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# Pairing effects in the normal phase of a two-dimensional Fermi gas

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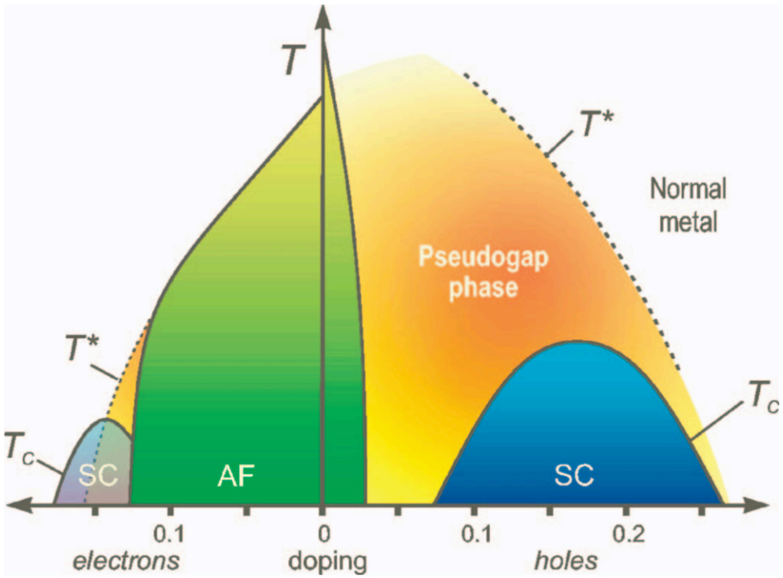
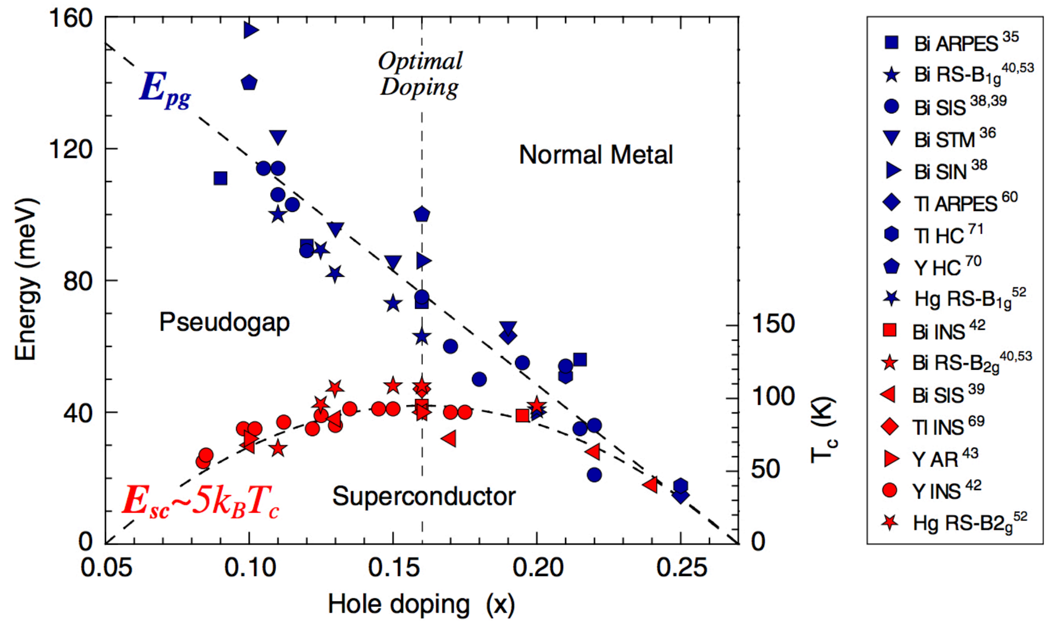
# Motivation

# Where all started: high-Tc superconductors



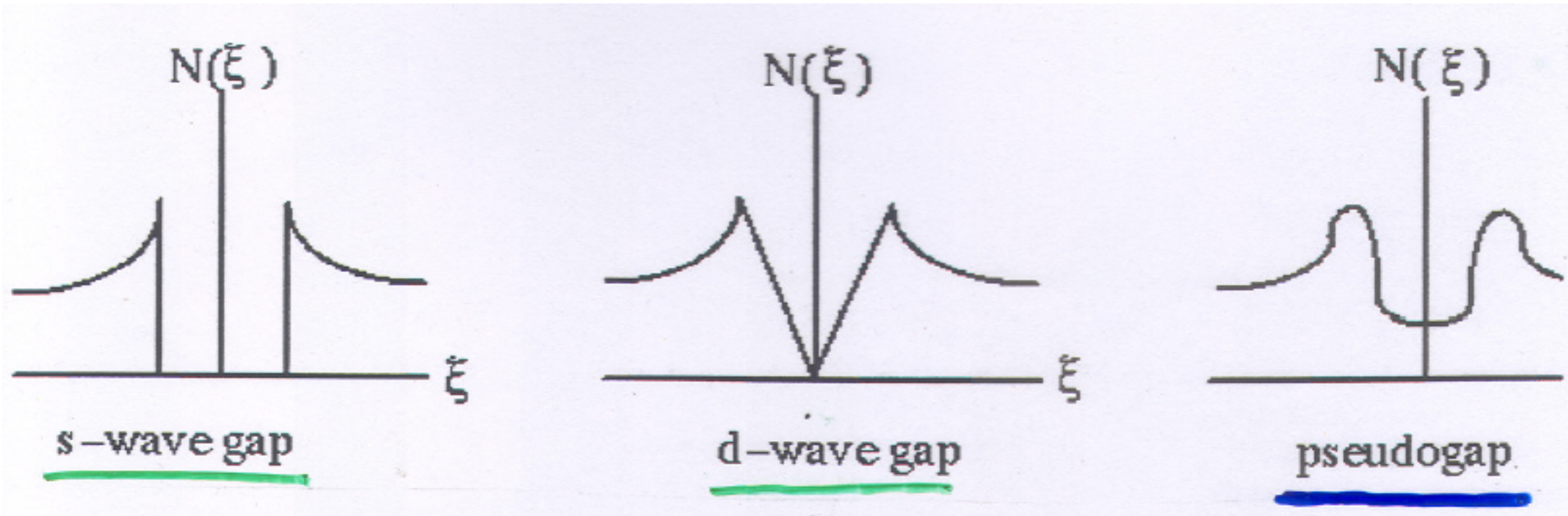

Rep. Prog. Phys. **71** (2008) 062501

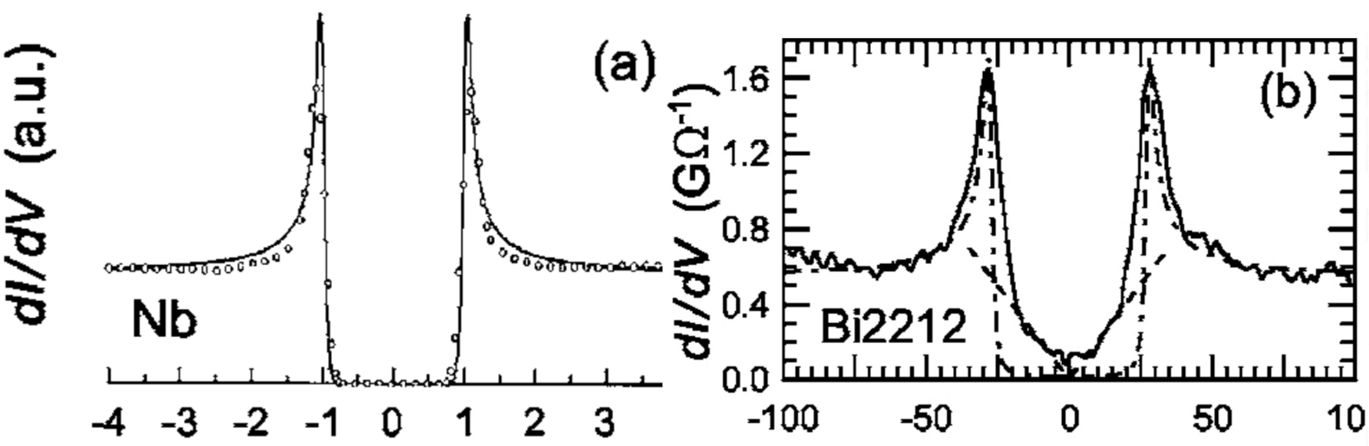


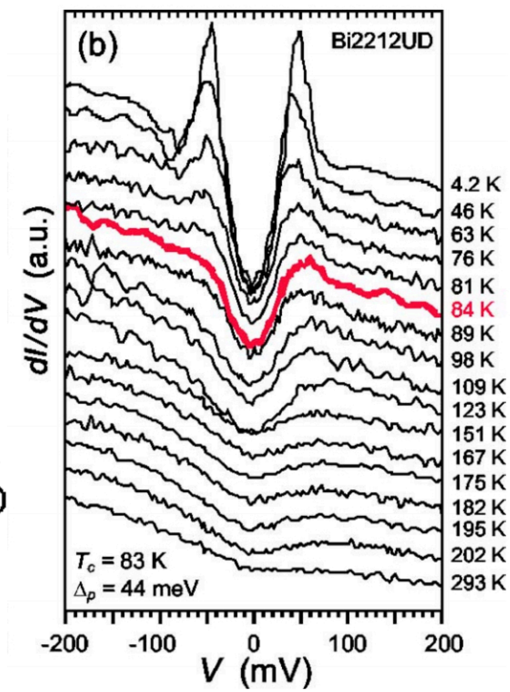

S. Huefner *et al.*, Rep. Prog. Phys. **71**, 062501 (2008)

Pseudogap: suppression of spectral weight about the Fermi energy.
Competing "phase" or precursor of superconducting gap?

# Pseudogap vs gap: density of states



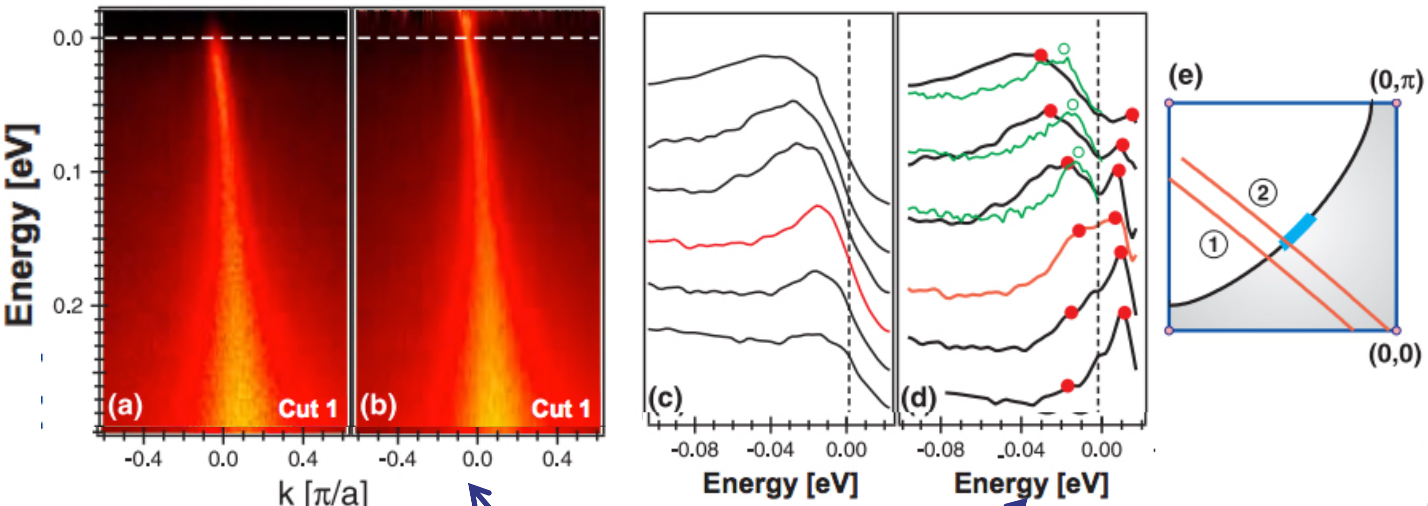



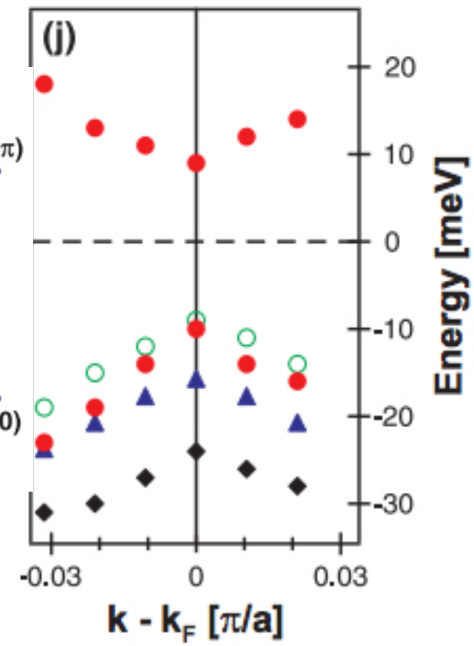




O. Fischer *et al.*, Rev. Mod. Phys. **79**, 353 (2007)

# Pseudogap and ARPES spectra

Underdoped $La_{1.895}Sr_{0.105}CuO_4$ at T=49K > Tc=30 K




M. Shi *et al*., EPL **88**, 27008 (2009)

**Fermi-function divided** spectra

Triangles and diamonds are the dispersions at 49K along cuts closer to the anti-nodal points.

Dispersions in the gapped region of the Brillouin zone (cut 1 in (e)). The **full circles** are the two branches of the dispersion at **49K**, **open circles** correspond to the same cut but at **12K**..

$$I(k,\omega) \propto A(k,\omega) f(\omega)$$

# Logical path

- Physics of high-Tc superconductors motivates the study of models where pseudogap and precursor effects are driven by a strong pairing attraction (one of the two main scenarios).

Normal phase of a superconductor/superfluid undergoing the BCS-BEC crossover.

- **Not an easy problem for theorists**: finite temperature, strong interaction, dynamical (frequency- dependent).

- Experiments with ultra-cold gases addressing this problem: three-dimensions (D. Jin's group), **two-dimensions** (M. Koehl's, S. Jochim's groups).

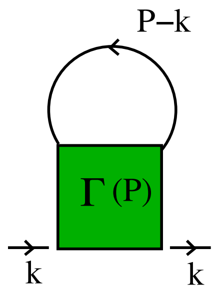
Still, difficulties in the experiments and its interpretation: averages over trap or momentum, final state effects…

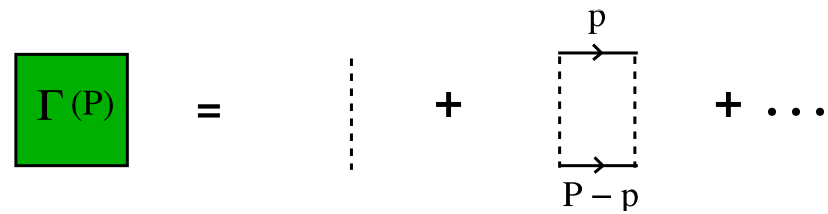
# Theoretical approach

# Inclusion of pairing fluctuations above Tc

T-matrix self-energy (sum of ladder diagrams):

$\Sigma(\mathrm{k})$ =




$$\Sigma(k) = -\int \frac{d\mathbf{P}}{(2\pi)^D} \frac{1}{\beta} \sum_{\Omega_\upsilon} \Gamma_0(P) G^0(P-k)$$

$k = (\mathbf{k}, \omega_n)$ ; $P = (\mathbf{P}, \Omega_\upsilon)$

$p = (\mathbf{p}, \omega_l)$

where, for a contact potential:

$$-\Gamma_0(P)^{-1} =$$

$$\frac{1}{\mathrm{v}_0} + \int \frac{d\mathbf{p}}{(2\pi)^D} \frac{1}{\beta} \sum_{\omega_l} G^0(p+P)\, G^0(-P)$$

3D

$$= \frac{m}{4\pi a} + \int \frac{d\mathbf{p}}{(2\pi)^3} \left[ \frac{1}{\beta} \sum_{\omega_l} G^0(p+P)\, G^0(-P) - \frac{m}{p^2} \right]$$

2D

$$= \int \frac{d\mathbf{p}}{(2\pi)^2} \left[ \frac{1}{\beta} \sum_{\omega_l} G^0(p+P)\, G^0(-P) - \frac{m}{p^2 + \varepsilon_0} \right]$$

# Single particle spectral function and density of states

Spectral function determined by **analytic continuation** to the real axis $i\omega_n \to \omega + i0^+$ of the temperature Green's function:

$$G(k) = \left[G^0(k)^{-1} - \Sigma(k)\right]^{-1}$$

$$G(k, i\omega_n) \to G(k, \omega + i0^+) \equiv G^R(k, \omega)$$

$$A(k,\omega) = -\frac{1}{\pi}\mathrm{Im}\, G^R(k,\omega) = \frac{(-1/\pi)\,\mathrm{Im}\,\Sigma(k,\omega)}{[\omega - \xi(k) - \mathrm{Re}\,\Sigma(k,\omega)]^2 + \mathrm{Im}\,\Sigma(k,\omega)^2}$$

$$\xi(k) = \frac{\hbar^2 k^2}{2m} - \mu$$

The continuation to real axis can be performed **exactly**, without resorting to approximate methods (such as MaxEnt, Padé …).

**density of states** $\quad N(\omega) = \int \frac{d\mathbf{k}}{(2\pi)^3} A(k,\omega)$

**momentum distribution** $\quad n_k = \int_{-\infty}^{+\infty} A(k,\omega) f(\omega)\, d\omega$

# Why T-matrix self-energy?

It recovers the correct asymptotic theories:

- For weak coupling and $T \ll T_F$: Galistkii in 3D; Bloom (1975), Engelbrecht & Randeria (1992) in 2D (provided T is not too low).

- For strong coupling and $T \ll \varepsilon_0$: non interacting Bose gas

- At high temperature: virial expansion till second order

## Why non-self-consistent?

- It allows for exact analytic continuation

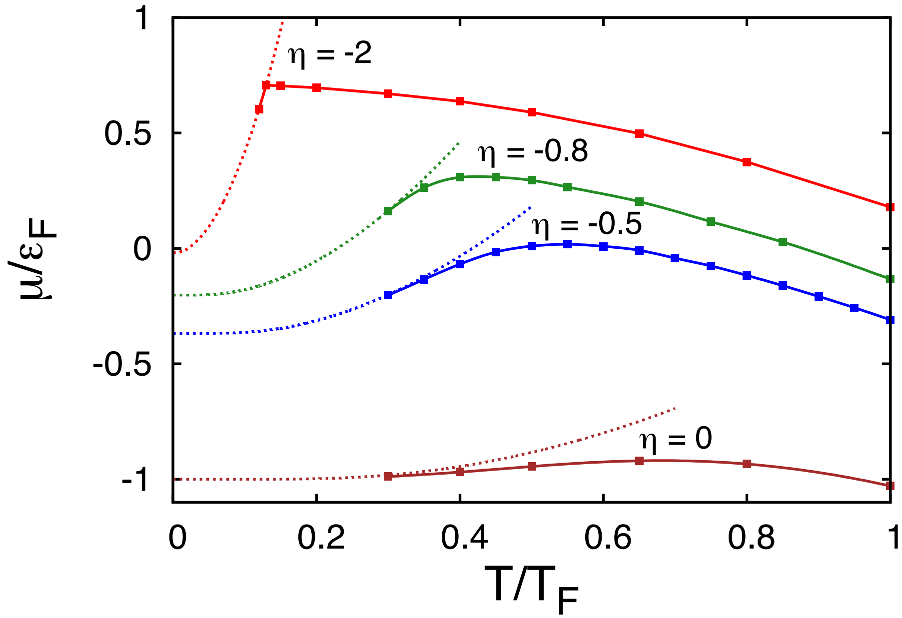
- In condensed matter physics, often implementing self-consistency worsens the calculation of **dynamical** properties like the spectral weight-functions [see e.g. B. Holm and U. von Barth, PRB **57**, 2018 (1998); D. Rohe and W. Metzner PRB **63**, 224509 (2001).]

# Shortcomings of T-matrix in two dimensions

At the critical temperature

$$\Gamma_0(\Omega_\nu = 0, q) \propto \frac{1}{q^2} \longrightarrow \int \frac{d^2 q}{(2\pi)^2} \Gamma_0(\Omega_\nu = 0, q) = \infty \longrightarrow \quad \Sigma(k) \text{ diverges at all } k$$

The **critical temperature can't be reached** (both in self-consistent and non-self-consistent schemes). Sometimes in the literature connected to Mermin-Wagner theorem forbidding long-range order. But at the BKT transition the large distance decay of the pair-pair correlation function changes form exponential to algebraic $\longrightarrow$ the particle-particle propagator Γ should diverge!

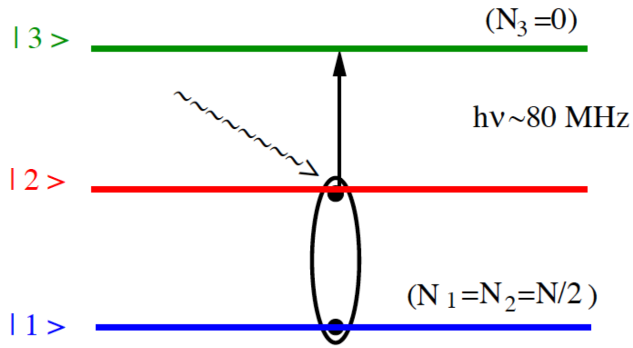



**Unphysical behavior** of the chemical potential **in non-self-consistent scheme at low T** because at given coupling strength η the curve μ(T) wants to avoid the curve $\Gamma_0(0,0;\mu,T)^{-1} = 0$.

$$\eta \equiv \ln\left(\frac{1}{k_F a_{2D}}\right) = \frac{1}{2}\ln\left(\frac{\varepsilon_0}{2E_F}\right)$$

# RF spectroscopy

## How does the spectral function enters in RF spectroscopy?




In the absence of final state interaction, linear response theory yields for the **RF** experimental signal:

$$RF(\omega_\delta) = \int d^3r \int \frac{d^3k}{(2\pi)^3} A(k, k^2/(2m) - \omega_\delta - \mu_2(r); r)\, f[k^2/(2m) - \omega_\delta - \mu_2(r)]$$

where $\omega_\delta$ is the detuning of the RF probe with respect to the frequency of the atomic transition $|2\rangle \rightarrow |3\rangle$ .

With the tomographic technique introduced at MIT, the trap average can be eliminated:

$$RF(\omega_\delta) = \int \cancel{d^3r} \int \frac{d^3k}{(2\pi)^3} A(k, k^2/(2m) - \omega_\delta - \mu_2(r); r)\, f[k^2/(2m) - \omega_\delta - \mu_2(r)]$$

but average over **k** remains.

# Momentum-resolved RF spectroscopy

Average over **k** can be eliminated (technique pioneered by D. Jin's group):

$$RF(\omega_\delta) = \int d^3r \int \cancel{\frac{d^3k}{(2\pi)^3}} A(k, k^2/(2m) - \omega_\delta - \mu_2(r); r) f[k^2/(2m) - \omega_\delta - \mu_2(r)]$$

but then trap average remains…

**Momentum resolved** RF spectrum proportional to:

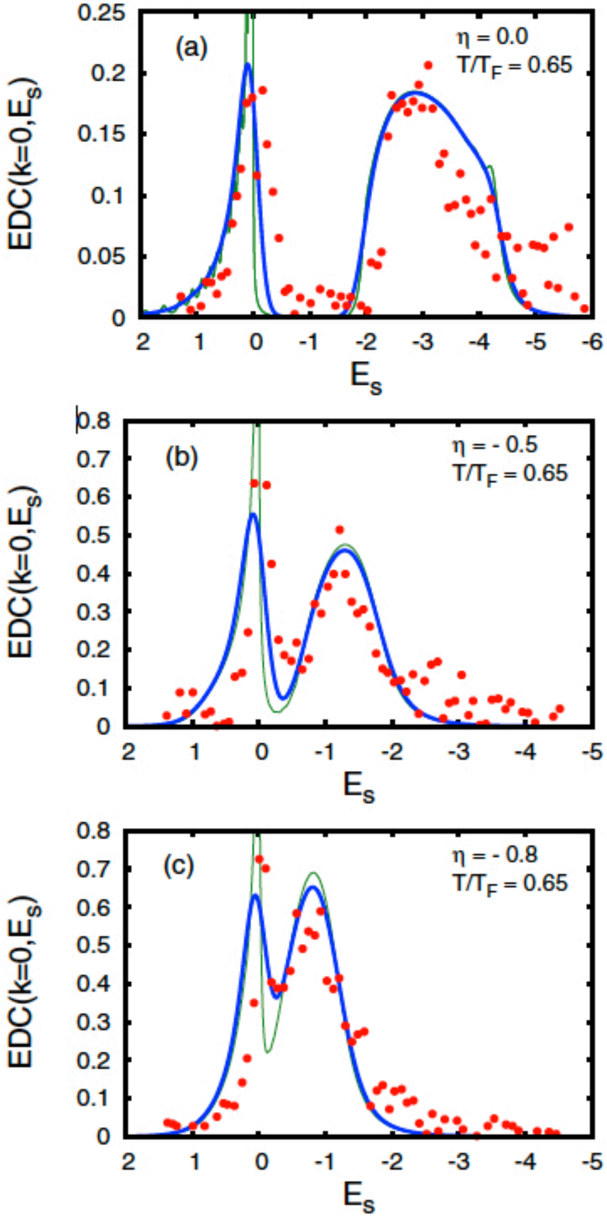
$$RF(k; E_s) = k^2 \int d^3r A(k, E_s - \mu_2(r); r) f[E_s - \mu_2(r)]$$

where $E_s = k^2/(2m) - \omega_\delta$ is the **"single-particle energy".**

It's the technique which is closest to ARPES spectra in solids:

$$I(k,\omega) \propto A(k,\omega) f(\omega)$$

# Validation of the theory against exp. data




Experiment (momentum-resolved RF):
M. Feld *et al.*, Nature (London) **480**, 75 (2011)

Theory:
F. Marsiglio *et al.*, PRB **91**, 054509 (2015)

Local coupling and temperature at trap center

| $\eta$ | $\eta(0)$ | $T/T_F(0)$ |
|---|---|---|
| −0.8 | −0.66 | 0.84 |
| −0.5 | −0.38 | 0.82 |
| 0.0 | −0.07 | 0.57 |

$$\eta \equiv -\ln\left(k_F a_{2D}\right)$$

obtain the value of $E_F$ at $T/T_F = 0.65$. This yields the value $E_F = 11$ kHz quoted above, which fixes the horizontal scale of the experimental spectra. The vertical scale of the experimental spectra is instead fixed by making the height of the right experimental peak to coincide with the theoretical prediction.

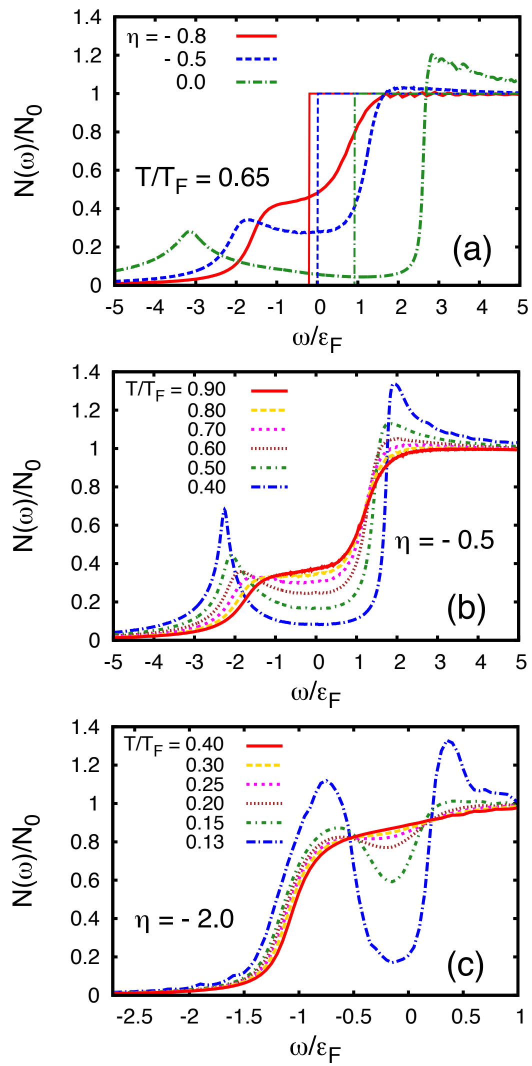
✓ Theory **validated** at these (not too low) temperatures.

pairing effects in the normal phase of 2D homogeneous Fermi gas

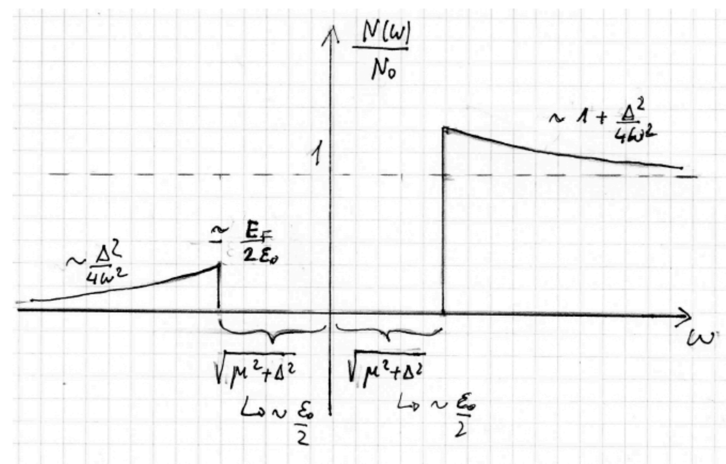
# Homogeneous system: density of states and pseudogap

| $\eta$ | $\varepsilon_0 / E_F$ |
|---|---|
| -2 | 0.035 |
| -0.8 | 0.4 |
| -0.5 | 0.75 |
| 0 | 2 |

$$\eta \equiv -\ln\left(k_F a_{2D}\right)$$

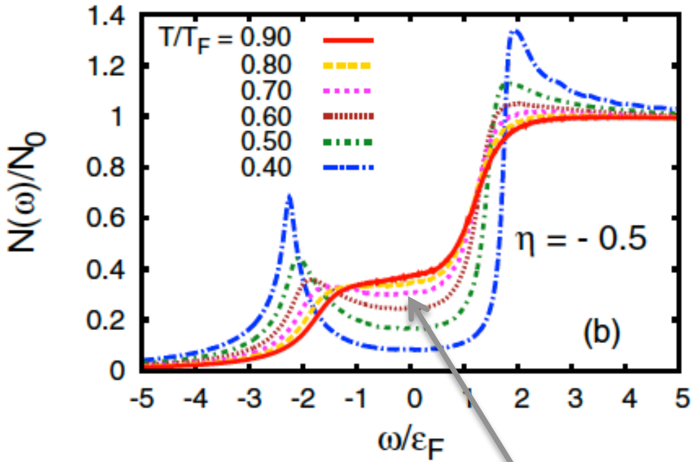



## BCS mean-field DOS in the BEC limit



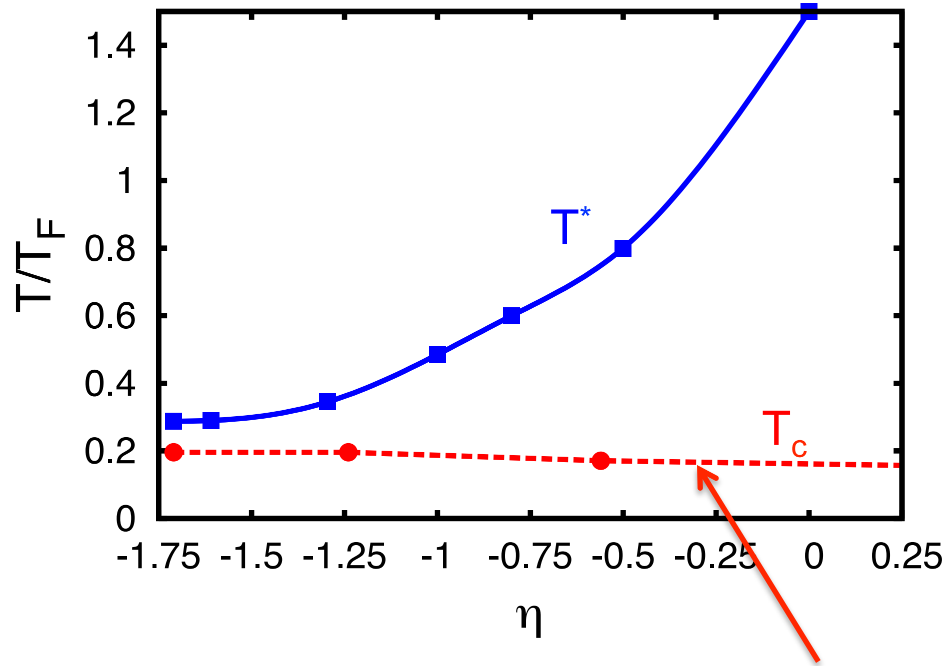

F. Marsiglio *et al.*, PRB **91**, 054509 (2015)

# Crossover pairing temperature $T^*$




Criterion: at $T^*$ a minimum appears in the DOS about $\omega = 0$.

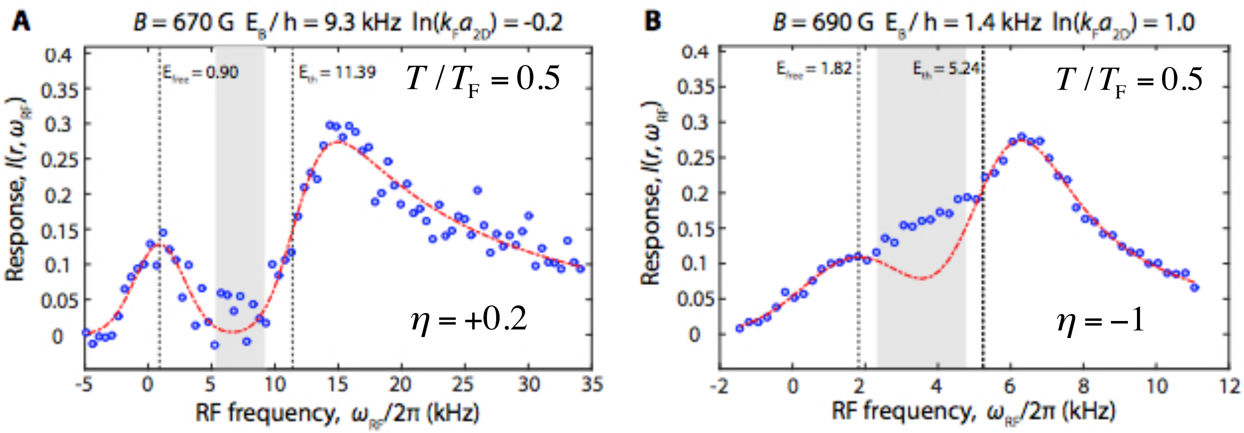
F. Marsiglio *et al*., PRB **91**, 054509 (2015)

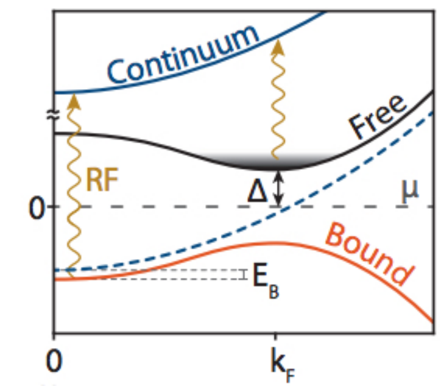



M. Ries et al., PRL **114,** 230401 (2015).

# Comparison with recent Heidelberg group experiment

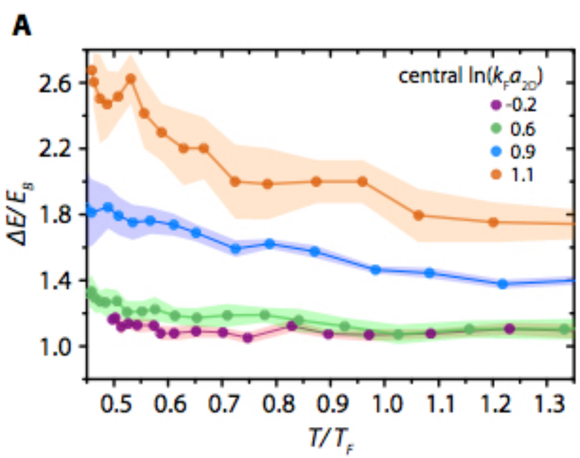
P. A. Murthy *et al.*, arXiv:1705.10577



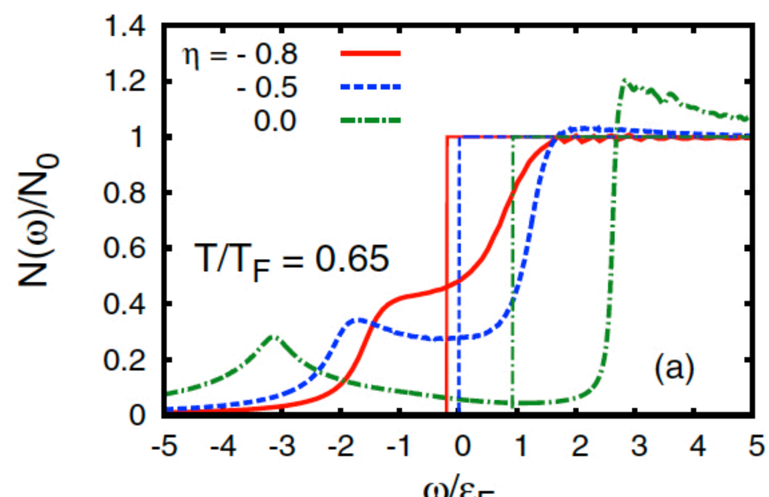




Pairing (pseudo)gap was found in an extended coupling range at fixed temperature $T=0.5\ T_F$.

P. A. Murthy *et al.*, arXiv:1705.10577

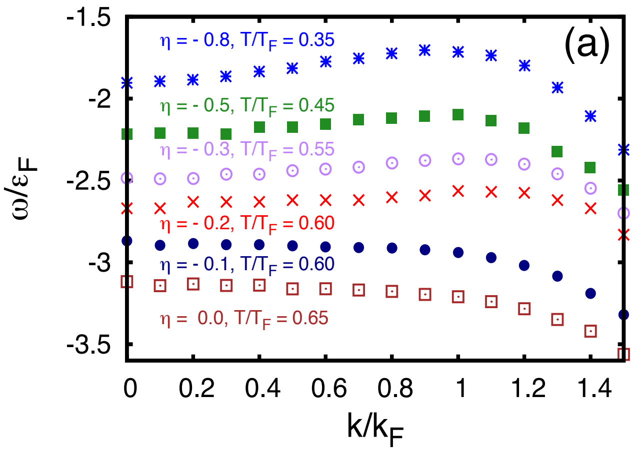


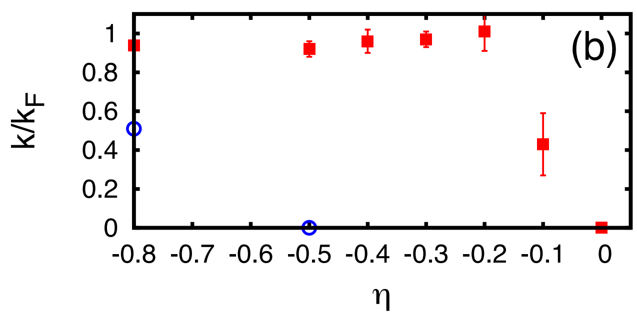

F. Marsiglio *et al.*, PRB **91**, 054509 (2015)

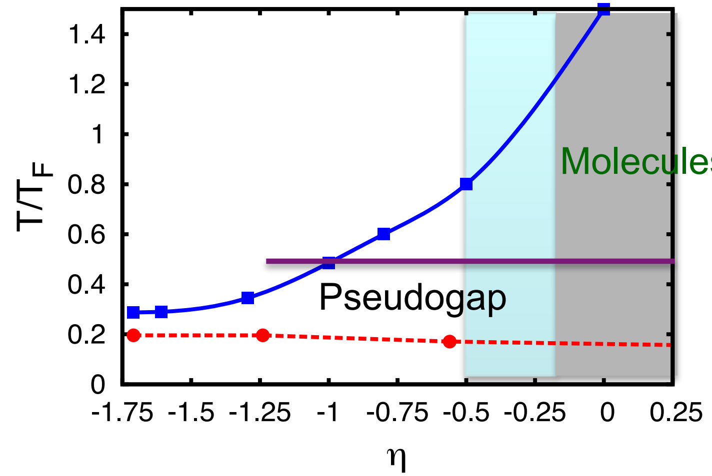


# Boundary of the pseudogap region

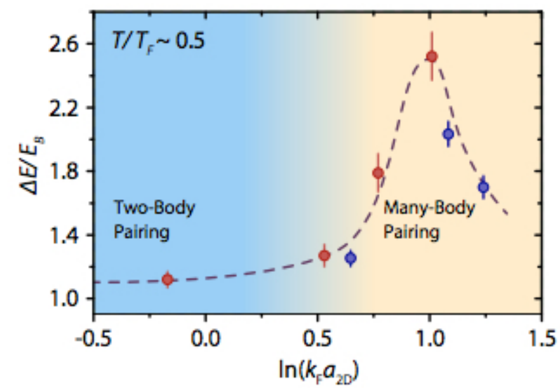
F. Marsiglio *et al.*, PRB **91**, 054509 (2015)



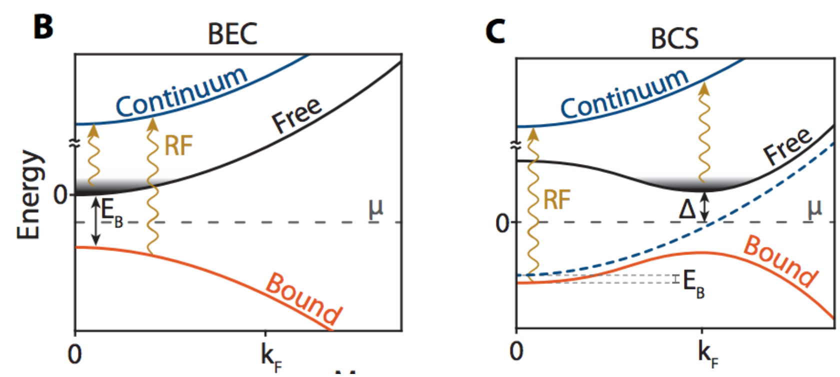



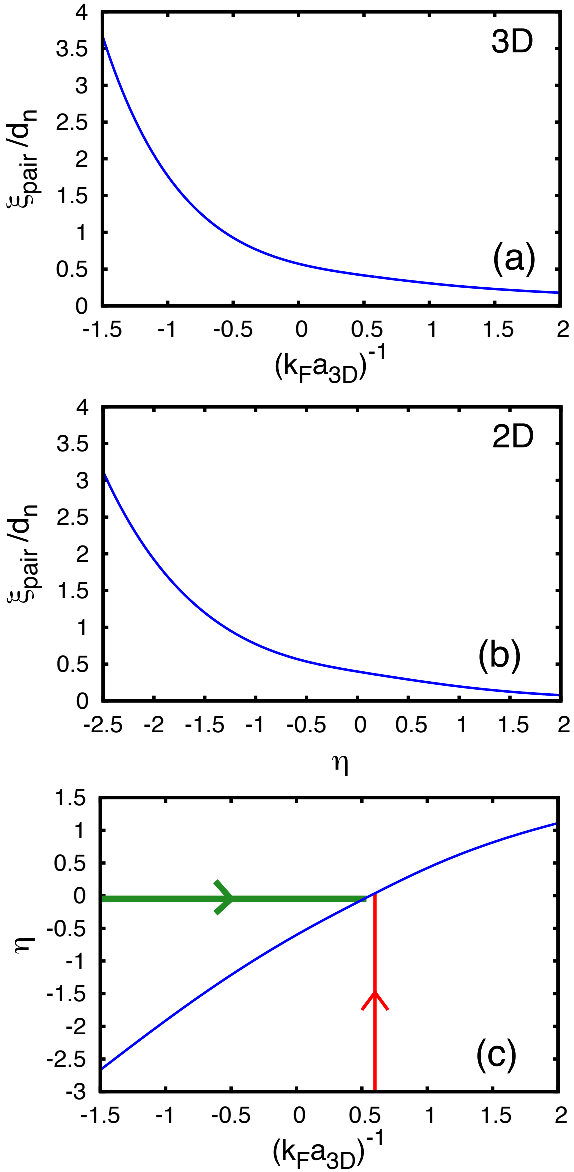

P. A. Murthy *et al.*, arXiv:1705.10577

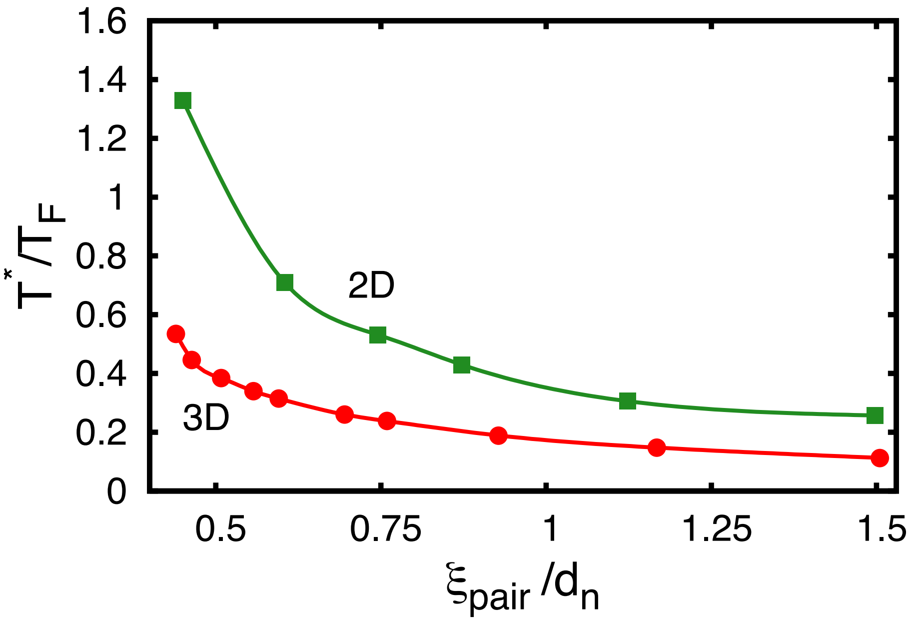



Dispersion back-bends at wave vector $k_L \approx k_F$ in the presence of an underlying Fermi surface. When $k_L \approx 0$: boundary between the pseudogap and molecular regimes.



# Mapping between 3D and 2D




How to compare results obtained in 2D with 3D results? Use the ratio between pair size $\xi_{pair}$ and average inter-particle distance $d_n$ [given by $4\pi n/3)^{-1/3}$ in 3D and by $(\pi n)^{-1/2}$ in 2D].

Strong enhancement of pairing fluctuations due to reduced dimensionality is evident from this comparison.

# Thank you!