Induced interactions & topological phases in 2D-3D Fermi-Bose mixtures

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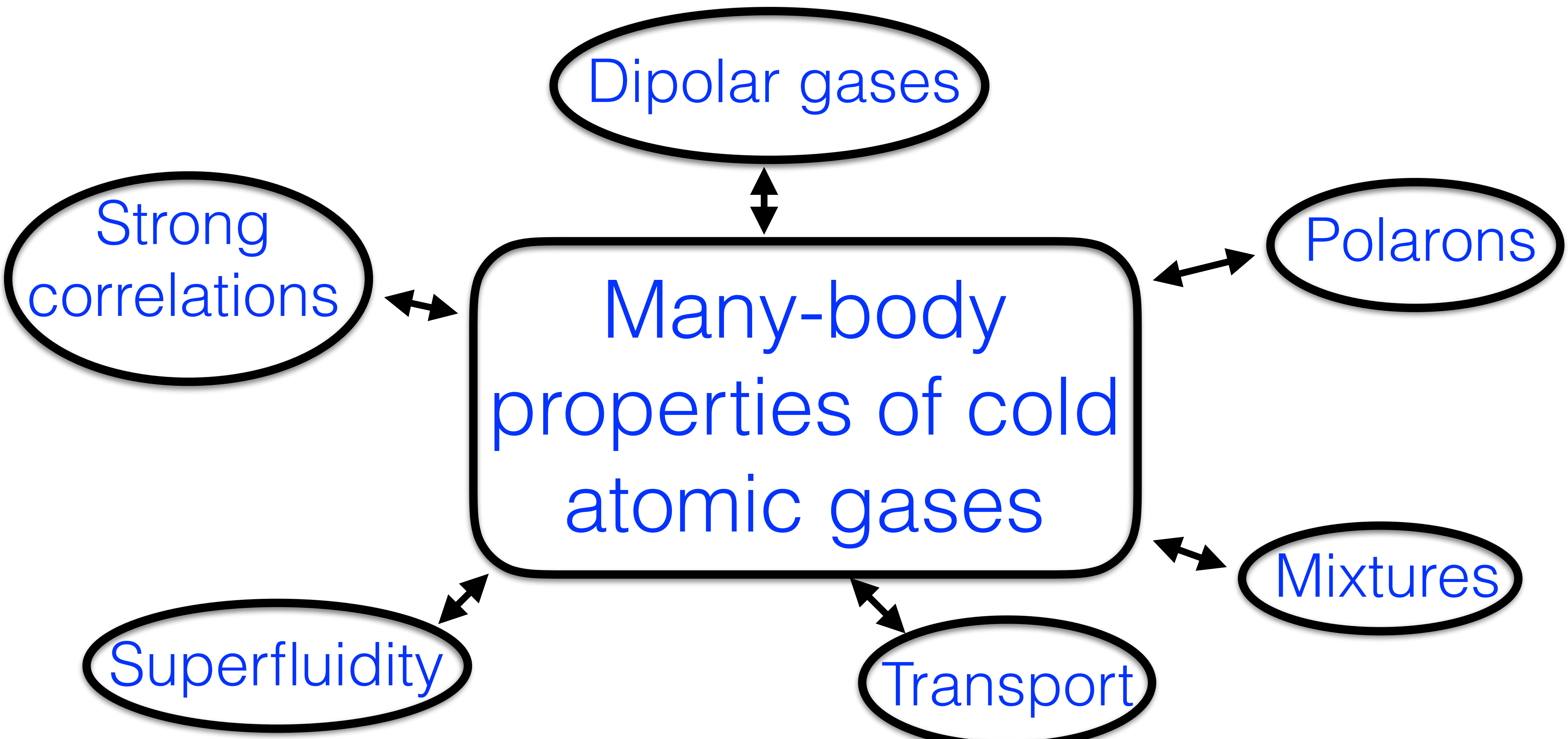
Aarhus University

Z. Wu & GMB, Phys. Rev. Lett. **117**, 245302 (2016)
D. Suchet, Z. Wu, F. Chevy & GMB, Phys. Rev. A **95**, 043643 (2017)
J. M. Midtgaard, Z. Wu, & GMB, arXiv:1705.10169


Dipolar gases
Strong correlations
Polarons
Many-body properties of cold atomic gases
Mixtures
Superfluidity
Transport

Bottom lines

❶ Fermions interact attractively via density oscillations in the BEC

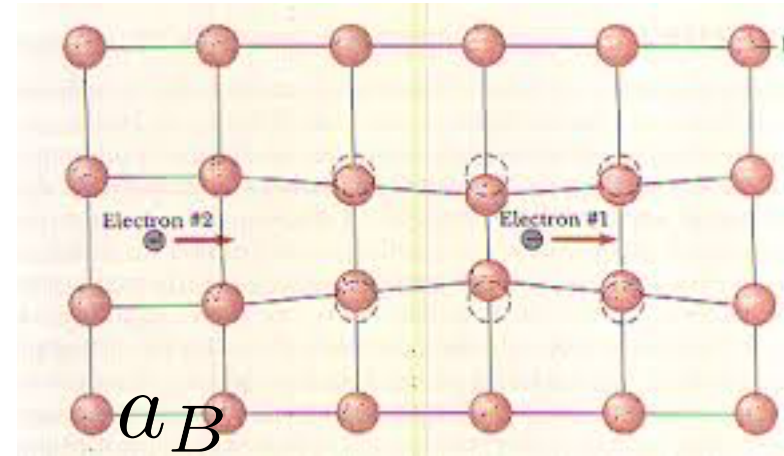
❷ Induced interaction can be tuned to form a p_x+ip_y superfluid with *maximum possible* T_c

❸ Bi-layer setup can realise a $\mathbb{Z}_2$ topolical superfluid with time-reversal symmetry

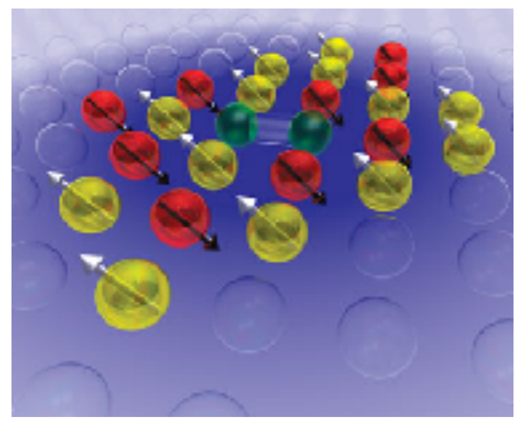
❹ Induced interaction can be probed unambiguously by bilayer dipole oscillations

Induced interaction

Cooper pairing via phonons in conventional superconductors



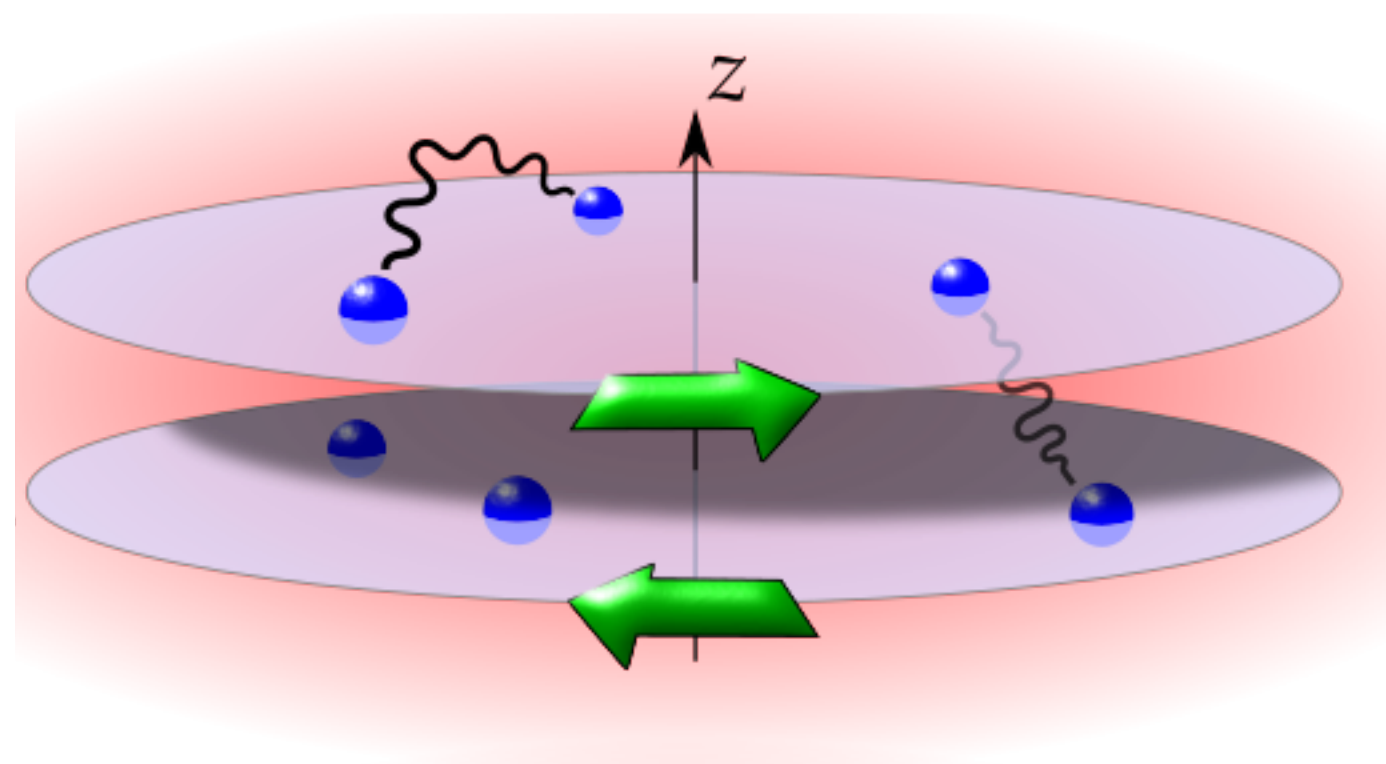
High T_c pairing via spin fluctuations?



Identical fermions ⇒

no direct interaction ⇒

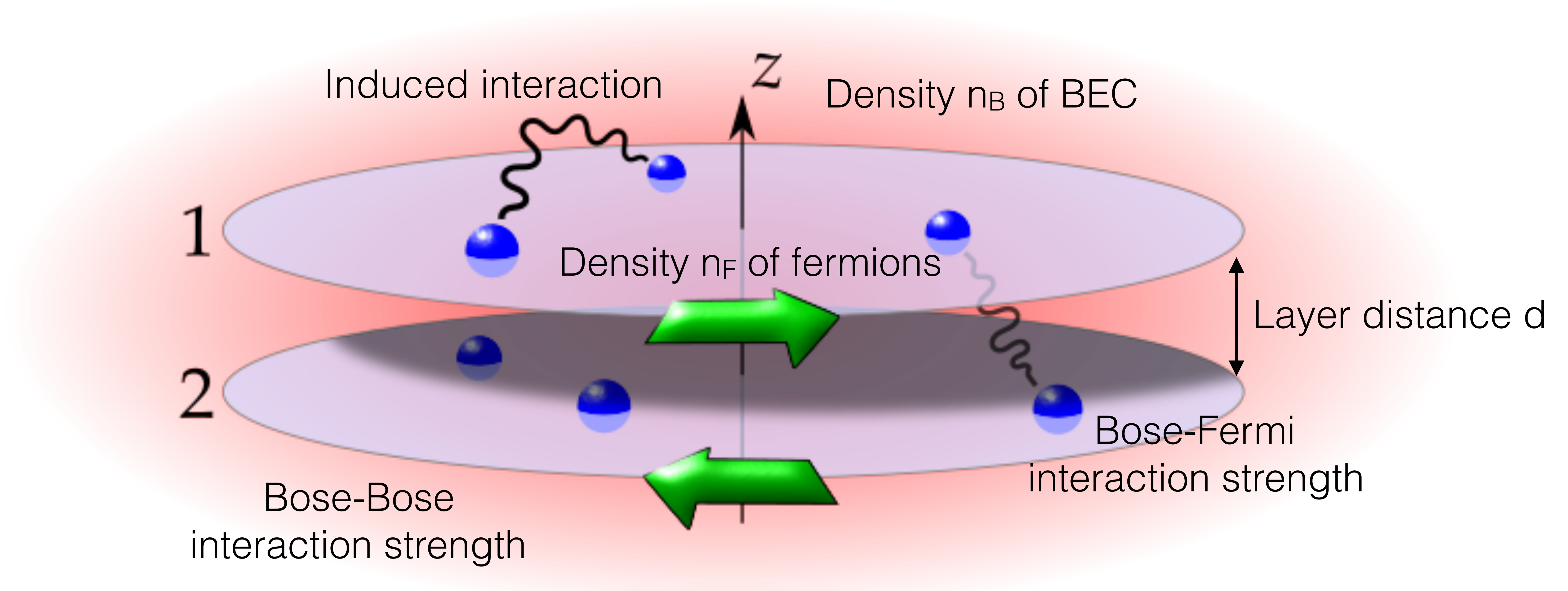
interact via the exchange of phonons in the BEC



System

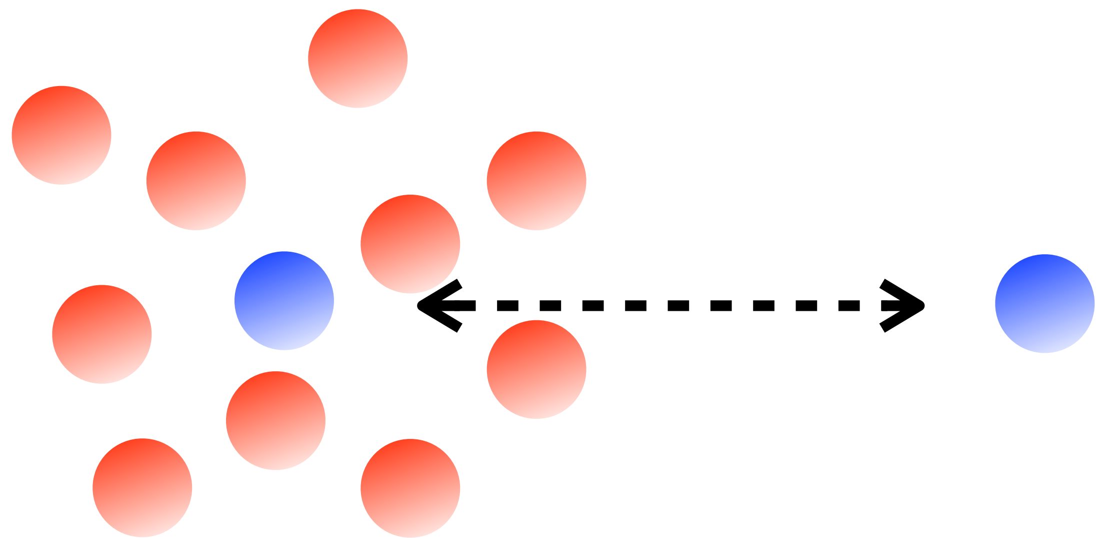
Spin polarised fermions move in 2D planes

Immersed in 3D weakly interacting BEC



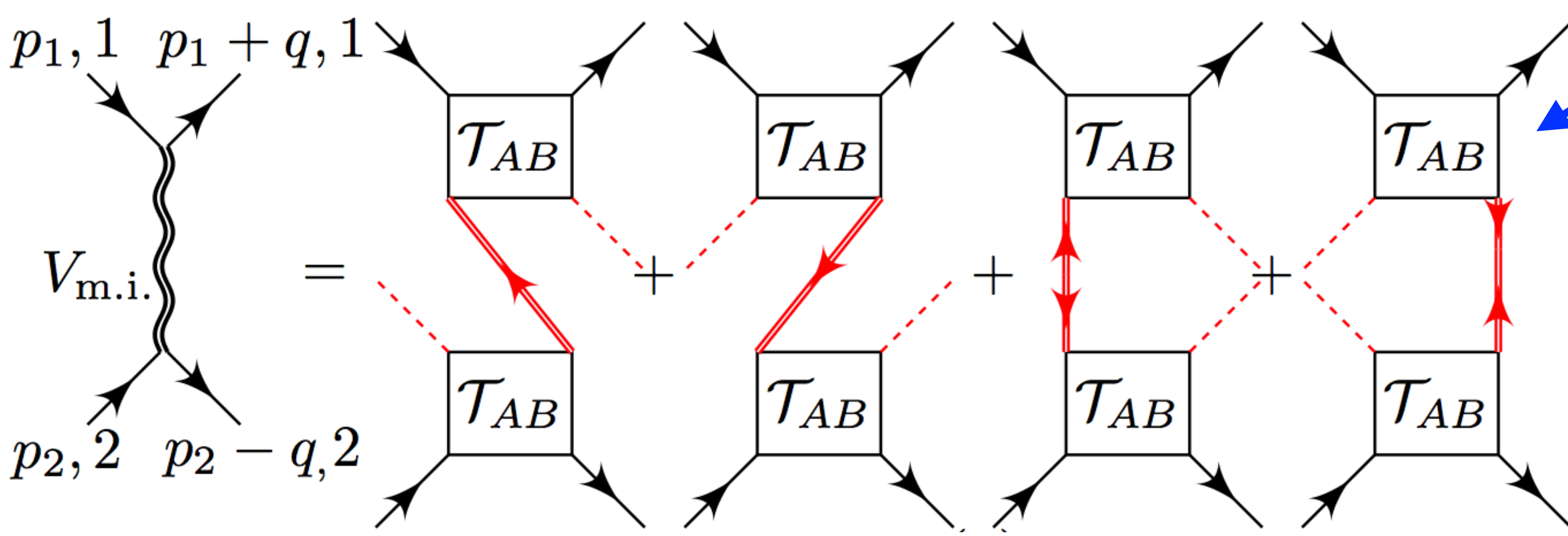
Many free parameters
Can tune system

Repulsive Attractive Bose-Fermi interaction

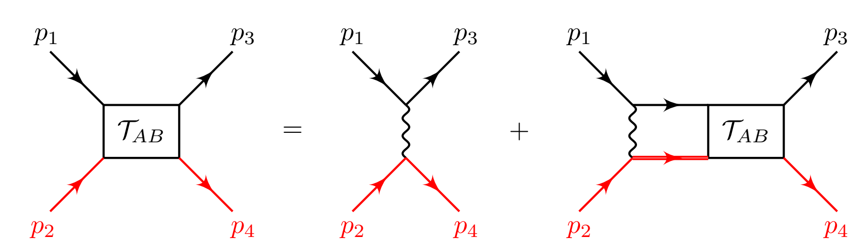


Attractive Fermi-Fermi interaction (static)

Exchange of phonon in the BEC:



2D-3D Bose-Fermi scattering:

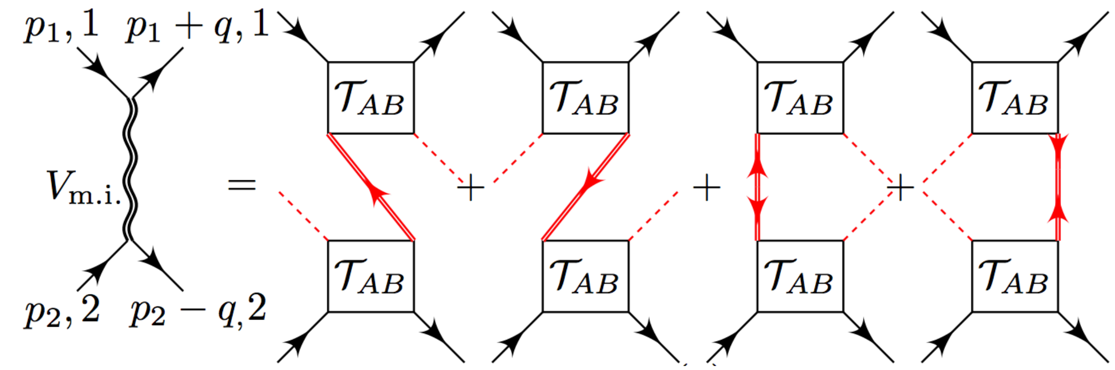


$$\mathcal{T}_{AB}(\mathbf{p}_\perp, i\omega_\nu) = \frac{g}{1 - g\Pi(\mathbf{p}_\perp, i\omega_\nu)},$$

Y. Nishida S. Tan Phys. Rev. Lett. **101**, 170401 (2008)
Y. Nishida, Ann. of Phys. **324**, 897 (2009)

Complicated mediated interaction:

$$
\begin{aligned}
V_{\text{m.i.}}(p_1,p_2;q) = & \, n_B \mathcal{T}_{AB}(p_1+q)\mathcal{T}_{AB}(p_2)\bar{G}^{\text{B}}_{11}(\mathbf{q}_\perp, i\omega_\nu) \\
& + n_B \mathcal{T}_{AB}(p_1)\mathcal{T}_{AB}(p_2-q)\bar{G}^{\text{B}}_{11}(-\mathbf{q}_\perp, -i\omega_\nu) \\
& + n_B \mathcal{T}_{AB}(p_1+q)\mathcal{T}_{AB}(p_2-q)\bar{G}^{\text{B}}_{12}(\mathbf{q}_\perp, i\omega_\nu) \\
& + n_B \mathcal{T}_{AB}(p_1)\mathcal{T}_{AB}(p_2)\bar{G}^{\text{B}}_{21}(\mathbf{q}_\perp, i\omega_\nu)
\end{aligned}
$$



Depends on frequency & COM momentum

Weak 2D-3D interaction:

Can control both strength and range

$$
V_{\text{ind}}(\mathbf{q}_\perp, i\omega_\nu) = \frac{g^2 n_B}{m_B}\int_{-\infty}^{\infty}\frac{dq_z}{2\pi}\frac{\mathbf{q}^2}{(i\omega_\nu)^2 - E^2_{\mathbf{q}}} \Leftrightarrow V(\mathbf{r},0) = -\frac{g^2 n_0 m_B}{\pi}\frac{e^{-\sqrt{2}r/\xi}}{r}
$$

Yukawa interaction

Attractive for any frequency ⇒ Pairing between identical fermions

Pairing between fermions in single layer

$$\Delta(p) = -T\sum_m \int \frac{d^3q}{(2\pi)^2} V_{\rm ind}(p-q)\frac{\Delta(q)}{\omega_m^2 + \mathcal{E}^2(q)}$$

$$\mathcal{E}(q) = \sqrt{\xi(q)^2 + \Delta(q)^2}$$

Include retardation effects

Can only be neglected when $v_F \ll c$

Bogoliubov speed of sound

Fermi velocity

Identical fermions: $p_x \pm ip_y$ pairing fully gaps the Fermi surface

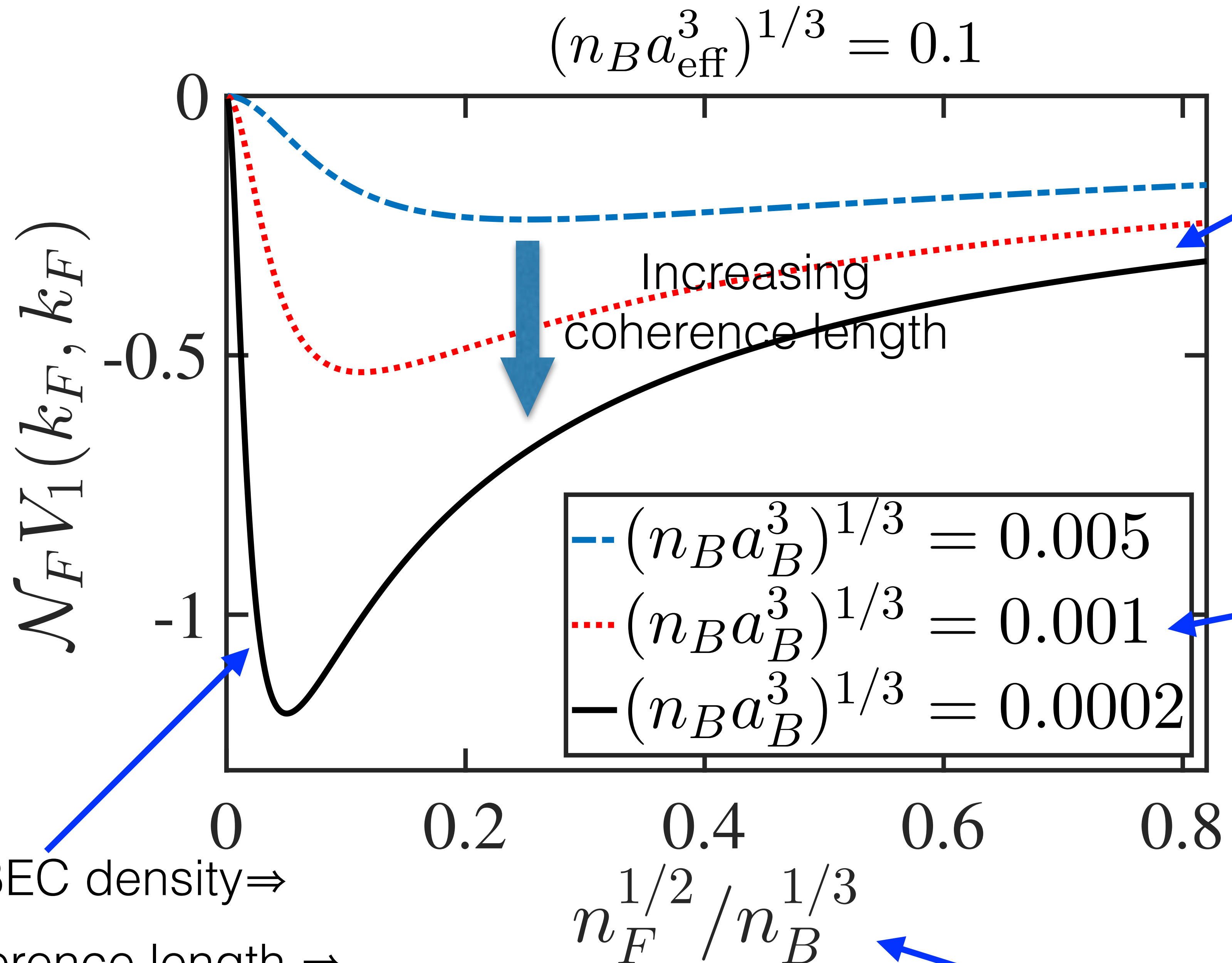
$$\Delta(\mathbf{p}) = \Delta(|\mathbf{p}|)\exp(\pm i\phi)$$

Breaks time-reversal symmetry ⇒ topological $\mathbb{Z}$ superfluid

Control both strength and range of interaction

$$V_1(|\mathbf{p}|,|\mathbf{q}|;i\omega_\nu)=\int_0^{2\pi}\frac{d\varphi}{2\pi}V_{\text{ind}}(\mathbf{p}-\mathbf{q};i\omega_\nu)e^{-i\varphi}$$

p-wave component of interaction



Small BEC density⇒

weak *p*-wave interaction

BEC gas parameter

Large BEC density⇒

short coherence length ⇒

weak p-wave interaction

Ratio of Bose and Fermi densities

Optimal density ratio n_F/n_B

Kosterlitz-Thouless transition in 2D

p_x+ip_y superfluid melts via vortex unbinding

Topological phase transition

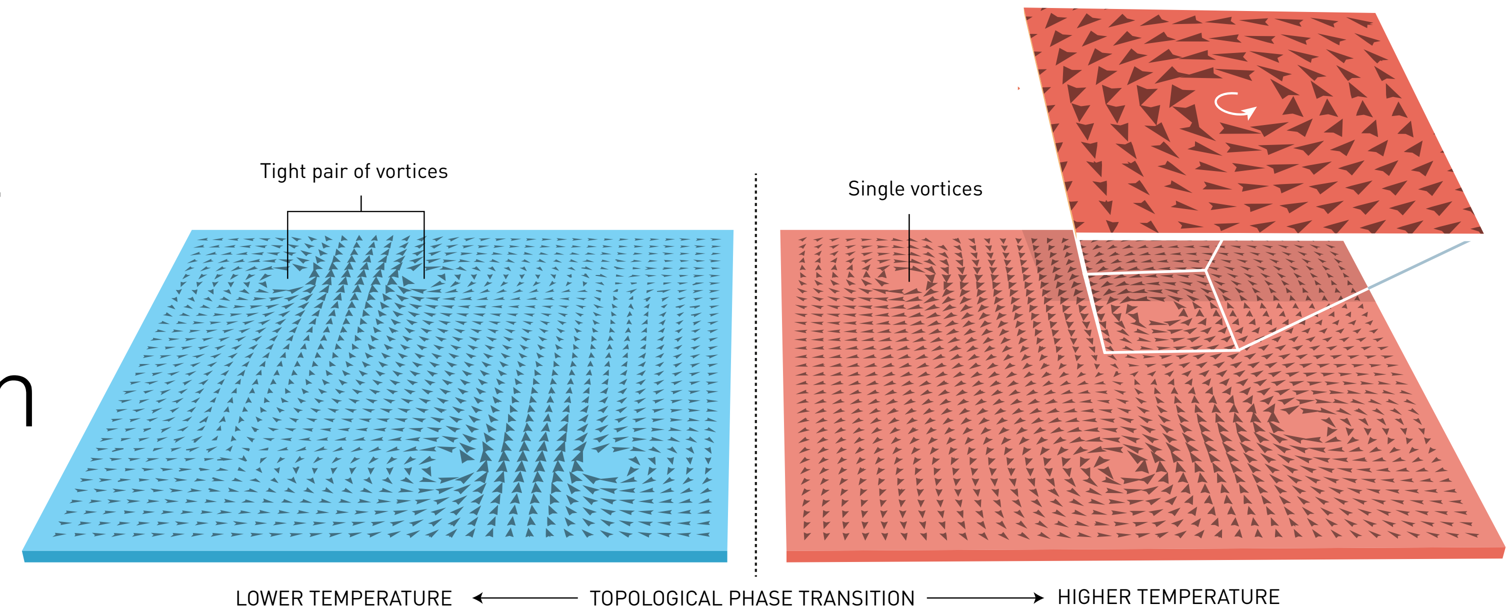
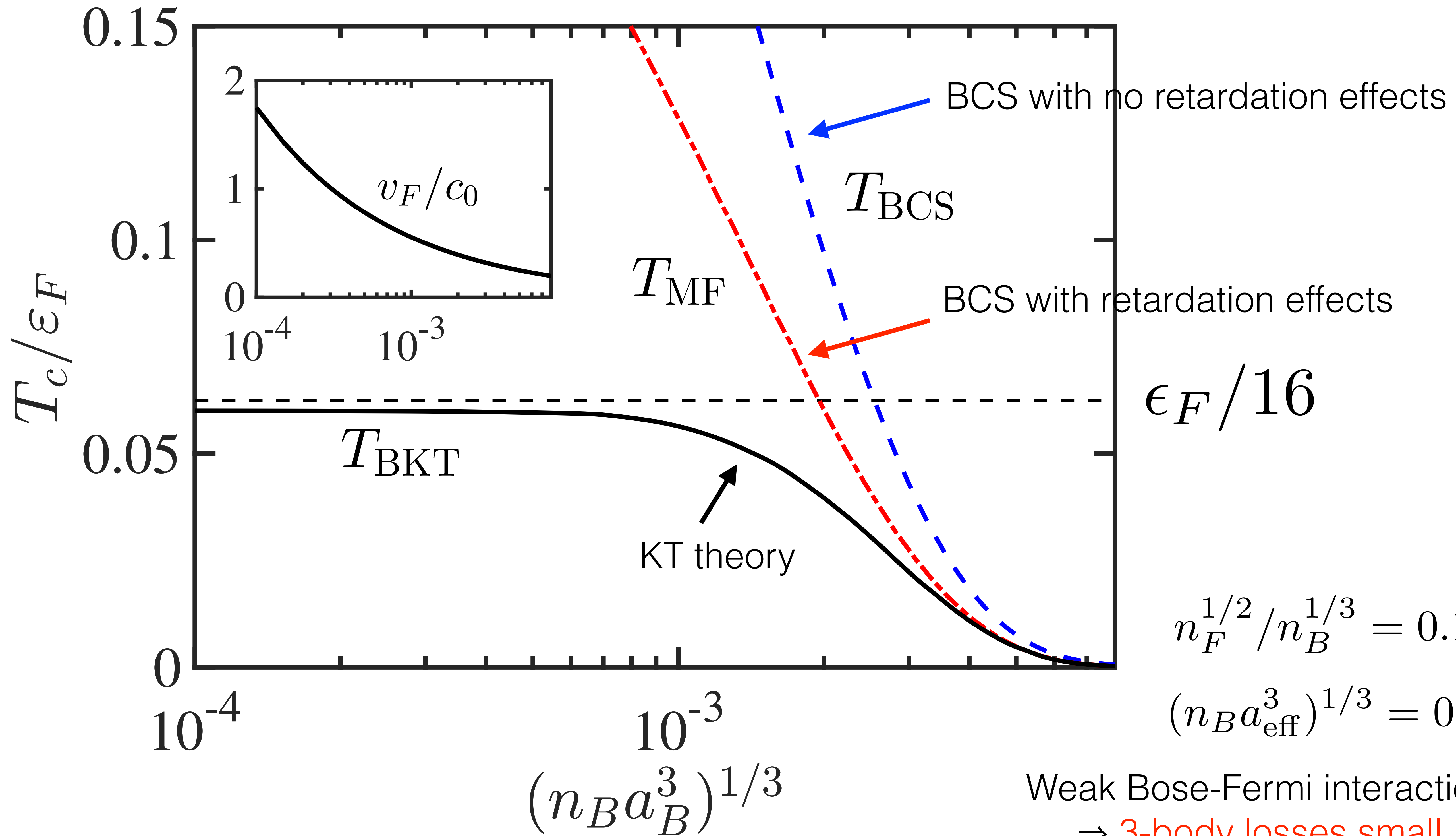


Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

$$T_{\mathrm{BKT}} = \frac{\pi}{8m_F^2}\rho_s\left(\{\Delta(i\omega_n)\}, T_{\mathrm{BKT}}\right)$$

Superfluid density $\rho_s = \rho_0 + \frac{T}{2}\sum_n \int \frac{d^2p}{(2\pi)^2}\mathbf{p}^2 \frac{\mathcal{E}^2(\mathbf{p}, i\omega_n) - \omega_n^2}{[\omega_n^2 + \mathcal{E}^2(\mathbf{p}, i\omega_n)]^2}$

Calculate melting of topological phase including <u>retardation</u> and <u>Kosterlitz-Thouless</u> physics



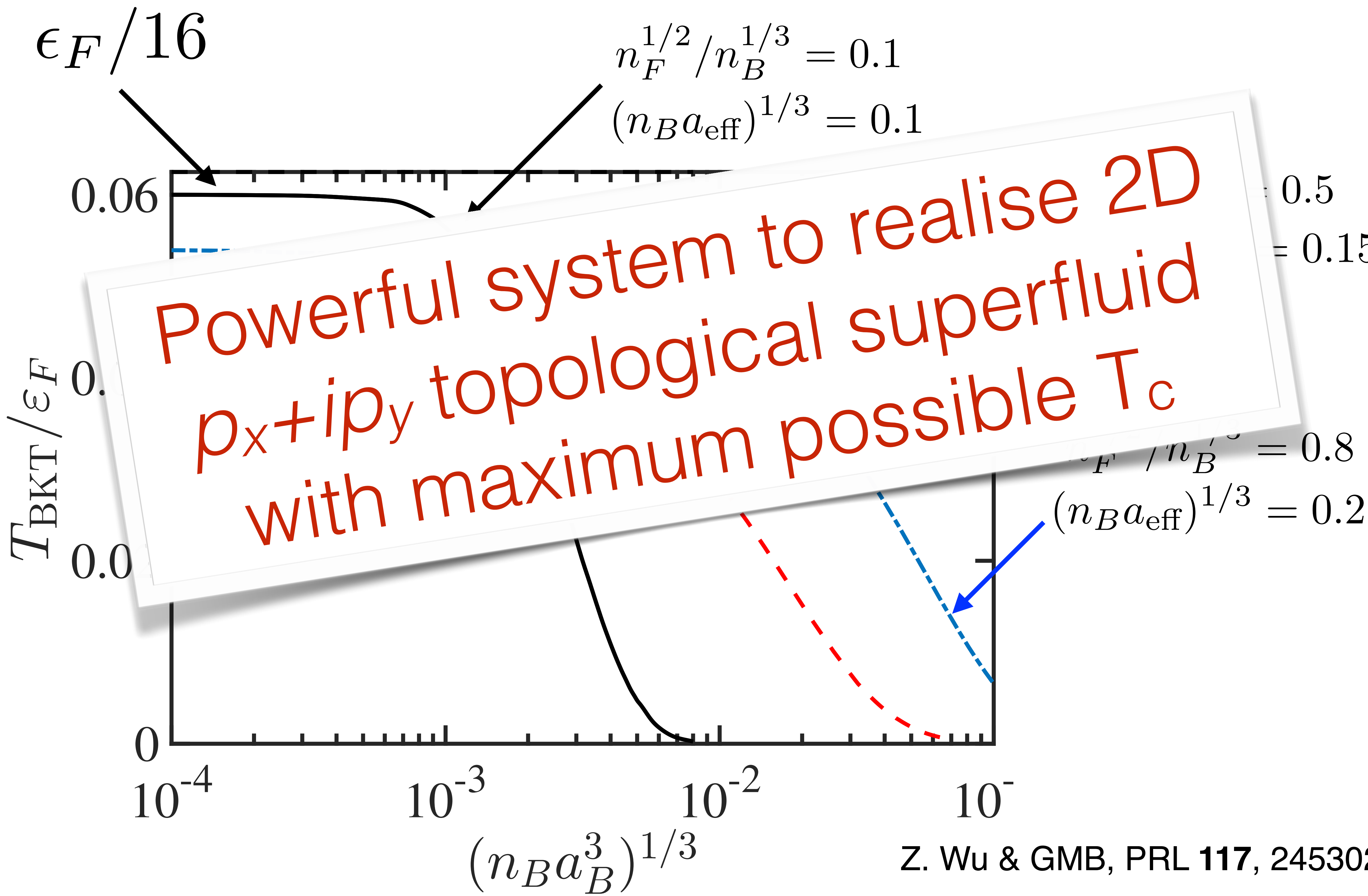
$n_F^{1/2}/n_B^{1/3} = 0.1$

$(n_B a_{\mathrm{eff}}^3)^{1/3} = 0.1$

Weak Bose-Fermi interaction
⇒ 3-body losses small

T_C saturates at maximum possible value $T_F/16$ allowed by KT theory

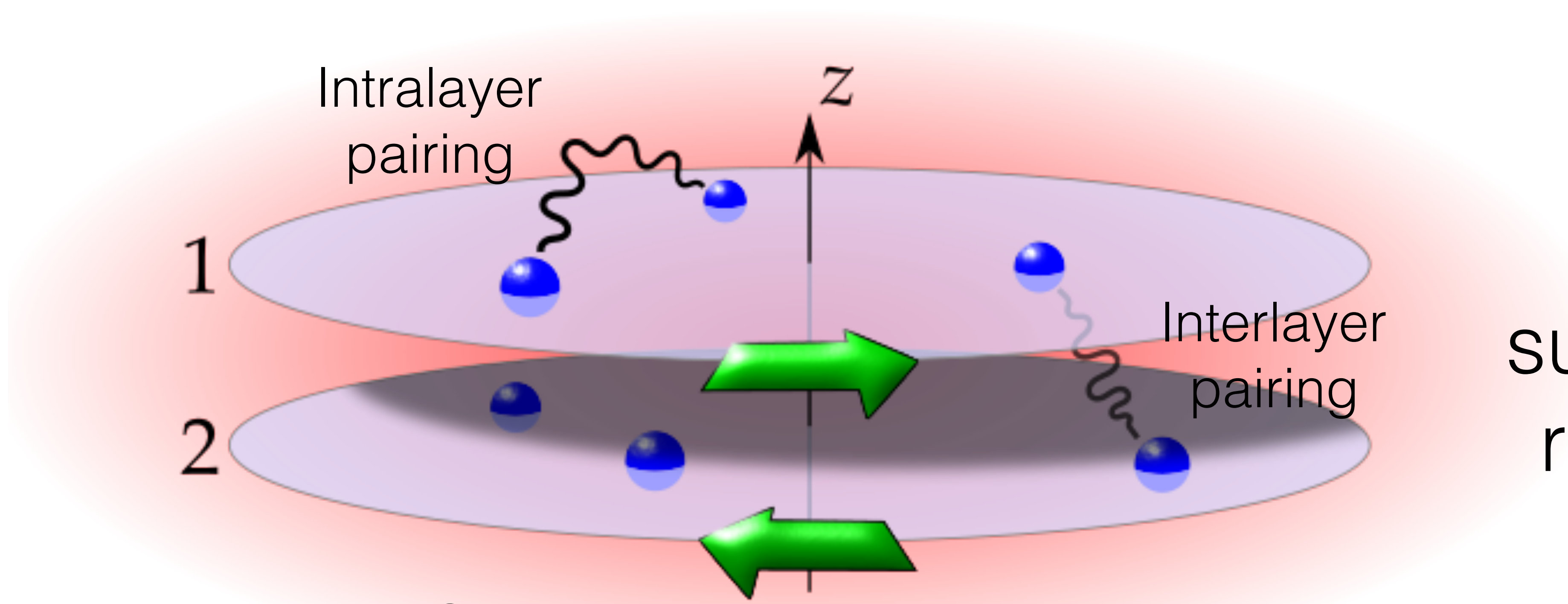
Flexible system: $T_C \simeq T_F/16$ for a *broad* range of parameters



Powerful system to realise 2D p_x+ip_y topological superfluid with maximum possible T_C

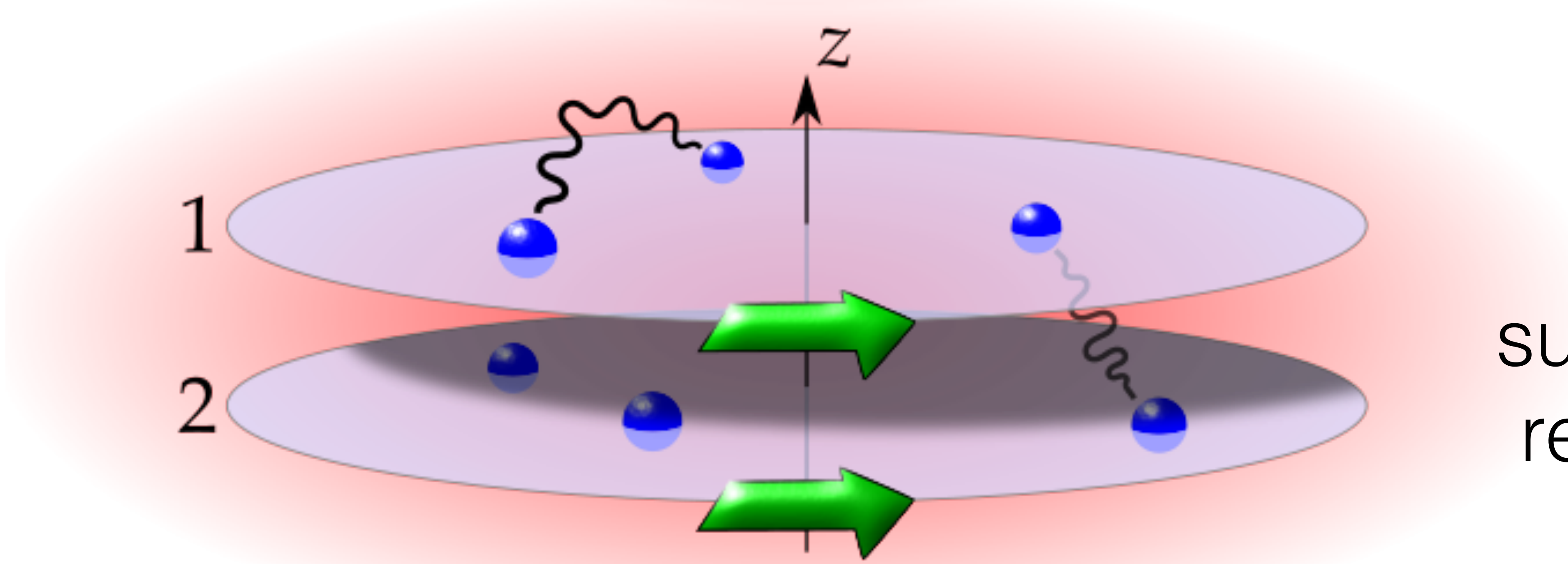
Z. Wu & GMB, PRL **117**, 245302 (2016)

Bi-layer setup



Counter propagating edge states

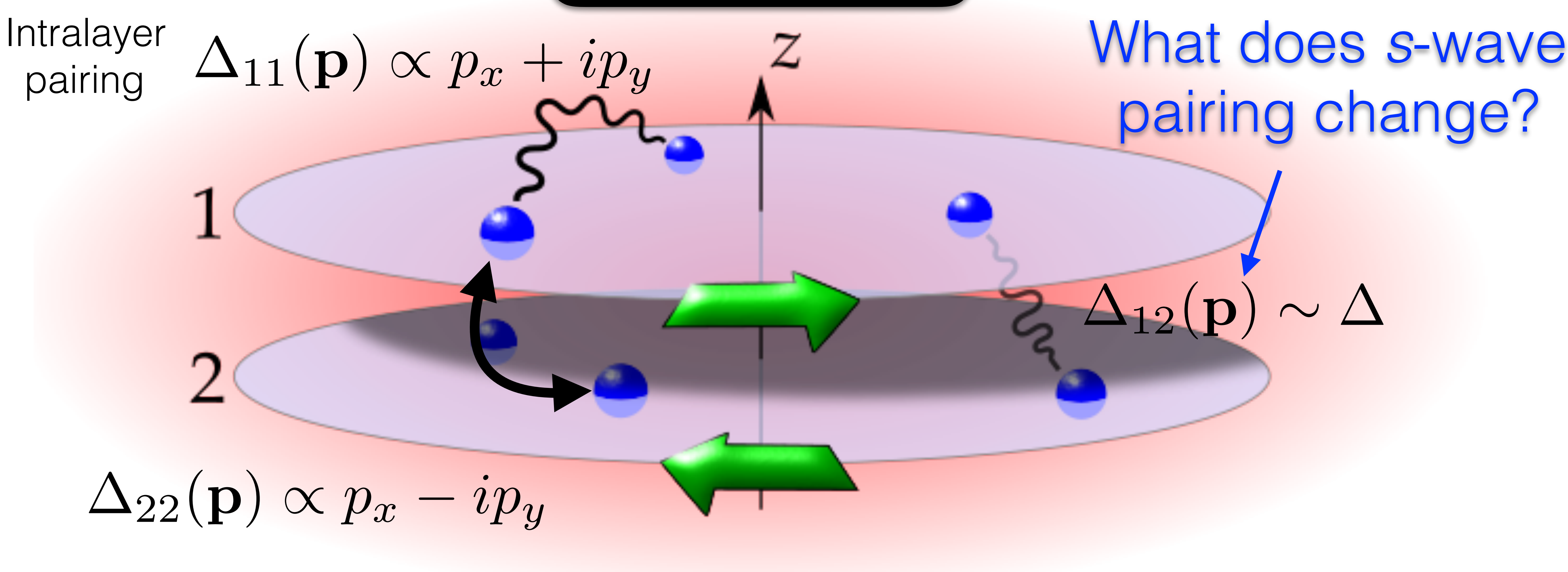
$\mathbb{Z}_2$ topological superfluid with time-reversal symmetry



Co-propagating edge states

$\mathbb{Z}$ topological superfluid. No time-reversal symmetry

$$\Delta_{22} = \Delta_{11}^*$$



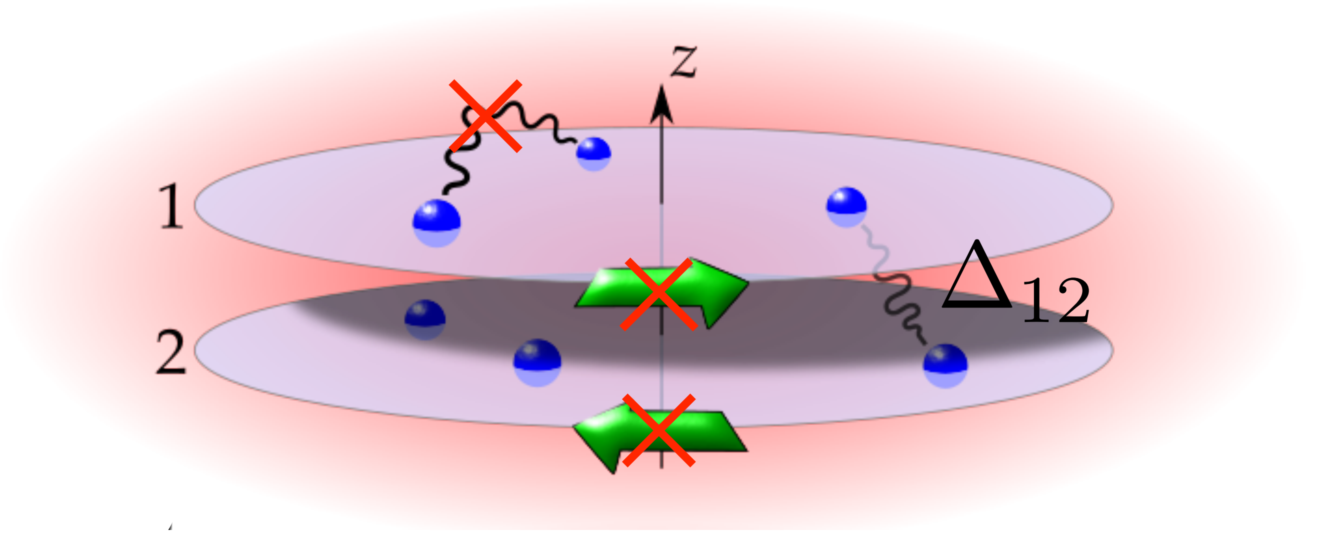
❶ Particle-hole symmetry

❷ Time-reversal symmetry $\mathcal{T}(a^1_{\mathbf{p}}, a^2_{\mathbf{p}})\mathcal{T}^{-1} = (a^2_{-\mathbf{p}}, -a^1_{-\mathbf{p}})$ $\mathcal{T}^2 = -1$

(Swaps particles between planes)

Class DIII superfluid with $\mathbb{Z}_2$ topological order
Gapless counter propagating edge modes
Protected by Kramer's theorem

Small layer distance or long coherence length:
Standard *s*-wave interlayer pairing
Topologically trivial



How do edge modes disappear with decreasing layer distance?

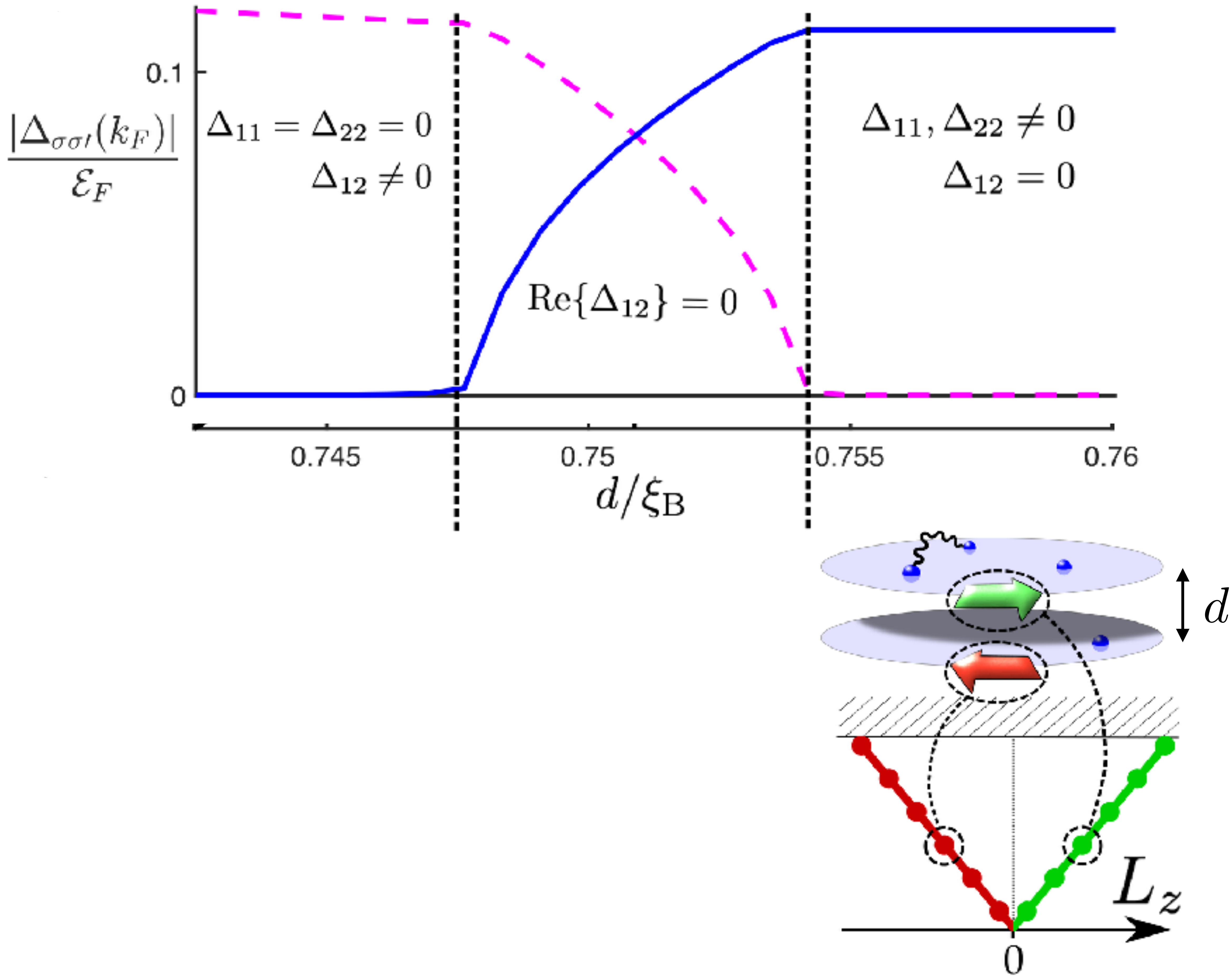
❶ Interlayer pairing preserves time-reversal symmetry
Δ_{12} real ⇒ bulk gap must close

❷ Interlayer pairing breaks time-reversal symmetry
Δ_{12} imaginary

BCS equation for intra- and interlayer pairing

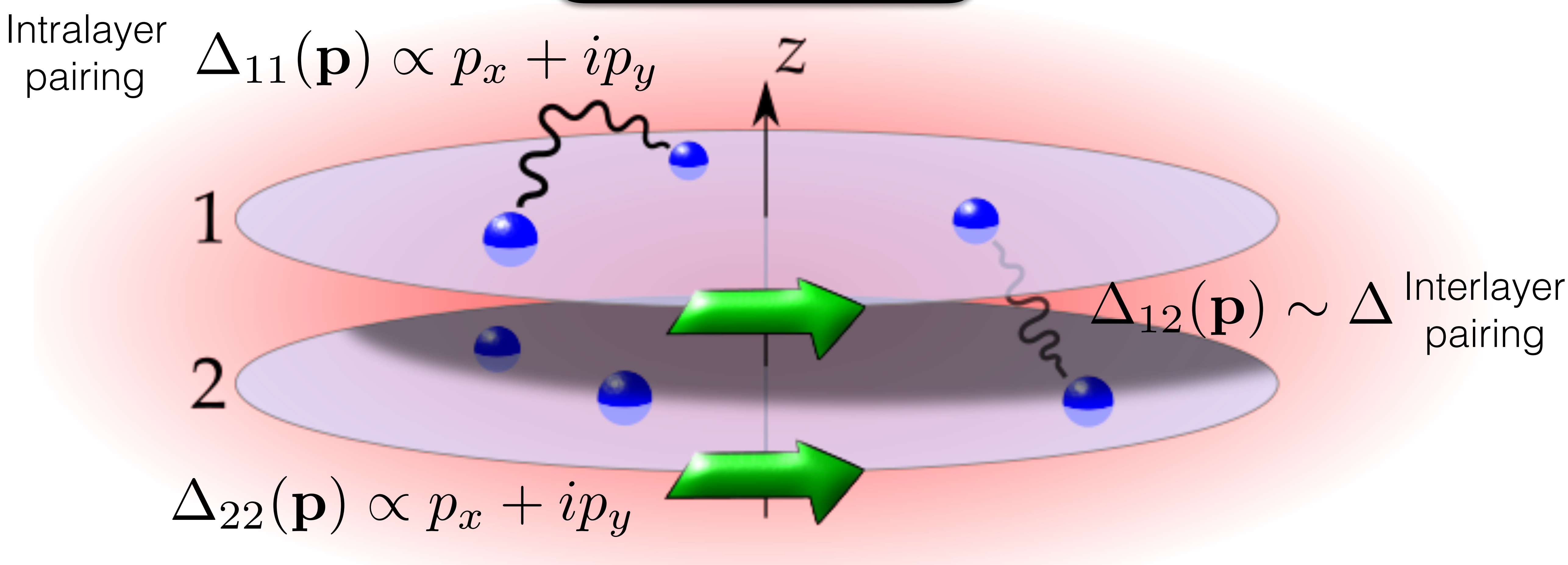
$$\mathcal{H}(\mathbf{p}) = \begin{bmatrix} \xi_{\mathbf{p}} & \Delta_{11}(\mathbf{p}) & 0 & \Delta_{12}(\mathbf{p}) \\ \Delta_{11}^{*}(\mathbf{p}) & -\xi_{\mathbf{p}} & -\Delta_{12}^{*}(\mathbf{p}) & 0 \\ 0 & -\Delta_{12}(\mathbf{p}) & \xi_{\mathbf{p}} & \Delta_{22}(\mathbf{p}) \\ \Delta_{12}^{*}(\mathbf{p}) & 0 & \Delta_{22}^{*}(\mathbf{p}) & -\xi_{\mathbf{p}} \end{bmatrix}$$

$$\Delta_{11} = -\Delta_{22}^{*} \Rightarrow \begin{cases} \Delta_{11}(\mathbf{k}) = -\sum_{\mathbf{p}} V^{11}(\mathbf{k}-\mathbf{p}) \frac{\Delta_{11}(\mathbf{p})}{|\Delta_{11}(\mathbf{p})|} \left[\frac{|\Delta_{11}(\mathbf{p})| + \Delta_{12}(\mathbf{p})}{4E_{\mathbf{p}}^{+}} \cdot \tanh\left(\frac{E_{\mathbf{p}}^{+}}{2T}\right) + \frac{|\Delta_{11}(\mathbf{p})| - \Delta_{12}(\mathbf{p})}{4E_{\mathbf{p}}^{-}} \cdot \tanh\left(\frac{E_{\mathbf{p}}^{-}}{2T}\right. \right. \\ \Delta_{12}(\mathbf{k}) = -\sum_{\mathbf{p}} V^{12}(\mathbf{k}-\mathbf{p}) \left[\frac{|\Delta_{11}(\mathbf{p})| + \Delta_{12}(\mathbf{p})}{4E_{\mathbf{p}}^{+}} \cdot \tanh\left(\frac{E_{\mathbf{p}}^{+}}{2T}\right) - \frac{|\Delta_{11}(\mathbf{p})| - \Delta_{12}(\mathbf{p})}{4E_{\mathbf{p}}^{-}} \cdot \tanh\left(\frac{E_{\mathbf{p}}^{-}}{2T}\right) \right] \\ \text{where } E_{\mathbf{p}}^{\pm} = \sqrt{\xi_{\mathbf{p}}^{2} + ||\Delta_{11}(\mathbf{p})| \pm \Delta_{12}(\mathbf{p})|^{2}} \end{cases}$$



$\frac{|\Delta_{\sigma\sigma'}(k_F)|}{\mathcal{E}_F}$
0.1
0
$\Delta_{11} = \Delta_{22} = 0$
$\Delta_{12} \neq 0$
$\mathrm{Re}\{\Delta_{12}\} = 0$
$\Delta_{11}, \Delta_{22} \neq 0$
$\Delta_{12} = 0$
0.745
0.75
0.755
0.76
d/ξ_{B}
d
L_z
0

$$\Delta_{22} = \Delta_{11}$$

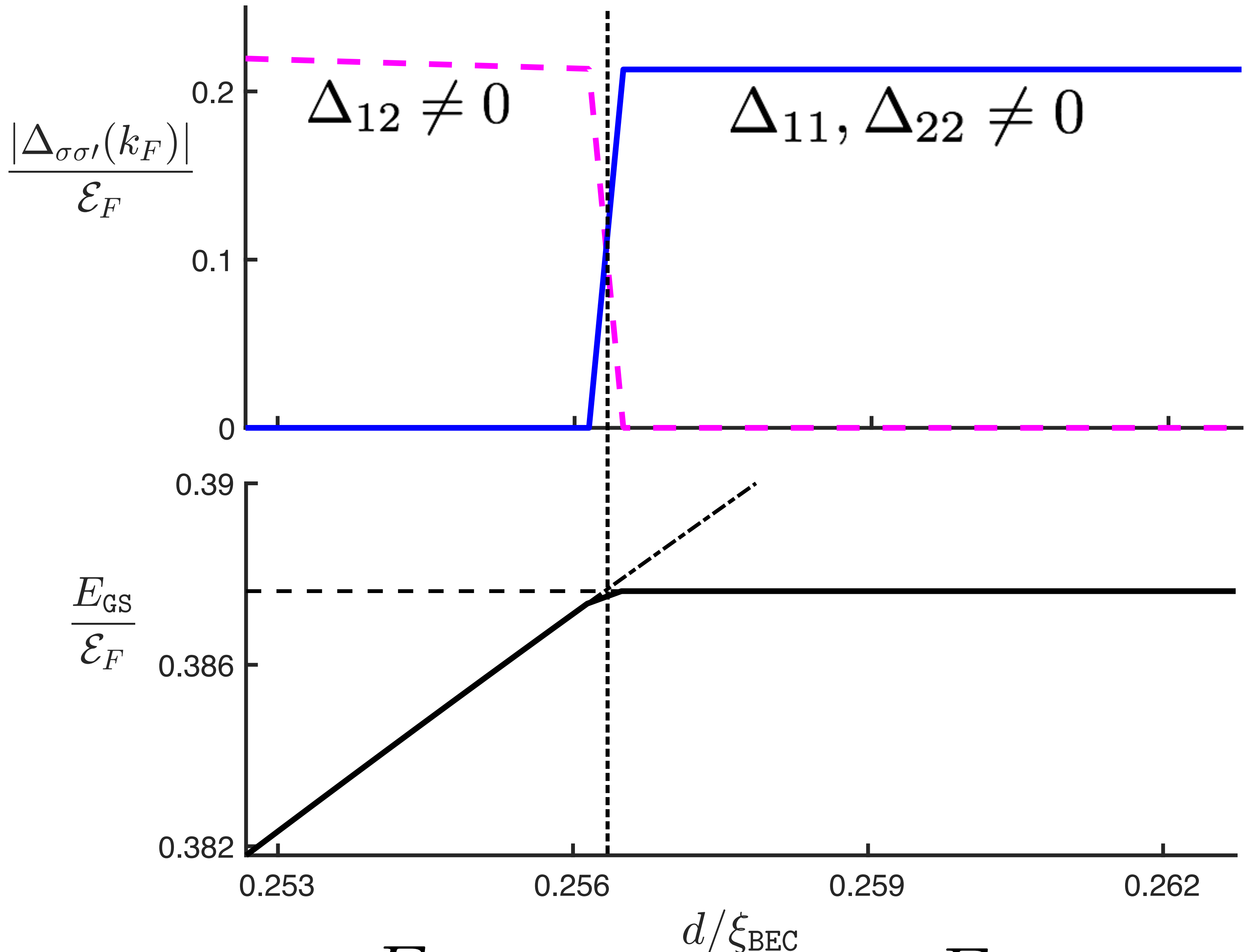


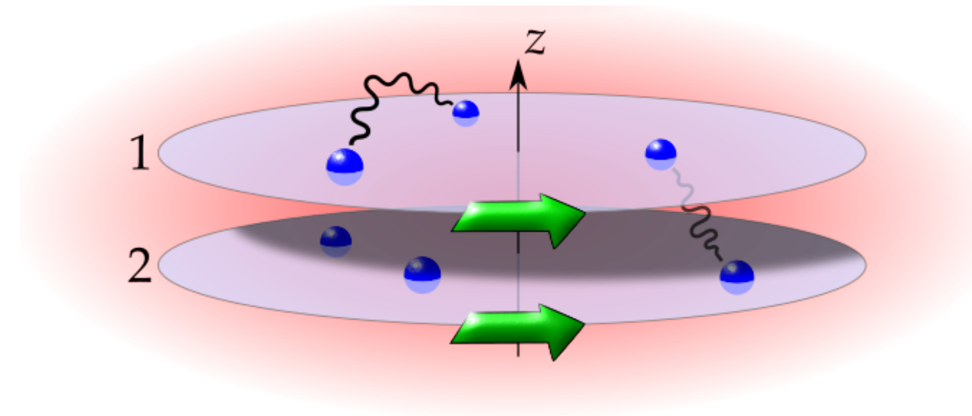
Only Particle-hole symmetry ⇒

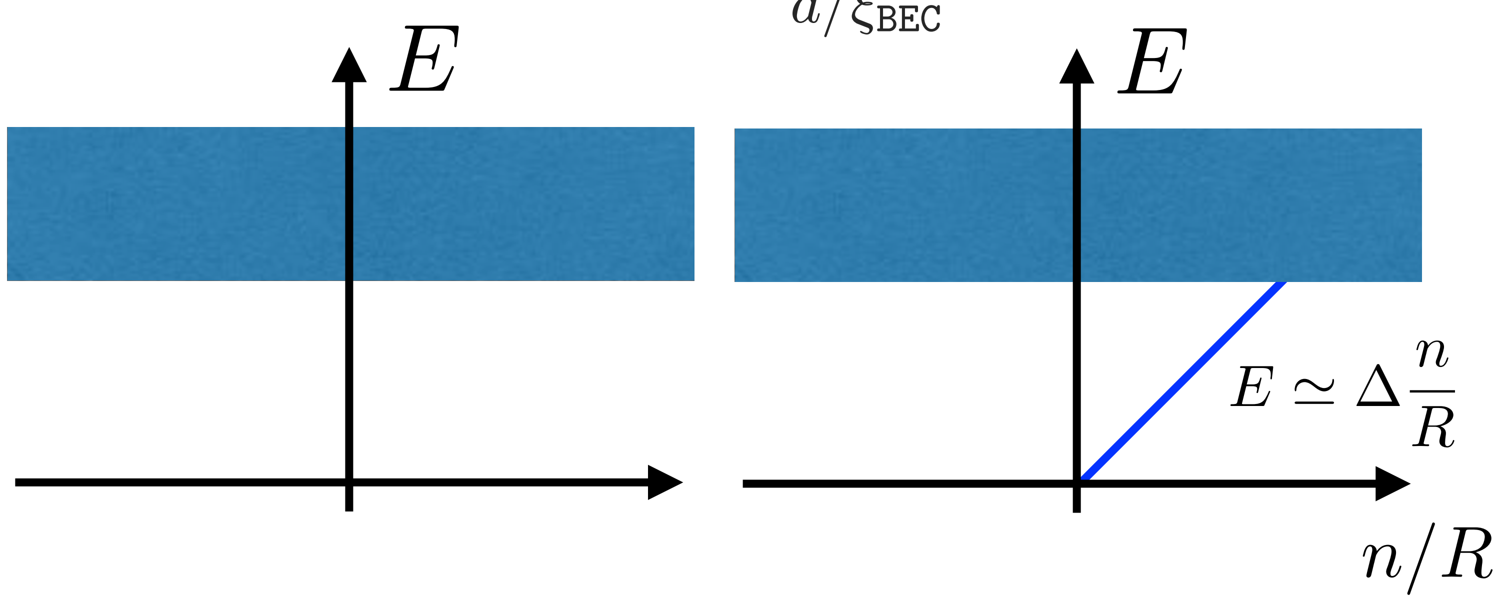
class D superfluid with $\mathbb{Z}$ topological order

End up in usual s-wave pairing for small layer distance

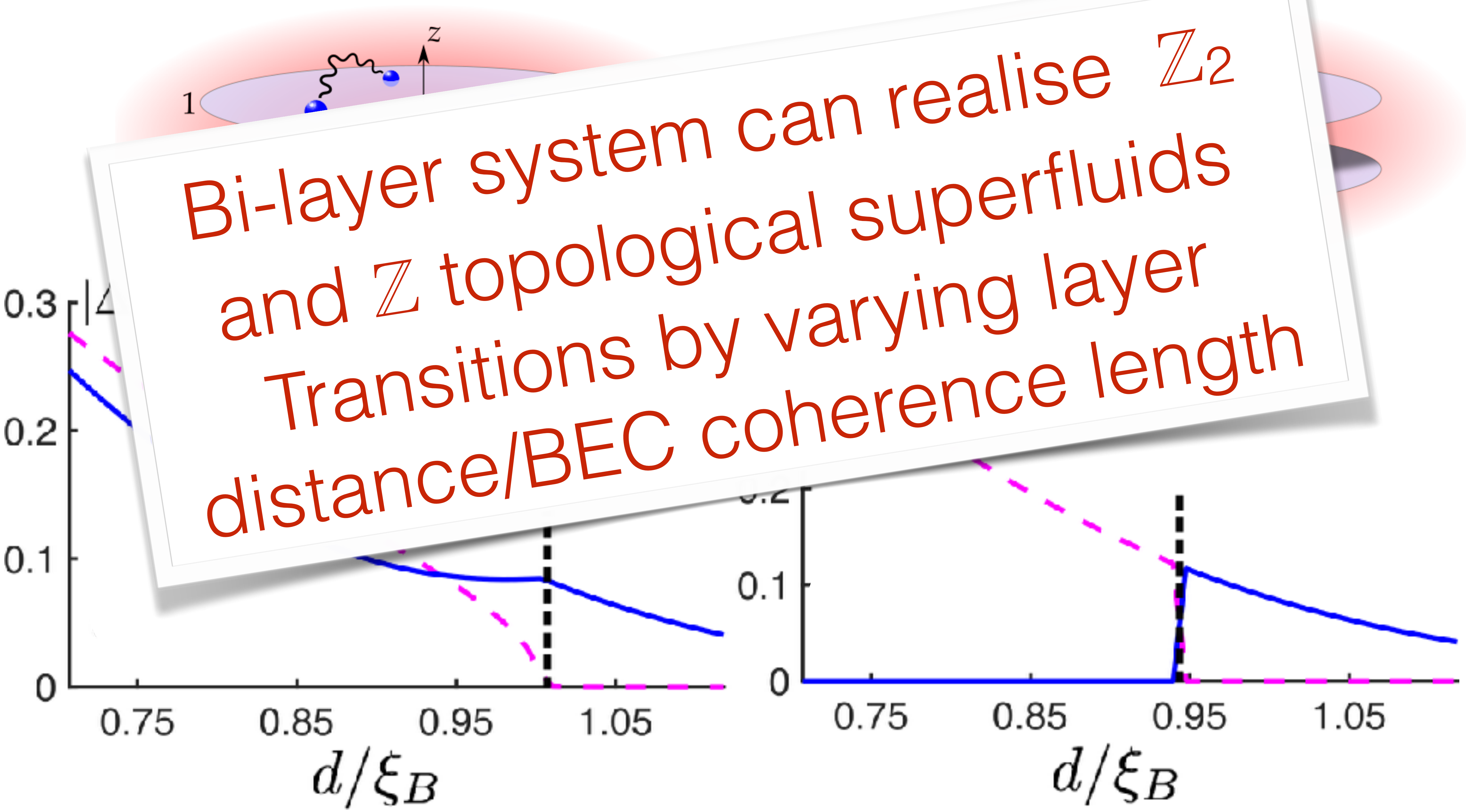
How do gapless edge modes disappear with decreasing layer distance?


$\frac{|\Delta_{\sigma\sigma'}(k_F)|}{\mathcal{E}_F}$
0.2
0.1
0
$\Delta_{12} \neq 0$
$\Delta_{11}, \Delta_{22} \neq 0$
$\frac{E_{GS}}{\mathcal{E}_F}$
0.39
0.386
0.382
0.253
0.256
0.259
0.262
d/ξ_{BEC}


z
1
2


E
E
$E \simeq \Delta \frac{n}{R}$
n/R

Vary BEC coherence length instead of layer distance

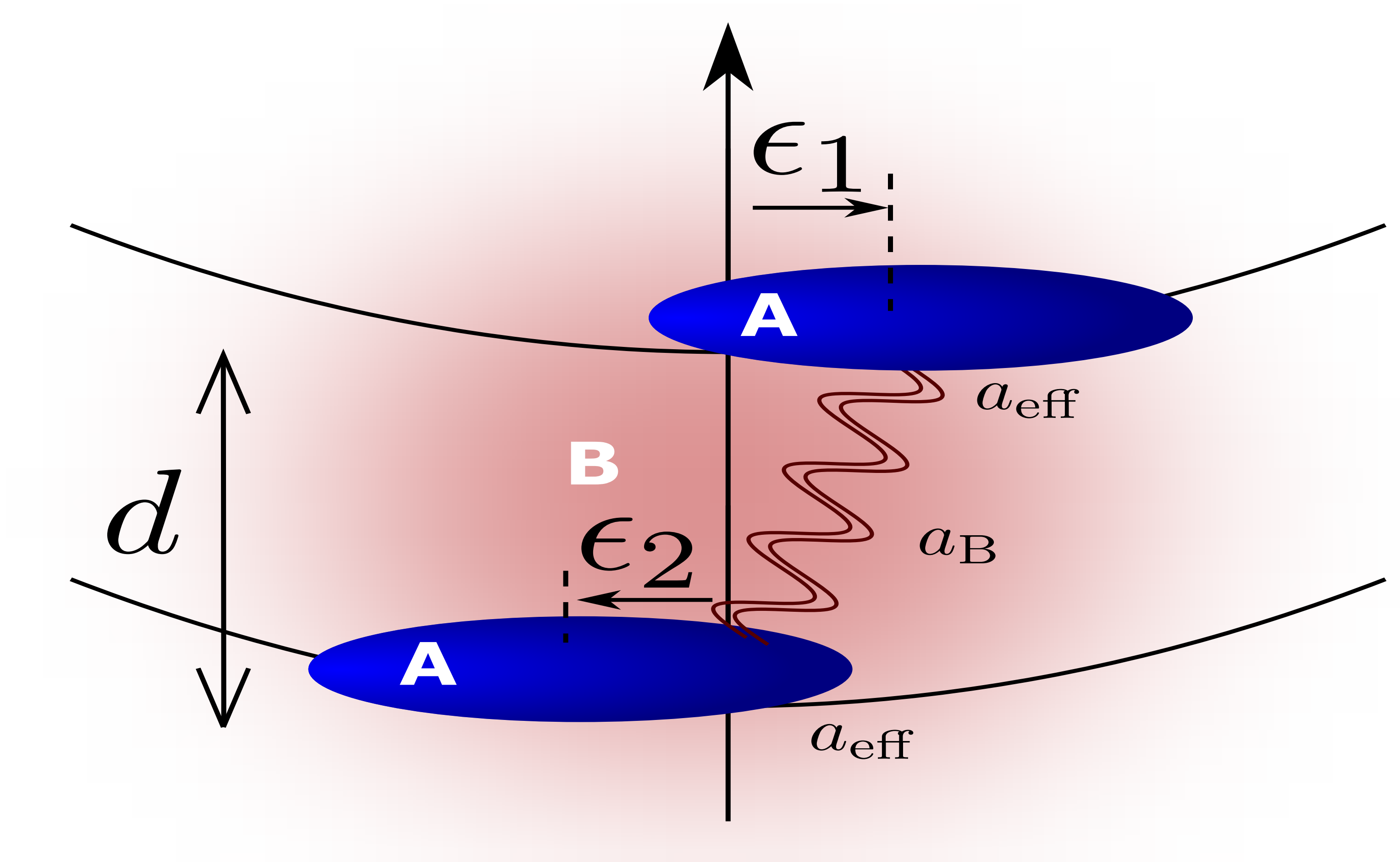


Bi-layer system can realise $\mathbb{Z}_2$ and $\mathbb{Z}$ topological superfluids

Transitions by varying layer distance/BEC coherence length

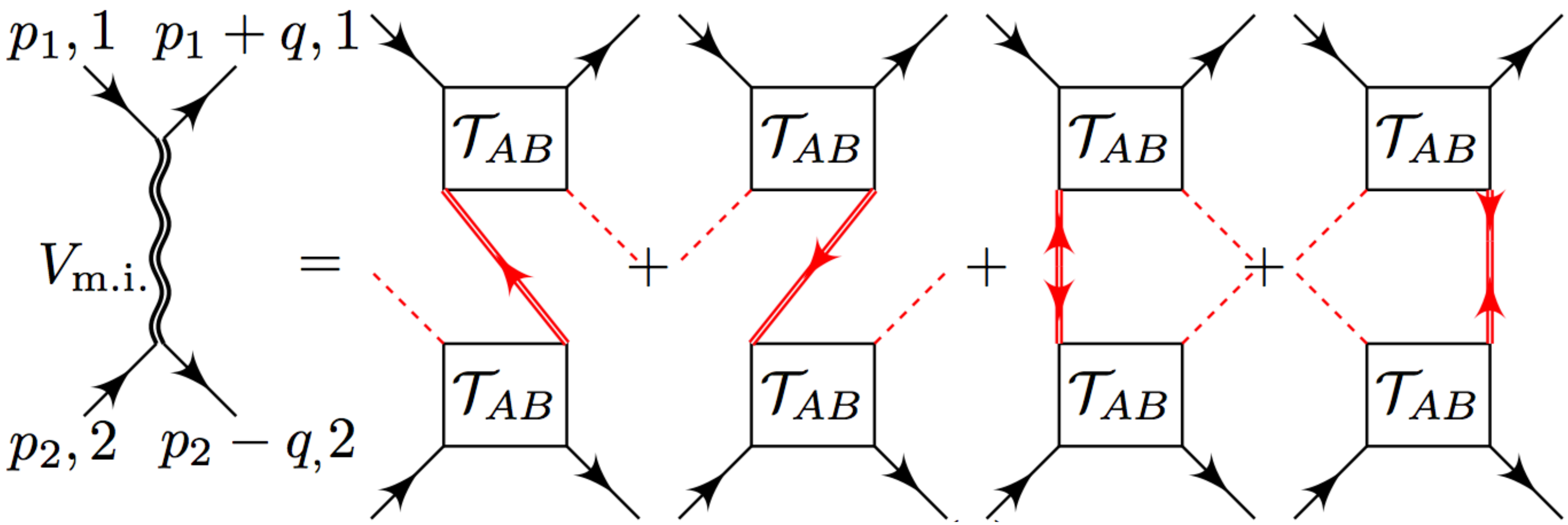
J. M. Midtgaard, Z. Wu, & GMB, arXiv:1705.10169

Probing induced interactions with bi-layer system



Idea: out-of-phase dipole oscillations have higher frequency than in-phase oscillations due to attractive induced interaction

2D-3D Fermi-Bose interaction can now be strong:

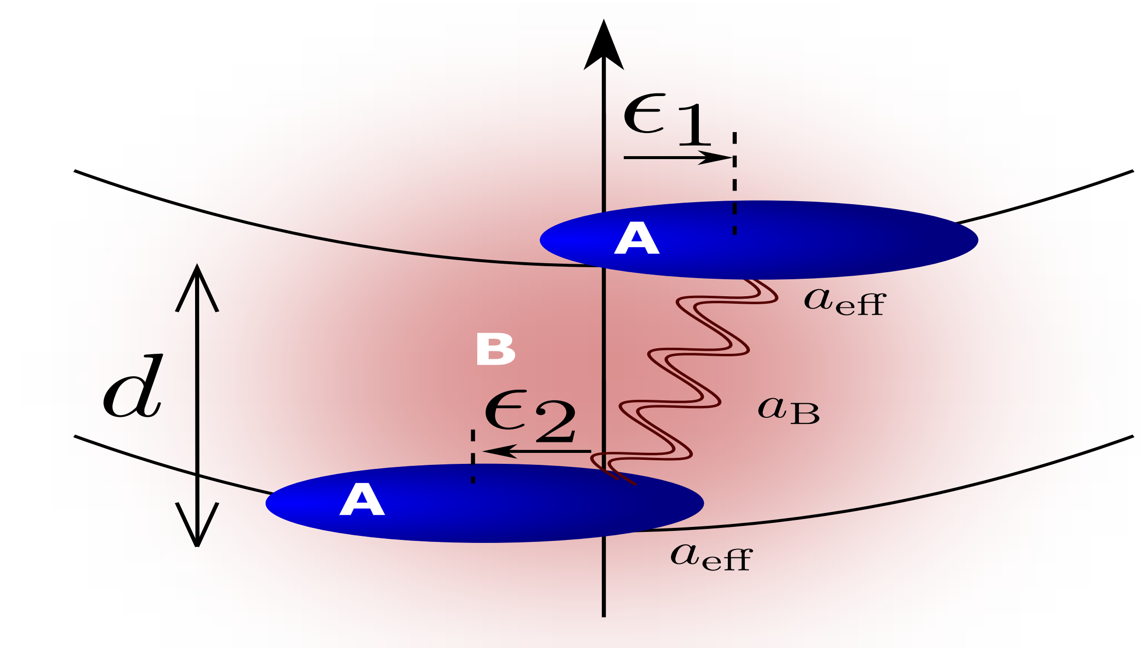


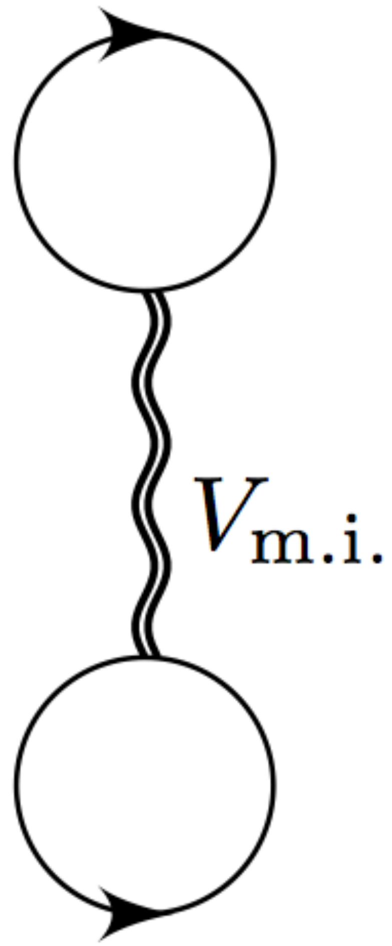
$$
\begin{aligned}
V_{\text{m.i.}}(p_1, p_2; q) =& n_B \mathcal{T}_{AB}(p_1+q)\mathcal{T}_{AB}(p_2)\bar{G}^{\text{B}}_{11}(\mathbf{q}_\perp, i\omega_\nu) \\
&+ n_B \mathcal{T}_{AB}(p_1)\mathcal{T}_{AB}(p_2-q)\bar{G}^{\text{B}}_{11}(-\mathbf{q}_\perp, -i\omega_\nu) \\
&+ n_B \mathcal{T}_{AB}(p_1+q)\mathcal{T}_{AB}(p_2-q)\bar{G}^{\text{B}}_{12}(\mathbf{q}_\perp, i\omega_\nu) \\
&+ n_B \mathcal{T}_{AB}(p_1)\mathcal{T}_{AB}(p_2)\bar{G}^{\text{B}}_{21}(\mathbf{q}_\perp, i\omega_\nu)
\end{aligned}
$$

Depends on COM as well as relative momentum/frequency

Get frequency shift from interaction energy

N. Matveeva, A. Recati, and S. Stringari, Eur. Phys. J. D **65,** 219 (2011)





$$\Delta E(\epsilon_1 - \epsilon_2) = -\frac{m_B n_B}{\pi}\int d^2r_1 d^2r_2 \frac{e^{-\sqrt{2}r/\xi_B}}{r}\bar{\Omega}_1(\mathbf{r}_1 - \epsilon_1\hat{\mathbf{x}})\bar{\Omega}_2(\mathbf{r}_2 - \epsilon_2\hat{\mathbf{x}})$$

$$\bar{\Omega}_j = \frac{1}{\beta}\sum_m \int \frac{d^2p}{(2\pi)^2}\mathcal{T}_{AB}(\mathbf{p}, i\omega_m)G_j^{\rm A}(\mathbf{p}, i\omega_m).$$

Yukawa interaction

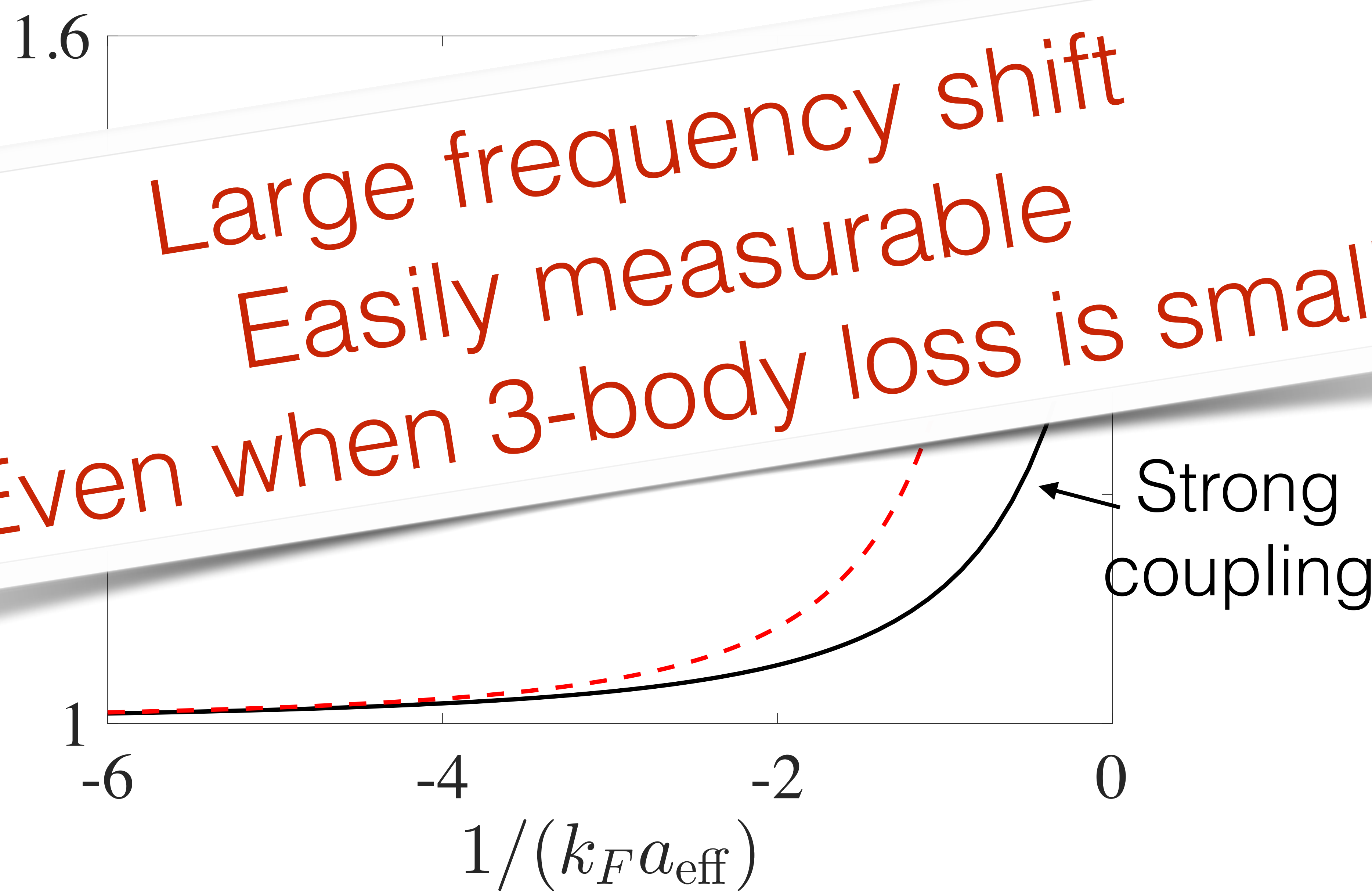
Weak Bose-Fermi interaction:

$$\Delta E(\epsilon_1 - \epsilon_2) = -g^2\frac{m_B n_B}{\pi}\int d^2r_1 d^2r_2 \frac{e^{-\sqrt{2}r/\xi_B}}{r} n_1(\mathbf{r}_1 - \epsilon_1\hat{\mathbf{x}} n_2(\mathbf{r}_2 - \epsilon_2\hat{\mathbf{x}})$$

Usual Hartree expression

Out-of-phase dipole oscillation

$$\omega_r = \omega_\perp \sqrt{1 + 2I/N_A m_A \omega_\perp^2} \qquad I = \frac{\partial^2}{\partial \epsilon_1^2} \Omega_{\text{m.i.}}(\epsilon_1 - \epsilon_2) \Big|_{\epsilon_1-\epsilon_2=0}$$



Large frequency shift
Easily measurable
Even when 3-body loss is small

D. Suchet, Z. Wu, F. Chevy & GMB, Phys. Rev. A **95**, 043643 (2017)

Conclusions

❶ Fermions interact attractively via density oscillations in the BEC

❷ Induced interaction be tuned to form a p_x+ip_y superfluid with *maximum possible*T_c

❸ Bi-layer setup can realise a $\mathbb{Z}_2$ topolical superfluid with time-reversal symmetry

❹ Induced interaction can be probed unambiguously by bilayer dipole oscillations

Collaborators

Zhigang Wu

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