Ultracold dipolar atoms in two dimensions: From Wigner crystal to pair superfluidity and ferromagnetism

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BOSE EINSTEIN CONDENSATION

CNR – Istituto Nazionale di Ottica
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Outline

- **Introduction to ultracold dipolar gases**

- **Single layer of dipolar fermions**
  - QPT from Fermi liquid to Wigner crystal

- **Dipolar Fermi polaron in bilayers**
  - Interlayer coupling between impurity and FL or WC

- **Bilayer of dipolar fermions and bosons**
  - Fermions: Novel type of BCS-BEC crossover
  - Bosons: Single-particle to pair superfluidity

- **Single layer of two-component dipolar fermions**
  - Ferromagnetic instability

Cold gases: interactions are s-wave and short range

typical range of interaction $R_0 \approx 10$ nm

typical interparticle distance $1/k_F \approx 100$ nm

s-wave scattering is sufficient to describe interactions

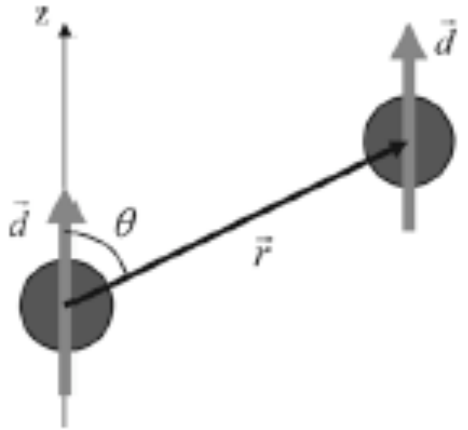
- **With dipoles interactions are anisotropic and long range**

Dipoles aligned along z

$$V(\mathbf{r}) = \frac{d^2}{r^3}\left(1 - 3\cos^2\theta\right)$$

➔ electric dipole d

➔ magnetic dipole d=μ



Strength of interaction

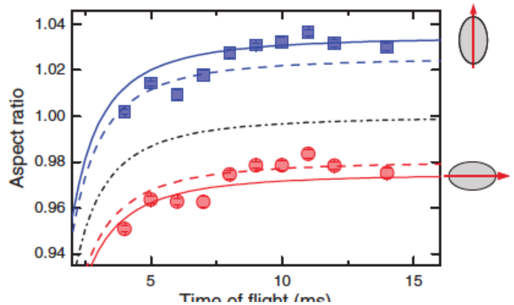
typical length $r_0 = md^2/\hbar^2$

Leads to new interesting many-body effects

Observation of Fermi surface deformation in a dipolar quantum gas

K. Aikawa,[1] S. Baier,[1] A. Frisch,[1] M. Mark,[1] C. Ravensbergen,[1,2] F. Ferlaino[1,2]*

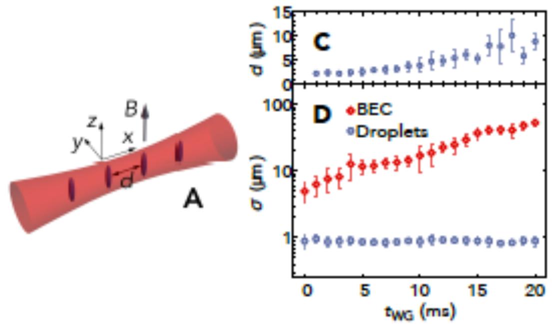
Science, 345 (2014)



Observation of quantum droplets in a strongly dipolar Bose gas

Igor Ferrier-Barbut, Holger Kadau, Matthias Schmitt, Matthias Wenzel, and Tilman Pfau

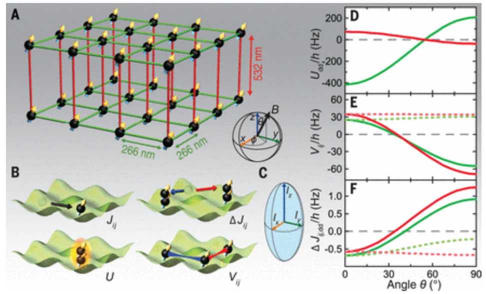
Phys. Rev. Lett., 116 (2016)



Extended Bose-Hubbard models with ultracold magnetic atoms

S. Baier,[1] M. J. Mark,[1,2] D. Petter,[1] K. Aikawa,[1]* L. Chomaz,[1,2] Z. Cai,[2] M. Baranov,[2] P. Zoller,[2,3] F. Ferlaino[1,2]†

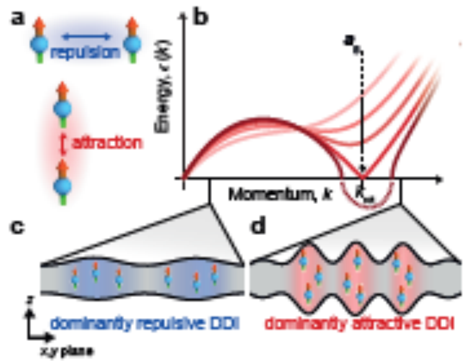
Science, 352 (2016)



Observation of the Roton Mode in a Dipolar Quantum Gas

L. Chomaz[1], R. M. W. van Bijnen[2], D. Petter[1], G. Faraoni[1,3], S. Baier[1], J. H. Becher[1], M. J. Mark[1,2], F. Wächtler[4], L. Santos[4], F. Ferlaino[1,2,*]

arXiv:1705.06914 (2017)



- Atomic species with large magnetic moment

  - **Chromium: Stuttgart – $\mu=6\mu_B$ ➔ d=0.06D - (r_0= 2.4 nm)**
  - **Dysprosium: Stanford, Stuttgart – $\mu=10\mu_B$ ➔ d=0.09D - (r_0= 21 nm)**
  - **Erbium: Innsbruck – $\mu=7\mu_B$ (r_0=10 nm)**

$k_F r_0$= 0.02 - 0.2

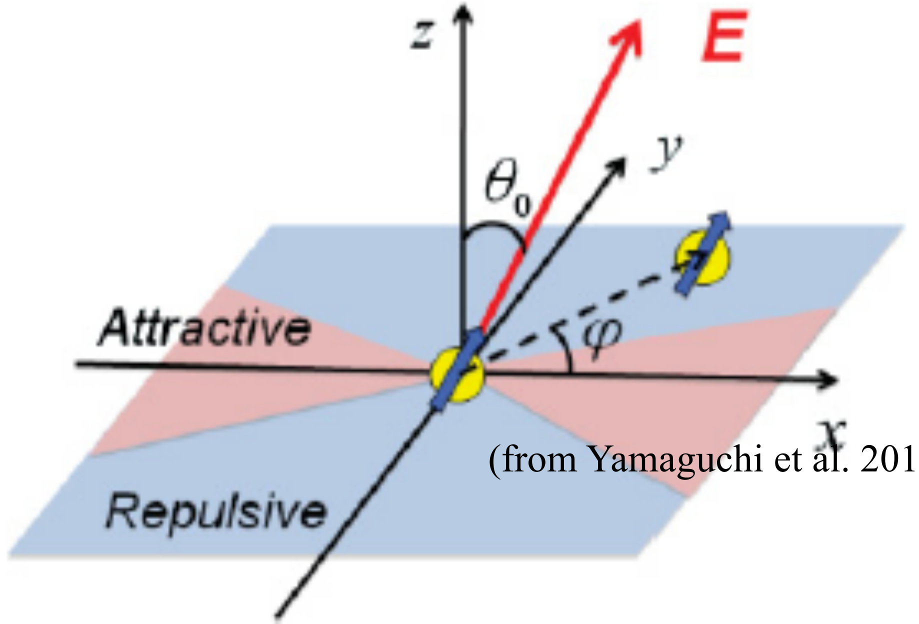
- Heteronuclear molecules with large electric moment

  - **^{40}K-^{87}Rb: JILA ➔ d=0.57D - (r_0= 611 nm)**
  - **^{23}Na-^{40}K: MIT, Hannover ➔ d=2.7D - (r_0=6800 nm)**
  - **^{6}Li-^{133}Cs: Heidelberg ➔ d=5.5D - (r_0= 62 µm)**
  - **….**

$k_F r_0$= 6 – 600

In 2D enhanced stability

$$V(\mathbf{r}) = \frac{d^2}{r^3}\left(1 - 3\sin^2\theta_0 \cos^2\varphi\right)$$



(from Yamaguchi et al. 2010)

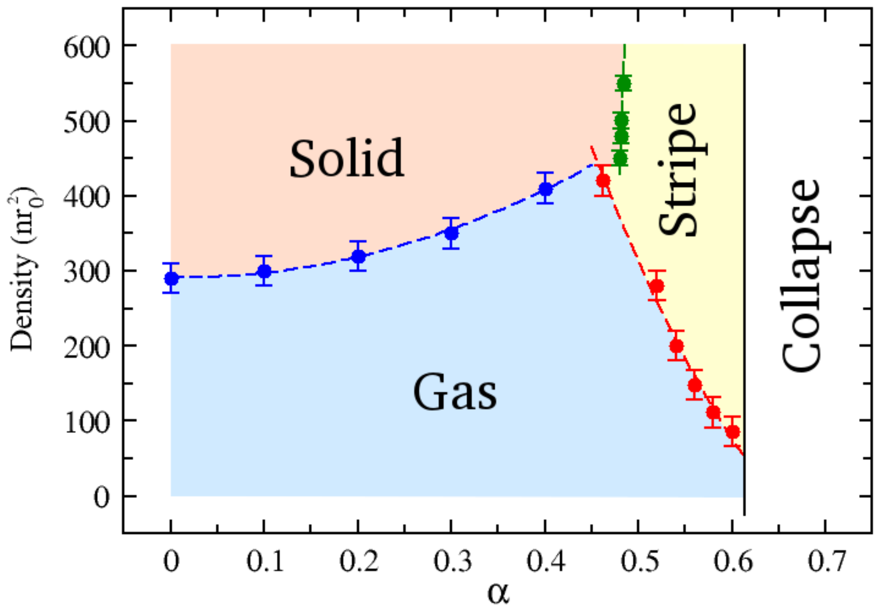
if θ_0=0
interaction purely repulsive

i. **avoids bad chemistry** KRb + KRb ➔ K_2 + Rb_2 + energy

ii. **avoids clusterization due to head to tail attraction**

Single-layer systems

- **perpendicular dipoles**
  - **fluid to solid transition**
    For bosons:
    Astrakharchik et al., Buechler et al.
  - **hexatic phase (Lechner et al.)**

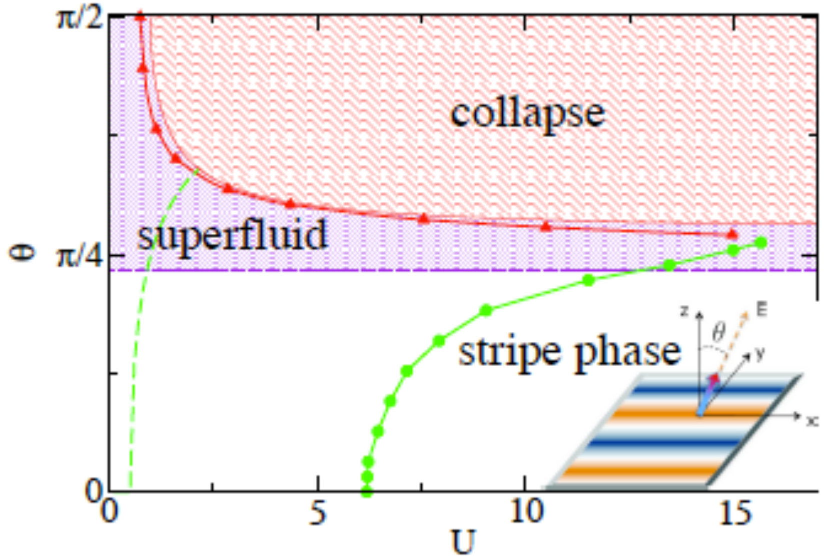


- **tilted dipoles**
  - **CDW (stripe) phase**

(Bruun and Taylor, Parish and Marchetti)

  - **p-wave Fermi superfluidity**

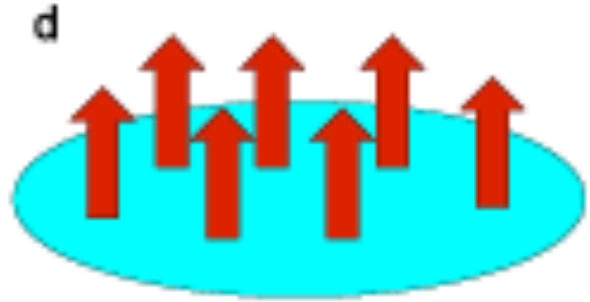
(Sieberer and Baranov)



Hamiltonian

($r_0 >> a_z$ transverse confinement)

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\nabla_i^2 + \sum_{i<j}\frac{d^2}{r_{ij}^3}$$



One dimensionless parameter: $k_F r_0$

$$k_F = \sqrt{4\pi n} \quad r_0 = \frac{md^2}{\hbar^2}$$

Use FN-DMC: projection method

$$\psi_0 e^{-\tau E_0} = \lim_{\tau\to\infty} e^{-\tau H}\psi_T = \lim_{n\to\infty} \underbrace{e^{-\delta\tau H}\ldots e^{-\delta\tau H}}_{n \text{ times}}\psi_T$$

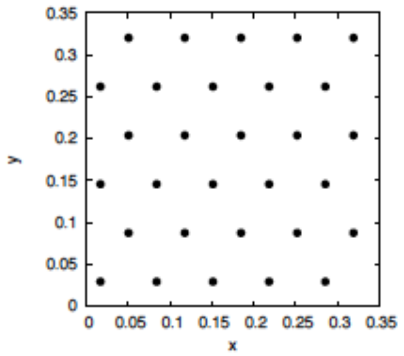
Nodal surface of ψ_T kept fixed during time evolution

➔ E_0 upper bound of ground-state energy

Fermi-liquid phase

$$\psi_T(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \prod_{i<j} f(r_{ij}) \ \det\left(e^{i\mathbf{k}_\alpha\cdot\mathbf{r}_i}\right)$$

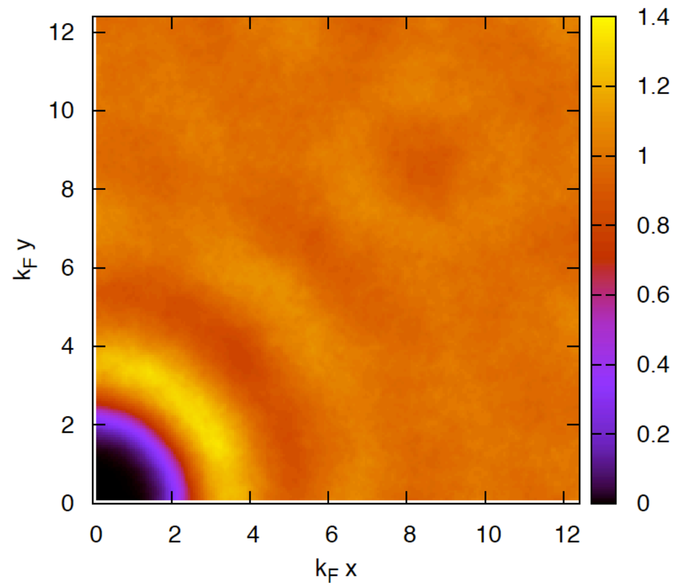
Crystal phase



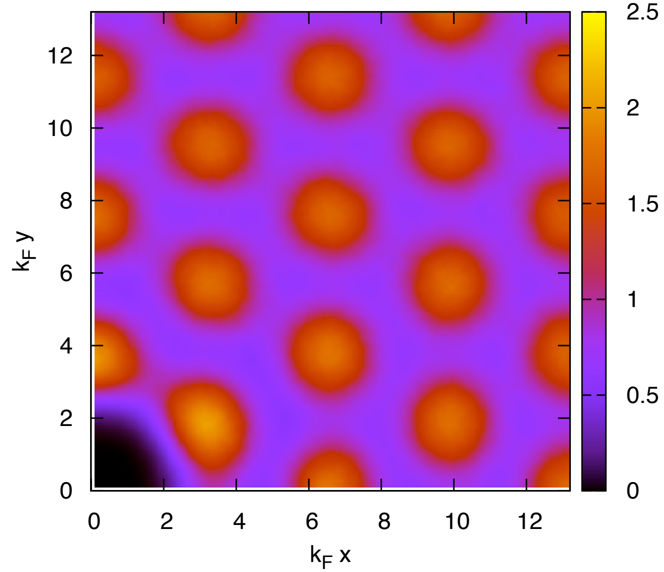
$$\psi_T(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \prod_{i<j} f(r_{ij}) \ \det\left(e^{-(\mathbf{r}_i-\mathbf{R}_m)^2/\alpha^2}\right)$$

$\mathbf{R}_\mathrm{m}$ are the lattice points of the WC

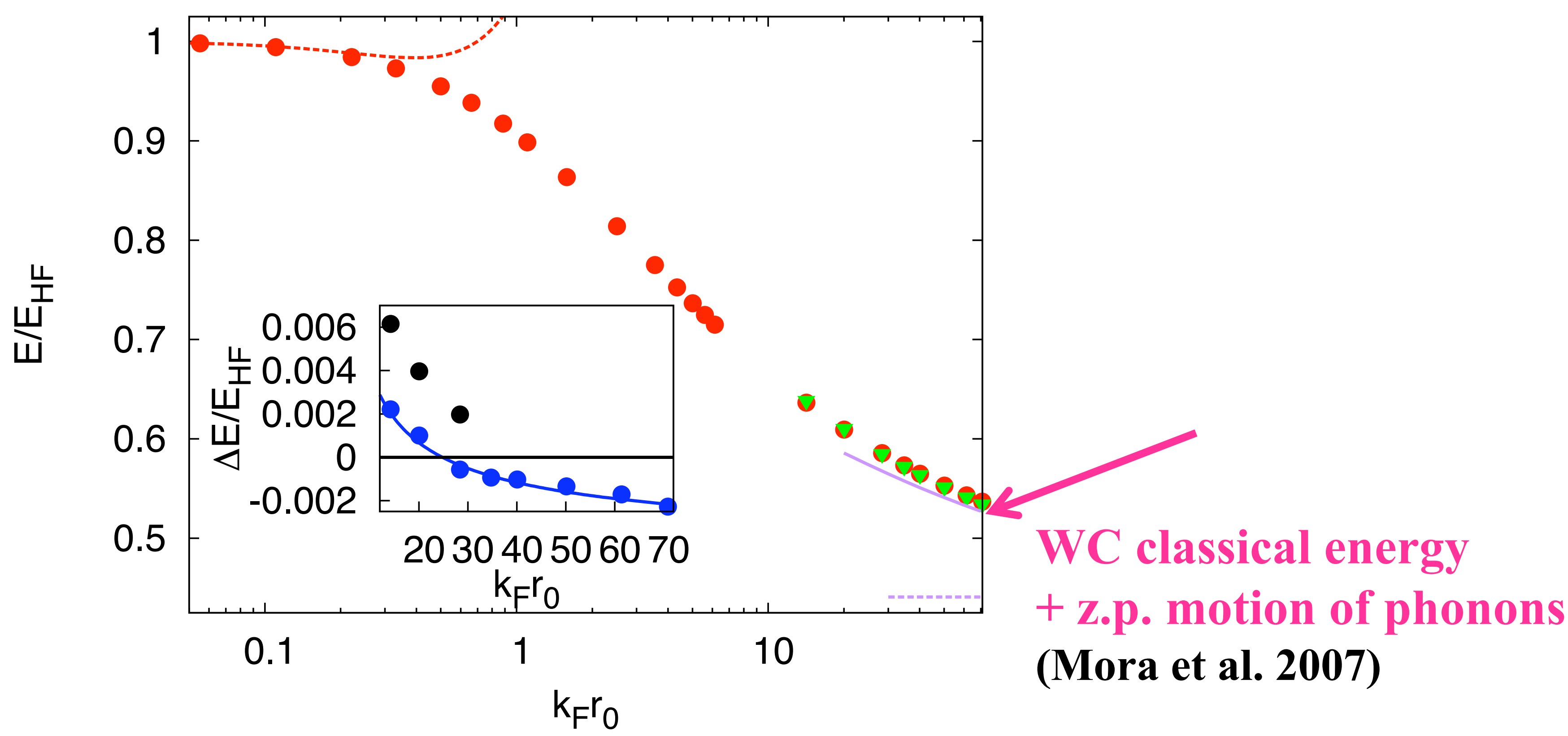
liquid



crystal



Equation of state



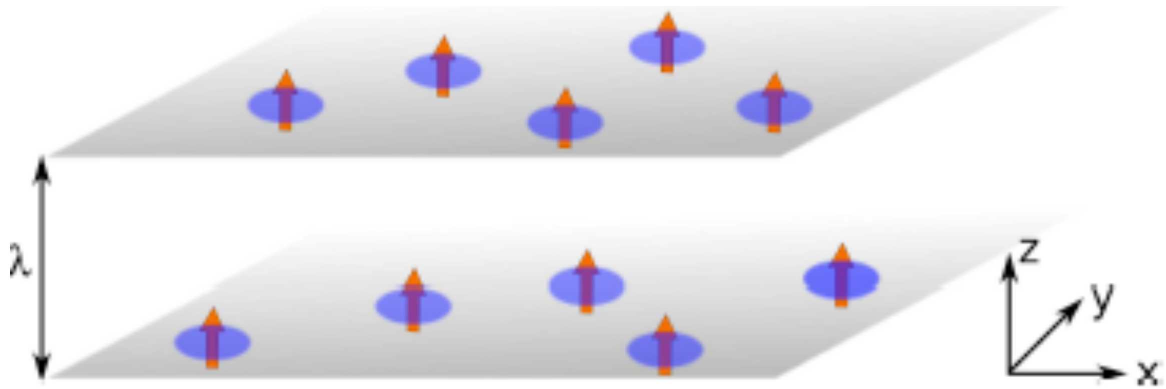
- **FL to WC transition at $k_F r_0$=25±3 (in bosons $k_F r_0 \approx 60$)**

Bilayer system (no interlayer tunneling)

- bound state of two particles (analogy with electron-hole exciton)

- Fermions: interlayer superfluidity and BCS-BEC crossover as a function of separation λ (Pikovski et al.)

  (analogy with electron-hole bilayer and two bilayer graphene – quest for high-Tc superconductivity)

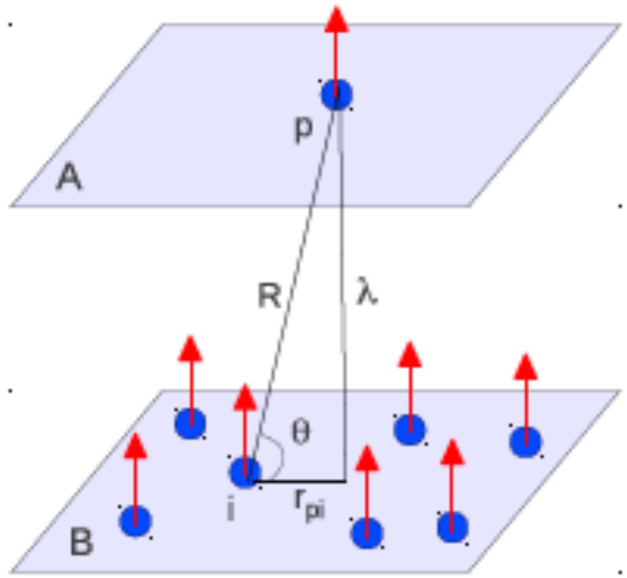


Polaron problem in bilayer system

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\nabla_i^2 + \sum_{i<j}\frac{d^2}{r_{ij}^3} + \sum_{i=1}^{N} V(r_{ip})$$

where

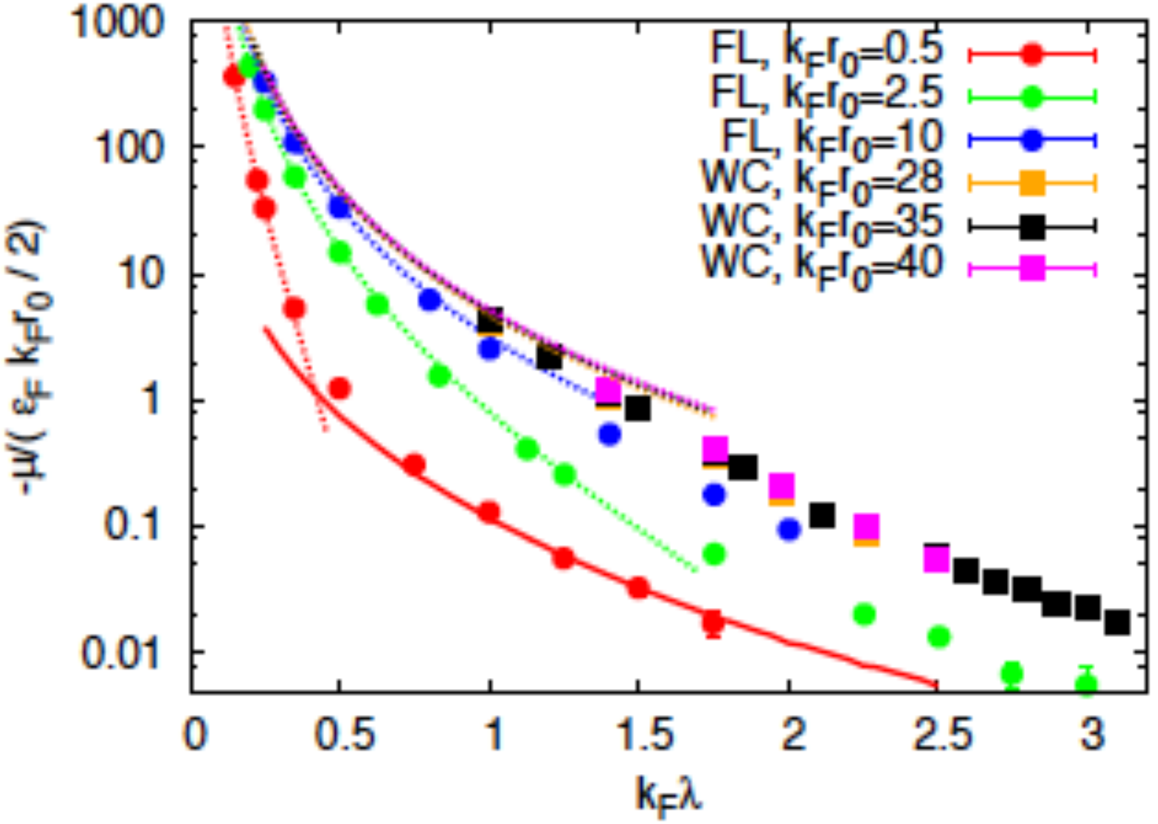
$$V(r_{ip}) = \frac{d^2(r_{ip}^2 - 2\lambda^2)}{(r_{ip}^2 + \lambda^2)^{5/2}}$$



- **Bound state always exists for 2 particles**
- **Many-body problem depends on:**
  a. $k_F r_0$ (interaction in lower layer)
  b. $k_F \lambda$ (interlayer coupling)

Polaron energy

$$\mu_P = E_{N+pol} - E_N$$

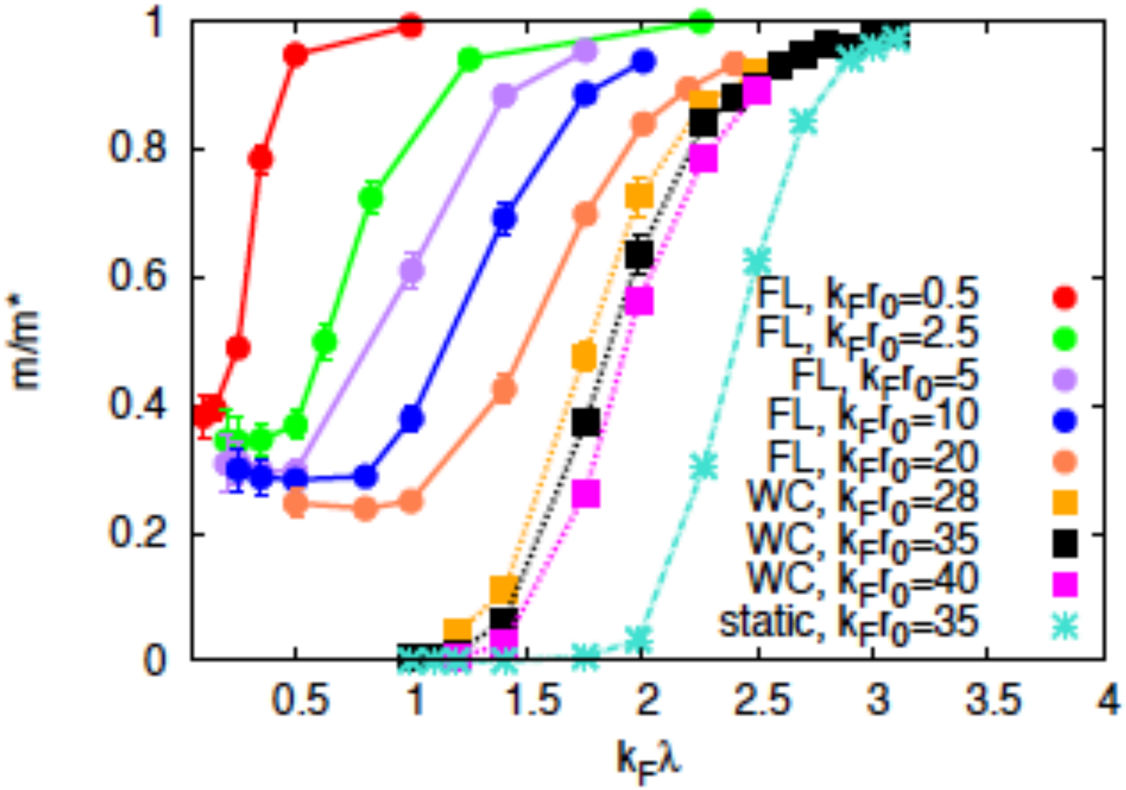


a) In units of Fermi energy varies by orders of magnitude as a function of $k_F\lambda$

b) At strong interlayer coupling (small $k_F\lambda$) ➔ 2-body binding energy

Polaron effective mass

$$\frac{m}{m^*} = \lim_{\tau \to \infty} \frac{\langle |\mathbf{r}_{\mathbf{imp}}(\tau) - \mathbf{r}_{\mathbf{imp}}(0)|^2 \rangle}{4D\tau}$$



a) very different behavior at large interlayer coupling in FL and WC phase

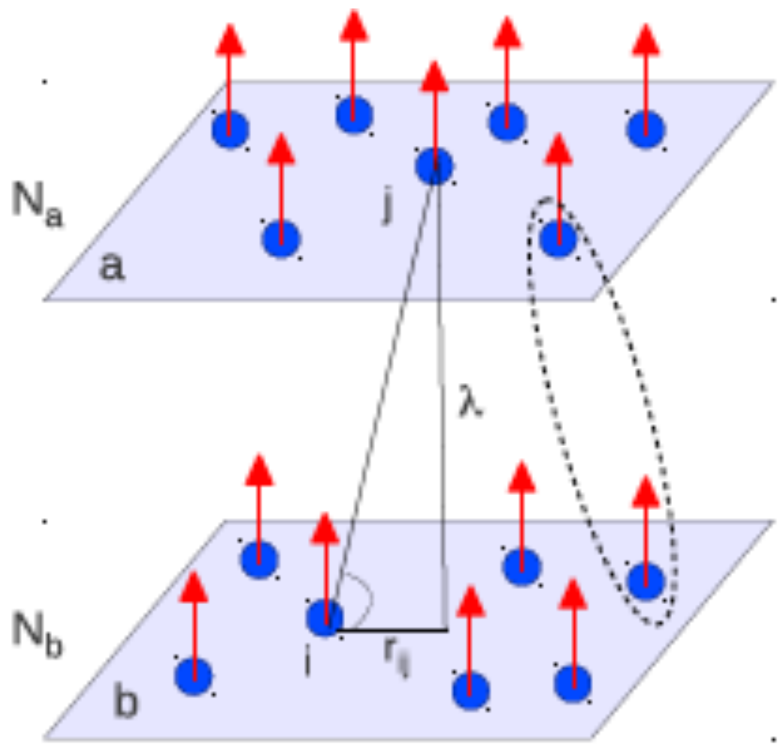
b) polaron “localization” in WC phase

Bilayer system with balanced populations ($N_a = N_b$)

$$H = -\frac{\hbar^2}{2m}\left(\sum_{i=1}^{N_a}\nabla_i^2 + \sum_{j=1}^{N_b}\nabla_\alpha^2\right) + \sum_{i<i'}\frac{d^2}{r_{ii'}^3} + \sum_{j<j'}\frac{d^2}{r_{jj'}^3} + \sum_{i,j} V(r_{ij})$$

where

$$V(r_{ij}) = \frac{d^2(r_{ij}^2 - 2\lambda^2)}{(r_{ij}^2 + \lambda^2)^{5/2}}$$



Fermions: Effective 2D system (always dimer bound state)

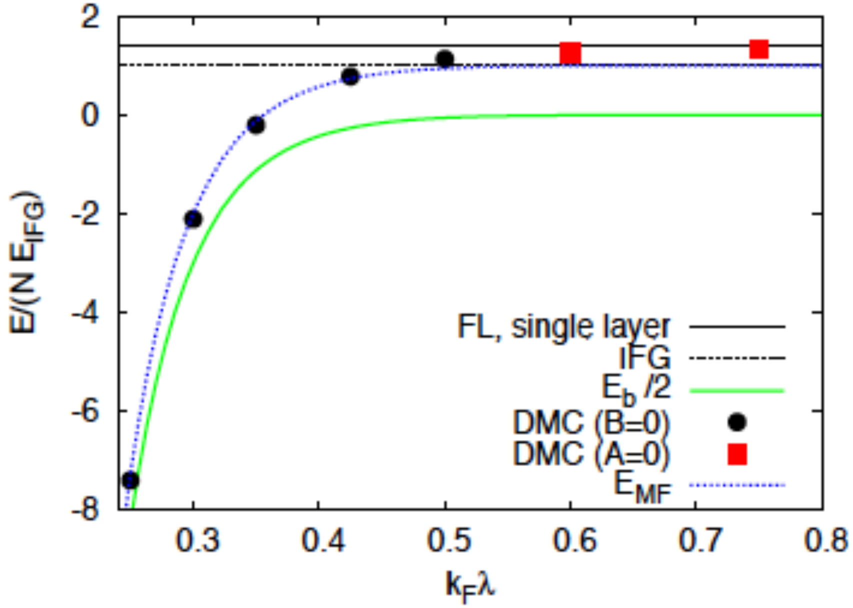
Mean-field result

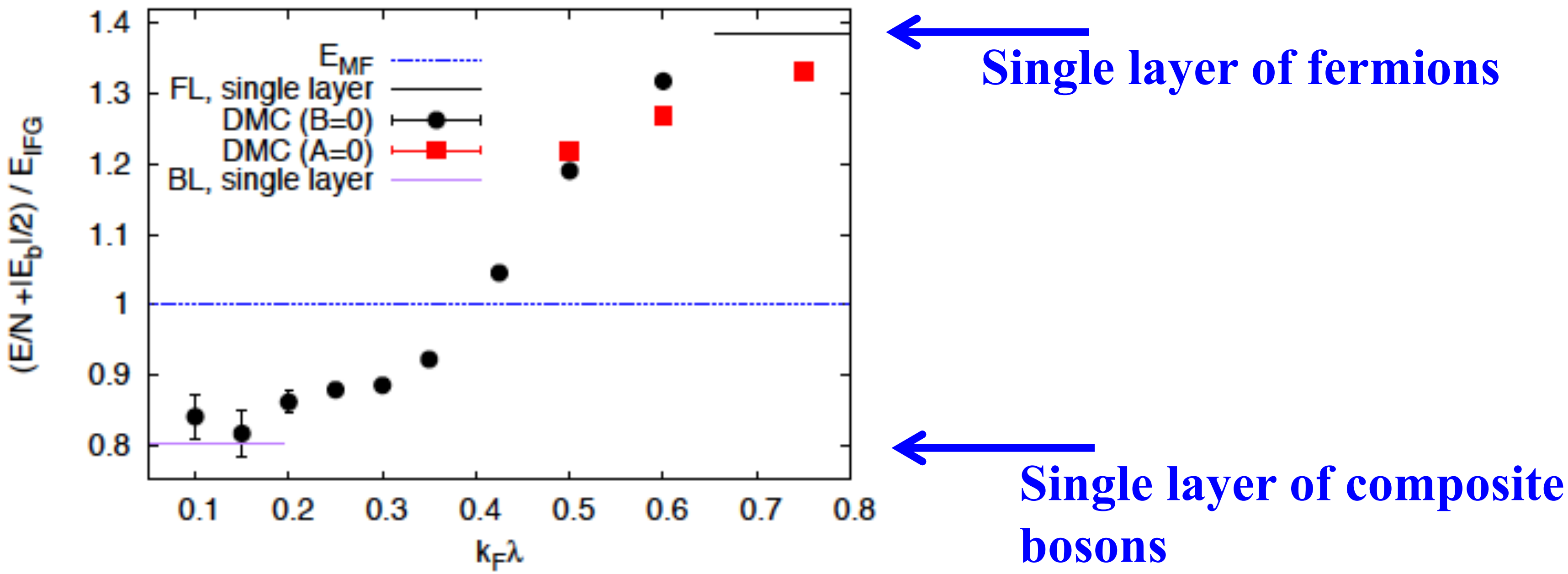
- $\mu = \varepsilon_F + \frac{E_b}{2}$
- $\Delta = \sqrt{2\varepsilon_F |E_b|}$

Equation of state

➢ weak intra-layer repulsion $k_F r_0 = 0.5$

➢ dimer binding energy E_b is the largest scale in the BEC regime



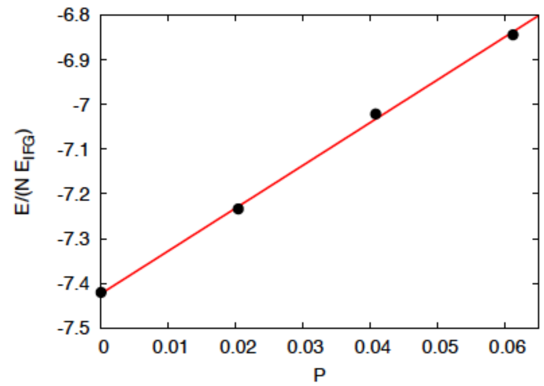


Pairing gap

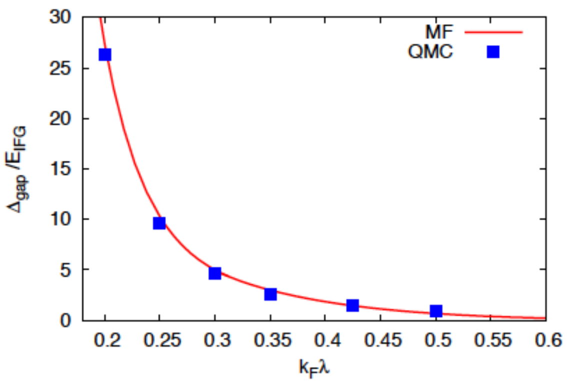
unbalanced populations:

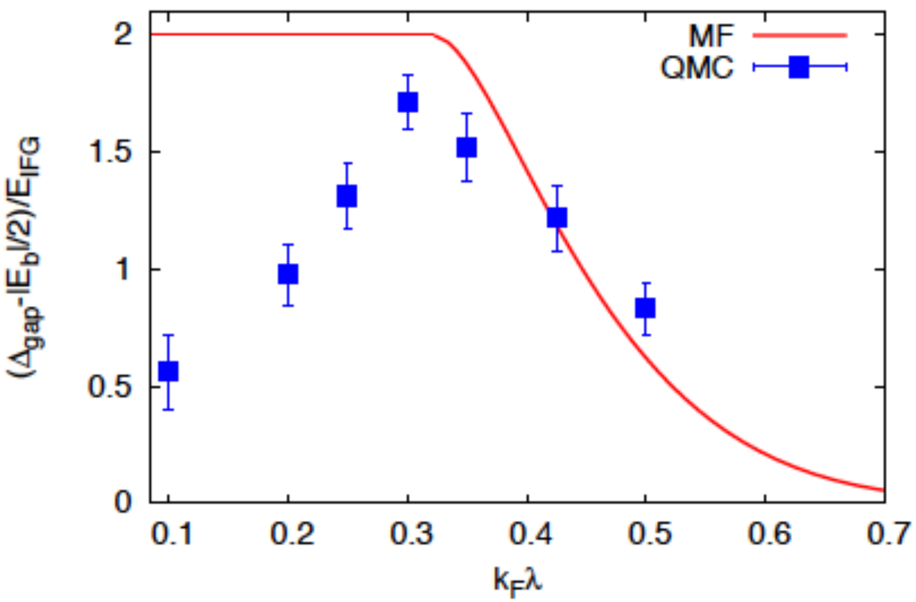
$$P = \frac{N_a - N_b}{N_a + N_b}$$

$$\frac{E(P)}{N} = \frac{E(P=0)}{N} + \Delta_{gap} P$$

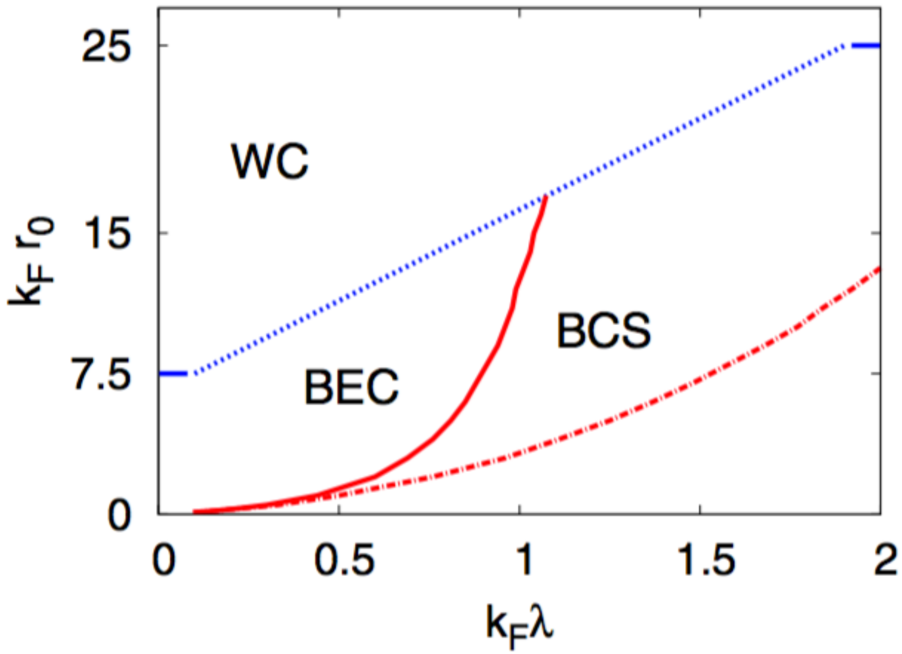


In the BEC regime E_b provides dominant contribution to gap





Schematic phase diagram



- **BCS to BEC separation when $\mu_{sl}\sim|E_b|/2$**
- **At small $k_F\lambda$ critical density of WC transition reduced by factor 8 with respect to Bose single layer ($k_F r_0\sim60$)**

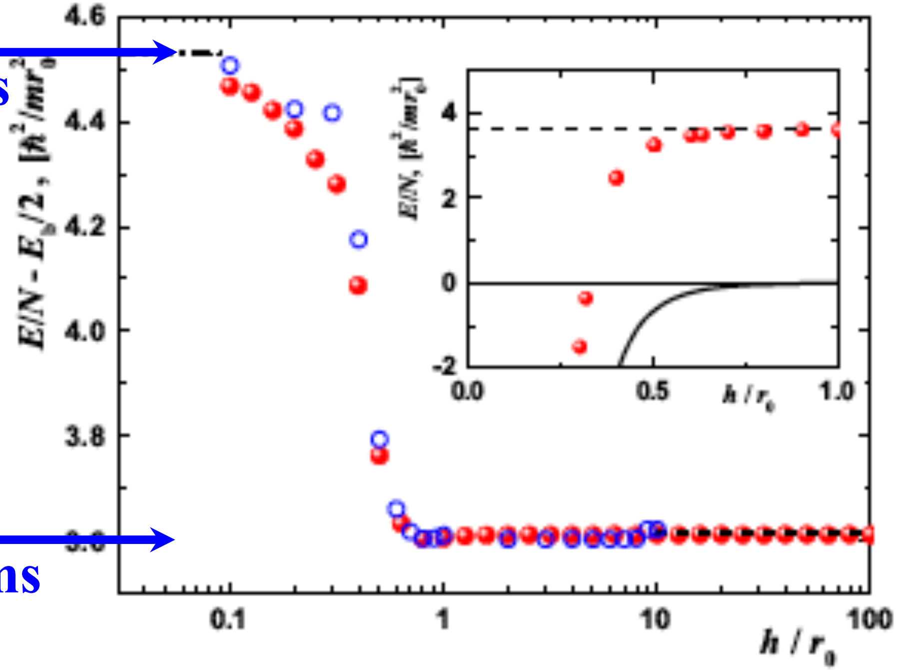
Bosons (DMC method provides exact ground state)

T=0 equation of state: in-plane interaction $nr_0^2=1$

Energy per particle as a function of interlayer distance h

At small interlayer distance: stable gas of pairs

Single layer of pairs
mass=2m
dipole moment=2d



Single layer of atoms

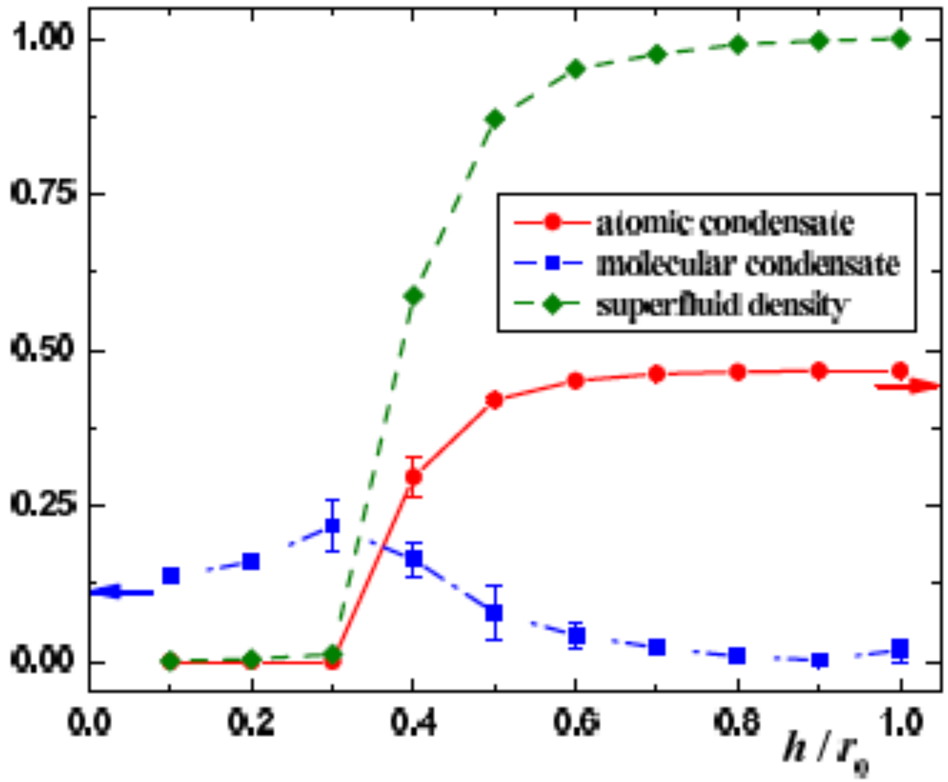
Quantum phase transition from single-particle to pair superfluidity

- **Atomic condensate from OBDM** $\left\langle \psi^{+}_{u(d)}(\vec{r})\psi_{u(d)}(\vec{r}\,')\right\rangle \to n_0$

- **Intrinsic molecular condensate from TBDM** $\left\langle \psi^{+}_{u}(\vec{r})\psi^{+}_{d}(\vec{r})\psi_{d}(\vec{r}\,')\psi_{u}(\vec{r}\,')\right\rangle - n_0^2 \to n_M$

- **Superfluid response of single atoms from winding number (super-counterfluid density)**

$$\rho_s = \lim_{\tau\to\infty} \frac{\langle(\mathbf{W}_u(\tau) - \mathbf{W}_d(\tau))^2\rangle}{6N\tau}$$

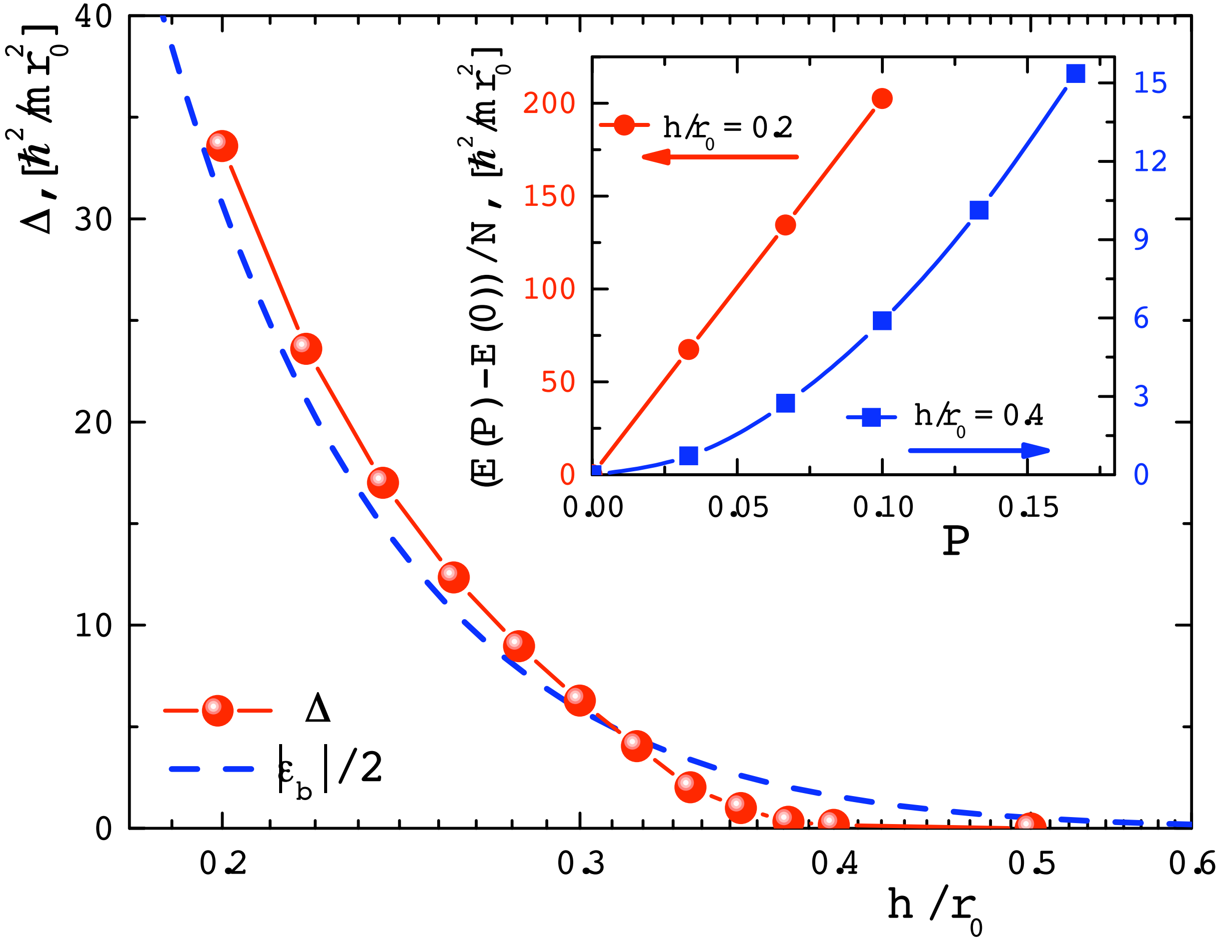


Pairing gap in single-particle excitations

$$\frac{E(P)}{N} = \frac{E(P=0)}{N} + \Delta_{gap} P$$

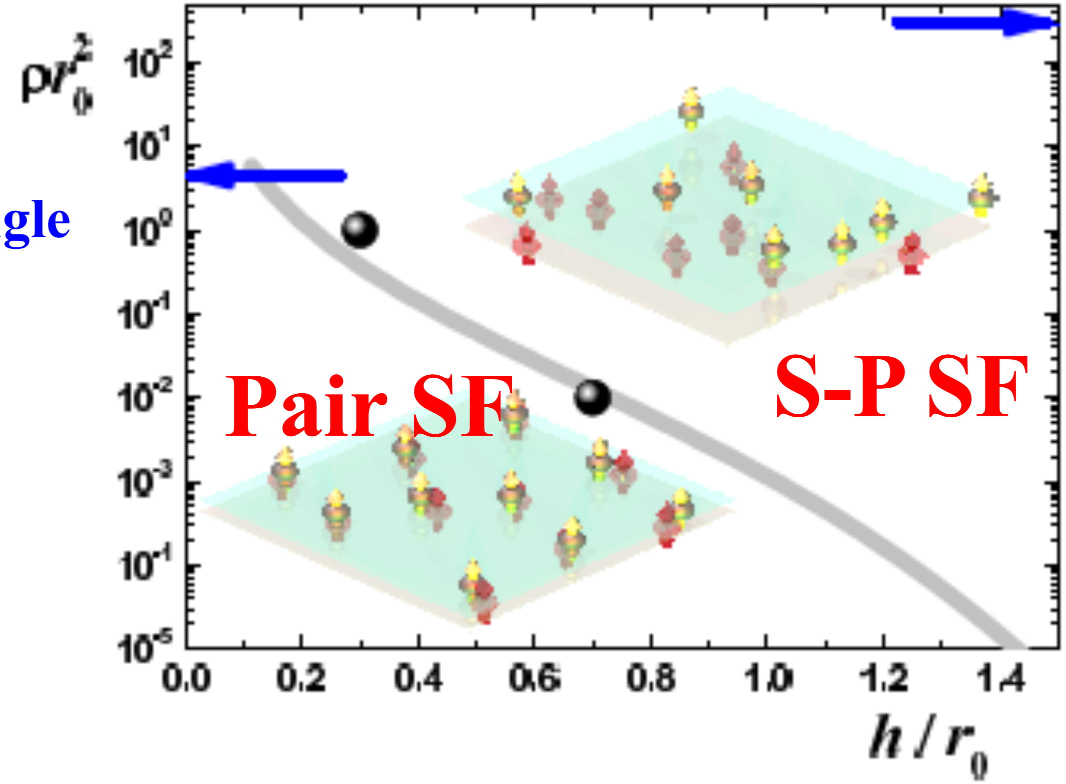
$\Delta_{gap} \neq 0$ in the pair superfluid

$$P = \frac{N_a - N_b}{N_a + N_b}$$



T=0 schematic phase diagram

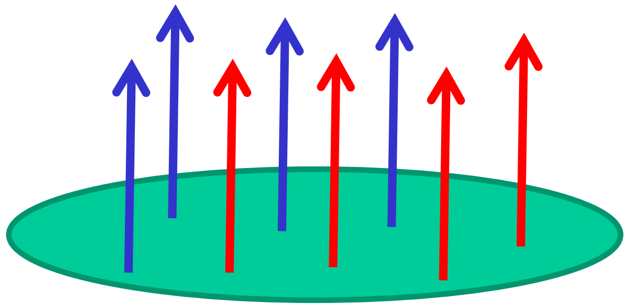
Freezing of single pair layer



Freezing of single atomic layer

Single-layer two-component Fermi gas ($N_a=N_b$)

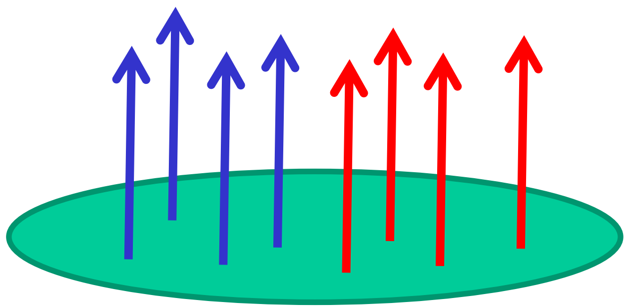
Itinerant ferromagnetism



Paramagnetic state

Spin symmetric Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\nabla_i^2 + \sum_{i<j}\frac{d^2}{r_{ij}^3}$$



Ferromagnetic state

- **No competition with pairing instability**
- **Ferromagnetism driven by exchange effects**

Analogy with Coulomb gas

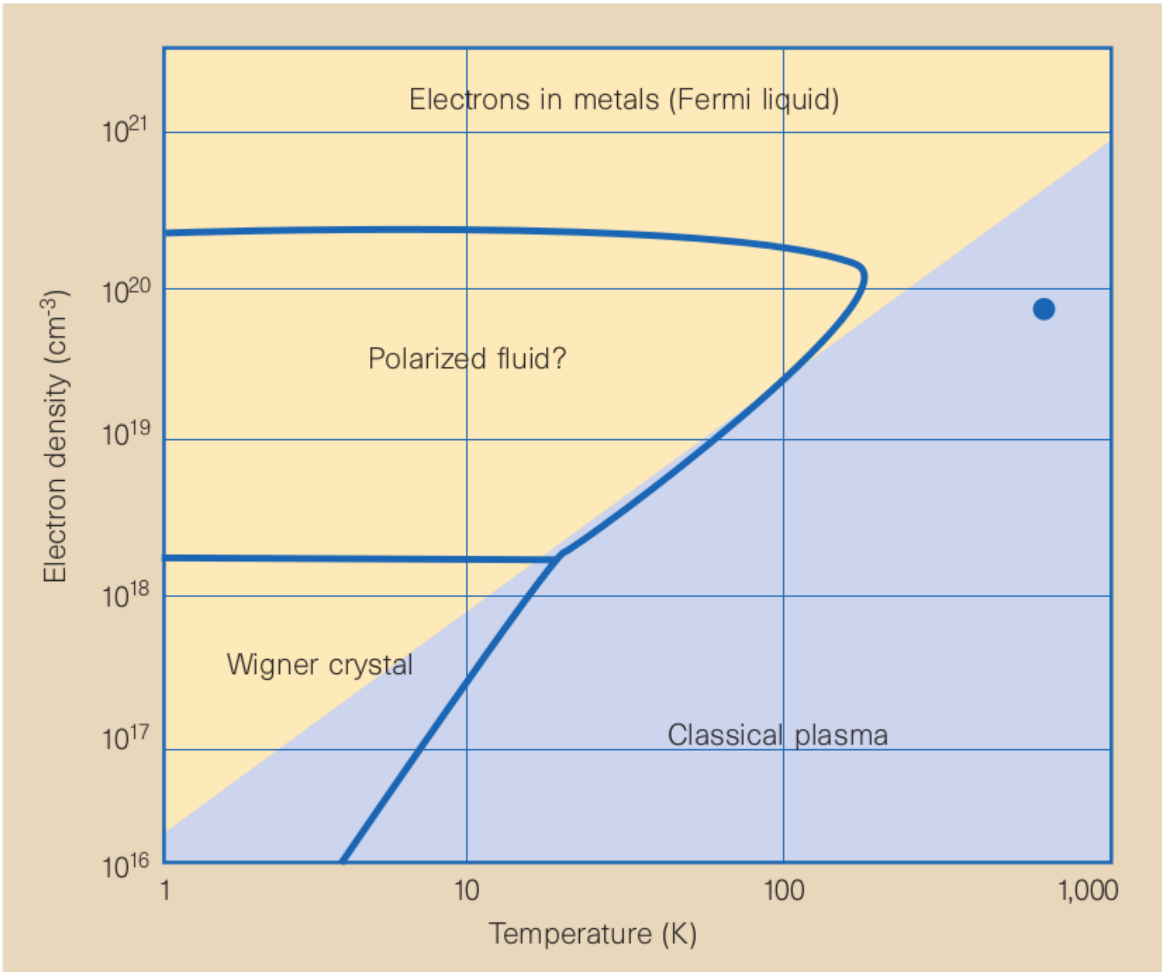
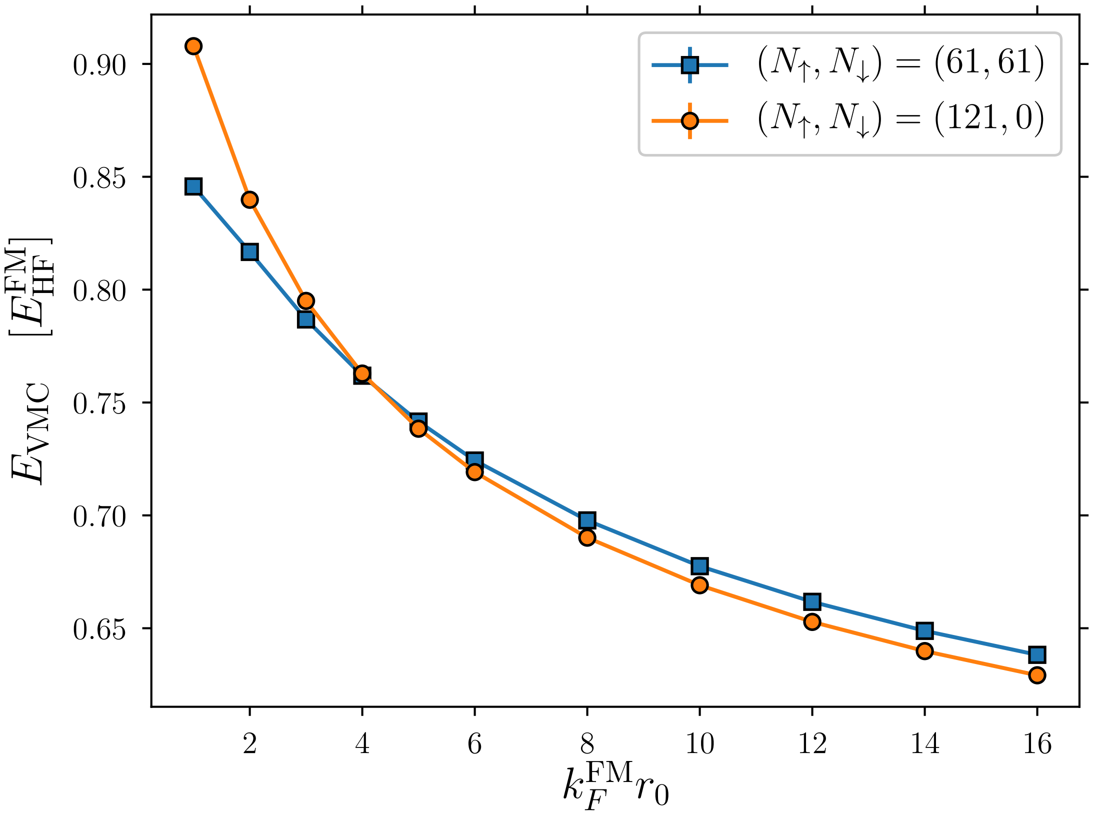


Figure 1 **Phase diagram of the electron gas. The two colours divide the classical (blue) from the quantum (yellow) regimes. The phase transition boundaries are estimates from ref. 6. The dot is the transition temperature measured by Young *et al.*[1].**

Preliminary results using VMC

Compare FM with PM ground state



- **Use DMC with fixed node approximation**
- **Add backflow to improve PM wave function**

Thank you for your attention!

Collaborators

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