

# Strong correlation effects in 2D topological quantum phase transitions



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*Frontiers in 2D Quantum Systems, ICTP*



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*Dresden*



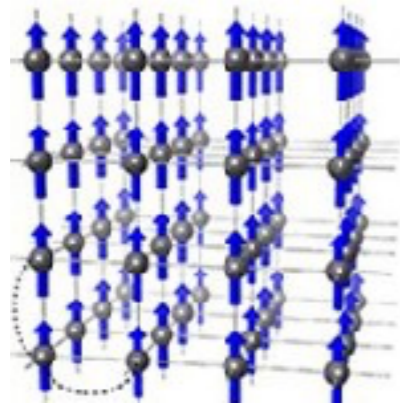
*J. Budich*

# Introduction.

Ginzburg-Landau theory: *symmetry breaking* classification of matter phases

key concept: *local order parameter*

Magnetism



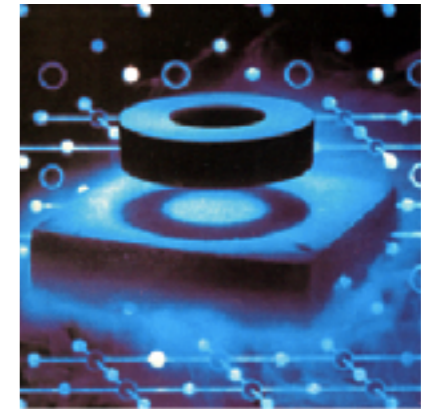
*magnetization  $M$*

Liquid-gas



*density difference  $n(L)-n(G)$*

“Superconductivity”



*pair amplitude  $\psi$*



Experimental detectability!

# Introduction.

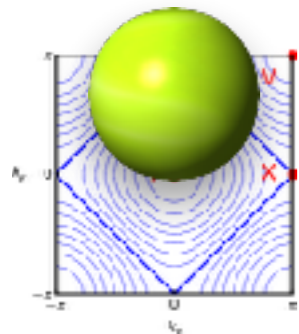
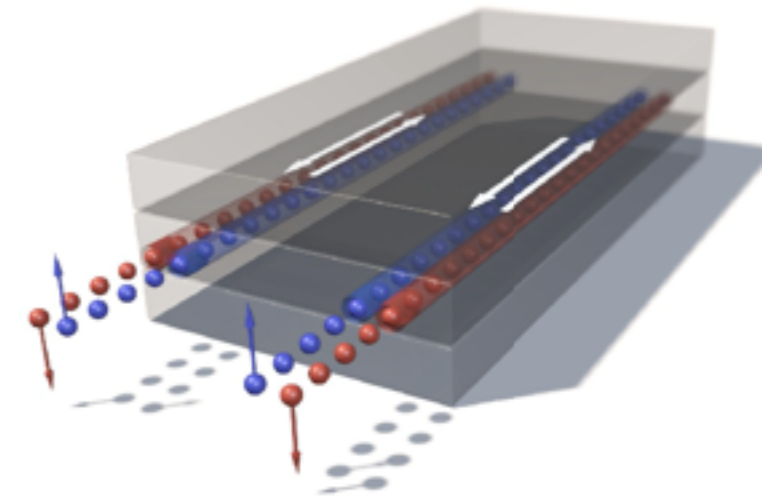
Haldane PRL88  
Kane, Mele PRL05  
Bernevig et al Science 2006  
.... many more

## TOPOLOGICAL INSULATORS

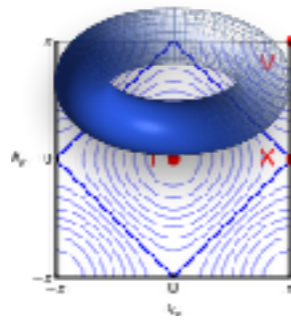
*quantum materials eluding the G-L paradigm!*

bulk (band) insulator + with *gapless* edge modes.

*Dirac semi-metal* + *Spin-Orbit Coupling*

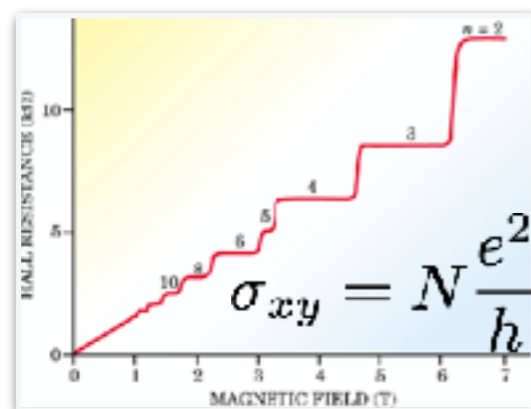


*Trivial*



*Non-Trivial*

States classified in terms of the  
*Topological Properties*  
of the Hilbert space of Bloch functions:



key concept: *global topological invariant*

# The quantum spin-Hall insulator

Initial focus on graphene but small SOC ( $gap \sim 10^{-3}meV$ )

*Idea:* look for systems with a larger SOC.

*Kane, Mele PRL 2005*

*Bernevig et al Science 2006*

*Konig et al Science 2007*

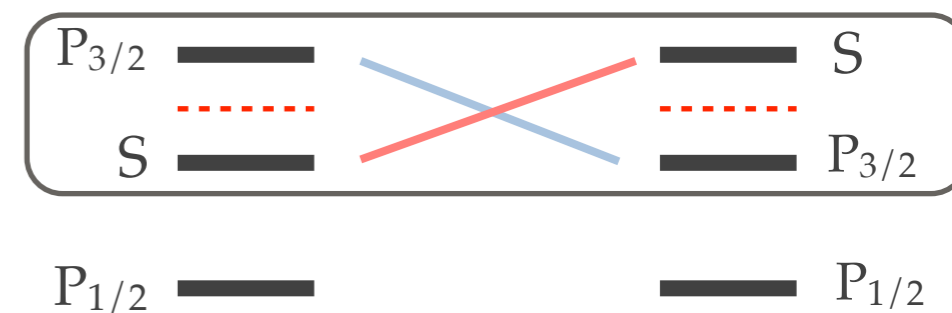
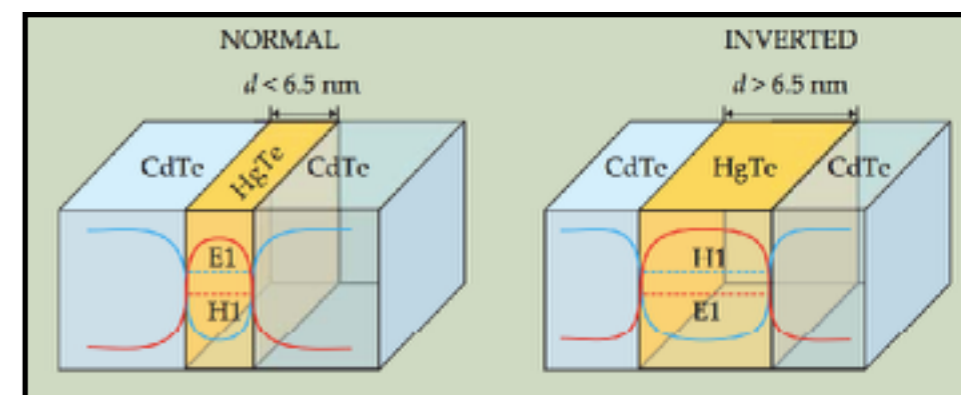
**BHZ model:** 2 QHI + Time Reversal Symmetry.

$$H = \begin{pmatrix} \mathbf{h}(\mathbf{k}) \uparrow & \mathbf{0} \\ \mathbf{0} & \mathbf{h}^*(-\mathbf{k}) \downarrow \end{pmatrix}$$

$$\mathbf{h}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau} \quad \text{Orbital pseudo-spin structure}$$

$$\mathbf{d}(\mathbf{k}) = [\lambda \sin k_x, \lambda \sin k_y, M - \varepsilon(\mathbf{k})]$$

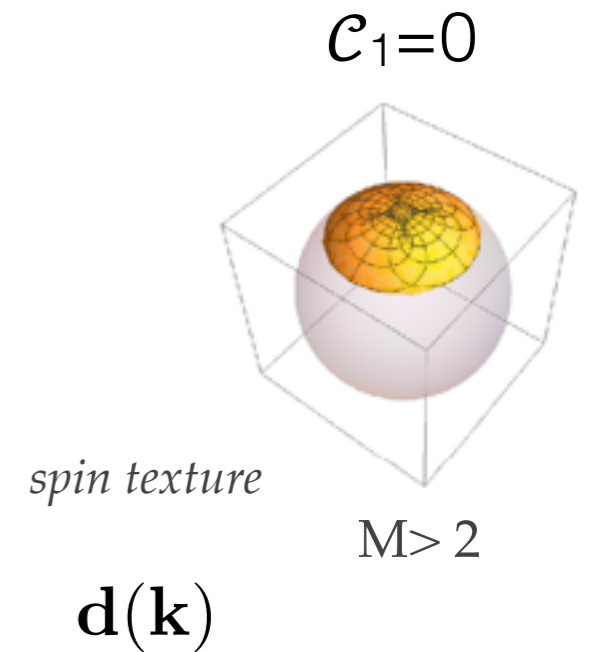
**CdTe/HgTe quantum wells.**



# Topological QPT

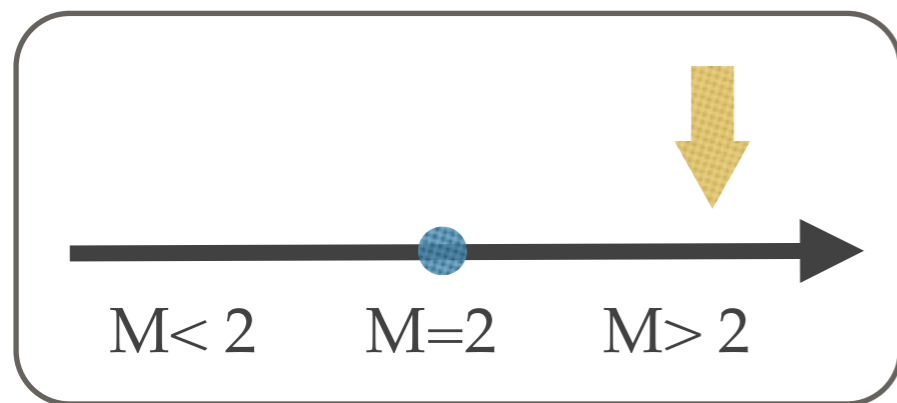
BHZ description of topological transition:

$$\mathbf{h}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau}$$

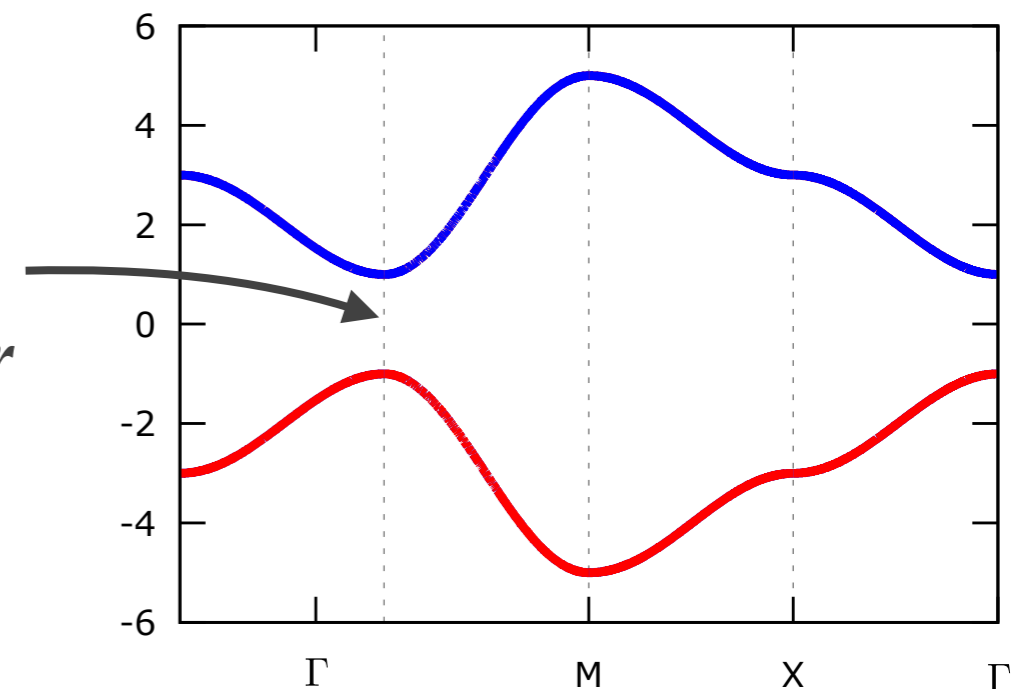


*Continuous* Topological Quantum Phase Transition

*band structure evolves smoothly with control parameters...*



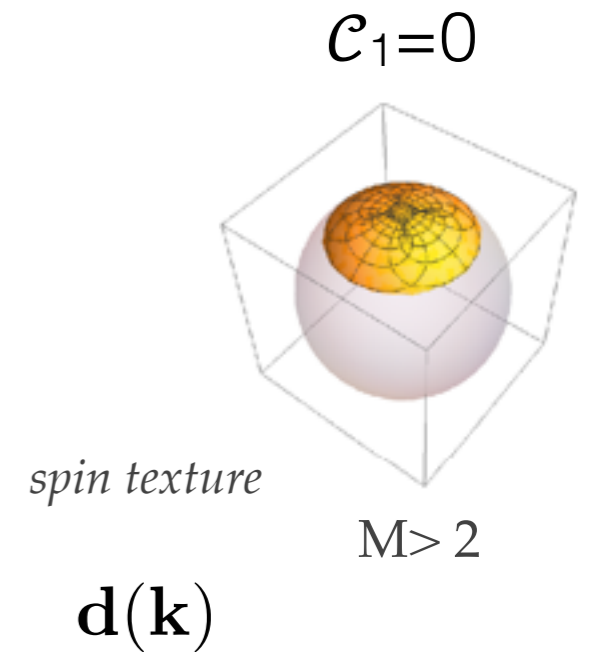
*trivial  
band insulator*



# Topological QPT

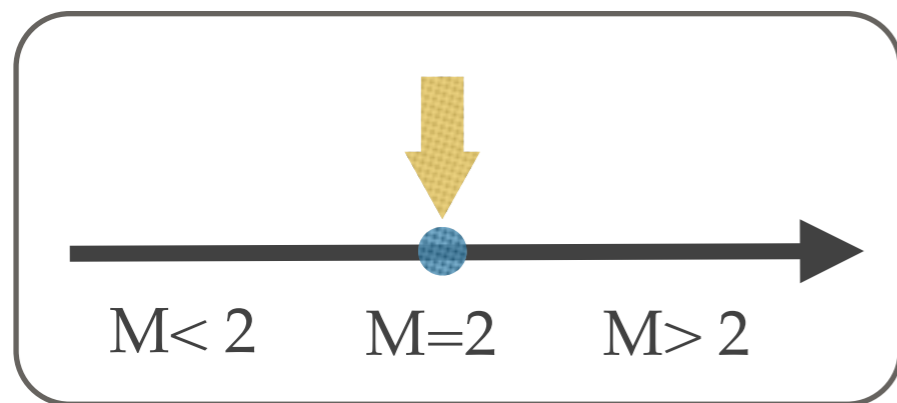
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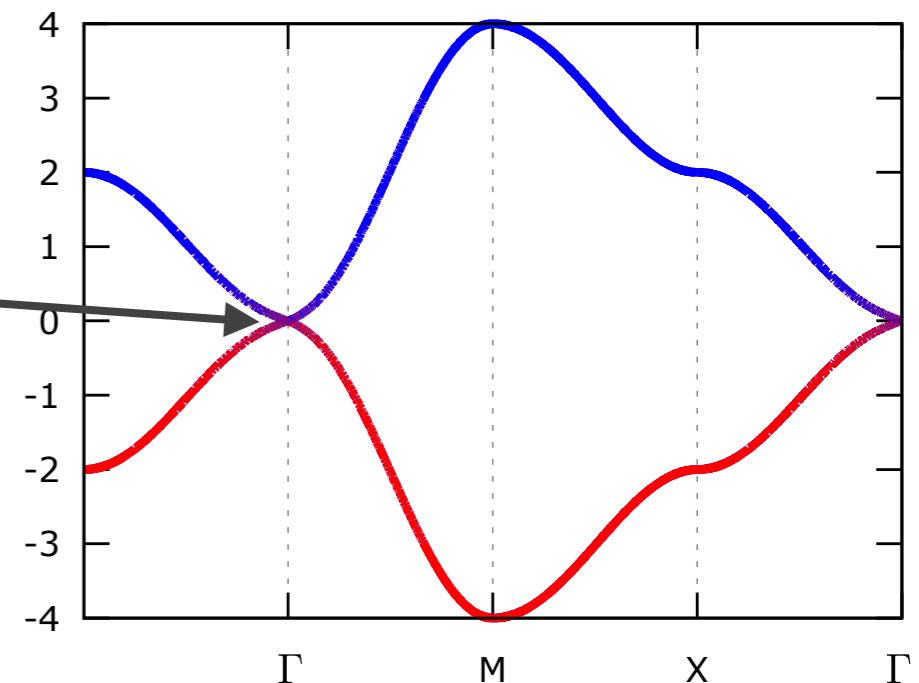


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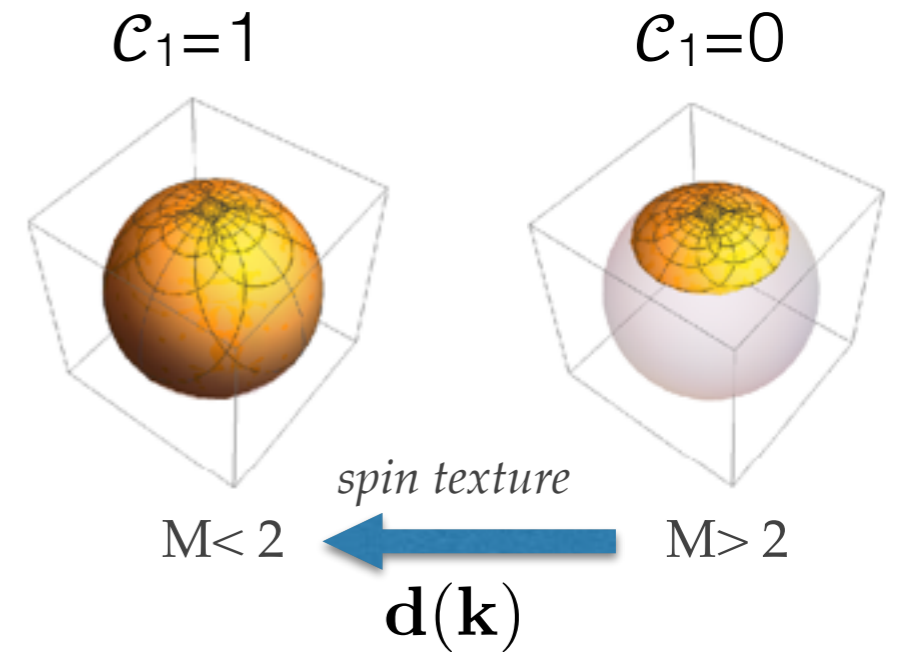
*Dirac cone  
semi-metal*



# Topological QPT

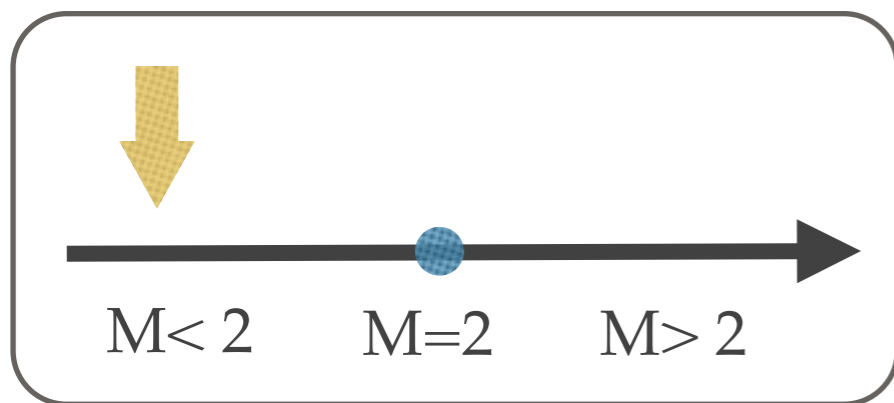
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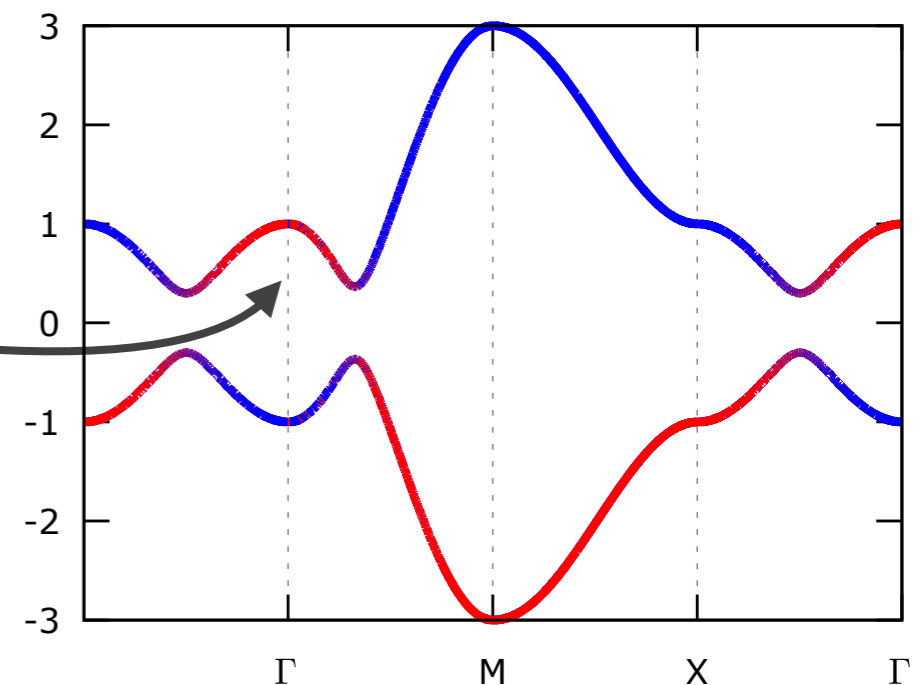


*Continuous* Topological Quantum Phase Transition

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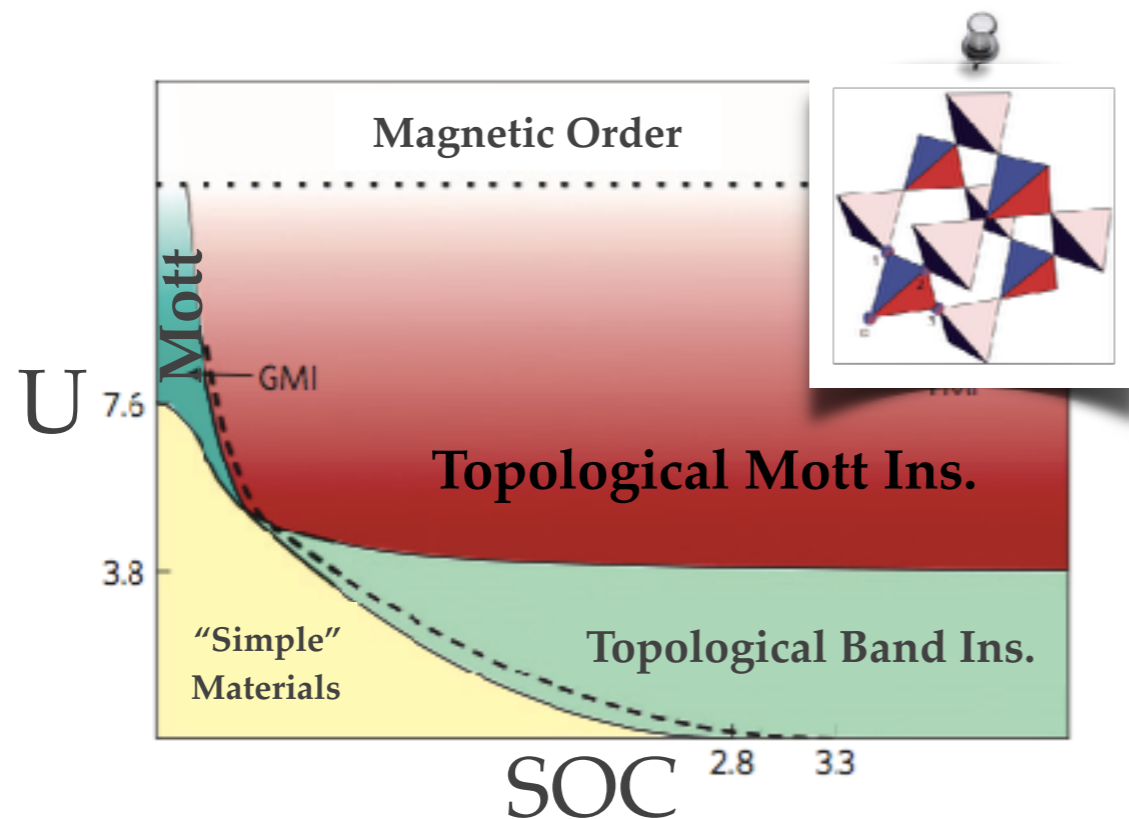


*band inversion*  
*QSH insulator*



# What about the interaction?

Quest for larger **SOC**...heavy elements compounds (5d/4,5f)



*Hexaborides Sm/PuB<sub>6</sub>,*

*Ir-based pyrochlores: Sr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>, etc..*

*Dzero et al. PRL 2010*

*D. Pesin, L. Balents, NP 2010*

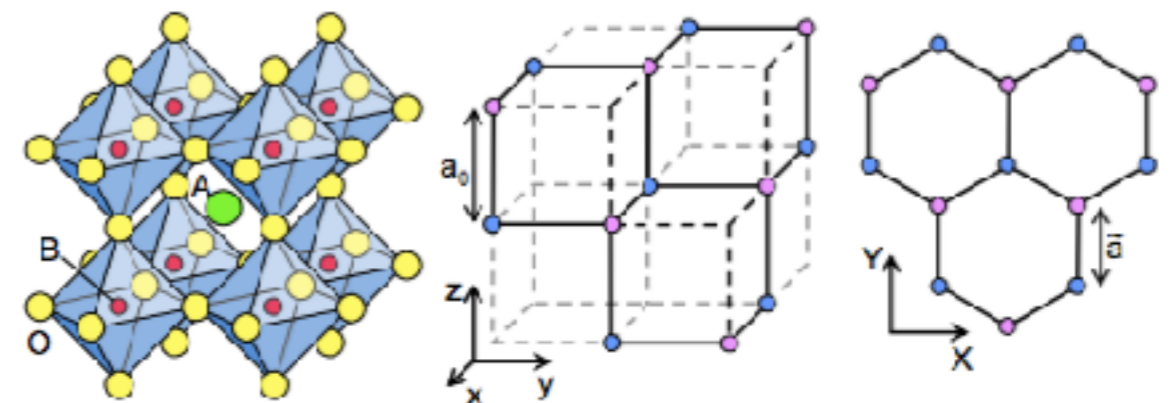
*Hohenadler, Assad. Journal of Phys. 2013*

*Deng et al PRL 2013*

*New materials?*

Engineering correlated TI:  
*Transition Metal Oxides Heterostructures*

*D. Xiao et al. Nat. Comm. 2011*



*LaAuO<sub>3</sub>*

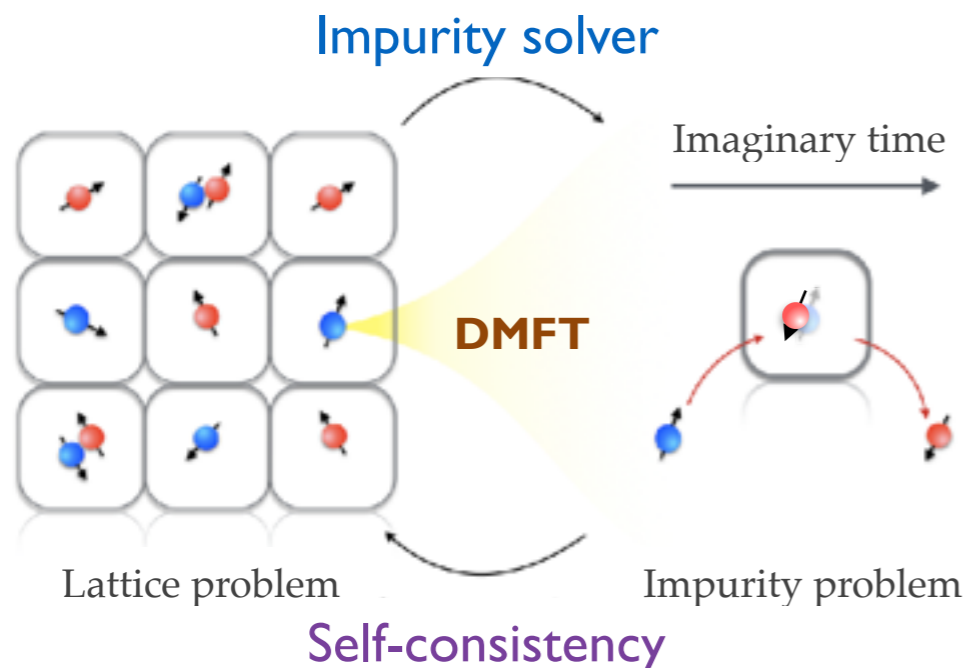


# DMFT solution

## Dynamical Mean-Field Theory

non-perturbative solution of the interacting problem

**Idea:** Reduce the interacting lattice problem to a *self-consistent* impurity problem



### Advantages:

- + local quantum physics (*beyond Hartree-Fock*).
- + non-perturbative in the interaction
- + access to topological invariant

### Drawbacks:

- neglects spatial fluctuations
- computational demanding...

solve using **Exact Diagonalization & CTQMC**

Obtain *dynamical* (non-scalar) self-energy.  
Describes the effects of interaction.

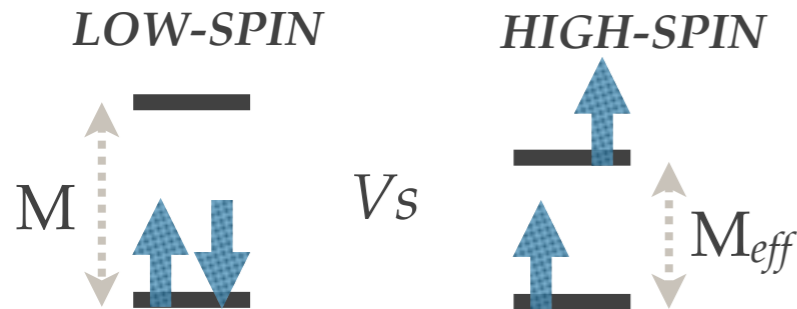
$$\hat{\Sigma}(\omega) = \text{Re}\Sigma(\omega)\tau_z + \text{Im}\Sigma(\omega)\tau_0$$

# BHZ - Interaction

AA et al PRL 2015

AA et al PRB 2016

## BHZ effective minimal model + multi-orbital interactions

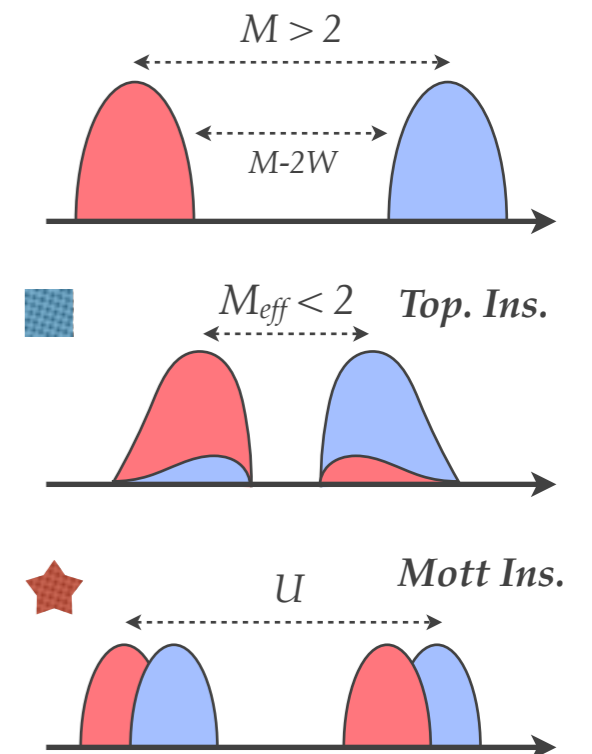
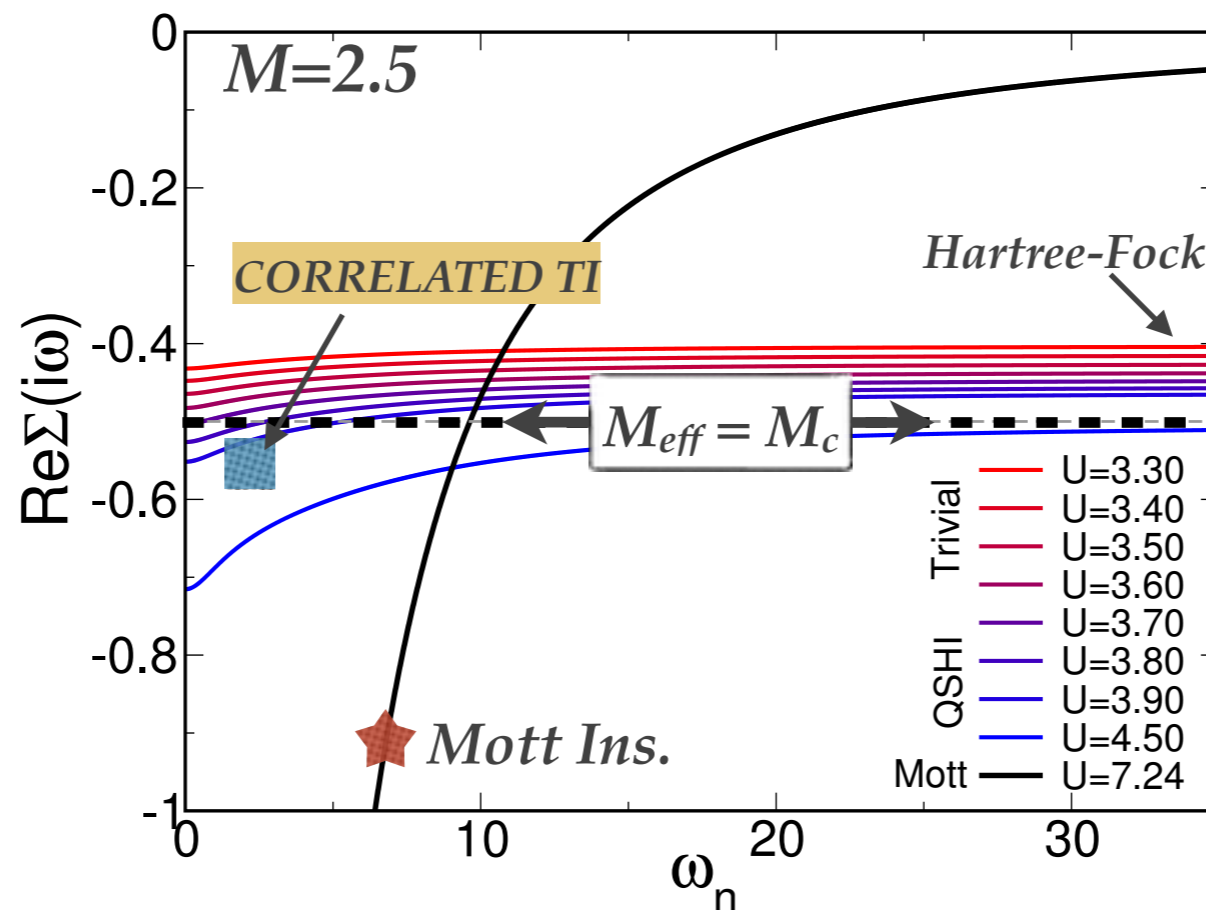


$$H_I = (U - J_H) \frac{N(N-1)}{2} - J_H \left( \frac{N^2}{4} + \frac{S_z^2}{2} - 2T_z^2 \right)$$

Effective reduction of the Mass term:  $M_{\text{eff}} = M + \text{Tr}[\tau_z \hat{\Sigma}(0)]/2$

Interaction driven TI

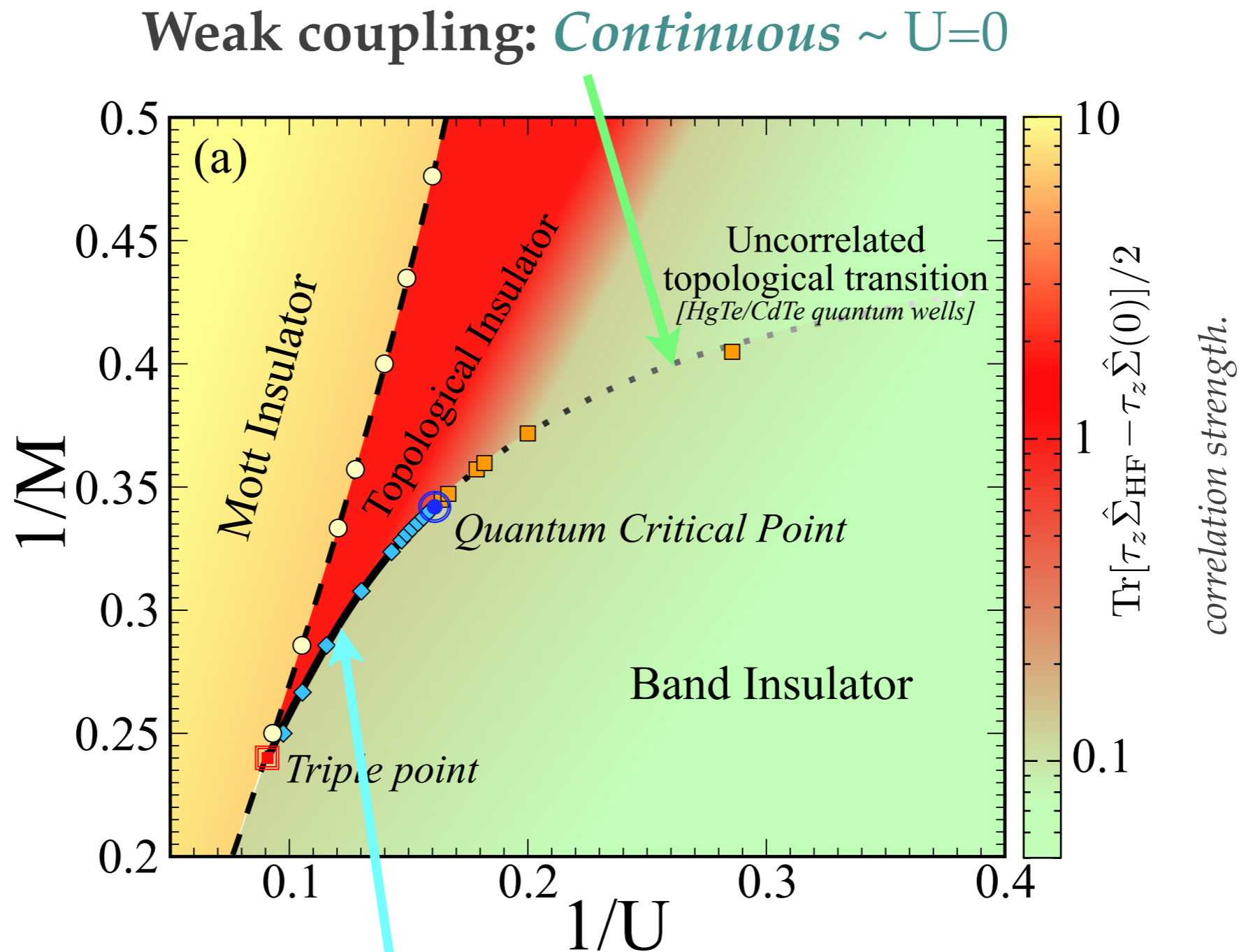
Mott phase at large U



# Correlated QSHI

AA et al PRL 2015

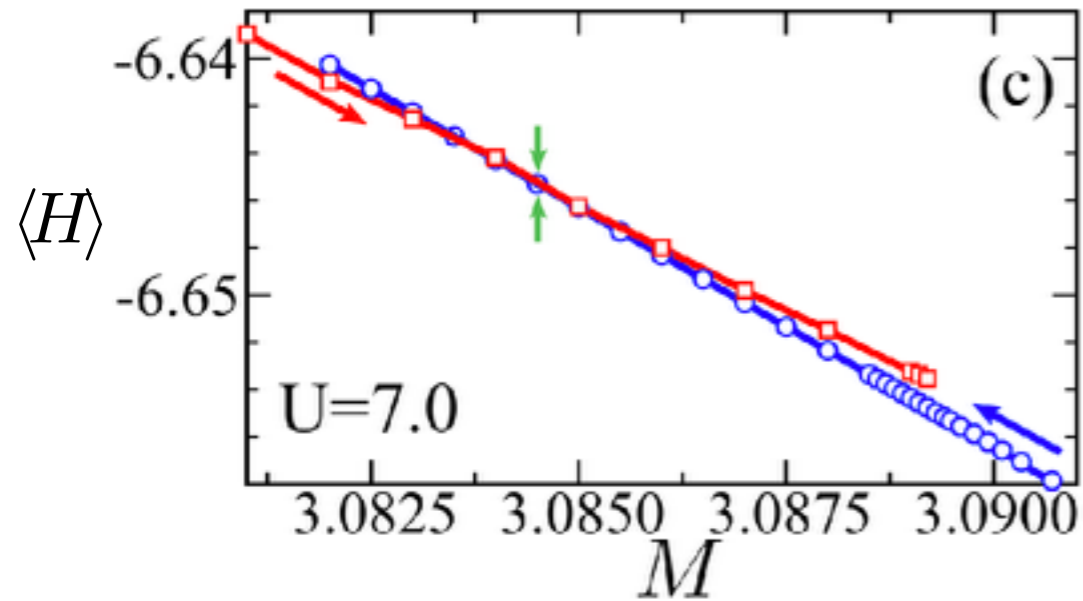
Phase diagram M-U (*flipped view*).



Strong coupling: **1<sup>st</sup> order TQPT** correlated many-body character

# Correlated QSHI

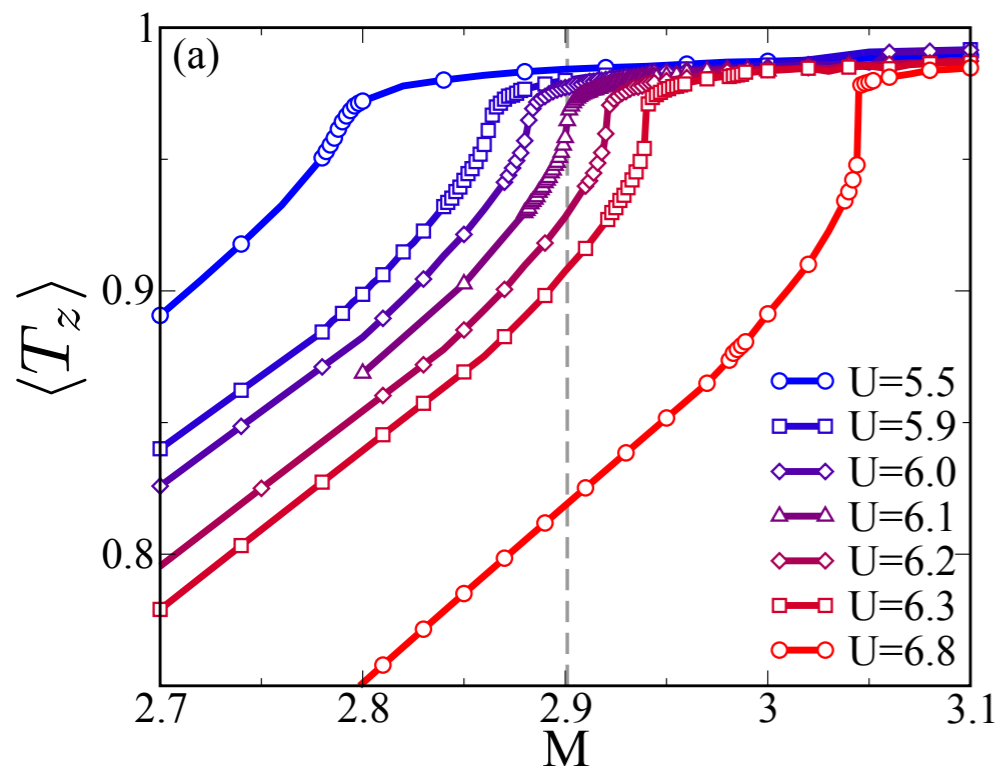
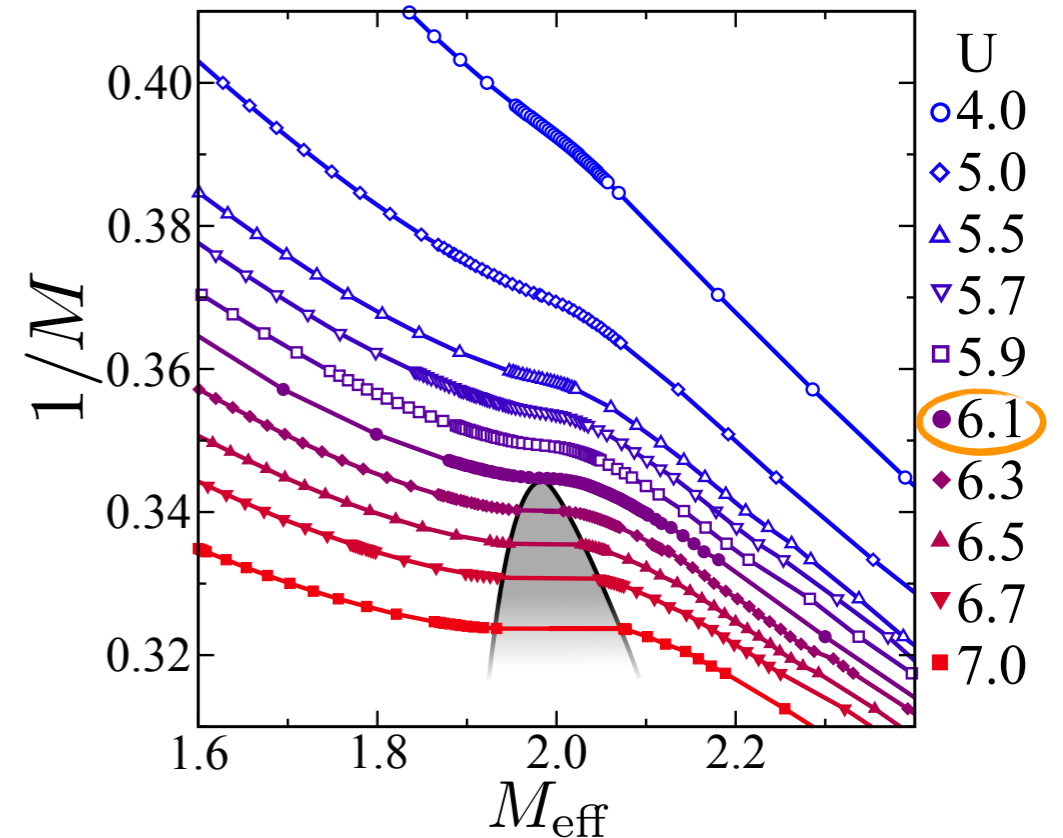
AA et al PRL 2015



Metastable states hallmark of 1<sup>st</sup> transition.

A clear picture from the *iso-U* curves

$$\Delta M_{\text{eff}} = M_{\text{eff}}(BI) - M_{\text{eff}}(QSH)$$



Diverging orbital compressibility at  $U=U_c$

$$\kappa = \partial \langle T_z \rangle / \partial M$$

Experimental accessible quantities marking the TQPT.

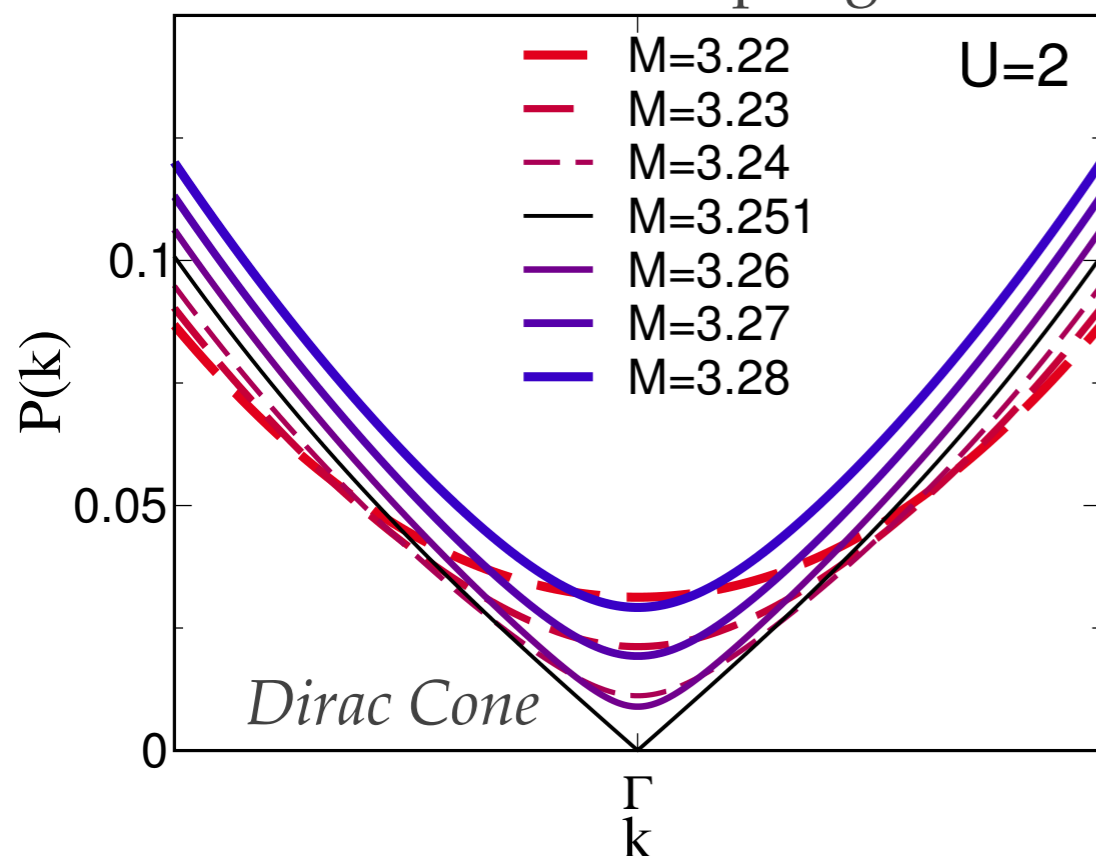
# Absence of gap closure

AA et al PRL 2015

AA et al PRB 2016

*Breakdown of the gap-less TQPT paradigm...*

Weak Coupling

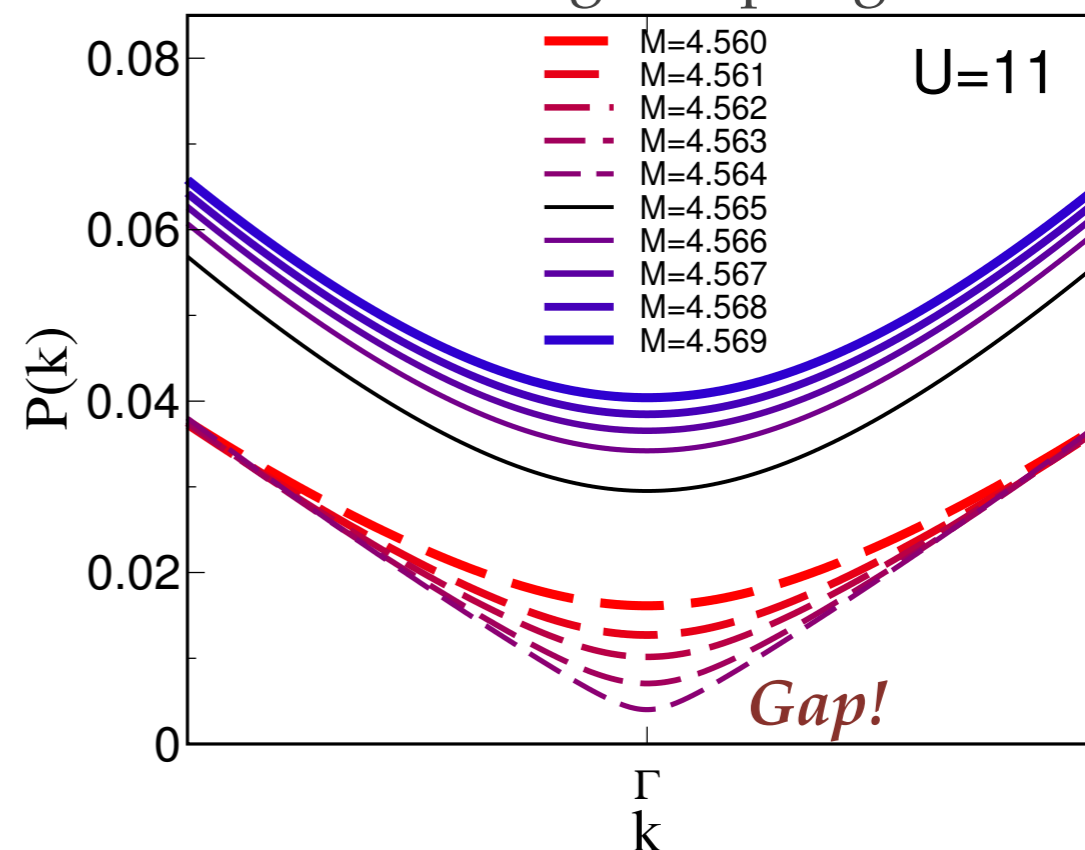


$$U < U_c$$

The transition to a topological state occurs thru band-gap closing.

*Dirac cone* formation.

Strong Coupling



$$U > U_c$$

*No* gap-closing

*No* suppression of any symmetries protecting the topological state.

# Correlated edge states

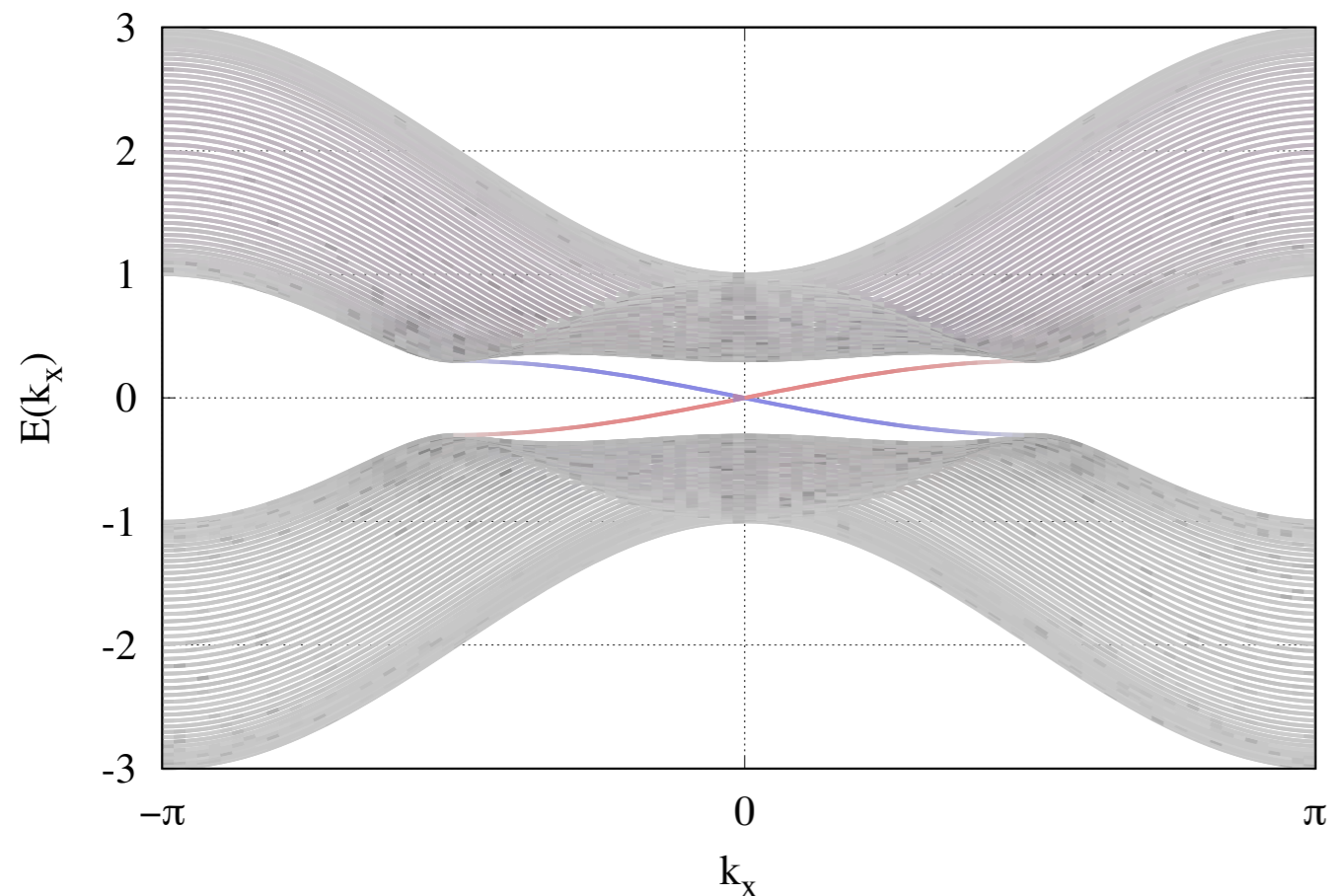
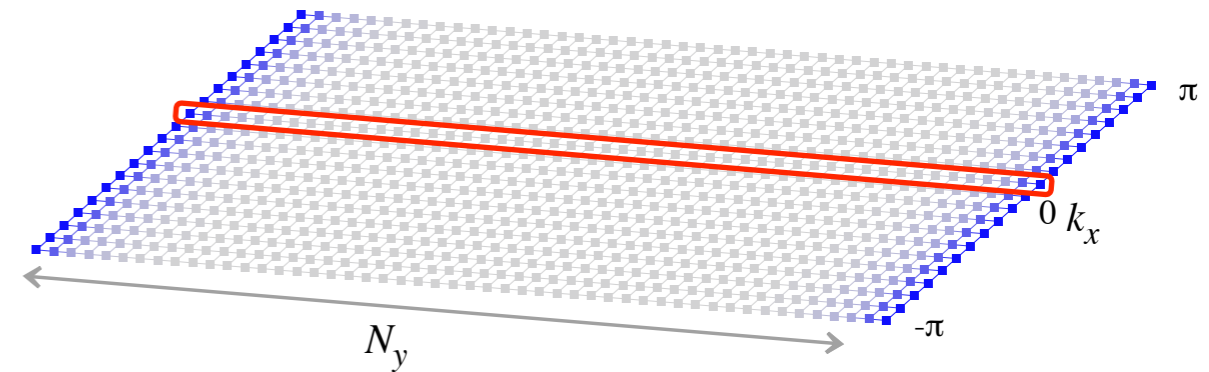
AA et al. PRB 2017

Consider a 2D stripe.

$$H = \sum_{k_x y y'} \Psi_{k_x y}^+ \mathbf{M}(k_x) \delta_{y y'} \Psi_{k_x y'} + \sum_{k_x y y'} \left( \Psi_{k_x y}^+ \mathbf{T} \delta_{y+1 y'} \Psi_{k_x y'} + H.c. \right)$$

$$\mathbf{M} = [M - 2t \cos k_x] \Gamma_5 + \lambda \sin k_x \Gamma_x$$

$$\mathbf{T} = -t \Gamma_5 + i \frac{\lambda}{2} \Gamma_y$$



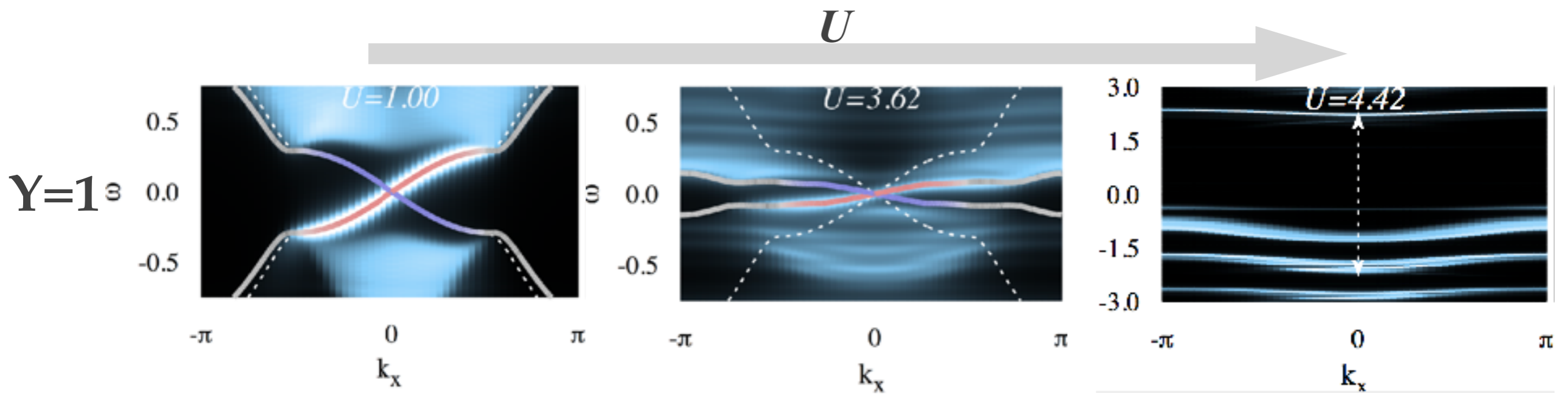
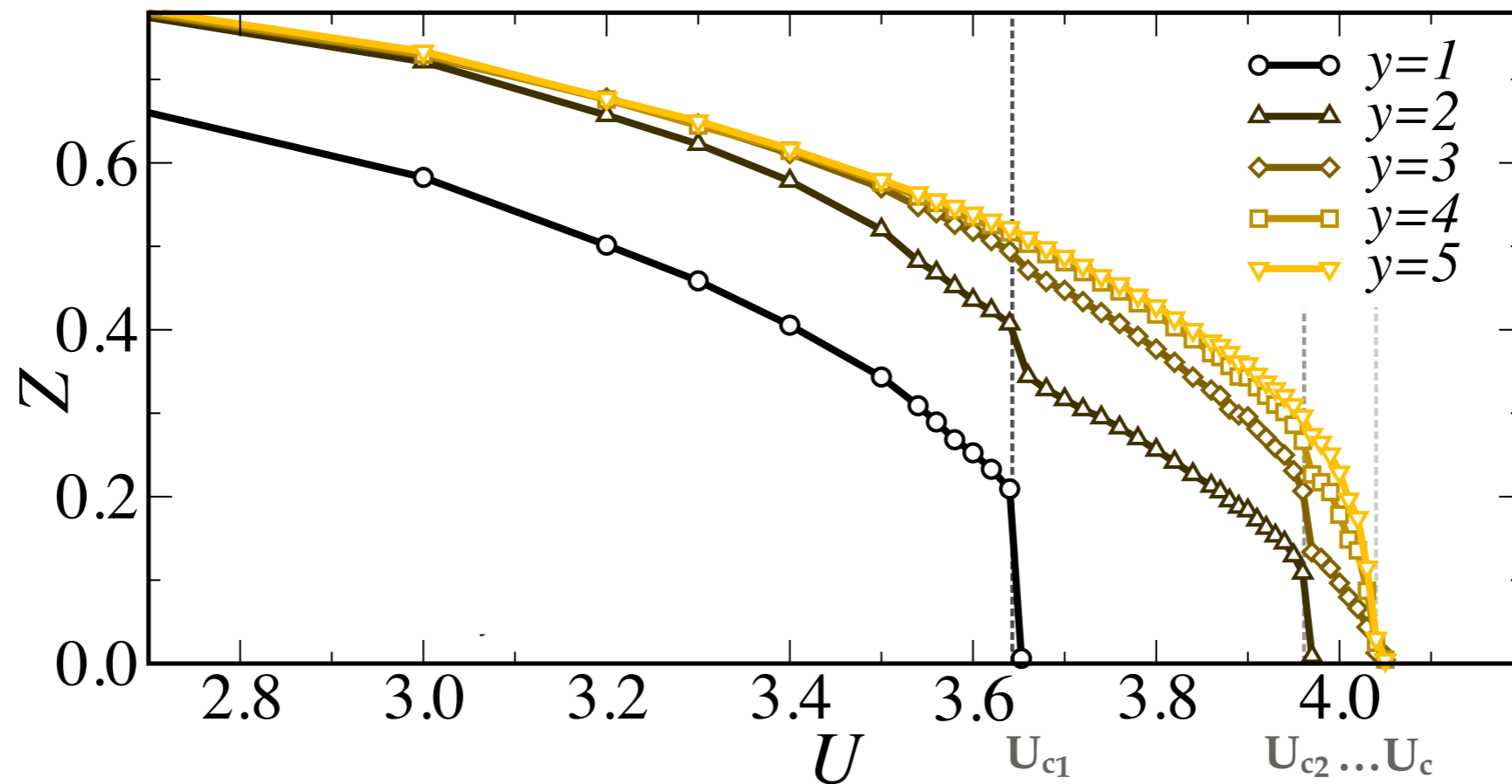
Helical gapless states localized at the edges.

What's the effects of strong correlation on the 2D stripe?

# Correlated edge states

AA et al. PRB 2017

Sequence of transitions to reach the Mott state.

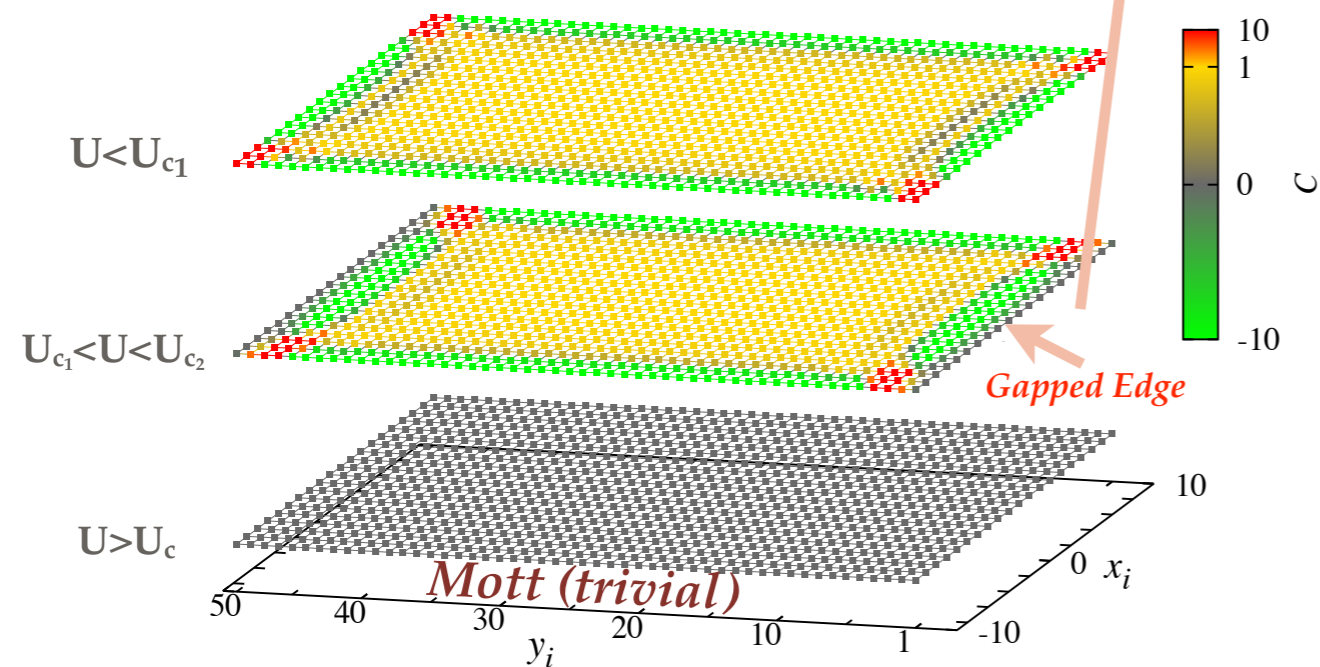
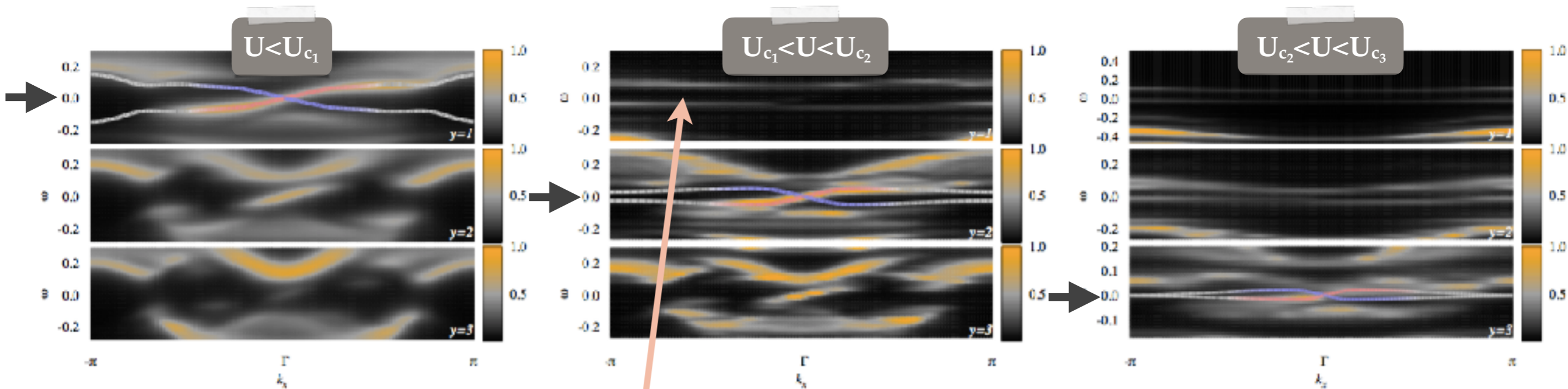


# Correlated edge states

AA et al. PRB 2017

What's the fate of the edge states?

*Bulk compression*  $\longrightarrow$  *Edge state reconstruction*



Topological properties with OBC:  
**Local Chern Marker**

$$\mathcal{C}_\sigma(\mathbf{r}) = 2\pi i \langle \mathbf{r} | \hat{x}_P^\sigma \hat{y}_Q^\sigma - \hat{y}_P^\sigma \hat{x}_Q^\sigma | \mathbf{r} \rangle$$

$$\mathbb{Z}_2 = (\mathcal{C}_\uparrow - \mathcal{C}_\downarrow) / 2$$



# Conclusions.

- Topological States can be favoured by strong interaction. *AA et al. PRL 2015*  
*AA et al. PRB 2016*
- *Emergent thermodynamic character: 1<sup>st</sup> order transition.*
- New paradigm for TQPT : **no** gap closing but **no** symmetry breaking!
- Correlation driven edge states reconstruction. *AA et al. PRB 2017*  
*AA et al. in preparation 2017*

## *Outlook...*

- *Break TRS or IS: correlation effects in Weyl SM.*
- *Interplay of strong interaction and SOC: from models to real materials.*
- *Topological Mott Insulators.*
- *Condensed matter realization of excitations beyond “standard model”*