Outline

- Digital CMOS design
- Arithmetic operators
  - Adders
  - Comparators
  - Shifters
  - Multipliers

Comparators

Comparing a natural number to a constant : =

Let consider a natural number a coded on 8 bits using Natural Binary Code

$$a_7 \quad a_6 \quad a_5 \quad a_4 \quad a_3 \quad a_2 \quad a_1 \quad a_0$$

$$= ?$$

$$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$\downarrow$$

$$0 \,/\, 1$$

Comparators

Comparing a natural number to zero : =

Boolean function

Null = 1 if

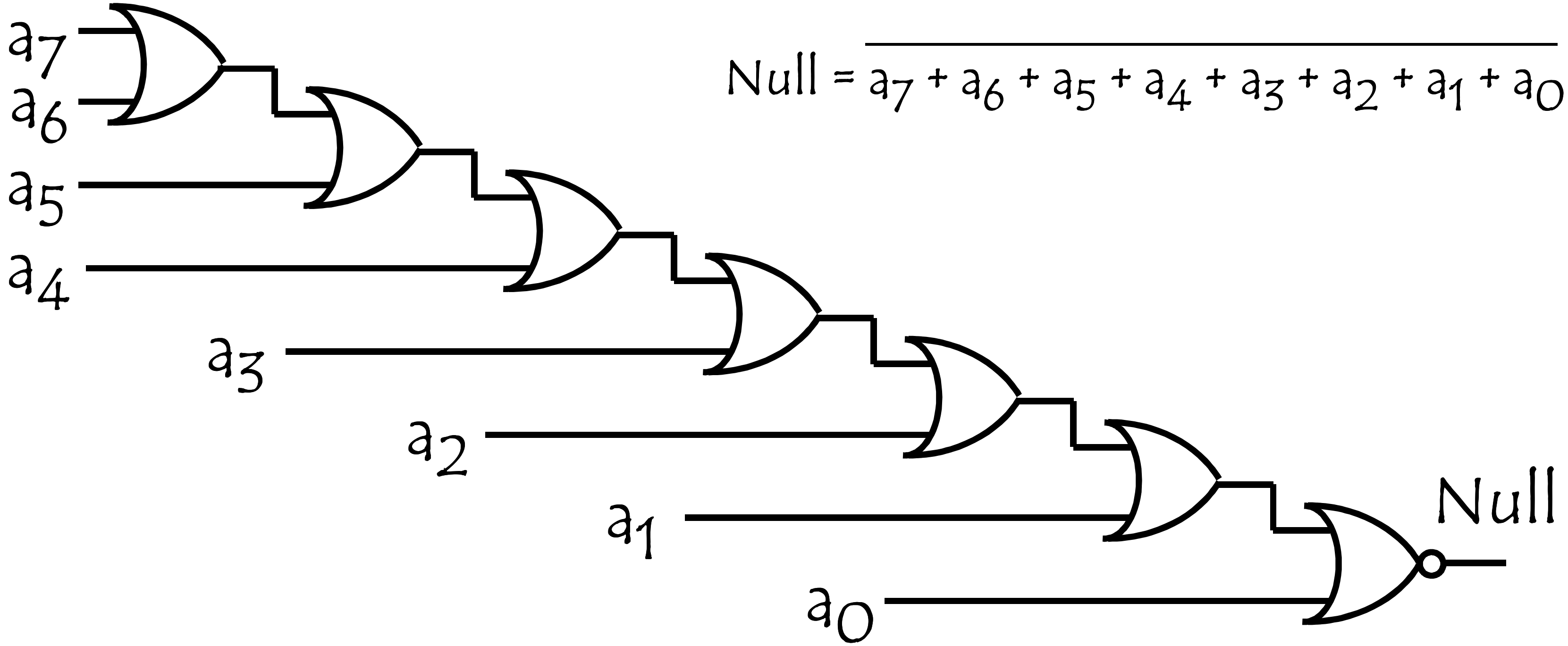
$$\overline{a_7} \cdot \overline{a_6} \cdot \overline{a_5} \cdot \overline{a_4} \cdot \overline{a_3} \cdot \overline{a_2} \cdot \overline{a_1} \cdot \overline{a_0} = 1$$

$$Null = \overline{a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0}$$

Comparators

Comparing a natural number to zero : =

Implementation

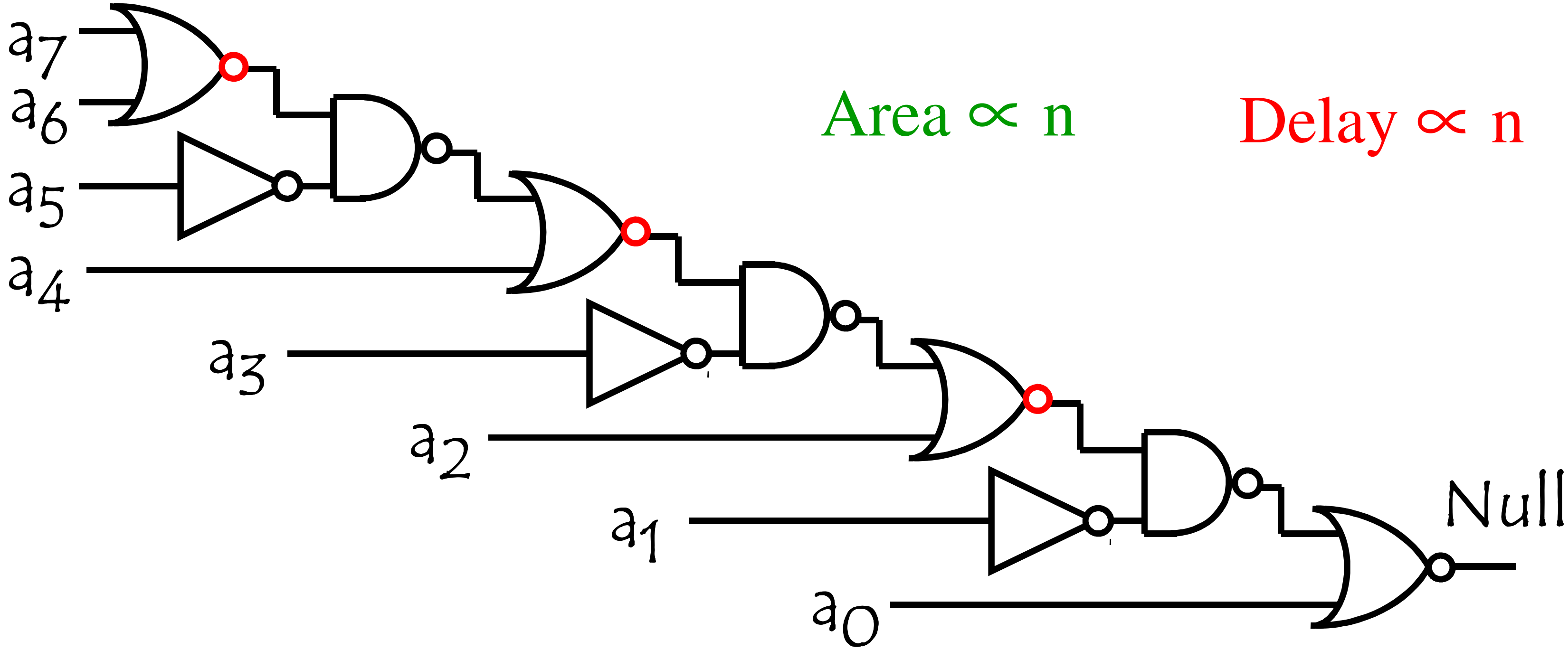


$$Null = \overline{a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0}$$

Comparators

Comparing a natural number to zero : =

Implementation



Area ∝ n

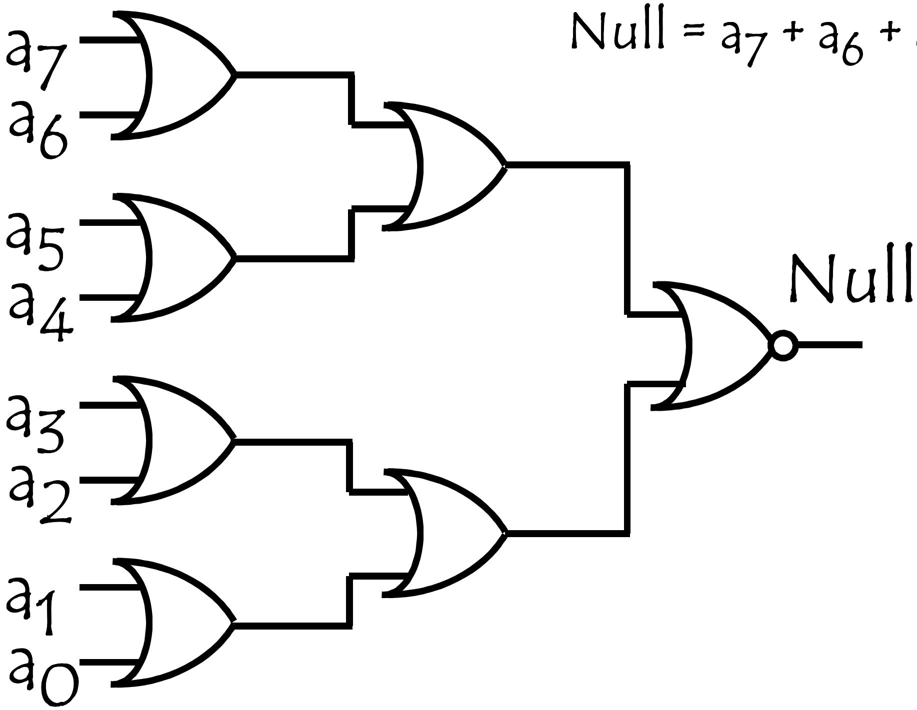
Delay ∝ n

Comparators

Comparing a natural number to zero : =

Implementation improvement

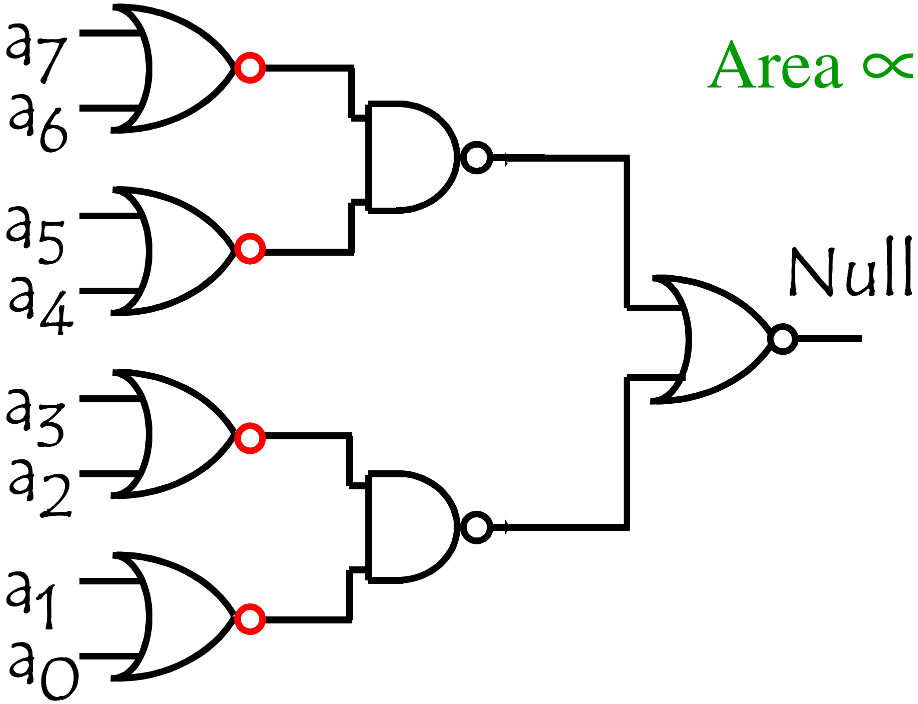
$$Null = \overline{a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0}$$



Comparators

Comparing a natural number to zero : =

Implementation improvement



Area ∝ n   Delay ∝ *log*(n)

Comparators

Comparing two natural numbers : =

Let consider two natural numbers a and b coded on 8 bits using Natural Binary Code

$$a_7 \; a_6 \; a_5 \; a_4 \; a_3 \; a_2 \; a_1 \; a_0$$
$$= ?$$
$$b_7 \; b_6 \; b_5 \; b_4 \; b_3 \; b_2 \; b_1 \; b_0$$
$$\downarrow$$
$$0 \,/\, 1$$

Comparators

Comparing two natural numbers : =

Boolean function

a Equal b if : $a_7 = b_7$ and $a_6 = b_6$ and … and $a_0 = b_0$

a Equal b if : $\overline{(a_7 \oplus b_7)} \; . \; \ldots \; . \; \overline{(a_0 \oplus b_0)} = 1$

$$Equal = \overline{(a_7 \oplus b_7) + \ldots + (a_0 \oplus b_0)}$$

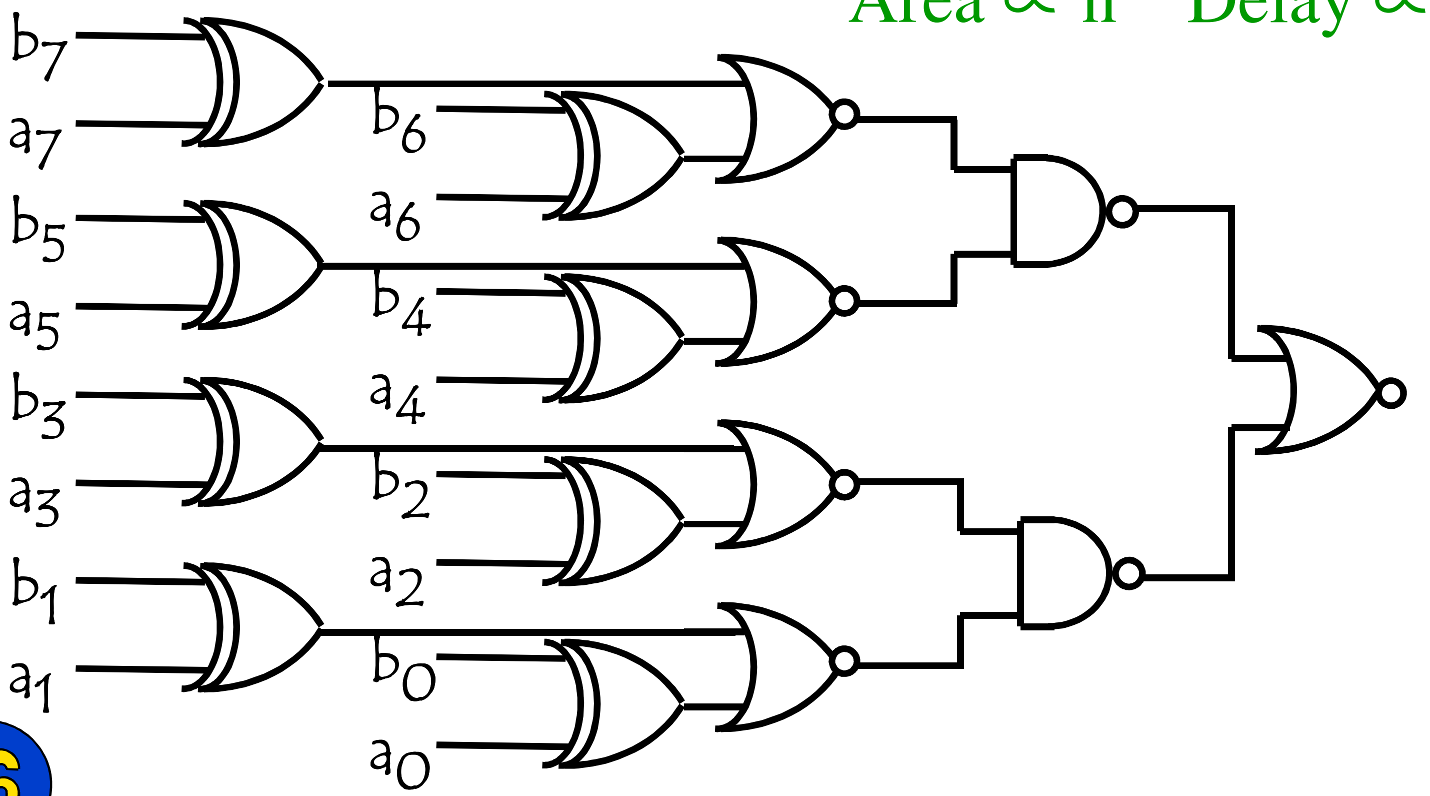
$$Equal = \overline{(\; e_7 \;) + \ldots + (\; e_0 \;)}$$

Comparators

Comparing two natural numbers : =

Implementation

Area ∝ n   Delay ∝ *log*(n)



Comparators

Comparing two natural numbers : <

Let consider two natural numbers a and b
coded on 8 bits using Natural Binary Code

$$a_7 \ a_6 \ a_5 \ a_4 \ a_3 \ a_2 \ a_1 \ a_0$$

$$< ?$$

$$b_7 \ b_6 \ b_5 \ b_4 \ b_3 \ b_2 \ b_1 \ b_0$$

$$\downarrow$$

$$0 \,/\, 1$$

Comparators

Comparing two natural numbers : <

Boolean function

$a < b$ if : $a_7 < b_7$ or ($a_7 = b_7$ and ($a_6 < b_6$ or ($a_6 = b_6$ and …)))

$$a_7 \; a_6 \; a_5 \; a_4 \; a_3 \; a_2 \; a_1 \; a_0$$

$$< ?$$

$$b_7 \; b_6 \; b_5 \; b_4 \; b_3 \; b_2 \; b_1 \; b_0$$

$$\downarrow$$

$$0 \,/\, 1$$

Comparators

Comparing two natural numbers : <

Boolean function

a < b if : $a_7 < b_7$ or ($a_7 = b_7$ and ($a_6 < b_6$ or ($a_6 = b_6$ and …)))

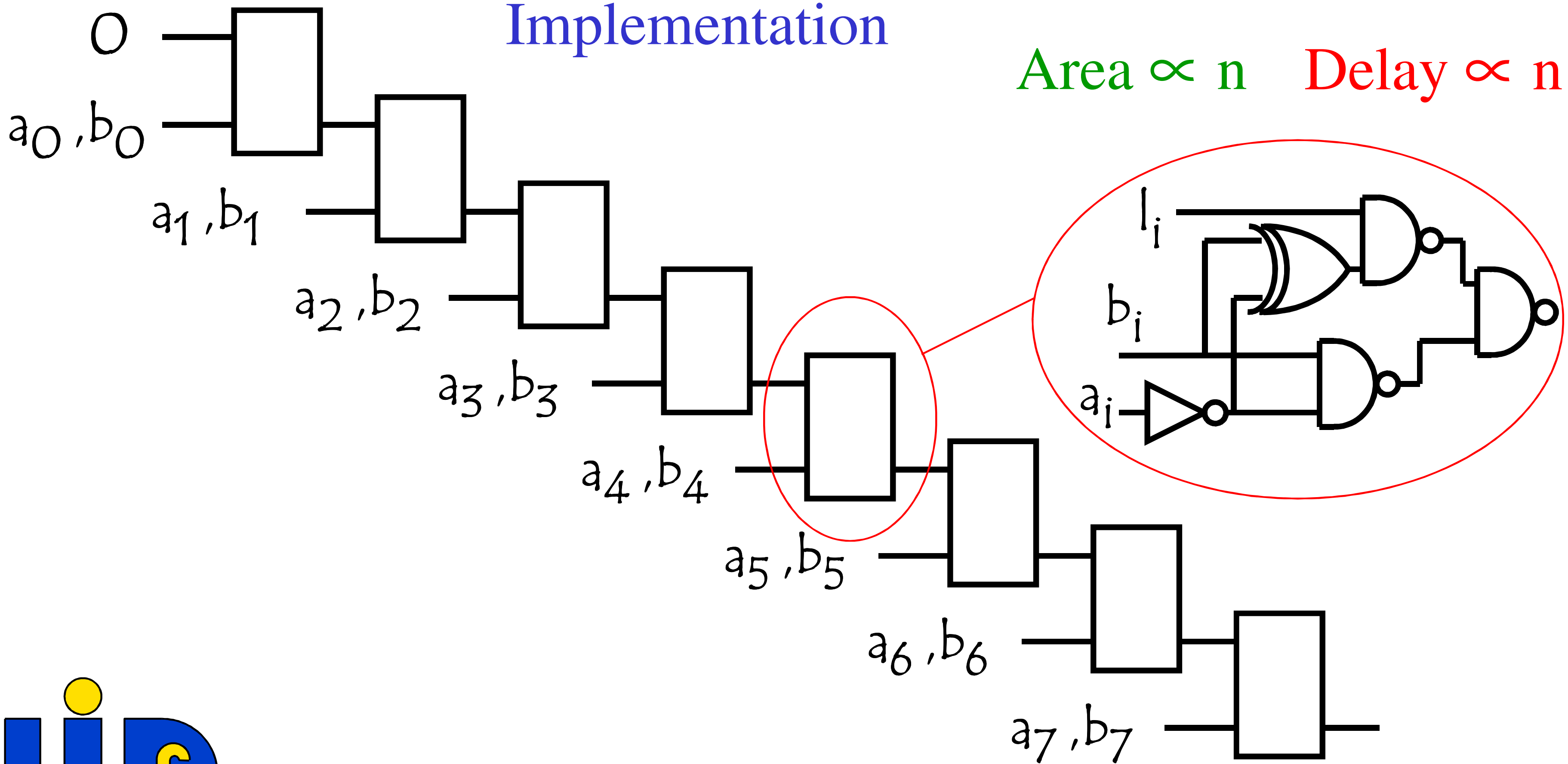
a < b if : $\overline{a_7} b_7 + ((\overline{a_7 \oplus b_7}) \,.\, (\overline{a_6} b_6 + ((\overline{a_6 \oplus b_6}) .\ \ldots)))$

Comparators

Comparing two natural numbers : <

Implementation

Area ∝ n   Delay ∝ n



Comparators

Comparing two natural numbers : <

Implementation Improvement

a < b if : $a_7 < b_7$ or $(a_7 = b_7$ and $(a_6 < b_6$ or $(a_6 = b_6$ and …)))

a < b if : $\overline{a_7} b_7 + ((\overline{a_7 \oplus b_7}) . (\overline{a_6} b_6 + ((\overline{a_6 \oplus b_6}). \ldots)))$

$\overline{a}_i b_i + (\overline{a}_i \oplus b_i) . l_i$

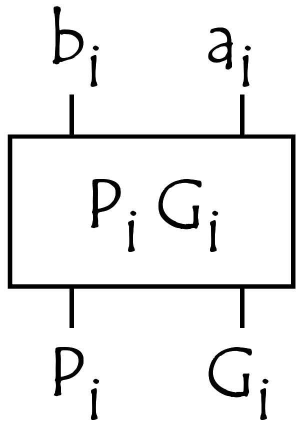
Propagation

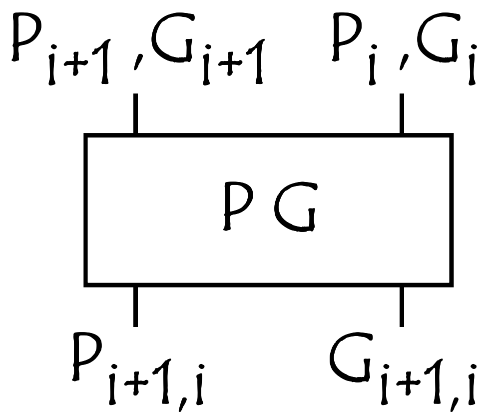
Generation

Comparators

Comparing two natural numbers : <

Implementation Improvement





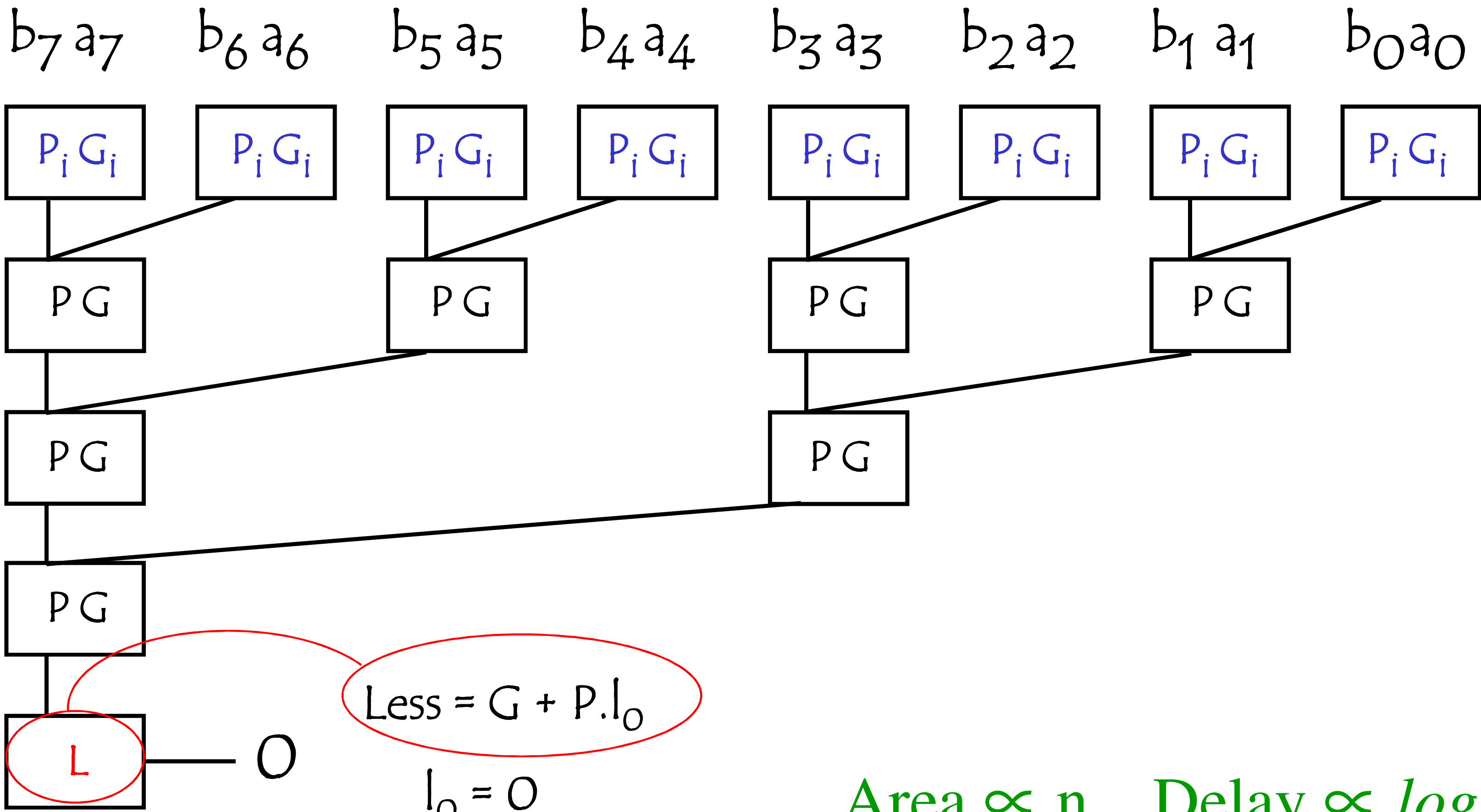
$$G_i = \overline{a_i}\, b_i$$

$$P_i = \overline{a_i} \oplus b_i$$

$$G_{i+1,\, i} = G_{i+1} + G_i . P_{i+1}$$

$$P_{i+1,\, i} = P_i \,.\, P_{i+1}$$

Comparators



Area $\propto$ n Delay $\propto$ *log*(n)

Comparators