Superfluid density and critical velocity near the fermionic Berezinskii-Kosterlitz-Thouless transition

Brendan Mulkerin

Theoretical Condensed Matter Physics
Swinburne University

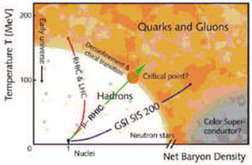
Todays talk

- Thanks A/Profs. Xia-ji Liu, Chris Vale, Paul Dyke, Jia Wang, Lianyi He, and Hui Hu for our collaborative work
- Thanks to our PhD students Umberto Toniolo, Sebastian Schaffer, Xiao-Long Chen, and Christopher Hoegaard for all their hard work
- BEC-BCS crossover in two dimensions
- The equation of state: Breathing mode in 2D gases
- BKT transition

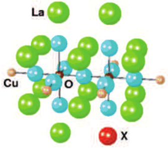
Why strongly interacting Fermi gases?

Strongly interacting Fermi gases with balanced populations very difficult to solve

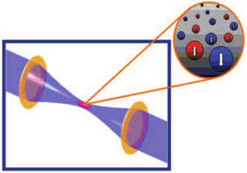
- Strongly correlated Fermi systems are a playground for many-body physics
- They are stable on long timescales and for strong interactions



neutron stars & nuclear matter



high-T_c materials



ultracold atoms

Figure: Xia-Ji Liu Physics Reports 524 (2), 37-83.

- They play a fundamental role in very different areas or physics
- Lower dimensions increase the fluctuations, quantum effects are larger

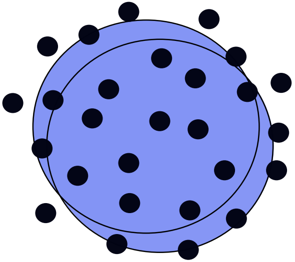
Two dimensional BCS-BEC crossover

2D scattering always allows a bound state and is energy dependent,

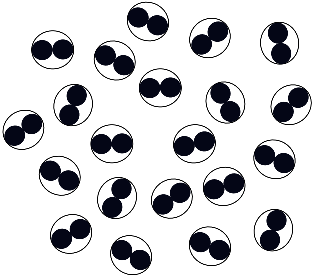
$$f(q) = \frac{4\pi}{\ln\left(1/a_{2\mathrm{D}}^2 q^2\right) + i\pi}, \quad \varepsilon_{\mathrm{B}} = \frac{\hbar^2}{m a_{2\mathrm{d}}^2}$$

No unitary regime but interactions can be changed from the BEC - BCS side through $\eta = \ln\left(k_{\mathrm{F}} a_{2\mathrm{D}}\right) = -\frac{1}{2}\ln\left(2E_{\mathrm{F}}/\varepsilon_{\mathrm{B}}\right)$

BCS side: weakly interacting pairs



BEC side: Tightly bound bosonic molecules



Fluctuations in 2D are larger:

This prevents long-range order [Mermin-Wagner-Hohenberg]

Experimental progress

Selected for a Viewpoint in *Physics*

PRL **116,** 045302 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2016

Thermodynamics of an Attractive 2D Fermi Gas

K. Fenech, P. Dyke, T. Peppler, M. G. Lingham, S. Hoinka, H. Hu, and C. J. Vale*
Centre for Quantum and Optical Sciences, Swinburne University of Technology, Melbourne 3122, Australia

(Received 17 August 2015; published 27 January 2016)

Thermodynamic properties of matter are conveniently expressed as functional relations between
…lly determine the compressibility, density, and
… the normal phase as a function of temperature
…alitatively different features to those found in

Selected for a Viewpoint in *Physics*

PRL **112,** 045301 (2014) PHYSICAL REVIEW LETTERS week ending 31 JANUARY 2014

Ground-State Pressure of Quasi-2D Fermi and Bose Gases

Vasiliy Makhalov, Kirill Martiyanov, and Andrey Turlapov*
Institute of Applied Physics, Russian Academy of Sciences, ulitsa Ulyanova 46, Nizhny Novgorod, 603000, Russia

(Received 13 May 2013; revised manuscript received 4 November 2013; published 27 January 2014)

Using an ultracold gas of atoms, we have realized a quasi-two-dimensional Fermi system with widely tunable *s*-wave interactions nearly in a ground state. Pressure and density are measured. The experiment covers physically different regimes: weakly and strongly attractive Fermi gases and a Bose gas of tightly bound pairs of fermions. In the Fermi regime of weak interactions, the pressure is systematically above a Fermi-liquid-theory prediction, maybe due to mesoscopic effects. In the opposite Bose regime, the pressure agrees with a bosonic mean-field scaling in a rang… acting regime, measurements disagree with a pur… testing of theoretical methods applicable across di…

DOI: 10.1103/PhysRevLett.112.045301

Selected for a Viewpoint in *Physics*

PRL **116,** 045303 (2016) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2…

Equation of State of Ultracold Fermions in the 2D BEC-BCS Crossover Region

I. Boettcher,[1,*] L. Bayha,[2] D. Kedar,[2] P. A. Murthy,[2] M. Neidig,[2] M. G. Ries,[2] A. N. Wenz,[2] G. Zürn,[2] S. Jochim,[2] and T. Enss[1]
[1]*Institute for Theoretical Physics, Heidelberg University, D-69120 Heidelberg, Germany*
[2]*Physikalisches Institut, Heidelberg University, D-69120 Heidelberg, Germany*

(Received 11 September 2015; published 27 January 2016)

…e equation of state of a two-dimensional Fermi gas with …sover from a weakly coupled Fermi gas to a Bose gas of …ı is varied. We demonstrate that interactions lead to a …y several orders of magnitude. We compare our data near …predictions for both fermions and bosons from quantum …heory. Our results serve as input for investigations of …he two-dimensional system.

Two-Dimensional Homogeneous Fermi Gases

Klaus Hueck,* Niclas Luick, Lennart Sobirey, Jonas Siegl, Thomas Lompe, and Henning Moritz
Institut für Laserphysik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany.

(Dated: May 30, 2017)

We report on the experimental realization of homogeneous two-dimensional (2D) Fermi gases trapped in a box potential. In contrast to harmonically trapped gases, these homogeneous 2D systems are ideally suited to probe local as well as non-local properties of strongly interacting many-body systems. As a first measurement, we use a local probe to extract the equation of state (EOS) of a non-interacting Fermi gas. We then perform matter wave focusing to extract its momentum distribution and directly observe Pauli blocking in a near unity occupation of momentum states. Finally, we measure the momentum distribution of strongly interacting homogeneous 2D gases in the crossover between attractively interacting fermions and deeply-bound bosonic molecules.

Experimentalists can directly measure the equation of state (E.O.S) and thermodynamic properties of the gas

Equation of state

The equation of state shows the non-trivial E.O.S. even in the normal state

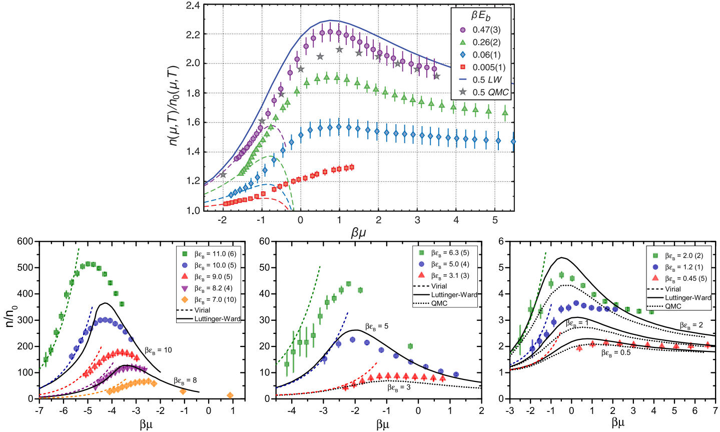


Figure: Fenech et al PRL 116 045302 (2016) (Top) and Boettecher et al PRL 116 045303 (2016) (Bottom).

We can use the E.O.S. to calculate the breathing mode anomaly

2D breathing mode anomaly

Using the 2D equation of state we can explore the breathing mode anomaly:

Delta function $V_{2\mathrm{D}}(\mathbf{r}-\mathbf{r}') = g_{2D}\delta(\mathbf{r}-\mathbf{r}')$ interaction is the most important interaction in a two-component Fermi gas scales as λ^{-2} in 2D, regularisation destroys this scaling:

$$g_{2D} \to \log(k_F a_{2D})$$

Including a harmonic trap, $H_{\mathrm{trap}} = \frac{1}{2}m\omega^2 r^2$, breaks the scale invariance,

$$\mathbf{r} \to \lambda\mathbf{r},\ H_{\mathrm{trap}} \to \lambda^2 H_{\mathrm{trap}}$$

However there is a hidden $SO(2,1)$ symmetry

This symmetry can excite a breathing mode, $\omega_B = 2\omega \to$ the quantum anomaly will break this hidden symmetry

Realisation

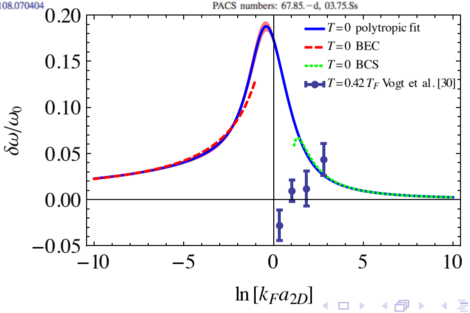
PRL 108, 070404 (2012) PHYSICAL REVIEW LETTERS week ending 17 FEBRUARY 2012

Scale Invariance and Viscosity of a Two-Dimensional Fermi Gas

Enrico Vogt, Michael Feld, Bernd Fröhlich, Daniel Pertot, Marco Koschorreck,* and Michael Köhl
Cavendish Laboratory, University of Cambridge, JJ Thomson Avenue, Cambridge CB3 0HE, United Kingdom
(Received 4 November 2011; published 17 February 2012)

We investigate collective excitations of a harmonically trapped two-dimensional Fermi gas from the collisionless (zero sound) to the hydrodynamic (first sound) regime. The breathing mode, which is sensitive to the equation of state, is observed with an undamped amplitude at a frequency 2 times the dipole mode frequency for a large range of interaction strengths and different temperatures. This provides evidence for a dynamical SO(2,1) scaling symmetry of the two-dimensional Fermi gas. Moreover, we investigate the quadrupole mode to measure the shear viscosity of the two-dimensional gas and study its temperature dependence.

DOI: 10.1103/PhysRevLett.108.070404 PACS numbers: 67.85.−d, 03.75.Ss



Theoretical results

Using the hydrodynamic formalism and equation of state:

Equation of state and the local density approximation, $\mu(\mathbf{r}) = \mu - V_{\text{trap}}(\mathbf{r})$,

$$n(\mathbf{r})\lambda^2 = f_n\left(\frac{\mu}{k_B T}\right), \quad \frac{P(\mathbf{r})\lambda^2}{k_B T} = f_p\left(\frac{\mu}{k_B T}\right), \quad \frac{df_p(x)}{dx} = f_n(x)$$

$$S^{(2)} = \frac{1}{2}\int d\mathbf{r}\left[\omega^2\rho_0\mathbf{u} - (\nabla\rho_0\cdot\mathbf{u})\left(\frac{\nabla V_{\text{trap}}}{M}\cdot\mathbf{u}\right) + 2\left(\rho_0\frac{\nabla V_{\text{trap}}}{M}\cdot\mathbf{u}\right)(\nabla\cdot\mathbf{u}) - \rho_0\left(\frac{\partial P}{\partial\rho}\right)_{\bar{s}}(\nabla\cdot\mathbf{u})^2\right]$$

No significant result at finite temperature $\rightarrow$ we do see damping

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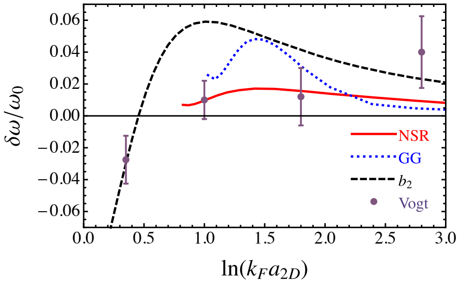

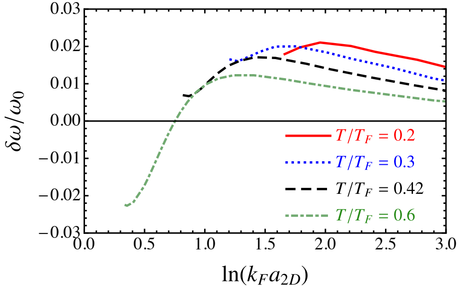

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There is a strange behaviour of the breathing mode in the high temperature regime

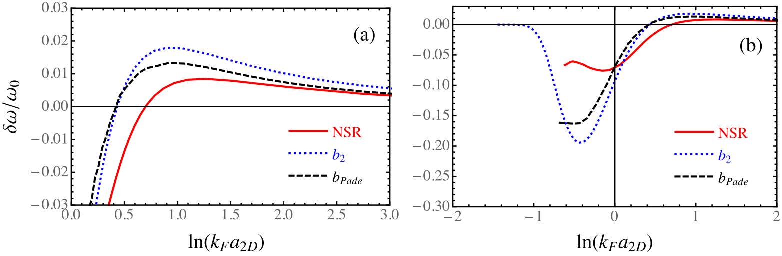


Figure: The breathing mode anomaly for $T/T_{\text{F}} = 0.8$

Path integral 1/3

Strongly interacting Fermi gases with balanced populations, well studied, very difficult to solve.

To begin with we have the thermodynamic potential found through the partition function

$$\Omega = -\beta^{-1} \ln \mathcal{Z},$$

where the partition function and action are given by,

$$\mathcal{Z} = \int \mathcal{D}\left[\psi, \bar{\psi}\right] e^{-S[\psi,\bar{\psi}]} \text{ and } S = \int_0^{\hbar\beta} d\tau \left[\int d\mathbf{r} \sum_\sigma \bar{\psi}_\sigma(x)\partial_\tau \psi_\sigma(x) + H \right],$$

and the action defined by a Hamiltonian is

$$S = \int_0^{\hbar\beta} d\tau \left[\int d\mathbf{r} \sum_\sigma \bar{\psi}_\sigma(x)\partial_\tau \psi_\sigma(x) + H \right],$$

Decouple through the Hubbard-Stratonovich transformation:

$$\mathcal{S}_{\text{eff}}\left[\Delta, \Delta^*\right] = \int dx \left[\frac{|\Delta(x)|^2}{U_0} - \operatorname{Tr}\ln\left[-G^{-1}\right] \right].$$

This is true for general dimension, where $\int dx = \int d^d\mathbf{r} d\tau$ and U_0 is regularised appropriately.

Path integral 2/3

Strongly interacting Fermi gases with balanced populations, well studied, very difficult to solve.

Expand the thermodynamic potential by taking the Bose field $\Delta(\mathbf{r},t)$ about its saddle point Δ_0,

$$\Delta(\mathbf{r},t) = \Delta_0 + \varphi(\mathbf{r},t),$$

The action is expanded in order of Δ_0 and the thermodynamic potential is

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_{\mathrm{GF}}.$$

Extend to the general case condensed pairs flow with a wavevector **Q**: $\Delta e^{i\mathbf{Q}\cdot\mathbf{r}}$

In this case, the mean-field thermodynamic potential is given by

$$\Omega_{\mathrm{MF}}\left(\mathbf{Q}\right) = \frac{\Delta^2}{U} + \sum_{\mathbf{k}}\left[\tilde{\xi}_{\mathbf{k}} - E_{\mathbf{k}} - \frac{2}{\beta}\ln\left(1 + e^{\beta E^{+}_{\mathbf{k}}}\right)\right],$$

and the mean-field gap equation,

$$\sum_{\mathbf{k}}\left[\frac{1-2f\left(E^{+}_{\mathbf{k}}\right)}{2E_{\mathbf{k}}} - \frac{1}{\hbar^2\mathbf{k}^2/M + \varepsilon_B}\right] = 0.$$

H. Hu, X.-J. Liu, P. D. Drummond, Europhys. Lett. 74, 574 (2006)

R. B. Diener, R. Sensarma, and M. Randeria, Phys. Rev. A 77, 023626 (2008)

Taylor, A. Griffin, N. Fukushima, and Y. Ohashi, Phys. Rev. A 74, 063626 (2006)

Path integral 3/3

The thermodynamic potential for gaussian pair fluctuations (GPF) is:

$$\Omega_{\mathrm{GF}}(\mathbf{Q}) = k_B T \sum_{\mathcal{Q}\equiv(\mathbf{q},i\nu_l)} \mathcal{S}(\mathcal{Q})\, e^{i\nu_l 0^+},$$

$$\mathcal{S}(\mathcal{Q}) = \frac{1}{2}\ln\left[1 - \frac{M_{12}^2(\mathcal{Q})}{M_{11}(\mathcal{Q})\, M_{11}(-\mathcal{Q})}\right] + \ln M_{11}(\mathcal{Q}),$$

See our recent paper PRA 96 053608 (2017) for the long definitions of M_{11} and M_{12}

This is difficult to solve below T_c, the Matsubara summation is tricky, instead we use:

$$\frac{1}{\beta}\sum_{|l|>l_0} \mathcal{S}_\eta(\mathbf{q}, i\nu_l) = -\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, \frac{\mathrm{Im}\mathcal{S}_\eta(\mathbf{q}, \omega + i\gamma)}{e^{\beta\omega}+1}$$

where $\mathcal{S}_\eta(\mathbf{q}, i\nu_l) \equiv \mathcal{S}(\mathbf{q}, i\nu_l)e^{i\nu_l\eta}$ and $\gamma = (2l_0+1)\pi/\beta$ for arbitrary positive integer l_0

L. He, H. Lü, G. Cao, H. Hu, and X.-J. Liu, Phys.Rev. A 92, 023620 (2015)

J. Tempere, S. N. Klimin, J. T. Devreese, and V. V. Moshchalkov, Phys. Rev. B 77, 134502 (2008)

Two-dimensional Fermi gas

To illustrate the importance of our full treatment of the GPF we find the E.O.S.

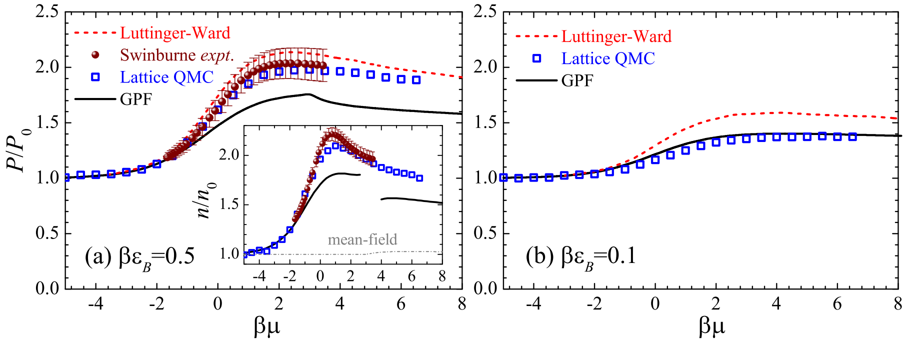


Figure: Comparing the results to experiment and Luttinger-Ward T-matrix theory PRA 96 053608 (2017)

The GPF underestimates the pressure but has a superfluid order parameter

Equation of state and superfluid density in 2D

The superfluid density can be found by adding a twist, giving the density to be

$$n_s = \frac{4m}{\hbar^2}\left[\frac{\partial^2\Omega(Q)}{\partial Q^2}\right]_{Q=0}$$

We can now look at the strongly correlated BCS side in 2D

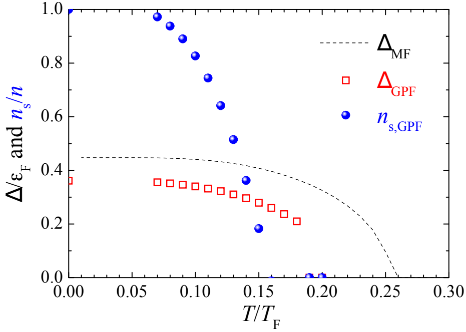


Figure: The behaviour of the order parameter and superfluid fraction in 2D for $\varepsilon_B/\varepsilon_F = 0.1$

There is a region where $n_s = 0$ and $\Delta_{\mathrm{GPF}} > 0$

Superfluid density in 2D

The Kosterlitz-Thouless criterion defines the BKT transition temperature:

$$k_B T_{BKT} = \frac{\pi}{2}\frac{\hbar^2}{4m} n_s(T)$$

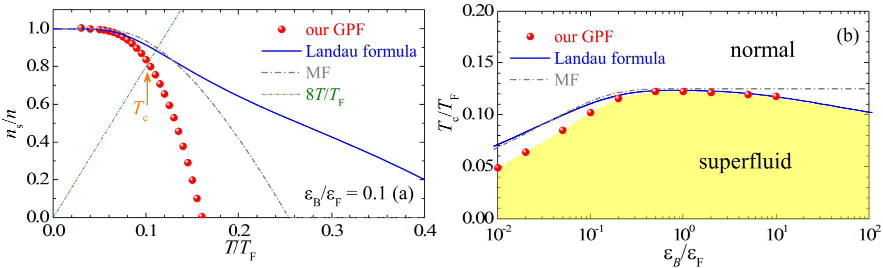


Figure: The order parameter and superfluid fraction in 2D for $\varepsilon_B/\varepsilon_F = 0.1$ and other theoretical attempts

Superfluid density in 2D

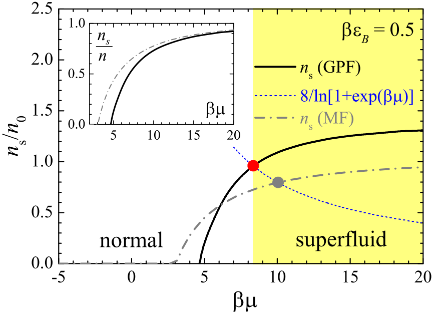
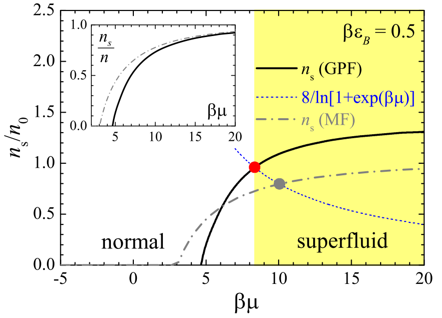


Figure: The superfluid fraction normalised by an ideal gas

Superfluid density in 2D



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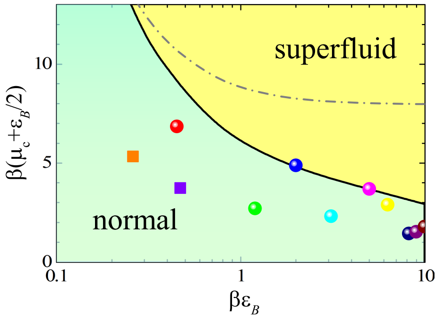

Figure: Critical chemical potential as a function of interaction strength for Swinburne (squares) and Heidelberg (circles)

We can define a critical chemical potential, which can be measured directly in experiment

Critical chemical potential → critical radius : $\mu_c = \mu - V(r_c)$

Unambiguously find the BKT transition

An unambiguous method find the BKT transition: use the LDA $\mu_g = \mu - V(r)$

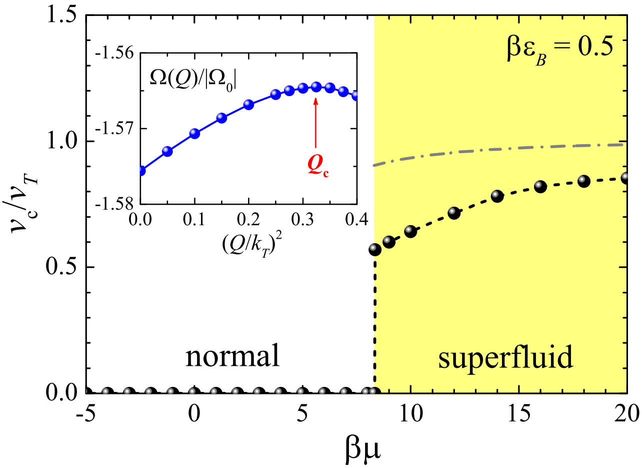

Figure: The critical velocity $v_c = \hbar Q/(2m)$ as a function of dimensionless chemical potential

Summary

- The breathing mode is damped as a function of temperature and is significant in the high temperature regime
- We have explicitly included pairing fluctuations in the calculation of the superfluid density
- Through stiring the gas we can find an unambiguos method to measure the BKT transition

Thank you

Thank you for your attention today