Insulator to Superfluid/Superconductor Transition in 2 dimensions

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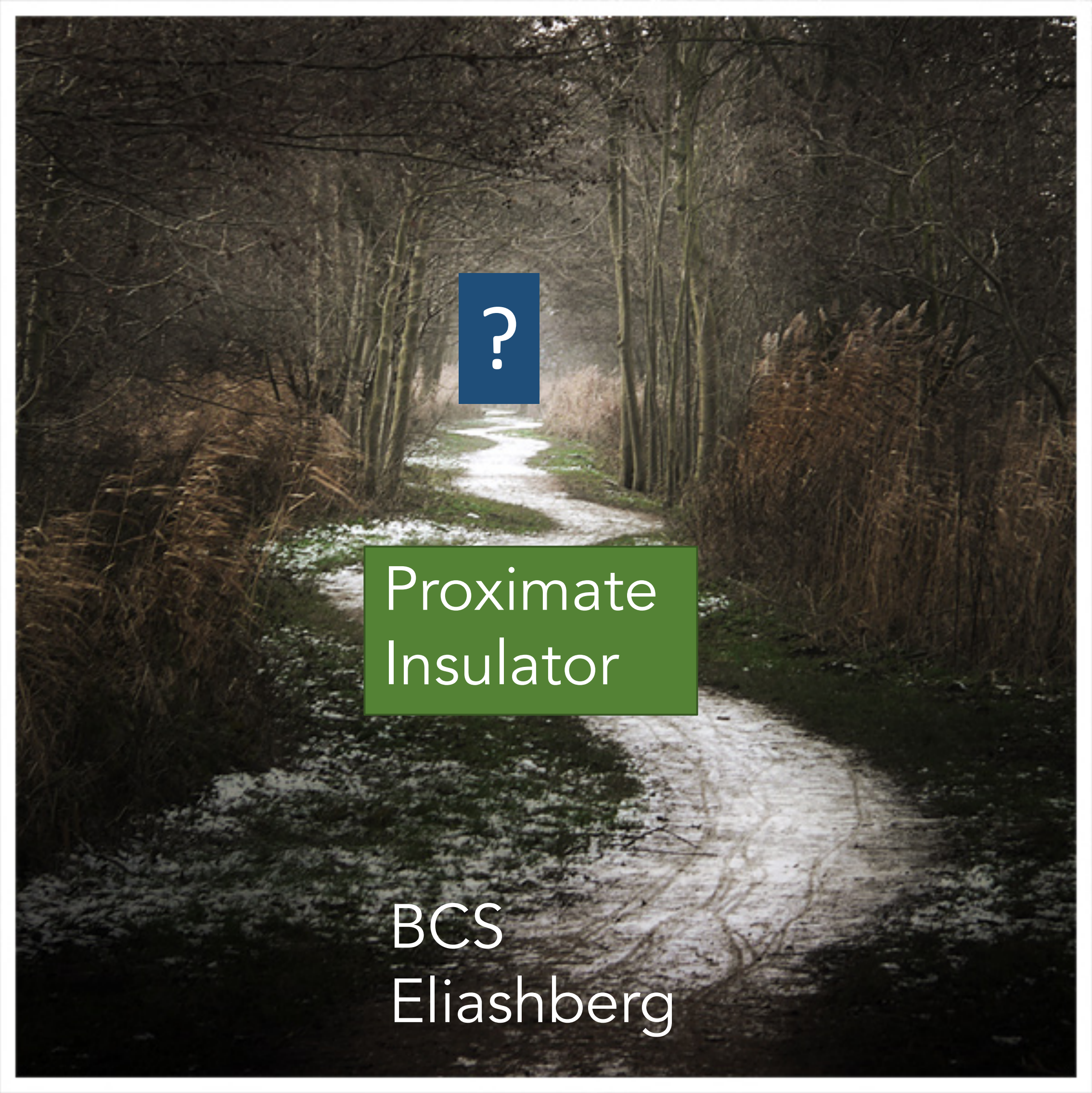
Can an insulator become a SC?
How does SC arise when there is ***no*** Fermi surface?

Roadmap:

Semi-Metal→ Topological SC

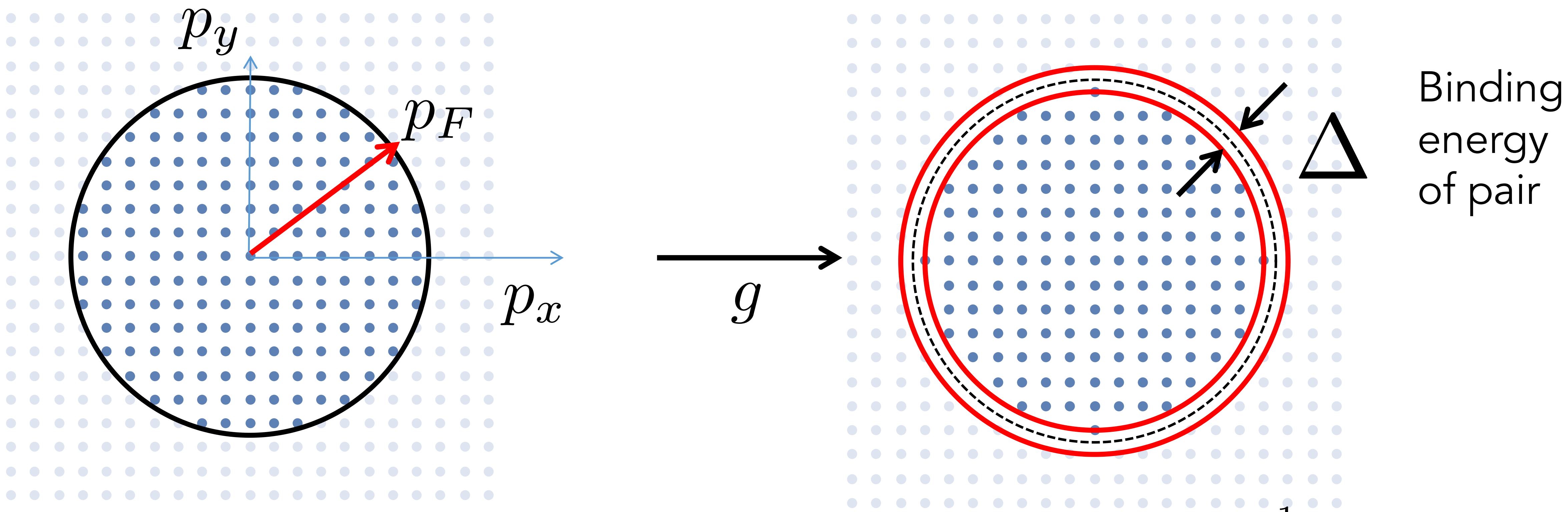
Insulator→ SC

Metal→ SC



Metal: Electron Waves

Instability of Fermi surface



$p = h/\lambda$ → wave length of electron wave

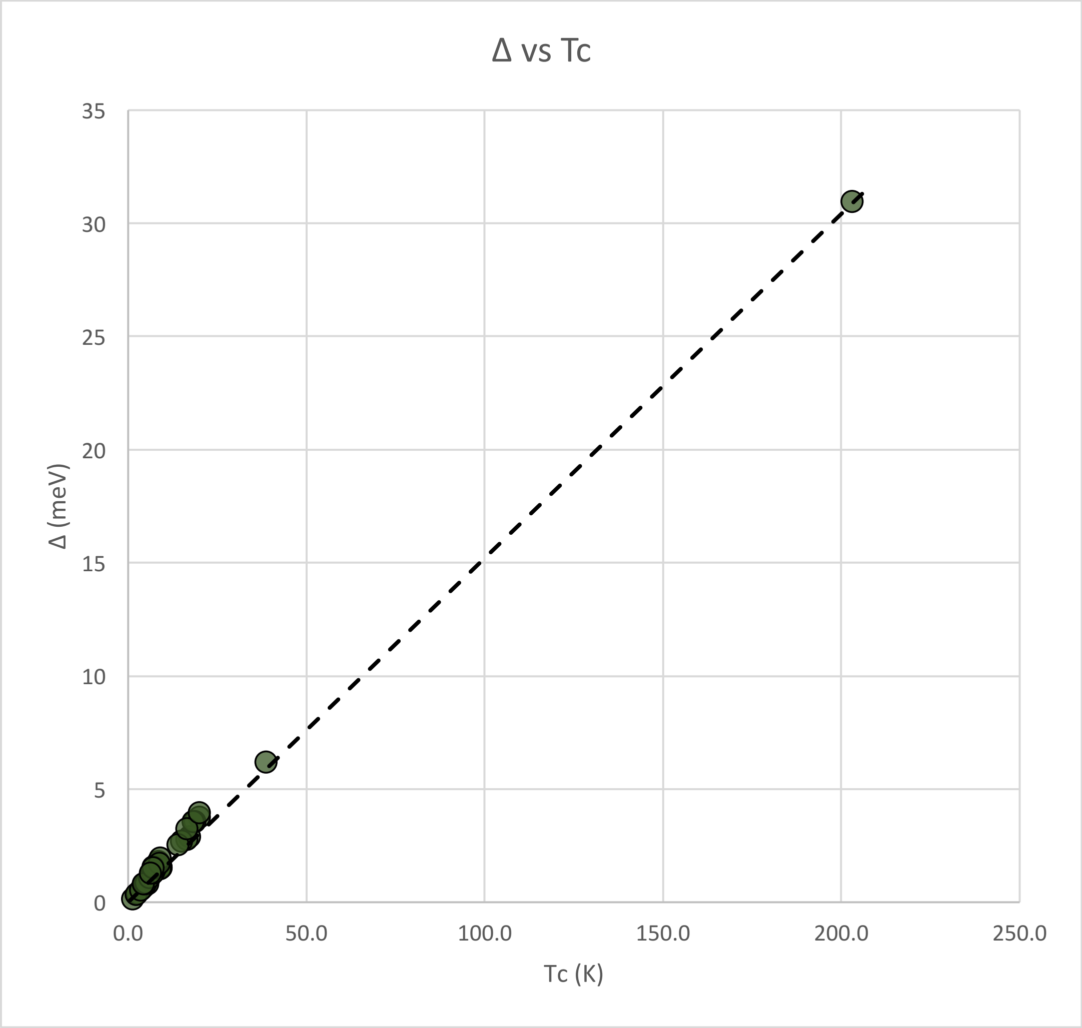
$$\Delta \sim E_m \, e^{-\frac{1}{g}}$$

$$E_F = \frac{p_F^2}{2m}$$ Fermi energy

$$\Delta < E_m < E_F$$

BCS Prediction $\Delta/T_c = 1.8$

Binding energy of pair or Energy gap in single particle spectrum @T=0

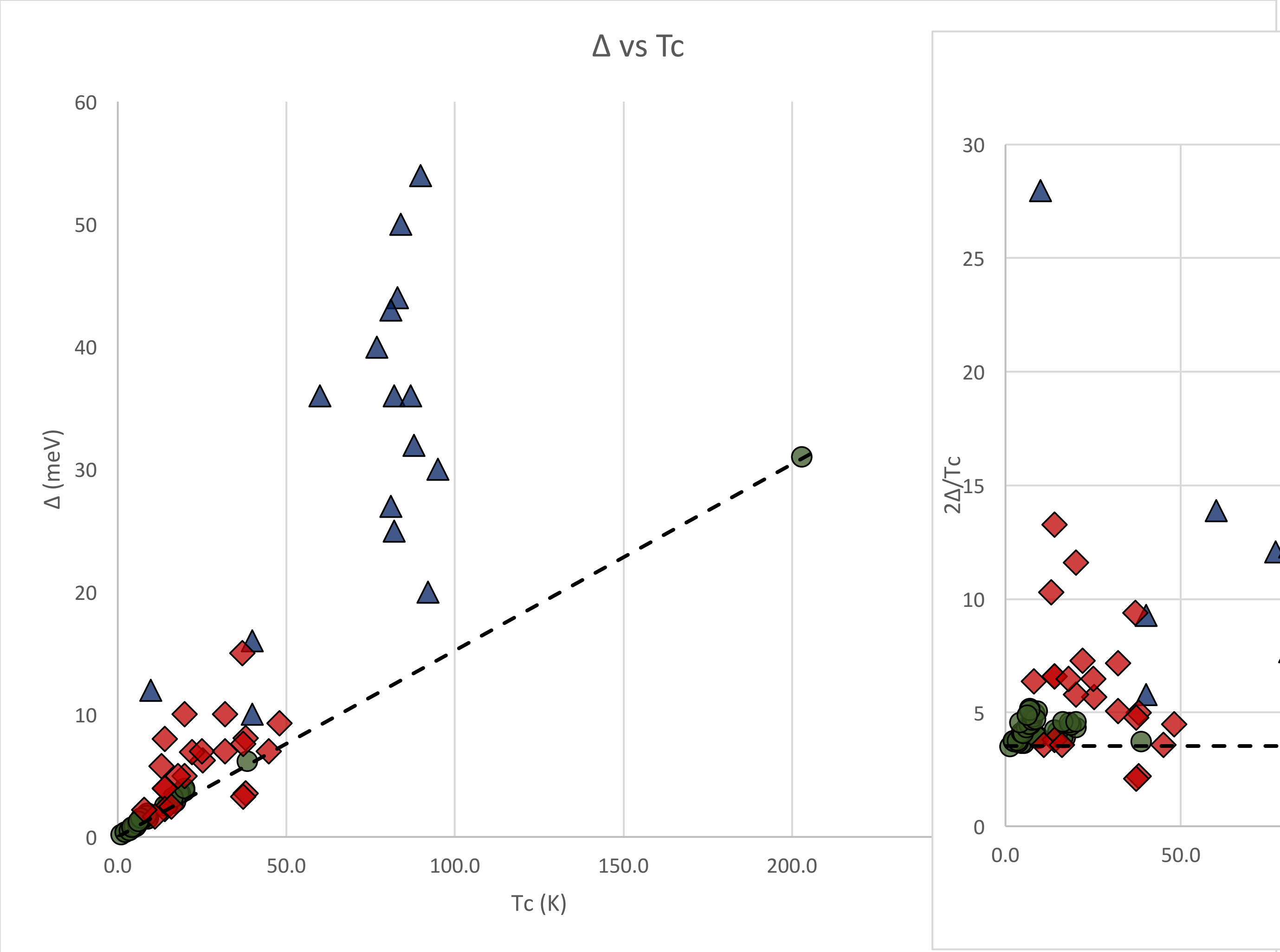


Transition temp

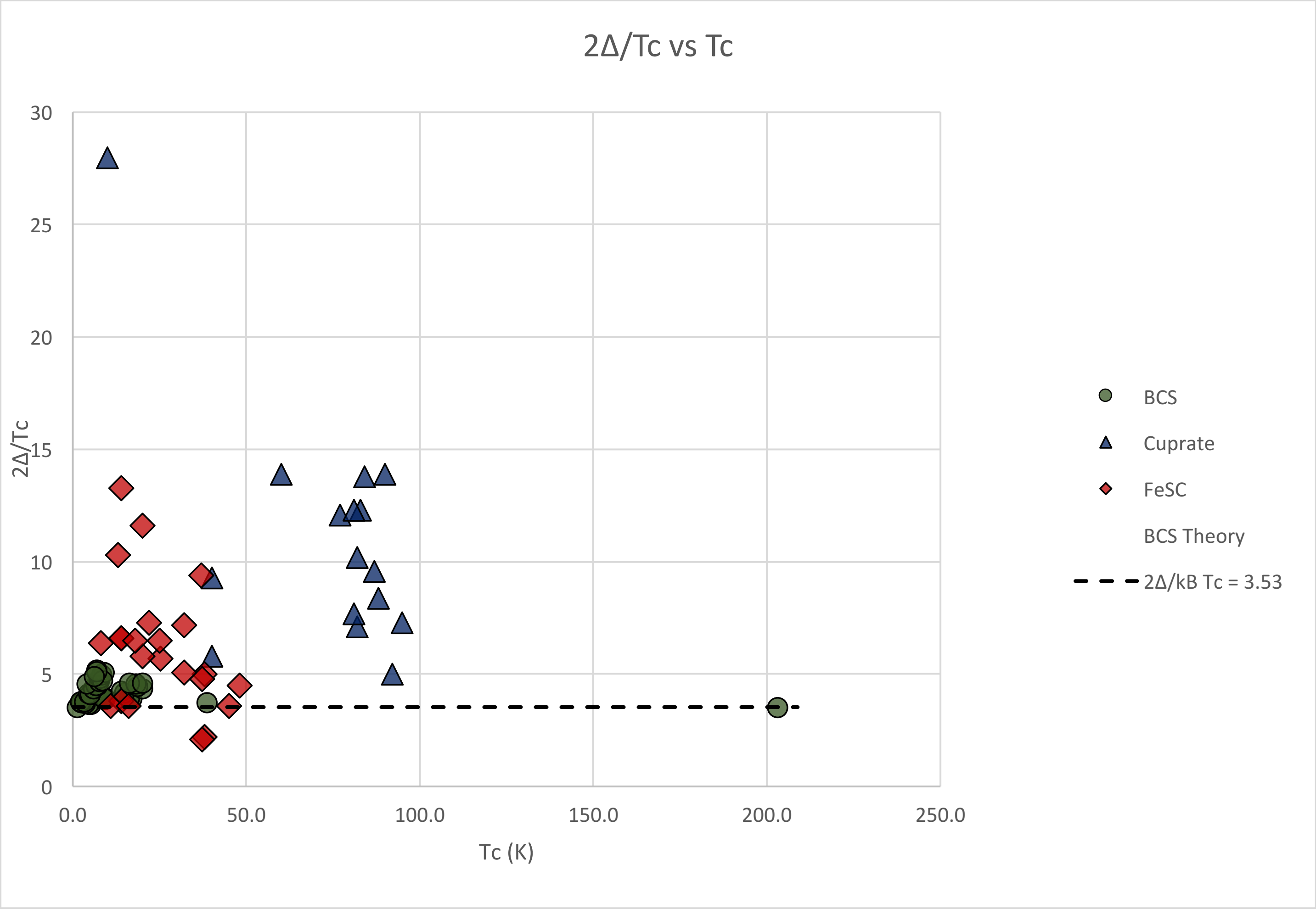
	Tc(K)
Al	1.2
Pb	7.2
Nb	9.2
Nb_3Ge	20.0
MgB2	38.6
H_3S	203.0

1 meV ~ 10K

GAP vs Tc



Δ vs Tc
Δ (meV)
60
50
40
30
20
10
0
0.0
50.0
100.0
150.0
200.0
Tc (K)



2Δ/Tc vs Tc
2Δ/Tc
30
25
20
15
10
5
0
0.0
50.0
100.0
150.0
200.0
250.0
Tc (K)
BCS
Cuprate
FeSC
BCS Theory
2Δ/kB Tc = 3.53

Another way the BCS paradigm can break down if

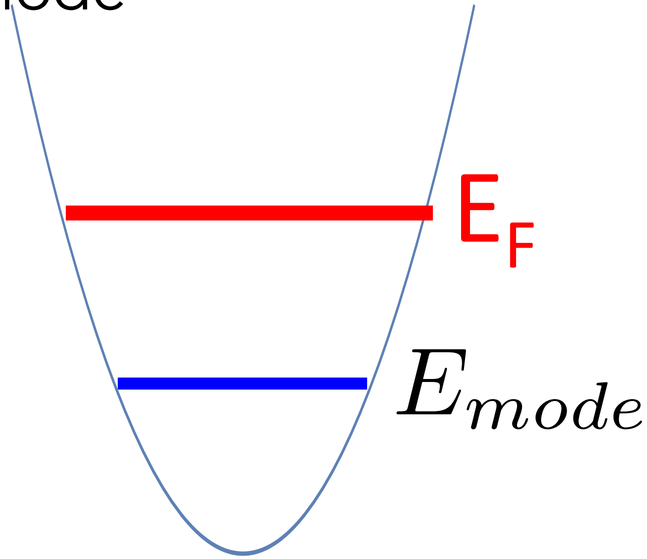
$$T_c \ll E_{mode} \ll E_F$$

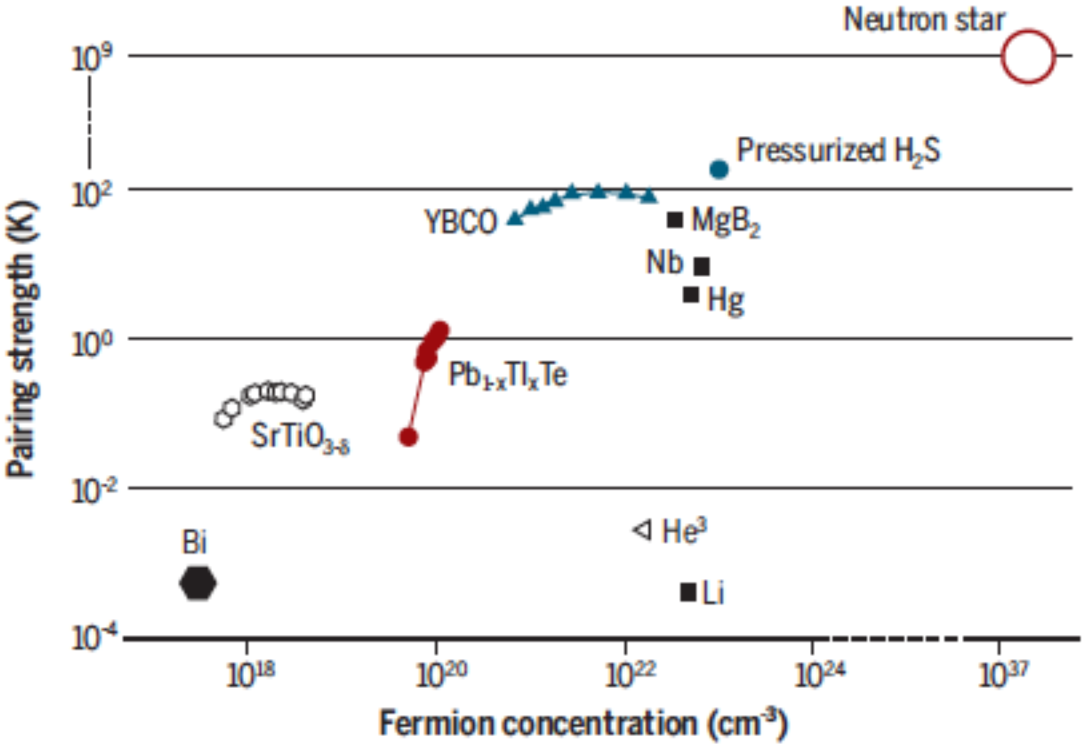
(1) Strong coupling to glue: Fermi sphere greatly perturbed

(2) non-adiabatic limit: electrons are slower than the mode

$$\tau_e > \tau_{mode}$$

$$E_F < E_{mode}$$





Bismuth: E_F=25 meV

$E_{mode=}$12 meV

Tc= 0.5 mK~0.05 meV

Prakash, Kumar, Thamizhavel, Ramakrishnan, Science 355, 52–55 (2017)

How can the BCS paradigm break down?

(1) Strong coupling to glue:

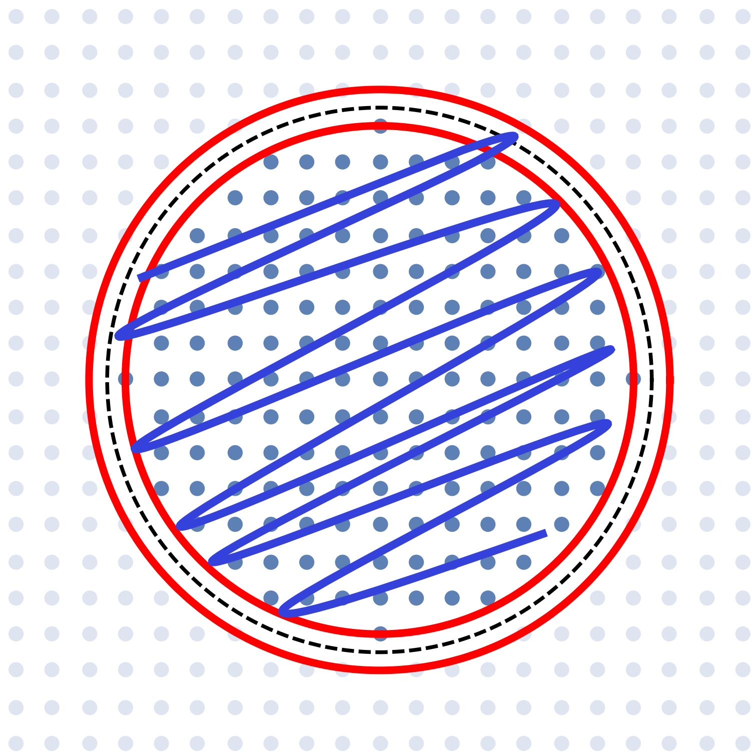
Fermi sphere greatly perturbed

$$T_c \not\propto \Delta$$

(2) Non-adiabatic limit:

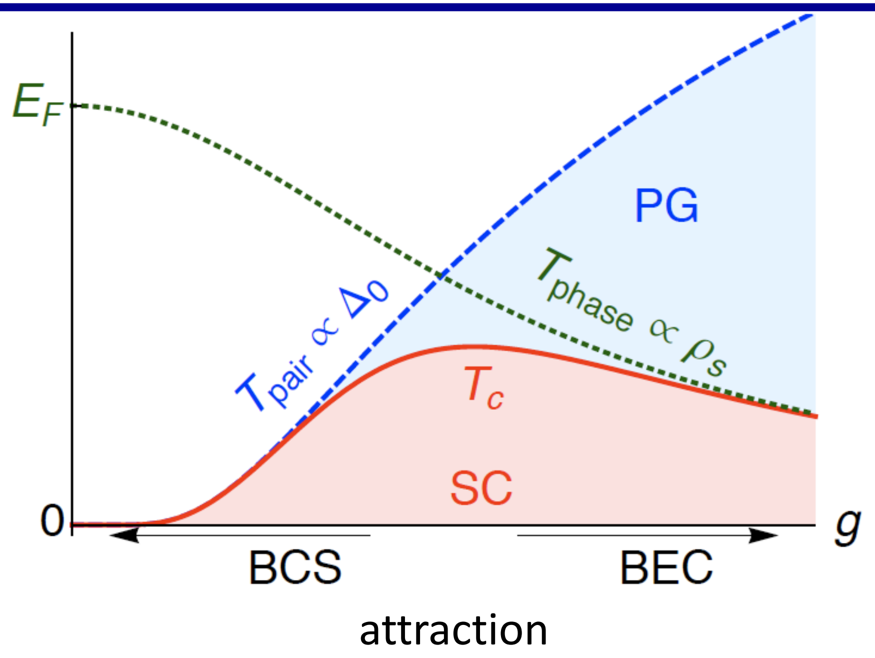
electrons are ***slower*** than the mode

$$\Delta \not\propto E_m \, e^{-\frac{1}{g}}$$

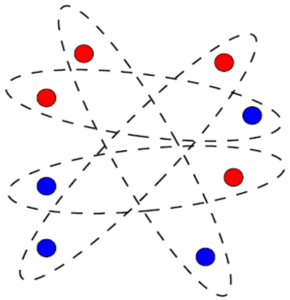


Strong glue: BCS-BEC Crossover

weak attraction:
pairing and coherence occur at the same temperature



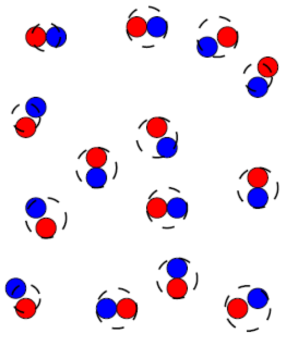
strong attraction:
pairing and coherence occur at different temperatures



BCS limit

- cooperative Cooper pairing
- pair size $\gg k_F^{-1}$

$$T_c = \min(\Delta_0, \rho_S)$$



BEC limit

- tightly bound molecules
- pair size $\ll k_F^{-1}$

M. Randeria and E. Taylor,
Ann. Rev. Cond. Mat. Phys. **5**, 209 (2014)

Superfluid density and stiffness

n_s= superfluid number density
ρ_s= sf mass density
ω_s= sf plasma frequency scale

$$\frac{4\pi n_s e^2}{m^*} = \omega_{p,s}^2 = \frac{c^2}{\lambda_L^2}$$

London penetration depth

$$F = \frac{1}{2} D_s \int d^d x |\nabla\theta|^2$$

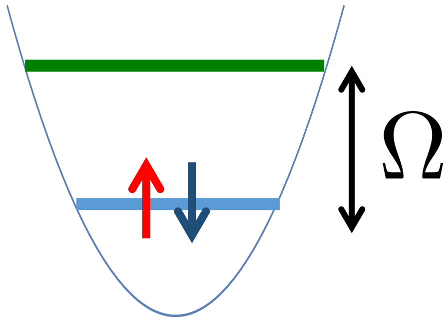
$$[D_s] = \frac{Energy}{L^{d-2}}$$

directly related to the spectral weight in the delta function in the optical conductivity

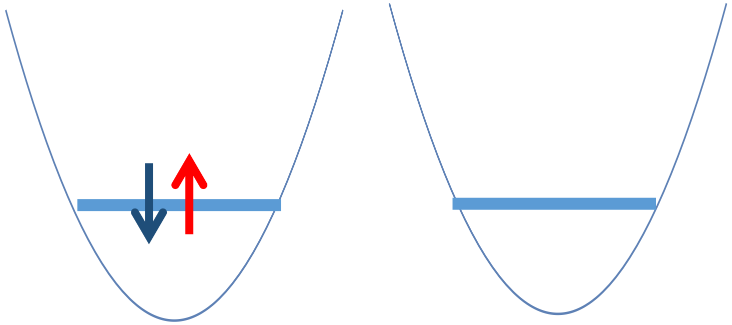
dim=2	Layered	dim=3
$D_s \sim \frac{\hbar^2}{m^*} n_s$	$D_s \sim \frac{\hbar^2 n_s}{m^* d}$	$D_s \sim \frac{\hbar^2 n_s^{2/3}}{m^*}$

Insulator: charge cannot move

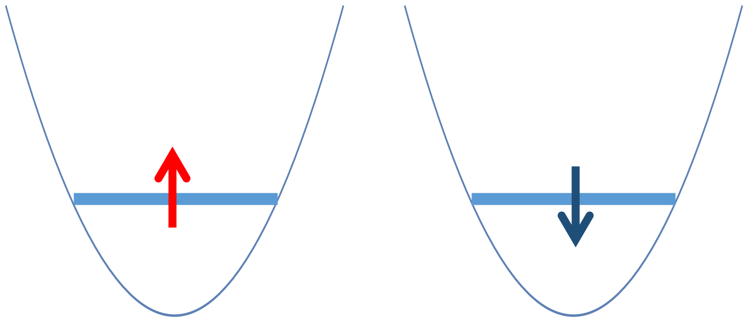
Band
Pauli



C-Mott
Attraction



S-Mott
Repulsion

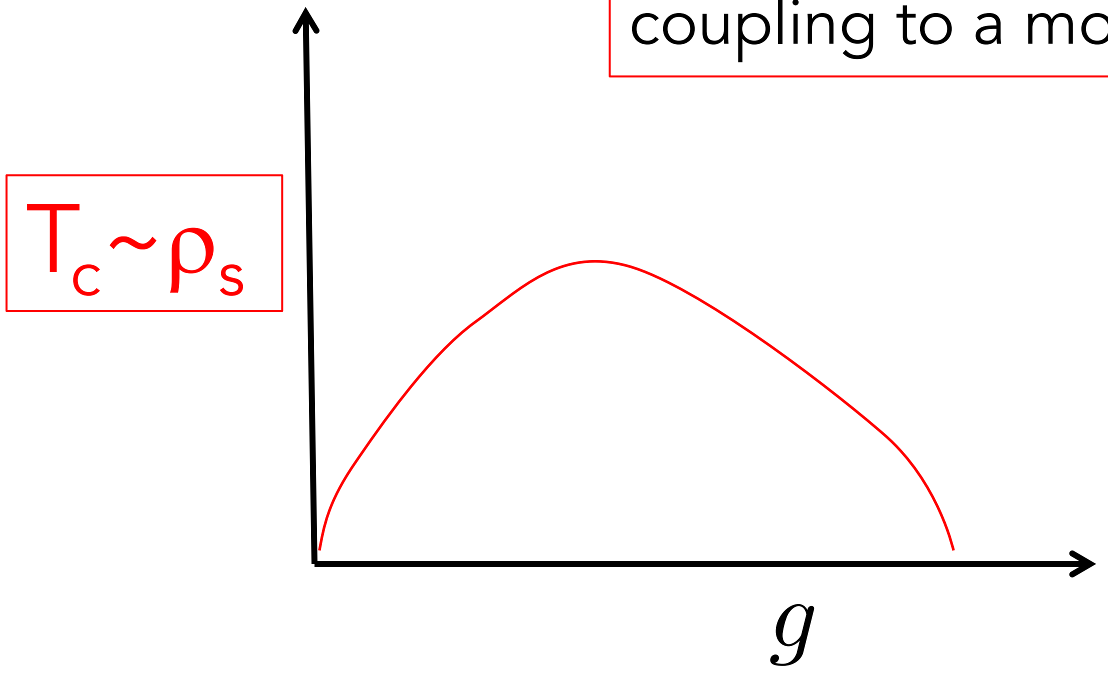


singlet $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

+ many other examples of insulators including disordered (localized) insulators

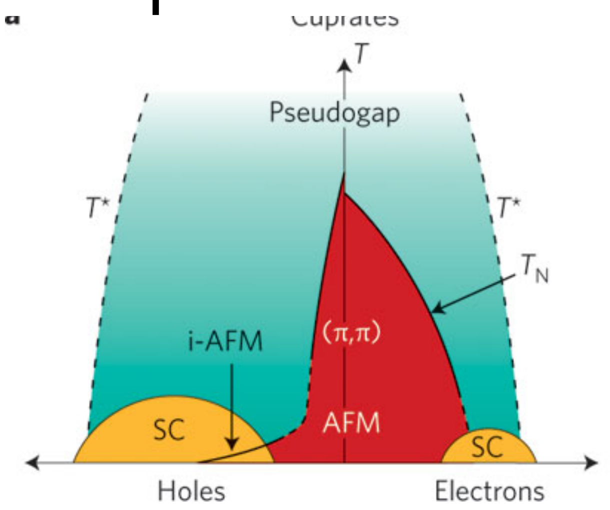
Superconductivity domes

Tc scale set by electronic energies not necessarily by coupling to a mode

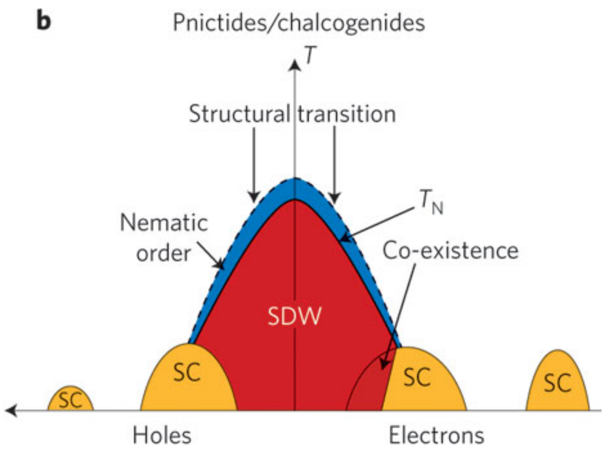


g: Driven by pressure, magnetic field, doping, gating

Cuprates

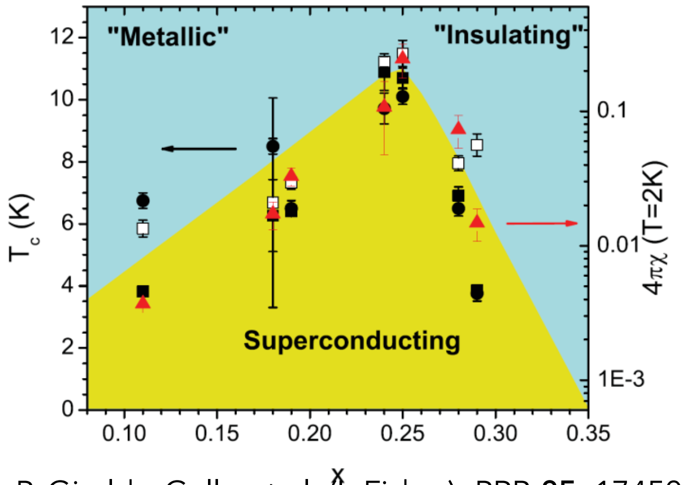


Pnictides



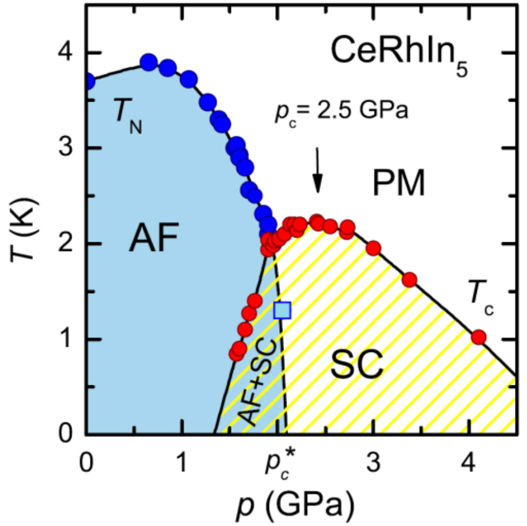
D. N. Basov & Andrey V. Chubukov, Nature Phys. **7**, 272 (2011)

$BaPb_{1-x}Bi_xO_3$



P. Giraldo-Gallo et al. (I. Fisher), PRB **85**, 174503 (2012),; Nature Comm. **6**, 8231 (2015)

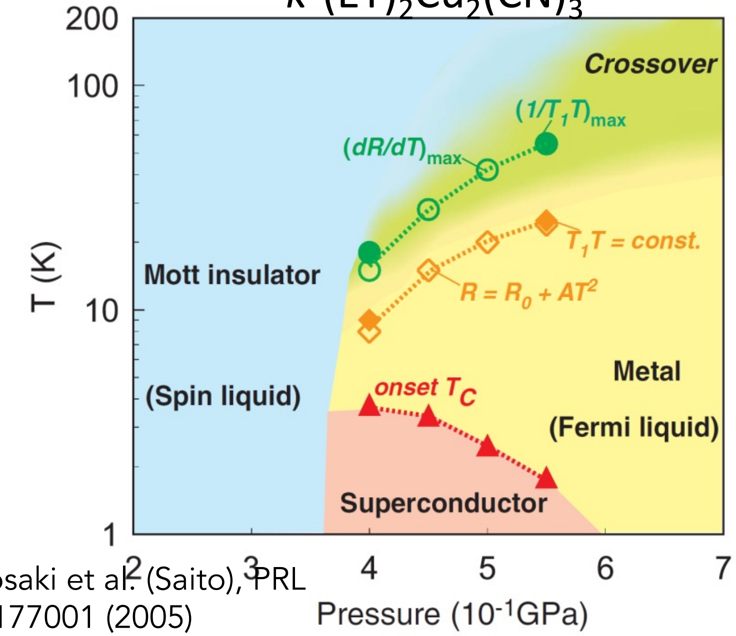
$CeRhIn_5$
(Heavy Fermion)



Georg Knebel, Dai Aoki, Jacques Flouquet, arXiv:0911.5223

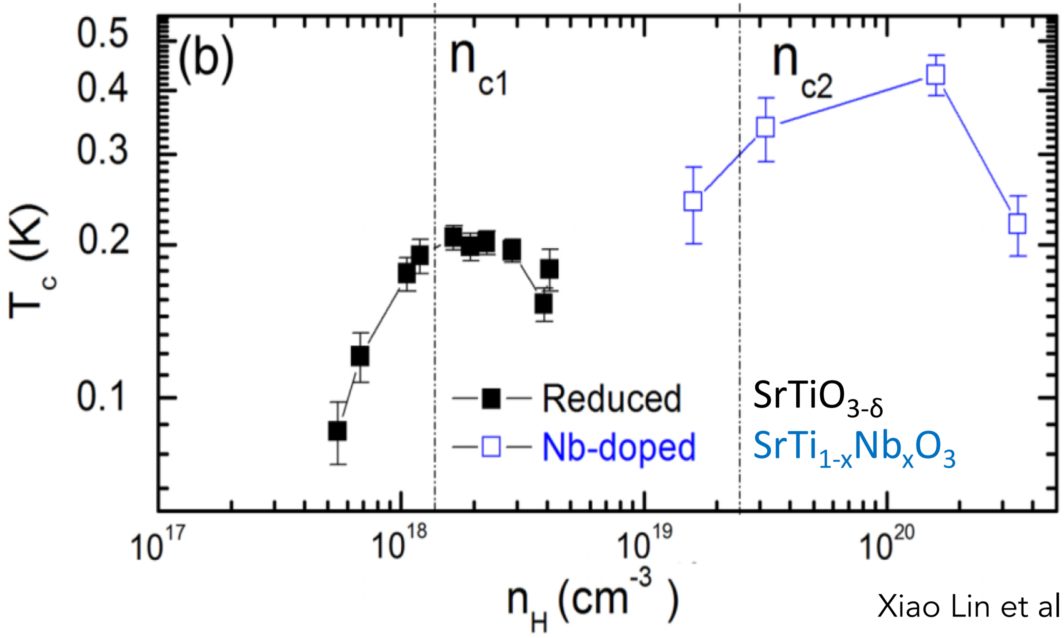
Organic Superconductors

κ-$(ET)_2Cu_2(CN)_3$

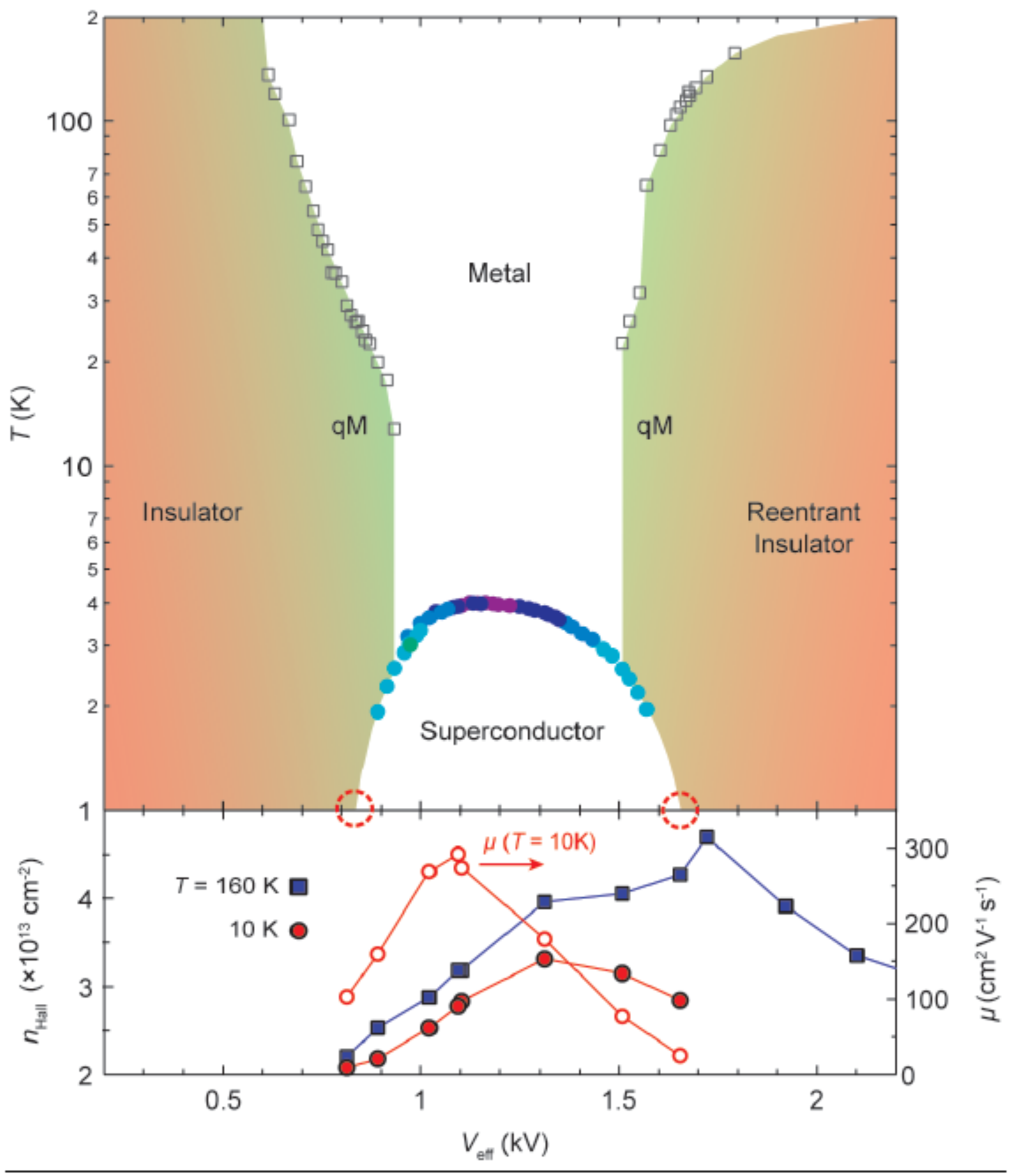


Kurosaki et al. (Saito), PRL **95**, 177001 (2005)

$SrTiO_3$

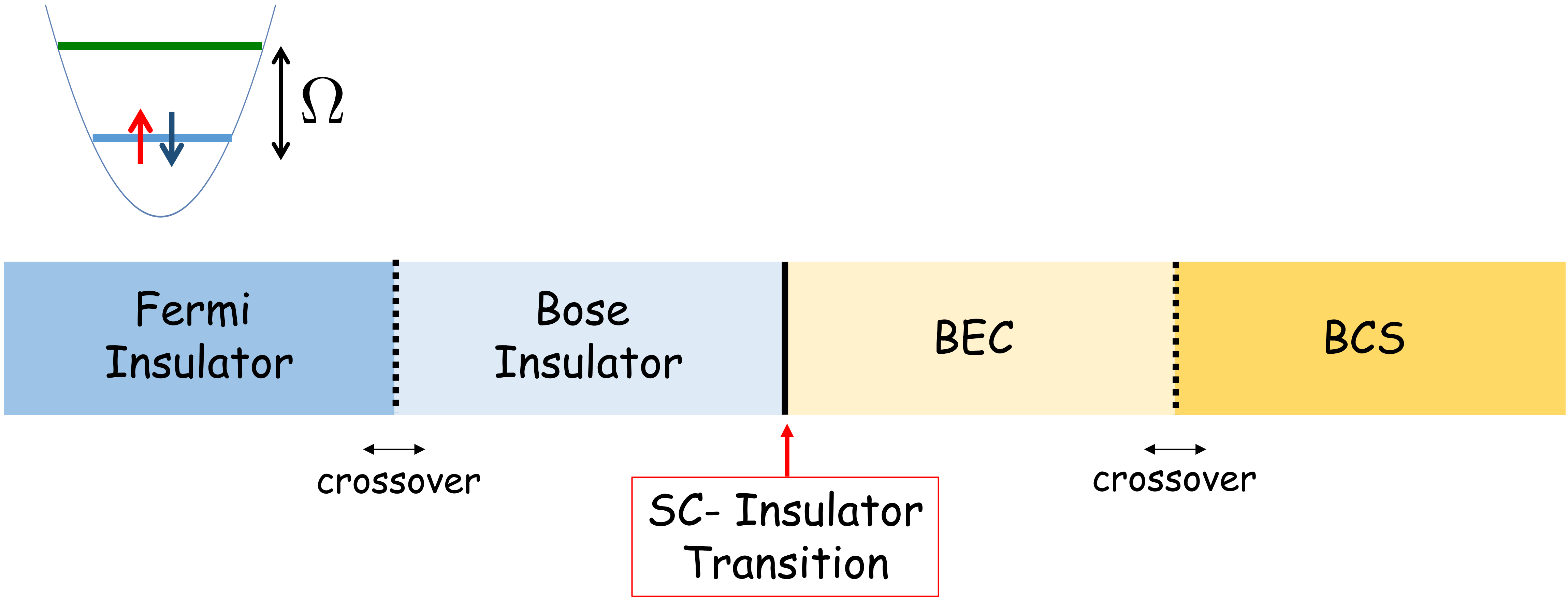


Xiao Lin et al. (K. Behnia), PRL **112**, 207002 (2014)

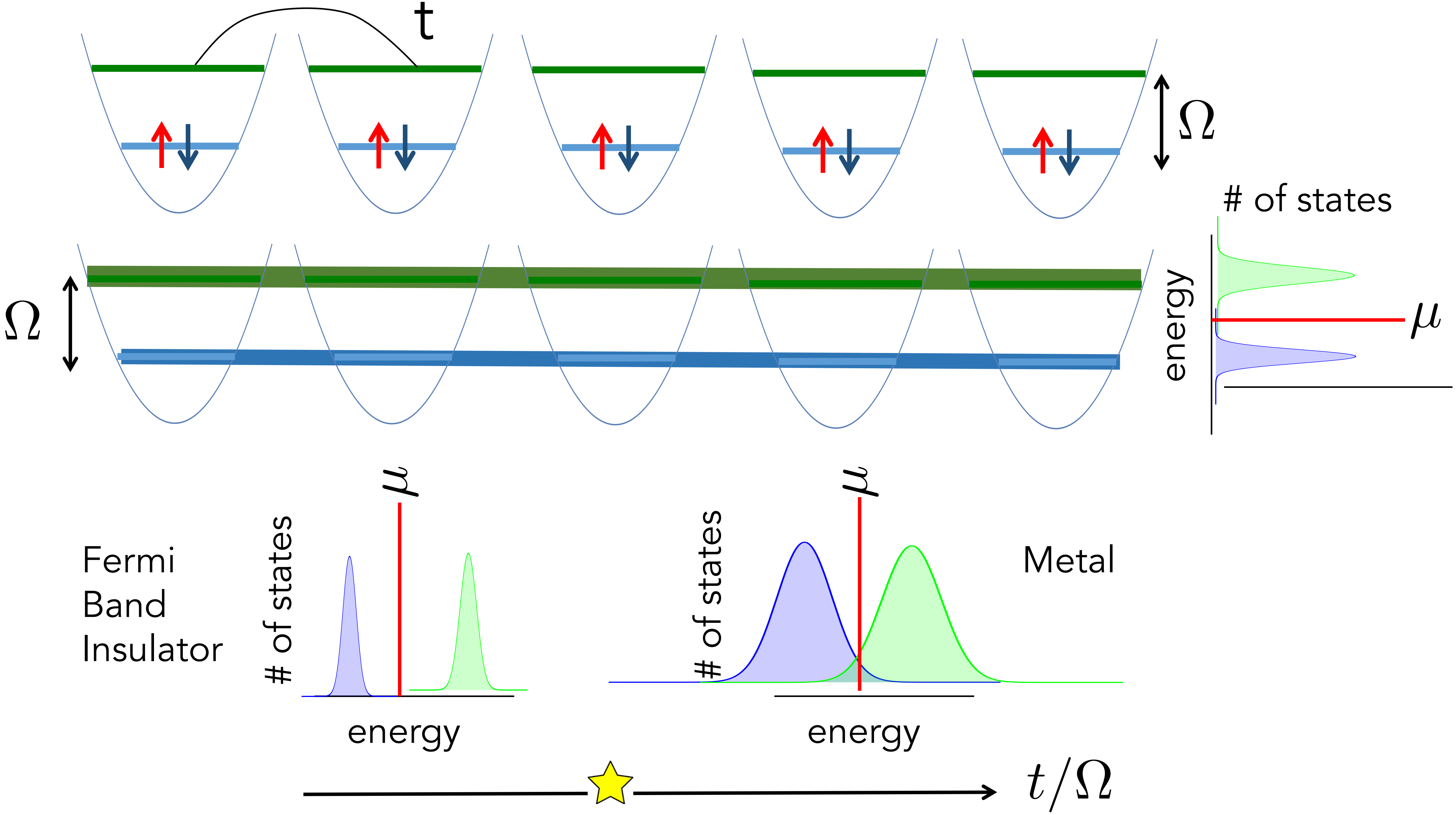


arXiv:1703.06369 A full superconducting dome of strong Ising protection in gated monolayer WS_2
J. M. Lu, O. Zheliuk, Q. H. Chen, I. Leermakers, N. E. Hussey, U. Zeitler, J. T. Ye

Band Insulator → SC

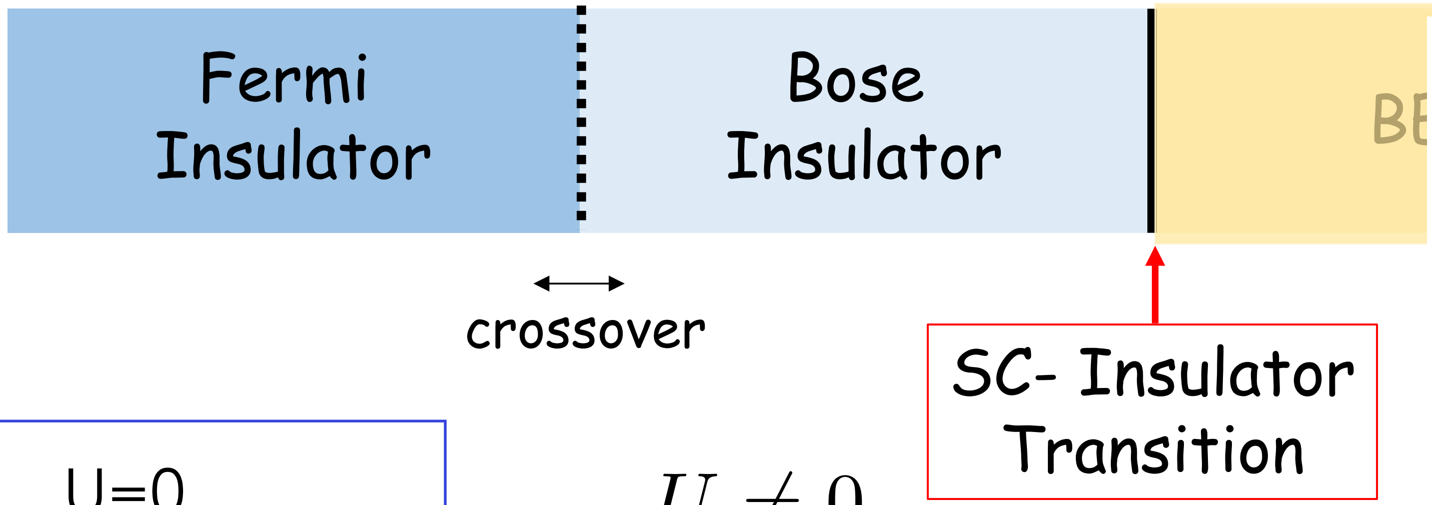


Loh, Randeria, Trivedi, Chen, Scalettar, : Superconductor-Insulator Transition and Fermi-Bose Crossovers"
Phys. Rev. X 6, 021029 (2016)



t
Ω
# of states
Ω
energy
μ
μ
μ
Fermi
Band
Insulator
# of states
energy
# of states
energy
Metal
t/Ω

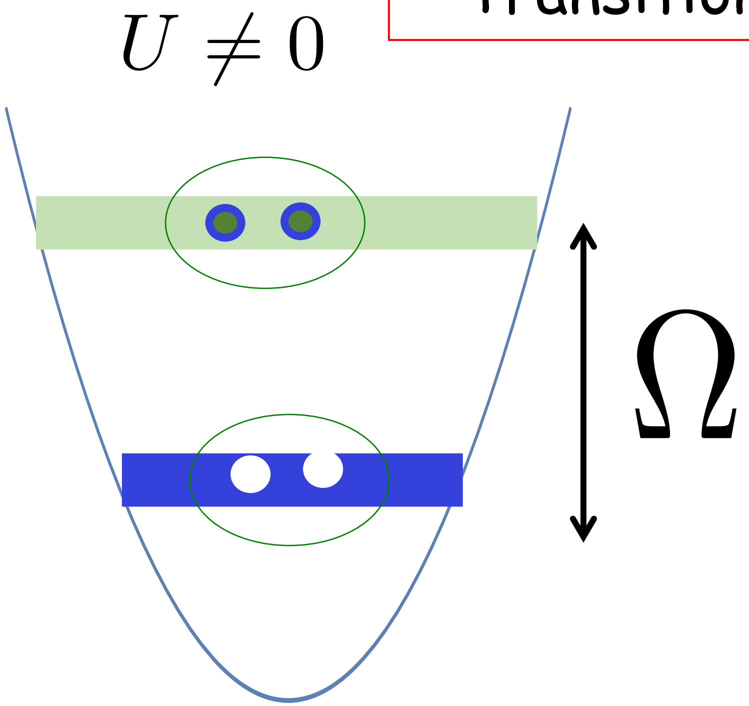
Fixed attraction U

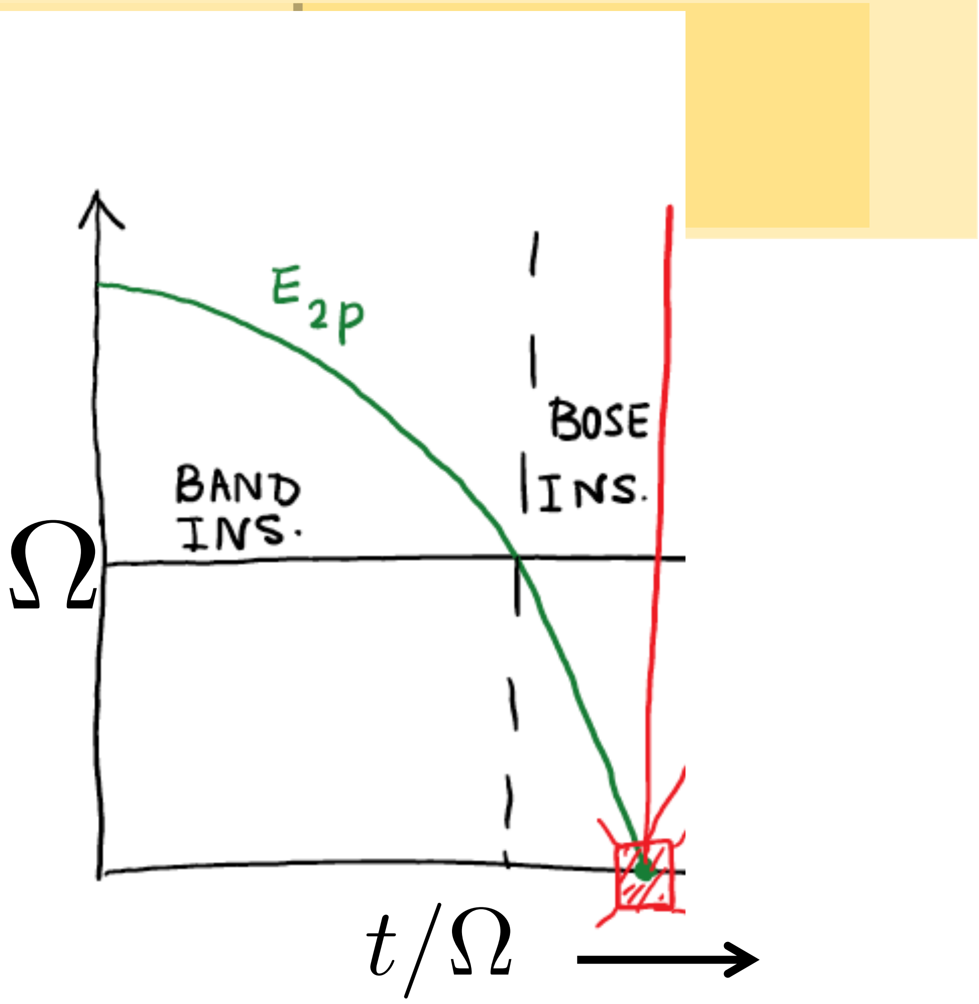


U=0

$$E_{1p} = \Omega$$

$$E_{2p} = 2\Omega$$

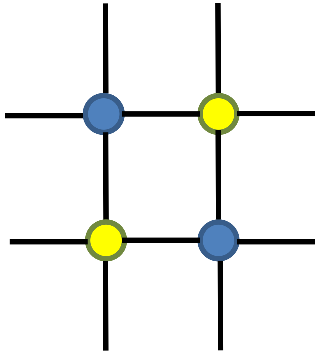




Increase hopping t between wells

2D Fermion model for band-insulator → SC transition

- Translationally invariant; no disorder
- (at least) 2 sites/orbitals per unit cell → 2 bands in insulator
- Attractive interactions → SC
  local attraction -|U| → no sign-problem in QMC
  → possible to realize in cold atoms expts.
- Non-bipartite lattice → suppress CDW order

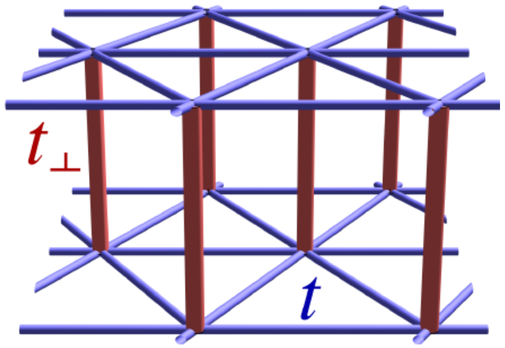


Checkerboard

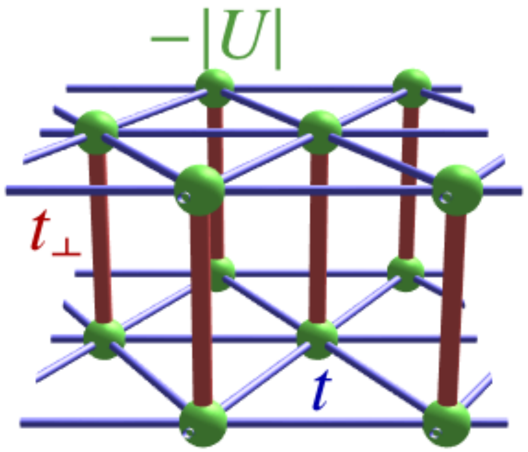
✗

Triangular
lattice bilayer

✓



Triangular bilayer attractive Hubbard model

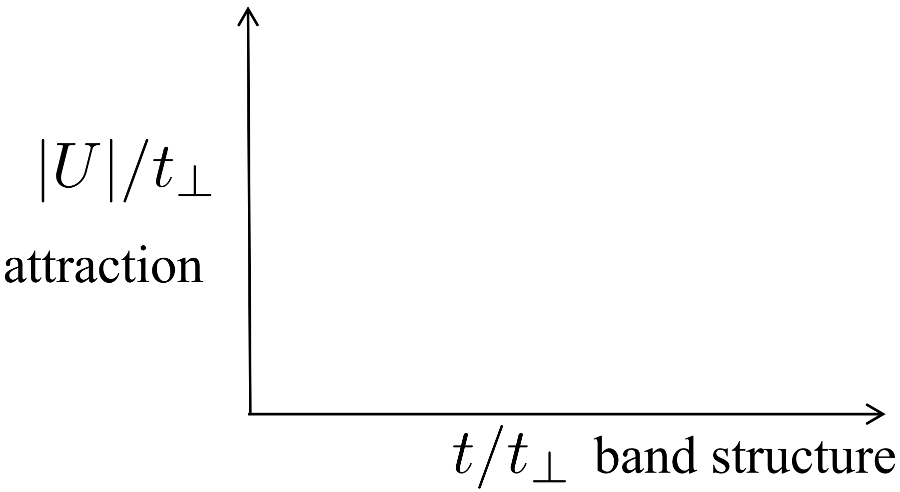


$$H = \boxed{-t \sum_{\langle ij \rangle_{\parallel} \sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + h.c. \right)}$$

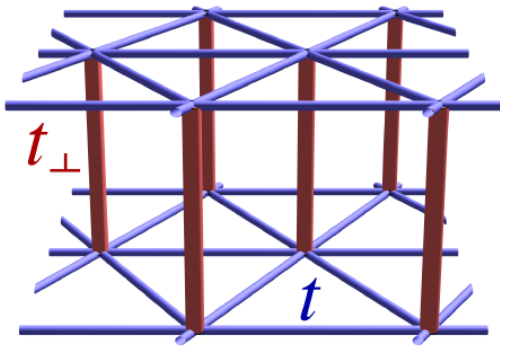
$$\boxed{- t_\perp \sum_{\langle ij \rangle_{\perp} \sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + h.c. \right)}$$

$$\boxed{- |U| \sum_i (n_{i\uparrow} - 1/2)(n_{i\downarrow} - 1/2)} \quad - \mu \sum_{i\sigma} n_{i\sigma}$$

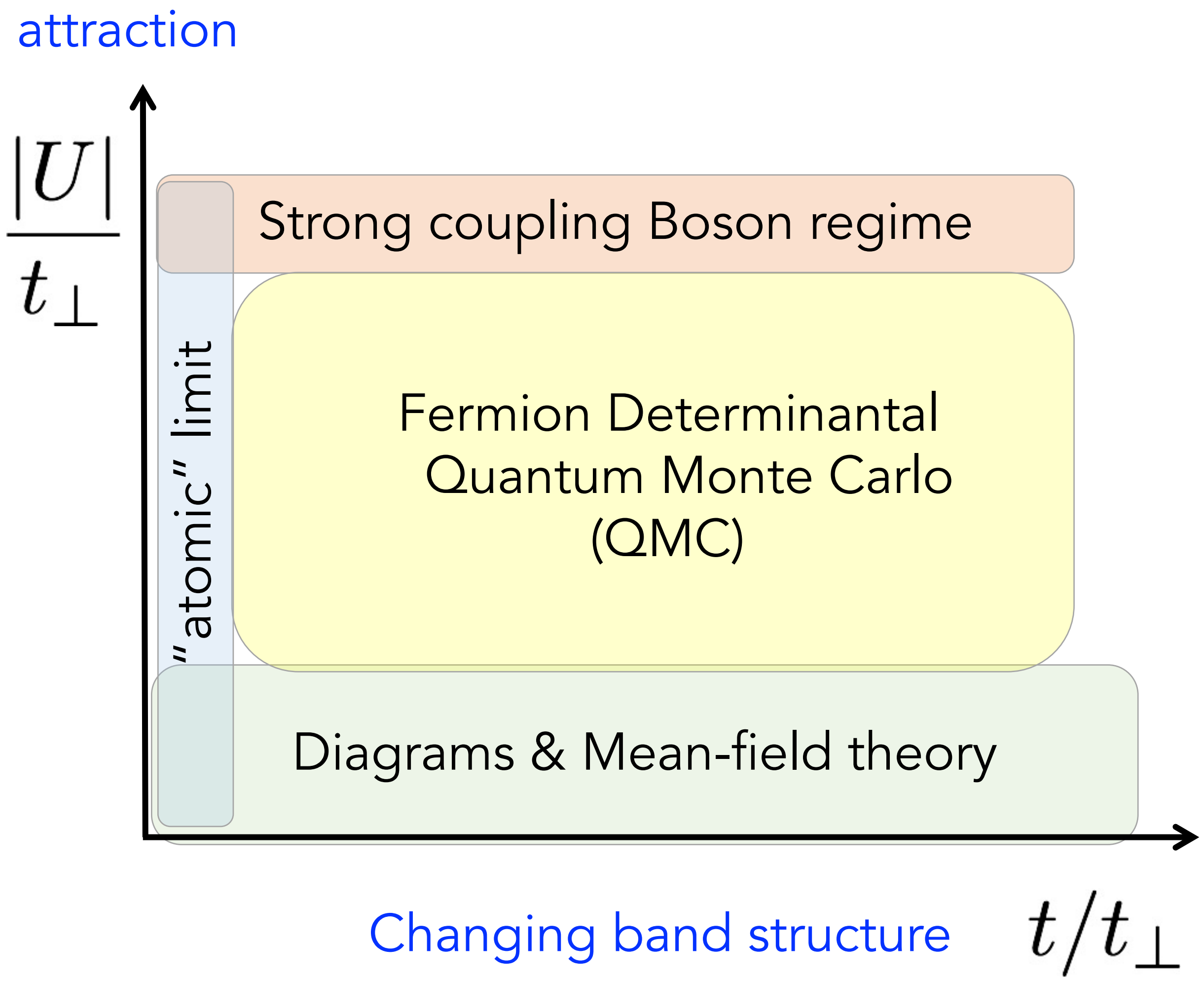
Fix T=0, $\quad t_\perp = 1 \quad \boxed{\langle n \rangle = 1}$



Methods



$t_\perp$
t

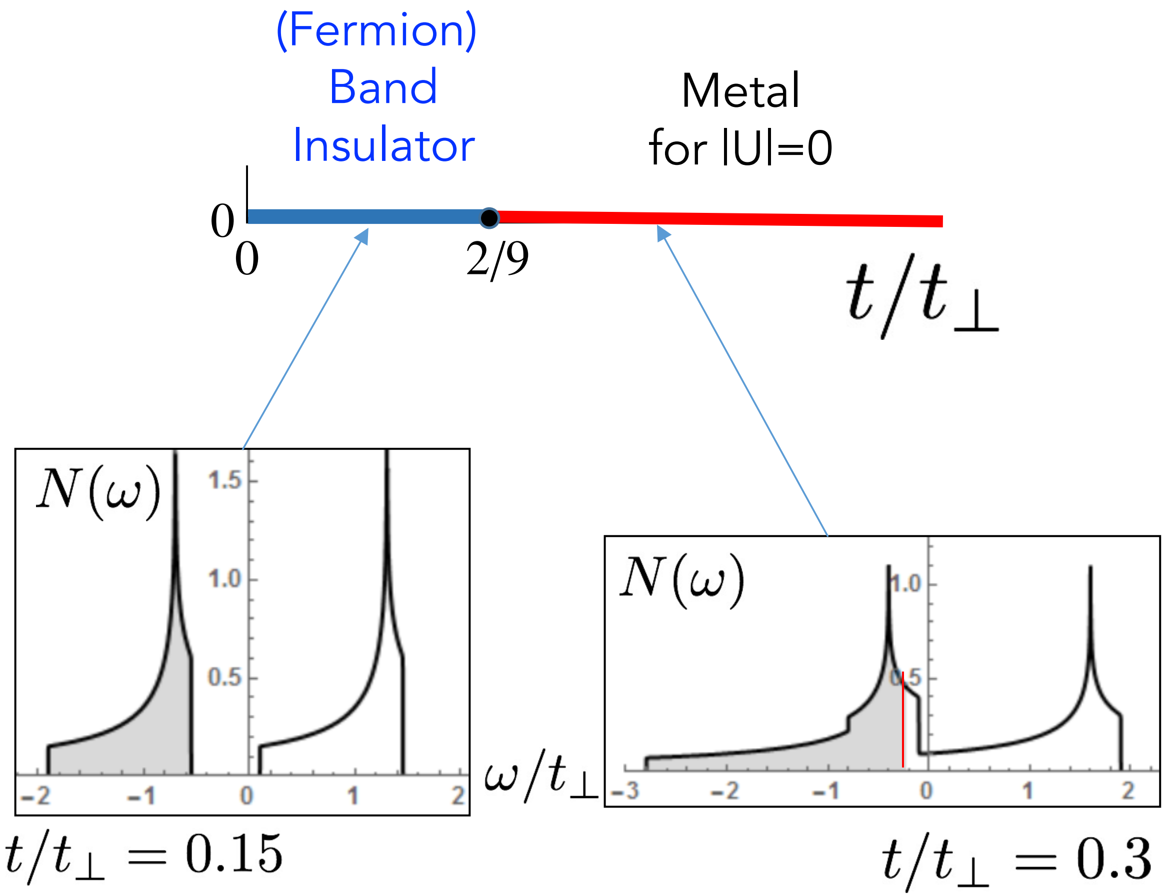


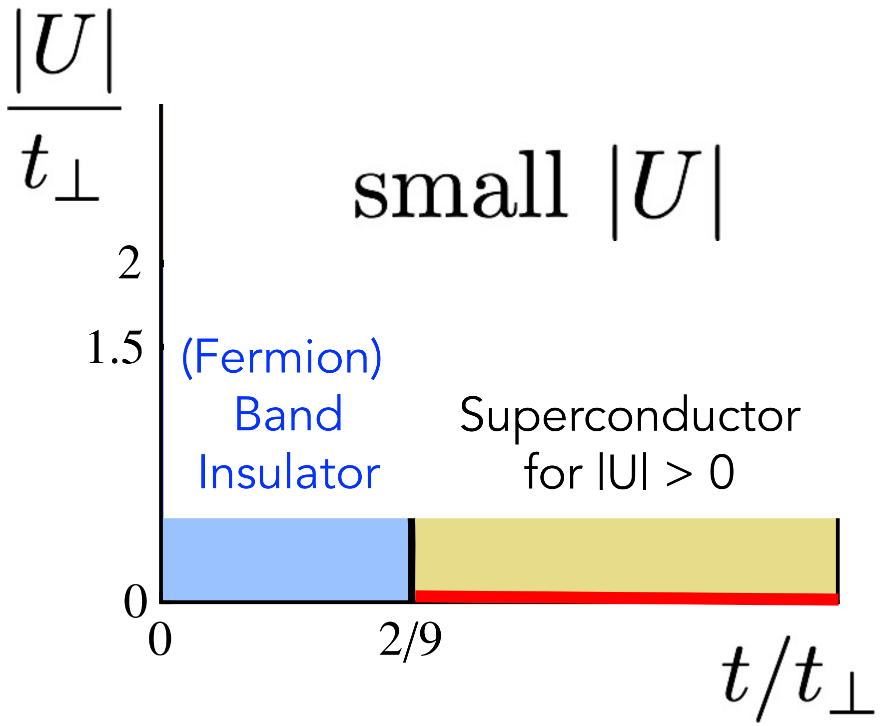
attraction
$\frac{|U|}{t_\perp}$
Strong coupling Boson regime
"atomic" limit
Fermion Determinantal
Quantum Monte Carlo
(QMC)
Diagrams & Mean-field theory
Changing band structure
$t/t_\perp$

Non-interacting

Non-interacting Limit

$$|U| = 0 \qquad \langle n \rangle = 1$$

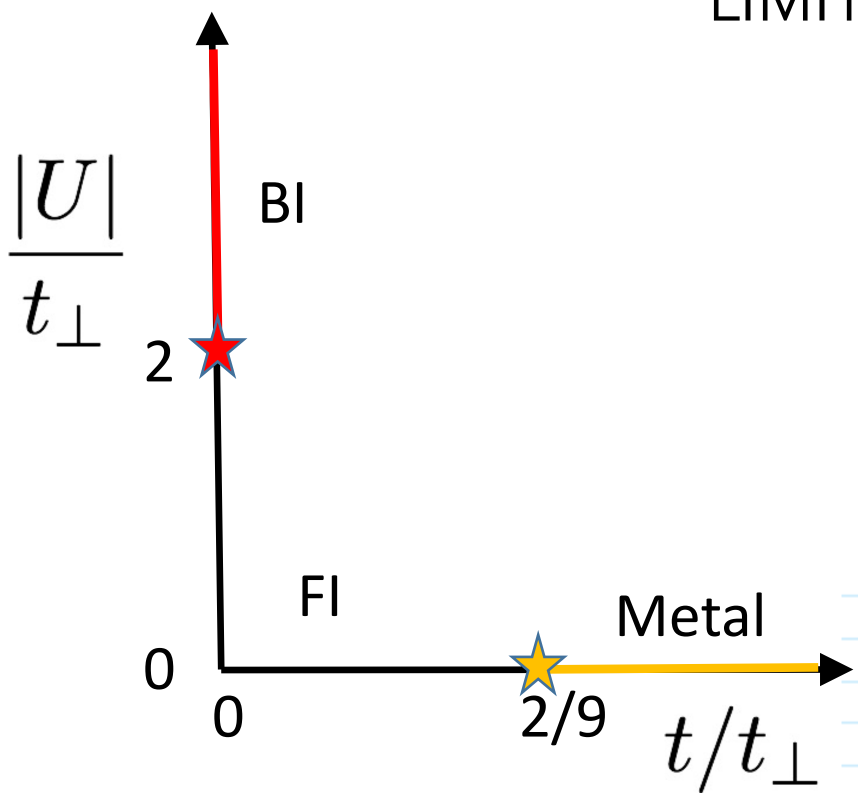




Compensated
(semi)metal with
Electron & Hole FS's

LIMITS: t=0 ATOMIC

Atomic



ATOMIC $t=0$

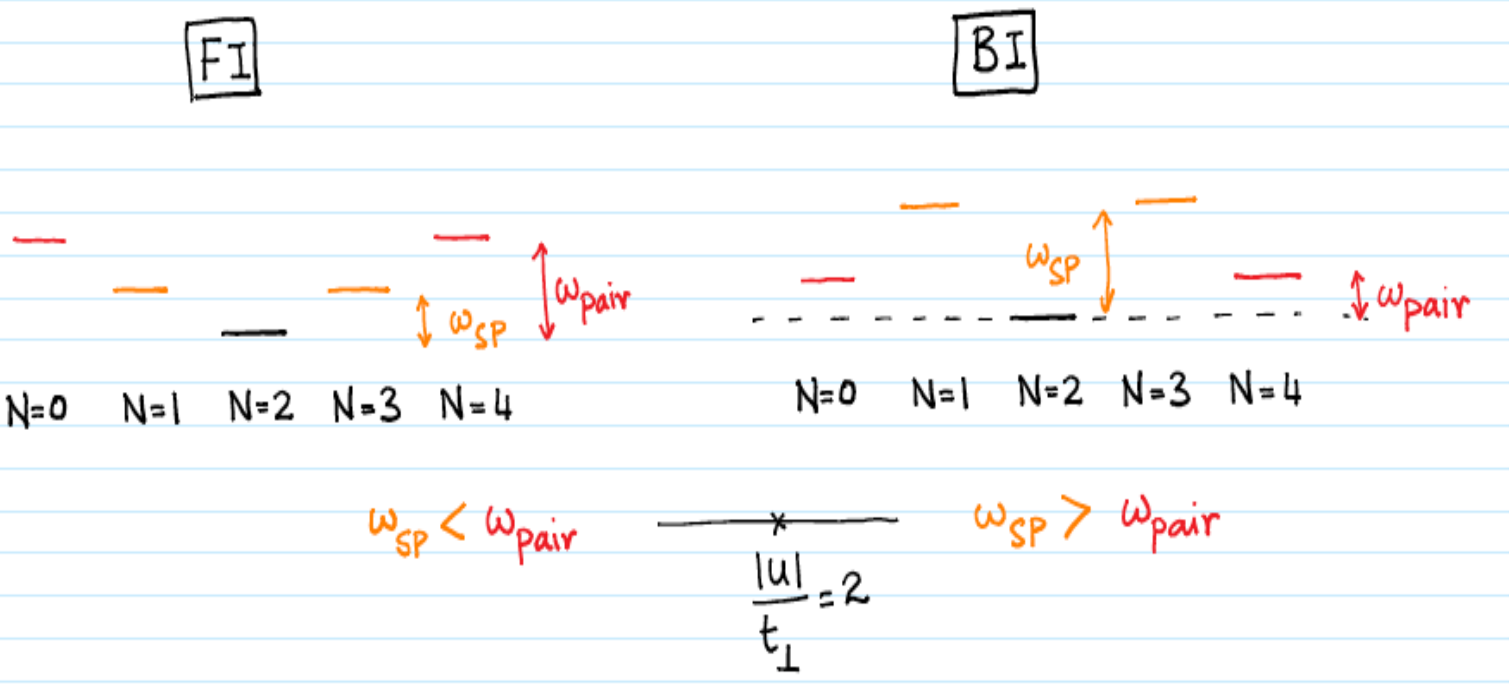
A, B

$t_\perp$

RUNG

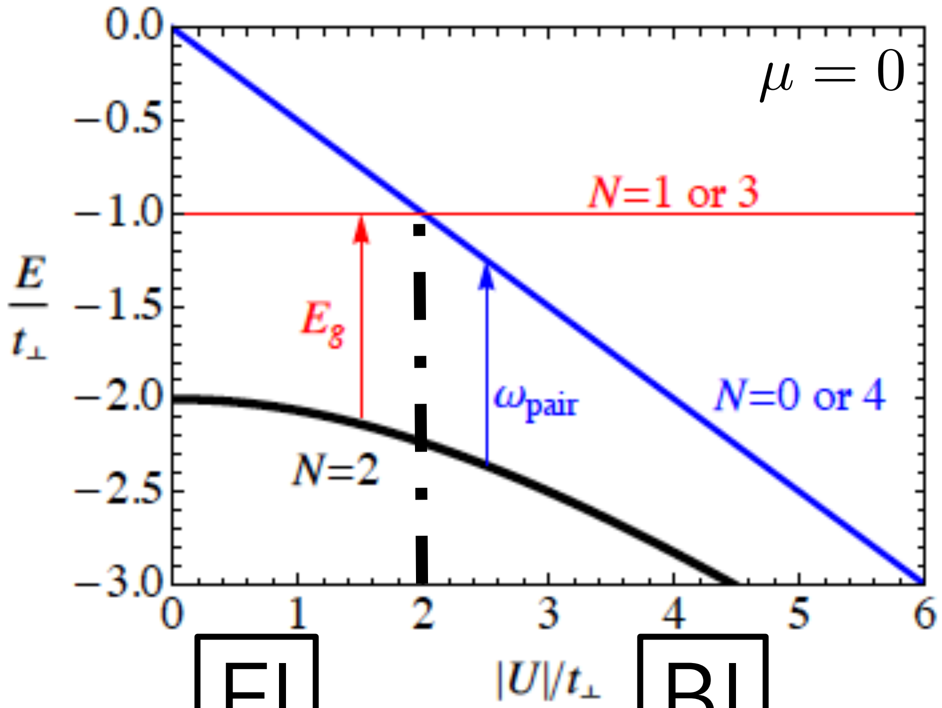
$N = 0, 1, 2, 3, 4$ fermions

gs: $|\psi_{N=2}\rangle = \{|\underset{A}{\uparrow\downarrow}\ \underset{B}{0}\rangle, |0\ \uparrow\downarrow\rangle, |\uparrow\ \downarrow\rangle, |\downarrow\ \uparrow\rangle\}$



Atomic

Atomic Limit: Insulators



FI

BI

$E_{\text{gap}} < \omega_{\text{pair}}$

$E_{\text{gap}} > \omega_{\text{pair}}$

Atomic Limit: $t_{\parallel} = 0$

N=0,1,2,3,4

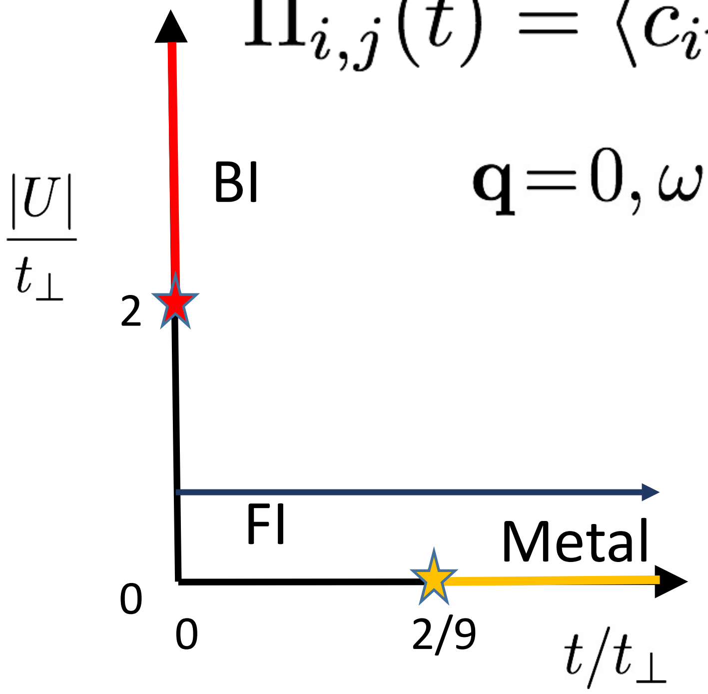
$$E_0 = -\tfrac{1}{2}\,|U| + 2\mu$$
$$E_1 = -t_\perp + \mu$$
$$E_2 = -\sqrt{\tfrac{1}{4}\,|U|^2 + 4t_\perp^2}$$
$$E_3 = -t_\perp - \mu$$
$$E_4 = -\tfrac{1}{2}\,|U| - 2\mu.$$

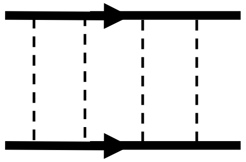
$$H = -t_\perp \sum_\sigma \left(c^\dagger_{A\sigma} c_{B\sigma} + h.c.\right) - \mu \sum_{i\sigma} n_{i\sigma} - |U| \sum_i (n_{i\uparrow} - 1/2)(n_{i\downarrow} - 1/2)$$

Pairing Instability in a Band Insulator

Weakly interacting

$$\Pi_{i,j}(t) = \langle c_{i\uparrow}(t)c_{i\downarrow}(t)c^{\dagger}_{j\downarrow}(0)c^{\dagger}_{j\uparrow}(0)\rangle$$

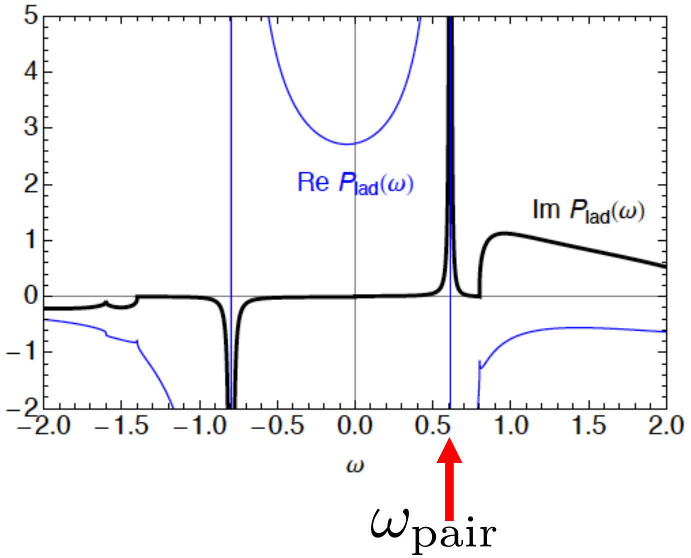


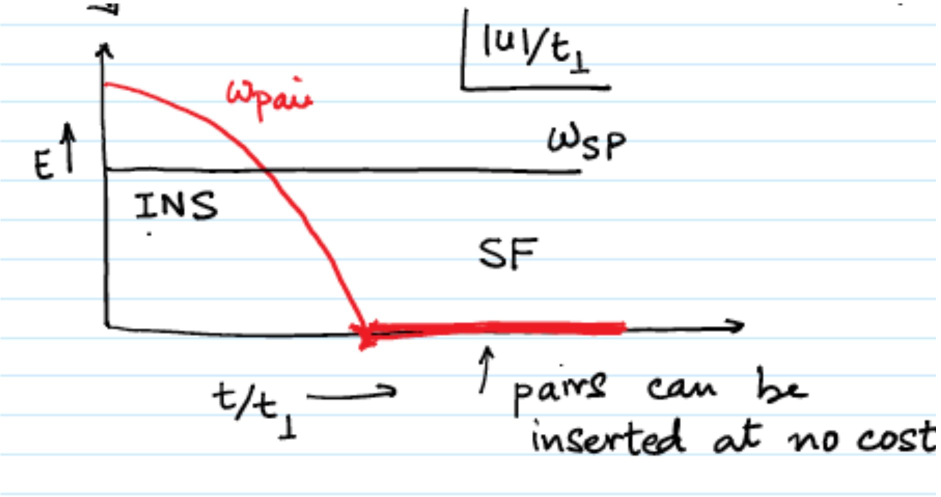
$\mathbf{q}=0, \omega \rightarrow$ 

$$\chi_0(\omega) = \frac{1}{N}\sum_{\mathbf{k}} \frac{1-2f_{\mathbf{k}}}{2\varepsilon_{\mathbf{k}} - \omega - i0^+}$$

$$\chi^{-1}(\omega) = \chi_0^{-1}(\omega) - |U|$$

pair susceptibility
$\mathrm{Im}\chi(q=0,\omega)$





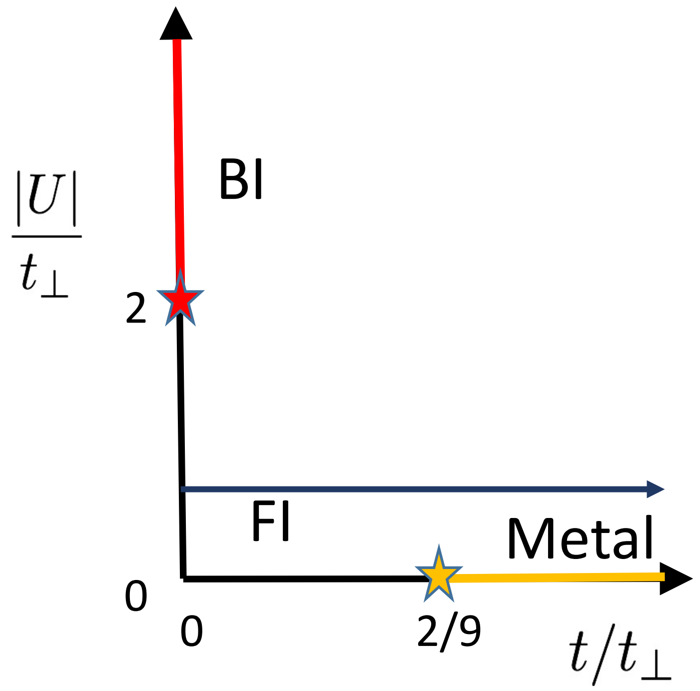
- Pole in $\chi(\omega)$ → Gap to pair excitations in Insulator

$\chi^{-1}(\omega_{\mathrm{pair}})=0$

= 2-particle gap
→ 0 at SIT

Pairing Instability in a Band Insulator

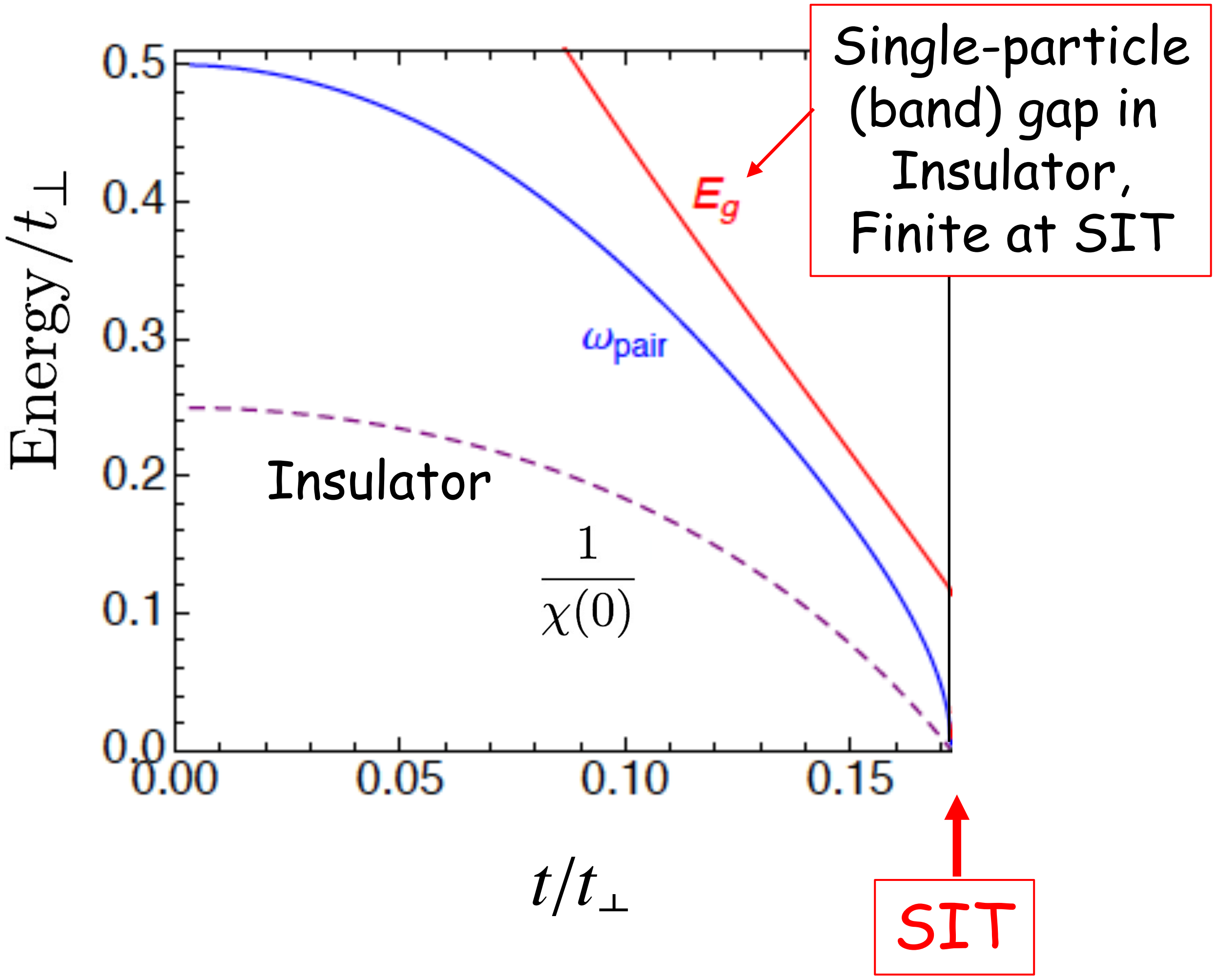
Weakly interacting



- Divergence of pairing $\chi(0)$

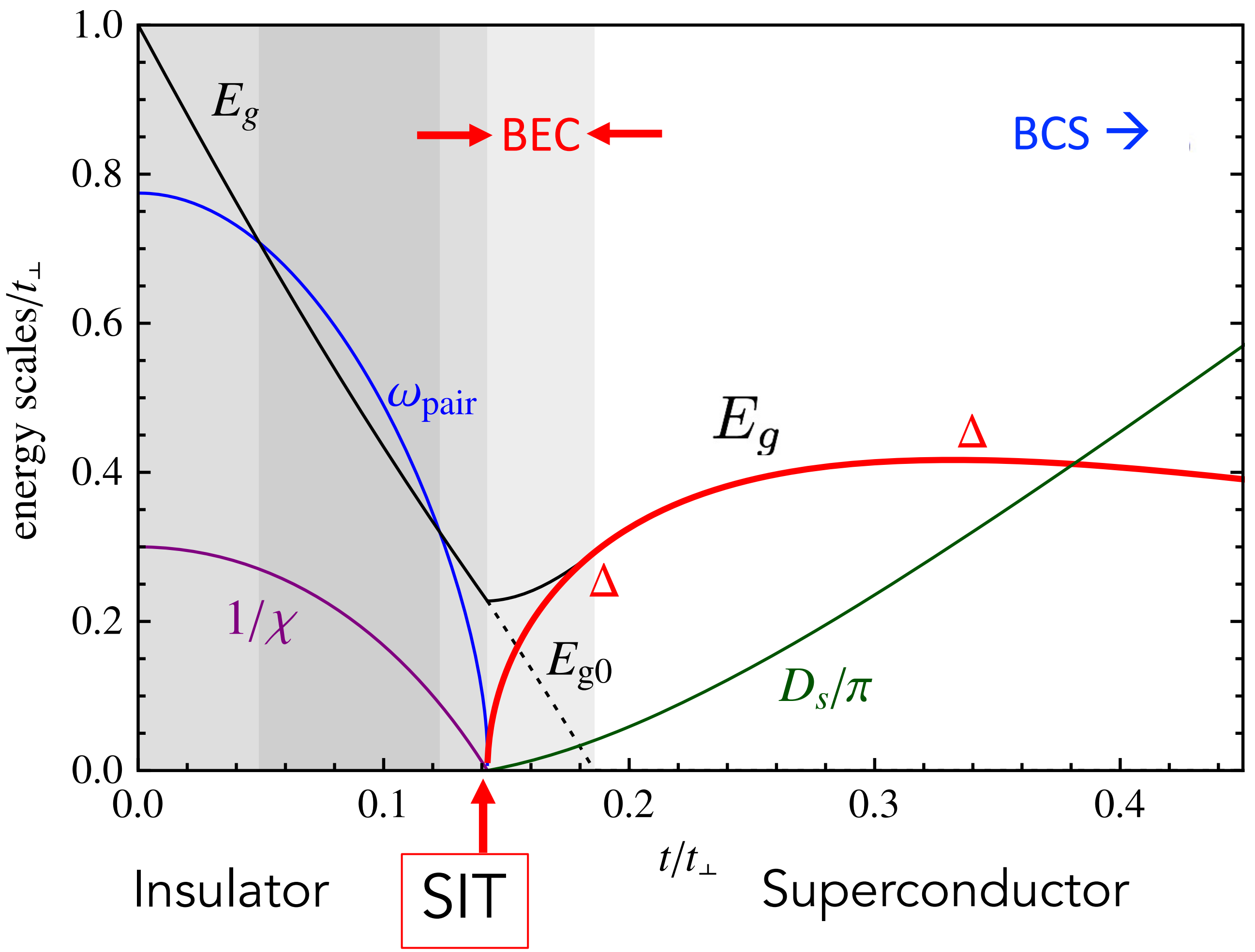
→ transition from insulator to SC

Note: $\omega_{\text{pair}} < E_g$ near the SIT



MFT

Mean Field Theory for SC state



Single-particle Energy Gap E_g

$$E_g = \begin{cases} E_{g0} & \text{Band gap in Insulator} \\ \sqrt{E_{g0}^2 + \Delta^2} & \leftarrow \boxed{\text{BEC regime near SIT}} \\ \Delta & \text{SC gap in BCS regime} \end{cases}$$

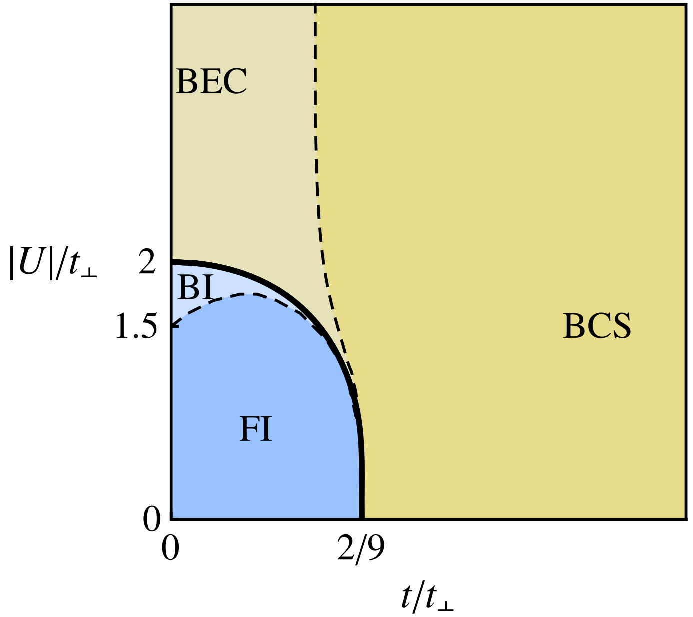
Superfluid Stiffness

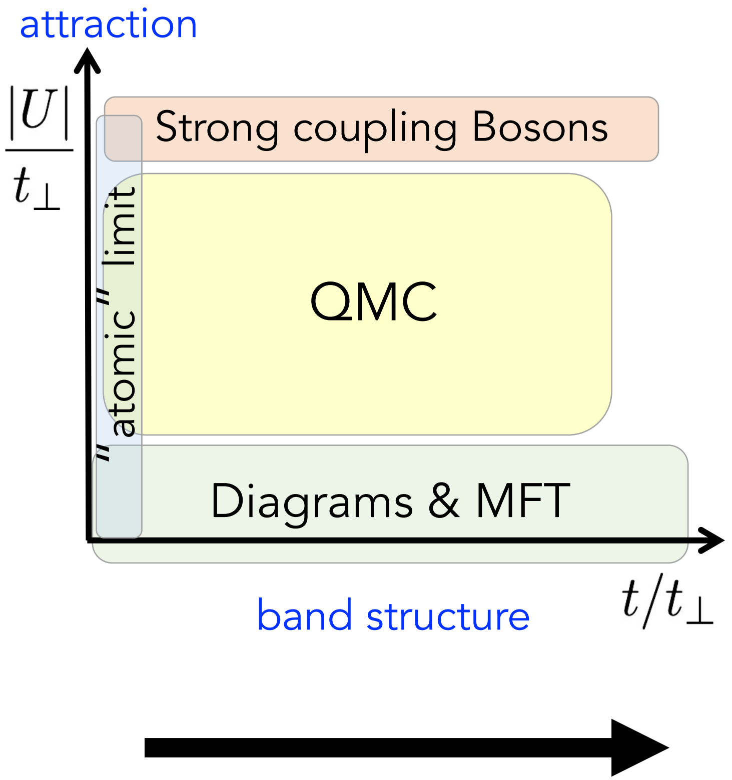
$$D_s < E_g$$

even outside the BEC regime

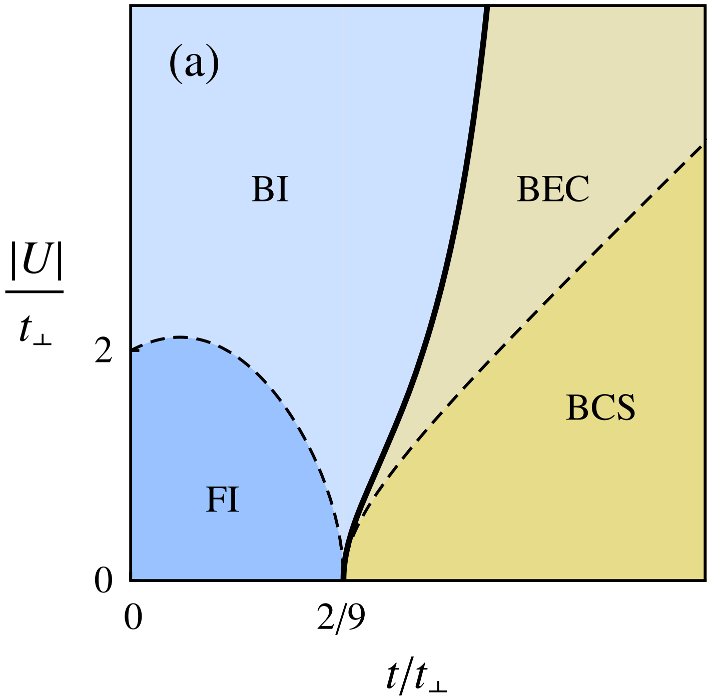
→ phase fluctuation dominate

Diagrams & MFT



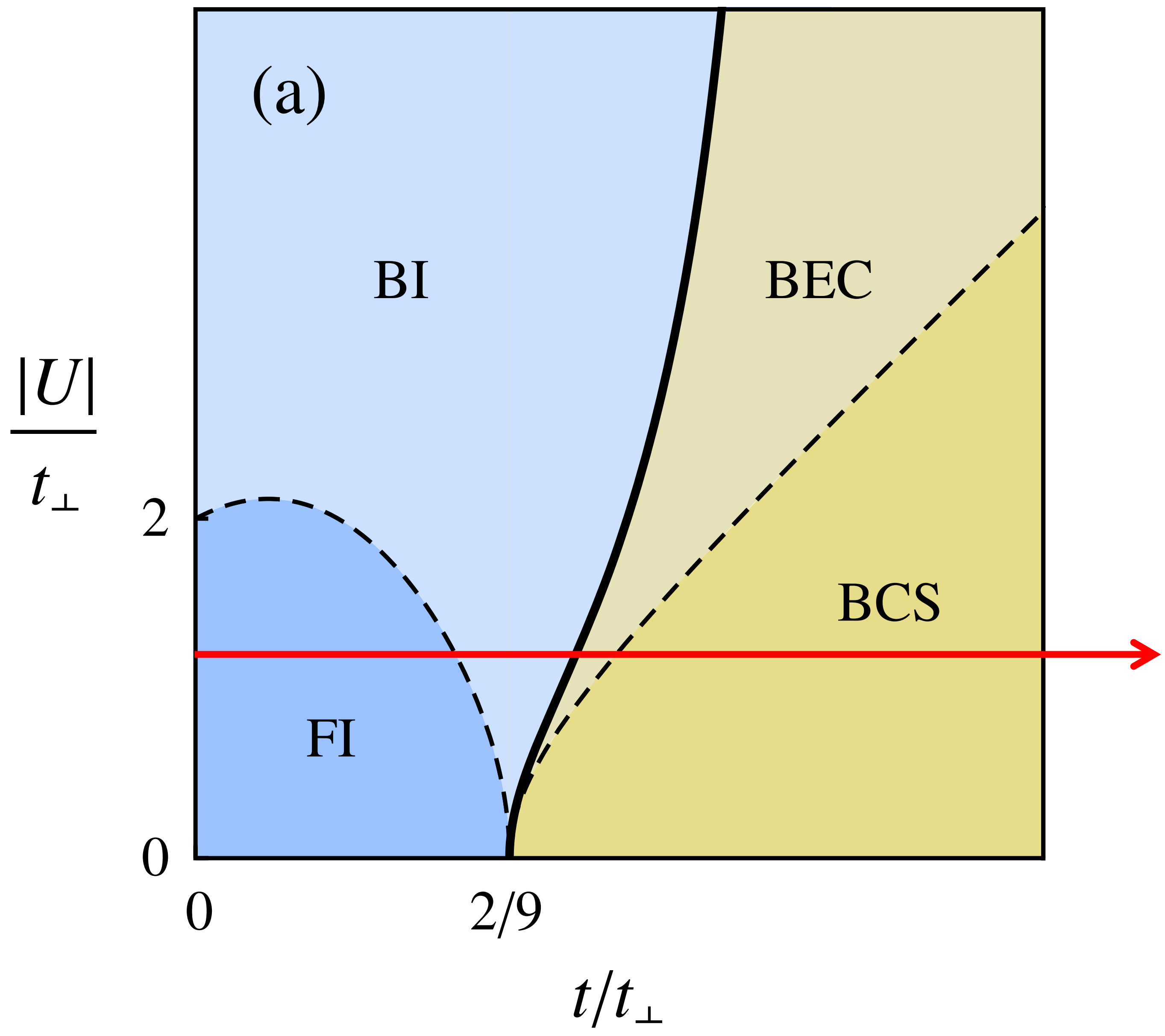


Actual Phase Diagram



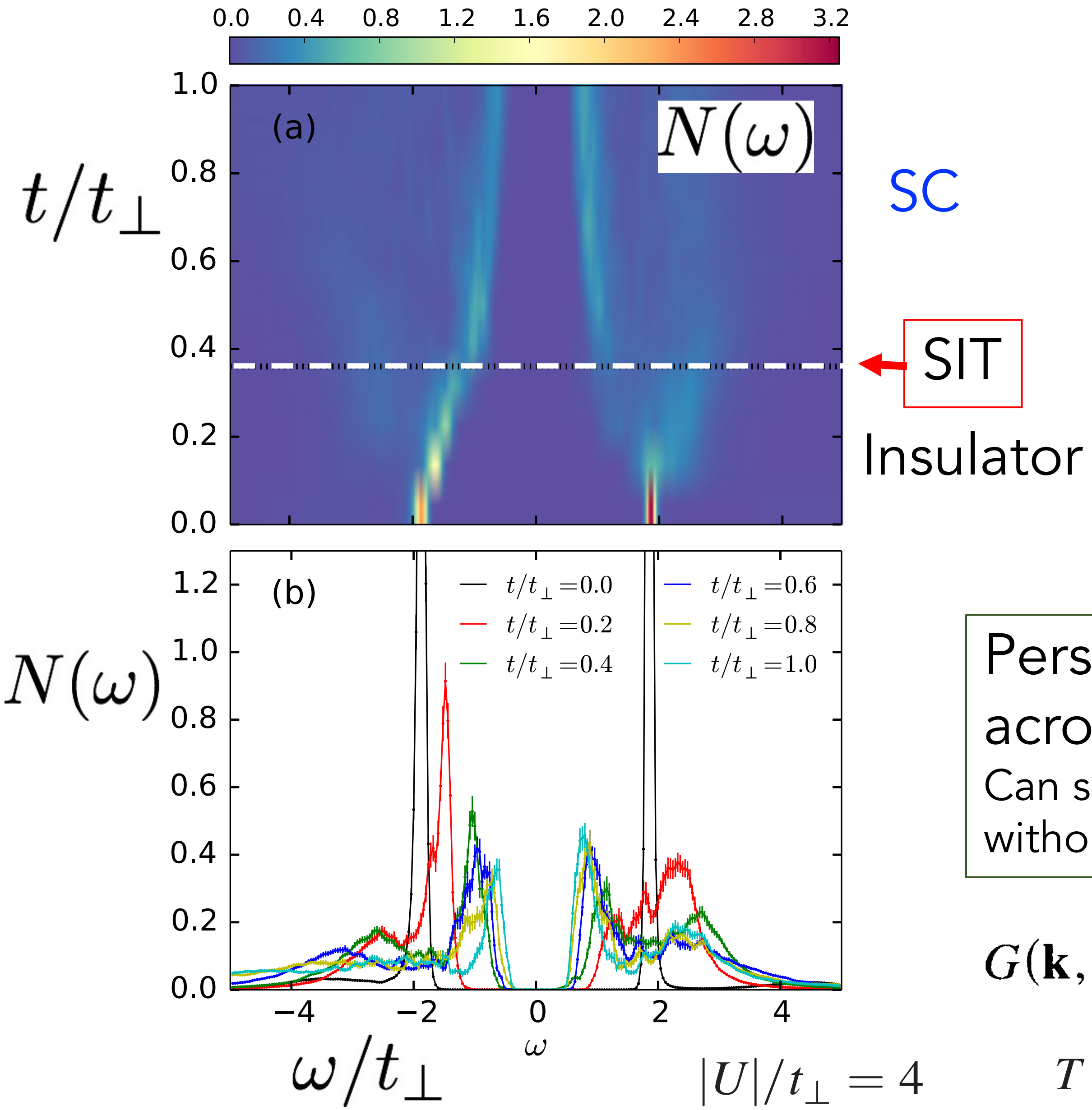
Determinental Quantum Monte Carlo

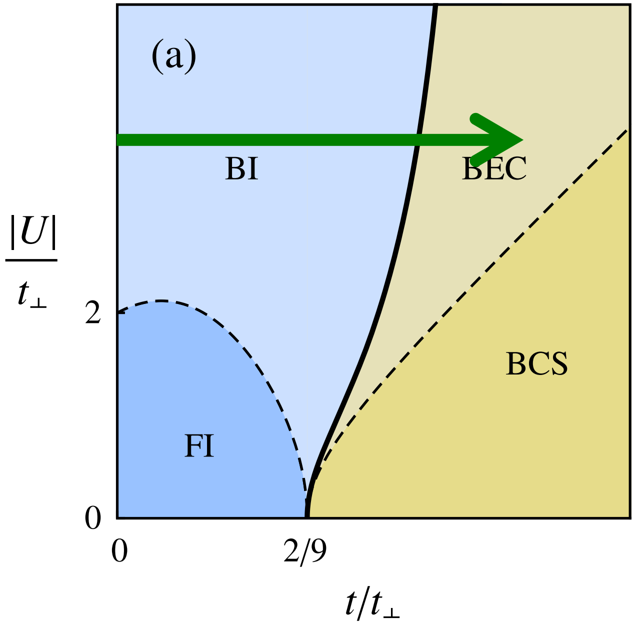
Attractive Hubbard -- Free of fermion sign-problem at all fillings


(a)
BI
BEC
BCS
FI
$\frac{|U|}{t_\perp}$
2
0
0
2/9
$t/t_\perp$

Single-Particle Density of States
From QMC + Maximum Entropy

Intermediate coupling: QMC





Persistence of single-particle gap across the SC-Insulator transition

Can see gap directly from imaginary time QMC data without analytic continuation

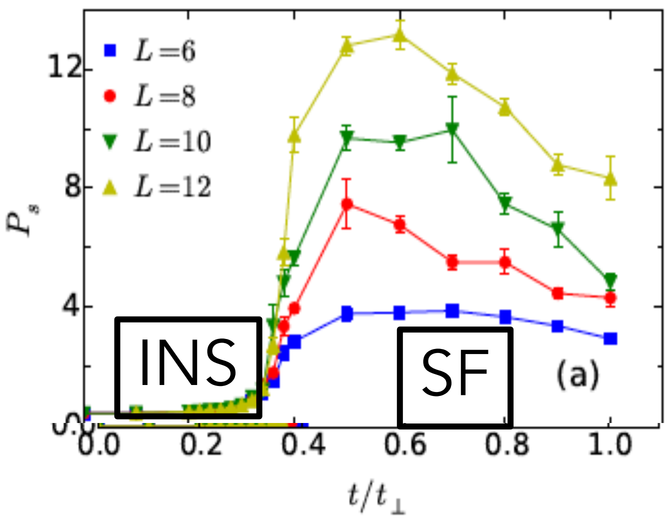
$$G(\mathbf{k},\tau) = -\int_{-\infty}^{\infty} d\omega \, \frac{\exp(-\omega\tau)}{1 + \exp(-\beta\omega)} \mathcal{A}(\mathbf{k},\omega)$$

$|U|/t_\perp = 4$ $T = 0.0803 t_\perp$ $12 \times 12 \times 2$ bilayer

QMC: Pairing structure factor

$$P_s = \frac{1}{N}\sum_{i,j}\langle c^\dagger_{i\uparrow}c^\dagger_{i\downarrow}c_{j\downarrow}c_{j\uparrow}\rangle$$

off diagonal long range order



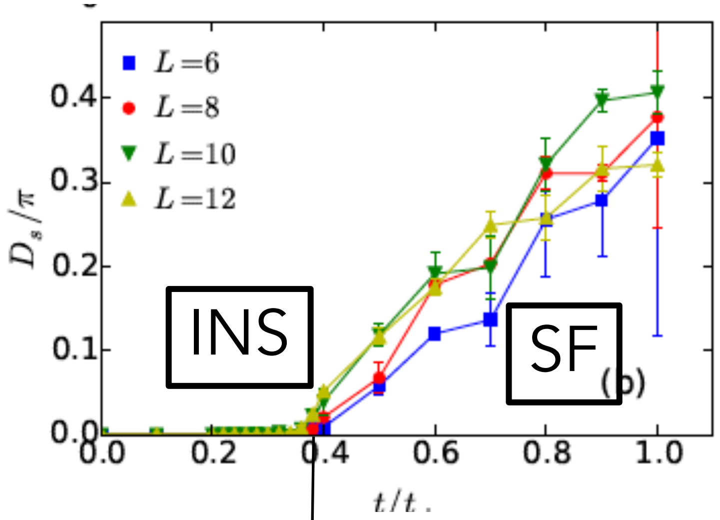
QMC: Superfluid density

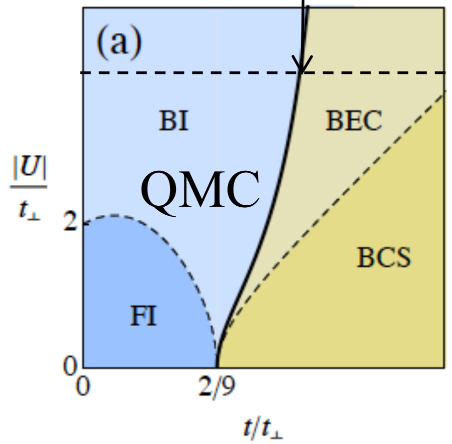
$$\frac{D_s}{\pi} = \langle -k_x\rangle - \Lambda_{xx}(q_x=0, q_y\to 0, i\omega=0)$$

$$\Lambda_{xx}(r_i, r_j, t) = \langle j_x(r_i,t) j_x^\dagger(r_j,0)\rangle$$

$T/t_\perp = 0.08$

$U/t_\perp = 4$





How to identify the BCS & BEC regimes in the crossover?

2 Predictions:

- Topology of "Minimum Gap Locus"–
  ARPES (angle resolved photoemission spectroscopy)
- Gap-edge singularity in DOS – tunneling

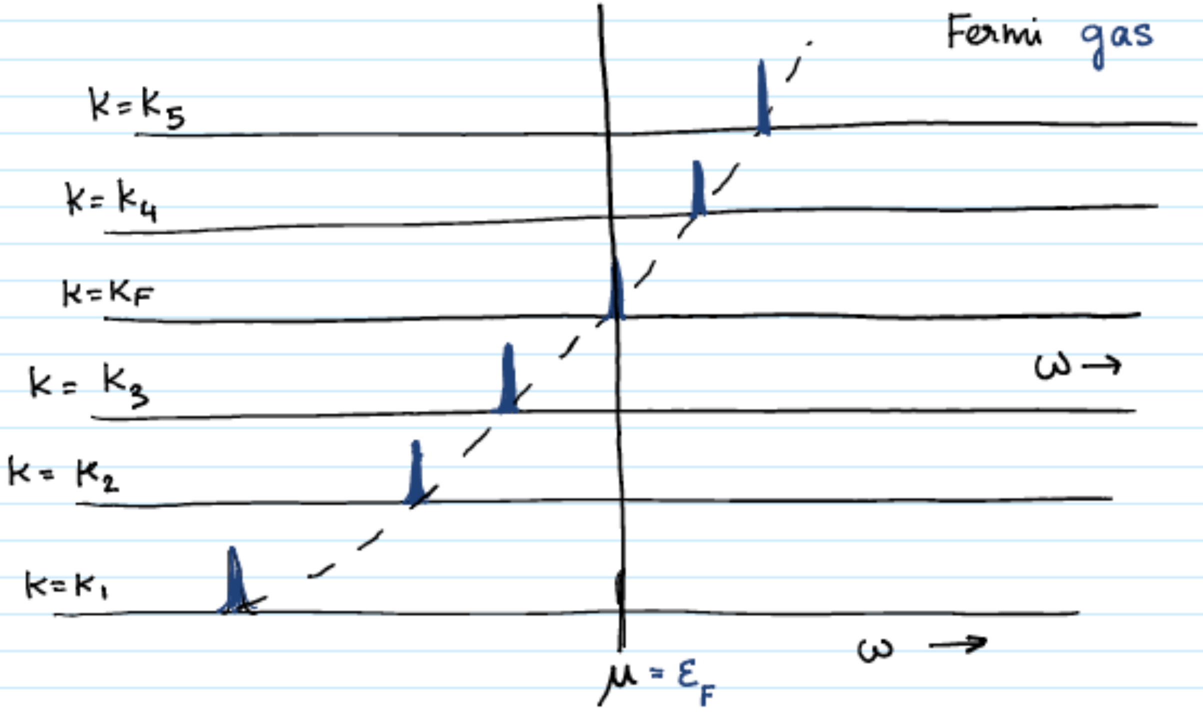
Fermion Spectral function

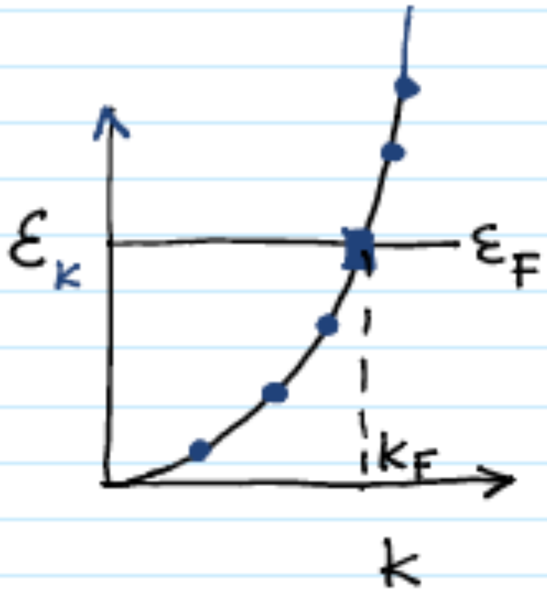
= probability to make an excitation
at momentum **k** and energy ω

Angle resolved
- photoelectron spectroscopy
- rf spectroscopy

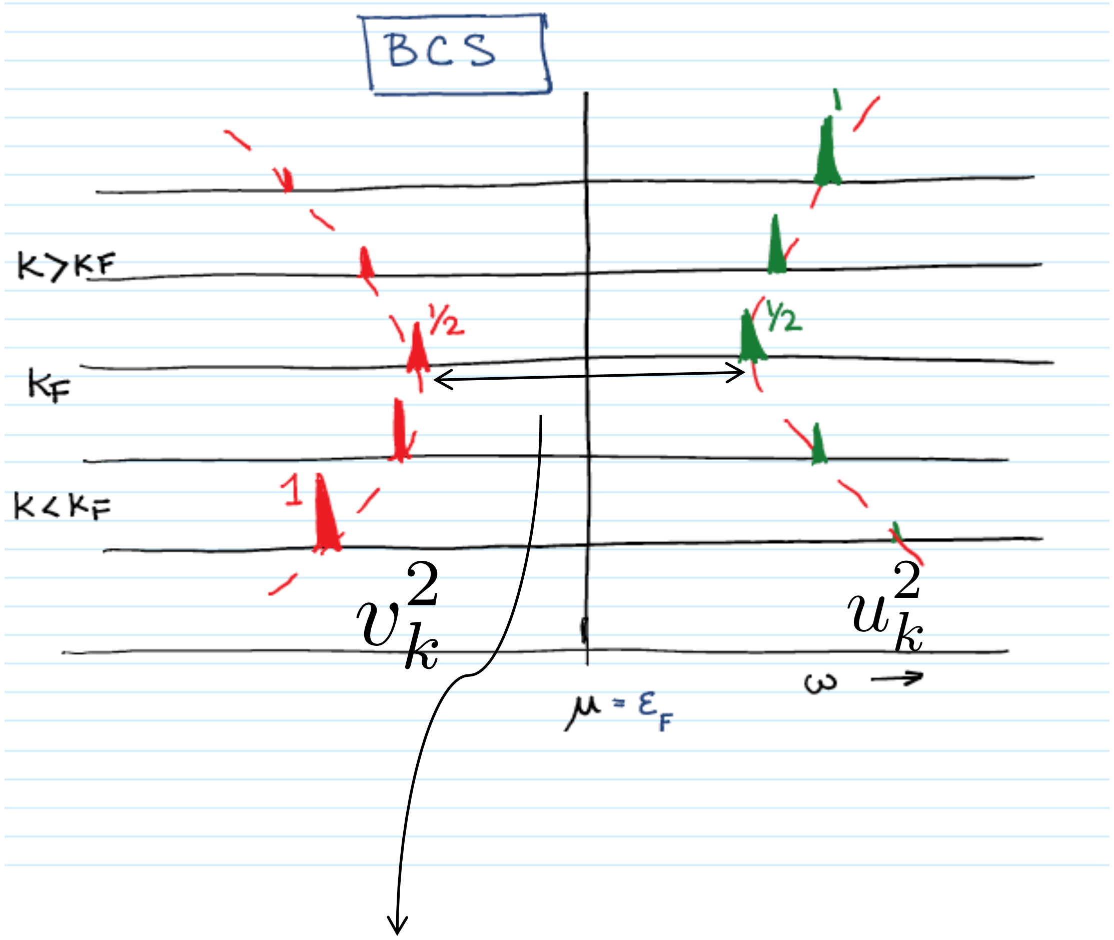
$$G_{ij}(t) = \langle c_i(t)\, c_j^{+}(0) \rangle$$ Single particle Green function

$$A(k,\omega) = -\frac{1}{\pi}\,\mathrm{Im}\, G_{ret}(k,\omega)$$ Spectral function





BCS: minimum gap locus at k_F

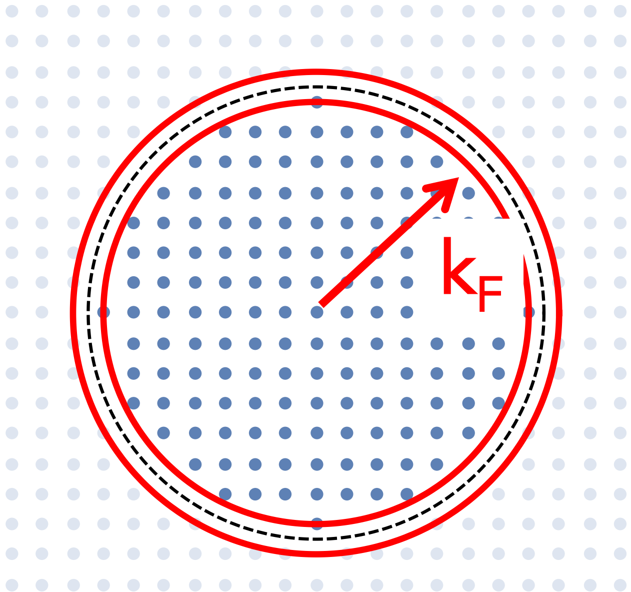


Minimum gap at $k=k_F$

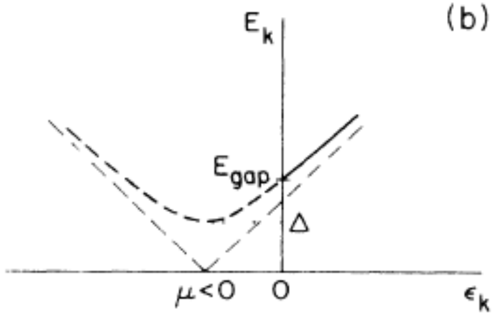
$$A(k,\omega) = u_k^2\delta(\omega - E_k) + v_k^2\delta(\omega + E_k)$$

$$E_k = \pm\sqrt{\xi_k^2 + |\Delta_k|^2}$$

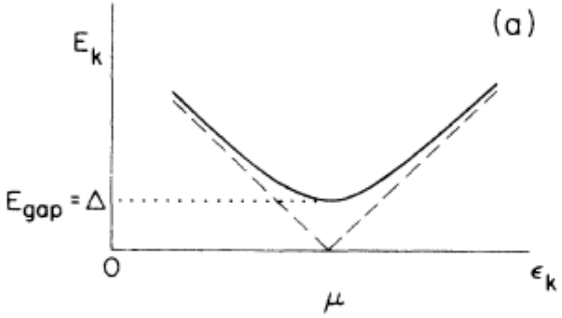
$$u_k^2 + v_k^2 = 1 \qquad v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$$



BEC regime ("strong pairing")



BCS regime ("weak pairing")



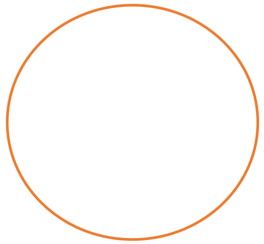
$$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}$$

Minimum gap locus in k-space

point

$$\epsilon_k = 0$$

or $k = 0$



contour

$$\epsilon_k = \mu$$

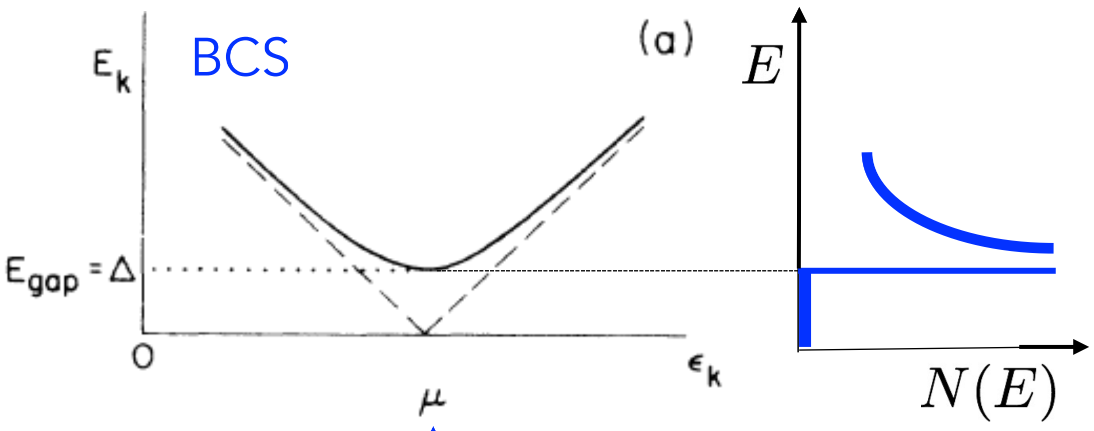
or $k = “k_F”$

Crossover from BCS to BEC regime

* Topology of "Minimum Gap Locus"
* Gap-edge singularity in DOS

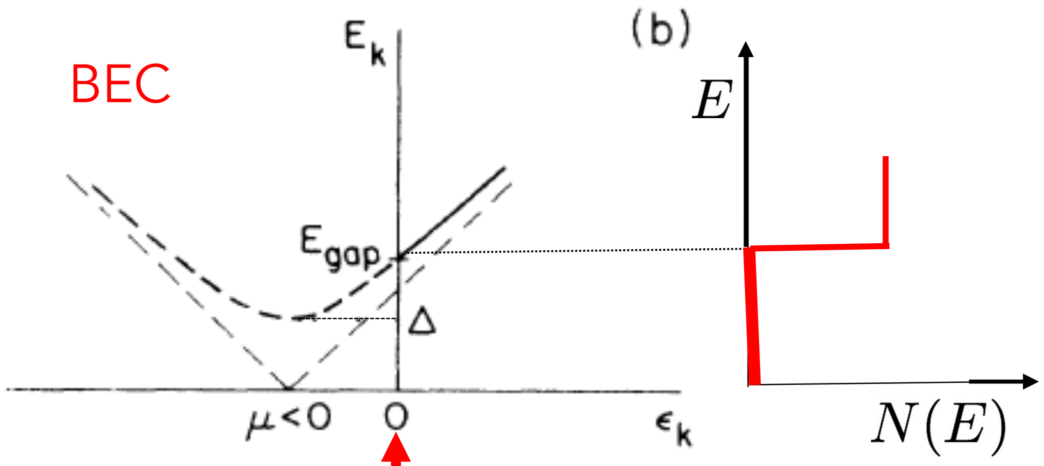
$$E_{\text{gap}} \equiv \min_{\varepsilon_k \geq 0} [(\varepsilon_k - \mu)^2 + |\Delta_{\mathbf{k}}|^2]^{1/2}$$

$$E_{\text{gap}} = \begin{cases} \Delta & \text{for } \mu > 0, \quad \leftarrow \text{BCS} \\ (\mu^2 + \Delta^2)^{1/2} & \text{for } \mu < 0, \quad \leftarrow \text{BEC} \end{cases}$$



Min gap locus is contour $\varepsilon_{\mathbf{k}} = \mu$

1/(square-root)
In DOS



Min gap locus is point $\mathbf{k} = 0$

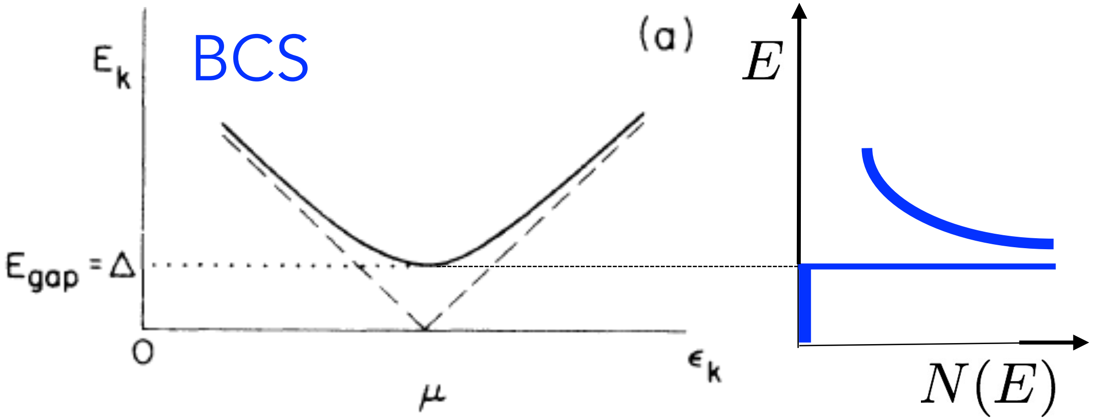
DOS has jump discontinuity (2D)

How to identify the BCS & BEC regimes in the crossover?

* Topology of "Minimum Gap Locus"

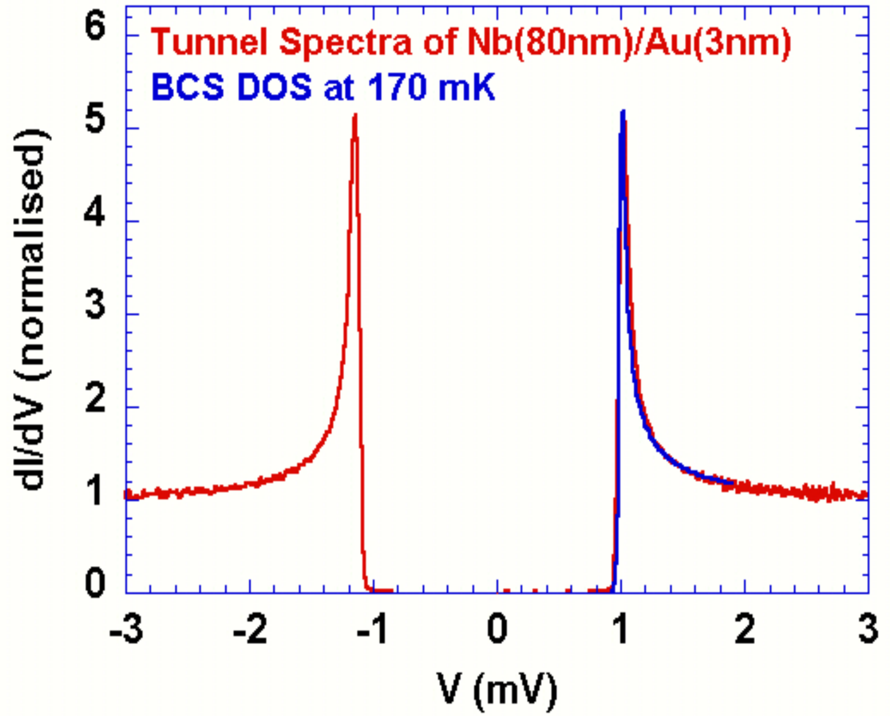
* **Gap-edge singularity in DOS**

$$E_{\mathrm{gap}} \equiv \min_{\varepsilon_k \geq 0} [(\varepsilon_k - \mu)^2 + |\Delta_{\mathbf{k}}|^2]^{1/2}$$

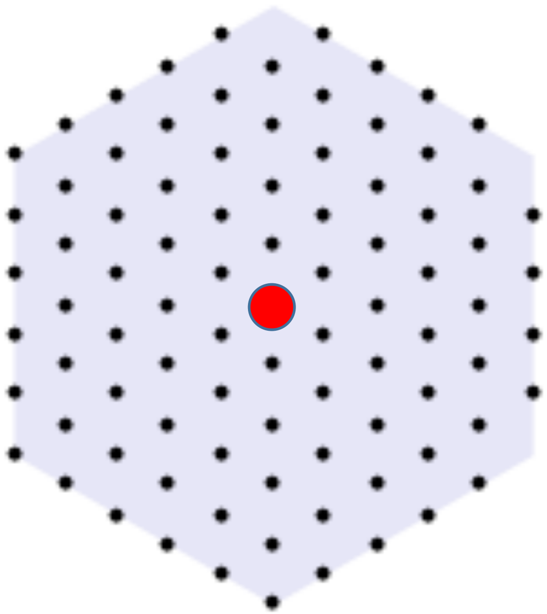


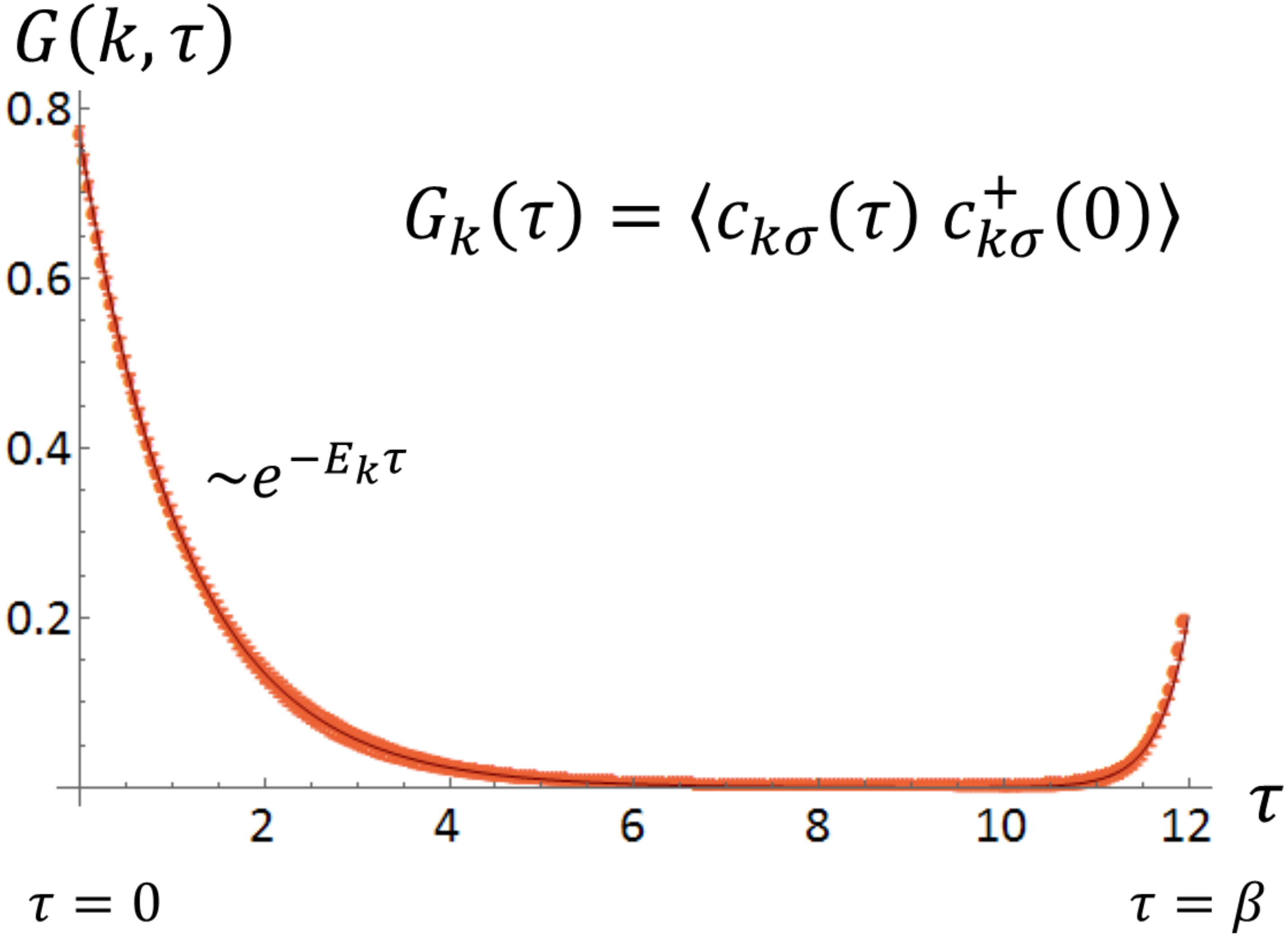
Min gap locus is <u>contour</u> $\varepsilon_{\mathbf{k}} = \mu$

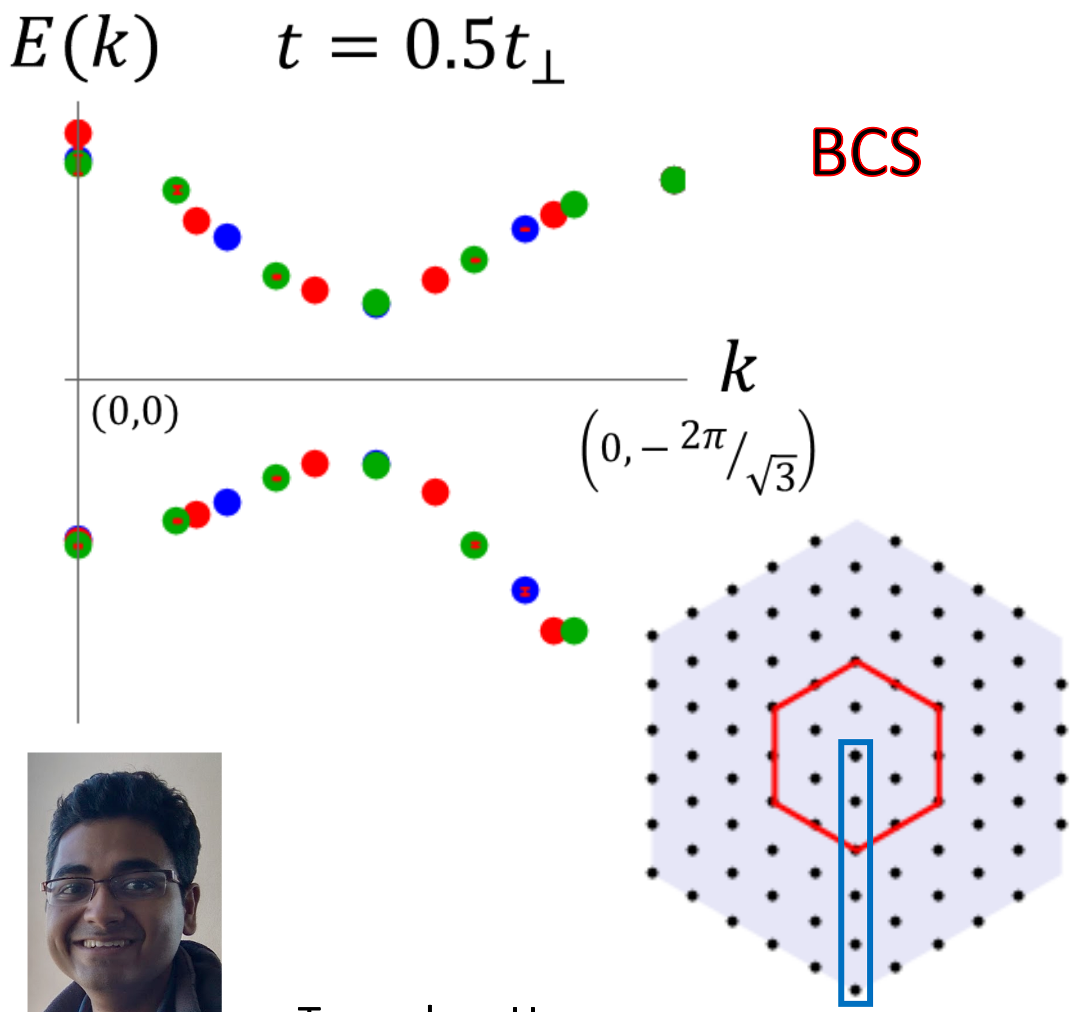
1/(square-root)
In DOS



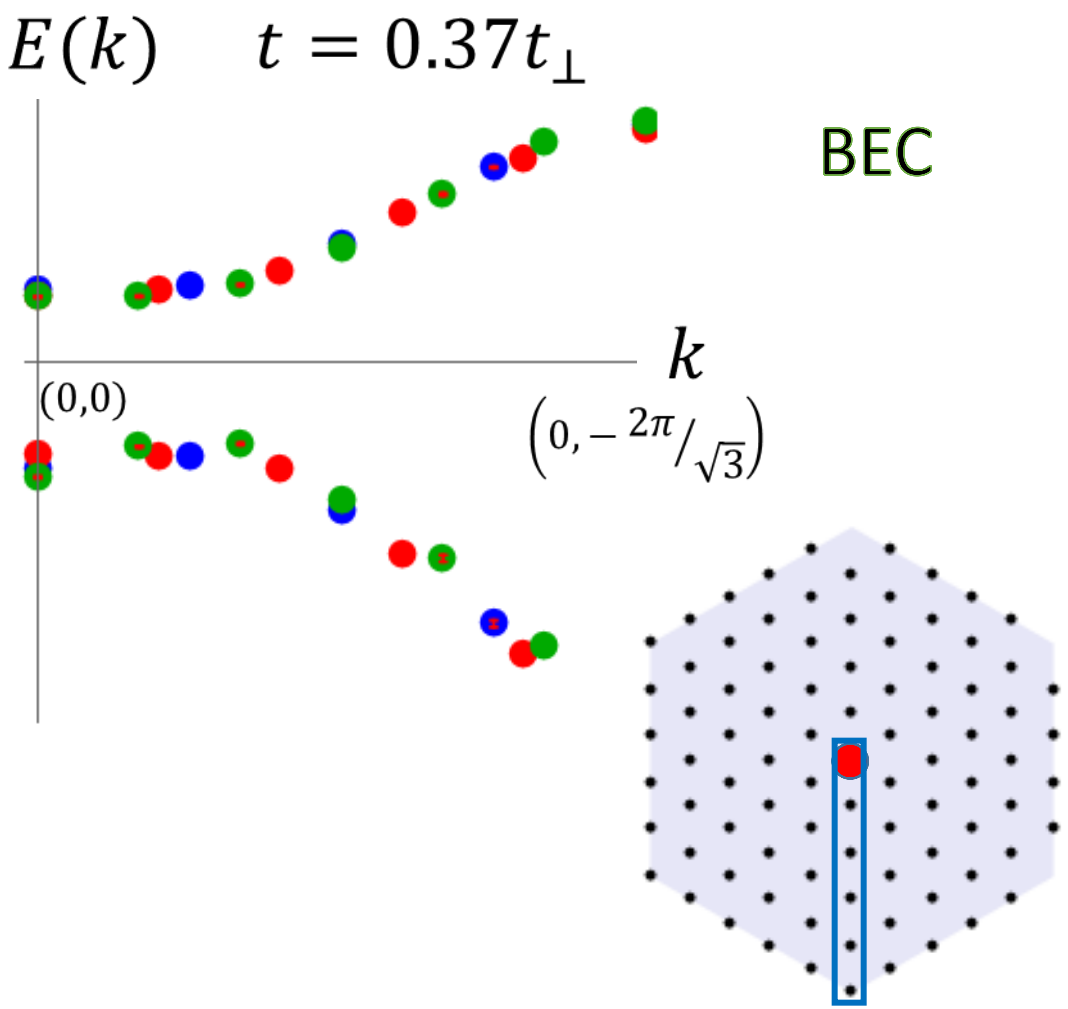
Single particle gap







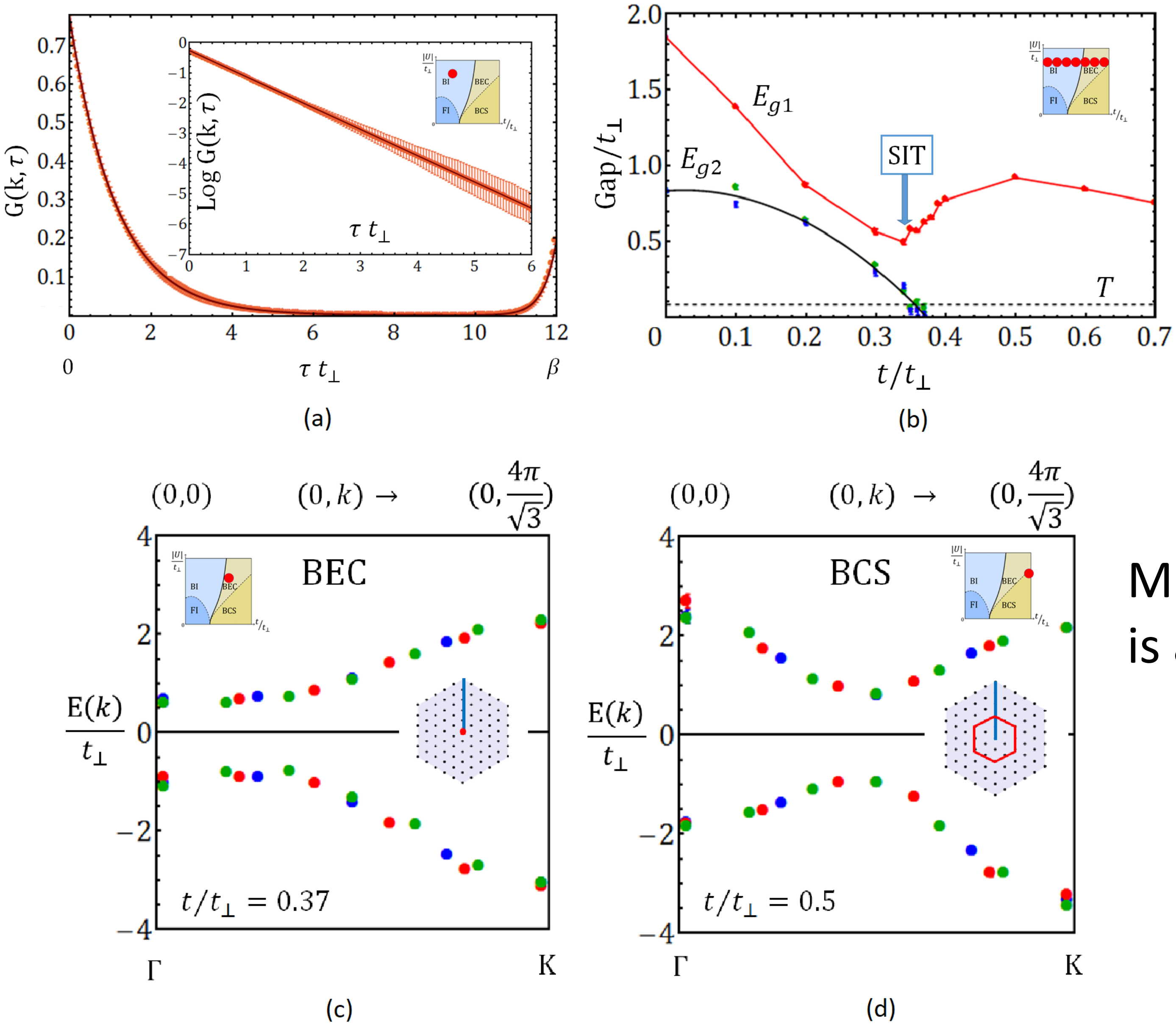
Tamaghna Hazra



● $12 \times 12 \times 2$ ● $10 \times 10 \times 2$ ● $8 \times 8 \times 2$

BCS-BEC Crossover : Topology of Minimum gap locus

Minimum gap locus is a <u>point</u> at k=0



Minimum gap locus is a <u>contour</u> at

$$\epsilon_k = \mu$$

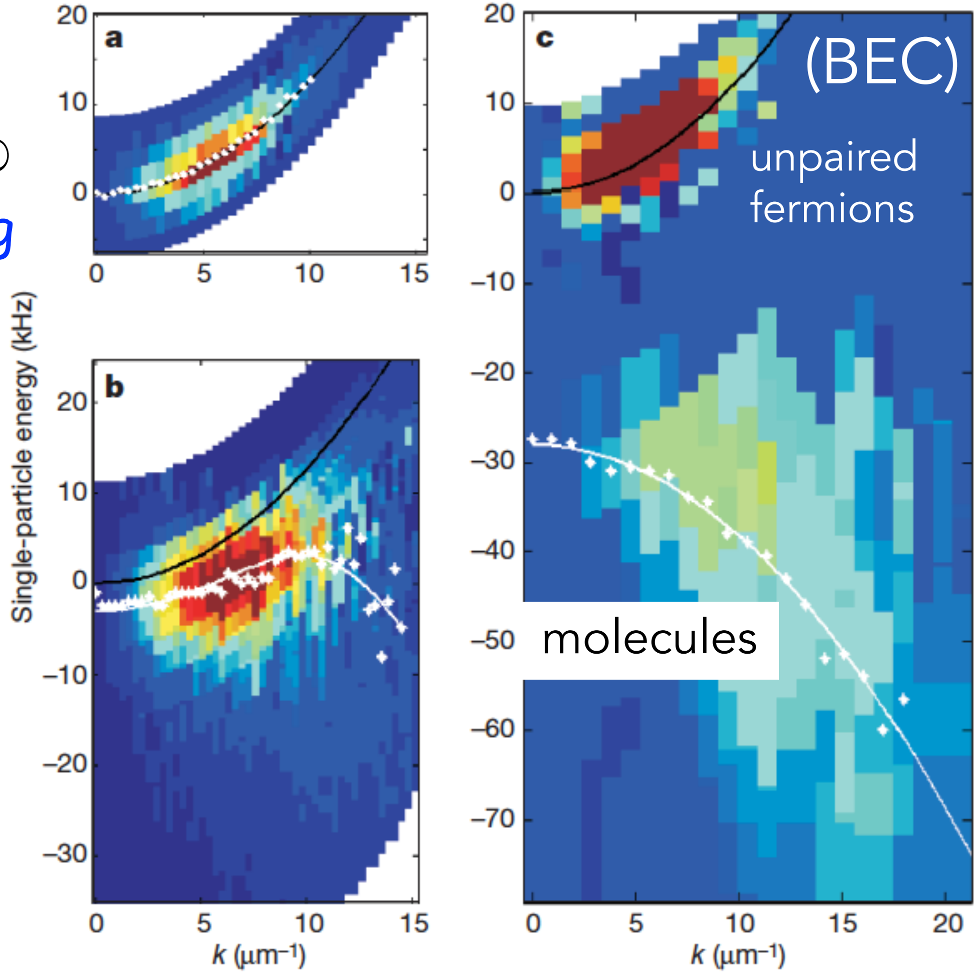
● $12 \times 12 \times 2$ ● $10 \times 10 \times 2$ ● $8 \times 8 \times 2$

Example from Cold Atoms Experiment

k-resolved RF Spectroscopy (~ ARPES) of strongly interacting Fermi Gases

$1/(k_F a) = -\infty$

non-interacting



$1/(k_F a) = 0$

unitarity

$1/(k_F a) \approx 1$

BEC regime

Stewart, Gaebler & Jin, Nature 454, 744 (2008)

Summary of Insulator-Superconductor Transition:

Fermi Insulator	Bose Insulator	BEC	BCS
Lowest energy excitation is a charge e fermion	Lowest energy excitation has charge 2e -- "boson" → goes soft at SIT	Minimum Gap locus is a Point	Minimum Gap locus is a Contour

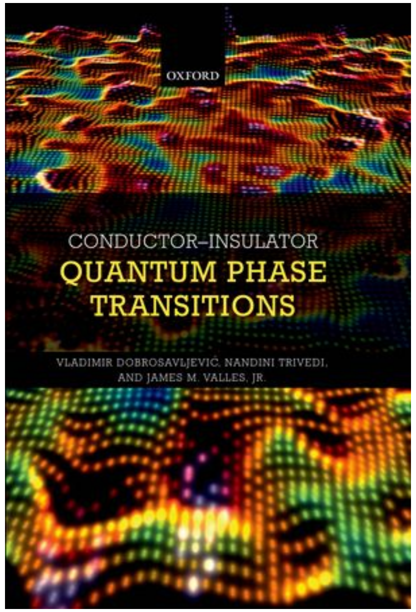
crossover (Fermi Insulator ↔ Bose Insulator)

SC- Insulator Transition (Bose Insulator ↔ BEC)

crossover (BEC ↔ BCS)

* Topology of minimum gap locus
* Gap-edge singularity in DOS

Disorder-driven Insulator-Superconductor Transition

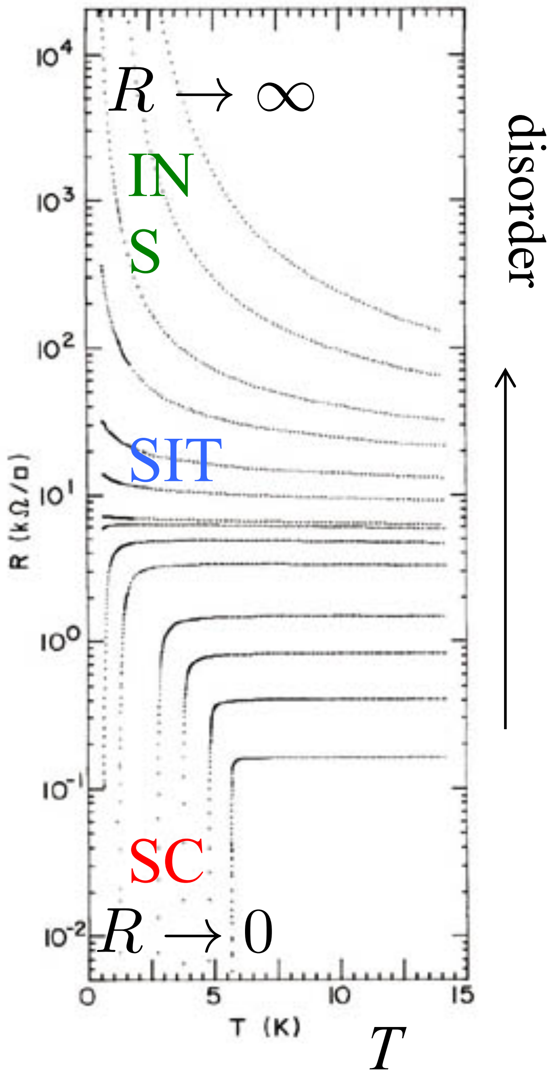


Conductor-Insulator Quantum Phase Transition
(ed. V. Dobrosavljevic, N. Trivedi,
and J. M. Valles) Oxford (2012)

– Quantum phase transition

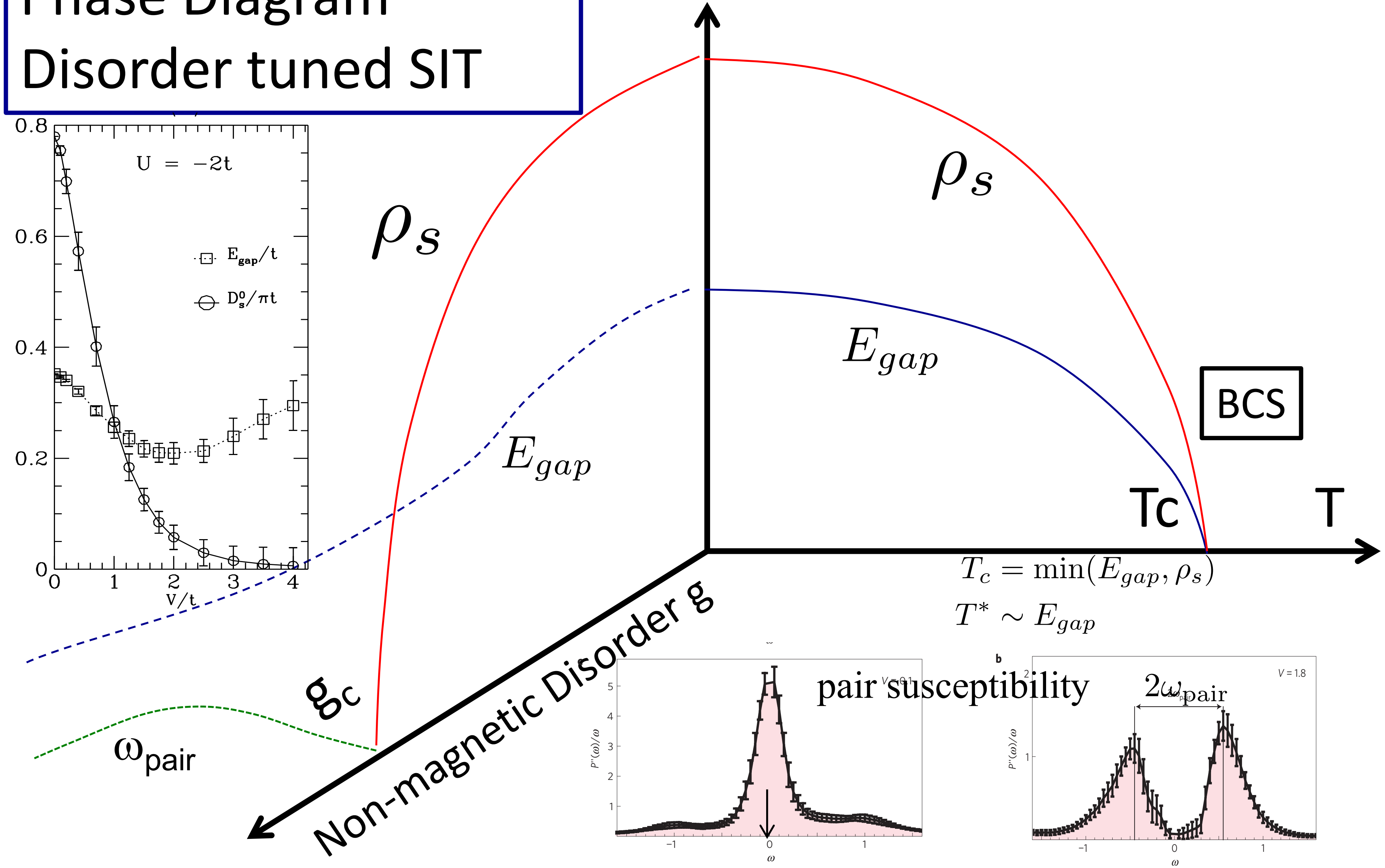
– Is the SIT "fermionic" or "bosonic"?

What is the nature of the insulator?



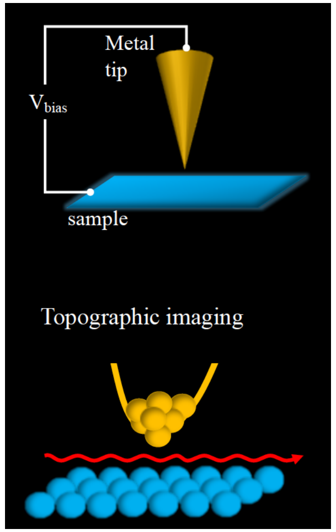
Haviland, Liu, Goldman,
PRL 62,2180 (1989).

Phase Diagram
Disorder tuned SIT



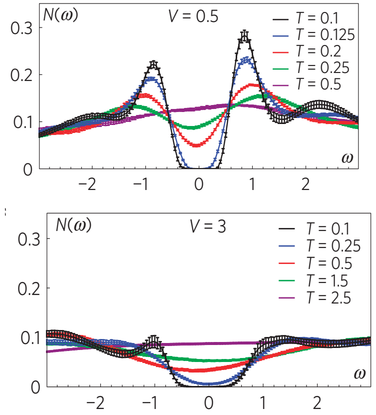
K. Bouadim, Y. L. Loh, M. Randeria, and N. Trivedi, Nat. Phys., 7, 884 (2011).
A. Ghosal, M. Randeria, and N. Trivedi, PRL 81, 3940 (1998); PRB 65, 014501 (2001).

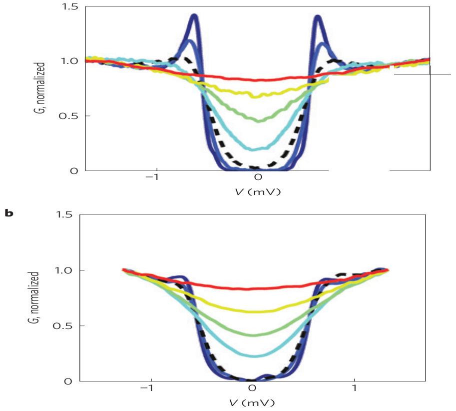
Local gap and Local superfluid density

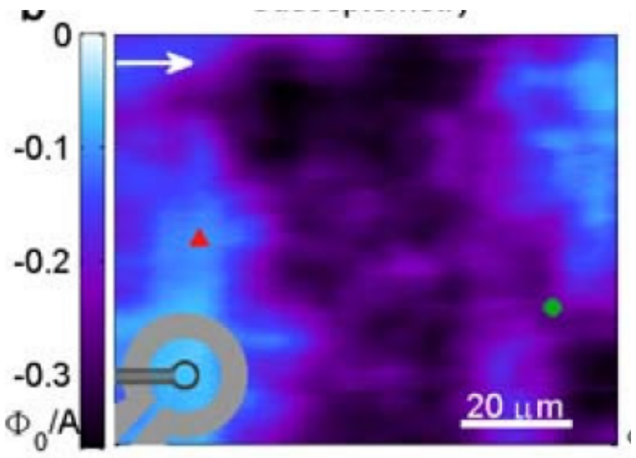


Scan-tunneling

DOS gapped on *both* sides of transition

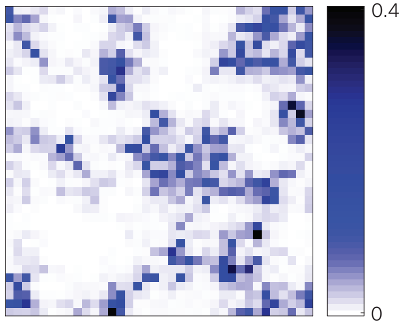


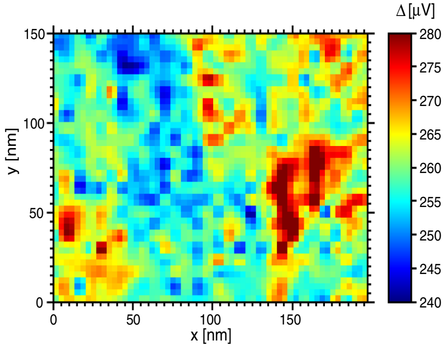




Scan-squid

Disorder leads to "SC puddles"





Theory: Ghosal *et al*, PRB 65 014501 (2001) PRL 81 3940 (1998)
Bouadim *et al*, Nat. Phys. 7, 884 (2011)

Experiment: Sacepe, *et al*, Nat. Phys **7** 239 (2011), PRL **101**, 157006 (2008)

Insulator to SC

(A) ***the insulator*** has the "seeds" of the kind of SC that will be born

(B) **non BCS SC:**

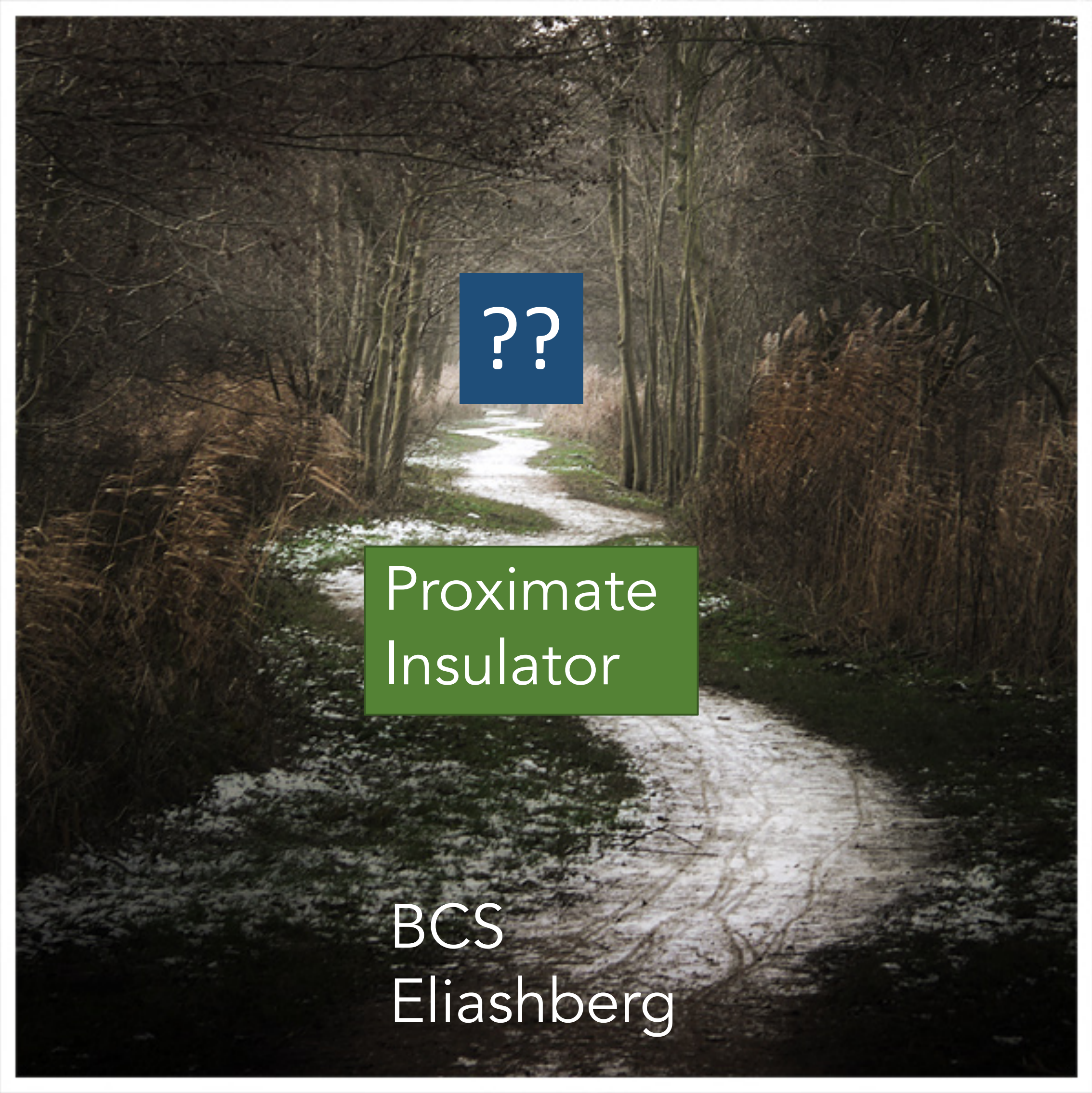
- Pairing and Coherence separated
- Dome shape of Tc
- Topological transition from BEC-BCS

Roadmap for Superconductivity:

Semi-Metal→ Topological SC

Insulator→ SC

Metal→ SC



Roadmap for Superconductivity:

Semi-Metal→ Topological SC

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