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## Hidden long-range order in a spin-orbit coupled two-dimensional Bose gas

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#### Long-range order in a 2D quantum gas?

Mermin-Wagner theorem:

Continuous symmetries cannot be spontaneously broken at finite temperature in systems with short-range interactions and dimensions  $d \le 2$ .

Bogoliubov (1962), Mermin, Wagner (1966), Hohenberg (1967), Coleman (1973)

Consequence: No off-diagonal long range order in two dimensions at finite *T*, no BEC

#### Berezinski-Kosterlitz-Thouless

Berezinski-Kosterlitz-Thouless superfluid for  $T < T_c$ - algebraic long range order:

$$\langle \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r'}) \rangle \sim |\mathbf{r} - \mathbf{r'}|^{\frac{1}{\alpha}}$$

Normal fluid for  $T > T_c$ 

- exponential decay of correlations:

$$\langle \hat{\psi}^{\dagger}(\mathbf{r}) \hat{\psi}(\mathbf{r}') \rangle \sim e^{\frac{|\mathbf{r}-\mathbf{r}'|}{\ell}}$$

What happens when you add spin-orbit coupling?

#### Synthetic spin-orbit coupling

$$H_{\rm sp} = -\frac{\hbar^2}{2m}\nabla^2 + \kappa_x p_x \sigma_x + \kappa_y p_y \sigma_y - \frac{\delta}{2}\sigma_x + \frac{\Omega}{2}\sigma_z$$

Bose gas with ID SOC: NIST Yin et al. Nature (2011) Fermi gas with ID SOC: MIT Cheuk et al. PRL (2012) Shanxi Wang et al. PRL (2012)

Bose gas with 2D SOC: Shanghai Wu et al. Science (2016) Fermi gas with 2D SOC: Shanxi Huang et al. Nat. Phys. (2016)



Data from Cheuk et al. PRL (2012)



Illustrations: Galitski and Spielman Nature (2013)

#### Bose-gas with SOC

$$H_{\rm sp} = -\frac{\hbar^2}{2m}\nabla^2 + \kappa_x p_x \sigma_x + \kappa_y p_y \sigma_y - \frac{\delta}{2}\sigma_x + \frac{\Omega}{2}\sigma_z$$

Add interactions:

$$\hat{\boldsymbol{\Psi}} = \begin{pmatrix} \hat{\Psi}_1(\mathbf{r}') \\ \hat{\Psi}_2(\mathbf{r}') \end{pmatrix}$$

$$\hat{H} = \int d^2 r \left[ \hat{\Psi}^{\dagger} \hat{H}_{sp} \hat{\Psi} + \frac{g_{11}}{2} (\hat{\Psi}_1^{\dagger} \hat{\Psi}_1)^2 + \frac{g_{22}}{2} (\hat{\Psi}_2^{\dagger} \hat{\Psi}_2)^2 + g_{12} \hat{\Psi}_1^{\dagger} \hat{\Psi}_1 \hat{\Psi}_2^{\dagger} \hat{\Psi}_2 \right],$$

Miscible: $g_{12} < g \equiv g_{11} = g_{12}$ Immiscible: $g_{12} > g \equiv g_{11} = g_{12}$ Spin-invariant: $g_{12} = g \equiv g_{11} = g_{12}$ 

#### Bose-gas with SOC phases



Miscible:  $g_{12} < g \equiv g_{11} = g_{12}$ Condense into superposition: Stripe phase

> Ho and Zhang, PRL <sup>-5</sup> (2011)<sub>-10</sub>

-5 0 5 10

Immiscible:  $g_{12} > g \equiv g_{11} = g_{12}$ Condense into single minimum: Plane-wave phase

-10

## More SOCed quantum gases (not 2D)

#### 3D Bose-Einstein condensate and 3D SOC

 Rich ground state phase diagram with first and second order phase transitions, tetracritical point R Liao, O Fialko, U Zülicke, JB, PRA (2015)



Bose gas in ring trap with Rashba SOC and spin-invariant interactions

- SOC can be "gauged away" and put into boundary conditions
- Vector NLS with "Manakov" solitons: magnetisation precession O Fialko, U Zülicke, JB, PRA (2012)



#### More SOCed quantum gases (not 2D)

#### Fermi superfluid with SOC – topological SF

- Solitons, vortices have Majorana quasiparticles
- Moving Majorana (dark) soliton has fixed phase relation Zhou, Brand, Liu, Hu, PRL (2016)



#### Phase transitions in Spin-orbit coupled 2D Bose gas

Earlier predictions: Jian, Zhai PRL (2011)

- Stripe phase:
  - stripe-order melting transition,
  - fractionalised vortex phase
- Plane-wave phase:
  - Isotropic SOC (Rashba): BKT transition temperature drops to zero [also Liao, Huang, Lin, Fialko, PRA (2014)]
  - Anisotropic SOC: regular BKT transiton

#### Finite temperature simulations

Simulations are done with effective field theory projected to lowenergy region (c-field region). States eliminated to give the effective field theory, with pseudopotential interaction  $d\Psi_{i} = \mathcal{P}\{-i\mathcal{L}_{i}\Psi_{i}dt + \Gamma(\mu - \mathcal{L}_{i})\Psi_{i}dt + dW_{i}\}$ Excitation energy  $--E_{max}$  $\hat{\Psi} = \begin{pmatrix} \hat{\Psi}_1(\mathbf{r}') \\ \hat{\Psi}_2(\mathbf{r}') \end{pmatrix} = \sqrt{n} e^{i\hat{\phi}_{\mathrm{t}}(\mathbf{r}')} \begin{pmatrix} e^{i\hat{\phi}_{\mathrm{r}}(\mathbf{r}')} \\ e^{-i\hat{\phi}_{\mathrm{r}}(\mathbf{r}')} \end{pmatrix}$ Incoherent Region  $\mathcal{P}_{\mathbf{L}}$ cut **Observables:** Pc c-field Region Total phase  $\phi_t(\mathbf{r})$ Relative phase  $\phi_r(\mathbf{r})$ 

Position Blakie, Bradley, Davis, Ballagh, Gardiner, Adv. Phys. (2008)

Phase correlation function

$$G_{\mathrm{t,r}}(\mathbf{r}',\mathbf{r}'') = \langle e^{i\hat{\phi}_{\mathrm{t,r}}(\mathbf{r}')-i\hat{\phi}_{\mathrm{t,r}}(\mathbf{r}'')} \rangle = e^{-\langle (\Delta\phi_{\mathrm{t,r}})^2 \rangle/2},$$

Thermal average is replaced by time average



#### **Relative phase correlations**



#### Relative phase: long-range order

Plateau of relative phase correlation function



#### **Bogoliubov theory**



In plane-wave phase the classical field starts in one minimum of the singleparticle dispersions

Bogoliubov quasiparticle dispersions:

Gapless mode for **total** phase fluctuations  $\omega_t$ 

Gapped mode for **relative** phase fluctuations  $\omega_r$ 

#### Bogoliubov theory

The thermal average of the phase fluctuations can be expressed as an integral of the Bogoliubov amplitudes:

$$\langle (\Delta \phi_{t,r})^2 \rangle = \int \frac{d^2 q}{\pi n} \left( N_{t,r}^{\mathbf{q}} + \frac{1}{2} \right) \left( u_{t,r}^{\mathbf{q}} + v_{t,r}^{\mathbf{q}} \right)^2 \sin^2 \frac{\mathbf{q} \cdot \mathbf{r}}{2}$$
$$\Delta \phi_{t,r} = \hat{\phi}_{t,r}(\mathbf{r}') - \hat{\phi}_{t,r}(\mathbf{r}'')$$

Integral for **total phase** has infrared divergence: leads to algebraic decay of correlation function (for all temperatures; artifact of Bog theory).

The **relative phase** fluctuations freeze out due to gap in dispersion relation.

#### Total phase correlation function from Bogoliubov theory



# Relative phase fluctuations from Bogoliubov theory



Bogoliubov theory reproduces plateau and short-range oscillations; fluctuations of the relative phase are anisotropic

#### Single-particle density matrix

Does off-diagonal long-range order of the relative phase imply Bose-Einstein condensation?

We would need to have a off-diagonal long-range order (a macroscopic eigenvalue) of the single-particle density matrix.

$$\boldsymbol{\rho}(\mathbf{r}',\mathbf{r}'') = n \begin{bmatrix} e^{-\frac{\langle (\Delta\phi_{t})^{2} \rangle}{2} - \frac{\langle (\Delta\phi_{t})^{2} \rangle}{2}} & e^{-\frac{\langle (\Delta\phi_{t})^{2} \rangle}{2} - \frac{\langle (\Delta+\phi_{t})^{2} \rangle}{2}} \\ e^{-\frac{\langle (\Delta\phi_{t})^{2} \rangle}{2} - \frac{\langle (\Delta+\phi_{t})^{2} \rangle}{2}} & e^{-\frac{\langle (\Delta\phi_{t})^{2} \rangle}{2} - \frac{\langle (\Delta\phi_{t})^{2} \rangle}{2}} \end{bmatrix}$$

Since the total phase correlation appears as a prefactor for the whole matrix, it vanishes for large r'' - r'. Hence, there is no macroscopic eigenvalue and no BEC.

#### Summary

- Ising-type transition from exponential decaying correlations to long-range order of the relative phase in spin-orbit coupled 2D Bose gas in plane-wave phase
- The occurrence of long-range order in the relative phase seems to appear at the BKT transition temperature
- No BEC Mermin Wagner is relevant
- Low-temperature correlation functions can be obtained from Bogoliubov theory

### The end!

