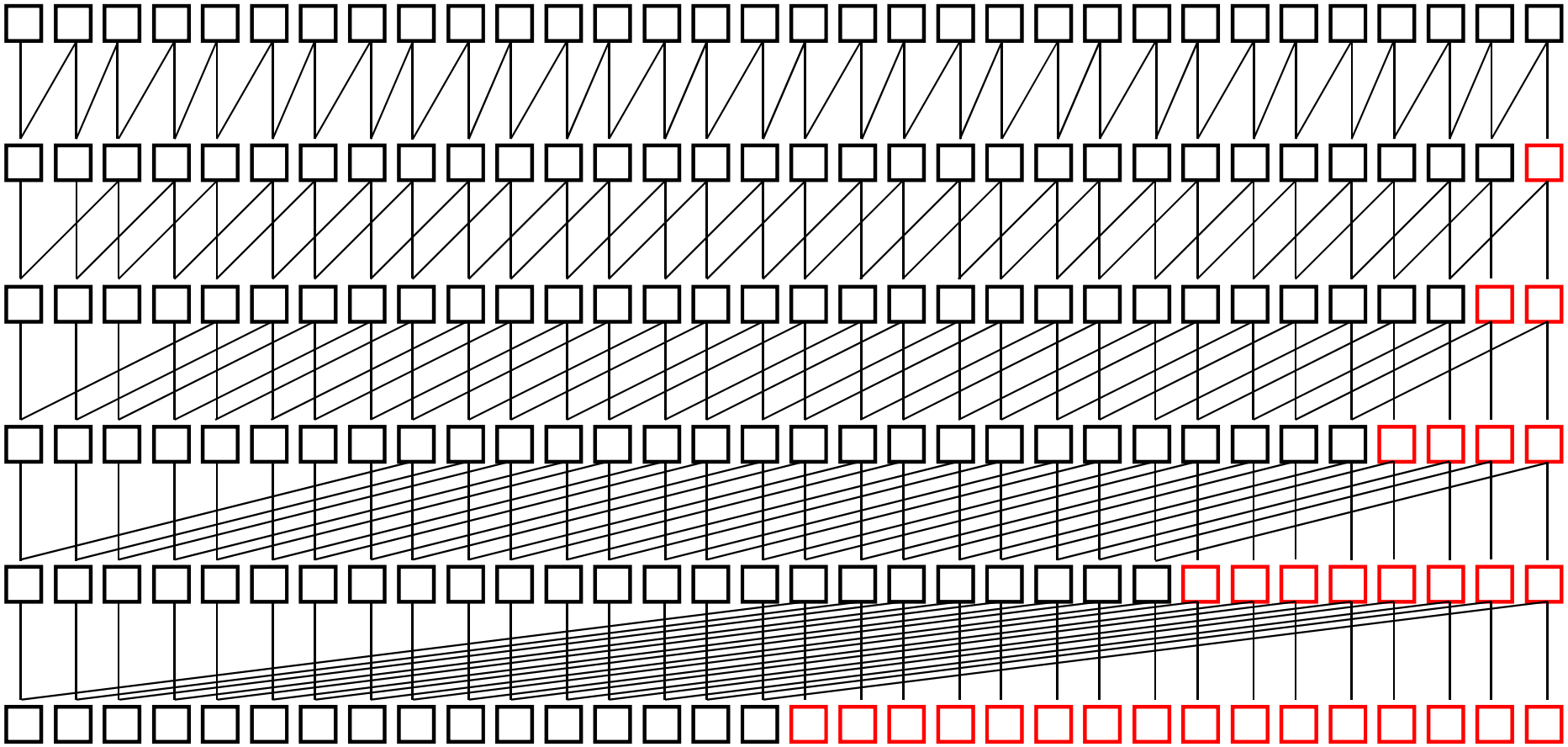


Adders



Slansky Adder

Adders



Kogge-Stone Adder

Adders

Adding two natural numbers (summary)

	Area	Delay
Ripple Carry (RCA)	n	n
Carry Select (CSLA)	$n^{log(3)}$	$log\ (n)$
Carry Lookahead (CLA)	$n\ log\ (n)$	$log\ (n)$